Chapter = 02

Scalars and Vectors

THEORY NOTES

SCALARS

DEFINITION:

"Scalars are those physical quantities which can be specified by a number having appropriate unit"

EXAMPLES: Mass, Temperature, Volume, Work, Energy

PROPERTIES:

a) **DIRECT PREFERENCE:**

It is a measure in which direction is unimportant or meaningless thus, they are quoted as a pure number with a unit

b) **ARITHMETICAL OPERATIONS:**

Scalars can be added, subtracted, multiplied or divide by simple arithmetical rules.

c) **REPRESENTATION:** Scalars are represented by a number with a suitable unit.

d) **EQUIVALENCE:**

Two or more than two scalars (measured in the same system of units) are equal only if they have same magnitude and sign.

VECTORS:

DEFINITION:

"Vectors are those physical quantities which can be specified by magnitude and direction with appropriate"

EXAMPLES: Force, Velocity, Displacement, Torque, Momentum

PROPERTIES:

a) **DIRECT PREFERENCE:**

It is a measure in which direction is important or must be usually be specified. Thus, they are quoted as a number with a unit and a direction.

b) **ARITHMETICAL OPERATIONS:**

Vectors can't be added, subtracted multiplied or divided by simple Mathematical rules. Addition of vectors must take account of direction. Multiplication of vectors is performed in two ways (i) Scalar Product (ii) Vector Product

c) **REPRESENTATION:**

Vectors are represented by an arrow-headed line segment.

⟶ Direction

Length = Magnitude

d) **EQUIVALENCE:**

Two or more Vectors are equal only if they same magnitude and direction.

RESOLUTION OF VECTORS:

Vectors can be resolved in two or more components. Thus, splitting of a vector into components is termed as "Resolution of Vectors".

RECTANGULAR COMPONENTS:

If the components of a vector are perpendicular to each, other then these usually these components are called:

i) The Horizontal component on X-Component
ii) The Vertical component on Y- Component.
iii)

PROCEDURE OF RESOLUTION:

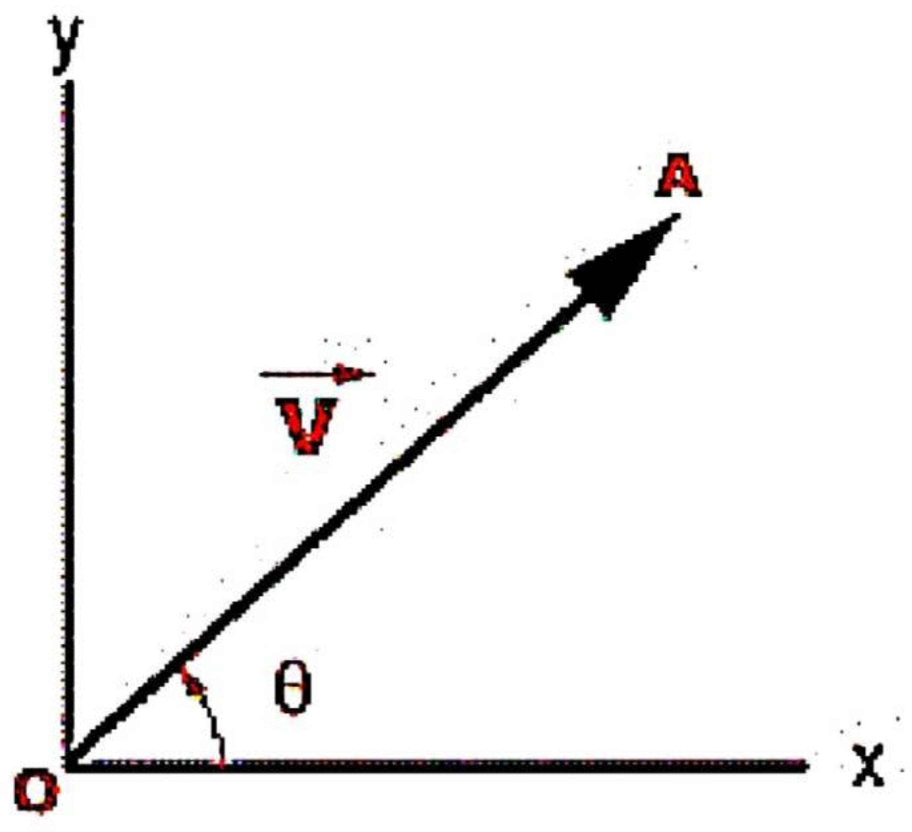


figure 01

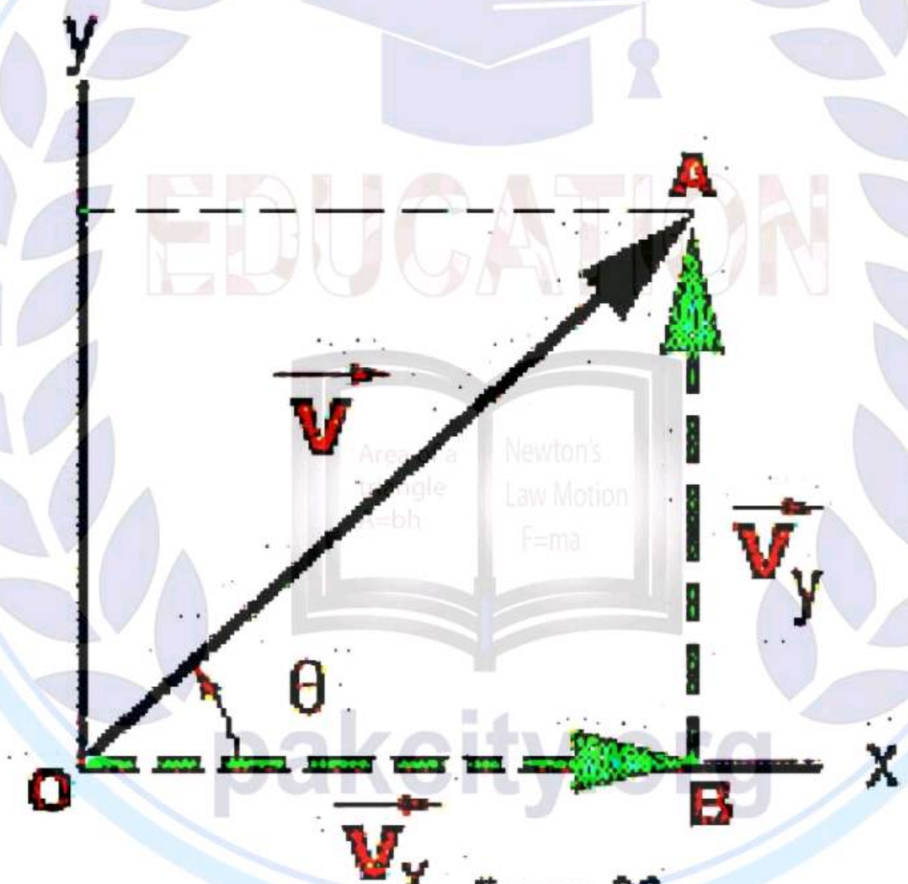


figure 02

Consider a vector $\vec{V}$ acting at a point making an angle **q** with positive X-axis. Vector $\vec{V}$ is represented by a line OA.From point A draw a perpendicular AB on X-axis.Suppose OB and BA represents two vectors.Vector OA is parallel to X-axis and vector BA is parallel to Y-axis.Magnitude of these vectors are V_x and V_y respectively.

By the method of head to tail we notice that the sum of these vectors is equal to vector .Thus Vx and Vy are the rectangular components of vector .

Now,

$\vec{V_x}$ = Horizontal Component
$\vec{V_y}$ = Vertical Component

From figure, $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{AB}$

$$\vec{V} = \vec{V_x} + \vec{V_y}$$

$$\vec{V} = \vec{V_x}\,\hat{\imath} + \vec{V_y}\hat{\jmath}$$

In ΔOAB

$$Cos\theta = \frac{\overrightarrow{OB}}{\overrightarrow{OA}} \text{ or } Cos\theta = \frac{\vec{V_x}}{\vec{V}}$$

or

$$\boxed{V_x = VCos\theta} \text{-----------(i)}$$

and

$$Sin\theta = \frac{\overrightarrow{AB}}{\overrightarrow{OA}} \text{ or } Sin\theta = \frac{\vec{V_y}}{\vec{V}}$$

or

$$\boxed{V_y = VSin\theta} \text{----------(ii)}$$

MAGNITUDE:

Squaring eq(i) and eq(ii) and then adding

$$V_x{}^2 + V_y^2 = V^2Cos^2\theta + V^2Sin^2\theta$$

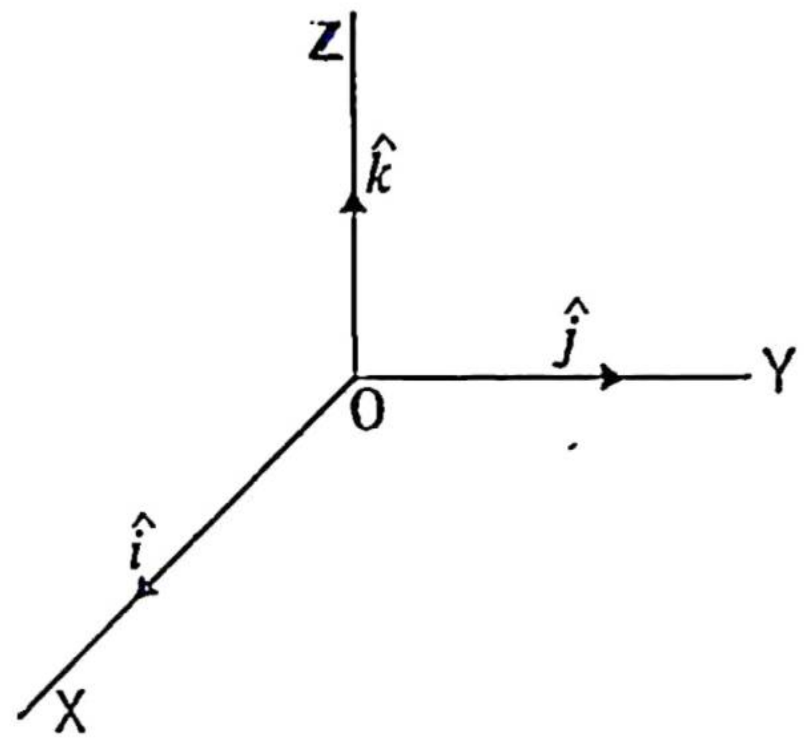
or $V_x{}^2 + V_y^2 = V^2(Cos^2\theta + Sin^2\theta)$ Since $Cos^2\theta + Sin^2\theta = 1$

or $V^2 = V_x{}^2 + V_y^2$

or

$$\boxed{V = \sqrt{V_x{}^2 + V_y^2}}$$

DIRECTION:



Dividing Eq(i) by Eq(ii)

$$\frac{V_x}{V_y} = \frac{VSin\theta}{VCos\theta}$$

or

$$\frac{V_x}{V_y} = Tan\theta$$

or

$$\boxed{\theta = Tan^{-1}\left(\frac{V_x}{V_y}\right)}$$

UNIT VECTOR:

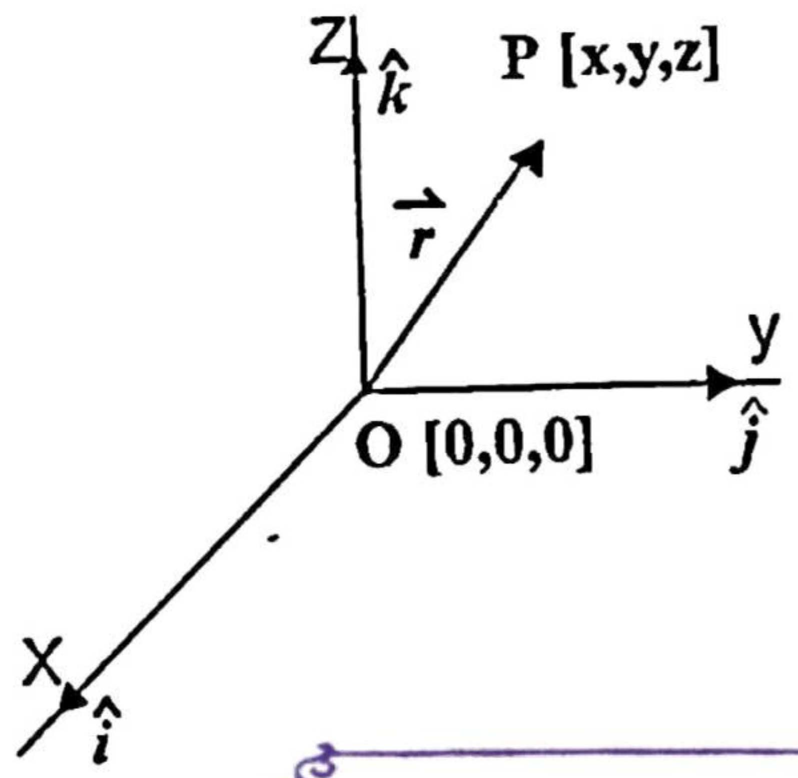
A vector whose magnitude equals to "one" is called unit vector and it just represents direction of vector. In three dimensional space the unit vectors along x,y and z axis are i, j and k respectively.

Mathematically a vector quantity $\vec{A}$ is defined through the equation.

$$\vec{A} = |\vec{A}|\hat{a} \qquad \text{or} \qquad \vec{A} = A\hat{a}$$

Where $|\vec{A}|$ represents magnitude or length of vector, $\hat{a}$ is the unit vector which represents direction of vector $\vec{A}$ e.g. If Force on a body 8 N along x - axis, then $\vec{F} = 8\hat{\imath}\ N$

POSITION VECTOR



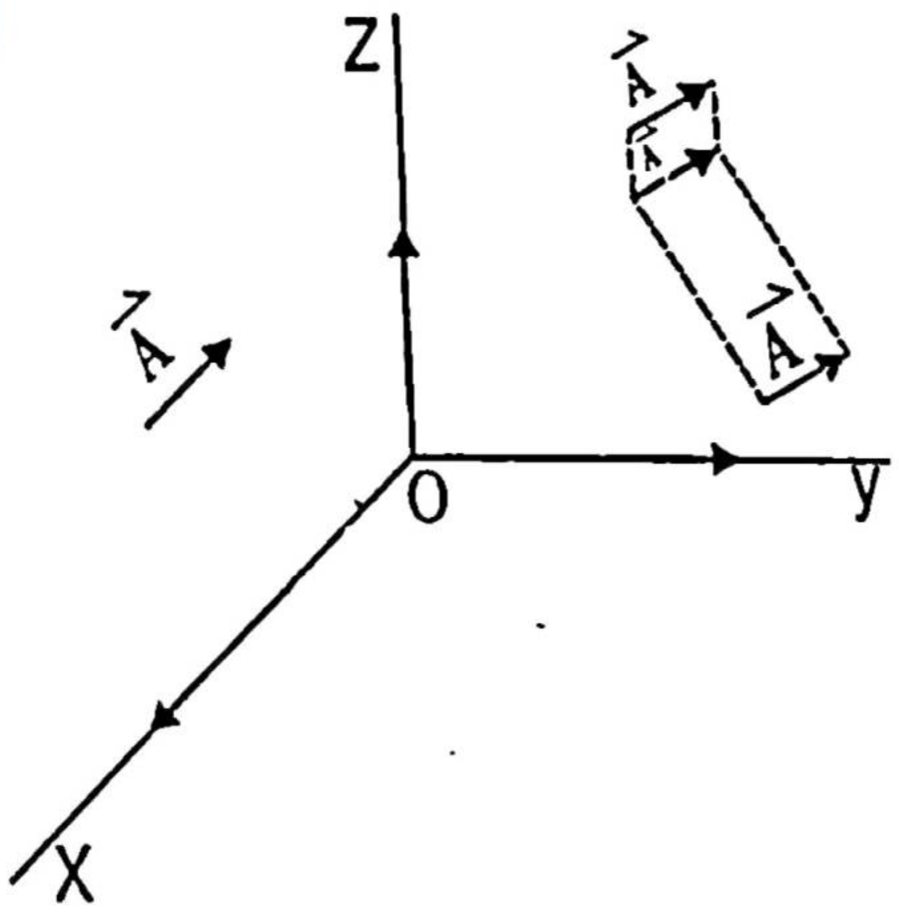
A vector which starts from origin or fixed point is called position vector. In three dimensional space it is usually written as $\vec{r}$ and in component form,

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

The magnitude of this position vector is represented by

$$r = \sqrt{x^2 + y^2 + z^2}$$

FREE VECTOR :



Such vector which can be displaced anywhere in space parallel to itself is called Free Vector. In this case magnitude and direction remain same. All vectors except positions are free vectors.

NULL VECTOR:

A vector whose magnitude equals to zero and has no direction and it may have any direction is called Null Vector.

Thls vector always appears as resultant of addition of two equal but opposite vectors i.e.

If A and B are equal in magnitudes and parallel but in opposite direction then

$$\vec{A} + \vec{B} = \vec{0}$$

Here $\vec{0}$ is null vector, and it may be written as

$$\vec{0} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

ADDITION OF VECTORS:

A vector can be added in another vector only which results in a new vector of the same kind. This may done in three different ways.

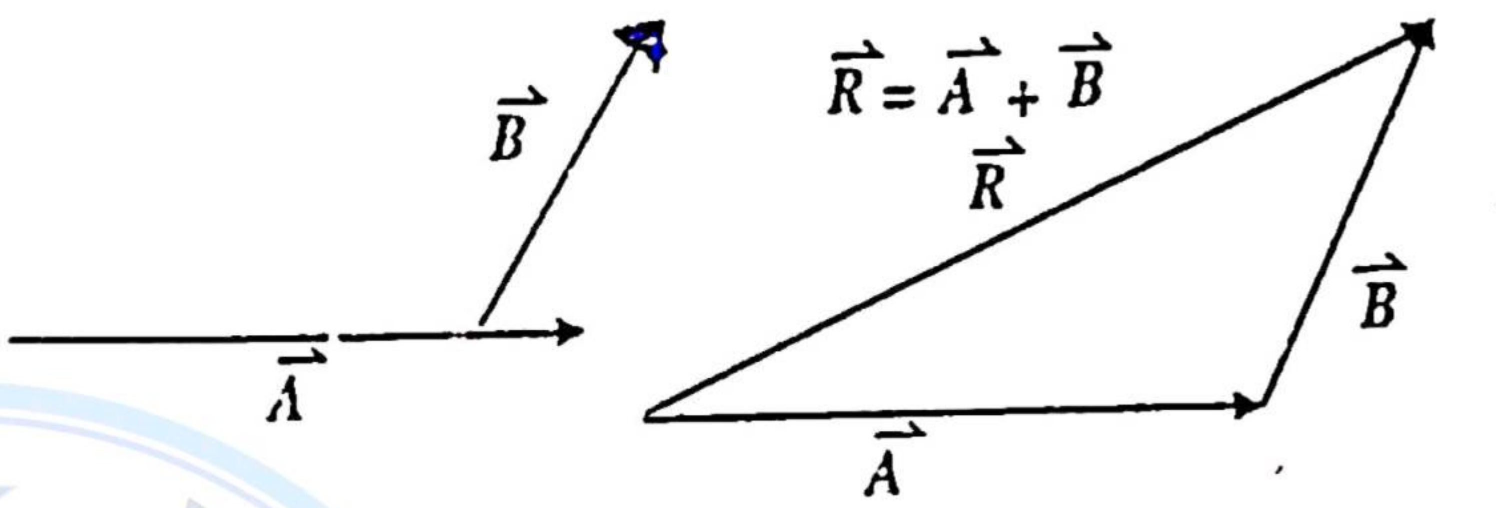
(i) By head - to - tail rule, or graphical method, or triangular law of vector addition.

(ii) Analytical method.

(iii) Addition of vectors by rectangular components method.

(i) GRAPHICAL METHOD (HEAD - TO - TAIL RULE):

In head - to - tail rule, number of vectors can be added by joining tail of successive vectors with the head of previous vector. The resultant vector is obtained by joining the tail of first with the head of last vector.



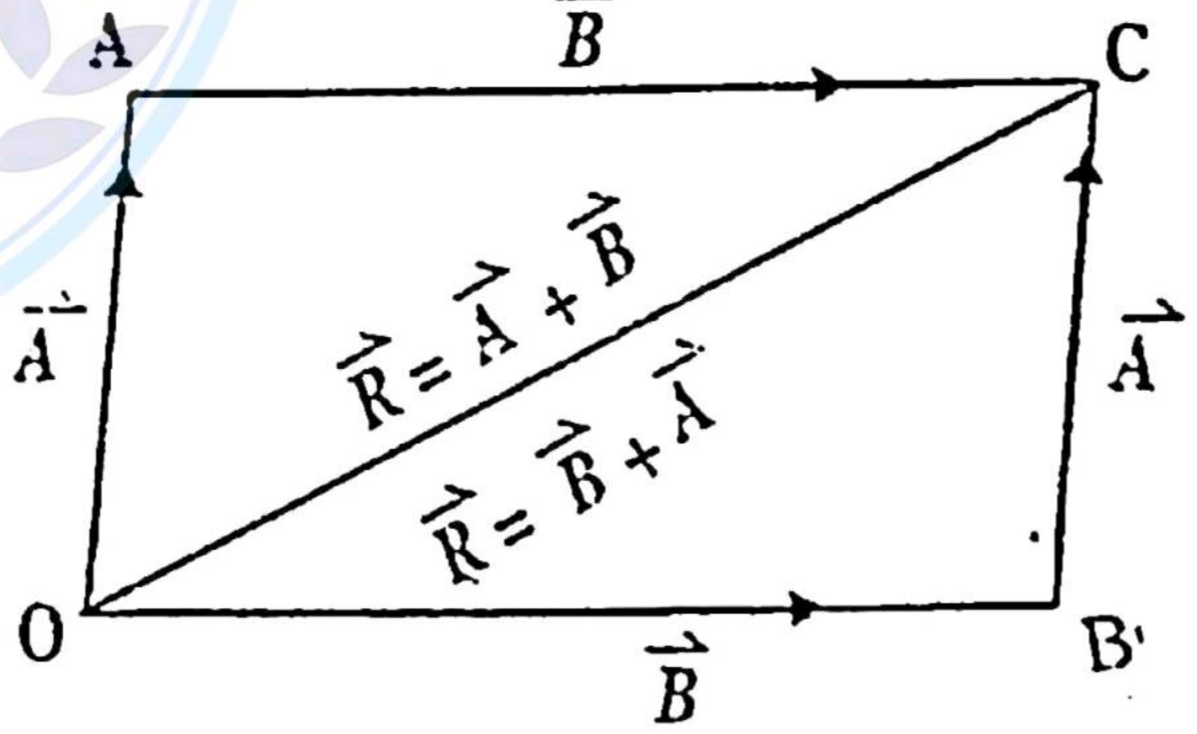
$$\vec{R} = \vec{A} + \vec{B}$$

This equation gives resultant vector and it is also known as "Triangle Law of Vector Addition".

PROPERTIES OF VECTOR ADDITION:

(a) Commutative law of vector addition:

Suppose two vectors $\vec{A}$ and $\vec{B}$ represent the two adjacent sides of a parallelogram then the diagonal OC represents the resultant vector $\vec{R}$ shown in fig.



Since $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = \vec{B} + \vec{A}$$

Therefore

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

This is known as commutative law of vector B addition or parallelogram law of vector addition.

(b) Associative law of vector addition:

Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are added (suppose using head to tail rule) in two different ways as shown in flg

The resultant vector $\vec{R}$ may be obtained in two different ways i.e.

$$\vec{R_1} = \vec{A} + \vec{B}$$

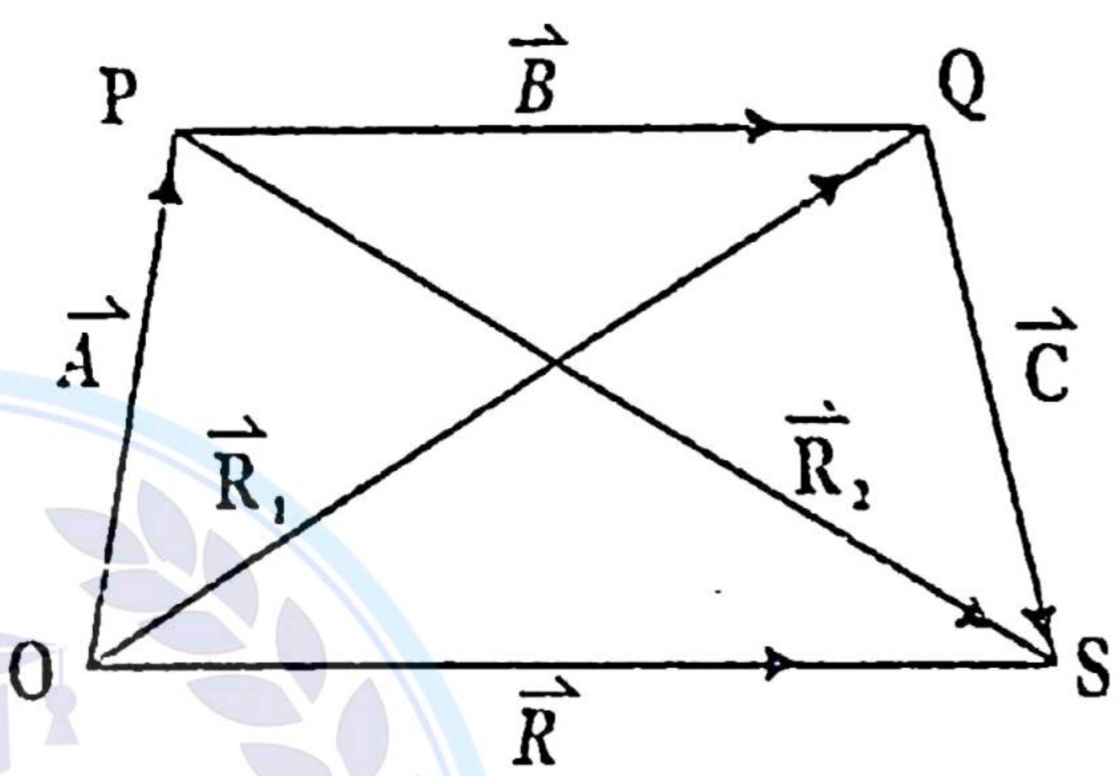
adding $\vec{C}$ on both sides

$$\vec{R_1} + \vec{C} = (\vec{A} + \vec{B}) + \vec{C}$$

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{R} \text{-----(i)}$$

Now,

$$\vec{R_2} = \vec{B} + \vec{C}$$



adding $\vec{A}$ on both sides

$$\vec{A} + \vec{R_1} = \vec{A} + (\vec{B} + \vec{C})$$

$$\vec{A} + (\vec{B} + \vec{C}) = \vec{R} \text{-----(ii)}$$

By comparing eq(i) and eq(ii) we get

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

This property is known as associative law of vector addition.

(ii) Analytical Method of Vector Addition:

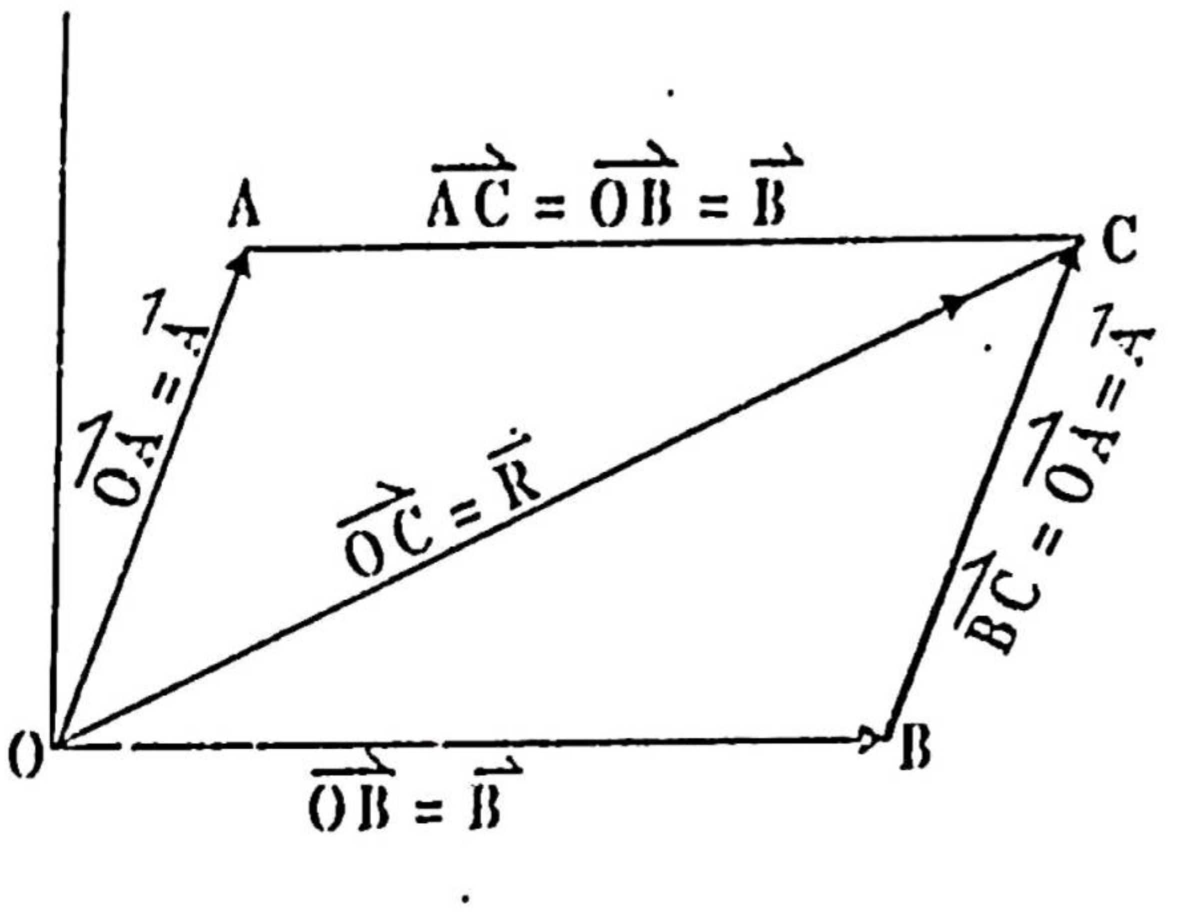
This is a mathematical method of vector addition and it is based upon laws of trigonometry.
According to law of cosines, in any triangle,

$a^2 = b^2 + c^2 - 2bc \cos \alpha$
$b^2 = a^2 + c^2 - 2ac \cos \beta$

Then the magnitude of resultant vector will be

$$R^2 = A^2 + B^2 - 2AB \cos \angle OAC$$

or $R = \sqrt{A^2 + B^2 - 2AB \cos \angle OAC}$



According to law of sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

and in our fig.

$$\frac{A}{\sin \angle ACO} = \frac{B}{\sin \angle AOC} = \frac{R}{\sin \angle OAC}$$

This formula gives direction of resultant vector R. However this method is restricted to the addition of two vectors only.

(iii) Rectangular Components Method:

The way of adding vectors with the help of their rectangular components is called addition of vectors by rectangular components method.

Suppose two Position vectors $\vec{V_1}$, and $\vec{V_2}$ having lengths or magnitudes V_1 and V_2 and making angles θ_1 and θ_2 respectively are to be added.

For this purpose we first adopt head-to-tail rule and then we draw perpendiculars form their heads on x and y axes to get their rectangular components as shown in fig

The rectangular components of V_1 are

$$V_{1x} = V_1 Cos\theta_1$$
$$V_{1y} = V_1 Sin\theta_1$$

The rectangular components of V_2 are

$$V_{2x} = V_2 Cos\theta_2$$
$$V_{2y} = V_2 Sin\theta_2$$

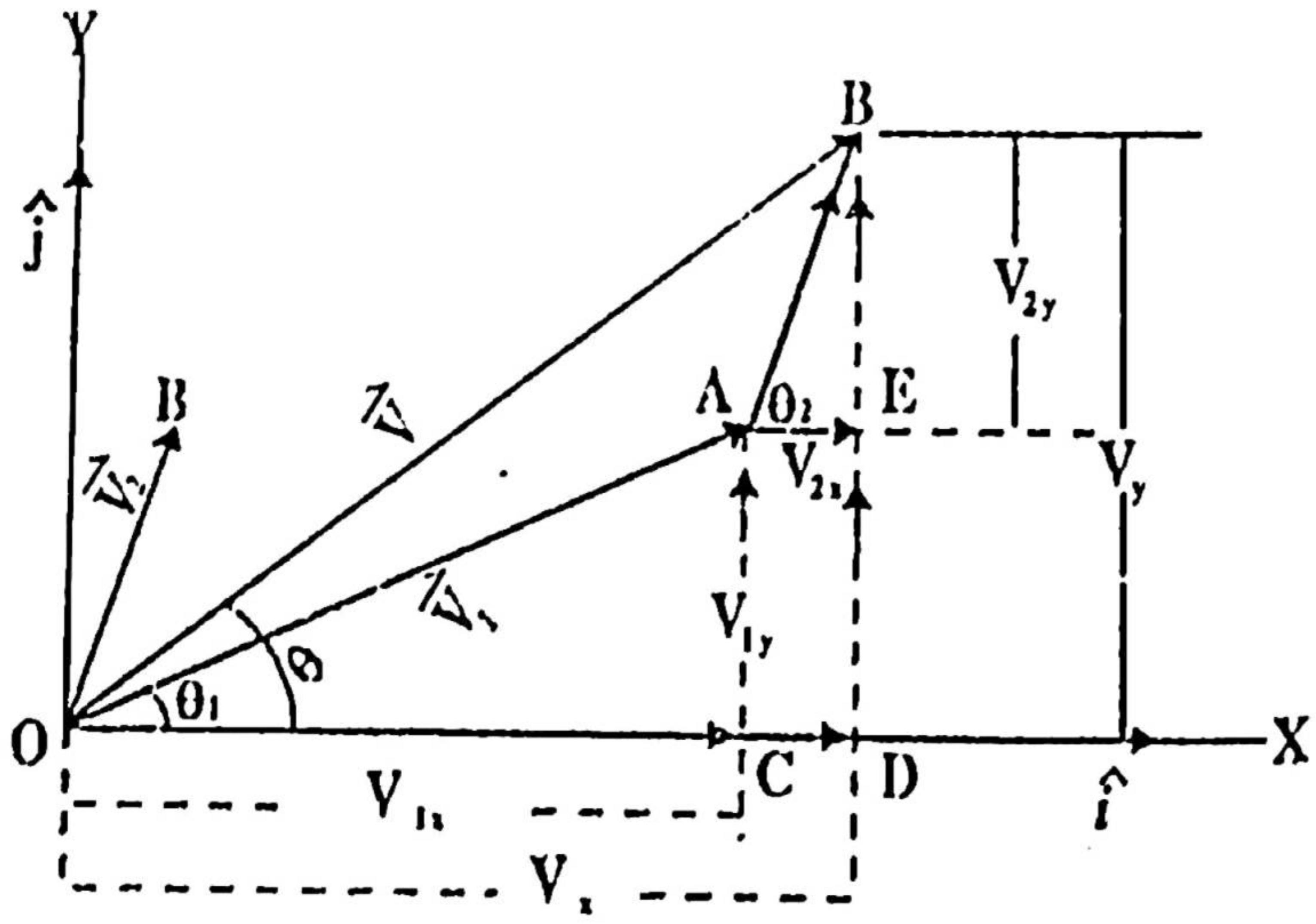
It is clear from the figure that

$\overrightarrow{V_{1x}} = \overrightarrow{OC}$
$\overrightarrow{V_{2x}} = \overrightarrow{AE}$
$\overrightarrow{V_x} = \overrightarrow{OD}$

and

$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$

or $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{AE}$



$\overrightarrow{V_x} = \overrightarrow{V_{1x}} + \overrightarrow{V_{2x}}$
$\overrightarrow{V_x}\hat{i} = \overrightarrow{V_{1x}}\hat{i} + \overrightarrow{V_{2x}}\hat{i}$
$\overrightarrow{V_x}\hat{i} = (\overrightarrow{V_{1x}} + \overrightarrow{V_{2x}})\hat{i}$
$\overrightarrow{V_x} = \overrightarrow{V_{1x}} + \overrightarrow{V_{2x}}$

or $$\boxed{\overrightarrow{V_x} = V_1 Cos\theta_1 + V_2 Cos\theta_2}$$

Similarly for y component

$\overrightarrow{V_{1y}} = \overrightarrow{CA}$
$\overrightarrow{V_{2y}} = \overrightarrow{EB}$
$\overrightarrow{V_y} = \overrightarrow{DB}$

and

$\overrightarrow{DB} = \overrightarrow{DE} + \overrightarrow{EB}$

or $\overrightarrow{OD} = \overrightarrow{CA} + \overrightarrow{AE}$

$\overrightarrow{V_y} = \overrightarrow{V_{1y}} + \overrightarrow{V_{2y}}$
$\overrightarrow{V_y}\hat{j} = \overrightarrow{V_{1y}}\hat{j} + \overrightarrow{V_{2y}}\hat{j}$
$\overrightarrow{V_y}\hat{j} = (\overrightarrow{V_{1y}} + \overrightarrow{V_{2y}})\hat{j}$
$\overrightarrow{V_y} = \overrightarrow{V_{1y}} + \overrightarrow{V_{2y}}$

or $$\boxed{\overrightarrow{V_y} = V_1 Sin\theta_1 + V_2 Sin\theta_2}$$

Now , the resultant can be calculated by the formula

$$\boxed{V = \sqrt{V_x{}^2 + V_y^2}}$$

PRODUCT OF TWO VECTORS

Vector can be multiplied in two different ways.

i) Scalars product.

ii) Vector product.

SCALAR PRODUCT

DEFINITION:

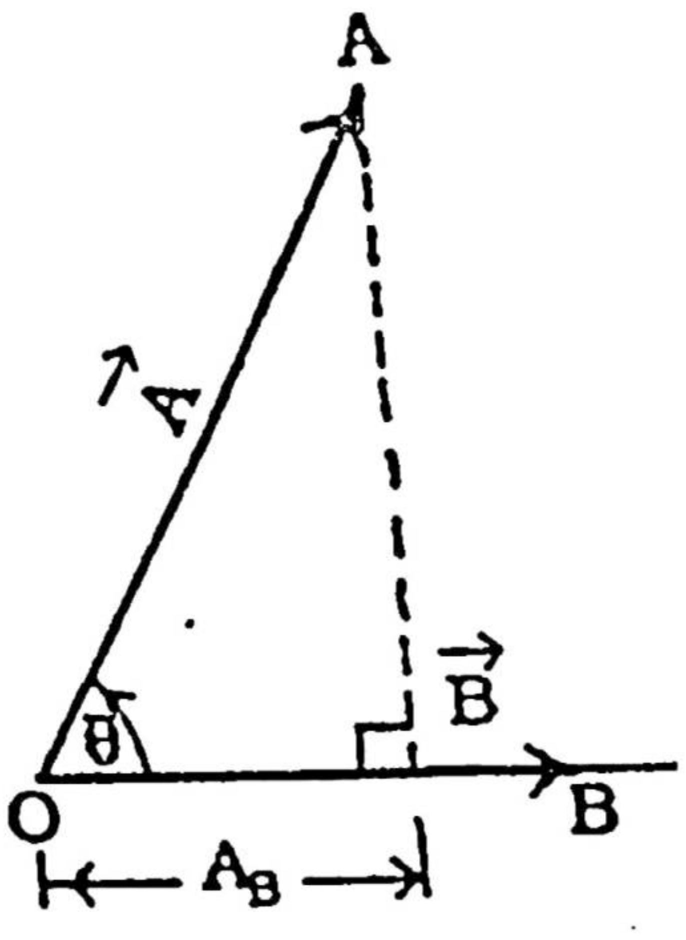
"The multiplication of two vectors to give a scalar."
Or in other words,
"it involves the multiplication of two vectors in such a way that their product is a scalar quantity".

REPRESENTATION:

This product is represented by placing a 'DOT' between two vectors.

EXAMPLES:

1) **Work:** Work is a scalar product of force (F) and displacement (d)
2) **Power:** Power is the scalar product of force (F) and d velocity (V)



EXPLANATION:

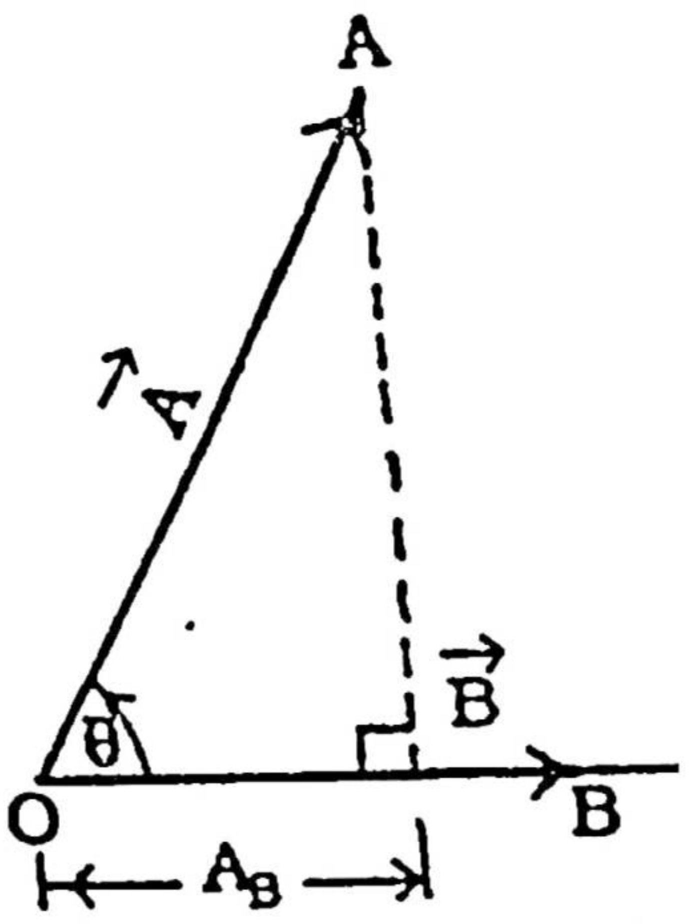
Scalar product of two vectors A and B is the product of magnitudes of two vectors and the cosine of the angle between them.

Let vector A and B. Draw a perpendicular form head of B on x axis.

According to the definition of scalar product , it is equal to the product of magnitude of first vector with the length of projection of second vector onto first vector.

Thus,

$$\vec{A}.\vec{B} = ABCos\theta$$



CHARACTERISTICS OF DOT PRODUCT:

1) **Commutative Law:**

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

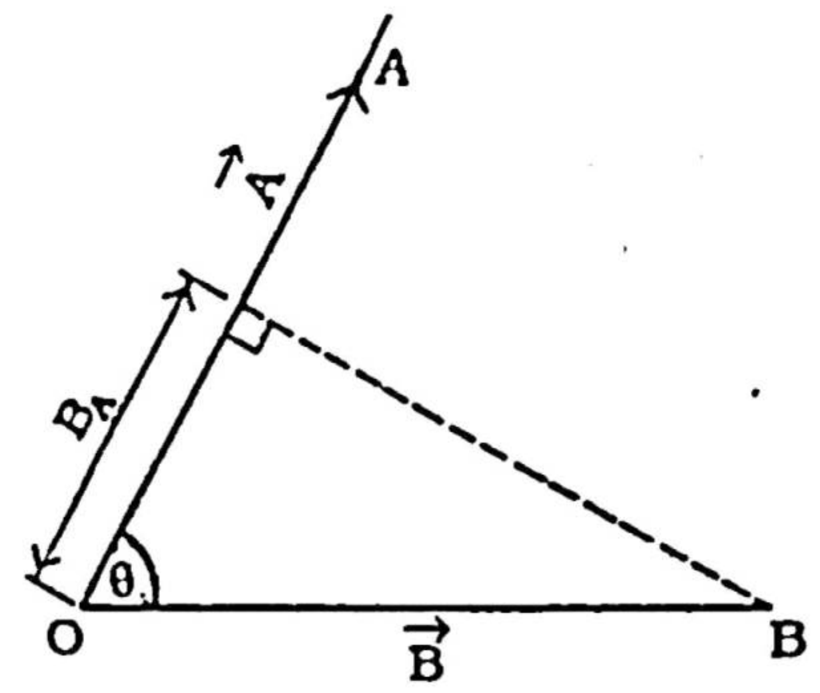
Proof:

Let vector A and B. Draw a perpendicular form head of B on x axis.

According to the definition of scalar product , it is equal to the product of magnitude of first vector with the length of projection of second vector onto first vector.

Thus,

$$\vec{A}.\vec{B} = ABCos\theta = A(BCos\theta)$$



In the same way, when

ACosθ = Magnitude of component of A onto B,

$$\vec{B}.\vec{A} = B(ACos\theta) = BACos\theta$$

Hence, it is clear that

$$ABCos\theta = BACos\theta$$

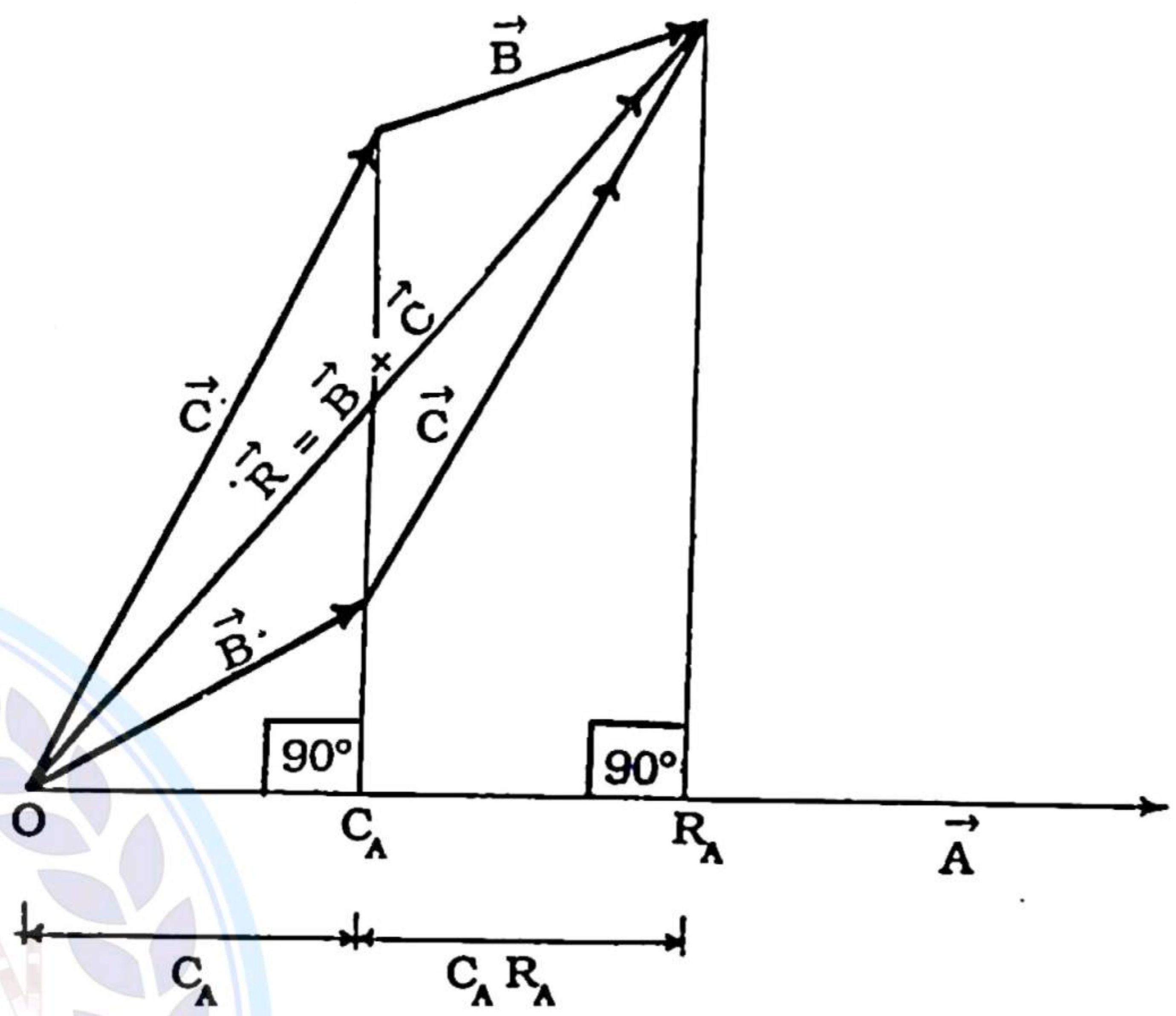
$$\boxed{\vec{A}.\vec{B} = \vec{B}.\vec{A}}$$

2) Distributive Law:

$$\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B}+\vec{A}.\vec{C}$$

Proof:

Let us consider Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ in different directions.let us first add B and C to get resultant vector $\vec{R}$,i.e.



$$\vec{R} = \vec{B} + \vec{C}$$

Now

$$\vec{A}.\vec{R} = A(R_A)$$
$$\vec{A}.\vec{R} = A(\overline{ON})$$
$$\vec{A}.\vec{R} = A(\overline{OM}+\overline{MN})$$
$$\vec{A}.\vec{R} = A(B_A + C_A)$$
$$\vec{A}.\vec{R} = AB_A + AC_A$$

or

$$\boxed{\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B}+\vec{A}.\vec{C}}$$

3. If $\vec{A}$ is parallel to $\vec{B}$ i.e. $\theta = 0°$ then

$$\vec{A}.\vec{B} = AB$$

4. If $\vec{A} = \vec{B}$ i.e. $\vec{A}$ is parallel and equal to B then

$$\vec{A}.\vec{A} = (A)\,(A)\,(\text{Cos}0°) = A^2$$

5. If $\vec{A}$ is perpendicular to $\vec{B}$ i.e. $\theta = 90°$ or one of the vector is null vector then

$$\vec{A}.\vec{B} = 0$$

6. The unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are perpendicular to each other therefore,

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

$$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$$

VECTOR PRODUCT:

DEFINITION:

It is the multiplication of two Vectors to give vectors
Or,
"It involves the multiplication of two vectors in such a way that the product is also a vector.

REPRESENTATION: It is represented by placing a Cross (X) between the two vectors.

EXPLANATION:

1) **Torque:** It is the vector product of vector r and force F.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

2) **Force:** Force on a particle of change 'q' moving with velocity V in a magnetic field B is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

EXPLANATION:

Vector product is the product of magnitudes of two vectors and the sine of the angle between than

$$\vec{A} \times \vec{B} = AB\,\text{Sin}\theta\,(\hat{n})$$

DIRECTION OF VECTOR PRODUCT:

Direction of vector product can be determined by right hand rule.

CHARACTERISTICS OF VECTOR PRODUCT:

1. COMMUTATIVE LAW:

$$\vec{A} \times \vec{B} = -\vec{A} \times \vec{B}$$

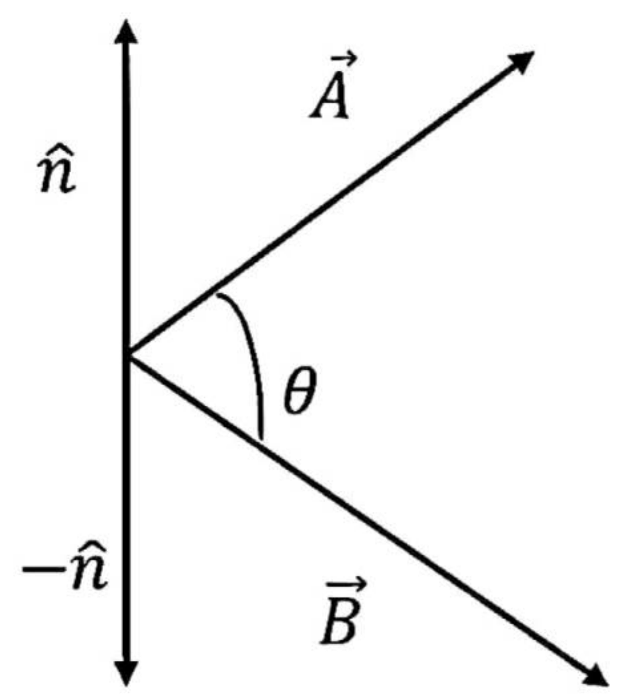
It means that vector product is not commutative.

Proof:
By the definition of vector product,

$$\vec{A} \times \vec{B} = AB \operatorname{Sin}\theta\,(\hat{n}) \text{--------------(i)}$$

where $\hat{n}$ is the unit vector normal pointing outwards the plane of vector $\vec{A} and\ \vec{B}$.

Similarly,



$$\vec{B} \times \vec{A} = BA \operatorname{Sin}(-\theta)\,(\hat{n})$$

or

$$\vec{B} \times \vec{A} = BA \operatorname{Sin}(\theta)\,(-\hat{n}) \qquad \text{Since } \operatorname{Sin}(-\theta) = -\operatorname{Sin}\theta$$

or

$$\vec{B} \times \vec{A} = -AB \operatorname{Sin}(\theta)\,(\hat{n})$$

Using eq(i) we get,

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$$

or

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

The above equation shows that vector product is not commutative.

2) Distributive Law:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

or

$$(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$$

3. If $\vec{A}$ is parallel to $\vec{B}$ i.e. $\theta = 0°$ then

$$\vec{A} \times \vec{B} = 0$$

4. If $\vec{A} = \vec{B}$ i.e. $\vec{A}$ is parallel and equal to B then

$$\vec{A} \times \vec{A} = (A)\ (A)\ (\text{Sin}0°) = 0$$

5. If $\vec{A}$ is perpendicular to $\vec{B}$ i.e. $\theta = 90°$ or one of the vector is null vector then

$$\vec{A} \times \vec{B} = AB\,\hat{n} \qquad \text{or} \qquad |\vec{A} \times \vec{B}| = AB$$

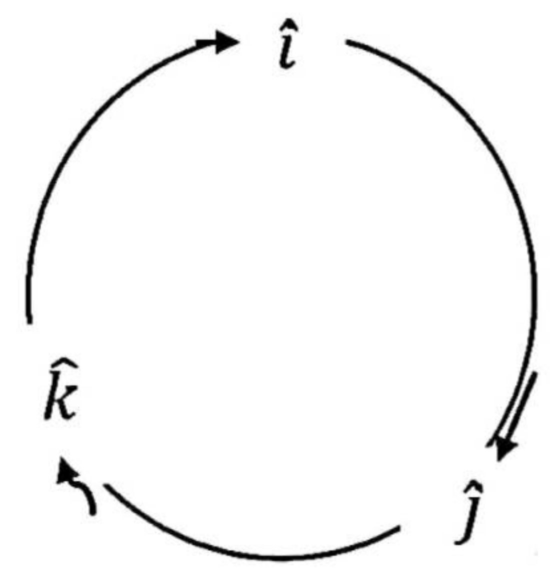
6. The unit vectors $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are perpendicular to each other therefore,

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k} \quad , \quad \hat{\jmath} \times \hat{\imath} = = -\hat{k}$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath} \quad , \quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath} \quad , \quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$$



M.C.Qs.

1. Which of the following is a vector quantity?
(a) Mass (b) Speed
(c) Temperature (d) Acceleration

2. Which one of the following is scalar?
(a) Acceleration (b) Velocity
(c) Force d) Work

3. In contrast to a scalar a vector must have a.
(a) Direction (b) Weight
(c) Quantity (d) None of these

4. Which is the following group of quantities represent the vectors:
(a) Acceleration, Force,Mass
(b) Mass,Displacement,velocity
(c) Acceleration, Electric flux, force
(d) Velocity, Electric field ,momentum

5. The following physical quantities are called vectors:
(a) Time and mass (b) Temperature and density (c) Force and Displacement (d) Length and volume

6. Vectors are physical quantities which are completely specified by:
(a) Magnitude-only (b) Direction only
(c) Magnitude and direction only
(d) None of these

7. Scalar quantities have:

(a) Only magnitudes (b) Only directions
(c) Both magnitude and direction
(d) None of these

8. A unit of a vector *A* is given by:
(a) $a = \boldsymbol{A}\,A$ (b) $a = \boldsymbol{A}/A$
(c) $a = A/\boldsymbol{A}$ (d) $a = \boldsymbol{A} + A$

9. A vector in space has ____________ components.
(a) one (b) Two
(c) Three (d) Four

10. When a vector is multiplied by a negative number its direction.
(a) is reversed (b) remains unchanged
(c) make and angle of 60^o
(d) may be changed or not

11. A vector which can be changed by display parallel to itself and applied at any point is known as:
(a) Parallel vector (b) Null vector
(c) Free vector (d) position vector

12. A vector in any given direction whose magnitude is unity is called:
(a) Normal vector (b) parallel vector
(c) Free vector (d) unit vector

13. The position vector of a point p is a vector that represents its position with respect to:
(a) Another vector
(b) Center of the earth
(c) Any point in space
(d) origin of the coordinate system

14. Negative of a vector has a direction ________that of the original vector.
(a) Same as (b) Perpendicular to
(c) Opposite to (d) Inclined to

15. The sum and difference of two vectors are equal in magnitude. The angle between the vectors is:
(a) 0^o (b) 90^0 (c) 120^o (d) 180^0

16. Two forces act together on an object The magnitude of their resultant is least when the angle between the forces is:
(a) 0° (b) 45° (c) 60° (d) 180°

17. The dot product of *i* and *j* is.
(a) more than 1 (b) 1 (c) 0 (d) any value

18. Scalar product obtains when.
(a) A Scalar is multiplied by a scalar.
(b) A scalar is multiplied by a vector
(c) Two vectors are multiplied to give a scalar
(d) Sum of two scalars is taken

19. If dot product of two vectors which are not perpendicular to each other is zero then either of the vector is obtain by adding two or more vectors is called:
(a) A unit vector (b) Opposite to the other
(c) A null vector (d) Position vector

20. The vector obtained by adding two or more vectors is called:
(a) Product Vector (b) Sum vector
(c) Resultant vector (d) Final vector

21. Scalar product of two vectors obeys.
(a) Commutative Law (b) Associate Law
(c) Both "a" and "b" (d) None of the above

22. If the dot product of two non-zero vectors A and B is zero. Their cross product will be of magnitude:
(a) AB Sin θ (b) B cos θ
(c) AB cos θ (d) AB

23. If the angle between the two vectors is zero degree then their:
(a) Dot product is zero
(b) Cross product is zero
(c) Either dot or cross product is zero
(d) Both dot & cross product is zero

24. *k* x *i* = ___________.
(a) *j* (b) – *j* (c) *k*
(d) -*k*

25. If a . b = 0 and also a x b = 0 then
(a) a and b are parallel to each other

(b) a and b are perpendicular to each
(c) a and b is a null vector
(d) Either a or b is a null vector

26. The magnitude of vector product is:
(a) Sum of the adjacent side
(b) Area of the parallelogram
(c) Product of the parallelogram
(d) Parameter of the parallelogram

27. If two vectors lie in xy-plane then their cross product lies.
(a) In the same plane (b) Adjacent plane
(c) Alone parallel to that plane
(d) Parallel to the plane

28. Two forces of 8N and 6N are acting simultaneously at right angle the resultant force will be:
(a) 14N (b) 2N (c) 10N (d) 12N

29. Two forces each of magnitude F act perpendicular to each other. The-angle made by the resultant force with the horizontal will be.
(a) 30° (b) 45° (c) 60° (d) 90°

30. When two equal forces F and F makes an angle 180° with each other the magnitude of their resultant is.
(a) F (b) 0 (c) 2F (d) 0.5F

PAST PAPER M.C.Qs.

2022

5. If A .B =0, A ×B =0 and A≠0 then B is:
*equal to A **_* Zero_** *Perpendicular to A *Anti-parallel to A

10. The dot product of force and velocity is called:
*work **_*power_** *momentum *energy

18. The magnitude of product i .(k x j):
*zero *1 **_*-1_** ` *k

24. If a vector has three components each equal to "a" the magnitude of vector will be
*$\sqrt{3a}$ *3a *a^3 ` **_*$\sqrt{3}a$_**

26. It is not a vector quantity.
*Force *Torque **_*Frequency_** *Weight

2021

21 .If i, j and k are unit vectors then k. (i x j) is equal to:
*zero **_*1_** *j *k

29. The y-component of a vector A= 15 units, when it forms an angle of 50 with positive x-axis is

*9.6 units ***11.5units** *-9.6units *-11.5units

2019

6.The magnitude of resultant of two forces of magnitudes 2N and 10N cannot be:

*** 4N** *6N *9N *13N

14.(i x j) . (j x i) is:

***-1** *k *1 *zero

2018

2. Two perpendicular vectors having magnitudes of 4 units and 3 units are added. There resultant has magnitude of :

***5 units** *7 units *12 units *25 units

2017

16. The magnitude of product k.(j x i):

*zero *1 ***-1** ` *k

2016

10.If i, j and k are unit vectors then k. (i x j) is equal to:

*zero ***one** *j *k

16. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

2015

3. The y-components of vector A =15 units when it forms an angle of 50° with positive x-axis is:

*9.6 units * -9.6 units *** 11.5 units** *-11.5 units

2014

7. If A =5i+j and B =2k then A - B is equal to:

*5i+j+2k *5i-j-2k ***5i+j-2k** *-5i-j+2k

16. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

2013

5. Two forces act together on an object; the magnitude of their resultant is minimum when the angle between them is:

*0^0 *45^0 *90^0 ***180^0**

2012

15. If A=a i and B=bj , then A x B is equal to:

***0** * ab k * -ab k * none of these

2011

11. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

2010

10. If A .B =0, A ×B =0 and A≠0 then B is:

*equal to A *** Zero** *Perpendicular to A *Anti-parallel to A

TEXTBOOK NUMERICALS

Q.6: The following forces act on a particle P: $F_1 = 2i + 3j - 5k$, $F_2 = -5i + j + 3k$, $F_3 = i - 2j + 4k$, $F_4 = 4i - 3j - 2k$, measured in newtons Find (a) the resultant of the force (b) the magnitude of the resultant force.

Data:

$F_1 = 2i + 3j - 5k$

$F_2 = -5i + j + 3k$

$F_3 = i - 2j + 4k$

$F_4 = 4i - 3j - 2k$

$\vec{F} = ?$

$|\vec{F}| = ?$

Solution:

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$\vec{F} = 2i + 3j - 5k + (-5i + j + 3k) + i - 2j + 4k + 4i - 3j - 2k$$

$$\vec{F} = (2i - 5i + i + 4i) + (3j + j - 2j - 3j) + (-5k + 3k + 4k - 2k)$$

$$\vec{F} = (2i - j + 0k)$$

$$\boxed{\vec{F} = 2i - j}$$

$$F = \sqrt{x^2 + y^2 + z^2}$$

$$F = \sqrt{(2)^2 + (-1)^2 + (0)^2}$$

$$\boxed{F = \sqrt{5}}$$

Result:

The resultant force is $2i - j$ and its magnitude is $\sqrt{5}$.

Q.7: If $A = 3i - j - 4k$, $B = -2i + 4j - 3k$ and $C = i + 2j - k$, find (a) $2A - B + 3C$, (b) $|A + B + C|$, (c) $|3A - 2B + 4C|$, (d) a unit vector parallel to $3A - 2B + 4C$

Data:

$A = 3i - j - 4k$

$B = -2i + 4j - 3k$

$C = i + 2j - k$

(a) $2A - B + 3C = ?$

(b) $|A + B + C| = ?$

(c) $|3A - 2B + 4C| = ?$

(d) a unit vector parallel to $3A - 2B + 4C = ?$

Solution:

(a) $2A - B + 3C = 2(3i - j - 4k) - (-2i + 4j - 3k) + 3(i + 2j - k)$

$$2A - B + 3C = 6i - 2j - 8k + 2i - 4j + 3k + 3i + 6j - 3k$$

$$2A - B + 3C = 11i + 0j - 8k$$

$$\boxed{2A - B + 3C = 11i - 8k}$$

(b) $A + B + C = 3i - j - 4k + (-2i + 4j - 3k) + i + 2j - k$

$$A + B + C = 2i + 5j - 8k$$

Now, $|A + B + C| = \sqrt{x^2 + y^2 + z^2}$

$$|A + B + C| = \sqrt{(2)^2 + (5)^2 + (-8)^2}$$

$$\boxed{|A + B + C| = \sqrt{93}}$$

(c) $3A - 2B + 4C = 3(3i - j - 4k) - 2(-2i + 4j - 3k) + 4(i + 2j - k)$

$$3A - 2B + 4C = 9i - 3j - 12k + 4i - 8j + 6k + 4i + 8j - 4k$$

$3A - 2B + 4C = 17\,i - 3\,j - 10\,k$

Now, $|3A - 2B + 4C| = \sqrt{x^2 + y^2 + z^2}$

$|3A - 2B + 4C| = \sqrt{(17)^2 + (-3)^2 + (-10)}$

$$\boxed{|3A - 2B + 4C| = \sqrt{398}}$$

(d) Unit Vector Perpendicular $= \hat{n} = \dfrac{vector}{magnitude}$

$$\boxed{\hat{n} = \frac{3A - 2B + 4C}{|3A - 2B + 4C|} = \frac{17\,i - 3\,j - 10\,k}{\sqrt{398}}}$$

Q.8: Two tugboats are towing a ship. Each exerts a force of 6000N, and the angle between the two ropes is 60°. Calculate the resultant force on the ship.

Data:

$\vec{F_1} = 6000\ N$

$\vec{F_2} = 6000\ N$

$\theta = 60°$

$\vec{F} = ?$

Solution:

According to the Parallelogram Method

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$F =$

$\sqrt{(6000)^2 + (6000)^2 + 2(6000)(6000)\cos 60°}$

$F = \sqrt{108000000}$

$$\boxed{F = 10392.3\ N}$$

Result: The resultant force on the ship is 10392.3 N

Q.9: The position vectors of points P and Q are given by r_1 = 2i +3j- k, r_2 = 4i – 3j + 2k. Determine $\overrightarrow{PQ}$ in terms of rectangular unit vector i, j and k and find its magnitude.

Data:

$r_1 = 2i + 3j - k$

$r_2 = 4i - 3j + 2k$

$\overrightarrow{PQ} = ?$

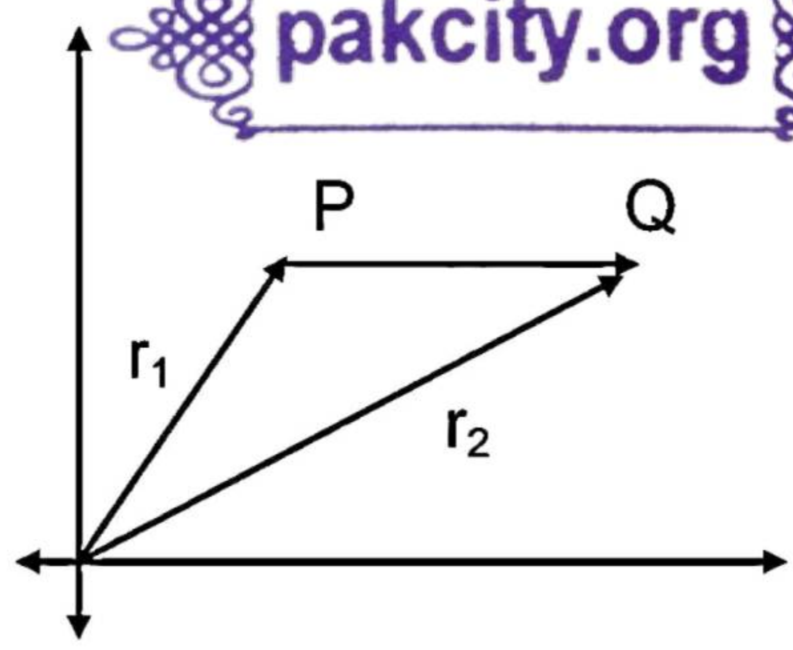
$|\overrightarrow{PQ}| = ?$

Solution:

According to the figure

$\overrightarrow{PQ} = r_2 - r_1$

$\overrightarrow{PQ} = 4i - 3j + 2k - (2i + 3j - k)$



$\overrightarrow{PQ} =$ $4i - 3j + 2k - 2i - 3j + k$

$$\boxed{\overrightarrow{PQ} = 2i - 6j + 3k}$$

Now, $|\overrightarrow{PQ}| = \sqrt{x^2 + y^2 + z^2}$

$|\overrightarrow{PQ}| = \sqrt{(2)^2 + (-6)^2 + (3)^2}$

$$\boxed{|\overrightarrow{PQ}| = 7\ units}$$

Result: $\overrightarrow{PQ} = 2i - 6j + 3k$ and $|\overrightarrow{PQ}| = 7\ units$

Q.10: Prove that the vectors A = 3i + j – 2k, B = -i + 3j + 4k and C = 4i – 2j – 6k can form the sides of a triangle. Find the length of the medians of the triangle.

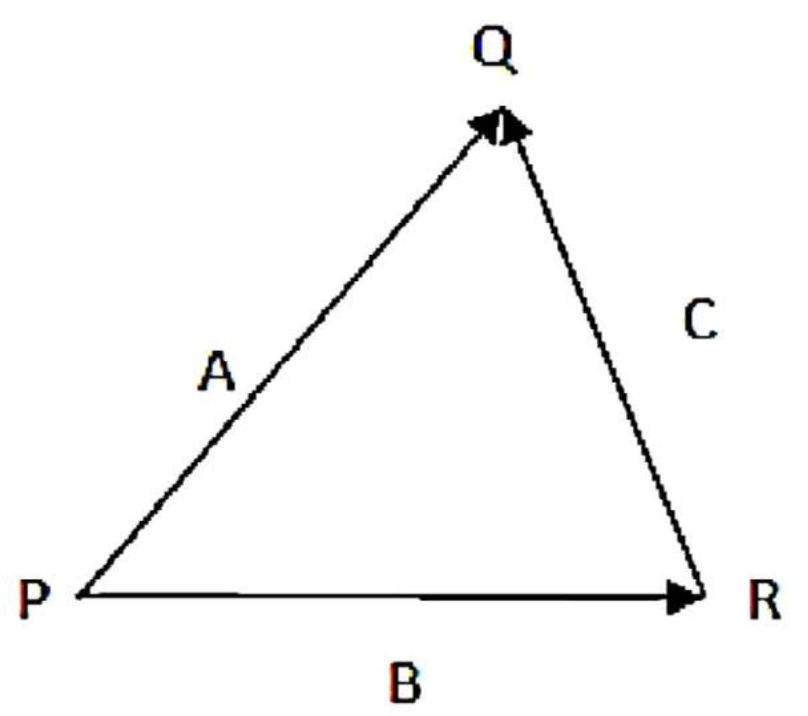
Data:

A = 3i + j – 2k

B = -i + 3j + 4k

C = 4i – 2j – 6k

$\Delta = ?$



Length of medians = ?

Solution:

A, B and C can form triangle if

$$\vec{B}+\vec{C}=\vec{A}$$

$-i + 3j + 4k + 4i - 2j - 6k = 3i + j - 2k$

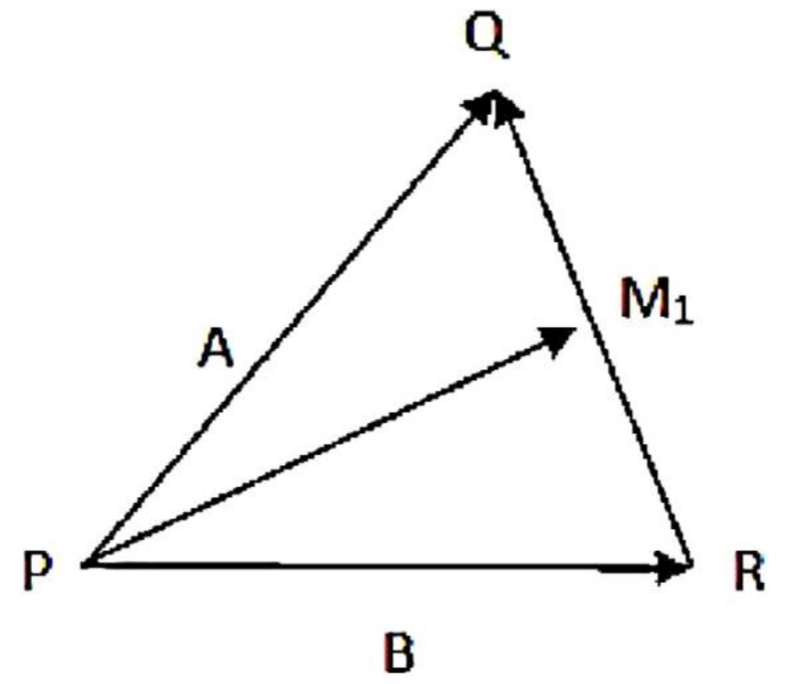
$3i + j - 2k = 3i + j - 2k$

So. L.H.S = R.H.S

Therefore, A, B and C can form triangle.

Now,

Length of 1st Median $\overrightarrow{PM_1}$:



According to the figure

$$\vec{B}+\frac{1}{2}\vec{C}=\overrightarrow{PM_1}$$

$$\overrightarrow{PM_1}=-i + 3j + 4k+\frac{1}{2}(4i - 2j - 6k)$$

$$\overrightarrow{PM_1}=-i + 3j + 4k+2i-j-3k$$

$$\boxed{\overrightarrow{PM_1}=i+2j+k}$$

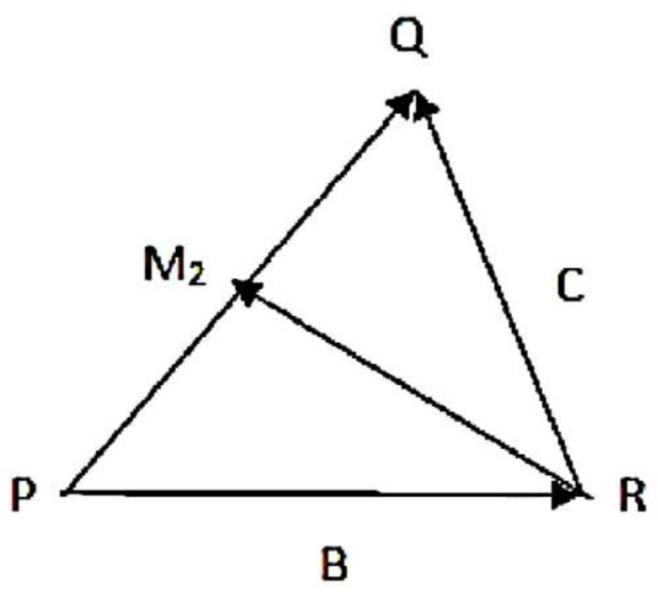
Now,

$$|\overrightarrow{PM_1}|=\sqrt{x^2+y^2+z^2}$$

$$|\overrightarrow{PM_1}|=\sqrt{(1)^2+(2)^2+(1)^2}$$

$$\boxed{|\overrightarrow{PM_1}|=\sqrt{6}\ units}$$

Length of 2nd Median $\overrightarrow{RM_2}$:



According to the figure

$$\vec{B}+\overrightarrow{RM_2}=\frac{1}{2}\vec{A}$$

$$\overrightarrow{RM_2}=-i + 3j + 4k-\frac{1}{2}(3i+j-2k)$$

$$\overrightarrow{RM_2}=-i + 3j + 4k-\frac{3}{2}i-\frac{1}{2}j+k$$

$$\overrightarrow{RM_2}=-\frac{5}{2}i+\frac{5}{2}j+5k$$

Now,

$$|\overrightarrow{RM_2}|=\sqrt{x^2+y^2+z^2}$$

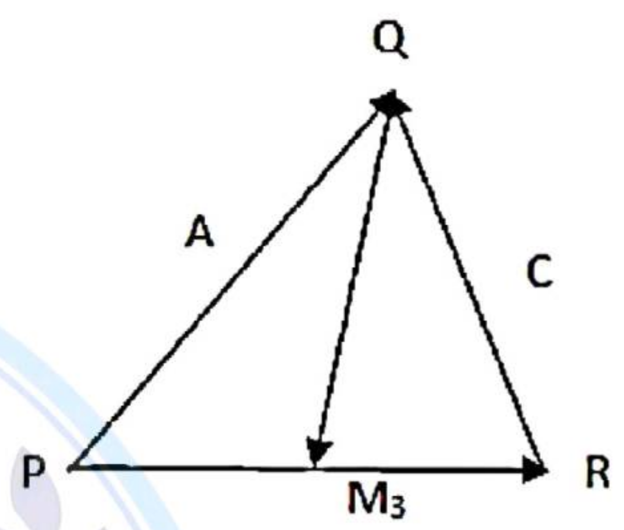
$$|\overrightarrow{RM_2}|=\sqrt{(-\frac{5}{2})^2+(\frac{5}{2})^2+(5)^2}$$

$$\boxed{|\overrightarrow{RM_2}|=\frac{1}{2}\sqrt{150}\ units}$$

Length of 3rd Median $\overrightarrow{QM_3}$:

According to the figure

$$\vec{A}+\overrightarrow{QM_3}=\frac{1}{2}\vec{B}$$



$$\overrightarrow{QM_3}=\frac{1}{2}\vec{B}-\vec{A}$$

$$\overrightarrow{QM_3}=\frac{1}{2}(-i + 3j + 4k)-(3i + j - 2k)$$

$$\overrightarrow{QM_3}=-\frac{1}{2}i+\frac{3}{2}j+2k-3i-j+2k$$

$$\overrightarrow{QM_3}=-\frac{7}{2}i-\frac{1}{2}j+4k$$

Now,

$$|\overrightarrow{QM_3}|=\sqrt{x^2+y^2+z^2}$$

$$|\overrightarrow{QM_3}|=\sqrt{(-\frac{7}{2})^2+(-\frac{1}{2})^2+(4)^2}$$

$$\boxed{|\overrightarrow{QM_3}|=\frac{1}{2}\sqrt{114}\ units}$$

Q.11: Find the rectangular components of a vector A, 15 unit long when it form an angle with respect to +ve x-axis of (i) 50°, (ii) 130° (iii) 230°, (iv) 310°.

Data:

$|\vec{A}| = 15\ units$

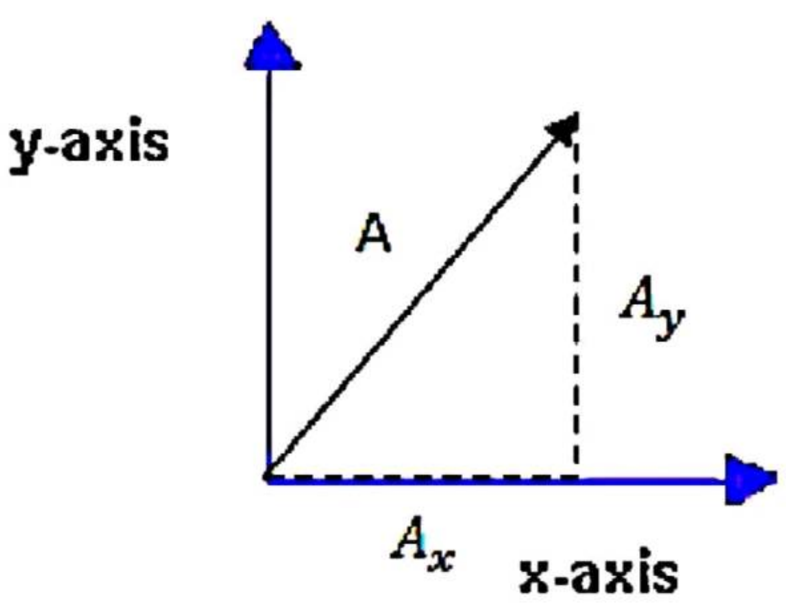
i) $\theta = 50°, A_x = ?\ and\ A_y = ?$

i) $\theta = 130°, A_x = ?\ and\ A_y = ?$

i) $\theta = 230°, A_x = ?\ and\ A_y = ?$

i) $\theta = 310°, A_x = ?\ and\ A_y = ?$

Solution:



As we know that

$A_x = A\ cos\theta$

and $A_y = A\ sin\theta$

i) $\theta = 50°$

$A_x = 15\ cos50° = 15 \times 0.642$

$\boxed{A_x = 9.64\ units}$

$A_y = 15\ sin50° = 15 \times 0.766$

$\boxed{A_y = 11.5\ units}$

ii) $\theta = 130°$

$A_x = 15\ cos130° = 15 \times (-0.642)$

$\boxed{A_x = -9.64\ units}$

$A_y = 15\ sin130° = 15 \times 0.766$

$\boxed{A_y = 11.5\ units}$

iii) $\theta = 230°$

$A_x = 15 \cos 230° = 15 \times (-0.642)$

$\boxed{A_x = -9.64\ units}$

$A_y = 15 \sin 230° = 15 \times (-0.766)$

$\boxed{A_y = -11.5\ units}$

iiv) $\theta = 310°$

$A_x = 15 \cos 310° = 15 \times (0.642)$

$\boxed{A_x = 9.64\ units}$

$A_y = 15 \sin 310° = 15 \times (-0.766)$

$\boxed{A_y = -11.5\ units}$

Q12: Two vectors 10 cm and 8 cm long form an angle of (a) 60° (b) 90° and (c) 120°. Find the magnitude of difference and the angle with respect to the larger vector.

Data:

$|\vec{A}| = 10\ cm$

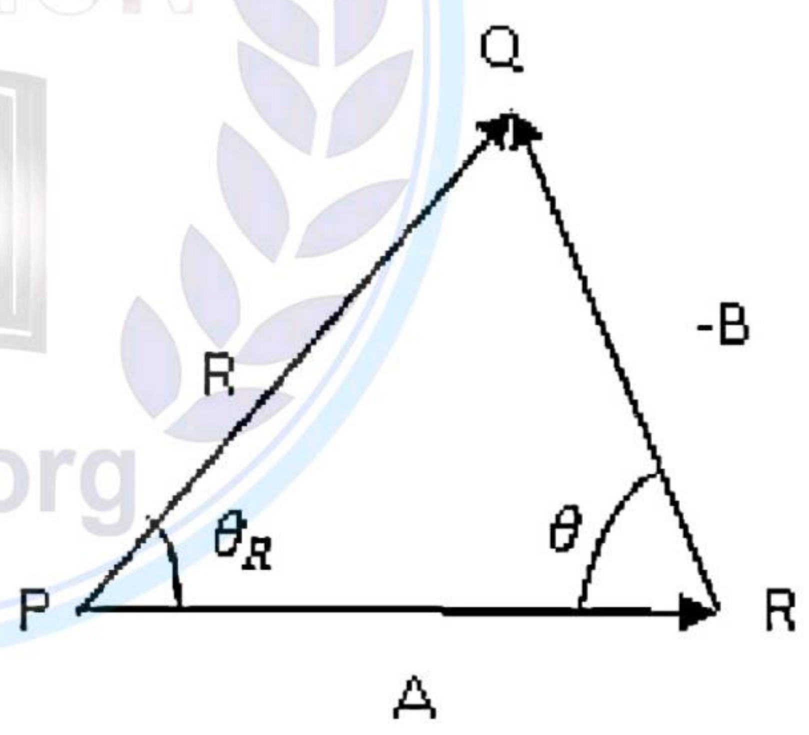
$|\vec{B}| = 8\ cm$

i) $\theta = 60°, |\vec{A} - \vec{B}| = ?\ and\ \theta_R = ?$

i) $\theta = 90°, |\vec{A} - \vec{B}| = ?\ and\ \theta_R = ?$

i) $\theta = 120°, |\vec{A} - \vec{B}| = ?\ and\ \theta_R = ?$

Solution:



i) Using Paralellogram Law

$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\ COS\ \theta}$

$|\vec{A}-\vec{B}| = \sqrt{(10)^2+(8)^2-2(10)(8)\ COS\ 60°}$

$\boxed{|\vec{A}-\vec{B}| = 9.2\ cm}$

Using Law of Sines

$\frac{\sin\theta_R}{B} = \frac{\sin\theta}{|\vec{A}-\vec{B}|}$

$\frac{\sin\theta_R}{8} = \frac{\sin 60}{9.2}$

$\sin\theta_R = 0.753$

$\theta_R = \sin^{-1}(0.753)$

$\boxed{\theta_R = 49°}$

ii) Using Paralellogram Law

$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\ COS\ \theta}$

$|\vec{A}-\vec{B}| = \sqrt{(10)^2+(8)^2-2(10)(8)\ COS\ 90°}$

$\boxed{|\vec{A}-\vec{B}| = 12.8\ cm}$

Using Law of Sines

$\frac{\sin\theta_R}{B} = \frac{\sin\theta}{|\vec{A}-\vec{B}|}$

$\frac{\sin\theta_R}{8} = \frac{\sin 90}{12.8}$

$\sin\theta_R = 0.625$

$\theta_R = \sin^{-1}(0.625)$

$\boxed{\theta_R = 38.6°}$

Or in Minutes:

$\theta_R = 38° + 0.6 \times 60'$

$\boxed{\theta_R = 38°\ 36'}$

iii) Using Paralellogram Law

$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\ COS\ \theta}$

$|\vec{A}-\vec{B}| = \sqrt{(10)^2+(8)^2-2(10)(8)\ COS\ 120°}$

$\boxed{|\vec{A}-\vec{B}| = 15.6\ cm}$

Using Law of Sines

$\frac{\sin\theta_R}{B} = \frac{\sin\theta}{|\vec{A}-\vec{B}|}$

$\frac{\sin\theta_R}{8} = \frac{\sin 120}{15.6}$

$\sin\theta_R = 0.444$

$\theta_R = \sin^{-1}(0.444)$

$\boxed{\theta_R = 26.3°}$

Or in Minutes:

$\theta_R = 26° + 0.3 \times 60'$

$\boxed{\theta_R = 26°\ 18'}$

Q.13: The angle between the vector A and B is 60° . Given that |A|=|B| = 1, calculate (a) |B - A|; (b) |B + A|

Data:

A|=|B| = 1

(a) |B - A|= ?

(b) |B + A|= ?

Solution:

(a) $|\vec{B}-\vec{A}| = \sqrt{B^2+A^2-2BA\ COS\ \theta}$

$|\vec{B}-\vec{A}| = \sqrt{(1)^2+(1)^2-2(1)(1)\ COS\ 60°}$

$|\vec{B}-\vec{A}| = \sqrt{1}$

$\boxed{|\vec{B}-\vec{A}| = 1}$

(b) $|\vec{B}+\vec{A}| = \sqrt{B^2+A^2+2BA\ COS\ \theta}$

$|\vec{B}+\vec{A}| = \sqrt{(1)^2+(1)^2+2(1)(1)\ COS\ 60°}$

$\boxed{|\vec{B}+\vec{A}| = \sqrt{3}}$

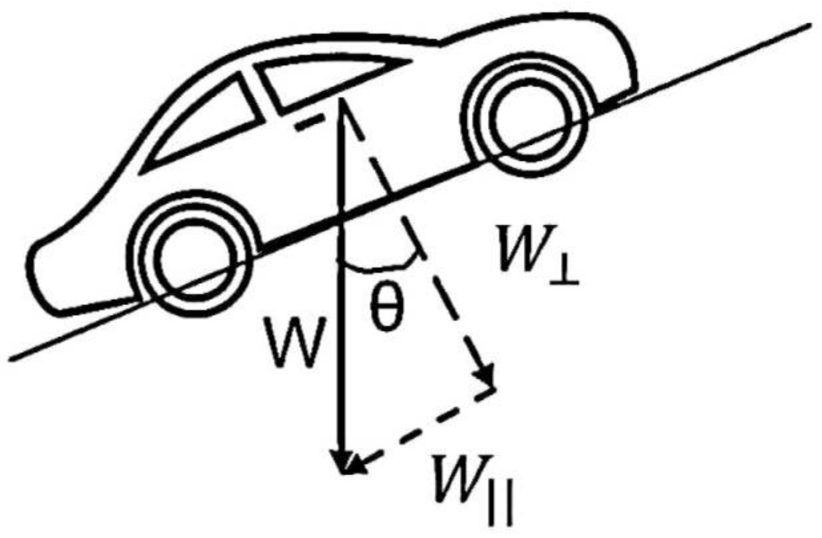
Q.14: A car weighing 10,000 N on a hill which makes an angle of 20° with the horizontal. Find the components of car's weight parallel and perpendicular to the road.

Data:

$|\vec{W}| = 10000\ N$

$\theta = 20°, W_{||} = ?$ and $W_{\perp} = ?$

Solution:



According to the given condition

$$W_{||} = W\sin\theta$$

$$W_{||} = 10000 \times \sin 20°$$

$$\boxed{W_{||} = 3420\ N}$$

and

$$W_{\perp} = W\cos\theta$$

$$W_{\perp} = 10000 \times \cos 20°$$

$$\boxed{W_{\perp} = 9396.9\ N}$$

Result: The component of car parallel to road is 3420N and perpendicular to road is 9396.9 N

Q.15: Find the angle between A = 2i + 2j – k and B = 6i – 3j + 2k.

Data:

A = 2i + 2j – k

B = 6i – 3j + 2k

$\theta = ?$

Solution:

According to the definition of Dot Product

$$\vec{A}.\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{A.B}{|A||B|} \text{ --- (i)}$$

$$\vec{A}.\vec{B} = (2\hat{\imath} + 2\hat{\jmath} - \hat{k}).(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$$

$$\vec{A}.\vec{B} = 12 - 6 - 2$$

$$\boxed{\vec{A}.\vec{B} = 4}$$

Now, $A = \sqrt{x^2 + y^2 + z^2}$

$$A = \sqrt{(2)^2 + (2)^2 + (-1)^2}$$

$$\boxed{A = 3}$$

and

$$B = \sqrt{x^2 + y^2 + z^2}$$

$$B = \sqrt{(6)^2 + (-3)^2 + (2)^2}$$

$$\boxed{B = 7}$$

Putting values in eq (i)

$$\cos\theta = \frac{4}{3\times7}$$

$$\theta = \cos^{-1}(\frac{4}{21})$$

$$\boxed{\theta = 79°}$$

Result:

The angle between given vectors is 79°

Q.16: Find the projection of the vector A = i – 2j + k onto the direction of vector B = 4i – 4j + 7k.

Data:

$\vec{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{B} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of A onto B = A_B = ?

Solution:

As we know that

$$\vec{A}.\vec{B} = A_B B$$

So $$A_B = \frac{\vec{A}.\vec{B}}{B} \text{ ------- (i)}$$

$$\vec{A}.\vec{B} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}).(4\hat{\imath} - 4\hat{\jmath} + 7\hat{k})$$

$$\vec{A}.\vec{B} = 4 + 8 + 7$$

$$\boxed{\vec{A}.\vec{B} = 19}$$

Now,

$$B = \sqrt{x^2 + y^2 + z^2}$$

$$B = \sqrt{(4)^2 + (-4)^2 + (7)^2}$$

$$\boxed{B = 9}$$

Putting values in eq (i)

$$\boxed{A_B = \frac{19}{9}}$$

Result: The projection of Vector A on Vector B is $\frac{19}{9}$ *units*.

Q.17: Find the angles α, β, γ which the vector A = 3i – 6j + 2k makes with the positive x, y, z axis respectively.

Data:
A = 3i – 6j + 2k
Angle w.r.t. x axis = α = ?
Angle w.r.t. x axis = β = ?
Angle w.r.t. x axis = γ = ?

Solution:
According to the definition of Dot Product

$$Cos\,\theta = \frac{A.B}{|A|\,|B|} \text{ --- (i)}$$

$$A = \sqrt{x^2 + y^2 + z^2}$$

$$A = \sqrt{(3)^2 + (-6)^2 + (2)^2}$$

$$\boxed{A = 7}$$

For α:

$$\vec{A}.\hat{\imath} = (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}).(\hat{\imath})$$

$$\vec{A}.\hat{\imath} = 3$$

$$\boxed{\vec{A}.\hat{\imath} = 3}$$

Putting values in eq (i)

$$Cos\,\alpha = \frac{3}{7}$$

$$\alpha = Cos^{-1}(\tfrac{3}{7})$$

$$\boxed{\alpha = 64.6°}$$

For β:

$$\vec{A}.\hat{\jmath} = (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}).(\hat{\jmath})$$

$$\vec{A}.\hat{\jmath} = -6$$

$$\boxed{\vec{A}.\hat{\jmath} = -6}$$

Putting values in eq (i)

$$Cos\,\beta = -\frac{6}{7}$$

$$\beta = Cos^{-1}(-\tfrac{6}{7})$$

$$\boxed{\beta = 149°}$$

For γ:

$$\vec{A}.\hat{k} = (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}).(\hat{k})$$

$$\vec{A}.\hat{k} = 2$$

$$\boxed{\vec{A}.\hat{k} = 2}$$

Putting values in eq (i)

$$Cos\,\gamma = \frac{2}{7}$$

$$\gamma = Cos^{-1}(\tfrac{2}{7})$$

$$\boxed{\gamma = 73.3°}$$

Result:
Angle w.r.t. x axis = α = 64.6°
Angle w.r.t. x axis = β = 149 °
Angle w.r.t. x axis = γ = 73.3°

Q.18: Find the work done in moving an object along a vector r = 3i + 2j – 5k if the applied force is F = 2i – j – k.

Data:
Displacement = r = 3i + 2j – 5k
Force = F = 2i - j - k
Work = W =?

Solution:
According to the def. of work

$$W = \vec{F}.\vec{S}$$

$$W = (2i - j - k).(3i + 2j - 5k)$$

$$W = 6 - 2 + 5$$

$$\boxed{W = 9 \text{ units}}$$

Result:
The work done is 9 units.

Q.19: Find the work done by a force of 30,000 N in moving an o bject through a distance of 45 m when: (a) the force is in the direction of motion: and (b) the force makes an angle of 40° to the direction of motion. Find the rate at which the force is working at a time when the

velocity is 2m/s.

Data:

Force = F = 30000 N

Distance = S = 45 m

a) $\theta = 0^0$ W =? and P = ?

b) $\theta = 40^0$ W =? and P = ?

Velocity = v = 2 m/s

Solution:

Accordint to the definition of work

$W = FS\cos\theta$

a) $W = 30000 \times 45 \times \cos 0°$

$\boxed{W = 1.35 \times 10^6 J}$

$P = FV\cos\theta$

$P = 30000 \times 2 \times \cos 0°$

$\boxed{P = 60000\ W}$

b) $W = 30000 \times 45 \times \cos 40°$

$\boxed{W = 1.03 \times 10^6 J}$

$P = FV\cos\theta$

$P = 30000 \times 2 \times \cos 40°$

$\boxed{P = 45962.6\ W}$

Result:

When $\theta = 0^0$, $W = 1.35 \times 10^6 J$ and $P = 60000\ W$

When $\theta = 40^0$, $W = 1.03 \times 10^6 J$ and $P = 45962.6\ W$

Q.20: Two vectors A and B are such that $|A| = 3$, $|B| = 4$, and A.B = -5, find (a) the angle between A and B (b) the length $|A + B|$ and $|A - B|$ (c) the angle between (A + B) and (A – B)

Data:

$|A| = 3$

$|B| = 4$

A.B = -5

a) θ = ? (b/w A and B)

b) $|A + B|$ =? and $|A - B|$ = ?

c) θ = ? (b/w A+B and A-B)

Solution:

According to the definition of Dot Product

$\vec{A}.\vec{B} = AB\,Cos\,\theta$

$Cos\,\theta = \frac{A.B}{|A|\,|B|}$ --- (i)

a) $Cos\,\theta = \frac{-5}{3\times4}$

$\theta = \cos^{-1}(-0.416)$

$\boxed{\theta = 114.5°}$

b) $|A + B| = \sqrt{A^2 + B^2 + 2\vec{A}.\vec{B}}$

$|A + B| = \sqrt{(3)^2 + (4)^2 + 2\times(-5)}$

$\boxed{|A + B| = \sqrt{15}}$

and, $|A - B| = \sqrt{A^2 + B^2 - 2\vec{A}.\vec{B}}$

$|A - B| = \sqrt{(3)^2 + (4)^2 - 2\times(-5)}$

$\boxed{|A - B| = \sqrt{35}}$

c) $Cos\,\theta = \frac{(\vec{A}+\vec{B}).(\vec{A}-\vec{B})}{|\vec{A}+\vec{B}||\vec{A}-\vec{B}|}$

$Cos\,\theta = \frac{A^2 - B^2}{\sqrt{15}\times\sqrt{35}} = \frac{(3)^2-(4)^2}{22.91}$

$\theta = \cos^{-1}(-0.305)$

$\boxed{\theta = 107.7°}$

Q.21: If A = 2i – 3j – k, B = i + 4j – 2k. Find (a) A x B (b) B x A (c) (A + B) x (A – B)

Data:

A = 2i – 3j – k

B = i + 4j – 2k

(a) A x B = ?

(b) B x A = ?

(c) (A + B) x (A – B) =?

Solution:

(a) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} -3 & -1 \\ 4 & -2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}(6+4) - \hat{\jmath}(-4+1) + \hat{k}(8+3)$

$\boxed{\vec{A}\times\vec{B} = 10\hat{\imath} + 3\hat{\jmath} + 11\hat{k}}$

b) $\vec{B}\times\vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 4 & -2 \\ 2 & -3 & -1 \end{vmatrix}$

$\vec{B}\times\vec{A} = \hat{\imath}\begin{vmatrix} 4 & -2 \\ -3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix}$

$\vec{B}\times\vec{A} = \hat{\imath}(-4-6) - \hat{\jmath}(-1+4) + \hat{k}(-3-8)$

$\boxed{\vec{B}\times\vec{A} = -10\hat{\imath} - 3\hat{\jmath} - 11\hat{k}}$

c) $\vec{A}+\vec{B} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k} + (\hat{\imath} + 4\hat{\jmath} - 2\hat{k})$

$\boxed{\vec{A}+\vec{B} = 3\hat{\imath} + \hat{\jmath} - 3\hat{k}}$

$\vec{A}-\vec{B} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k} - (\hat{\imath} + 4\hat{\jmath} - 2\hat{k})$

$\vec{A}-\vec{B} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k} - \hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

$\boxed{\vec{A}+\vec{B} = \hat{\imath} - 7\hat{\jmath} + \hat{k}}$

Now,

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix}$

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

$= \hat{\imath}\begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix}$

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

$= \hat{\imath}(1-21) - \hat{\jmath}(3+3) + \hat{k}(-21 - 1)$

$\boxed{(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = -20\hat{\imath} - 6\hat{\jmath} - 22\hat{k}}$

Q.22: Determine the unit vector perpendicular to the plane of A = 2i – 6j – 3k and B = 4i + 3j – k.

Data:

$\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$

$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ ----- (i)

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$

$\vec{A}\times\vec{B} = \hat{\imath}(6+9) - \hat{\jmath}(-2+12) + \hat{k}(6+24)$

$\boxed{\vec{A}\times\vec{B} = 15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}}$

$|\vec{A}\times\vec{B}| = \sqrt{x^2+y^2+z^2}$

$|\vec{A}\times\vec{B}| = \sqrt{(15)^2+(-10)^2+(30)^2}$

$\boxed{|\vec{A}\times\vec{B}| = 35}$

Putting in eq (i)

$\boxed{\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{15\hat{\imath}-10\hat{\jmath}+30\hat{k}}{35} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}}$

Result: The unit vector perpendicular to the plane of A and B is $\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$.

Q.23: Using the definition of vector product, prove the law of sines for plane triangles of sides a, b and c.

Proof:

According to the definition of vector product

$\vec{A}\times\vec{B} = AB\ Sin\theta\ \hat{n}$

or

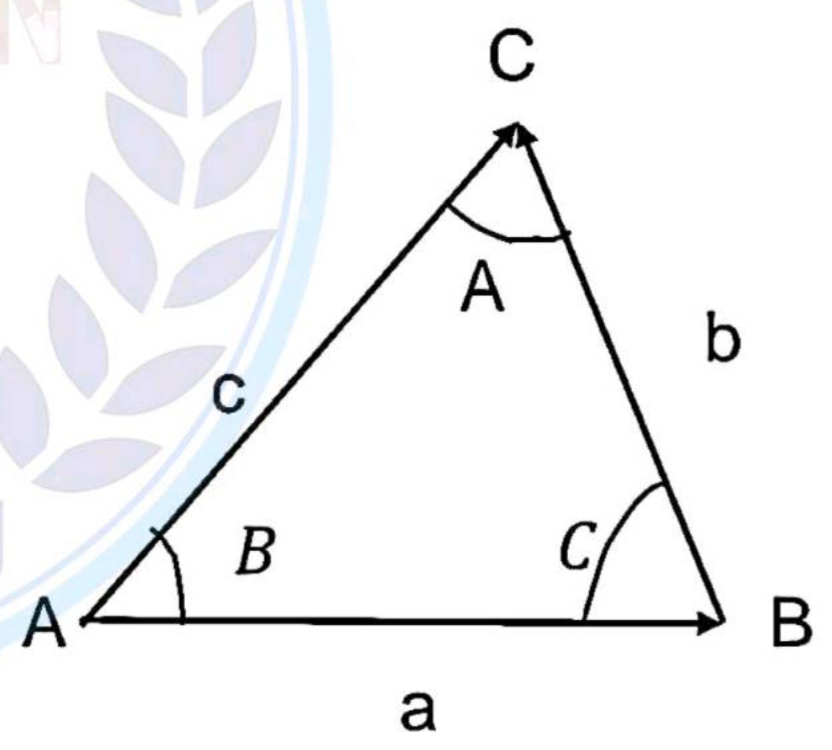
$|\vec{A}\times\vec{B}| = AB\ SinC$ ---(i)

Also,

$|\vec{B}\times\vec{C}| = BC\ SinA$----(ii)

and

$|\vec{C}\times\vec{A}| = AC\ SinB$ ----(iii)



As we know that Area of triangle is given by

$\Delta = \frac{1}{2}|\vec{A}\times\vec{B}| = \frac{1}{2}|\vec{B}\times\vec{C}| = \frac{1}{2}|\vec{C}\times\vec{A}|$

Putting values from eq(i) , (ii) and (iii)

$$\frac{1}{2}AB\ SinC\ = \frac{1}{2}BC\ SinA = \frac{1}{2}AC\ SinB$$

$$AB\ SinC\ = BC\ SinA = AC\ SinB$$

Dividing by ABC

$$\frac{AB\ SinC}{ABC} = \frac{BC\ SinA}{ABC} = \frac{AC\ SinB}{ABC}$$

$$\boxed{\frac{SinC}{C} = \frac{SinA}{A} = \frac{SinB}{B}}$$ (Proved)

Q.24: If r_1 and r_2 are the position vectors (both lie in xy plane) making angle θ_1 and θ_2 with the positive x-axis measured counter clockwise, find their vector product when

(i) $|r_1| = 4$ cm $\theta_1 = 30°$ $|r_2| = 3$ cm $\theta_2 = 90°$

(ii) $|r_1| = 6$ cm $\theta_1 = 220°$ $|r_2| = 3$ cm $\theta_2 = 40°$

(iii) $|r_1| = 10$ cm $\theta_1 = 20°$ $|r_2| = 9$ cm $\theta_2 = 110°$

Data:

(i) $|r_1| = 4$ cm $\theta_1 = 30°$ $|r_2| = 3$ cm $\theta_2 = 90°$

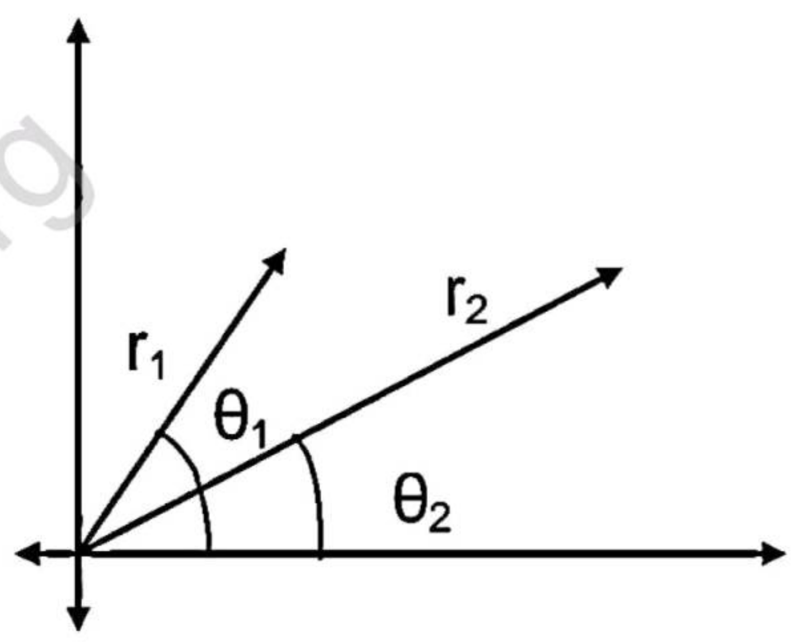
$|\vec{r_1} \times \vec{r_2}| = ?$

(ii) $|r_1| = 6$ cm $\theta_1 = 220°$ $|r_2| = 3$ cm $\theta_2 = 40°$

$|\vec{r_1} \times \vec{r_2}| = ?$

(iii) $|r_1| = 10$ cm $\theta_1 = 20°$ $|r_2| = 9$ cm $\theta_2 = 110°$

$|\vec{r_1} \times \vec{r_2}| = ?$



Solution:

According to the definition of cross product

$\vec{r_1} \times \vec{r_2} = |r_1||r_2| \cos\theta$ --- (i)

a) $\theta = \theta_2 - \theta_1$

$\theta = 90 - 30$

$\theta = 60°$

Putting values in eq (i)

$|\vec{r_1} \times \vec{r_2}| = 4 \times 3 \times \cos 60°$

$|\vec{r_1} \times \vec{r_2}| = 12 \times \frac{\sqrt{3}}{2}$

$\boxed{|\vec{r_1} \times \vec{r_2}| = 6\sqrt{3}\ cm^2}$

b) $\theta = \theta_1 - \theta_2$

$\theta = 220 - 40$

$\theta = 180°$

Putting values in eq (i)

$|\vec{r_1} \times \vec{r_2}| = 6 \times 3 \times \cos 180°$

$|\vec{r_1} \times \vec{r_2}| = 18 \times (-1)$

$\boxed{|\vec{r_1} \times \vec{r_2}| = -18\ cm^2}$

c) $\theta = \theta_2 - \theta_1$

$\theta = 110 - 20$

$\theta = 90°$

Putting values in eq (i)

$|\vec{r_1} \times \vec{r_2}| = 10 \times 9 \times \cos 90°$

$|\vec{r_1} \times \vec{r_2}| = 90 \times 0$

$\boxed{|\vec{r_1} \times \vec{r_2}| = 0\ cm^2}$

PAST PAPER NUMERICALS

2021

Q.2 (iv) Determine the unit vector perpendicular in the plane of A= 2i-6j-3k and B =4i+3j-k

Data:

$\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$

$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ ----- (i)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{\imath}\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{\imath}(6+9) - \hat{\jmath}(-2+12) + \hat{k}(6+24)$$

$$\boxed{\vec{A} \times \vec{B} = 15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}}$$

$$|\vec{A} \times \vec{B}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{A} \times \vec{B}| = \sqrt{(15)^2 + (-10)^2 + (30)^2}$$

$$\boxed{|\vec{A} \times \vec{B}| = 35}$$

Putting in eq (i)

$$\boxed{\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}}{35} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}}$$

Result: The unit vector perpendicular to the plane of A and B is $\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$.

2019

Q.2 (xiii)Two vectors A and B are such that |A|=4, |B|=6 and A.B=13.5.Find the magnitude of difference of vectors and angle between A and B.

Data:

|A|=4

|B|=6

A.B=13.5

Angle $= \theta = ?$

Magnitude of Difference =|A-B|=?

Solution:

$$|A - B| = \sqrt{|A|^2 + |B|^2 - 2A.B}$$

$$|A-B| = \sqrt{(4)^2+(6)^2-2(13.5)}$$
$$|A-B| = \sqrt{25} = 5\ unit$$
$$Cos\,\theta = \frac{A.B}{|A||B|}$$
$$Cos\,\theta = \frac{13.5}{(4)(6)}$$
$$Cos\,\theta = 0.5625$$
$$\boxed{\theta = Cos^{-1}(0.5625) = 55.77^0}$$

Result: The magnitude of difference is 5 *unit* and angle between them is 55.77^0

2018

Q.2(i) If the vector $\vec{A} = a\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{B} = \hat{\imath} + a\hat{\jmath} + \hat{k}$ are perpendicular to each other then find the value 'a'.

Data:

$\vec{A} = a\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{B} = \hat{\imath} + a\hat{\jmath} + \hat{k}$

a = ?

Solution:

According to the given condition

$$\vec{A}.\vec{B} = 0\ (\text{b/c}\ \vec{A} \perp \vec{B})$$
$$(a\hat{\imath} + \hat{\jmath} - 2\hat{k}).(\hat{\imath} + a\hat{\jmath} + \hat{k}) = 0$$
$$a + a - 2 = 0$$
$$2a = 2$$
$$\boxed{a = 1}$$

Result: The value of a for which $\vec{A} \perp \vec{B}$ is 1.

2017

Q.2(i) Textbook Numerical 22

2016

Q.2 (xi) Determine the unit vector perpendicular to the plane of A = 3i + 4j -k and B =4i+3j-2k vectors.

Data:

$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ -- (i)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & -1 \\ 4 & 3 & -2 \end{vmatrix}$$
$$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} 4 & -1 \\ 3 & -2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & -1 \\ 4 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix}$$
$$\vec{A}\times\vec{B} = \hat{\imath}(-8+3) - \hat{\jmath}(-6+4) + \hat{k}(9-16)$$
$$\boxed{\vec{A}\times\vec{B} = -5\hat{\imath} + 2\hat{\jmath} - 7\hat{k}}$$
$$|\vec{A}\times\vec{B}| = \sqrt{x^2+y^2+z^2}$$
$$|\vec{A}\times\vec{B}| = \sqrt{(-5)^2+(2)^2+(-7)^2}$$
$$\boxed{|\vec{A}\times\vec{B}| = \sqrt{78}}$$

Putting in eq (i)

$$\boxed{\hat{n} = \frac{\overrightarrow{\vec{A}\times B}}{|\vec{A}\times\vec{B}|} = \frac{-5\hat{\imath}+2\hat{\jmath}-7\hat{k}}{\sqrt{78}} = -\frac{5}{\sqrt{78}}\hat{\imath} + \frac{2}{\sqrt{78}}\hat{\jmath} - \frac{7}{\sqrt{78}}\hat{k}}$$

Result: The unit vector perpendicular to the plane of A and B is $-\frac{5}{\sqrt{78}}\hat{\imath} + \frac{2}{\sqrt{78}}\hat{\jmath} - \frac{7}{\sqrt{78}}\hat{k}$.

2015

Q.2 xii) Two sides of a triangle are formed by vectors A =3i + 6j -2k and B = 4i - j + 3k . Determine the area of the triangle.

Data:

A =3i + 6j -2k

B = 4i - j + 3k

Area of Triangle = Δ =?

Solution:

Area of Triangle is given by

$$\Delta = \frac{1}{2}|\vec{A}\times\vec{B}| \text{ ---- (i)}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 6 & -2 \\ 4 & -1 & 3 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} 6 & -2 \\ -1 & 3 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 3 & -2 \\ 4 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 6 \\ 4 & -1 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{\imath}(18-2) - \hat{\jmath}(9+8) + \hat{k}(-3-24)$$

$$\boxed{\vec{A}\times\vec{B} = 16\hat{\imath} - 17\hat{\jmath} - 27\hat{k}}$$

Now,

$$|\vec{A}\times\vec{B}| = \sqrt{x^2+y^2+z^2}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(16)^2+(-17)^2+(-27)^2} = 35.70$$

Putting in eq (i)

$$\boxed{\Delta = \frac{1}{2}(35.70) = 17.85\ sq.units}$$

Result: The area formed by these vectors is 17.85 sq.units

2014

(x) Determine the unit vector perpendicular to the plane containing A and B.

$\vec{A} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ **and** $B = \hat{\imath} + 4\hat{\jmath} - \widehat{2k}$

Data:

$\vec{A} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k}$

$\vec{B} = \hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\hat{n} = ?$

Solution:

Unit Vector Perpendicular $= \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$ -- (i)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{\imath}\begin{vmatrix} -3 & -1 \\ 4 & -2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{\imath}(6+4) - \hat{\jmath}(-4+1) + \hat{k}(8+3)$$

$$\boxed{\vec{A}\times\vec{B} = 10\hat{\imath} + 3\hat{\jmath} + 11\hat{k}}$$

$$|\vec{A}\times\vec{B}| = \sqrt{x^2+y^2+z^2}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(10)^2+(3)^2+(11)^2}$$

$$\boxed{|\vec{A}\times\vec{B}| = \sqrt{230}}$$

Putting in eq (i)

$$\boxed{\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{10\hat{\imath}+3\hat{\jmath}+11\hat{k}}{\sqrt{230}}}$$

Result: The unit vector perpendicular to the plane of A and B is $\frac{10\hat{\imath}+3\hat{\jmath}+11\hat{k}}{\sqrt{230}}$.

2013

Q.2 (i) If $\vec{A} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{B} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$. Find the projection of A on to B.

Data:

$\vec{A} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{B} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

Projection of A onto B = A_B = ?

Solution:

As we know that

$\vec{A}.\vec{B} = A_B B$

So $A_B = \frac{\vec{A}.\vec{B}}{B}$ -------- (i)

$$\vec{A}.\vec{B} = (3\hat{\imath} + \hat{\jmath} - 2\hat{k}).(-\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

$$\vec{A}.\vec{B} = -3 + 3 - 8$$

$$\boxed{\vec{A}.\vec{B} = -8}$$

Now,

$$B = \sqrt{x^2+y^2+z^2}$$

$$B = \sqrt{(-1)^2+(3)^2+(4)^2}$$

$$\boxed{B = \sqrt{26}}$$

Putting values in eq (i)

$$\boxed{A_B = -\frac{8}{\sqrt{26}}}$$

Result: The projection of Vector A on Vector B is $-\frac{8}{\sqrt{26}}\ units$.

Q.2 (iv) Prove that $|\vec{A}\times\vec{B}|^2 + (\vec{A}.\vec{B})^2 = A^2B^2$

Proof:

Taking L.H.S

L.H.S $= \left|\vec{A}\times\vec{B}\right|^2 + \left(\vec{A}.\vec{B}\right)^2$ ---- (i)

Since

$$\left|\vec{A}\times\vec{B}\right| = ABSin\theta$$

And

$$\vec{A}.\vec{B} = ABCos\theta$$

Putting values in eq (i)

L.H.S $=(ABSin\theta)^2 + (ABCos\theta)^2$

L.H.S $=A^2B^2(Sin^2\theta + Cos^2\theta)$

Since $Sin^2\theta + Cos^2\theta = 1$

Therefore L.H.S $=A^2B^2(1)$

Or L.H.S $=A^2B^2$

Or $\boxed{L.H.S = R.H.S}$

2012

Q.2 (iii) Two vectors A and B are such that A=4 , B=6 and |A-B|=5. Find |A+B|

Data:

Magnitude of $\vec{A} = |\vec{A}| = 4$

Magnitude of $\vec{B} = |\vec{B}| = 6$

Magnitude of $|\vec{A}-\vec{B}| = 5$

Magnitude of $|\vec{A}+\vec{B}| =?$

Solution:

As we know that

$$|\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 - 2AB\ cos\theta}$$

$$5 = \sqrt{(4)^2 + (6)^2 - 2(4)(6)\ cos\theta}$$

S.O.B.S

$$25 = 16 + 36 - 48\ cos\theta$$

$$-27 = -48 \times cos\theta$$

$$cos\theta = \frac{27}{48}$$

$$\theta = \cos^{-1}\left(\frac{27}{48}\right)$$

$$\theta = 55.7^0$$

Now,

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\ cos\theta}$$

$$|\vec{A}+\vec{B}| = \sqrt{(4)^2 + (6)^2 + 2(4)(6)\cos(55.7)}$$

$$|\vec{A}+\vec{B}| = \sqrt{79.5}$$

Result: The magnitude of $\vec{A}+\vec{B}$ is $\sqrt{79.5}$ units.

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Q.2(viii) If one of the rectangular components of force 50 N is 25N; find the value of the other.

Data:

Magnitude of Force $= F = 50\ N$

One component of Force $= F_1 = 25\ N$

Second component of Force $= F_2 = ?$

Solution:

The magnitude of force is given by

$$F = \sqrt{F_1^2 + F_2^2}$$

$$50 = \sqrt{(25)^2 + F_2^2}$$

$$50 = \sqrt{625 + F_2^2}$$

S.O.B.S.

$$(50)^2 = 625 + F_2^2$$

$$F_2^2 = 2500 - 625$$

$$F_2^2 = 1875$$

Taking Square root O.B.S.

$$\boxed{F_2 = 43.3\ N}$$

Result: The second component of force is 43.3 N.