Chapter NO. 9:-

Fundamentals of Trigonometry

Definitions

Trigonometry:-

Trigonometry is an important branch of mathematics. The word **Trigonometry** has been derived from three words: **Teri** (three), **Goni** (angles), **and** **Metron** (measurements). Literally it means of **measurement of triangle.**

Concept of an Angle:-

Two rays with a common starting point form an **angle**. One of rays of angle is called **initial side** and the other as **terminal side.**

Sexagesimal System:- (D°M'S")

If the initial ray $\overrightarrow{OA}$ rotates in anti-clockwise direction in such a way that it coincides with itself, the angle then formed is said to be of 360 degrees (360°).

⇒ 1 degree (1°) is divided into 60 minutes (60′) and 1 minute (1′) is divided into 60 seconds (60″).

As this system of measurement of angle owes its origin to the English and because 90, 60 are multiple of 6 and 10, so it is known as English system or Sexagesimal system.

Important Points

★ اگر DMS کو decimal میں change کرنا ہے تو 60 سے divide کرنا ہوگا۔

★ اگر decimal کو DMS میں change کرنا ہے تو 60 سے multiply کرنا ہوگا۔

★ جب degree کو radian میں change کرنا ہے تو $\frac{\pi}{180}$ سے multiply کریں گے

★ جب radian کو degree میں change کرنا ہے تو $\frac{180}{\pi}$ سے multiply کریں گے۔

★ $1^\circ = 0.01745$ radian

★ $l = r\theta$

★ $360^\circ = 2\pi$ rad.

★ $180^\circ = \pi$ rad.

Circular System:-

There is another system of angular measurement, called the circular system. It is most useful for the study of higher mathe-matics. Specially, in calculus, angles are measured in radians.

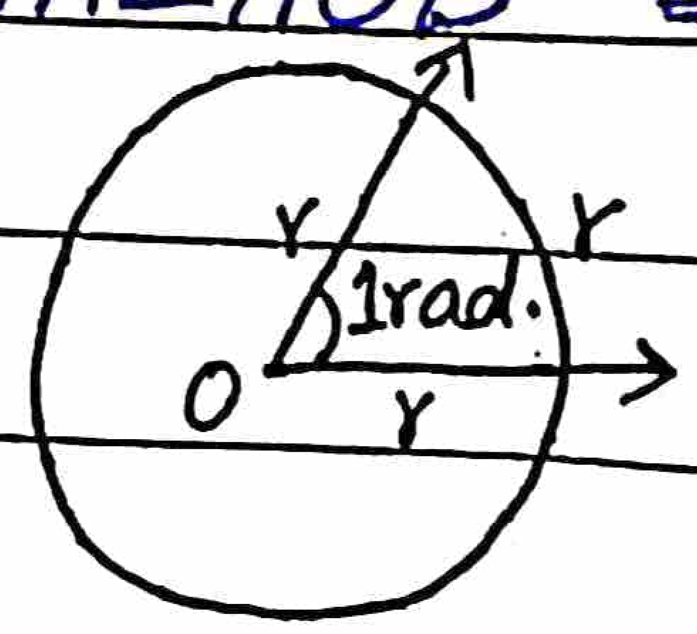
Radian:-

Radian is the measure of the angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle.

Examples:-

Consider a circle of radius r. Construct an angle $\angle AOB$ at the centre of the circle whose rays cut off an arc $\overset{\frown}{AB}$ on the circle whose length is equal to the radius r.

Thus $m\angle AOB = 1$ radian

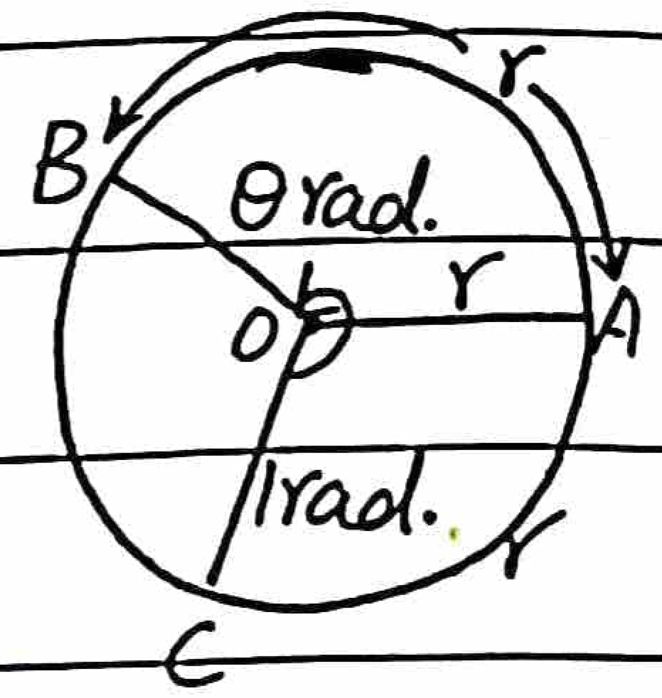


Alternative Proof:-

$$l = r\theta$$

$$\Rightarrow \frac{m\angle AOB}{m\angle AOC} = \frac{m\overset{\frown}{AB}}{m\overset{\frown}{AC}}$$

$$\Rightarrow \frac{\theta \text{ radian}}{1 \text{ radian}} = \frac{l}{r}$$



$$\theta = \frac{l}{r}$$

$$L = r\theta$$

Hence proved

Exercise 9.1

Q.NO.1:-

Express the following sexagesim measures of angles in radians:

i) $30°$

$$= \left(\cancel{30}^{1} \times \frac{\pi}{\cancel{180}_{6}}\right) \text{radian}$$

$$= \frac{\pi}{6} \text{ radian}$$

ii) $45°$

$$= \left(\cancel{45}^{\cancel{15}\,1} \times \frac{\pi}{\cancel{180}_{\cancel{60}\,4}}\right) \text{radian}$$

$= \frac{\pi}{4}$ radian

(iii) 60°

$= \left(\cancel{60} \times \frac{\pi}{{}^{3}\cancel{180}}\right)$ radian

$= \frac{\pi}{3}$ radian

(iv) 75°

$= \left(\cancel{75}^{\cancel{25}\,5} \times \frac{\pi}{{}^{\cancel{60}\,12}\cancel{180}}\right)$ radian

$= \frac{5\pi}{12}$ radian

(v) 90°

$= \left(\cancel{90}^{1} \times \frac{\pi}{{}^{2}\cancel{180}}\right)$ radian

$= \frac{\pi}{2}$ radian

(vi) 105°

$= \left(\cancel{105}^{\cancel{21}\,7} \times \frac{\pi}{{}^{\cancel{36}\,12}\cancel{180}}\right)$ radian

$= \frac{7\pi}{12}$ radian

(vii) 120°

$= \left(\cancel{120}^{\,2} \times \dfrac{\pi}{\cancel{180}_{\,3}} \right)$ radian

$= \dfrac{2\pi}{3}$ radian

(viii) 135°

$= \left(\cancel{135} \times \dfrac{\pi}{\cancel{180}} \right)$ radian

$= \dfrac{3\pi}{4}$ radian

(ix) 150°

$= \left(\cancel{150}^{\,5} \times \dfrac{\pi}{\cancel{180}_{\,6}} \right)$ radian

$= \dfrac{5\pi}{6}$ radian

(x) $10^\circ 15'$

$= 10^\circ + \dfrac{15}{60}$

$= 10^\circ.25^\circ$

$= \dfrac{\cancel{1025}^{\,41}}{\cancel{100}_{\,4}} \times \dfrac{\pi}{180}$

$= \dfrac{4\pi}{720}$

(xi) $35^\circ 20'$

$= 35^\circ + \frac{20}{60}$

$= 35^\circ + 0.33^\circ$

$= 35.33^\circ$

$= \left(35.33 \times \frac{\pi}{180}\right)$ radian

$= \left(\frac{3533}{100} \times \frac{\pi}{180}\right)$ radian

$= \left(\frac{3533\pi}{18000}\right)$ radian

$= 19978.6$ radian

(xii) $75^\circ 6' 30''$

$= 75^\circ + \left(\frac{6}{60}\right)^\circ + \left(\frac{30}{3600}\right)^\circ$

$= 75^\circ + 0.1^\circ + 0.0083^\circ$

$= 75.1083^\circ$

$= \left(75.1083 \times \frac{\pi}{180}\right)$ radian

$= \left(\frac{751083}{1000} \times \frac{\pi}{180}\right)$ radian

$= 1.31$ radian

(xiii) $120' 40''$

$= \left(\frac{120}{60}\right)^\circ + \left(\frac{40}{3600}\right)^\circ$

$= 2^\circ + 0.01^\circ$

$= 2.01^\circ$

$= \left(2.01 \times \frac{\pi}{180}\right)$ radian

$= 0.035$ radian

(xiv) $154^\circ 20''$

$= 154^\circ + \left(\frac{20}{3600}\right)^\circ$

$= (154^\circ) + \left(\frac{1}{180}\right)^\circ$

$= \left(\frac{27720 + 1}{180}\right)^\circ$

$= \left(\frac{27721}{180}\right)^\circ$

$= \left(\frac{27721}{180} \times \frac{\pi}{180}\right)$ radian

$= \frac{27721\pi}{32400}$ radian

(xv) 0°

$= \left(0 \times \frac{\pi}{180}\right)$ radian

$= 0$ radian

(xvi) $3''$

$$= \left(\frac{\cancel{3}^{1}}{\cancel{3600}_{1200}}\right)^\circ$$

$$= \left(\frac{1}{1200}\right)^\circ$$

$$= \left(\frac{1}{1200} \times \frac{\pi}{180}\right) \text{radian}$$

$$= \frac{\pi}{216000} \text{ radian.}$$

Q.NO.2:-

Convert the following radian measures of angles into the measures of sexagesimal system:

(i) $\frac{\pi}{8}$

$$= \left(\frac{\cancel{\pi}}{\cancel{8}_{\cancel{4}\,2}} \times \frac{\cancel{180}^{\cancel{90}\,45}}{\cancel{\pi}}\right) \text{radian}$$

$$= \frac{45^\circ}{2}$$

(ii) $\frac{\pi}{6}$

$$= \left(\frac{\cancel{\pi}}{\cancel{6}_{1}} \times \frac{\cancel{180}^{30}}{\cancel{\pi}}\right)^\circ$$

$$= 30^\circ$$

(iii) $\frac{\pi}{4}$

$$= \left(\frac{\cancel{\pi}}{\cancel{4}_1} \times \frac{\overset{\cancel{90}\,45}{\cancel{180}}}{\cancel{\pi}}\right)^\circ$$

$$= 45^\circ$$

(iv) $\frac{\pi}{3}$

$$= \left(\frac{\cancel{\pi}}{\cancel{3}_1} \times \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}}\right)^\circ$$

$$= 60^\circ$$

(v) $\frac{\pi}{2}$

$$= \left(\frac{\cancel{\pi}}{\cancel{2}_1} \times \frac{\overset{90}{\cancel{180}}}{\cancel{\pi}}\right)^\circ$$

$$= 90^\circ$$

(vi) $\frac{2\pi}{3}$

$$= \left(\frac{2\cancel{\pi}}{\cancel{3}_1} \times \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}}\right)^\circ$$

$$= 120^\circ$$

(vii) $\frac{3\pi}{4}$

$$\left(\frac{3\pi}{\cancel{4}_1} \times \frac{\overset{\cancel{90}\,45}{\cancel{180}}}{\pi}\right)^\circ$$

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$= 135^\circ$

vii) $\dfrac{3\pi}{4}$

$= \left(\dfrac{3\cancel{\pi}}{\cancel{4}_{1}} \times \dfrac{\cancel{180}^{\cancel{90}\,45}}{\cancel{\pi}}\right)^\circ$

$=$

viii) $\dfrac{5\pi}{6}$

$= \left(\dfrac{5\cancel{\pi}}{\cancel{6}_{1}} \times \dfrac{\cancel{180}^{30}}{\cancel{\pi}}\right)^\circ$

$= 150^\circ$

ix) $\dfrac{7\pi}{12}$

$= \left(\dfrac{7\cancel{\pi}}{\cancel{12}_{6\,2}} \times \dfrac{\cancel{180}^{90\,45}}{\cancel{\pi}}\right)^\circ$

$= \left(\dfrac{\cancel{135}^{105}}{\cancel{3}}\right)^\circ$

$= 105^\circ$

x) $\dfrac{9\pi}{5}$

$= \left(\dfrac{9\cancel{\pi}}{\cancel{5}} \times \dfrac{\cancel{180}^{36}}{\cancel{\pi}}\right)^\circ$

$= 324^\circ$

(xi) $\frac{11\pi}{27}$

$= \left(\frac{11\cancel{\pi}}{\cancel{27}_{3}} \times \frac{\overset{20}{\cancel{180}}}{\cancel{\pi}}\right)^{\circ}$

$= 73.333^{\circ}$

$= 73^{\circ}\,20'$

(xii) $\frac{13\pi}{16}$

$= \left(\frac{13\cancel{\pi}}{16} \times \frac{180}{\cancel{\pi}}\right)^{\circ}$

$= \left(\frac{2340}{16}\right)^{\circ}$

$= 146.25^{\circ}$

$= 146^{\circ}\,15'$

(xiii) $\frac{17\pi}{24}$

$= \left(\frac{17\cancel{\pi}}{24} \times \frac{180}{\cancel{\pi}}\right)^{\circ}$

$= \left(\frac{3060}{24}\right)^{\circ}$

$= 127.5^{\circ}$

$= 127^{\circ}\,30'$

(xiv) $\frac{25\pi}{36}$

$= \left(\frac{25\cancel{\pi}}{36} \times \frac{180}{\cancel{\pi}}\right)^\circ$

$= \left(\frac{4500}{36}\right)^\circ$

$= 125^\circ$

(xv) $\frac{19\pi}{32}$

$= \frac{19\cancel{\pi}}{32} \times \frac{180}{\cancel{\pi}}$

$= \left(\frac{3420}{32}\right)^\circ$

$= 106^\circ 52' 30''$

Q.NO.3:-

What is the circular measure of the angle between the hands of a watch at 4 O'clock?

Angle of 12 hour $= 2\pi$ rad.

Angle of 1 hour $= \frac{\cancel{2}^{1}\pi}{\cancel{12}_{6}}$ rad.

$= \frac{\pi}{6}$ rad.

Angle of 4 hour $= \frac{\pi}{\cancel{6}_3} \times \cancel{4}^2 \text{ rad.}$

$= \frac{2\pi}{3} \cdot \text{rad.}$

Q.NO.4:-

Find θ, when:

(i) $l = 1.5cm$, $r = 2.5cm$

$l = r\theta$

$\frac{l}{r} = \theta$

$\frac{1.5}{2.5} = \theta$

$\theta = 0.6$ radian

(ii) $l = 3.2m$, $r = 2m$

$l = r\theta$

$\frac{l}{r} = \theta$

$\frac{3.2}{2} = \theta$

1.6 radian $= \theta$

Q.NO.5:-

Find l, when:

(i) $\theta = \pi$ radian , $r = 6\,cm$

$l = r\theta$

$l = (6)(\pi)$

$l = 6\pi$

$l = 18.8cm$

(ii) $\theta = 65^\circ 20'$, $r = 18mm$

$\theta = 65^\circ 20'$

$= 65 + \frac{20}{60}$

$= 65 + 0.33$

$\theta = \left(65.33 \times \frac{\pi}{180}\right)$ radian

$\theta = 1.14$ radian

$l = r\theta$

$l = (18)(1.14)$

$l = 20.52\,mm$

Q.NO.6:-

Find r, when:

i) $l = 5cm$, $\theta = \frac{1}{2}$ radian

$l = r\theta$

$\frac{l}{\theta} = r$

$\frac{5}{1/2} = r$

$5 \times 2 = r$

$10cm = r$

(ii) $l = 56cm$, $\theta = 45°$

$\theta = 45°$

$= \left(\frac{45° \times \pi}{180}\right) rad.$

$= \frac{\pi}{4} rad.$

$l = r\theta$

$\frac{l}{\theta} = r$

$\frac{56}{\pi/4} = r$

$\frac{56 \times 4}{\pi} = r$

$\frac{224}{\pi} = r$

$71.30 cm = r$

Q.NO.7:-

What is the length of the arc intercepted on a circle of radius 14 cms by the arms of a central angle of 45°?

$l = ?$, $r = 14cm$, $\theta = 45°$

$l = r\theta$

$l = (14)\left(\frac{\pi}{4}\right)$

$l = 10.99 \text{ cm}$

Q.NO.8:-

Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35 cm.

$r = ?$, $\theta = 1 \text{ radian}$, $l = 35cm$

$l = r\theta$

$\frac{l}{\theta} = r$

$35 \text{ cm} = r$

Q.NO.9:-

A railway train is running on a circular track of radius 500 meters at the rate of 30 km per hour. Through what angle will it turn in 10 sec?

$r = 500 \text{ cm}$

$v = 30 \text{ kmh}^{-1}$

$v = \frac{30 \times 1000}{3600}$

$v = 8.33 \text{ ms}^{-1}$

$t = 10 \text{ sec}$

$l = s = vt$

$l = (8.33)(10)$

$l = 83.3 \text{ m}$

$\theta = ?$

$l = r\theta$

$\frac{l}{r} = \theta$

$\frac{83.3}{500} = \theta$

$0.16 \text{ radian} = \theta$

Q.NO.10:-

A horse is tethered to a peg by a rope of 9 meter length and it can move in a circle with the peg as centre. If the horse moves along the circumference of the circle, keep the rope right, how far will it have gone when the rope has turned through an angle of 70°?

$r = 9m$

$\theta = 70^\circ$

$\theta = \left(\dfrac{70 \times \pi}{180}\right)$ radian

$\theta = 1.22$ radian

$l = r\theta$

$l = (9)(1.22)$

$l = 10.98m$

Q.NO.11:-

The pendulum of a clock is 20 cm long and it swings through an angle of 20° each second. How far does the tip of the pendulum move in 1 s?

$r = 20cm$

$\theta = 20^\circ$

$\theta = \left(\dfrac{20 \times \pi}{180}\right)$ radian

$\theta = 0.349$

$l = r\theta$

$l = (20)(0.349)$

$l = 6.98\ cm$

Q.NO. 12:-

Assuming the average dista[nce] of the earth from the sun to be 148×10^6 km and the a[ngle] subtended by the sun at the eye of a person on the ea[rth] of measure 9.3×10^{-3} radians. Fin[d] the diameter of the sun.

$r = 148 \times 10^6 km$

$\theta = 9.3 \times 10^{-3}$ radian

$l = ?$

$l = r\theta$

$l = (9.3 \times 10^{-3})(148 \times 10^6)$

$l = 1376.4 \times 10^{6-3}$

$l = 1376.4 \times 10^3$

$l = 1376400$

Q.NO. 13:-

A circular wire of radi[us] 6 cm is cut straightened and then bent so as to lie al[ong] the circumference of a hoop of radius 24 cm. Find the measure of the angle which it subtends at the centre of the hoop.

$l = c = 2\pi r$

$= 2\pi(6)$

$l = 12\pi \text{ cm}$

$r = 24 \text{cm}$

$\theta = ?$

$l = r\theta$

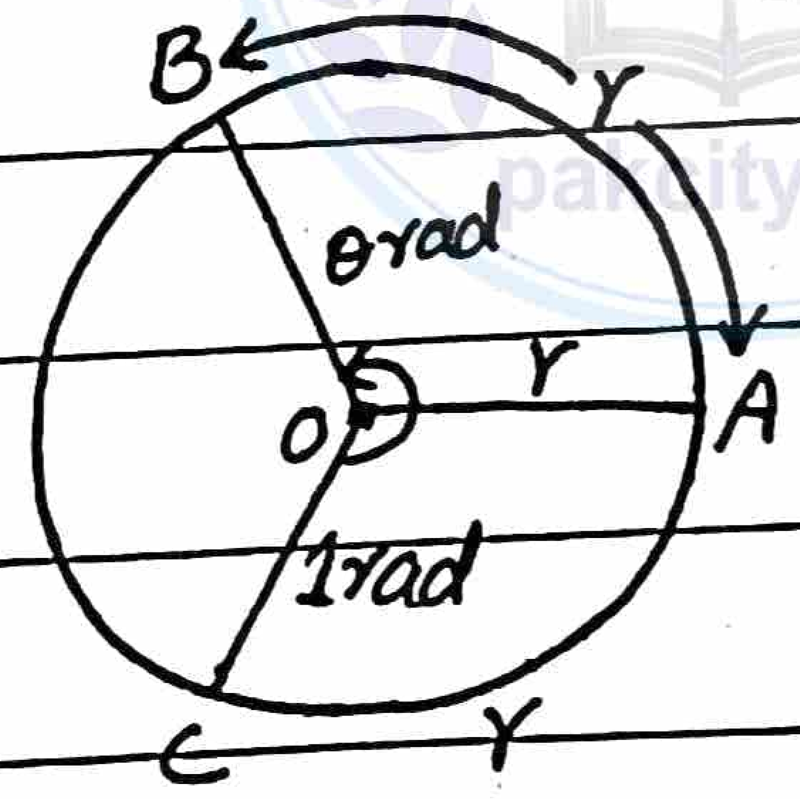
$\frac{l}{r} = \theta$

$\frac{\cancel{12}^{1}\pi}{\cancel{24}_{2}} = \theta$

$\left(\frac{\pi}{2}\right) \text{radian} = \theta$

Alternative Proof:-

$l = r\theta$



$\Rightarrow \frac{m\angle AOB}{m\angle AOC} = \frac{m\overset{\frown}{AB}}{m\overset{\frown}{AC}}$

$$\Rightarrow \frac{\theta \text{ radian}}{1 \text{ radian}} = \frac{l}{r}$$

$$\theta = \frac{l}{r}$$

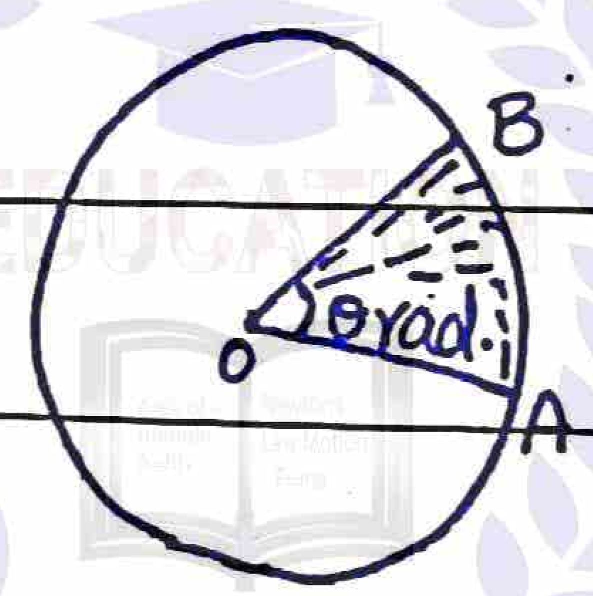
$$L = r\theta$$

Hence proved

Q.NO. 14:-

Show that the area of a sector of a circular region of radius r is $\frac{1}{2}r^2\theta$, where θ is the circular measure of the central angle of the sector.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$



Consider

Area of sector = A

Area of circle = πr^2

Angle of sector = θ radian

Angle of secircle = 2π radian

Area of sector : Area of circle = Angle of sector : Angle of circle

$$A : \pi r^2 = \theta : 2\pi$$

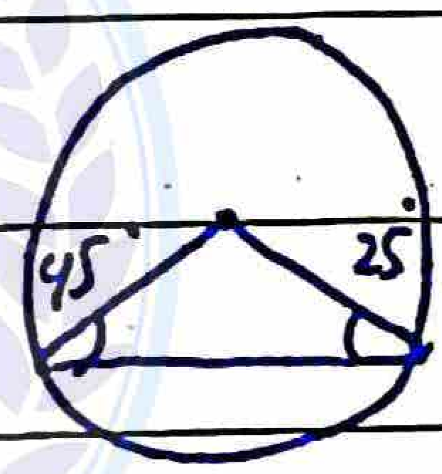
$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{\theta}{2\pi} \times \pi r^2$$

$$A = \frac{1}{2} r^2 \theta$$

Q.NO.15:-

Two cities A and B lies on the equator such that their longitudes are 45°E and 25°W respectively. Find the distance between the two cities, taking radius of the earth as 6400 kms.



$$\theta = 45° + 25°$$

$$\theta = 70°$$

$$\theta = \left(70 \times \frac{\pi}{180}\right) \text{radian}$$

$$\theta = 1.22 \text{ radian}$$

$$r = 6400 \text{ km}$$

$$l = ?$$

$$l = r\theta$$

$l = (6400)(1.22)$

$l = 9819\,km$

Q.NO.16:-

The moon subtends an angle of $0.5°$ at the eye of an observer on earth. The distance of the moon from the earth is 3.844×10^5 km approx. What is the length of the diameter of the moon?

$\theta = 0.5°$

$\theta = \left(\frac{0.5 \times \pi}{180}\right)$ radian

$\theta = 8.72 \times 10^{-3}$

$r = 3.844 \times 10^5\,km$

$l = ?$

$l = r\theta$

$l = (3.844 \times 10^5) \times (8.72 \times 10^{-3})$

$l = 33.51968 \times 10^2$

$l = 3351.968\,km$

Q.NO.17:-

The angle subtended by earth at the eye of a spaceman, landed on the moon is 1° ~~4~~54′ The radius of the earth is 6400 km. Find the approximate distance between the moon and the earth.

$$\theta = 1^\circ 54^\circ$$

$$\theta = 1^\circ + \left(\frac{54}{60}\right)$$

$$\theta = 1.9^\circ$$

$$\theta = \left(1.9 \times \frac{\pi}{180}\right) \text{radian}$$

$$\theta = 0.033 \text{ radian}$$

$$l = r\theta$$

$$\frac{l}{\theta} = r$$

$$l = 2r$$

$$l = 2(6400)$$

$$l = 12800$$

$$\frac{l}{\theta} = r$$

$$\frac{12800}{0.033} = r$$

$$387878 \text{ km} = r$$

Definitions

Angle in the Standard Position:-

An angle is said to be in **standard position** if its vertex lies at the origin of rectangular coordinate system and its initial side along the positive x-axis.

Trigonometric functions:-

The ratios depend only on the size of the angle and not on the triangle formed. Therefore, these ratios are called **trigonometric functions** of angle θ.

We observe useful relationships between these **six trigonometric functions** as follows:

$$\cos\theta = \frac{1}{\sin\theta} \; ; \quad \sec\theta = \frac{1}{\cos\theta} \; ;$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; ; \quad \cot\theta = \frac{\cos\theta}{\sin\theta} \; ;$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Exercise 9.2

Q.NO.1:-

Find the signs of the following:

(i) $\sin 160^\circ$

Positive

(ii) $\cos 190^\circ$

Negative

(iii) $\tan 115^\circ$

Negative

(iv) $\sec 245^\circ$

Negative

(v) $\cot 80^\circ$

Positive

(vi) $\operatorname{cosec} 297^\circ$

Negative

Q.NO.2:- cos/sec (+ve), otherwise all (-ve)

Fill in the blanks.

(i) $\sin(-310^\circ) = \underline{\ -\ } \sin 310^\circ$

(ii) $\cos(-75^\circ) = \underline{\ +\ } \cos 75^\circ$

(iii) $\tan(-182^\circ) = \underline{\ -\ } \tan 182^\circ$

(iv) $\cot(-137^\circ) = \underline{\ -\ } \cot 137^\circ$

(v) $\sec(-216^\circ) = \underline{\ +\ } \sec 216^\circ$

(vi) $\operatorname{cosec}(-15^\circ) = \underline{\ -\ } \operatorname{cosec} 15^\circ$

Q.NO.3:-

In which quadrant are the terminal arms of the angle lie when

(i) $\sin\theta < 0$ and $\cos\theta > 0$

(-ve) (+ve)

IV

(ii) $\cot\theta > 0$ and $\csc\theta > 0$

(+ve) (+ve)

I

(iii) $\tan\theta < 0$ and $\cos\theta > 0$

(-ve) (+ve)

IV

(iv) $\sec\theta < 0$ and $\sin\theta < 0$

(-ve) (-ve)

III

(v) $\cot\theta > 0$ and $\sin\theta < 0$

(+ve) (-ve)

III

(vi) $\cos\theta < 0$ and $\tan\theta < 0$

(-ve) (-ve)

II

Q. NO. 4:-

Find the values of the remaining trigonometric functions:

(i) $\sin\theta = \frac{12}{13}$ and the terminal arm of the angle is in quad. I.

$$\sin\theta = \frac{12}{13} = \frac{perp}{hyp}$$

By using pythagoras theorem

$$(hyp)^2 = (Base)^2 + (prep)^2$$

$$(13)^2 = (Base)^2 + (12)^2$$

$$169 = (Base)^2 + 144$$

$$169 - 144 = (Base)^2$$

$$25 = (Base)^2$$

$$\sqrt{25} = \sqrt{(Base)^2}$$

$$5 = Base$$

$$\sin\theta = \frac{12}{13}$$

$$\cos\theta = \frac{Base}{hyp} = \frac{5}{13}$$

$$\tan\theta = \frac{Prep}{Base} = \frac{12}{5}$$

$$\operatorname{cosec}\theta = \frac{13}{12}$$

$\sec\theta = \frac{13}{5}$

$\cot\theta = \frac{5}{12}$

(ii) $\cos\theta = \frac{9}{41}$ and the terminal arm of the angle is in quad. IV.

$\cos\theta = \frac{9}{41} = \frac{Base}{hyp}$

By using pythagoras theorem

$(hyp)^2 = (Base)^2 + (prep)^2$

$(41)^2 = (9)^2 + (prep)^2$

$1681 = 81 + (prep)^2$

$1681 - 81 = (prep)^2$

$\sqrt{1600} = \sqrt{(prep)^2}$

$40 = prep$

$\cos\theta = \frac{9}{41}$

$\sin\theta = -\frac{40}{41}$

$\tan\theta = -\frac{40}{9}$

$\operatorname{cosec}\theta = \frac{-41}{40}$

$\sec\theta = \frac{41}{9}$

$\cot\theta = \frac{-9}{40}$

Q.NO.3.

(iii) $\cos\theta = \frac{-\sqrt{3}}{2}$ and the terminal arm of the angle is in quad. III.

$\cos\theta = \frac{Base}{hyp} = -\frac{\sqrt{3}}{2}$

By pythagoras theorem

$(hyp)^2 = (Base)^2 + (prep)^2$

$(2)^2 = (\sqrt{3})^2 + (prep)^2$

$4 = 3 + (prep)^2$

$4 - 3 = (prep)^2$

$\sqrt{1} = \sqrt{(prep)^2}$

$1 = prep$

$\sin\theta = \frac{-1}{2}$

$\cos\theta = \frac{-\sqrt{3}}{2}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\csc\theta = -2$

$\sec\theta = \frac{-2}{\sqrt{3}}$

$\cot\theta = \sqrt{3}$

(iv) $\tan\theta = \frac{-1}{3}$ and the terminal arm of the angle is in quad. II

$\tan\theta = \frac{Perp}{Base} = \frac{-1}{3}$

By pythagoras theorem

$(Hyp)^2 = (Base)^2 + (prep)^2$

$(Hyp)^2 = (3)^2 + (1)^2$

$(Hyp)^2 = 9 + 1$

$\sqrt{(Hyp)^2} = \sqrt{10}$

$Hyp = \sqrt{10}$

$\sin\theta = \frac{1}{\sqrt{10}}$

$\cos\theta = \frac{-3}{\sqrt{10}}$

$\tan\theta = \frac{-1}{3}$

$\csc\theta = \sqrt{10}$

$\sec\theta = \frac{-\sqrt{10}}{3}$

$\cot\theta = -3$

(v) $\sin\theta = \frac{-1}{\sqrt{2}}$ and the terminal arm of the angle is not in quad. III.

$$\sin\theta = \frac{prep}{hyp} = \frac{-1}{\sqrt{2}}$$

By pythagoras theorem

$$(hyp)^2 = (Base)^2 + (prep)^2$$

$$(\sqrt{2})^2 = (Base)^2 + (1)^2$$

$$2 = (Base)^2 + 1$$

$$2 - 1 = (Base)^2$$

$$\sqrt{1} = \sqrt{(Base)^2}$$

$$1 = Base$$

$$\sin\theta = \frac{-1}{\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\tan\theta = -1$$

$$\operatorname{cosec}\theta = -\sqrt{2}$$

$$\sec\theta = \sqrt{2}$$

$$\cot\theta = -1$$

Q.NO:5:-

If $\cot\theta = \frac{15}{8}$ and the

terminal arm of the angle is not is quad. I, find the value of cos θ and cosec θ.

$$\cot\theta = \frac{Base}{prep} = \frac{15}{8}$$

By pythagoras theorem

$$(hyp)^2 = (Base)^2 + (prep)^2$$

$$(hyp)^2 = (15)^2 + (8)^2$$

$$(hyp)^2 = 225 + 64$$

$$\sqrt{(hyp)^2} = \sqrt{289}$$

$$hyp = 17$$

$$\cos\theta = \frac{Base}{hyp} = \frac{-15}{17}$$

$$\operatorname{cosec}\theta = \frac{hyp}{prep} = \frac{-17}{8}$$

Q.NO.6:-

If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ and $m>0$ $\left(0<\theta<\frac{\pi}{2}\right)$, find the values of the remaining trigonometric ratios.

$$\operatorname{cosec}\theta = \frac{hyp}{prep}$$

$$= \frac{m^2+1}{2m}$$

By pythagoras theorem

$$(hyp)^2 = (Base)^2 + (prep)^2$$

$$(m+1)^2 = (Base)^2 + (2m)^2$$

$$m^4 + 2m^2 + 1 = (Base)^2 + 4m^2$$

$$m^4 + 2m^2 + 1 - 4m^2 = (Base)^2$$

$$m^4 - 2m^2 + 1 = (Base)^2$$

$$\sqrt{(m^2-1)^2} = \sqrt{(Base)^2}$$

$$m^2 - 1 = Base$$

$$\sin\theta = \frac{2m}{m^2+1}$$

$$\cos\theta = \frac{m^2-1}{m^2+1}$$

$$\tan\theta = \frac{2m}{m^2-1}$$

$$\mathrm{cosec}\,\theta = \frac{m^2+1}{2m}$$

$$\sec\theta = \frac{m^2+1}{m^2-1}$$

$$\cot\theta = \frac{m^2-1}{2m}$$

Q.NO.8

If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quad., find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$.

$$\cot\theta = \frac{5}{2} = \frac{Base}{perp}$$

By pythagoras theorem

$$(H)^2 = (B)^2 + (P)^2$$

$$(hyp)^2 = (5)^2 + (2)^2$$

$$(hyp)^2 = 25 + 4$$

$$\sqrt{(hyp)^2} = \sqrt{29}$$

$$\sin\theta = \frac{prep}{hyp}$$

$$= \frac{2}{\sqrt{29}}$$

$$\cos\theta = \frac{Base}{hyp} = \frac{5}{\sqrt{29}}$$

Now,

$$= \frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{3 \cdot \frac{2}{\sqrt{29}} + 4\,\frac{5}{\sqrt{29}}}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}}$$

$$= \frac{6/\sqrt{29} + 20/\sqrt{29}}{5/\sqrt{29} - 2/\sqrt{29}}$$

$$= \frac{26/\sqrt{29}}{3/\sqrt{29}}$$

$$= \frac{26}{3}$$

Q.NO.7:-

If $\tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in III quad., find the values of $\frac{\cos^2\theta - \sec^2\theta}{\cos^2\theta + \sec^2\theta}$

$$\tan\theta = \frac{1}{\sqrt{7}}$$

By pythagoras theorem

$$(hyp)^2 = (Base)^2 + (perp)^2$$

$$(hyp)^2 = (\sqrt{7})^2 + (1)^2$$

$$(hyp)^2 = 7+1$$

$$\sqrt{(hyp)^2} = \sqrt{8}$$

$$hyp = \sqrt{8}$$

$$\sec\theta = \frac{hyp}{base}$$

$$\sec\theta = \frac{\sqrt{8}}{\sqrt{7}}$$

$$\operatorname{cosec}\theta = \frac{hyp}{perp}$$

$$\operatorname{cosec}\theta = \frac{\sqrt{8}}{1}$$

$$= \sqrt{8}$$

$$= \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

$$= \frac{(\sqrt{8})^2 - \left(\frac{\sqrt{8}}{\sqrt{7}}\right)^2}{(\sqrt{8})^2 - \left(\frac{\sqrt{8}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{\frac{56-8}{\cancel{7}}}{\frac{56+8}{\cancel{7}}}$$

$$= \frac{\cancel{48}\,3}{\cancel{64}\,4}$$

$$= \frac{3}{4}$$

Tables:-

	$\pi/6$	$\pi/4$	$\pi/3$
θ	30°	45°	60°
sin	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$
tan	$1/\sqrt{3}$	1	$\sqrt{3}$
cosec	2	$\sqrt{2}$	$2/\sqrt{3}$
sec	$2/\sqrt{3}$	$\sqrt{2}$	2
cot	$\sqrt{3}$	1	$1/\sqrt{3}$

		$\pi/2$	π	$3\pi/2$	2π
θ	0°	90°	180°	270°	360°
sin	0	1	0	-1	0
cos	1	0	-1	0	1
tan	0	∞	0	∞	0
cosec	∞	1	∞	-1	∞
sec	1	∞	-1	∞	1
cot	∞	0	∞	0	∞

* Exercise 9.3

Q.NO.1:-

Verify the following:

i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

L.H.S $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$

$\left(\frac{\sqrt{3}}{4}\right)^2 - \frac{1}{4} = \frac{1}{2}$

$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\frac{3-1}{4} = \frac{1}{2}$

$\frac{\cancel{2}1}{\cancel{4}2} = \frac{1}{2}$

$$\frac{1}{2} = \frac{1}{2}$$

Hence proved

(ii) $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$

$\sin^2 30^\circ + \sin^2 60^\circ + \tan^2 45^\circ = 2$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 = 2$$

$$\frac{1}{4} + \frac{3}{4} + 1 = 2$$

$$\frac{1+3+4}{4} = 2$$

$$\frac{\cancel{8}^{2}}{\cancel{4}} = 2$$

$$2 = 2$$

Hence proved

(iii) $2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

$$2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(\sqrt{2}) = \frac{3}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{3}{\sqrt{2}}$$

$$\frac{4 + (\sqrt{2})^2}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\frac{4+2}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\frac{\cancel{6}^{3}}{\cancel{2}\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

Hence proved

(iv) $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4$

$\sin^2 30^\circ : \sin^2 45 : \sin^2 60 : \sin^2 90^\circ = 1:2:3:4$

$\left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^{\cancel{2}} : \left(\frac{\sqrt{3}}{2}\right)^{\cancel{2}} : (1)^2 = 1:2:3:4$

$\frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1 = 1:2:3:4$

Multiply by 4

$\frac{1}{\cancel{4}} \times \cancel{4} : \frac{1}{\cancel{2}} \times \cancel{4}^{2} : \frac{3}{\cancel{4}} \times \cancel{4} : 4 = 1:2:3:4$

$1:2:3:4 = 1:2:3:4$

Hence proved

Q.NO.2:-

Evaluate the following:

(i) $\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

$$= \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \cdot \tan 30^\circ}$$

$$= \frac{\sqrt{3} - 1/\sqrt{3}}{1 + \sqrt{3} \cdot 1/\sqrt{3}}$$

$$= \frac{\frac{(\sqrt{3})^2 - 1}{\sqrt{3}}}{1 + \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}}$$

$$= \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$= \frac{\frac{\cancel{2}}{\sqrt{3}}}{\cancel{2}}$$

$$= \frac{1}{\sqrt{3}}$$

(ii) $$\frac{1 - \tan^2 \frac{\pi}{3}}{1 + \tan^2 \frac{\pi}{3}}$$

$$= \frac{1 - \tan^2 60^\circ}{1 + \tan^2 60^\circ}$$

$$= \frac{1 - (\cancel{\sqrt{3}})^{\cancel{2}}}{1 + (\cancel{\sqrt{3}})^{\cancel{2}}}$$

$$= \frac{1-3}{1+3}$$

$$= \frac{-\cancel{2}\,1}{\cancel{4}\,2}$$

$$= \frac{-1}{2}$$

Q.NO.4:-

Find x, if $\tan^2 45$

$$\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$$

$$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3})$$

$$1 - \frac{1}{4} = x\left[\frac{\sqrt{3}}{(\sqrt{2})^2}\right]$$

$$\frac{4-1}{4} = x\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{3}{4} = x\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{3}{\cancel{4}_2} \times \frac{\cancel{2}}{\sqrt{3}} = x$$

$$\frac{3}{2\sqrt{3}} = x$$

$$\frac{\sqrt{3}\cdot\cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}} = x$$

$$\frac{\sqrt{3}}{2} = x$$

Q.NO.3:-

Verify the following when

$\theta = 30^\circ, 45^\circ$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2(30^\circ) = 2\sin(30^\circ)\cos(30^\circ)$

$\sin 60^\circ = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

For $\theta = 45^\circ$

$\sin 2(45^\circ) = 2\sin 45^\circ \cos 45^\circ$

$\sin 90^\circ = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$1 = 2\left[\frac{1}{(\sqrt{2})^{\cancel{2}}}\right]$

$1 = \cancel{2}\left(\frac{1}{\cancel{2}}\right)$

$1 = 1$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

For $\theta = 30^\circ$

$\cos 2(30^\circ) = \cos^2(30^\circ) - \sin^2(30^\circ)$

$\cancel{\frac{1}{2}} =$

$\cos 60^\circ = \cos^2(30^\circ) - \sin^2(30^\circ)$

$\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^{\cancel{2}} - \left(\frac{1}{2}\right)^2$

$$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$$

$$\frac{1}{2} = \frac{3-1}{4}$$

$$\frac{1}{2} = \frac{1}{2}$$

For $\theta = 45^\circ$

$$\cos 2(45^\circ) = \cos^2(45^\circ) - \sin^2(45^\circ)$$

$$\cos 90^\circ = \cos^2 45^\circ - \sin^2 45^\circ$$

$$0 = \left(\frac{1}{\sqrt{2}}\right)^{2} - \left(\frac{1}{\sqrt{2}}\right)^{2}$$

$$0 = \frac{1}{2} - \frac{1}{2}$$

$$0 = 0$$

(iii) $\cos 2\theta = 2\cos^2\theta - 1$

For $\theta = 30^\circ$

$$\cos 2(30^\circ) = 2\cos^2(30^\circ) - 1$$

$$\cos 60^\circ = 2\cos^2 30^\circ - 1$$

$$\frac{1}{2} = 2\left(\frac{\sqrt{3}}{2}\right)^{2} - 1$$

$$\frac{1}{2} = \cancel{2}^{1}\left(\frac{3}{\cancel{4}^{2}}\right) - 1$$

$$\frac{1}{2} = \frac{3}{2} - 1$$

$$\frac{1}{2} = \frac{3-2}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

For $\theta = 45^\circ$

$$\cos 2(45^\circ) = 2\cos^2(45^\circ) - 1$$

$$\cos 90^\circ = 2\cos^2 45^\circ - 1$$

$$0 = \cancel{2}\left(\frac{1}{\sqrt{\cancel{2}}}\right)^{\cancel{2}} - 1$$

$$0 = 1 - 1$$

$$0 = 0$$

(iv) $\cos 2\theta = 1 - 2\sin^2\theta$

For $\theta = 30^\circ$

$$\cos 2(30^\circ) = 1 - 2\sin^2(30^\circ)$$

$$\cos 60^\circ = 1 - 2\sin^2 30^\circ$$

$$\frac{1}{2} = 1 - 2\left(\frac{1}{2}\right)^2$$

$$\frac{1}{2} = 1 - \cancel{2}\left(\frac{1}{\cancel{4}}\right)$$

$$\frac{1}{2} = 1 - \frac{1}{2}$$

$$\frac{1}{2} = \frac{2-1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

For $\theta = 45°$

$$\cos 2(45°) = 1 - 2\sin^2(45°)$$

$$\cos 90° = 1 - 2\sin^2 45°$$

$$0 = 1 - \cancel{2}\left(\frac{1}{\sqrt{\cancel{2}}}\right)^{\cancel{2}}$$

$$0 = 1 - 1$$

$$0 = 0$$

(v) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

For $\theta = 30°$

$$\tan 2(30°) = \frac{2\tan(30°)}{1 - \tan^2(30°)}$$

$$\tan 60° = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\sqrt{3} = \frac{2/\sqrt{3}}{1 - \frac{1}{3}}$$

$$\sqrt{3} = \frac{2/\sqrt{3}}{\frac{3-1}{3}}$$

$$\sqrt{3} = \frac{2/\sqrt{3}}{2/3}$$

$$\sqrt{3} = \frac{\cancel{2}}{\sqrt{3}} \times \frac{3}{\cancel{2}}$$

$$\sqrt{3} = \frac{\sqrt{3}}{\sqrt{3}}$$

$$\sqrt{3} = \frac{\sqrt{3}\cdot\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$$

$$\sqrt{3} = \sqrt{3}$$

For $\theta = 45^\circ$

$$\tan 2(45^\circ) = \frac{2\tan 45^\circ}{1-\tan^2 45^\circ}$$

$$\tan 90^\circ = \frac{2\tan 45^\circ}{1-\tan^2 45^\circ}$$

$$\infty = \frac{2(1)}{1-(1)^2}$$

$$\infty = \frac{2}{1-1}$$

$$\infty = \frac{2}{0}$$

$$\infty = \infty$$

Q.NO.68-

Find the values of trigono[metric] functions of the fallowing an[gles]

(i) 360°

Co- terminal : $\theta \pm 360^\circ k$

$= 390^\circ$

$= 30^\circ + 360^\circ(1)$

$= 30^\circ$

$\sin 30^\circ = \frac{1}{2}$, $\operatorname{cosec} 30^\circ = 2$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\cot 30^\circ = \sqrt{3}$

i) -330°

$\therefore \theta + 360^\circ K$

$= 30^\circ + 360^\circ(1)$

$= 30^\circ$

$\sin 30^\circ = \frac{1}{2}$, $\operatorname{cosec} 30^\circ = 2$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\cot 30^\circ = \sqrt{3}$

iii) 765°

$= 45^\circ + 360^\circ(2)$

$= 45^\circ$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\operatorname{cosec} 45^\circ = \sqrt{2}$

$\operatorname{cosec} 45^\circ = \frac{1}{\sqrt{2}}$, $\sec 45^\circ = \sqrt{2}$

$\operatorname{Tan} 45^\circ = 1$, $\cot 45^\circ = 1$

(iv) -675°

$= 45^\circ + 360^\circ(-2)$

$= 45^\circ$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\operatorname{cosec} 45^\circ = \sqrt{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\sec 45^\circ = \sqrt{2}$

$\tan 45^\circ = 1$, $\cot 45^\circ = 1$

(v) $\frac{-17}{3}\pi$

$= -1020^\circ$

$= 60^\circ + 360^\circ(-3)$

$= 60^\circ$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$

$\cos 60^\circ = \frac{1}{2}$, $\sec 60^\circ = 2$

$\tan 60^\circ = \sqrt{3}$, $\cot 60^\circ = \frac{1}{\sqrt{3}}$

(vi) $\frac{13\pi}{3}$

$= 780^\circ$

$= 60^\circ + 360^\circ(2)$

$= 60^\circ$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$

$\cos 60^\circ = \frac{1}{2}$; $\sec 60^\circ = 2$

$\tan 60^\circ = \sqrt{3}$; $\cot 60^\circ = \frac{1}{\sqrt{3}}$

(vii) $\frac{25}{6}\pi$

$= 750^\circ$

$= 30^\circ + 360^\circ(2)$

$= 30^\circ$

$\sin 30^\circ = \frac{1}{2}$; $\operatorname{cosec} 30^\circ = 2$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$; $\cot 30^\circ = \sqrt{3}$

(viii) $\frac{-71}{6}\pi$

$= -2130^\circ$

$= 30^\circ + 360^\circ(-6)$

$= 30^\circ$

$\sin 30^\circ = \frac{1}{2}$; $\operatorname{cosec} 30^\circ = 2$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$; $\cot 30^\circ = \sqrt{3}$

(ix) -1035°

$= 45^\circ + 360^\circ(-3)$

$= 45^\circ$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$; $\operatorname{cosec} 45^\circ = \sqrt{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$; $\sec 45^\circ = \sqrt{2}$

$\tan 45^\circ = 1$; $\cot 45^\circ = 1$

Q.NO:5:-

Find the values of the trigonometric functions of the following quadrantal angles:

(i) $-\pi$

$= -180^\circ$

$= -180^\circ + 360^\circ(-1)$

$= 180^\circ$

$\sin 180^\circ = 0$; $\operatorname{cosec} 180^\circ = \infty$

$\cos 180^\circ = -1$; $\sec 180^\circ = -1$

$\tan 180^\circ = 0$; $\tan 180^\circ = \infty$

ii) -3π

$= -3(180^\circ)$

$= -540$

$= 180^\circ + 360^\circ(-2)$

$= 180^\circ$

$\sin 180^\circ = 0$; $\operatorname{cosec} 180^\circ = \infty$

$\cos 180^\circ = -1$; $\sec 180^\circ = -1$

$\tan 180^\circ = 0$; $\cot 180^\circ = \infty$

iii) $\frac{5}{2}\pi$

$= 450^\circ$

$= 90^\circ + 360^\circ(1)$

$= 90^\circ$

$\sin 90^\circ = 1$; $\operatorname{cosec} 90^\circ = 1$

$\cos 90^\circ = 0$; $\sec 90^\circ = \infty$

$\tan 90^\circ = \infty$; $\cot 90^\circ = 0$

(iv) $-\frac{9}{2}\pi$

$= -810^\circ$

$= 270^\circ + 360^\circ(-3)$

$= 270^\circ$

$\sin 270^\circ = -1$, $\operatorname{cosec} 270^\circ = -1$

$\cos 270^\circ = 0$; $\sec 270^\circ = \infty$

$\tan 270^\circ = \infty$; $\cot 270^\circ = 0$

(v) -15π

$= -2700$

$= 180^\circ + 360^\circ(-8)$

$= 180^\circ$

$\sin 180^\circ = 0$; $\operatorname{cosec} 180^\circ = \infty$

$\cos 180^\circ = -1$; $\sec 180^\circ = -1$

$\tan 180^\circ = 0$; $\cot 180^\circ = \infty$

(vi) 1530°

$= 90^\circ + 360^\circ(4)$

$= 90^\circ$

$\sin 90^\circ = 1$; $\operatorname{cosec} 90^\circ = 1$

$\cos 90^\circ = 0$; $\sec 90^\circ = \infty$

$\tan 90^\circ = \infty$; $\cot 90^\circ = 0$

(vii) -2430°

$= 90^\circ + 360^\circ(-7)$

$= 90^\circ$

$\sin 90^\circ = 1$; $\operatorname{cosec} 90^\circ = 1$

$\cos 90^\circ = 0$; $\sec 90^\circ = \infty$

$\tan 90^\circ = \infty$; $\cot 90^\circ = 0$

(viii) $\frac{235}{2}\pi$

$= 21150^\circ$

$= 270^\circ + 360^\circ(58)$

$= 270^\circ$

$\sin 270^\circ = -1$; $\text{cosec } 270^\circ = -1$

$\cos 270^\circ = 0$; $\sec 270^\circ = \infty$

$\tan 270^\circ = \infty$; $\tan 270^\circ = 0$

(ix) $\frac{407}{2}\pi$

$= 36630^\circ$

$= 270^\circ + 360^\circ(101)$

$= 270^\circ$

$\sin 270^\circ = -1$; $\text{cosec } 270^\circ = -1$

$\cos 270^\circ = 0$; $\sec 270^\circ = \infty$

$\tan 270^\circ = \infty$; $\cot 270^\circ = 0$

* Exercise 9.4

Prove the following identities, state the domain of θ in each case:

Q.No.1:-

$\tan\theta + \cot\theta = \text{cosec}\,\theta \sec\theta$

L.H.S:-

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \operatorname{cosec}\theta \sec\theta$$

So L.H.S = R.H.S

Q.NO.2:-

$$\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta = 1$$

L.H.S:-

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} \cancel{\sin\theta} \cdot \cancel{\cos\theta}$$

$$= 1$$

So L.H.S = R.H.S

Q.NO.3:-

$$\cos\theta + \tan\theta \sin\theta = \sec\theta$$

L.H.S:-

$$= \cos\theta + \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$\frac{1}{\cos\theta} = \sec\theta$$

So L.H.S = R.H.S

Q.NO.4:-

$$\operatorname{cosec}\theta + \tan\theta\sec\theta = \operatorname{cosec}\theta\sec^2\theta$$

L.H.S

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos^2\theta}$$

$$= \frac{1}{\sin\theta\cos^2\theta}$$

$$= \frac{1}{\sin\theta}\cdot\frac{1}{\cos^2\theta}$$

$$= \operatorname{cosec}\theta\sec^2\theta$$

So L.H.S = R.H.S

Q.NO.5:-

$$\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$$

L.H.S:-

$$= \sec^2\theta - \operatorname{cosec}^2\theta$$

$$= \cancel{1} + \tan^2\theta - \cancel{1} - \cot^2\theta$$

$$= \tan^2\theta - \cot^2\theta$$

So L.H.S = R.H.S

Q.NO.6:-

$$\cot^2\theta - \cos^2\theta = \cot^2\theta \cos^2\theta$$

L.H.S:-

$$= \cot^2\theta - \cos^2\theta$$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$$

$$= \cos^2\theta\left[\frac{1}{\sin^2\theta} - 1\right]$$

$$= \cos^2\theta\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

$$= \cos^2\theta \cot^2\theta$$

So L.H.S = R.H.S

Q.NO.7:-

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

L.H.S:-

$$= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$$

$$= \sec^2 - \tan^2\theta$$

$$= 1 + \cancel{\tan^2\theta} - \cancel{\tan^2\theta}$$

$$= \underline{1}$$

So L.H.S = R.H.S

Q.NO.8:-

$$2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

L.H.S:-

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$= 2(1-\sin^2\theta) - 1$

$= 2 - 2\sin^2\theta - 1$

$= 1 - 2\sin^2\theta$

So L.H.S = R.H.S

Q.NO.9:-

$$\cos^2\theta - \sin^2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

R.H.S:-

$$= \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$$= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1}$$

$$= \cos^2\theta - \sin^2\theta$$

So L.H.S = R.H.S

Q.NO.10:-

$$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{\cot-1}{\cot+1}$$

R.H.S:-

$$=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$=\frac{\frac{\cos\theta}{\sin\theta}-1}{\frac{\cos\theta}{\sin\theta}+1}$$

$$=\frac{\frac{\cos\theta-\sin\theta}{\cancel{\sin\theta}}}{\frac{\cos\theta+\sin\theta}{\cancel{\sin\theta}}}$$

$$=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

So L.H.S = R.H.S

Q.NO.11:-

$$\frac{\sin\theta}{1+\cos\theta}+\cos\theta=\operatorname{cosec}\theta$$

L.H.S:-

$$=\frac{\sin\theta}{1+\cos\theta}+\cos\theta$$

$$=\frac{\sin^2\theta+(1+\cos\theta)\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$=\frac{\sin^2\theta+\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$$

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$$= \frac{\cancel{1+\cos\theta}}{\sin\theta\,(\cancel{1+\cos\theta})}$$

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta$$

So $L.H.S = R.H.S$

Q.NO.12B-

$$\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$$

L.H.S

$$= \frac{\cot^2\theta - 1}{1+\cot^2\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1+\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cancel{\sin^2\theta}}}{\frac{\sin^2\theta + \cos^2\theta}{\cancel{\sin^2\theta}}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1}$$

$$= \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - (1 - \cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$= 2\cos^2\theta - 1$

So $L.H.S = R.H.S$

Q.NO. 13:-

$$\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$

R.H.S:-

$= (\operatorname{cosec}\theta + \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1+\cos)^2}{1-\cos^2\theta}$

$= \frac{(1+\cos\theta)^{\cancel{2}}}{\cancel{(1+\cos\theta)}(1-\cos\theta)}$

$= \frac{1+\cos\theta}{1-\cos\theta}$

So $L.H.S = R.H.S$

Q.NO. 14:-

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

L.H.S:-

$$= (\sec - \tan\theta)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cancel{(1-\sin\theta)}(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

So L.H.S = R.H.S

Q.NO.15:-

$$\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$$

L.H.S:-

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{2\tan\theta}{\sec^2\theta}$$

$$= \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\frac{1}{\cos^2\theta}}$$

$= 2\sin\theta \cos\theta$

so L.H.S = R.H.S

Q.NO.16:-

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

L.H.S:-

$$= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

So L.H.S = R.H.S

Q.NO.17:-

$$(\tan\theta + \cot\theta)^2 = \sec^2\theta \csc^2\theta$$

L.H.S:-

$$= (\tan\theta + \cot\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \left(\frac{1}{\cos\theta \cdot \sin\theta}\right)^2$$

$$= \frac{1}{\cos^2\theta} \cdot \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \ \csc^2\theta$$

So $L.H.S = R.H.S$

Q.NO.18:-

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$

L.H.S:-

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$\therefore \ 1 + \tan^2\theta = \sec^2\theta$

$\therefore \ 1 = \sec^2\theta - \tan^2\theta$

$$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)\cancel{[1 - \sec\theta + \tan\theta]}}{\cancel{1 - \sec\theta + \tan\theta}}$$

$$= \sec\theta + \tan\theta$$

So $L.H.S = R.H.S$

Q.NO. 19:-

$$\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+c}$$

$$\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta}$$

$$\frac{1}{\operatorname{cosec}\theta-\cot\theta}+\frac{1}{\operatorname{cosec}\theta+\cot\theta}=\frac{1}{\sin\theta}+\frac{1}{\sin\theta}$$

$$\frac{\operatorname{cosec}\theta+\cot\theta+\operatorname{cosec}\theta-\cot\theta}{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)}=\frac{1+1}{\sin\theta}$$

$$\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-\cot^2\theta}=\frac{2}{\sin\theta}$$

$$\frac{2\operatorname{cosec}\theta}{1+\cancel{\cot^2\theta}-\cancel{\cot^2\theta}}=2\cdot\frac{1}{\sin\theta}$$

$$2\operatorname{cosec}\theta=2\operatorname{cosec}\theta$$

So L.H.S = R.H.S

Q.NO.20:-

$$\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)(1+\sin\theta\cos$$

L.H.S:-

$$=\sin^3\theta-\cos^3\theta$$

$$\therefore a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$=(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)$$

$$=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$$

So L.H.S = R.H.S

Q.NO. 21:-

$\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

L.H.S:-

$= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\sin^2\theta\cos^2\theta) + (\cos^2\theta)^2]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

So L.H.S = R.H.S

Q.NO. 22:-

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

L.H.S

$= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 - (\sin^2\theta)(\cos^2\theta)]$

$= 1[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2(\sin^2\theta)(\cos^2\theta) - 3(\sin^2\theta)(\cos^2\theta)]$

$= 1(\sin^2\theta + \cos^2\theta)^3 - 3(\sin^2\theta\cos^2\theta)$

$= 1(1) - 3(\sin^2\theta\cos^2\theta)$

$= 1 - 3(\sin^2\theta\cos^2\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta$

So L.H.S = R.H.S

Q.NO.23:-

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

L.H.S:-

$$=\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$

$$=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$=\frac{2}{1-\sin^2\theta}$$

$$=\frac{2}{\cos^2\theta}$$

$$=2\cdot\frac{1}{\cos^2\theta}$$

$$=2\sec^2\theta$$

So L.H.S = R.H.S

Q.NO.24:-

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{2}{1-2\sin^2\theta}$$

L.H.S:-

$$=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$=\frac{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta) + (\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta)}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta + \sin^2\theta \cancel{- 2\cos\theta\sin\theta}}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{(\cos^2\theta + \sin^2\theta) + (\cos^2\theta + \sin^2\theta)}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{1 + 1}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2}{1 - \sin^2\theta - \sin^2\theta}$$

$$= \frac{2}{1 - 2\sin^2\theta}$$

So L.H.S = R.H.S

Unit NO. 11:-

Trigonometric Functions and their graphs.

Definition

Period:-

Period of a trigonometric function is the smallest +ve number which, when added to