1st Year Physics

Chapter # 05

Circular Motion

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Ch # 05

Circular Motion

circular motion:

The motion of an object in a circular path is called circular motion.

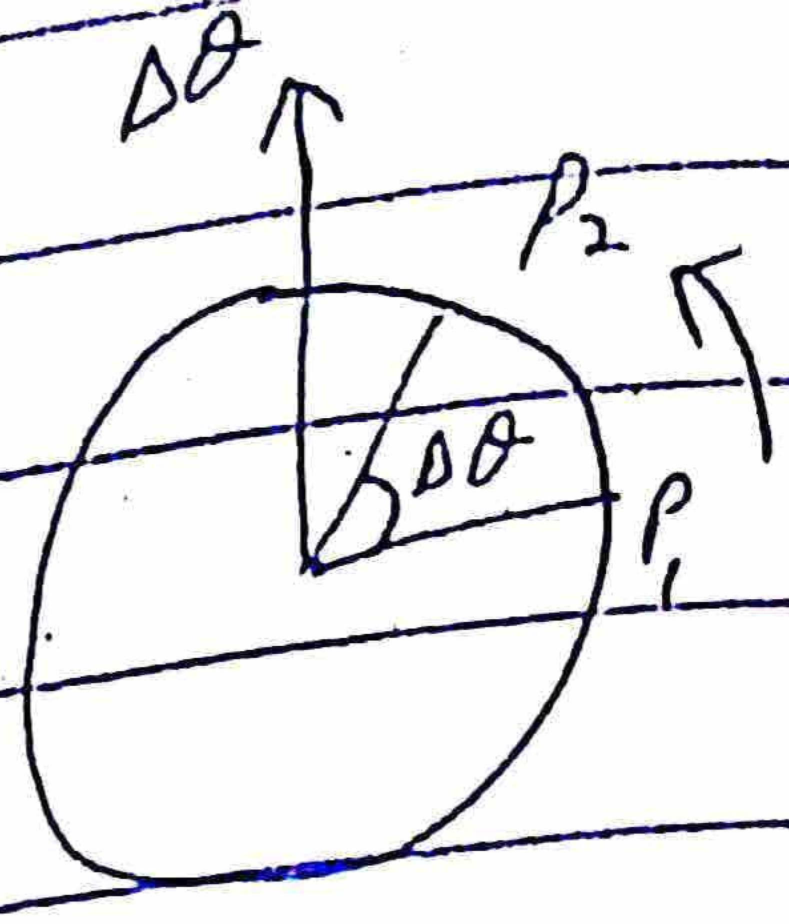
For example, motion of moon around the earth, motion of earth around the sun, motion of a stone attached to string etc.

Angular displacement:

The displacement covered by a body in circular path is called angular displacement. It is represented by $\vec{\Delta\theta}$. Its units are degree, radian or revolution. The SI unit is radian.

It is a vector quantity.

Direction:

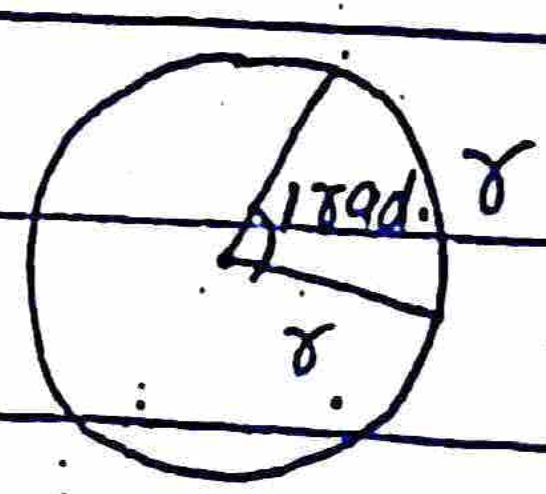


Grasp the axis of rotation in right hand with fingers curling in the direction of rotation the thumb points in the direction of angular displacement.

Degree:

If the circumference of the circle is divided into 360 equal parts then the angle made at the centre for each part will be 1 degree.

Radian:



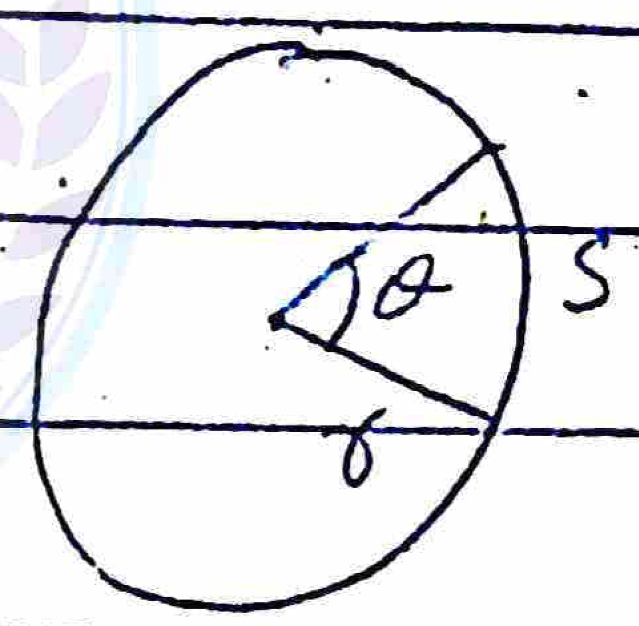
The angle made at the centre of the circle by an arc whose length is equal to the radius of the circle is called one radian.

Revolution:

The angle made at the centre of the circle when body completes one round trip is called one revolution.

Prove that $S = r\theta$

Consider a body rotating in a circle of radius "r", the arc length is S making angle θ at the centre of circle. According to defination of radian:



$$\theta = \frac{\text{arc length}}{\text{radius}} \text{ rad.}$$

$$\theta = \frac{S}{r} \text{ rad.}$$

$$\boxed{S = r\theta}$$

Show the relation between degree, radian and revolution.
OR Prove that $1 \text{ rad.} = 57.3^\circ$

$$\theta = \frac{s}{r}$$

$$\text{one revolution} = \frac{2\pi \cancel{r}}{\cancel{r}}$$

$$\boxed{1 \text{ rev.} = 2\pi \text{ rad.}}$$

$$1 \text{ revolution} = 2\pi \text{ rad.} = 360^\circ$$

$$2\pi \text{ rad.} = 360^\circ$$

$$1 \text{ rad} = \frac{360^\circ}{2\pi}$$

$$\boxed{1 \text{ rad.} = 57.3^\circ}$$

And

$$\boxed{1 \text{ rev.} = 360^\circ}$$

Angular velocity:

The rate of change of angular displacement is called angular velocity. It is represented by $\vec{\omega}$. Its units may be degree/min, degree/s, rev./min, rad./min or rad./s.

The SI unit is $rad.\,s^{-1}$.

It is a vector quantity.

$$\vec{\omega} = \frac{\Delta \vec{\theta}}{\Delta t}$$

Direction:

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation the thumb points in the direction of angular velocity.

Average angular velocity:

The total angular displacement divided by total time is called average angular velocity.

$$\vec{\omega}_{av.} = \frac{\vec{\Delta\theta}}{\Delta t}$$

Instantaneous angular velocity:

The angular velocity of a rotating body at any instant of time is called instantaneous angular velocity. Mathematically.

$$\vec{\omega}_{ins} = \lim_{\Delta t \to 0} \frac{\vec{\Delta\theta}}{\Delta t}$$

Here $\lim_{\Delta t \to 0}$ shows a very short interval of time.

Angular acceleration:

The rate of change of angular velocity of a body is called angular acceleration. It is represented by $\vec{\alpha}$. It is a vector quantity. Its SI unit is $rad.s^{-2}$.

$$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$$

$$\vec{\alpha} = \frac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$$

Direction:

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation the thumb points in the direction of angular acceleration.

Average angular acceleration:

The total change in angular velocity divided by total time is called average angular acceleration.

$$\vec{\alpha}_{av.} = \frac{\Delta \vec{\omega}}{\Delta t}$$

Instantaneous angular acceleration:

The angular acceleration of a rotating body at any instant of time is called instanteneous angular acceleration.

$$\vec{\alpha}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t}$$

Here $\lim_{\Delta t \to 0}$ shows a very short interval of time.

Show that $v = r\omega$

For a body rotating in a circle of radius r.

$$S = r\theta$$

For a short interval.

$$\Delta S = r\,\Delta\theta$$

Dividing by Δt on both sides:

$$\frac{\Delta S}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

Taking $\lim_{\Delta t \to 0}$ on both sides

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\boxed{v = r\omega}$$

Show that $a = r\alpha$

For a body rotating in a circle of radius r.

$$v = r\omega$$

For a short interval.

$$\Delta v = r\,\Delta \omega$$

Dividing by Δt on both sides:

$$\frac{\Delta v}{\Delta t} = r\,\frac{\Delta \omega}{\Delta t}$$

Taking $\lim_{\Delta t \to 0}$ on both sides

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

$$\boxed{a = r\alpha}$$

Equations of motion:

Linear Form	Angular Form
1- $v_f = v_i + at$	1- $\omega_f = \omega_i + \alpha t$
2- $S = v_i t + \frac{1}{2} a t^2$	2- $\theta = \omega_i t + \frac{1}{2} \alpha t^2$
3- $2aS = v_f^2 - v_i^2$	3- $2\alpha\theta = \omega_f^2 - \omega_i^2$

Centripetal Force

Defination:

The force needed to bend the normally straight path of the particle into a circular path is called centripetal force. It is represented by F_c. mathematically,

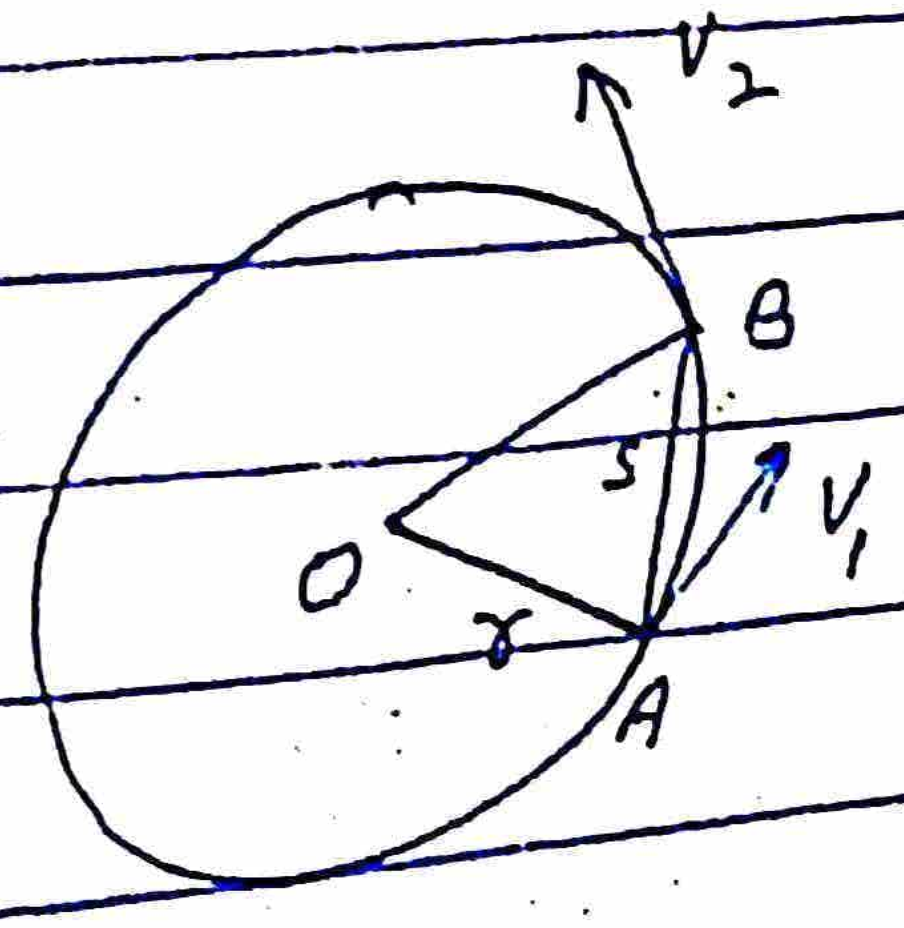
$$F_c = \frac{mv^2}{r}$$

Example:

i- Satellites require centripetal force to revolve around the earth.

ii- Earth require centripetal force to revolve around the sun.

Explanation:



Consider a body of mass "m" revolving

in a circle of radius "r". The object has uniform velocity; the difference of v_1 and v_2 is due to only its location at points A and B.

$$|v_1| = |v_2| = v$$

The rate of change of velocity of the body is called acceleration. So,

$$a = \frac{\Delta v}{\Delta t}$$

We know that

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\Delta t = \frac{s}{v}$$

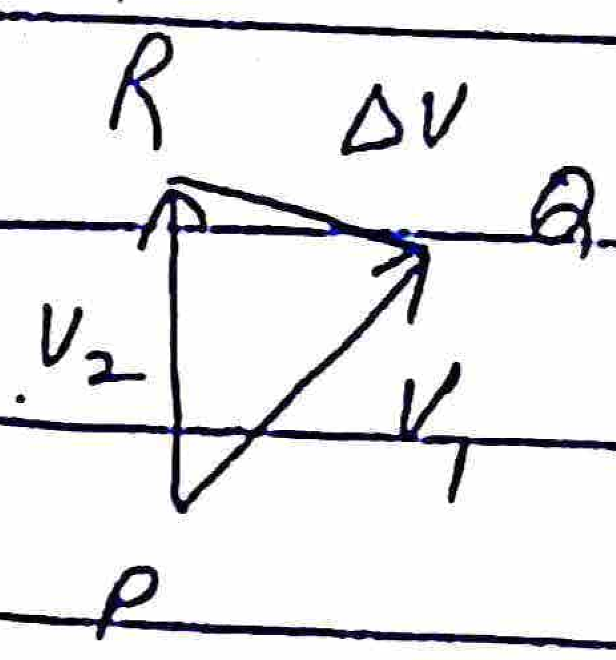
Here s is the distance covered from point A to point B in time Δt.

Now equation for acceleration will be:

$$a = \frac{\Delta v}{\frac{s}{v}}$$

$$a = \frac{V \Delta V}{S} \longrightarrow (1)$$

We consider a triangle PQR equivalent as triangle OAB.



So, by comparing these triangles:

$$\frac{|QR|}{|PQ|} = \frac{|AB|}{|OA|}$$

$$\frac{\Delta V}{V_1} = \frac{S}{r}$$

As $|V_1| = |V_2| = V$

So,

$$\frac{\Delta V}{V} = \frac{S}{r}$$

$$\Delta V = \frac{SV}{r}$$

Put in eq. (1)

$$a = \frac{V\left(\frac{SV}{r}\right)}{S} = \frac{\cancel{S}V^2}{r\cancel{S}}$$

$$a = \frac{v^2}{r}$$

This is the mathematical expression for centripetal acceleration, it is the instantaneous acceleration of a body rotating in circle.

$$a_c = \frac{v^2}{r}$$

According to Newton's second law:

$$F_c = m a_c$$

$$\boxed{F_c = \frac{mv^2}{r}}$$

This is the mathematical formula for centripetal force.

As $v = r\omega$

So,

$$F_c = \frac{m(r\omega)^2}{r}$$

$$= \frac{m r^2 \omega^2}{r}$$

$$\boxed{F_c = mr\omega^2}$$

Direction: The direction of centripetal force is towards the centre of the circle. The direction of centripetal acceleration is also same as force.

Moment of Inertia

Defination:

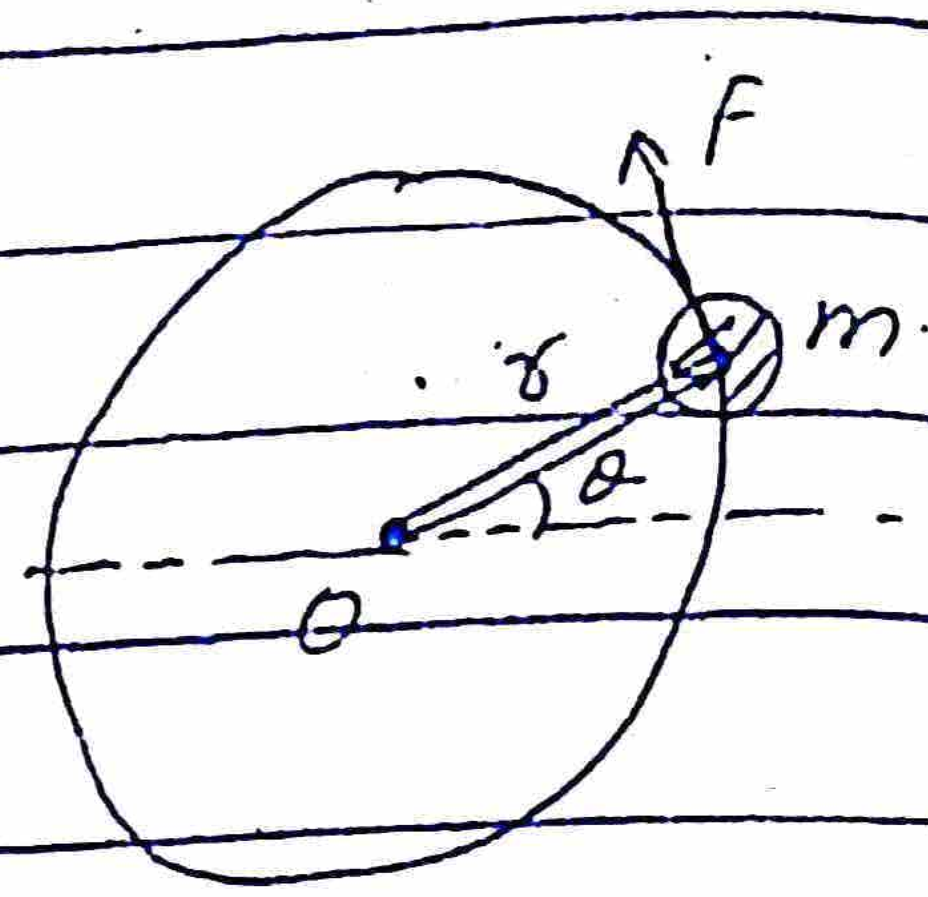
If a body is rotating in a circle then the product of its mass and square of the radius of circle is called moment of inertia. It is represented by I.

$$I = mr^2$$

It is a scalar quantity. Its SI unit is $kg\,m^2$.

Significance:

Moment of inertia plays the same role in angular motion as mass plays in linear motion.



Explanation:

Consider a body of mass "m" attached to a light rod of length "r" rotating in a circle by applied force F. According to Newton's second law:

$$F = ma$$

The linear acceleration (a) and angular acceleration (α) are related as:

$$a = r\alpha$$

So,

$$F = mr\alpha$$

As the body is rotating in circle. So, torque produced will be:

$$\tau = rF$$

$$= r(mr\alpha)$$

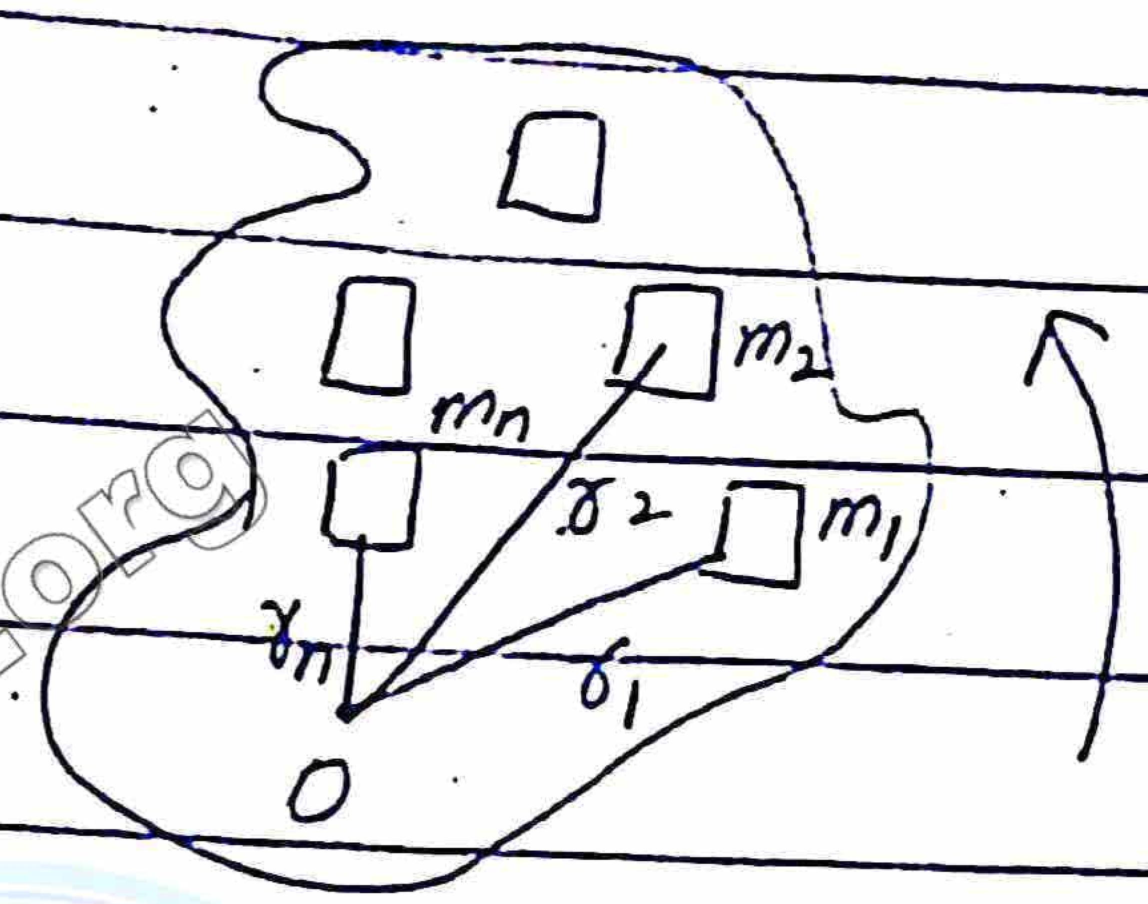
$$\tau = mr^2\alpha$$

Here mr^2 is called moment of inertia (I). So,

$$\boxed{\tau = I\alpha}$$

For a rigid body:

Consider a rigid body composed of n number of small masses



$m_1, m_2, \ldots m_n$ having distance $r_1, r_2, \ldots r_n$ from the axis of rotation, O. The total torque in the rigid body will be equal to the sum of torque for all masses.

$$\tau_{total} = \tau_1 + \tau_2 + \cdots + \tau_n$$
$$= m_1 r_1^2 \alpha_1 + m_2 r_2^2 \alpha_2 + \cdots + m_n r_n^2 \alpha_n$$

The angular acceleration will be same for all the pieces.

$$\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$$

so,

$$\tau_{total} = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \cdots + m_n r_n^2 \alpha$$

$$= (m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2)\alpha$$

$$\tau_{total} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

The term in brackets is the moment of inertia for the rigid body.

$$\boxed{\tau_{total} = I\alpha}$$

Angular Momentum

Defination:

A particle is said to posses an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The cross product of position vector $\vec{r}$ and linear momentum $\vec{P}$ is called angular momentum $\vec{L}$.

It is represented by $\vec{L}$, it is a vector quantity.

$$\vec{L} = \vec{r} \times \vec{P}$$

Its unit is $kg\,m^2 s^{-1}$ or Js.

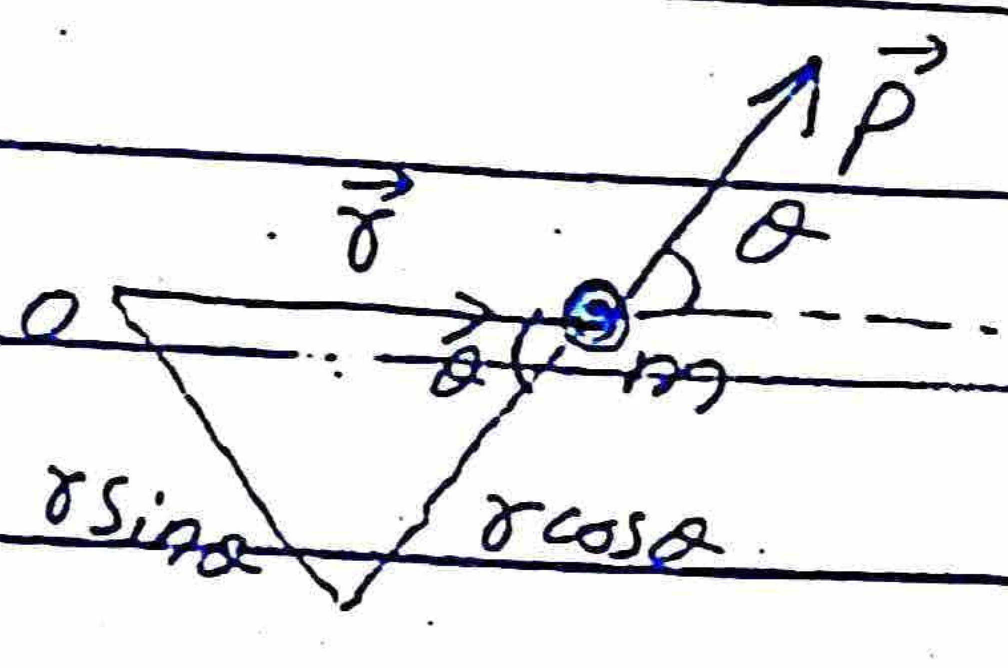
Direction:

The direction of angular momentum can be found by right hand rule.

"Curl the fingers of right hand from $\vec{r}$ to $\vec{P}$ then erect thumb will show the direction of $\vec{L}$."

Explanation:

Consider a body of mass "m" moved through linear momentum $\vec{P}$ making angle



θ with position vector $\vec{r}$ as shown in figure. Then we resolve position vector $\vec{r}$ into its rectangular components $r\sin\theta$ and $r\cos\theta$. Now angular momentum is:

$$\vec{L} = \vec{r} \times \vec{P}$$

$$L = rP\sin\theta$$

For maximum angular momentum, $\theta = 90°$

$$L = rP\sin 90°$$

$$= rP(1)$$

$$L = rP$$

Linear momentum is given by $P = mv$

So, $L = rmv$

As $v = r\omega$

So,

$$L = rm(r\omega)$$

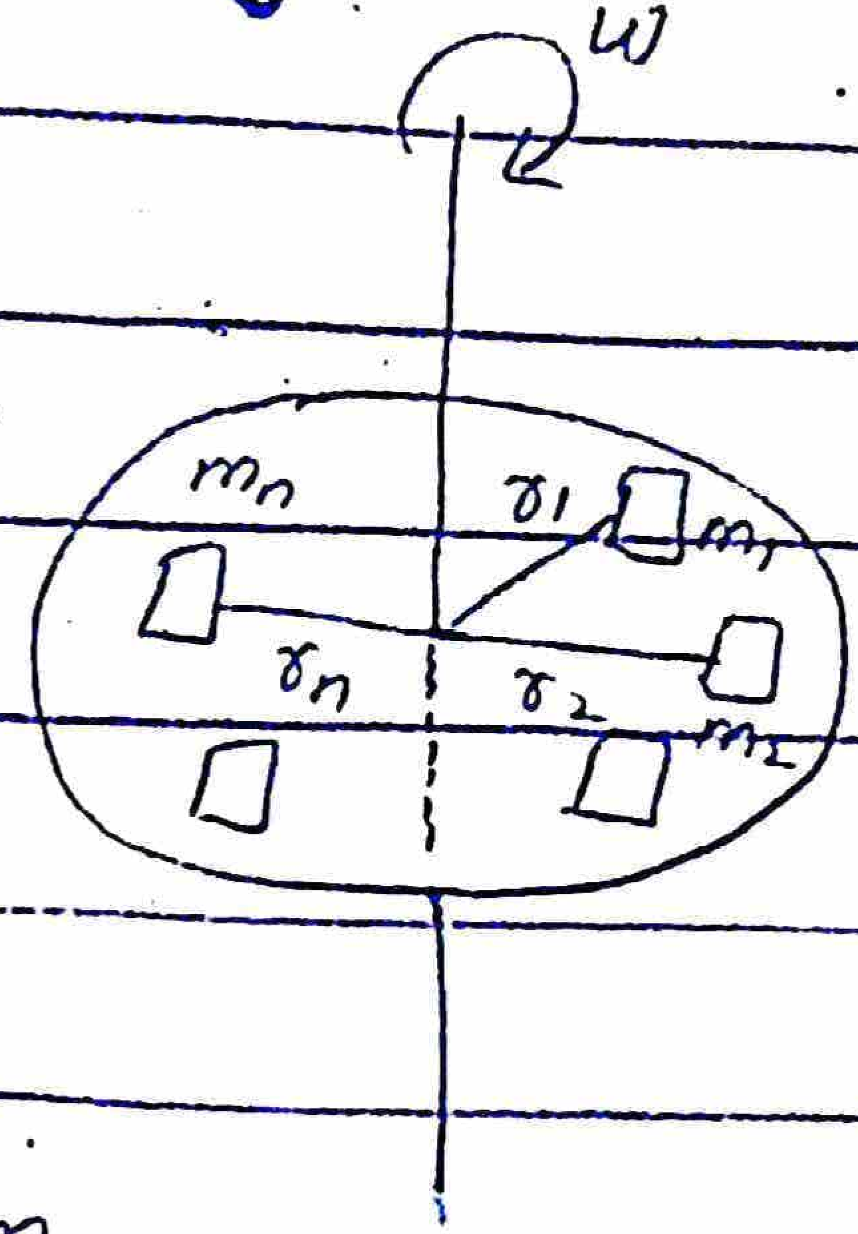
$$L = mr^2\omega$$

Here term mr^2 is named as moment of inertia(I).

$$\boxed{L = I\omega}$$

For a rigid body:

Consider a rigid body composed of many small pieces having masses $m_1, m_2, \ldots m_n$ having distance $r_1, r_2, \ldots r_n$ from the axis of rotation.



The total angular momentum of rigid body will be equal to the sum of angular momentum for all the pieces.

$$L = L_1 + L_2 + \cdots + L_n$$

$$L = m_1 r_1^2 \omega_1 + m_2 r_2^2 \omega_2 + \cdots + m_n r_n^2 \omega_n$$

The angular velocity will be same for all the pieces.

So,

$$\omega_1 = \omega_2 = \cdots = \omega_n = \omega$$

So,

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + \cdots + m_n r_n^2 \omega$$

$$L = (m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2)\,\omega$$

$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$

The term in the brackets is called moment of inertia for rigid body.

$$\boxed{L = I\omega}$$

Types:

There are two types of angular momentum:

i- Spin angular momentum:

The angular momentum of a spining body about its own axis is called spin angular momentum. It is represented by L_s.

ii- Orbital angular momentum:

The angular momentum

of a rotating body in an orbit is called orbital angular momentum. It is represented by L_o.

Point object:

The object whose size is very small as compared to the orbital radius is called point object.

Law of conservation of angular momentum:

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots\ldots = \text{constant}$$

M.C.Q

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

Why does diver change his body position before and after diving in the pool?

The diver changes his body position before and after diving in the pool to conserve the angular momentum. When the diver stretches his legs and arms moment of inertia will be greater and angular velocity will be smaller. While when the diver closes his legs and arms moment of inertia will be smaller and angular velocity will be greater. Such that the product of moment of inertia (I) and angular velocity (w) remains constant.

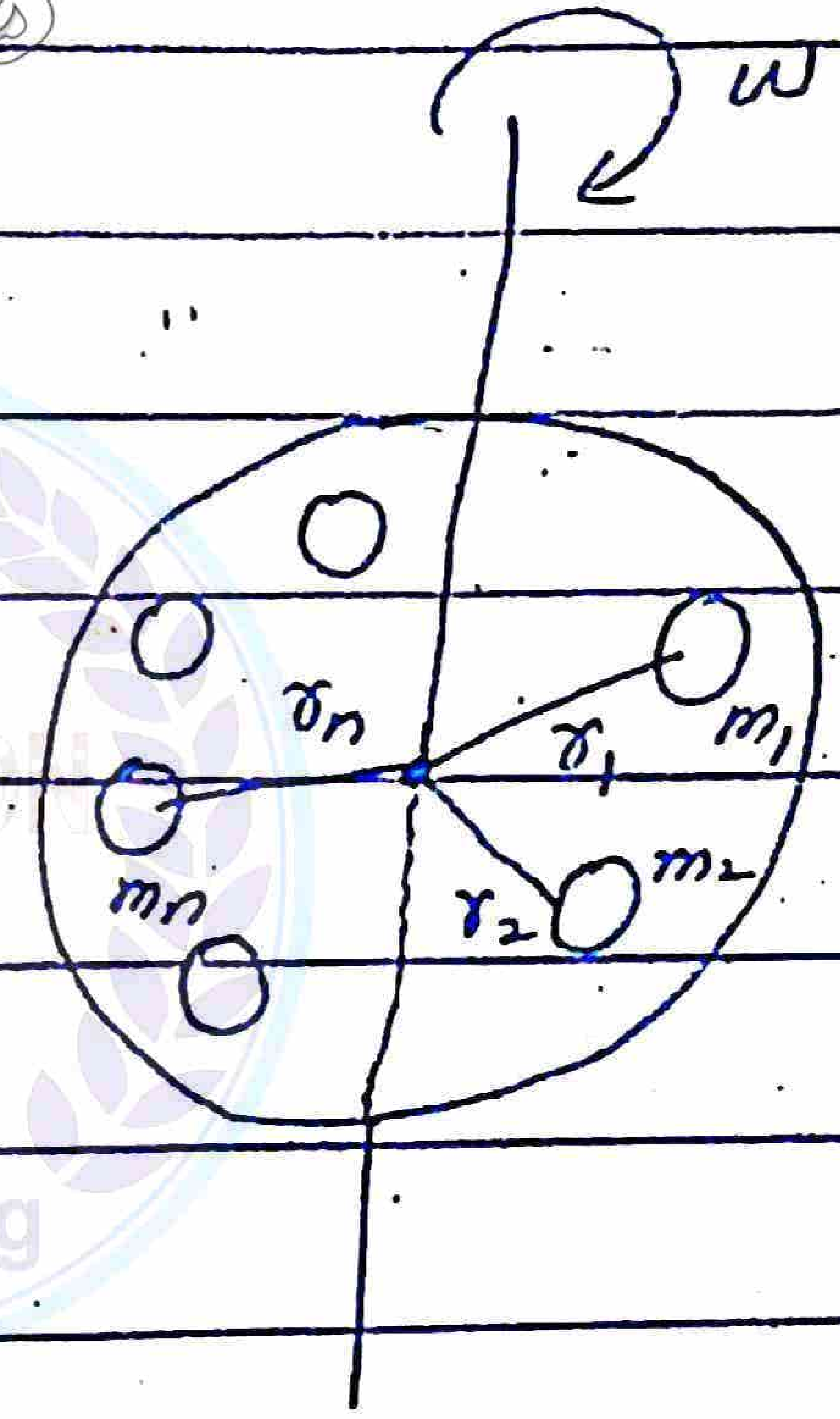
$$I_1 w_1 = I_2 w_2$$

Rotational Kinetic Energy:

The kinetic energy of a rotating body about its own axis is called rotational kinetic energy. mathematically,

$$K.E_{rot.} = \frac{1}{2} I \omega^2$$

Explanation:



Consider a rigid body composed of n number of small pieces having masses $m_1, m_2, --- m_n$ with distance $r_1, r_2, ---- r_n$ from the axis of rotation. The total kinetic energy of the rigid body will be equal to the sum of K.E for all pieces.

$$(K.E)_{total} = (K.E)_1 + (K.E)_2 + \cdots + (K.E)_n$$

$$(K.E)_{total} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \cdots + \frac{1}{2} m_n v_n^2$$

But $v = r\omega$ in case of rotational motion

$$K.E_{rot} = \frac{1}{2} m_1 r_1^2 \omega_1^2 + \frac{1}{2} m_2 r_2^2 \omega_2^2 + \cdots + \frac{1}{2} m_n r_n^2 \omega_n^2$$

The angular velocity for all the pieces will be same

$$\omega_1 = \omega_2 = \cdots = \omega_n = \omega$$

$$K.E_{rot.} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \cdots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$= \frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2\right)\omega^2$$

$$K.E_{rot} = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2.$$

The term in the brackets is called moment of inertia for rigid body.

$$\boxed{K.E_{rot} = \frac{1}{2} I \omega^2}$$

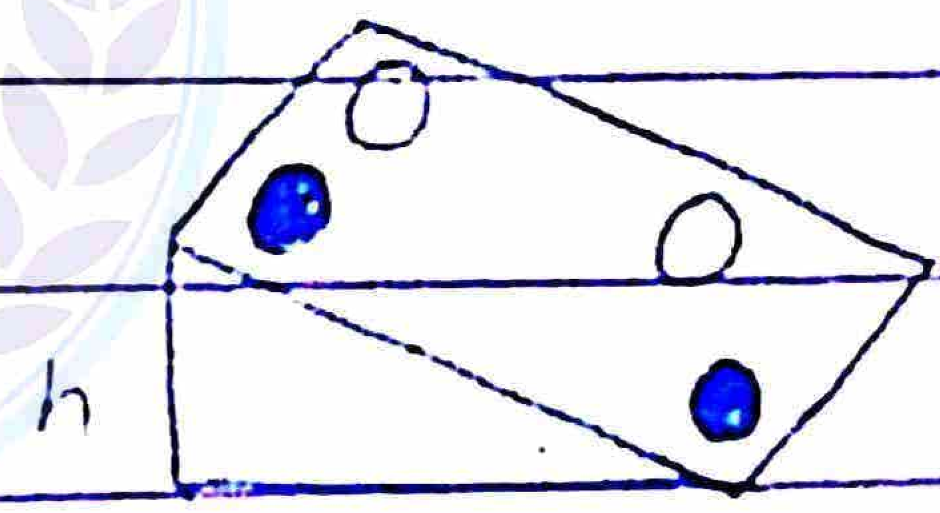
Rotational Kinetic Energy Of A Disc And A Hoop:

The rotational K.E of a rotating body is:

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

we consider a disc and a hoop of same mass "m" moved through an incline of height "h" as shown in figure.

now we will compare the rotational K.E of disc and hoop.



For disc:

For the disc moment of inertia is:

$$I = \frac{1}{2} m r^2$$

$$K.E_{rot} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$= \frac{1}{4}mr^2\omega^2$$

$$K.E_{rot} = \frac{1}{4}m(r\omega)^2$$

But $v = r\omega$

$$K.E_{rot} = \frac{1}{4}mv^2$$

For hoop:

For the hoop, moment of inertia is:

$$I = mr^2$$

So,

$$K.E_{rot} = \frac{1}{2}(mr^2)\omega^2$$

$$= \frac{1}{2}mr^2\omega^2$$

$$K.E_{rot} = \frac{1}{2}m(r\omega)^2$$

But

$$v = r\omega$$

So, $K.E_{rot} = \frac{1}{2}mv^2$

Result: The rotational K.E for hoop is greater than rotational K.E of disc.

$$K.E_{hoop} > K.E_{disc}$$

Comparison of speed:

Now we will compare the speed for disc and hoop through the inclined.

The disc and hoop has P.E due to their height that is converted into rotational and translational K.E when they move through inclined.

For disc:

$$P.E = K.E_{tran.} + K.E_{rot.}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{mv^2}{2} + \frac{mv^2}{4}$$

$$= \frac{2mv^2 + mv^2}{4}$$

$$mgh = \frac{3mv^2}{4}$$

$$4gh = 3v^2$$

$$\frac{4gh}{3} = v^2$$

$$v_{disc} = \sqrt{\frac{4gh}{3}}$$

For hoop:

$$P.E = K.E_{trans.} + K.E_{rot.}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$= \frac{mv^2}{2} + \frac{mv^2}{2}$$

$$= \frac{mv^2 + mv^2}{2}$$

$$mgh = \frac{2mv^2}{2}$$

$$gh = v^2$$

$$v_{hoop} = \sqrt{gh}$$

Result: The speed of disc is greater than speed of hoop.

$$v_{disc.} > v_{hoop}$$

Artificial Satellites

Satellites are objects that orbit around the earth. They are put into orbit by rockets and are held in orbits by the gravitational pull of the earth.

Value of acceleration:

The low flying earth satellites have acceleration $9.8ms^{-2}$ towards the centre of the earth. If they do not, they would fly off in a straight line tangent to the orbit.

Equation of acceleration:

When the satellite is moving in a circle, it has an acceleration $\frac{v^2}{r}$. In a circular orbit around the earth, the centripetal acceleration is supplied by gravity and we have,

$$g = \frac{v^2}{R}$$

Critical velocity:

The minimum velocity necessary to put a satellite into the orbit is called critical velocity. If v is the orbital velocity and R is the radius of the earth (6400km). Then

$$v = \sqrt{gR}$$

$$= \sqrt{9.8\,ms^{-2} \times 6.4\times10^{6}\,m}$$

$$= 7.9\,kms^{-1}.$$

Time period:

The period T is given by.

$$T = \frac{2\pi}{v} = 2 \times 3.14 \times \frac{6400\,km}{7.9\,kms^{-1}}$$

$$= 5060\,s = 84\ min\ approx.$$

M.C.Q

The higher the satellite, the slower will be the required speed and longer it will take to complete one revolution around the earth.

GPS:

Close orbiting satellites orbit the Earth at a height of about 400km. Twenty four such satellites from the Global poistioning system. An airline pilot, sailor or any other person can now use a pocket size instruement or mobile phone to find his position on the Earth's surface to within 10m accuracy.

Weightlessness in satellite:

When a satellite is falling free in space, everything within this freely falling system will appear to be weightless. It doesnot matter where the object is, whether it is falling under the force of attraction of the Earth, the sun, or some distant star. This property is called weightlessness in satellite

Gravity free system:

Since the space ship is in free fall, all the object within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellite. Such a system is called gravity free system.

Real And Apparent Weight:

Real weight:

The gravitational pull of earth acting on a body is called its real weight. It is denoted by w. Its SI unit is newton (N)

Apparent weight:

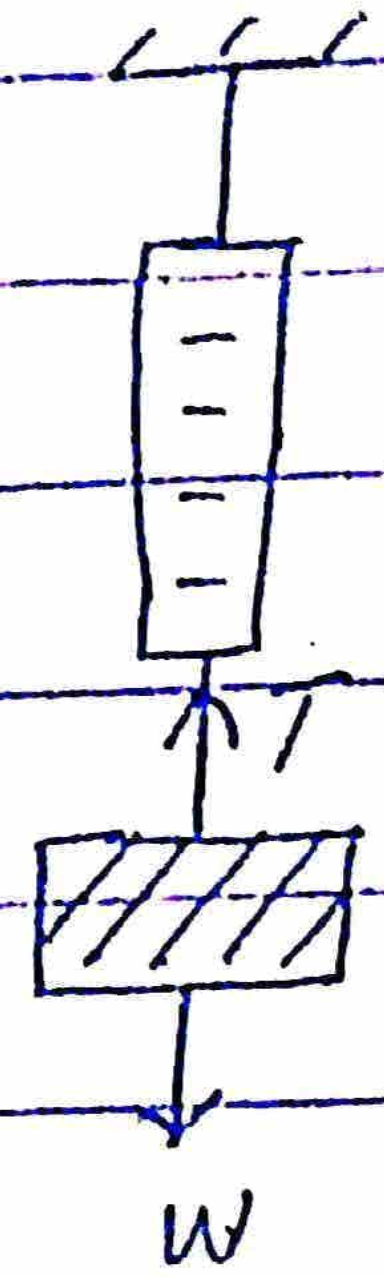
The reading shown on the spring balance for a body is called its apparent weight. It is denoted by T. Its SI unit is newton (N)

Relation:

The apparent weight may be equal, greater or smaller than the real weight. Moreover apparent weight may also be zero.

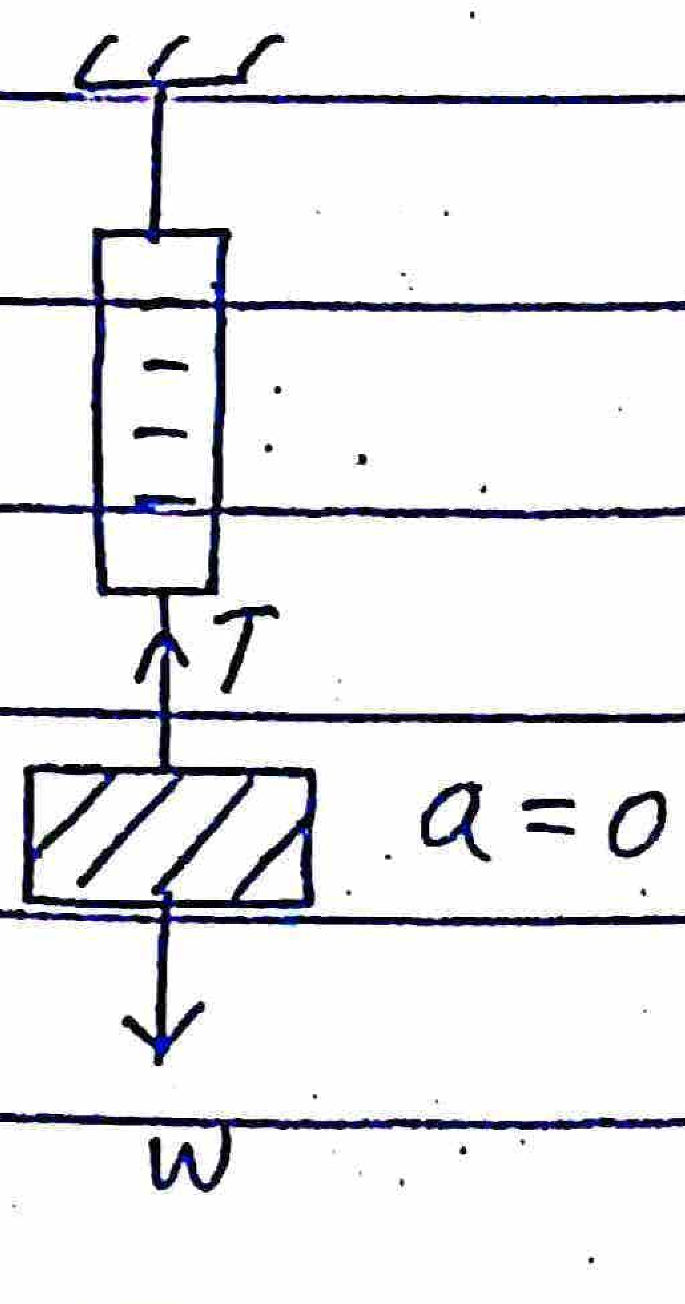
Explanation:

Consider a body of mass "m" attached to a spring balance which hanging vertically in a lift as shown in the figure. The real weight "w" is acting downward while apparent weight T is acting upward. Now we will discuss different cases.



Case. 1 : When lift is at rest:

When the lift is at rest, the acceleration will be zero.



Net force = T − W

According to Newton's second law:

$$F = ma$$

as $a = 0$

so, $F = 0$

Comparing the above equations for forces

$$T - W = 0$$

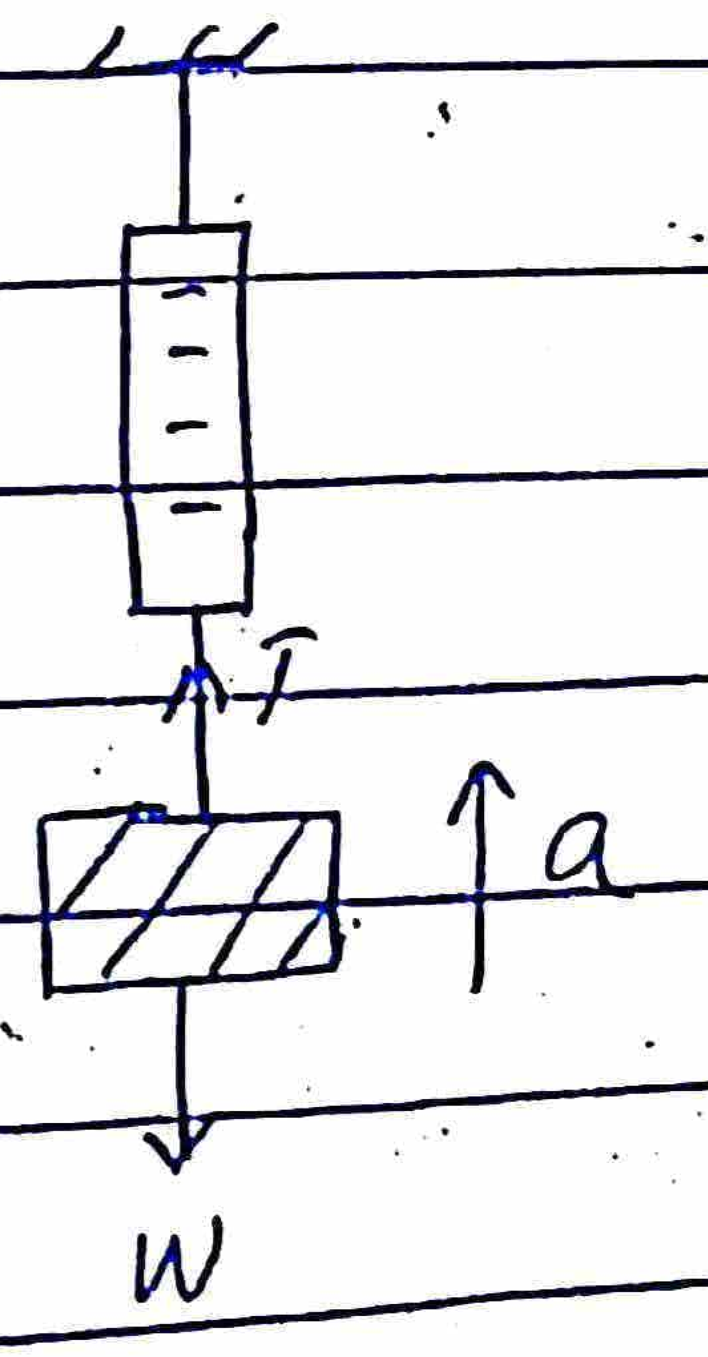
$$T = W$$

Result:

When the lift is at rest, apparent weight will be equal to real weight.

Case. 2: When the lift is moving upward:

When the lift is moving upward with an acceleration "a"



Net force $= T - W$

According to newton's second law

$$F = ma$$

comparing the above equation for forces:

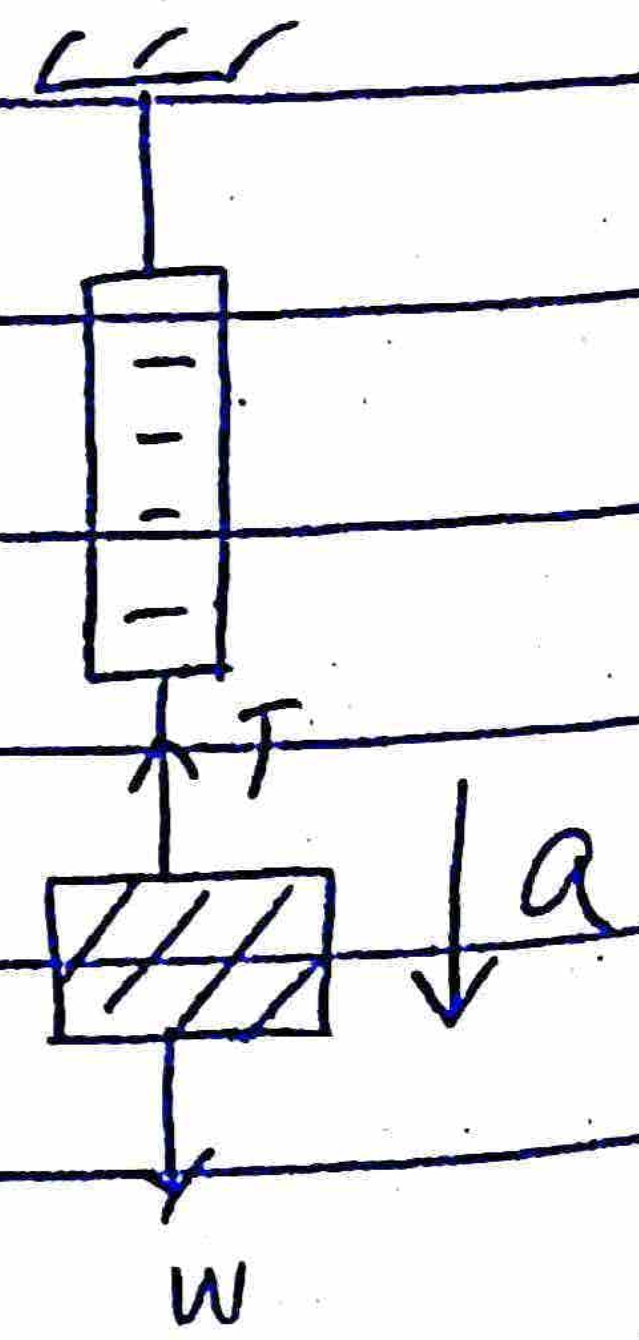
$$T - W = ma$$

$$T = W + ma$$

Result:

when the lift is moving upward, the apparent weight of the object is greater than real weight by a factor "ma".

Case. 3: When the lift is moving downward:



when the lift is moving downward with an acceleration "a"

Net force $= W - T$

According to Newton's second law

$$F = ma$$

comparing the above equations for forces:

$$W - T = ma$$

$$W = ma + T$$

$$W - ma = T$$

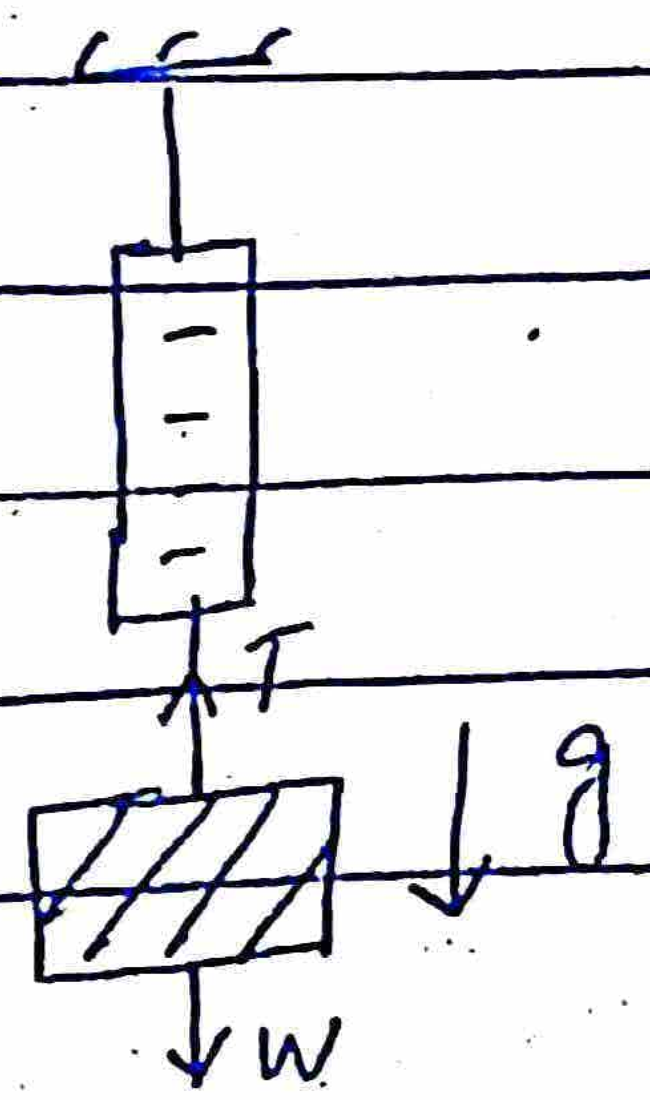
$$T = W - ma$$

Result:

when the lift is moving downward, the apparent weight is smaller than real weight by a factor "ma".

case.4: when the lift is freely falling:

When the lift is freely falling under the action of gravity



Net force = $W - T$

According to Newton's second law:

$$F = ma$$

here $a = g$

$$F = mg$$

comparing the above equations for forces:

$$W - T = mg$$

$$W = mg + T$$

$$W - mg = T$$

$$T = W - mg$$

$$= mg - mg$$

$$T = 0$$

Result:

When the lift is freely falling due to gravity, the apparent weight of the object will be zero.

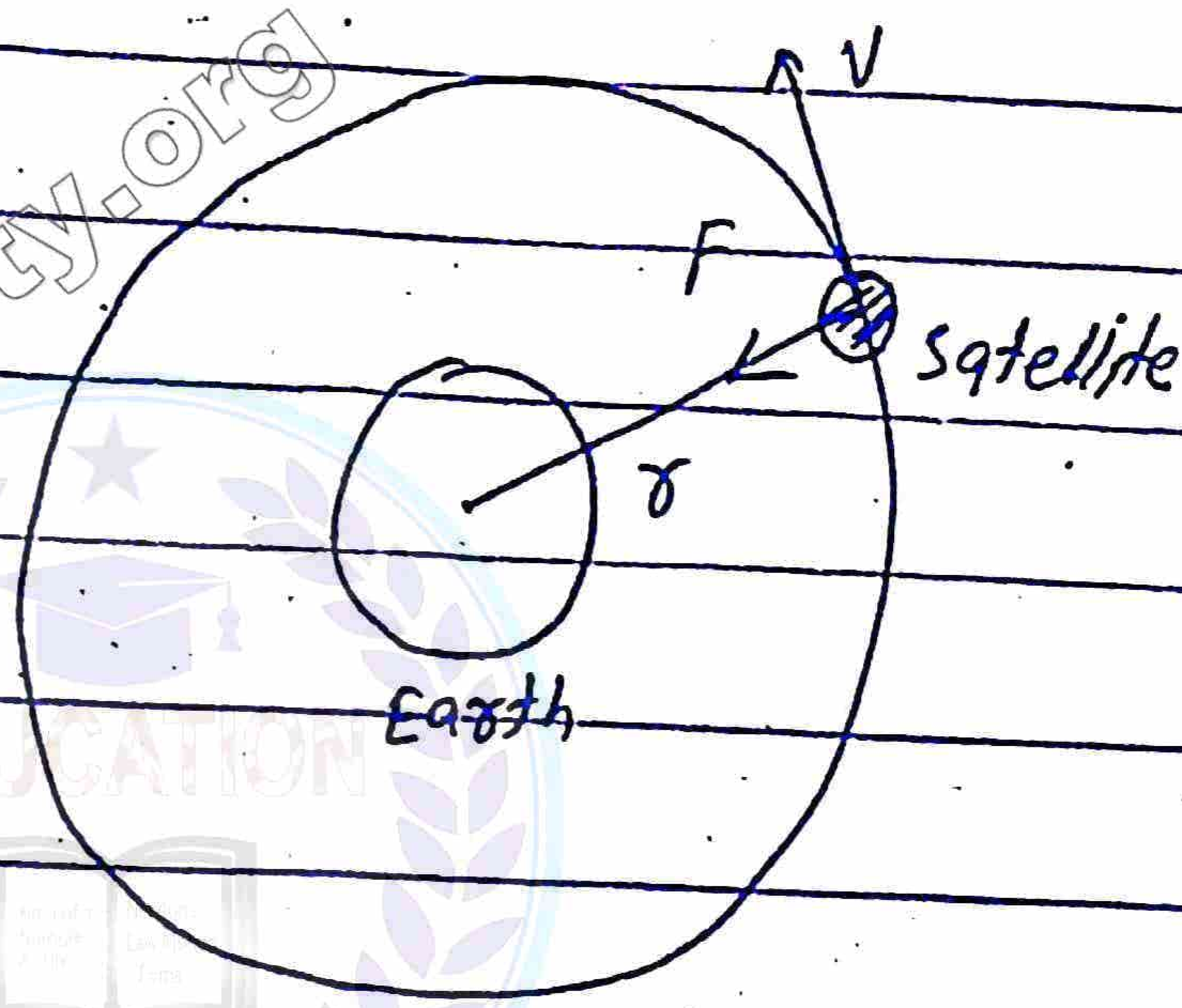
Orbital velocity:

The velocity required by the satellite to revolve around the earth is called orbital velocity. mathematically,

$$v = \sqrt{\frac{GM}{r}}$$

Explanation:

Consider a satellite of mass m_s revolving around the earth with velocity v in an orbit of radius "r"



The necessary centripetal force for the satellite to revolve in orbit is provided by the gravitational force of earth.

Gravitational force = centripetal force

$$G\frac{\cancel{m_s}M}{r^2} = \frac{\cancel{m_s}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2$$

$$v = \sqrt{\frac{GM}{r}}$$

This is the mathematical equation for orbital velocity. Here G is the gravitational constant, M is the mass of earth and r is the orbital radius.

Therefore, the orbital velocity depends upon the radius of the orbit, it is independent of the mass of the satellite.

Artificial Gravity

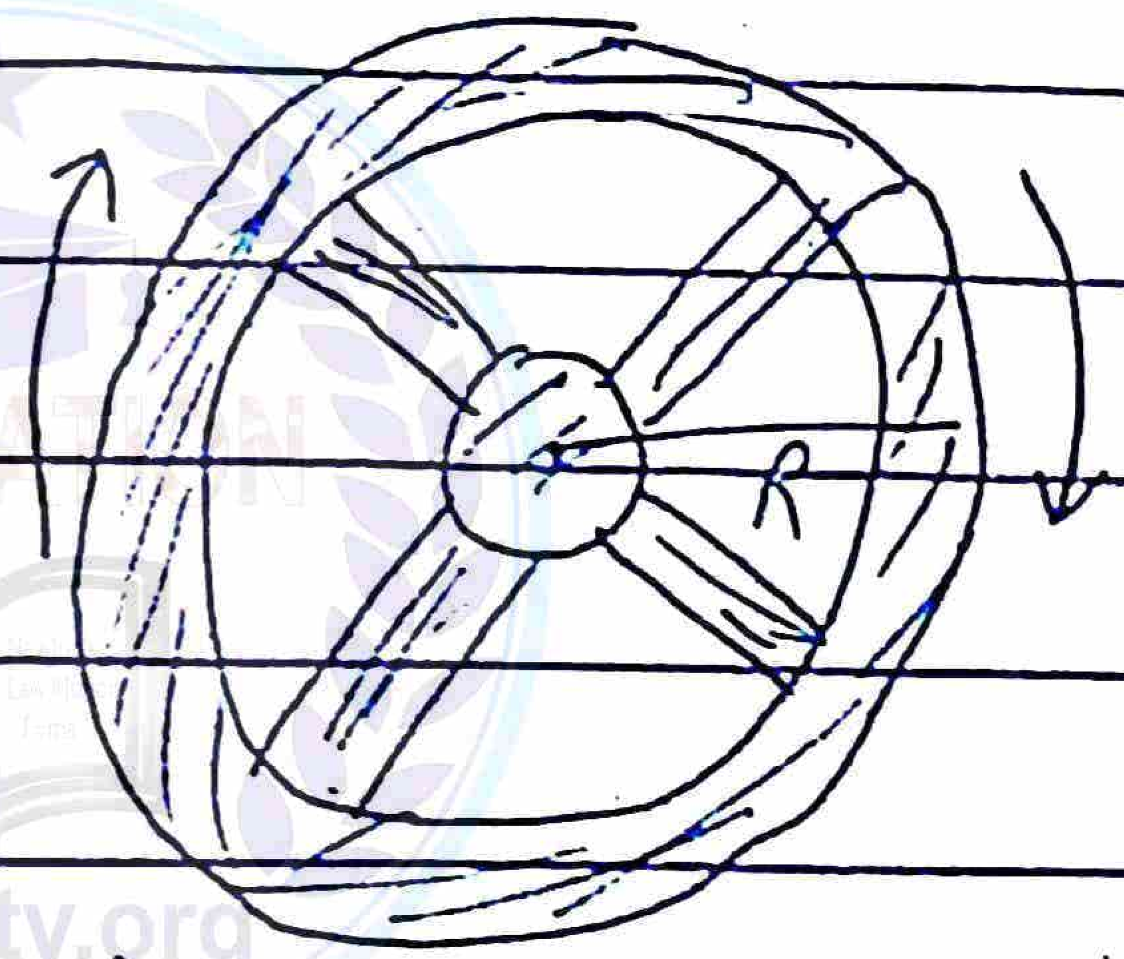
Introduction:

The scientists / astronants face difficulties in space due to unavailability of gravity.

So, the space craft is rotated with a specific frequency to create gravity called artificial gravity. Due to this artificial gravity, astronauts become able to remain on the floor of space craft.

Explanation:

Consider a space craft rotated about its own axis, the radius of space craft is R as shown in the figure.



The centripetal acceleration is given by:

$$a_c = \frac{v^2}{R}$$

but $v = R\omega$

so, $a_c = \frac{R^{\cancel{2}}\omega^2}{\cancel{R}}$

$$a_c = R\omega^2$$

Here ω is the angular frequency given as

$$\omega = 2\pi f$$

so,

$$a_c = R(2\pi f)^2$$

$$a_c = R(4\pi^2 f^2)$$

$$\frac{a_c}{R} = 4\pi^2 f^2$$

$$\frac{1}{4\pi^2}\left(\frac{a_c}{R}\right) = f^2$$

$$f^2 = \frac{1}{4\pi^2}\left(\frac{a_c}{R}\right)$$

$$\sqrt{f^2} = \sqrt{\frac{1}{4\pi^2}\left(\frac{a_c}{R}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

To create the artificial gravity, the frequency should be so as it makes the centripetal acceleration equal to gravitational acceleration.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

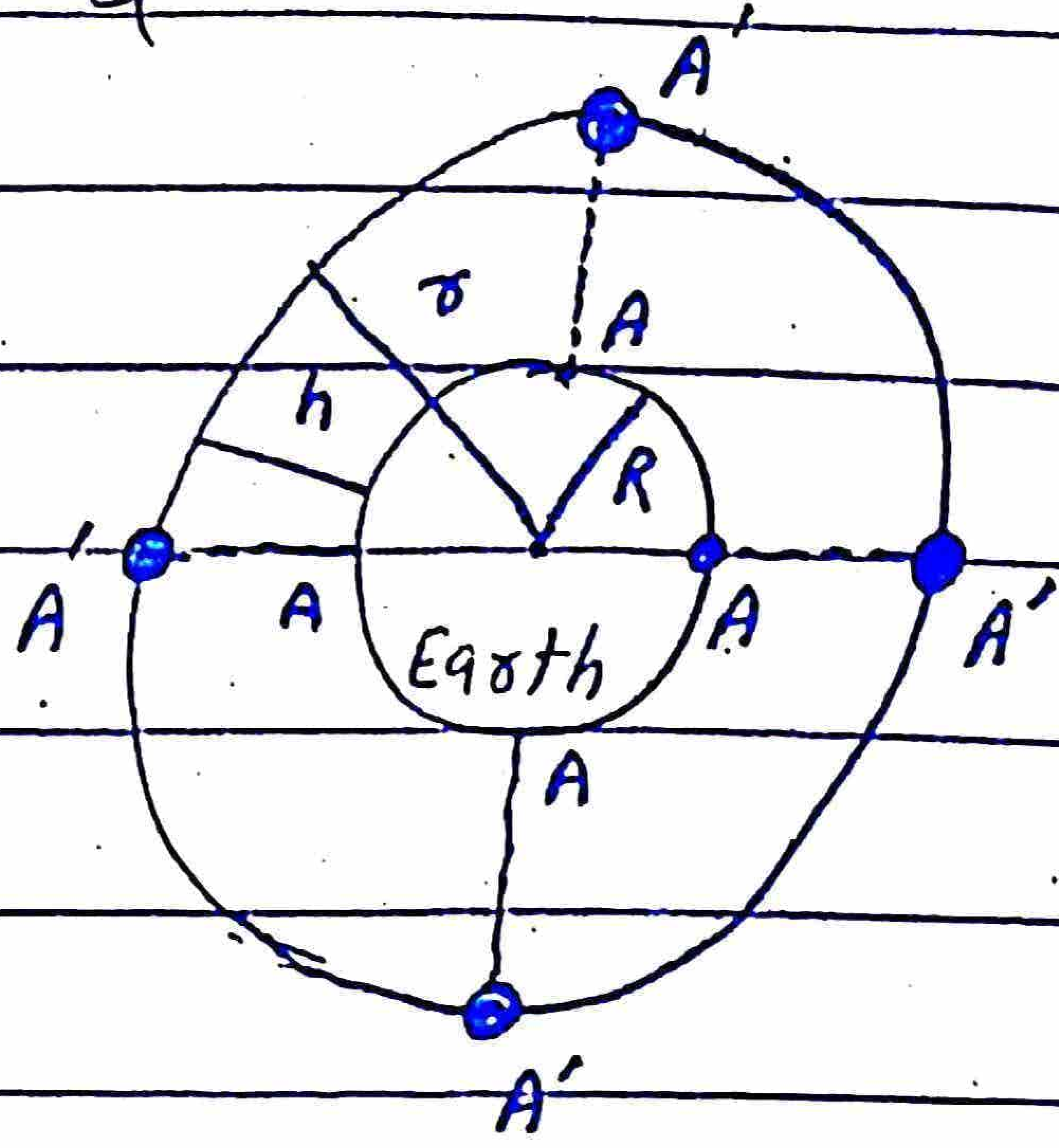
This is the required expression for frequency to create artificial gravity.

Geostationary Orbits

Defination:

The satellites that revolve around the earth with velocity same as the spining motion of earth are called geostationary satellites. These satellites thus remain at the same position as earth. The orbit of such satellites is called geostationary orbit.

Consider a geostationary satellite orbiting around the earth in geostationary orbit of radius r as shown in the figure.



The earth and satellite are orbiting such that satellite remains at the same point A'. The orbital velocity is given by:

$$V = \sqrt{\frac{GM}{r}}$$

we know that

$$V = \frac{S}{t}$$

For one complete rotation of satellite around the earth the distance will be the circumference for orbit and time will be equal to time period.

$$V = \frac{2\pi r}{T}$$

As geostationary satellites also have velocity same as rotation of earth. So,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\left(\frac{2\pi r}{T}\right)^2 = \left(\sqrt{\frac{GM}{r}}\right)^2$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$4\pi^2 r^3 = GMT^2$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

By putting the values of terms used in right side:

Gravitational constant $= G = 6.67 \times 10^{-11}\ Nm^2 kg^{-2}$
mass of earth $= M = 6 \times 10^{24}$ kg
Time period $= T = 24$ hours

$$T = 24 \times 60 \times 60\ s$$

$$T = 86400\ s$$

So,

$$r = 4.23 \times 10^4\ km$$

This is the value for radius of geostationary orbit.

Now

$$R + h = 4.23 \times 10^4\ km$$

$$h = 4.23 \times 10^4 - R$$

$$h = (4.23 \times 10^4) - 6400$$

$$h = 35900\ km$$

$$h \approx 36000\ km$$

This is the height of geostationary orbit from the surface of earth.

Communication Satellites.

Satellite communication system:

A satellite communication system can be set up by placing several geostationery satellites in orbit over different points on the surface of the Earth. One such satellite covers 120° of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites.

2) Which waves are used in satellite communication:

Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the earth.

3- Energy:

Energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites.

4- Earth stations:

There are over 200 Earth stations which transmit signals to satellites and recieve signals via satellite from other countries.

5: INTELSAT:

The largest satellite system is managed by 126 countries international Telecommunication satellite organization (INTELSAT). An INTELSAT VI satellite. It operates at microwave frequencies of 4, 6, 11 and 14 GHz and has a capacity of 30,000 two way telephone circuits plus three TV channels.

Q1: What are newton's and EINSTEN'S Views of Gravitation?

According to Newton.

According to newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly propotional to the product of their masses and inversaly propotional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{r^2}$$

According to Einsten's theory:

According to Einsten's theory, space time is curved, especially locally near massive bodies. In Einsten's theory we dont speak of the force of gravity acting on the bodies, instead we say that bodies and light rays move along geodiscs (equivalent to straight lines in plane geometry) in curved space time. Thus a body at rest or moving slowly near the great mass would follow a geodesic toward that body.

Q2: Which theory of gravitation is better either Newton's or Einstein's?

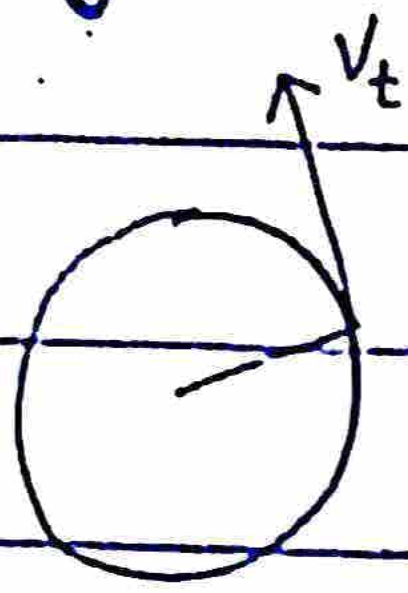
Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity. Einstein's theory also says that gravity follows an inverse square law (except in strong gravitational field), but it tells us why this should be so. Thats why Einsten's theory is better than Newton's, even though it includes Newton's theory within itself and gives the same answers as

Newton's theory everywhere except where the gravitational field is very strong.

Short Questions

S.1

Tangential velocity	Angular velocity
i- When a body is rotating in a circle, then the velocity of body along the tangent is called tangential velocity.	i- The rate of change of angular displacement is called angular velocity.
ii- It is represented as V_t	ii- It is represented by ω.
iii- Its SI unit ms^{-1}	iii- Its SI unit $rad.s^{-1}$.



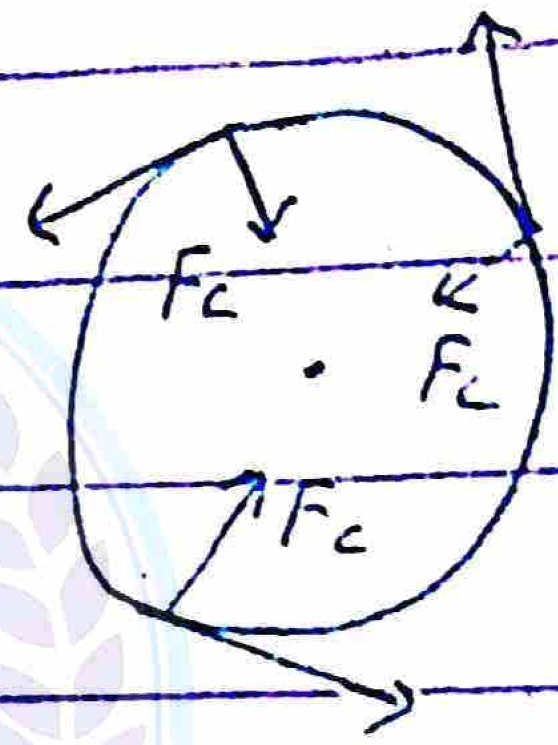
For a wheel of known radius (r), the tangential velocity and angular velocity will be related as:

$$v = r\omega$$

5.2 The force needed to bend the normally straight path of the particle into a circular path is called as centripetal force.

$$F_c = \frac{mv^2}{r}$$

When a body moves in a circle its direction continuously changes. So, centripetal force is required to keep it in the circle.



5.3

If a body is rotating in a circular path then the product of its mass and square of the radius of circle is called moment of Inertia.

It is represented by I.

$$I = mr^2.$$

It is a scalar quantity. Its SI unit is kgm^2.

Significance: Moment of inertia plays the same role in angular motion as mass plays in linear motion.

5.4: Angular momentum:

A particle is said to posses an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis. It is represented $\vec{L}$, it is a vector quantity.

$$\vec{L} = \vec{r} \times \vec{p}$$

Its unit is kgm^2s^{-1} or Js.

Law of conservation of angular momentum:

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum remains constant.

$$\vec{L}_{total} = \vec{L}_1 + \vec{L}_2 + \dots\dots = \text{Constant}$$

5.5 Show that $L = mvr$

The cross product of position vector $\vec{r}$ and linear momentum $\vec{p}$ is called angular momentum $\vec{L}$. It is represented by $\vec{L}$.

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = rp\sin\theta$$

For maximum angular momentum,

$$\theta = 90^\circ$$

$$L = rp\sin 90^\circ$$

$$= rp(1)$$

$$L = rp$$

Linear momentum is given by

$$P = mv$$

$$L = rmv$$

$$L = mvr$$

5.6

The minimum velocity necessary to put a satellite into the orbit is called critical velocity. If v is the orbital velocity and R is the radius of the Earth (6400km). Then

$$v = \sqrt{gR}$$

$$= \sqrt{9.8ms^{-2} \times 6.4 \times 10^{6} m}$$

$$= 7.9 kms^{-1}.$$

5.7 Direction of angular momentum:

The direction of angular momentum can be found by right hand rule.

"Curl the fingers of right hand from $\vec{r}$ to $\vec{p}$ then erect thumb will show the direction of $\vec{L}$"

Direction of angular velocity:

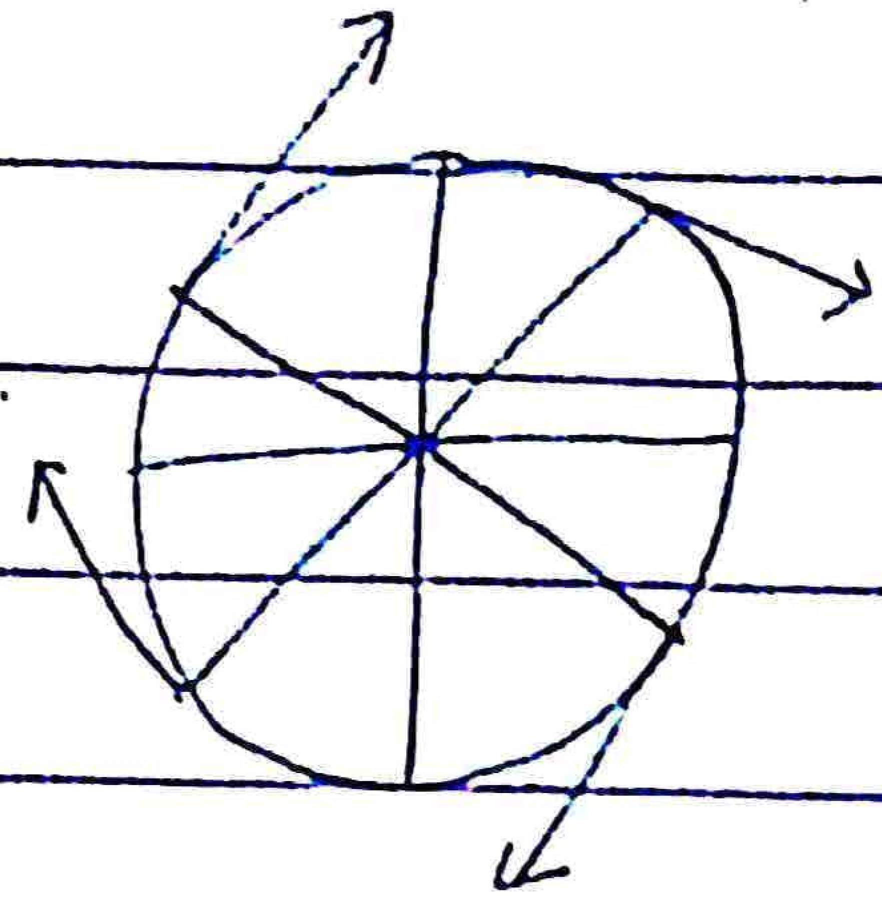
Grasp the axis of rotation in right hand with fingers curling in the direction of rotation the thumb points in the direction of angular velocity.

S.8

An object orbiting the Earth moves with an acceleration equal to gravitational acceleration (g). Therefore, it is called freely falling.

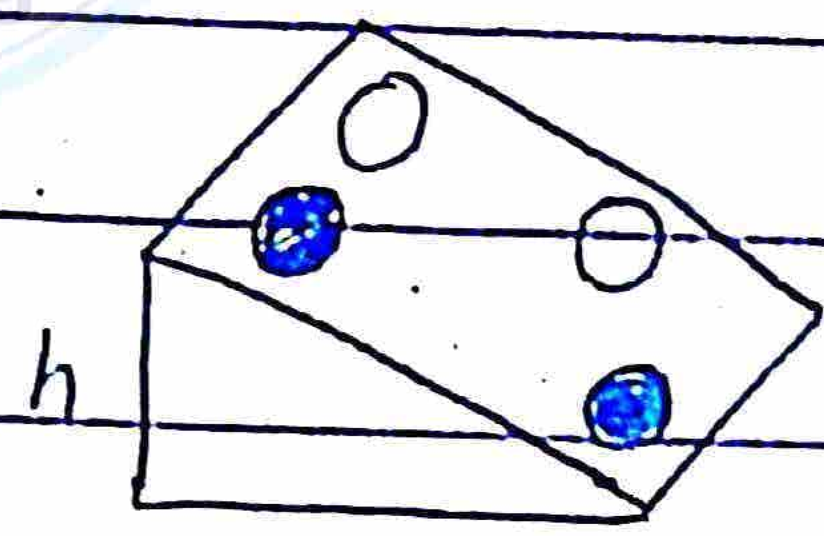
Any object moving with gravitational acceleration is said to be weightless. Because its apparent weight becomes zero due to freely falling.

S.9 The mud will remain stick to the tyre due to attractive forces between them. When the speed of tyre becomes very large and centrifugal forces becomes enough to free the mud. Then the mud will fly along the tangent with the tyre.



S.10

When a disc and a hoop start moving down from the top of an inclined plane having heiht "h" there speeds are given as:



$$v_{disc} = \sqrt{\frac{4gh}{3}} \quad , \quad v_{hoop} = \sqrt{gh}$$

$$v_{disc} > v_{hoop}$$

Therefore, disc will be moving faster on reaching the bottom.

5.11 The diver changes his body position before and after diving in the pool to conserve the angular momentum. When the diver stretches his legs and arms moment of inertia will be greater and angular velocity will be smaller. While when the diver closes his legs and arm moment of inertia will be smaller and angular velocity will be greater. Such that product of moment of inertia (I) and angular velocity (ω) remains constant.

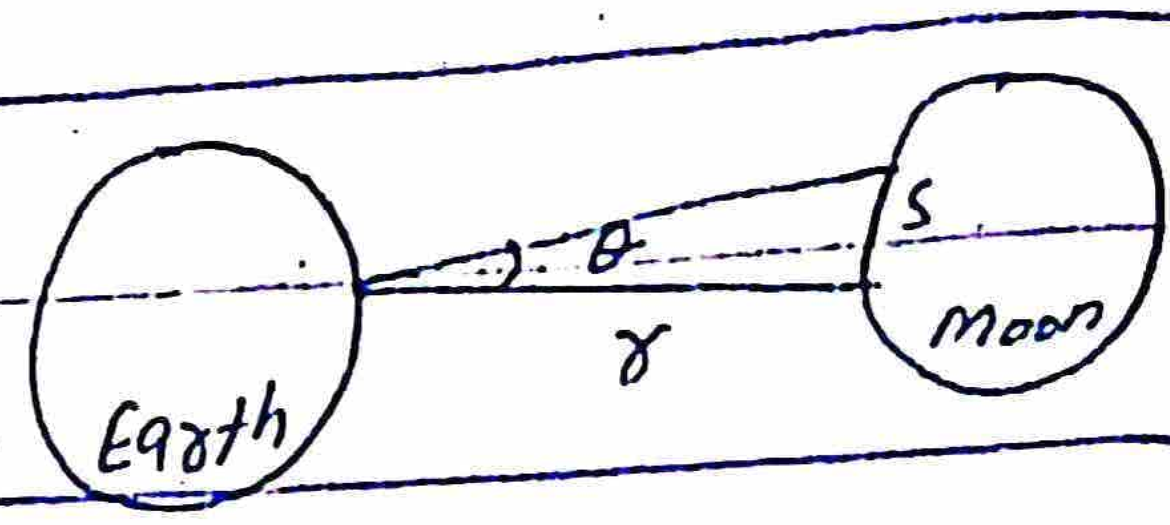
$$I_1 \omega_1 = I_2 \omega_2$$

5.13 A satellite communication system can be set up by placing several geostationary satellite in orbit over different points on the surface of the earth. One such satellite covers 120° of longitude, so that whole of the populated Earth's surface can be covered by three correctly poistioned satellites.

Numerical Problems

S.1 Data

diameter of beam
$S = 2.5\ m$



angle $= \theta = ?$
distance of moon from earth $= r = 3.8 \times 10^{8}\ m$

Solution:

$$S = r\theta$$

$$\frac{S}{r} = \theta$$

$$\theta = \frac{2.5}{3.8 \times 10^{8}}$$

$$\boxed{\theta = 6.6 \times 10^{-9}\ rad.}$$

S.2 Data

initial angular velocity $= \omega_i = 0$
final angular velocity $= \omega_f = 45\ rev.\ min^{-1}$

$$\omega_f = \frac{45\ rev.}{min}$$

$$= \frac{45(2\pi)\ rad.}{60\ s}$$

$$\omega_f = 1.5\pi\ rad.\ s^{-1}$$

time = $t = 1.6\ s$

angular acceleration = $\alpha = ?$

Solution:

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\alpha = \frac{1.5\pi - 0}{1.6}$$

$$\boxed{\alpha = 2.95\ rad\cdot s^{-2}}$$

5.3 Data

moment of inertia = $I = 0.8\ kg\,m^2$

angular velocity = $\omega = 100\ rad.\ s^{-1}$

angular momentum = $L = ?$

torque = $\tau = ?$

Solution:

$$L = I\omega$$

$$= (0.8)(100)$$

$$\boxed{L = 80\ Js}$$

$$\tau = I\alpha$$

As the angular velocity is constant. so, angular acceleration will be zero

Here $\alpha = 0$

So,

$$\tau = I(0)$$

$$\boxed{\tau = 0}$$

5.4 Data

$m = 5\ kg$, $F = 0.6\ N$

$r = 0.2\ m$

torque $= \tau = ?$

angular acceleration $= \alpha = ?$

Solution:

$$\tau = rF$$

$$= (0.2)(0.6)$$

$$\boxed{\tau = 0.12\ Nm}$$

Now

$$\tau = I\alpha$$

For cylinder $I = \frac{1}{2}mr^2$

$$\tau = \frac{1}{2}mr^2\alpha$$

$$2\tau = mr^2\alpha$$

$$\frac{2\tau}{mr^2} = \alpha$$

$$\alpha = \frac{2(0.12)}{5(0.2)^2}$$

$$\alpha = \frac{0.24}{0.2}$$

$$\boxed{\alpha = 1.2 \text{ rad.} s^{-2}}$$

5.5 Data

angular momentum $= L = ?$

mass of star $= m = 2 \times 10^{30}$ kg

radius $= r = 7 \times 10^5$ km $= 7 \times 10^5 \times 10^3$ m

$r = 7 \times 10^8$ m

time period $= T = 20$ days.

$T = 20 \times 24 \times 60 \times 60$ s

$T = 1728000$ s

$K.E = ?$

Solution:

For star $\quad I = \frac{2}{5} m r^2$

$$I = \frac{2}{5}(2 \times 10^{30})(7 \times 10^8)^2$$

$$I = 3.92 \times 10^{47} \text{ kgm}^2$$

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2\pi}{1728000} = 3.64 \times 10^{-6} \text{ rad.} s^{-1}$$

Now $L = I\omega$

$$= (3.92 \times 10^{47})(3.64 \times 10^{-6})$$

$$\boxed{L = 1.4 \times 10^{42}\ Js}$$

Now

$$K.E = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2}(3.92 \times 10^{47})(3.64 \times 10^{-6})^2$$

$$\boxed{K.E = 2.5 \times 10^{36}\ J}$$

S.6 Data

mass of car = $m = 1000\ kg$

speed = $v = 144\ kmh^{-1}$

$$v = \frac{144\ km}{h} = \frac{144000\ m}{3600\ s}$$

$$v = 40\ ms^{-1}$$

radius = $r = 100\ m$

centripetal force = $F_c = ?$

Solution:

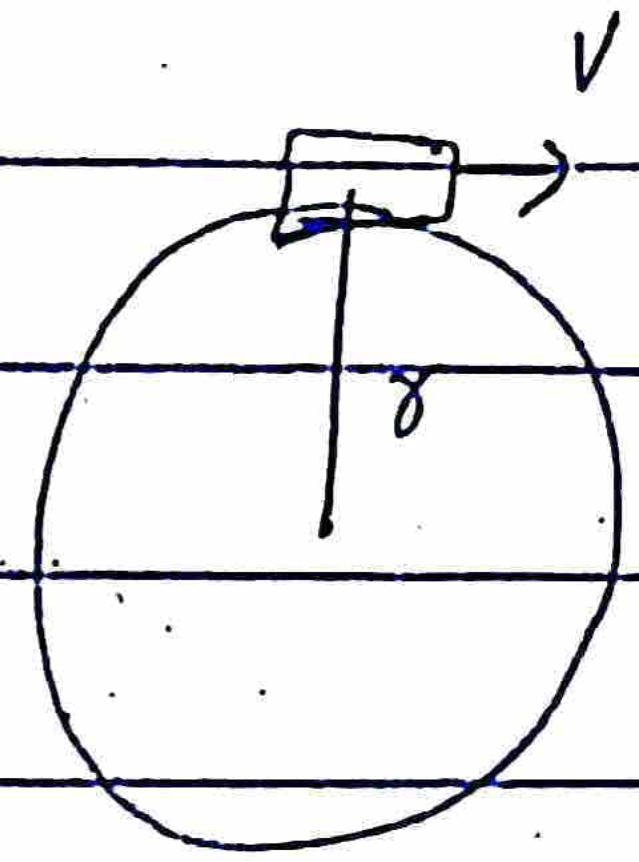
$$F_c = \frac{mv^2}{r}$$

$$= \frac{(1000)(40)^2}{100}$$

$$F_c = 16000\ N$$

$$\boxed{F_c = 1.6 \times 10^{4}\ N}$$

5.7 Data



Speed $= v = ?$

radius of loop

$r = 1\ Km = 1000\ m$

$g = 9.8\ ms^{-2}$

Solution:

If the pilot does not fall down then its weight should be equal to centripetal force.

$$F_c = W$$

$$\frac{mv^2}{r} = mg$$

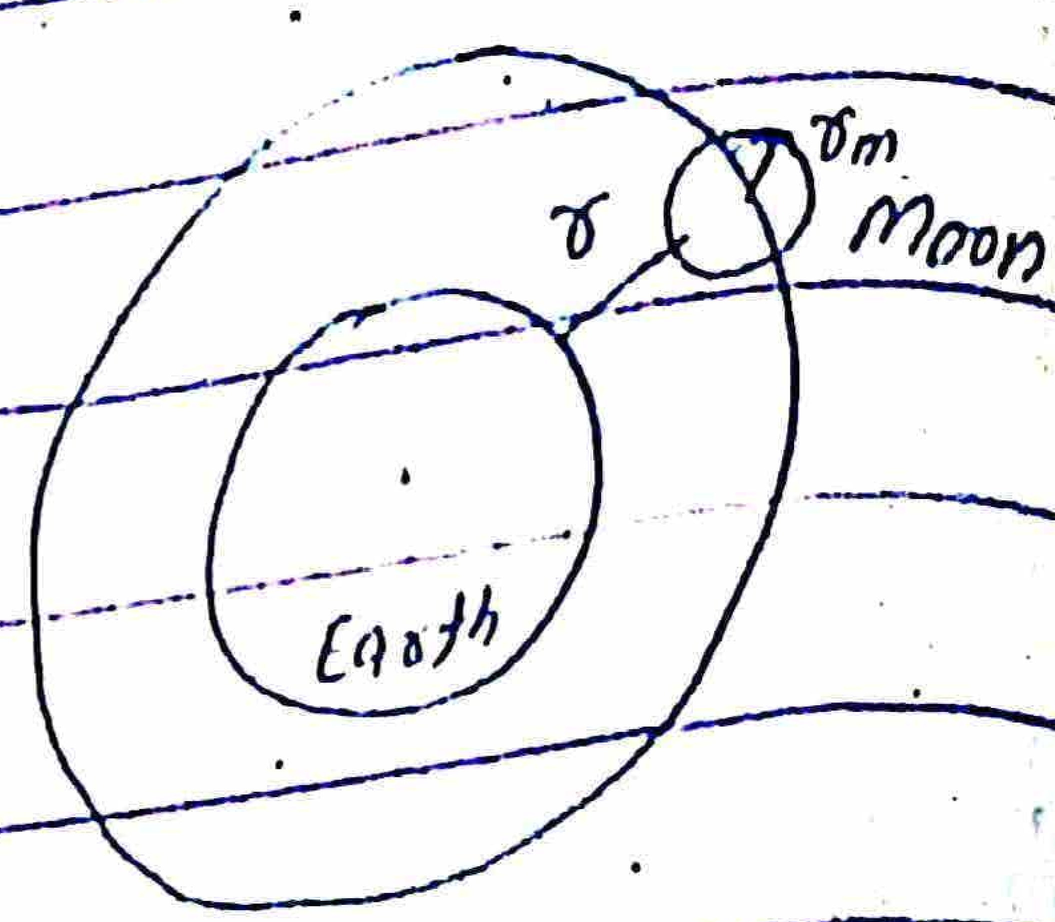
$$v^2 = gr$$

$$v = \sqrt{gr}$$

$$= \sqrt{(9.8)(1000)}$$

$$\boxed{v = 99\ ms^{-1}}$$

5.8 Data



$$\frac{L_s}{L_o} = ?$$

distance between Earth and moon $= r = 3.85 \times 10^8$ m

radius of moon $= r_m = 1.74 \times 10^6$ m

Solution:

$$\frac{L_s}{L_o} = \frac{I_s \omega}{I_o \omega}$$

$$\frac{L_s}{L_o} = \frac{I_s}{I_o}$$

$$= \frac{\frac{2}{5} \cancel{m} r_m^2}{\cancel{m} r^2}$$

$$\frac{L_s}{L_o} = \frac{2 r_m^2}{5 r^2}$$

$$= \frac{2 (1.74 \times 10^6)^2}{5 (3.85 \times 10^8)^2}$$

$$= \frac{6.05 \times 10^{12}}{7.41 \times 10^{17}}$$

$$\boxed{\frac{L_s}{L_o} = 8.2 \times 10^{-6}}$$

5.9 Data

time period $= T = 1$ day $= 24$ hours

Radius of Earth $= R$

radius of Earth after contraction $= R' = \frac{R}{2}$

time period after contraction $= T' = ?$

Solution: According to law of conservation of momentum, the angular momentum before and after contraction will be same.

$$L = L'$$

$$I\omega = I'\omega'$$

$$\left(\frac{2}{5}\cancel{M}R^2\right)\left(\frac{2\pi}{T}\right) = \left(\frac{2}{5}\cancel{M}R'^2\right)\left(\frac{2\pi}{T'}\right)$$

$$\frac{R^2}{T} = \frac{R'^2}{T'}$$

$$\frac{R^2}{T} = \frac{\left(\frac{R}{2}\right)^2}{T'}$$

$$\frac{\cancel{R^2}}{T} = \frac{\cancel{R^2}}{4T'}$$

$$4T' = T$$

$$T' = \frac{T}{4}$$

$$T' = \frac{24}{4}$$

$$\boxed{T' = 6 \text{ hours}}$$

5.10 Data

speed $= v = ?$

height of satellite $= h = 900$ Km

$h = 900000$ m

mass of Earth $= M = 6 \times 10^{24}$ kg

radius of Earth $= R = 6400$ Km

$R = 6.4 \times 10^{6}$ m

gravitational constant $= G = 6.67 \times 10^{-11}\ Nm^2 kg^{-2}$

Solution:

$$v = \sqrt{\frac{GM}{R+h}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(6\times10^{24})}{(6.4\times10^{6})+900000}}$$

$$= \sqrt{\frac{4\times10^{14}}{7300000}}$$

$$v = 7402\ ms^{-1}$$

$$v = 7.402\times10^{3}\ ms^{-1}$$

$$\boxed{v = 7.4\ Kms^{-1}}$$