Chapter = 05

# Torque, Angular Momentum and Equilibrium

# THEORY NOTES



## TORQUE

**DEFINITION:**

The turning effect of force is called torque Mathematically it is defined as the product of force and the force arm that is the perpendicular distance between the point of application of force and the fixed point or fulcrum about which body rotates.

**EXPLANATION:**

From above definition,
Torque = Force x Momemt arm

$$\vec{\tau} = \vec{r} \times \vec{F}$$

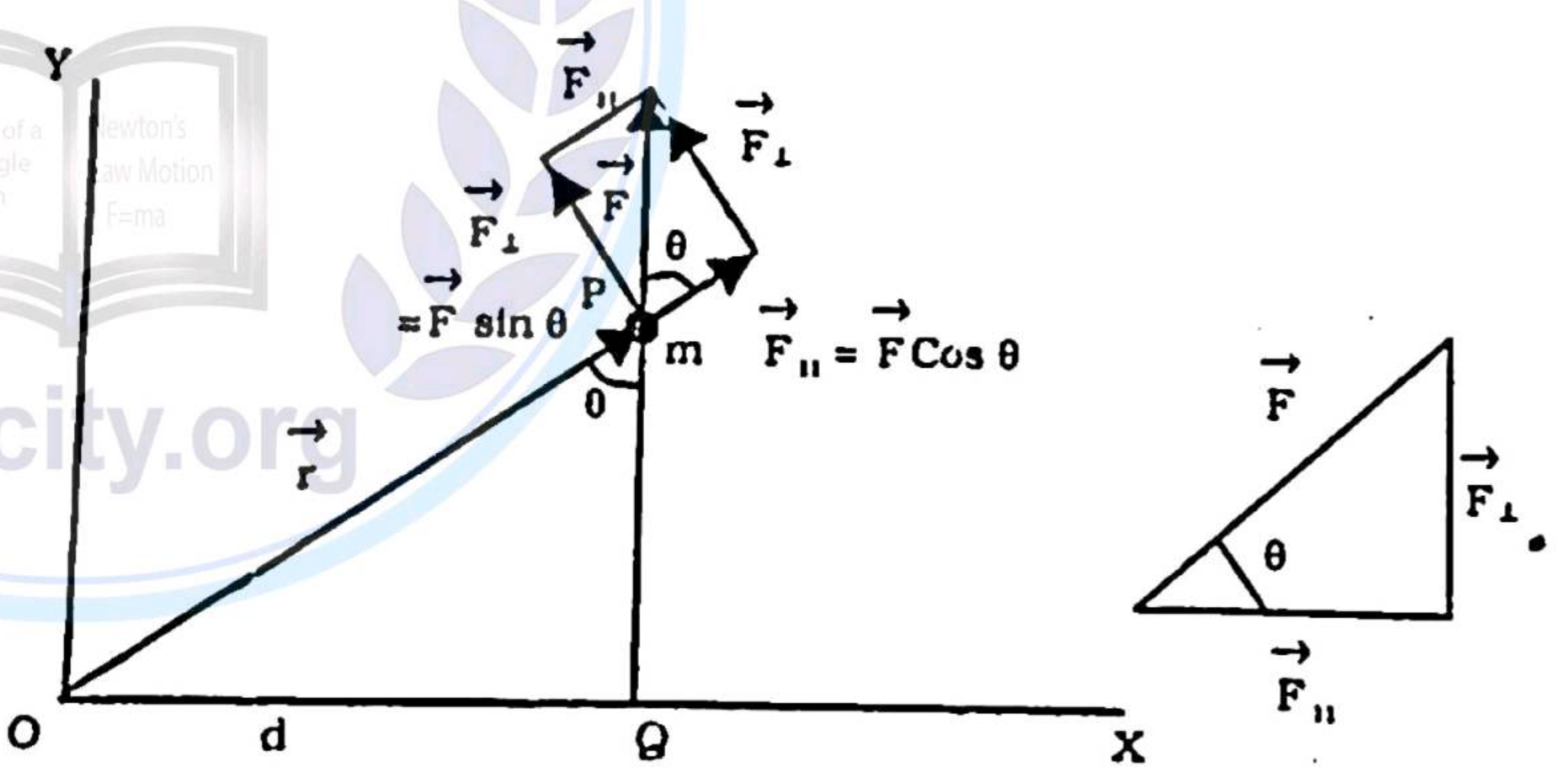
rFSinθ
Where 'θ' is the angle between 'F' and 'r'

**MAGNITUDE:**

Let us consider a particle of mass 'm' which is acted upon by a force F . Let r be the position vector of the particle which is also the position vector of the point of application of the force. The force P can be resolved into its rectangular components i.e.

(i) $F_{\|}$, i.e. parallel to the vector r

(ii) $F_{\perp}$, i.e. perpendicular to the vector r as shown in fig.




It is clear from fig. that $F_{\|}$ is the pulling component and $F_{\perp}$ is the rotating compo- nent, i.e. $F_{\|}$ only is responsible to rotate the body about point O, therefore the magnitude of torque produced b the force F about point O will be

$$\tau = (r)(F_{\perp})$$
$$\tau = (r)(FSin\theta)$$

Where $\theta$ is the smaller angle between the positive directions of $\vec{r}$ and $\vec{F}$.

As Torque is the Vector quantity then

$$\vec{\tau} = (r)(FSin\theta)\hat{n}$$

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Where $\hat{n}$ is the unit vector perpendicular to $\vec{r}$ and $\vec{F}$.

**IN RECTANGULAR COMPONENT FORM:**

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

and $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$

then

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$



## COUPLE:

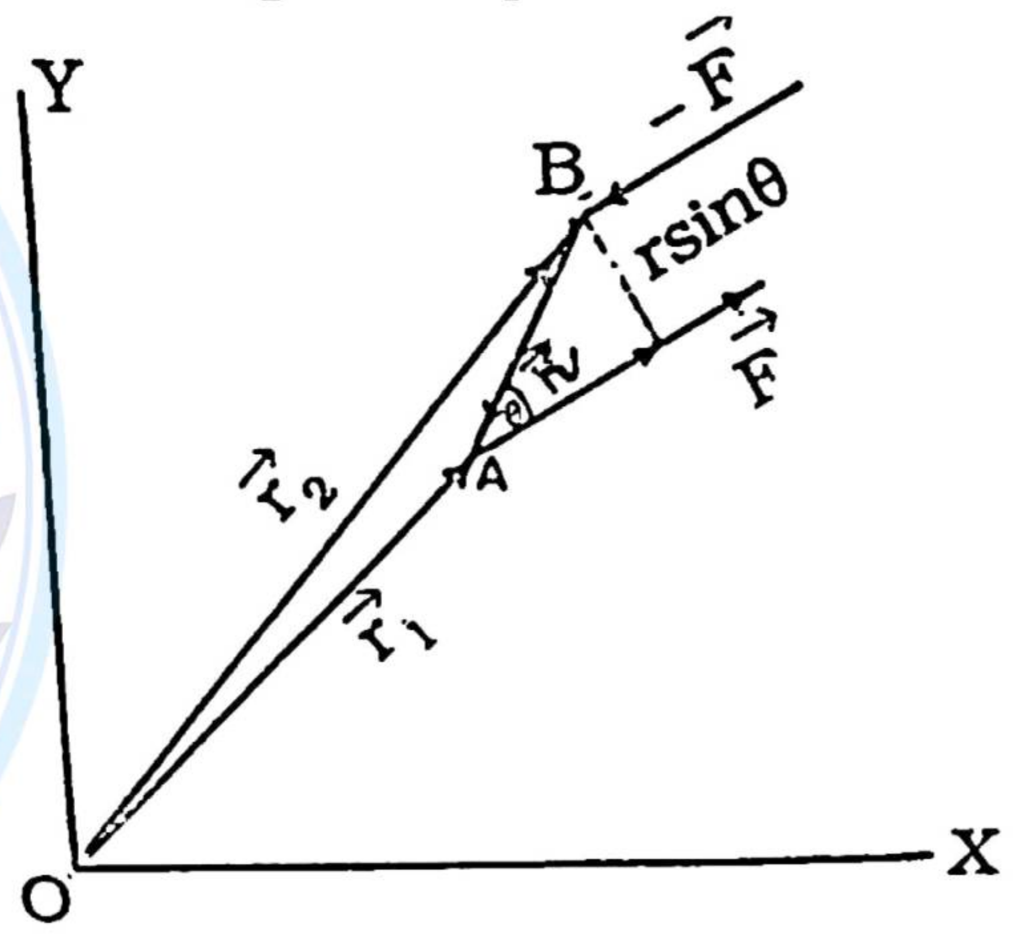
**DEFINITION:**

Two force which are equal in magnitude but opposite in direction and not acting along with the same direction constitute a couple"

**EXPLANATION:**

Let the forces constituting the couple are represented by F and –F acting at the points 'A' and 'B'. The moment of force F about 'O' is

$$\vec{\tau}_1 = \vec{r_1} \times \vec{F}$$




And, The moment of force – F about the same point is

$$\vec{\tau}_2 = \vec{r_2} \times -\vec{F}$$

The total moment of the two forces is given by,

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$
$$\vec{\tau} = \vec{r_1} \times \vec{F} + \vec{r_2} \times -\vec{F}$$
$$\vec{\tau} = \vec{r_1} \times \vec{F} - \vec{r_2} \times \vec{F}$$

Taking common

$$\vec{\tau} = (\vec{r_1} - \vec{r_2}) \times \vec{F}$$

A/c to the figure , $\vec{r} = \vec{r_1} - \vec{r_2}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The Magnitude of $\vec{\tau}$ is given by

$$\tau = (r)(FSin\theta)$$

or $\tau = Fd$

Where d $= rSin\theta$ is the perpendicular distance between the line of action of forces.

Centre of Mass 

**DEFINITION:**

The centre of mass of a body, or a system of particles, is a point on the body that moves in the same way that a single particle would move under the influence of the same external forces. The whole mass of the body is supposed to be concentrated at this point. This point is called Centre of Mass.

**EXPLANATION**

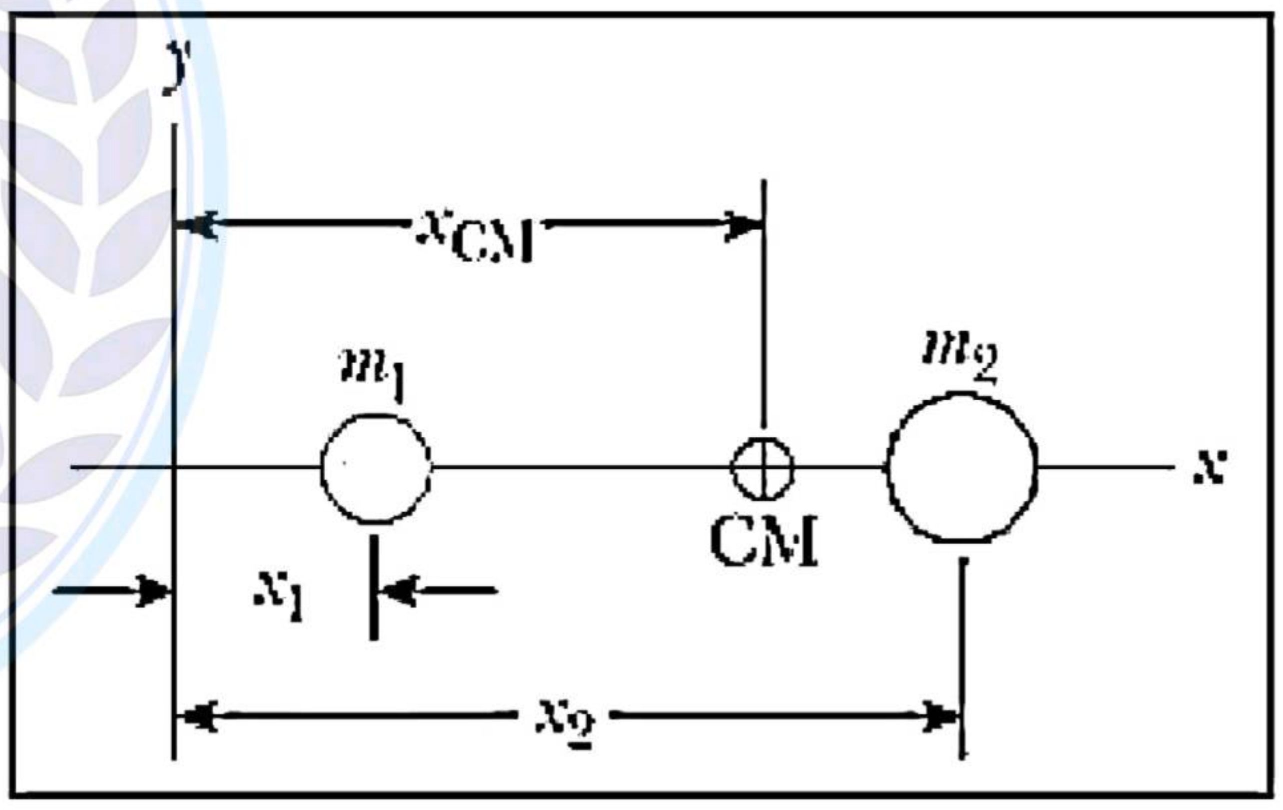
During translational motion each point of a body moves in the same manner i.e., different particles of the body do not change their position w.r.t each other. Each point on the body undergoes the same displacement as any other point as time goes on. So the motion of one particle represents the motion of the whole body. But in rotating or vibrating bodies different particles move in different manners except one point called centre of mass. The centre of mass of a body or a system of particle is a point which represents the movement of the entire system. It moves in the same way that a single particle would move under the influence of same external forces.

**CENTRE OF MAS AND CENTRE OF GRAVITY**

In a completely uniform gravitational field, the centre of mass and centre of gravity of an extended body coincides. But if gravitational field is not uniform, these points are different.

**DETERMINATION OF CENTRE OF MASS**

Consider a system of two particles having masses $m_1$, $m_2$ having position coordinates $x_1$ and $x_2$. Now the centre of mass of this system is the arithmetic mean between positions of the masses.




Mathematically we can write as,

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

and for "n" number of particles

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n}$$

Or

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

Similarly for y co-ordinate of centre of mass,

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

and for z co-ordinate of centre of mass,

$$z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$$

## EQUILIBRIUM:



A body is said to be equilibrium if it is at rest or is moving with uniform velocity.

1) **STATIC EQUILIBRIUM:**

A body at rest is said to be in static equilibrium

2) **DYNAMIC EQUILIBRIUM:**

A body in uniform motion along a straight line is said to be in dynamic equilibrium.

In both the cases the bodies do not posses any acceleration neither linear nor angular.

## CONDITIONS OF EQUILIBRIUM:

There are two conditions of equilibrium.

### A) FIRST CONDITION OF EQUILIBRIUM: (TRANSLATION EQUILIBRIUM):

**STATEMENT:**

A body will be in equilibrium if the resultant of all the forces on it is equal to zero."

**EXPLANATION:**

Let $\vec{F_1}, \vec{F_2}, \ldots \vec{F_n}$ be the external forces acting on a body. Thus, according to the first condition.

$$\vec{F_1} + \vec{F_2} + \ldots + \vec{F_n} = 0$$

or

$$\sum_{i=1}^{n} \vec{F_i} = 0$$

**A FORCES ACTING ALONG X-AXIS**

Let $\overrightarrow{F_{ix}}, \overrightarrow{F_{2x}}, \ldots\ldots\ldots \overrightarrow{F_{nx}}$ be forces acting along x-axis

Thus, from first condition,

$$\overrightarrow{F_{1x}} + \overrightarrow{F_{2x}} + \ldots\ldots + \overrightarrow{F_{nx}} = 0$$

Or $(F_{1x} + F_{2x} + \ldots\ldots F_{nx})\,\hat{\imath} = 0$

or

$$\sum_{i=1}^{n} \overrightarrow{F_{ix}} = 0$$

**B FORCES ACTING Y-AXIS:**

Let $\overrightarrow{F_{iy}}, \overrightarrow{F_{2y}}, \ldots\ldots\ldots \overrightarrow{F_{ny}}$ be forces acting along x-axis

Thus, from first condition,

$$\overrightarrow{F_{1y}} + \overrightarrow{F_{2y}} + \ldots\ldots + \overrightarrow{F_{ny}} = 0$$

Or $(F_{1y} + F_{2y} + \ldots\ldots F_{ny})\,\hat{\jmath} = 0$

$$\sum_{i=1}^{n} \overrightarrow{F_{iy}} = 0$$

I.e. For body to be in equilibrium, the sum of x-components of all the forces and the sum of y-components of all the forces must be equal to zero separately. Therefore First condition of equilibrium can be written as

$$\sum_{i=1}^{n} \overrightarrow{F_{ix}} = 0\ ,\ \sum_{i=1}^{n} \overrightarrow{F_{iy}} = 0 \ and\ over\ all\ \sum_{i=1}^{n} \vec{F_i} = 0$$

**B) SECOND CONDITION OF EQUILIBRIUM:(ROTATIONAL EQUILIBRIUM)**

**STATEMENT:**

"If the vector sum of the torques acting on a body is zero, the body is said to in rotational equilibrium."

**EXPLANATION:**

Let $\tau_1, \tau_2$ , ……, $\tau_n$ are the torques on the body then

$$\tau_1 + \tau_2 + \ldots\ldots + \tau_n = 0$$

Or $$\sum_{i=1}^{n} \vec{\tau} = 0$$

This is the required condition.

For this condition it is necessary that" Sum of clockwise torque is equal to sum of anticlockwise torque" Where Anticlockwise Torque is taken Positive and Clockwise Torque is taken Negative.

## ANGULAR MOMENTUM:



**DEFINITION**: Angular momentum of an object moving in a circle is the cross product of linear momentum and position vector from the origin."



**EXPLANATION:**

A body having rotatory motion possesses angular velocity and angular momentum.

Consider a particle of mass 'm' let r be its position vector and P be the linear momentum with r espect to origin.

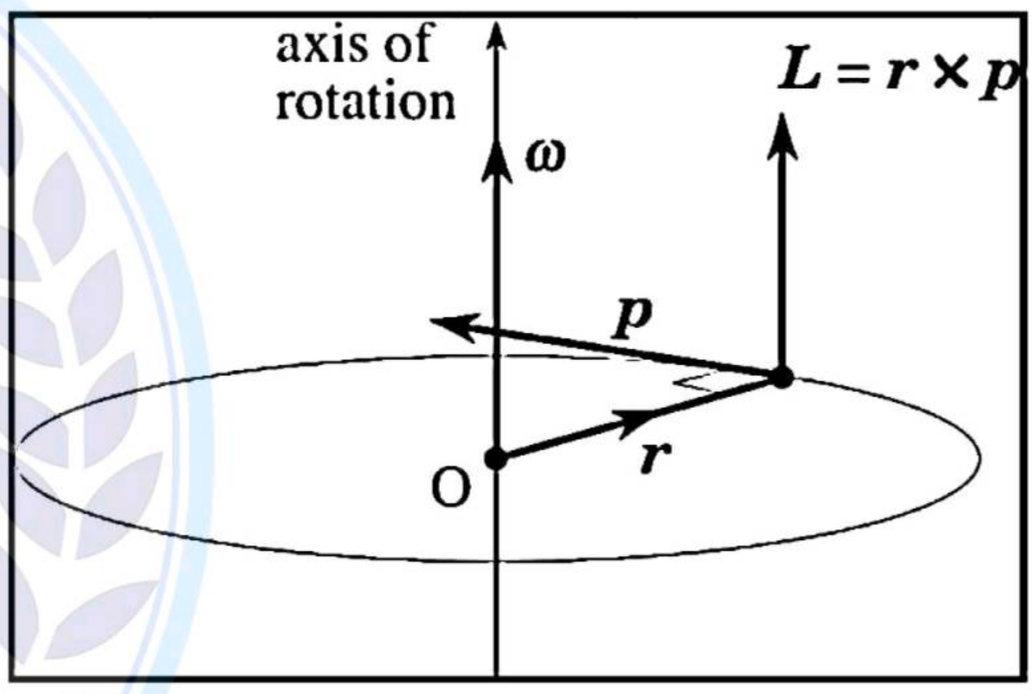
From Definition,

Angular momentum = Position Vector X Linear Momentum

$$\vec{l} = \vec{r} \times \vec{P}$$

$$\vec{l} = \vec{r} \times \vec{mv} \quad [P = mv]$$

$$\vec{l} = m\vec{r} \times \vec{v}$$




where, $\vec{v}$ be the velocity of the particle.

The direction of angular momentum is normal to the plane formed by $\vec{r}$ and $\vec{P}$ as given by right and rule.

$$l = rp\sin\theta$$

or $$l = mvr\sin\theta$$

Where 'θ' is the angle between $\vec{r}$ and $\vec{P}$

When $\theta = 90^o$ [Sin90 = 1]

$$l = rp = mvr$$

In Cartesian Coordinate system when,

$$\vec{l} = \vec{r} \times \vec{P} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \times (P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k})$$

$$\vec{l} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

**DIMENSION AND UNIT:**

The dimension of angular momentum is

$[l] = [r]\ [p] = [r]\ [m]\ [v] = L.M.L/T = L^2MT^{-1}$

In S.I system its units is NmS = J.S.

## LAW OF CONSERVATION OF ANGULAR MOMENTUM



**STATEMENT:**

The angular momentum of a particle is conserved (constant) if the torque acting on it is zero."

**EXPLANATION:**

If F is the force acting on a particle of mass 'm' moving with velocity $\vec{V}$ and $\vec{P}$ is the linear momentum, then,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times \frac{\vec{P_f} - \vec{P_i}}{\Delta t} \quad \because \text{Force is the rate of change of momentum}$$

$$= \frac{\vec{r} \times \vec{P_f} - \vec{r} \times \vec{P_i}}{\Delta t}$$

$$= \frac{\vec{L_f} - \vec{L_i}}{\Delta t} \qquad \because \vec{L} = \vec{r} \times \vec{P}$$

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

This equation states that the torque acting on a particle is the time rate of change of its angular momentum.

If net torque acting on the particle is zero, Then,

$$\frac{\Delta \vec{L}}{\Delta t} = 0$$

or $\Delta\vec{L} = 0$ => $\boxed{\vec{L} = Constant}$

thus, angular momentum of a particle is conserved, i.e. law of conservation of angular momentum.

# M.C.Qs.



**1. Torque is also known as:**
(a) Angular speed (b) Angular momentum
(c) Moment of inertia (d) Moment of force

**2. The rate of change of angular momentum is called:**
(a) Force (b) Torque
(c) Momentum (d) Equilibrium

**3. A force of 8 N is applied to the spanner perpendicularly at a distance of 0.12 m from the centre of nut ,the moment of force acting on the nut is:**
(a) 0.96 Nm (b) 1.5 Nm
(c) 2.1 Nm (d)3 Nm

**4. Torque is zero, if angle θ between force and momentum arm is:**
(a) 0° (b) 60° (c) 90° (d) $180^0$

**5. The motion of the body can describe by the motion of it's:**
(a) Center of gravity (b) Origin
(c) Center of mass (d) None of these

**6. The ratio of SI unit of angular momentum to linear momentum is:**
(a) J.s (b) N/J
(c) J.N (d) J/N

**7. The two forces constitute couple are:**
(a) Equal in magnitude
(b) Opposite in direction
(c) Not acting along the same line
(d) All of these

**8. The angular momentum of a particle changes from 0 to 720 is in 4 sec, the magnitude of torque acting will be:**
(a) 1440J (b) 360J
(c) 180J (d) 4.5J

**9. The point at which whole weight of the body is concentrated is called.**
(a) Centre of mass (b) Centre of gravity
(c) Origin (d) Centre of action

**10. The magnitude of couple depends upon:**
(a) The distance of F from origin
(b) The distance of –F from origin
(c) Distance between F and –F
(d) None of these

**11. If linear momentum of body is doubled and parallel to the axis of rotation then angular momentum will be:**
(a) Doubled (b) Halved
(c) Quadrupled (d) Zero

**12. The centre of mass coincides with centre of gravity of body, if it is placed:**
(a) In a non-uniform gravitation field.
(b) In a uniform gravitation field

(c) At the centre of earth
(d) At the poles

**13. The magnitude of the angular momentum is given by:**
(a) L = rp cos θ (b) L = rp/sin θ
(c) L = rp sin θ (d) both A & B

**14. The angular momentum of tyre of car of mass 10kg and radius 0.5m and the car moving with velocity of 10m/s is:**
(a) 50 (b) 25 (c) 100 (d) Zero

**15. If the net torque acting on a body is zero then the ___ of the body is conserved:**
(a) Force (b) Liner momentum
(c) Torque (d) Angular momentum

**16. According to law of conservation of angular momentum.**
(a) $\frac{dl}{dt} = 0$ (b) $\frac{dl}{dt} = constant$
(c) $\frac{dl}{dt} = \frac{df}{dx}$ (d) Both a and b

**17. The product of moment of inertia and angular acceleration is:**
(a) Angular momentum (b) Torque
(c) Couple (d) None of these

**18. If the force of F = 2i + 4j – 3k acting on a body pivoted at 2m from the axis of rotation along x-axis. Then the magnitude of rotational analogue of the force is:**
(a) 12N-m (b) 10N-m
(c) 8N-m (d) 6N-m

**19. A body will be in translation equilibrium if the vector sum of external forces acting on a body is :**
(a) Maximum (b) Minimum
(c) Square (d) Zero

**20. If the axis of rotation passes through the body itself the corresponding rotator motion is called the:**
(a) Spin –motion (b) Orbital motion
(c) Vibratory motion (d) To and fro motion

**21. For which of the following does the centre of mass lie outside the body?**
(a) Pen (b) Dice
(c) Rectangular tile (d) Bangle

**22. Two bodies of masses 5kg and 15kg are located in the cartesian plane at (1,0) and (0,1). What is the location of their centre of mass?**
(a) 1/4, 1/4 (b) 3/4, 3/4
(c) 3/4, 1/4 (d) 1/4, 3/4

**23. A dancer on ice starts spinning faster, when she folds her arms. This is due to**
(a) decrease in friction at the skates
(b) increase in angular momentum
(c) decrease in angular momentum
(d) constant angular momentum and decrease in moment of inertia



**24. When a torque acting on a system is increased, then which one of the following quantities will increase**
(a) linear momentum (b) Angular momentum
(c) force (d) Displacement

**25. The resultant force acting in the couple is**
________
(a) Zero (b) Infinite
(c) Twice the magnitude of the single force
(d) Half the magnitude of the single forcea

# PAST PAPER M.C.Qs.



## 2022

1. The SI unit of angular momentum is:

***J-s** *J/s *s/J *J-s²

14. The point which describes the motion of the whole system or body is known as the:

***center of mass** *inertia *centre of gravity *moment of inertia

## 2021

(vii) The rate of change of angular momentum is called

*Power *** Torque** *Momentum *Force

(viii) A 400N force acting perpendicularly to an object at the distance of 200cm from the axis of rotation, the moment of force generated is:

*100N m *200 Nm * 400Nm *** 800 Nm**

(xl) A body will be in complete equilibrium is:

*1s condition of equilibrium only *2nd condition of equilibrium only

***Both 1" and 2nd condition of equilibrium** *Neither 1st nor 2d condition of equilibrium

(xlii) The SI unit of angular momentum is:

***J-s** *J/s *s/J *J-s²



## 2019

9. The point which describes the motion of the whole system or body is known as the:

***center of mass** *inertia

*centre of gravity *moment of inertia

15. Two forces of equal magnitude but opposite direction in direction and not acting along the same straight line form a:

*circle ***couple**

*power *torque

## 2018

14. The angular momentum of particle moving in circle is conserved if:

***net torque acting on particle is zero** *the acceleration of particle is zero

* net angular displacement of particle is zero * net force acting on particle is zero



17.A force of 8 N is applied to the spanner perpendicularly at a distance of 0.15 m from the centre of nut ,the moment of force acting on the nut is:

***1.2 Nm** *1.5 Nm * 2.1 Nm * 3 Nm

**2017**

4. Torque is maximum when force:

*is parallel to moment arm
*as anti parallel to moment arm
*makes angle 60 degree with moment arm
**_* is perpendicular to moment arm_**

14. The ratio of SI unit of angular momentum to linear momentum is:

*J.s
*N/J
*J.N
**_*J/N_**

**2016**

3.The rate of change of angular momentum is also known as:

*Linear momentum
**_*Torque_**
*Force
*Energy

**2015**

6. When torque acting on a system is zero, this will be constant:

*force
**_*angular momentum_**
* linear momentum
* velocity

**2013**

7. The centre of mass of a body:

*always coincides with the centre of gravity
* never coincides with the centre of gravity
**_*coincides with the centre of gravity only in uniform field_**
*is lower than the centre of gravity



9. The angular momentum of a particle changes from 0 to 720 is in 4 sec, the magnitude of torque acting will be:

*1440J
*360J
**_*180J_**
*4.5J

16. The sum of torques acting on a body is zero, and then this will be constant:

**_*angular momentum_**
*force
*linear momentum
*pressure

**2012**

12. The magnitude of couple depends upon:

*The distance of F from origin
* The distance of –F from origin
**_* Distance between F and –F_**
* None of these

**2011**

12. The rate of change of angular momentum with respect to time is:

*force
* angular velocity
*angular acceleration
**_*torque_**

13. Two forces of equal magnitude but opposite in direction and not acting on the same line constitute:

**_*a couple_**
* power
*a circle
*a force

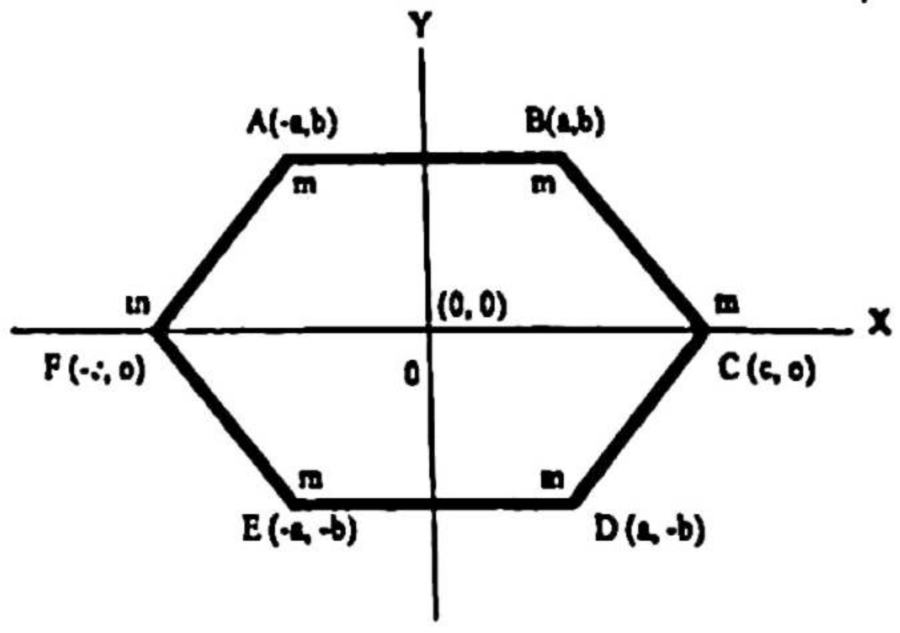
**2010**

14. Torque is defined as the time rate of change of:

**_* angular momentum_**
* linear momentum
* angular velocity
* angular acceleration

# TEXTBOOK NUMERICALS

**Q.1: Locate the centre of mass of a system of particles each of mass 'm', arranged to correspond in position to the six corners of a regular (planar) hexagon.**




**Data:**

Mass of each particle $= m$

Centre of Mass $= C(x, y) = ?$

**Solution:**

The x co-ordinate of centre of mass is given by

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$x = \frac{m(-a) + m(a) + m(c) + m(a) + m(-a) + m(-c)}{m + m + m + m + m + m}$$

$$x = \frac{-ma + ma + mc + ma - ma + mc}{6m}$$

$$x = \frac{0}{6m}$$

$$\boxed{x = 0}$$

The y co-ordinate of centre of mass is given by

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$y = \frac{m(b) + m(b) + m(0) + m(-b) + m(-b) + m(0)}{m + m + m + m + m + m}$$

$$y = \frac{mb + mb - mb - mb}{6m}$$

$$y = \frac{0}{6m}$$

$$\boxed{y = 0}$$

**Result:** The centre of mass of given hexagon is at origin (0,0).



**Q.2: Find the position of centre of mass of five equal-mass particles located at the five corners of a square-based right pyramid with sides of length '$l$' and altitude 'h'.**

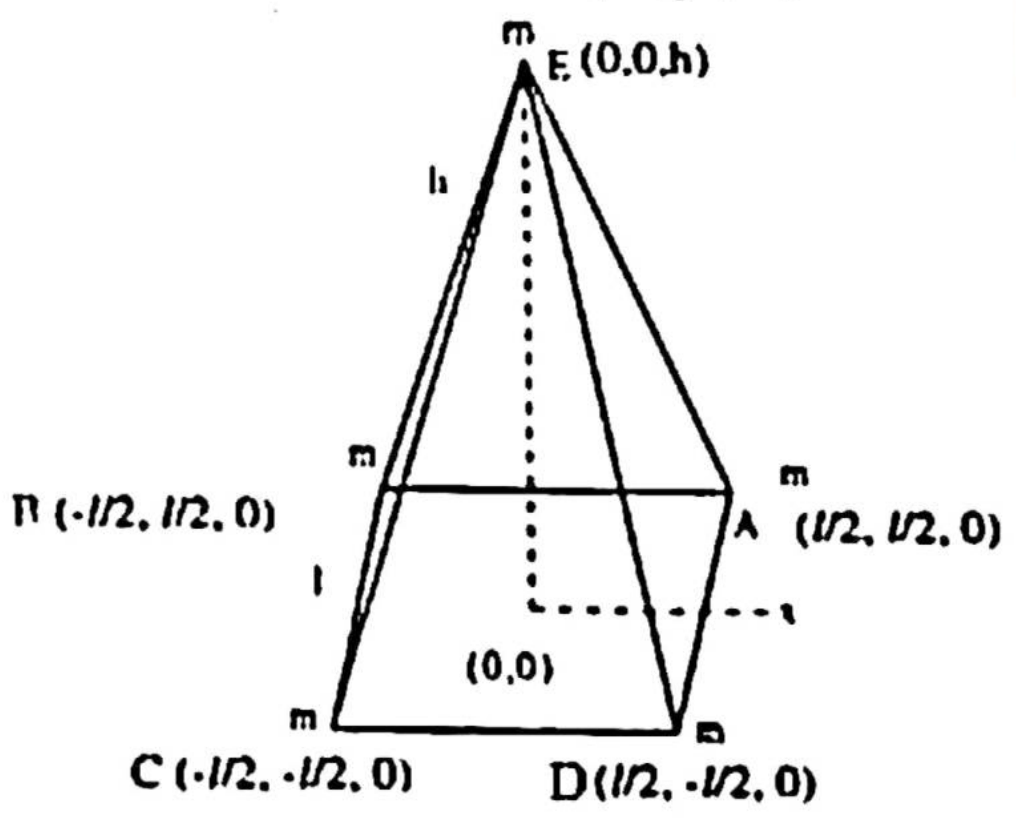
**Data:**

Mass of each particle $= m$

Length of base $= l$

Height of Pyramid $= h$

Centre of Mass $= C(x, y, z) = ?$




**Solution:**

The x co-ordinate of centre of mass is given by

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$x = \frac{m(l/2) + m(-l/2) + m(-l/2) + m(l/2) + m(0)}{m + m + m + m + m}$$

$$x = \frac{m(l/2) - m(l/2) - m(l/2) + m(l/2)}{5m}$$

$$x = \frac{0}{5m}$$

$$\boxed{x = 0}$$

The y co-ordinate of centre of mass is given by

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$y = \frac{m(l/2) + m(l/2) + m(-l/2) + m(-l/2) + m(0)}{m + m + m + m + m}$$

$$y = \frac{m(l/2) + m(l/2) - m(l/2) - m(l/2)}{5m}$$

$y = \frac{0}{5m}$

$\boxed{y = 0}$

The z co-ordinate of centre of mass is given by

$z = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \cdots + m_n z_n}{m_1 + m_2 + m_3 + \cdots + m_n}$

$z = \frac{m(0) + m(0) + m(0) + m(0) + m(h)}{m + m + m + m + m}$

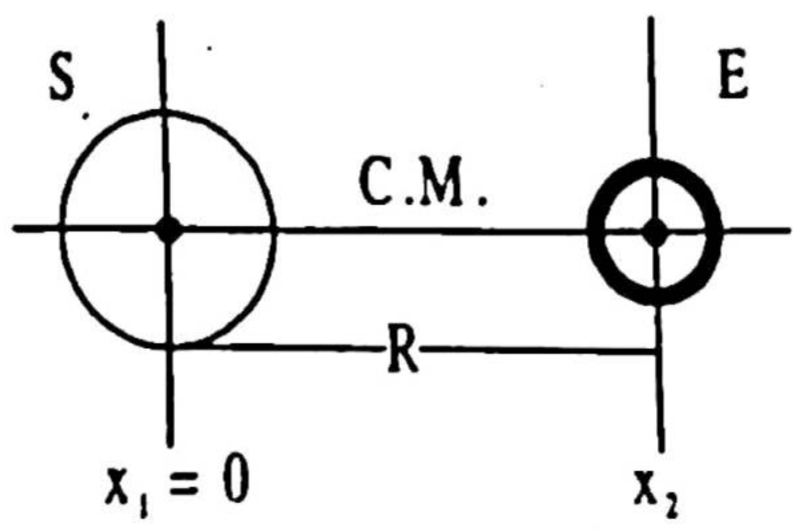
$z = \frac{mh}{5m}$

$\boxed{z = h/5}$

**Result:** The centre of mass of given Pyramid is at 1/5th of its height (0,0,h/5).



**Q.3: The mass of the sun is 329.390 times the earth's mass and the mean distance from the centre of the sun to the centre of the earth is 1.496x10⁸ km. Treating the earth and sun as particles with each mass concentrated at the respective geometric centre, how far from the centre of the sun is the C.M (centre of mass) of the earth-sun system? Compare this distance with the mean radius of the sun (6.9960x10⁵ km)**




**Data:**

Mass of Sun $= M_s = 329.390\ M_e$

Mean distance b/w sun & earth $= R =$ 1.496x10⁸km

Mean radius of the sun $= R_s = 6.9960\text{x}10^5$ km

Distance of Centre of mass of system $= x = ?$

Ration of distance to radius of sun $= \frac{x}{R_s} = ?$

**Solution:**

The x co-ordinate of centre of mass is given by

$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$x = \frac{M_s x_1 + M_e x_2}{M_s + M_e}$

$x = \frac{329.390\ M_e \times 0 + M_e \times 1.496\text{x}10^8}{329.390\ M_e + M_e}$

$x = \frac{M_e \times 1.496\text{x}10^8}{330.390\ M_e}$

$\boxed{x = 4.52 \times 10^5 km}$

Now,

$\frac{x}{R_s} = \frac{4.52 \times 10^5\ km}{6.9960\text{x}10^5\ \text{km}}$

$\boxed{\frac{x}{R_s} = 0.64}$

**Result:** The centre of mass of Sun-Earth system is $4.52 \times 10^5 km$ far from the sun and this distance is 0.64 times of radius of sun.



**Q.4: A particle of mass 4 kg moves along the x-axis with a velocity v = 15t m/s, where t = 0 is the instant that the particle is at the origin.**

**(a) At t= 2 s , what is the angular momentum of particle about a point P located on +ve y axis 6 m from the origin?**

**(b) What torque about P acts on the particle?**

**Data:**

Mass of particle $= m = 4\ kg$

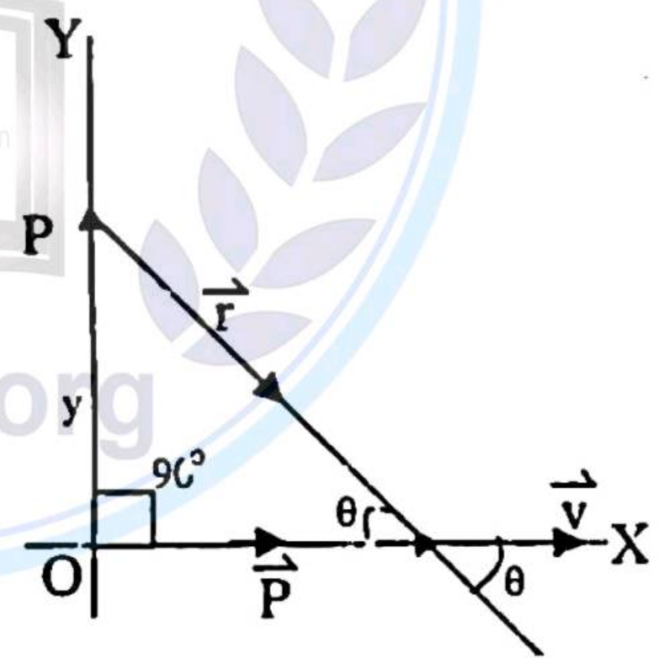
Velocity of particle $= v = 15$ t m/s

Angular Momentum $= l = ?$

Distance of Particle $= r = 6$ m

Time $= t = 2$ s

Torque $= \tau = ?$




**Solution:**

The angular momentum of particle is given by

$$l = mvr$$
$$l = m \times 15 \times t \times r$$
$$l = 4 \times 15 \times 2 \times 6$$
$$\boxed{l = 720\ kgm^2/s}$$

Now, Torque is given by

$$\tau = \frac{\Delta l}{\Delta t} = \frac{720-0}{2}$$
$$\boxed{\tau = 360\ Nm}$$

**Result:** The angular momentum of particle is $720\ kgm^2/s$ and torque is $360\ Nm$.

**Q.5: A particle of mass 'm' is located at the vector position r and has a linear momentum vector p. The vector r and p are non zero. If the particle moves only in the x, y plane, prove that $L_x = L_y = 0\ and\ L_z \neq 0$**

**Proof:**

According to the def. of Angular Momentum

$$\vec{l} = \vec{r} \times \vec{P}$$

$$\vec{l} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{\imath}\begin{vmatrix} y & z \\ P_y & P_z \end{vmatrix} - \hat{\jmath}\begin{vmatrix} x & z \\ P_x & P_z \end{vmatrix} + \hat{k}\begin{vmatrix} x & y \\ P_x & P_y \end{vmatrix}$$

$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{\imath}(yP_z - zP_y) - j(xP_z - zP_x) + \hat{k}(xP_y - yP_x)$$

As Motion is in the x-y plane therefore z=0 and Pz=0

$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{\imath}(y(0) - (0)P_y) - j(x(0) - (0)P_x) + \hat{k}(xP_y - yP_x)$$

$$l_x\hat{\imath} + l_y\hat{\jmath} + l_z\hat{k} = \hat{k}(xP_y - yP_x)$$

By equating the components we get

$$L_x = L_y = 0\ and\ L_z = \hat{k}(xP_y - yP_x)$$

Hence it is proved that $L_x = L_y = 0\ and\ L_z \neq 0$



**Q.6: A light rigid rod 1m in length rotates in the xy-plane about a pivot through the rod's centre. Two particles of mass 2kg and 3kg are connected to its ends. Determined the angular momentum of the system about the origin at the instant the speed of each particle is 5m/s.**

**Data:**

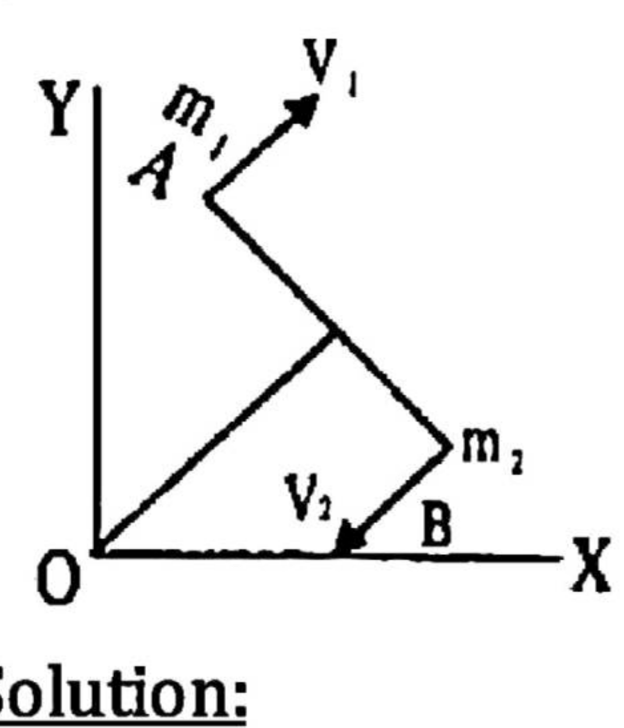
length of rod $= r = 1\ m$

Mass of 1st particle $= m_1 = 2\ kg$

Mass of 2nd particle $= m_2 = 3\ kg$

Angular Momentum $= l = ?$

Velocity of particles $= v_1 = v_2 = 5$ m/s




**Solution:**

The angular momentum of 1st particle is given by

$$\vec{l}_1 = \vec{r_1} \times \vec{P_1}$$
$$l_1 = r_1 P_1 \sin 90°$$
$$l_1 = \frac{r}{2} m_1 v_1$$
$$l_1 = \frac{1}{2} \times 2 \times 5$$
$$\boxed{l_1 = 5\ kgm^2/s}$$

Now, The angular momentum of 2nd particle is given by

$$\vec{l}_2 = \vec{r_2} \times \vec{P_2}$$
$$l_2 = r_2 P_2 \sin 90°$$
$$l_2 = \frac{r}{2} m_2 v_2$$
$$l_2 = \frac{1}{2} \times 3 \times 5$$
$$\boxed{l_2 = 7.5\ kgm^2/s}$$

Now, total angular momentum is

$$\boxed{l = l_1 + l_2 = 5 + 7.5 = 12.5\ kgm^2/s}$$

**Result:** The angular momentum of the system is $12.5\ kgm^2/s$.

**Q.7: A uniform beam of mass 'M' supports two masses $m_1$ and $m_2$. If the knife edge of the support is under the beam's centre of gravity and $m_1$ is at a distance 'd' from the centre, determine the position of $m_2$ such that the system is balanced.**



**Data:**

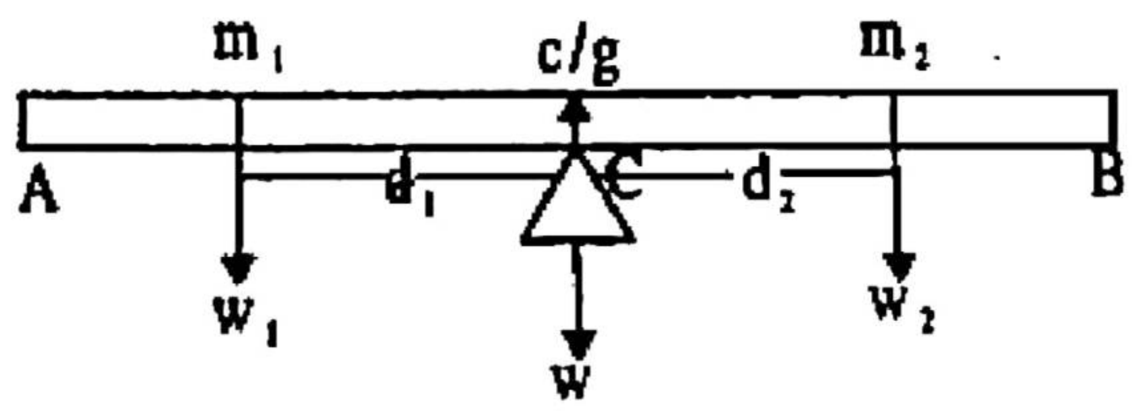
Mass of Beam $= M$

Mass of $1^{st}$ particle $= m_1$

Mass of $2^{nd}$ particle $= m_2$

Distance of $m_1$ from centre $= d_1 = d$

Distance of 2 from centre $= d_2 = D = ?$




**Solution:**

Using 2nd condition of equilibrium

$$\sum \tau = 0$$

$$W_1 \times d_1 - W_2 \times d_2 + W \times 0 = 0$$

$$W_1 \times d_1 = W_2 \times d_2$$

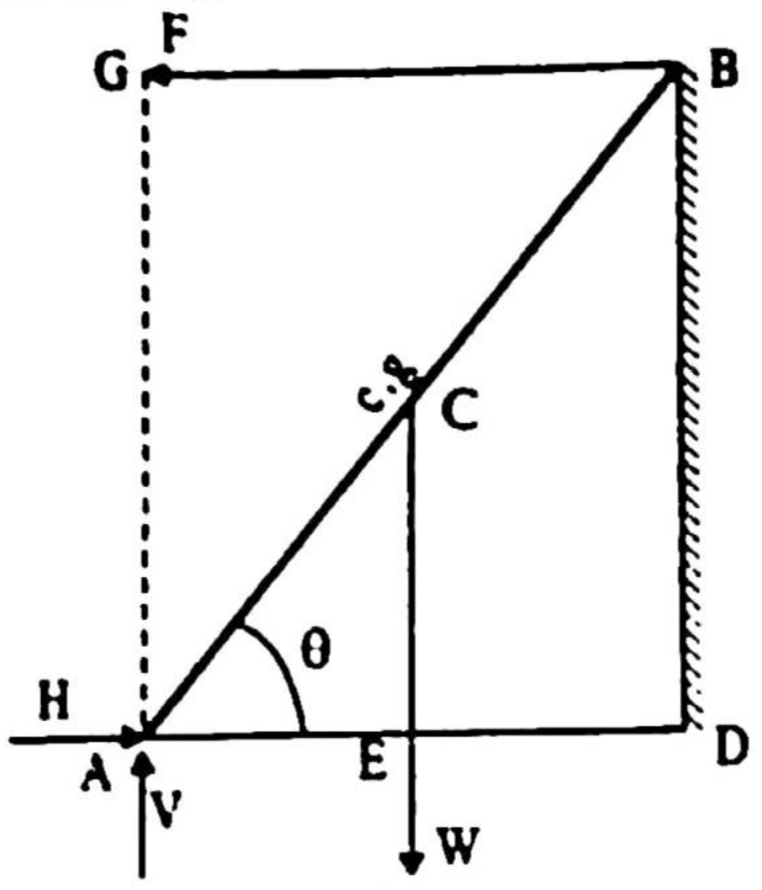
$$m_1 g \times d = m_2 g \times D$$

$$D = \frac{m_1 g \times d}{m_2 g}$$

$$\boxed{D = \frac{m_1}{m_2} d}$$

**Result:** The position of $m_2$ such that the system is balanced is $\frac{m_1}{m_2} d$ from the centre.

**Q.8: A uniform ladder of length $l$ and weight W = 50 N rests against a smooth vertical wall. If the coefficient of friction between the ladder and the ground is 0.40, find the minimum angle $\theta_{min}$ such that the ladder may not slip.**




**Data:**

Length $= \overrightarrow{AB} = L$

Weight $= W = 50N$

Co-efficient of friction $= \mu = 0.4$

Minimum angle $= \theta = ?$

**Solution:**

Using first condition of equilibrium

$\sum F_x = 0 ; \sum F_y = 0$

$H = F$ ----(i)

And

$V = W$----(ii)

Dividing equation (i) by Equation (ii)

$\frac{H}{V} = \frac{F}{W}$

Since

$\mu = \frac{H}{V}$

Therefore

$\mu = \frac{F}{W}$

$0.4 = \frac{F}{50}$

$\boxed{F = 20N}$

Using 2nd condition of equilibrium

$\sum \tau = 0$

$F \times \overrightarrow{AG} + H \times 0 + v \times 0 - W \times \overrightarrow{AE} = 0$

$F \times \overrightarrow{BD} = W \times \overrightarrow{AE}$ --- (iii)

From $\Delta ABD$

$$\sin\theta = \frac{\overrightarrow{BD}}{\overrightarrow{AB}}$$

$$\sin\theta = \frac{\overrightarrow{BD}}{L}$$

$$\boxed{\overrightarrow{BD} = L \sin\theta}$$

and From $\Delta ACE$

$$\cos\theta = \frac{\overrightarrow{AE}}{\overrightarrow{AC}}$$

$$\cos\theta = \frac{\overrightarrow{AE}}{L/2}$$

$$\boxed{\overrightarrow{AE} = \frac{1}{2} L \cos\theta}$$

Putting values of $\overrightarrow{BD}$ and $\overrightarrow{AE}$ in eq (iii)

$$F\, L \sin\theta = W \frac{L}{2} \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{W}{2F}$$

Or

$$Tan\theta = \frac{W}{2F}$$

$$Tan\theta = \frac{50}{2 \times 20}$$

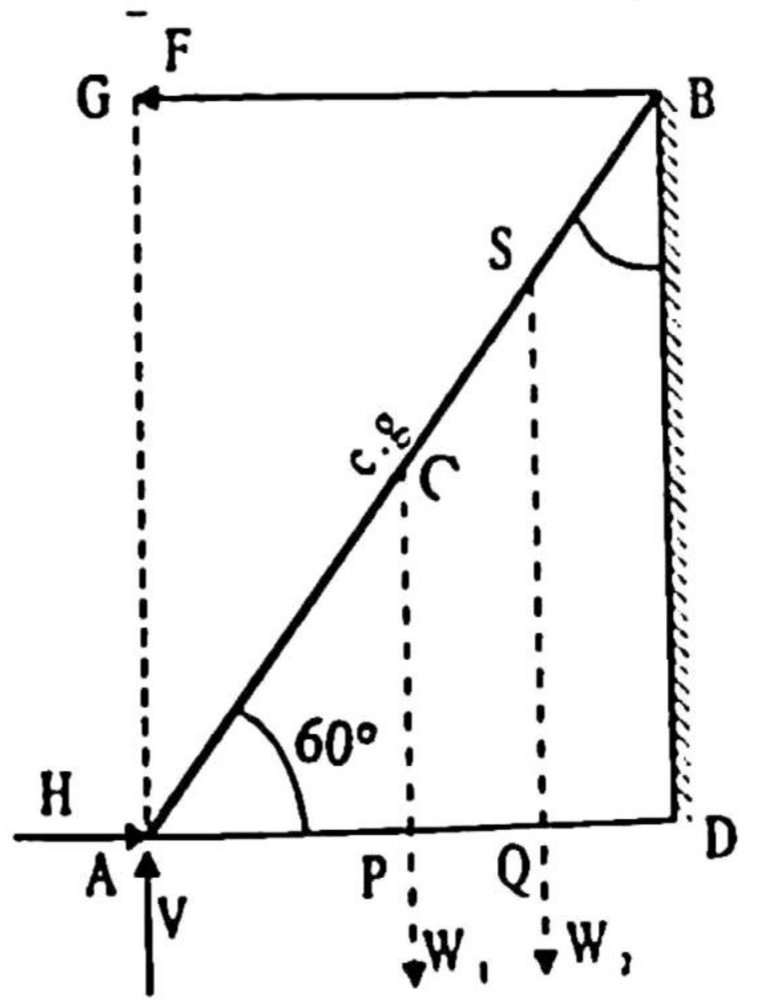
$$\theta = Tan^{-1}\frac{5}{4}$$

$$\boxed{\theta = 51.3^0}$$

**Result:** The minimum angle should be $51.3^0$ .

**Q.9: A ladder with a uniform density and a mass 'm' rests against a frictionless vertical wall at an angle of 60° . The lower end rests on a flat surface where the coefficient of friction (static) is 0.40. A student with a mass (M = 2m) attempts to climb the ladder. What fraction of the length 'L' of the ladder will the student have reached when the ladder begins to slip?**






**Data:**

Angle $= \theta = 60^0$

Length $= \overrightarrow{AB} =$ L

Mass of Ladder $= m$

Mass of Student $= M = 2m$

Weight of Ladder $= W_1$

Weight of Student $= W_2$

Co-efficient of friction $= \mu = 0.4$

Fraction of the ladder covered $= \frac{\overrightarrow{AS}}{\overrightarrow{AB}} = ?$

**Solution:**

Using first condition of equilibrium

$\sum F_x = 0 ; \sum F_y = 0$

$H = F$ ----(i)

And

$V = W_1 + W_2$

$V = mg + 2mg$

$\boxed{V = 3\ mg}$

Dividing equation (i) by Equation (ii)

$\frac{H}{V} = \frac{F}{3\ mg}$

Since

$\mu = \frac{H}{V}$

Therefore

$\mu = \frac{F}{3\ mg}$

$0.4 = \frac{F}{3\ mg}$

$\boxed{F = 1.2\ mg}$

Using 2nd condition of equilibrium

$\sum \tau = 0$

$F \times \overrightarrow{AG} + H \times 0 + v \times 0 - W_1 \times \overrightarrow{AP} - W_2 \times \overrightarrow{AQ} = 0$

$F \times \overrightarrow{BD} = W_1 \times \overrightarrow{AP} + W_2 \times \overrightarrow{AQ}$ --- (iii)

From $\Delta ABD$

$sin\theta = \frac{\overrightarrow{BD}}{\overrightarrow{AB}}$

$\boxed{\overrightarrow{BD} = \overrightarrow{AB}\sin\theta}$

and from $\Delta ACP$

$Cos\theta = \frac{\overrightarrow{AP}}{\overrightarrow{AC}}$

$Cos\ \theta = \frac{\overrightarrow{AE}}{\overrightarrow{AB}/2}$

$\boxed{\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AB}\cos\theta}$

from $\Delta ASQ$

$Cos\theta = \frac{\overrightarrow{AQ}}{\overrightarrow{AS}}$

$\boxed{\overrightarrow{AQ} = \overrightarrow{AS}\cos\theta}$

Putting values of $\overrightarrow{BD}$ and $\overrightarrow{AP}$ and $\overrightarrow{AQ}$ in eq (iii)

$F \times \overrightarrow{AB}\sin\theta = W_1 \times \frac{1}{2}\overrightarrow{AB}\cos\theta + W_2 \times \overrightarrow{AS}\cos\theta$

Putting values

$1.2\ mg \times \overrightarrow{AB}\sin 60° = mg \times \frac{1}{2}\overrightarrow{AB}\cos 60° +$

$2mg \times \overrightarrow{AS}\cos 60°$

$1.2 \times \overrightarrow{AB} \times 0.866 = \frac{1}{2}\overrightarrow{AB} \times 0.5 + 2 \times \overrightarrow{AS} \times 0.5$

$1.0392 \times \overrightarrow{AB} = \overrightarrow{AB} \times 0.25 + \overrightarrow{AS}$

$1.0392 \times \overrightarrow{AB} - \overrightarrow{AB} \times 0.25 = \overrightarrow{AS}$

$\overrightarrow{AB}(1.0392 - 0.25) = \overrightarrow{AS}$

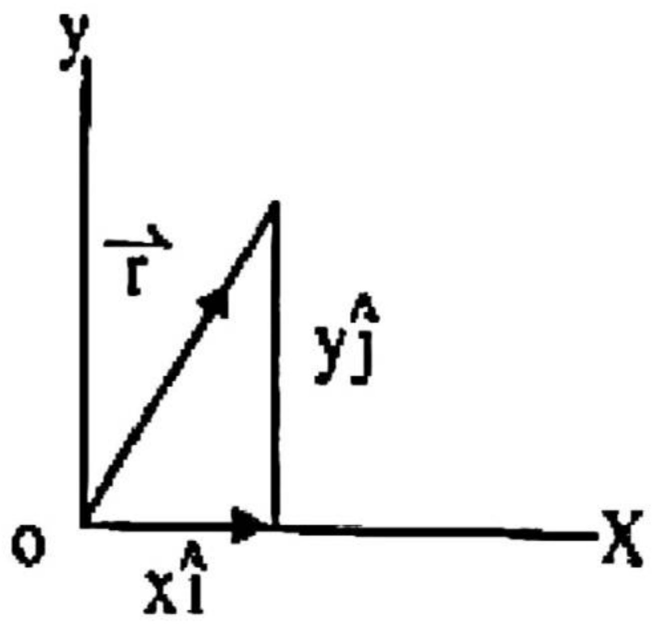
$\overrightarrow{AB}(0.789) = \overrightarrow{AS}$

Or $\frac{\overrightarrow{AS}}{\overrightarrow{AB}} = 0.789$

In Percentange: $\boxed{\frac{\overrightarrow{AS}}{\overrightarrow{AB}}\% = 78.9\ \%}$

**Result:** The student have reached 78.9 % of length of ladder.

**Q.10: A particle of mass 0.3 kg moves in the xy-plane. At the instant its coordinates are (2, 4)m, its velocity is (3i + 4j)m/s. At this instant determine the angular momentum of the particle relative to the origin.**




**Data:**

Mass of particle = m = 0.3 kg

Co-ordinates of position = $\vec{r}$ = (2, 4) =2i + 4j

Velocity of particle = $\vec{v}$ = 3i + 4 j

Angular momentum of particle = $\vec{l}$ =?

**Solution:**

According to the def. of Angular Momentum

$\vec{l} = m(\vec{r} \times \vec{v})$ ---- (i)

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \hat{\imath}\begin{vmatrix} 4 & 0 \\ 4 & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix}$$

$\vec{r} \times \vec{v} = \hat{\imath}(0 - 0) - \hat{\jmath}(0 - 0) + \hat{k}(8 - 12)$

$\boxed{\vec{r} \times \vec{v} = -4\hat{k}}$

Putting in eq (i)

$\vec{l} = 0.3 \times (-4\hat{k})$

Or $\boxed{\vec{l} = -1.2\ \hat{k}}$

**Result:** The Magnitude of angular momentum is 1.2 j.s and direction is along –ve z axis.

**Q.11: A uniform horizontal beam of length 8m and weighing 200N is pivoted at the wall with its far end supported by a cable that makes an angle of 53° with the horizontal. If a person weighing 600N stands 2m from the wall, find the tension and the reaction force at the pivot.**

**Data:**

Length of Beam = $\overline{AB} = 8m$

Weight of Beam = $W_1 = 200\ N$

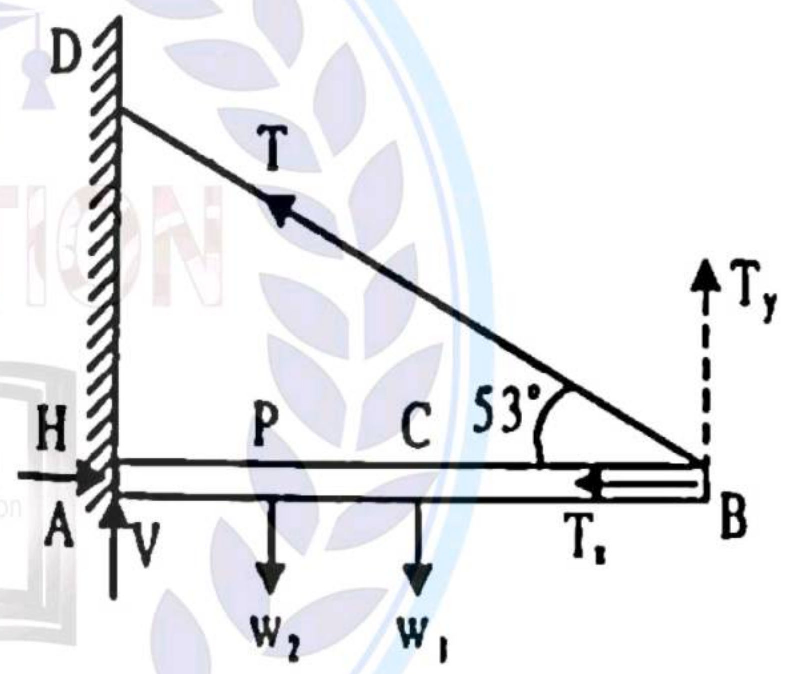
Weight of Person = $W_2 = 600\ N$

Distance of Person from wall = $\overrightarrow{AP} = 2m$

Angle = = $\theta = 53°$

Tension = = $T$ =?

Reaction force at pivot = = $R$ =?




**Solution:**

Using first condition of equilibrium

$\sum F_x = 0 ; \sum F_y = 0$

$H = T_x$

$H = T\cos\theta$

$H = T\cos 53°$

$H = 0.6\,T$ --- (i)

$V + T_y = W_1 + W_2$

$V + T\sin\theta = 200 + 600$

$V + T\sin 53 = 800$

$V + 0.798\,T = 800$ ---(ii)

Using 2nd condition of equilibrium

$\sum\tau = 0$

Taking Moment about A where Tx is parallel to moment arm so its torque is zero.

$T_y \times \overrightarrow{AB} + H \times 0 + V \times 0 - W_1 \times \overrightarrow{AC} - W_2 \times \overrightarrow{AP} = 0$

$0.798\,T \times 8 - 200 \times \frac{\overrightarrow{AB}}{2} - 600 \times 2 = 0$

$6.4\,T - 200 \times 4 - 600 \times 2 = 0$

$6.4\,T \times 8 - 2000 = 0$

$\boxed{T = \frac{2000}{6.4} = 312.5\,N}$

For H Putting in eq(i)

$H = 0.6 \times 312.5$

$\boxed{H = 187.5\,N}$

For V Putting in eq(ii)

$V + 0.798 \times 312.5 = 800$

$\boxed{V = 553.4\,N}$

Now,Reaction force at pivot is given by

$R = \sqrt{H^2 + V^2}$

$R = \sqrt{(187.5)^2 + (553.4)^2}$

$\boxed{R = 584.3\,N}$

**Result:** The tension in the scable is 312.5 Nand the reaction force at the pivot is 584.3 N

# PAST PAPER NUMERICALS



**2022** Q.2 (ix) Textbook Numerical 5

**2019** Q.2 (ix) Textbook Numerical 8

**2018, 2017** No Numerical

## 2016

Q.2 (iv) Textbook Numerical 8

2015 No Numerical

## 2014

**Q.2(i) A particle of mass 500 gm rotates in a circular orbit of radius 25 cm at a constant rate of 1.5 revolutions per second. Find the angular momentum with respect to centre of the orbit .**

**Data:**

Mass of particle =500 gm = 0.5 kg

Radius of circular orbit =25 cm = 0.25 m

Angular velocity = 1.5 rev / sec = 1.5 × 2π = $3\pi\ rad / sec$

Angular momentum =?

**Solution:**

As we know that

$L = m\,v\,r$

Or $L = m\,(r\omega)\,r = mr^2\omega$

$L = (0.5)(0.25)^2(3 \times 3.14)$

$\boxed{L = 0.294\,kg\,m^2/s}$

**Result:** The angular momentum of particle is $0.294\,kg\frac{m^2}{s}$.

## 2013

No Numerical

## 2012

**Q.2 (xi) A particle of mass 0.5 kg moves along xy-plane. At that instant, the coordinates are (3, 4)m and its velocity is (4i +5j) m/sec. Determine the angular momentum relative to origin at that time.**

**Data:**

Mass of particle = m = 0.5 kg

Co-ordinates of position $= \vec{r} = (3, 4) = 3i + 4j$

Velocity of particle $= \vec{v} = 4i + 5\,j$

Angular momentum of particle $= \vec{l} = ?$

**Solution:**

According to the def. of Angular Momentum

$$\vec{l} = m(\vec{r} \times \vec{v}) \text{ ---- (i)}$$

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & 0 \\ 4 & 5 & 0 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \hat{\imath}\begin{vmatrix} 4 & 0 \\ 5 & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & 0 \\ 4 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}(15-16)$$

$$\boxed{\vec{r} \times \vec{v} = -\hat{k}}$$

Putting in eq (i)

$$\vec{l} = 0.5 \times (-\hat{k})$$

Or $$\boxed{\vec{l} = -0.5\,\hat{k}}$$

**Result:** The Magnitude of angular momentum is 0.5 j.s and direction is along –ve z axis.

## 2011

**Q.2 (x)** **Textbook Numerical 8**

## 2010

**Q.2 (xv)** **Textbook Numerical 8**