Mathematics

Unit NO. 5:-
"Partial Fractions"

1) Definitions

Inconsistent Equation:-

The equation having no solution is called inconsistent equation.

Fraction:-

The quotient of two numbers is called fraction.

Partial Fractions:-

To express a single rational function as a sum of two or more single rational functions which are called **Partial Fractions**.

Examples:-

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

Conditional Equations:-

It is an equation in which two algebric expressions are equal for particular value/s of the variable.

Examples:-

$2x = 3$ is a conditional equation and it is true only if $x = 3/2$.

Identity:-

It is an equation which holds good for all values of the variable.

Examples:-

$(a+b)x = ax + bx$ is an identity as its two sides are equal for all values of x.

Rational Fraction:-

The expression which is written in the form of $\frac{P(x)}{q(x)}$, $Q(x) \neq 0$, $p(x)$ and $q(x)$ $\in$ polynomials is called **rational fraction**.

Examples:-

$$\frac{x+1}{x^2+2x+2}$$

Proper Rational Fraction:-

A rational function $\frac{P(x)}{q(x)}$ is called a **Proper Rational Fraction** if the degree of the polynomial $P(x)$ in the numerator is less than the degree of the polynomial $Q(x)$ in the denominator.

Example:-

$$\frac{3}{x+1}, \quad \frac{2x-5}{x^2+4}$$

Improper Rational Fraction:-

A rational fraction $\frac{P(x)}{q(x)}$ is called an **Improper Rational Fraction** if the degree of the polynomial $P(x)$ in the numerator is equal to or greater than the degree of the polynomial $Q(x)$ in the denominator.

Example:-

$$\frac{x}{2x-3}, \quad \frac{(x-2)(x+1)}{(x-1)(x+4)}$$

* Exercises 5.1

Resolve into Partial Fractions.

Q. No. 1:-

$$\frac{1}{x^2-1}$$

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \rightarrow (i)$$

By multiplying $(x+1)(x-1)$ on b.s of (i)

$$1 = A(x-1) + B(x+1) \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$1 = A(1-1) + B(1+1)$$

$$1 = 0 + 2B$$

$$\boxed{\frac{1}{2} = B}$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$1 = A(-1-1) + 0$$

$$1 = -2A$$

$$\boxed{-\frac{1}{2} = A}$$

Put A and B in eq (i)

$$\frac{1}{(x+1)(x-1)} = -\frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$

Q. NO. 28-

$$\frac{x^2+1}{(x+1)(x-1)}$$

$$= \frac{x^2+1}{x^2-1} \qquad \because a^2-b^2=(a+b)(a-b)$$

$$\begin{array}{r|l} & 1 \\ \hline x^2-1 & x^2+1 \\ & \pm x^2 \mp 1 \\ \hline & 2 \end{array}$$

$$\frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1} \rightarrow (i)$$

Taking

$$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \rightarrow (ii)$$

By multiplying $(x+1)(x-1)$ on b.s of (ii)

$$2 = A(x-1) + B(x+1) \rightarrow (iii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (iii)

$$2 = 0 + B(1+1)$$

$$2 = 2B$$

$$\frac{\cancel{2}\,1}{\cancel{2}\,1} = B$$

$$\boxed{1 = B}$$

Put $x+1=0 \Rightarrow x=-1$ in eq (iii)

$$2 = A(-1-1) + 0$$

$2 = -2A$

$\frac{-\cancel{2}^{1}}{\cancel{2}_{1}} = A$

$\boxed{-1 = A}$

Put A and B in eq (ii)

$$\frac{2}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$

$$\frac{x^2+1}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}$$

Q.No.3:-

$$\frac{2x+1}{(x-1)(x+2)(x+3)}$$

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3} \rightarrow i$$

By multiplying $(x-1)(x+2)(x+3)$ on b.s of i

$2x+1 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2) \rightarrow ii$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$2(1)+1 = A(1+2)(1+3) + 0 + 0$

$2+1 = A(3)(4)$

$3 = 12A$

$\frac{\cancel{3}^{1}}{\cancel{12}_{4}} = A$

$\boxed{\frac{1}{4} = A}$

Put $x+2=0 \Rightarrow x=-2$ in eq (ii)

$2(-2)+1 = 0+B(-2-1)(-2+3)+0$

$-4+1 = B(-3)(1)$

$-3 = -3B$

$\frac{\cancel{+3}}{\cancel{+3}} = B$

$\boxed{1 = B}$

Put $x+3=0 \Rightarrow x=-3$ in eq (ii)

$2(-3)+1 = 0+0+C(-3-1)(-3+2)$

$-6+1 = C(-4)(-1)$

$-5 = 4C$

$\boxed{\frac{-5}{4} = C}$

Put A, B and C in eq (i)

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{1}{4(x-1)} + \frac{1}{x+2} - \frac{5}{4(x+3)}$$

Q.NO.4:

$$\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)}$$

$$\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)} = \frac{3x^2-4x-5}{(x-2)(x^2+5x+2x+10)}$$

$$\frac{3x^2-4x-5}{(x-2)(x+5)(x+2)} = \frac{A}{x-2} + \frac{B}{x+5} + \frac{C}{x+2} \rightarrow (i)$$

Multiplying $(x-2)(x+5)(x+2)$ on b.S of (i)

$3x^2-4x-5 = A(x+2)(x+5)+B(x-2)(x+2)+C(x-2)(x+5)$

Put $x+2=0 \Rightarrow x=-2$ in eq (ii)

$3(-2)^2-4(-2)-5 = 0+0+C(-2-2)(-2+5)$

$3(4)+8-5 = C(-4)(3)$

$12+8-5 = -12C$

$15 = -12C$

$-\frac{\cancel{15}\,5}{\cancel{12}\,4} = C$

$\boxed{-\frac{5}{4} = C}$

Put $x+5=0 \Rightarrow x=-5$ in eq (ii)

$3(-5)^2-4(-5)-5 = 0+B(-5-2)(-5+2)+0$

$3(25)+20-5 = B(-7)(-3)$

$75+20-5 = 21B$

$90 = 21B$

$\frac{\cancel{90}\,30}{\cancel{21}\,7} = B$

$\boxed{\frac{30}{7} = B}$

Put $x-2=0 \Rightarrow x=2$ in eq (ii)

$3(2)^2-4(2)-5 = A(2+2)(2+5)+0+0$

$8(4)-8-5 = A(4)(7)$

$12-8-5 = 28A$

$-1 = 28A$

$$\boxed{\frac{-1}{28} = A}$$

Put A, B and C in eq (ii)

$$\frac{3x^2-4x-5}{(x-2)(x+5)(x+2)} = \frac{-1}{28(x-2)} + \frac{30}{7(x+5)} + \frac{5}{4(x+2)}$$

Q.NO.5:-

$$\frac{1}{(x-1)(2x-1)(3x-1)}$$

$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{3x-1} \rightarrow (i)$$

Multiplying $(x-1)(2x-1)(3x-1)$ on b.s of (i)

$$1 = A(2x-1)(3x-1) + B(x-1)(3x-1) + C(x-1)(2x-1) \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$1 = A[2(1)-1][3(1)-1] + 0 + 0$$

$$1 = A(2-1)(3-1)$$

$$1 = A(1)(2)$$

$$\boxed{\frac{1}{2} = A}$$

Put $2x-1=0$ $x = \frac{1}{2}$ in eq (ii)

$$1 = 0 + B\left[\frac{1}{2} - 1\right]\left[3\left(\frac{1}{2}\right) - 1\right] + 0$$

$$1 = B\left[\frac{1-2}{2}\right]\left[\frac{3}{2} - 1\right]$$

$$1 = B\left(\frac{-1}{2}\right)\left(\frac{3-2}{2}\right)$$

$$1 = B\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)$$

$$1 = B\left(\frac{-1}{4}\right)$$

$$\boxed{-4 = B}$$

Put $3x-1=0 \Rightarrow x = \frac{1}{3}$ in eq (ii)

$$1 = 0+0+C\left[\frac{1}{3} - \cancel{3}1\right]\left[2\left(\frac{1}{3}\right)-1\right]$$

$$1 = C\left[\frac{1-3}{3}\right]\left[\frac{2}{3} - 1\right]$$

$$1 = C\left[\frac{-2}{3}\right]\left[\frac{2-3}{3}\right]$$

$$1 = C\left[\frac{-2}{3}\right]\left[\frac{-1}{3}\right]$$

$$1 = C\left[\frac{2}{9}\right]$$

$$\boxed{\frac{9}{2} = C}$$

Put A, B and C in eq (i)

$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{1}{2(x-1)} - \frac{4}{2x-1} + \frac{9}{2(3x-1)}$$

Q.NO.6:-

$$\frac{x}{(x-a)(x-b)(x-c)}$$

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} \rightarrow (i)$$

Multiplying $(x-a)(x-b)(x-c)$ on b.s of (i)

$$x = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b) \rightarrow (ii)$$

Put $x-a=0 \Rightarrow x=a$ in eq (ii)

$$a = A(a-b)(a-c) + 0 + 0$$

$$\boxed{\frac{a}{(a-b)(a-c)} = A}$$

Put $x-b=0 \Rightarrow x=b$ in eq (ii)

$$b = 0 + B(b-a)(b-c) + 0$$

$$\boxed{\frac{b}{(b-a)(b-c)} = B}$$

Put $x-c=0 \Rightarrow x=c$ in eq (ii)

$$c = 0 + 0 + C(c-a)(c-b)$$

$$\boxed{\frac{c}{(c-a)(c-b)} = C}$$

Put A, B and C in eq (i)

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{a}{(x-a)(a-b)(a-c)} + \frac{b}{(x-b)(b-a)(b-c)} + \frac{c}{(x-c)(c-a)(c-b)}$$

Q. NO 07:

$$\frac{6x^3+5x^2-7}{2x^2-x-1}$$

$$3x+4$$

$$2x^2-x-1 \;\Big|\; 6x^3+5x^2-7$$

$$\cancel{\pm 6x^3} \mp 3x^2 \mp 3x$$

$$\cancel{8x^2}+3x-7$$

$$\cancel{\pm 8x^2} \mp 4x \mp 4$$

$$7x-3$$

$$\frac{6x^3+5x^2-7}{2x^2-x-1} = 3x+4+\frac{7x-3}{(2x+1)(x-1)} \rightarrow (i)$$

Taking

$$\frac{7x-3}{(2x+1)(x-1)} = \frac{A}{2x+1}+\frac{B}{x-1} \rightarrow (ii)$$

Multiplying $(2x+1)(x-1)$ on b.S of (ii)

$$7x-3 = A(x-1)+B(2x+1) \rightarrow (iii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (iii)

$$7(1)-3 = 0+B[2(1)+1]$$

$$4 = 0+3B$$

$$\boxed{\frac{4}{3}=B}$$

Put $2x+1=0 \Rightarrow x=-\frac{1}{2}$ in eq (iii)

$$7\left(\frac{-1}{2}\right)-3 = A\left[\frac{-1}{2}-1\right]+0$$

$$\frac{-7}{2}-3 = A\left[\frac{-1}{2}-1\right]$$

$$\frac{-7-6}{2} = A\left[\frac{-1-2}{2}\right]$$

$$\frac{-13}{\cancel{2}} = A\left(\frac{-3}{\cancel{2}}\right)$$

$$\frac{+13}{+3} = A$$

$$\boxed{\frac{13}{3} = A}$$

Put A and B in (ii)

$$\frac{7x-3}{(2x+1)(x-1)} = \frac{13}{3(2x+1)} + \frac{4}{3(x-1)}$$

$$\frac{6x^3+5x^2-7}{2x^2-x-1} = 3x+4+\frac{13}{3(2x+1)}+\frac{4}{3(x-1)}$$

Q. NO. 8:-

$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}$$

$$\begin{array}{r|l} & 1 \\ 2x^3+x^2-3x & \cancel{2x^3}+\cancel{x^2}-5x+3 \\ & \cancel{\pm 2x^3} \cancel{\pm x^2} \mp 3x \\ \hline & -2x+3 \end{array}$$

$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x} = 1 - \frac{2x+3}{2x^3+x^2-3x} \rightarrow (i)$$

Taking

$$\frac{2x+3}{x(2x^2+x-3)} = \frac{2x+3}{x(2x^2+3x-2x-3)}$$

$$\frac{2x+3}{x[2x(x+3)-2(x+3)]} = \frac{2x+3}{x(2x-2)(x+3)}$$

$$\frac{2x+3}{x(2x-2)(x+3)} = \frac{A}{x} + \frac{B}{2x-2} + \frac{C}{x+3} \rightarrow (ii)$$

Multiplying $x(2x-2)(x+3)$ on b.s of (ii)

$$2x+3 = A(2x-2)(x+3)+Bx(x+3)+Cx(2x-2)$$

Put $x=0$ in eq (iii)

$$2(0)+3 = A[2(0)-2][0+3]+B(0)(0+3)+C(0)[2(0)$$

$$3 = A(-2)(3)$$

$$3 = -6A$$

$$\frac{-\cancel{3}^{1}}{\cancel{6}_{2}} = A$$

$$\boxed{\frac{-1}{2} = A}$$

Put $2x-2=0 \Rightarrow x=1$ in eq (iii)

$$2(1)+3 = 0+B(1)(1+3)+0$$

$$2+3 = 4B$$

$$5 = 4B$$

$$\boxed{\frac{5}{4} = B}$$

Put $x+3=0 \Rightarrow x=-3$ in eq (iii)

$$2(-3)+3 = 0+0+C(-3)[2(-3)-2]$$

$$-6+3 = C(-3)(-8)$$

$$-3 = 24C$$

$$-\frac{\cancel{3}^{\,1}}{\cancel{24}_{\,8}} = C$$

$$\boxed{-\frac{1}{8} = C}$$

Put A, B and C in eq (ii)

$$\frac{2x+3}{x(2x-2)(x+3)} = \frac{-1}{2x} + \frac{5}{4(2x-2)} - \frac{1}{8(x+3)}$$

$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x} = 1 - \frac{1}{2x} + \frac{5}{4(2x-2)} - \frac{1}{8(x+3)}$$

Q. NO. 9:-

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$$

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)} = \frac{(x^2-3x-x+3)(x-5)}{(x^2-4x-2x+8)(x-6)}$$

$$\frac{(x^2-4x+3)(x-5)}{(x^2-6x+8)(x-6)} = \frac{x^3-4x^2+3x-5x^2+20x-15}{x^3-6x^2+8x-6x^2+36x-48}$$

$$= \frac{x^3-9x^2+23x-15}{x^3-12x^2+44x-48}$$

$$x^3-12x^2+44x-48 \overline{\big)\, x^3-9x^2+23x-15}\ (1$$
$$\underline{\pm x^3 \mp 12x^2 \pm 44x \mp 48}$$
$$3x^2-21x+33$$

$$\frac{x^3-9x^2+23x-15}{x^3-12x^2+44x-48} = 1+\frac{3x^2-21x+33}{x^3-12x^2+44x-48} \rightarrow (i)$$

Taking

$$\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)} = \frac{A}{x-2}+\frac{B}{x-4}+\frac{C}{x-6} \rightarrow (ii)$$

Multiplying $(x-2)(x-4)(x-6)$ on b.s of (ii)

$$3x^2-21x+33 = A(x-4)(x-6)+B(x-2)(x-6)+C(x-2)(x-4) \rightarrow (iii)$$

Put $x-2=0 \Rightarrow x=2$ in eq (iii)

$$3(2)^2-21(2)+33 = A(2-4)(2-6)+0+0$$

$$3(4)-42+33 = A(-2)(-4)$$

$$12-42+33 = 8A$$

$$\boxed{\frac{3}{8} = A}$$

Put $x-4=0 \Rightarrow x=4$ in eq (iii)

$$3(4)^2-21(4)+33 = 0+B(4-2)(4-6)+0$$

$$3(16)-84+33 = B(2)(-2)$$

$$48-84+33 = -4B$$

$$\frac{+3}{+4} = B$$

$$\boxed{\frac{3}{4} = B}$$

Put $x-6=0 \Rightarrow x=6$ in eq (iii)

$3(6)^2 - 21(6) + 33 = 0 + 0 + C(6-2)(6-4)$

$3(36) - 126 + 33 = C(4)(2)$

$108 - 126 + 33 = 8C$

$15 = 8C$

$$\boxed{\frac{15}{8} = C}$$

Put A, B and C in eq (ii)

$$\frac{3x^2 - 21x + 33}{(x-2)(x-4)(x-6)} = \frac{3}{8(x-2)} + \frac{3}{4(x-4)} + \frac{15}{8(x-6)}$$

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)} = 1 + \frac{3}{8(x-2)} + \frac{3}{4(x-4)} + \frac{15}{8(x-6)}$$

Q. NO. 10:-

$$\frac{1}{(1-ax)(1-bx)(1-cx)}$$

$$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{A}{1-ax} + \frac{B}{1-bx} + \frac{C}{1-cx} \rightarrow (i)$$

Multiplying $(1-ax)(1-bx)(1-cx)$ on b.s of (i)

$1 = A(1-bx)(1-cx) + B(1-ax)(1-cx) + C(1-ax)(1-bx) \rightarrow (ii)$

Put $1-ax=0 \Rightarrow x = \frac{1}{a}$ in eq (ii)

$$1 = A\left[1 - b\left(\frac{1}{a}\right)\right]\left[1 - c\left(\frac{1}{a}\right)\right]$$

$$1 = A\left[1 - \frac{b}{a}\right]\left[1 - \frac{c}{a}\right]$$

$$1 = A\left[\frac{a-b}{a}\right]\left[\frac{a-c}{a}\right]$$

$$\boxed{\frac{a^2}{(a-b)(a-c)} = A}$$

Put $1 - bx = 0 \Rightarrow x = \frac{1}{b}$ in eq (ii)

$$1 = 0 + B\left[1 - a\left(\frac{1}{b}\right)\right]\left[1 - c\left(\frac{1}{b}\right)\right] + 0$$

$$1 = B\left[1 - \frac{a}{b}\right]\left[1 - \frac{c}{b}\right]$$

$$1 = B\left[\frac{b-a}{b}\right]\left[\frac{b-c}{b}\right]$$

$$\boxed{\frac{b^2}{(b-a)(b-c)} = B}$$

Put $1 - cx = 0 \Rightarrow x = \frac{1}{c}$ in eq (ii)

$$1 = 0 + 0 + C\left[1 - a\left(\frac{1}{c}\right)\right]\left[1 - b\left(\frac{1}{c}\right)\right]$$

$$1 = C\left[1 - \frac{a}{c}\right]\left[1 - \frac{b}{c}\right]$$

$$1 = C\left[\frac{c-a}{c}\right]\left[\frac{c-b}{c}\right]$$

$$\boxed{\frac{c^2}{(c-a)(c-b)} = C}$$

Put A, B and C in eq (i)

$$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{a^2}{(a-b)(a-c)(1-ax)} + \frac{b^2}{(b-a)(b-c)(1-bx)} + \frac{c^2}{(c-a)(c-b)(1-cx)}$$

Q.NO. 118-

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$$

Let $x^2 = y$

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)} = \frac{A}{y+b^2} + \frac{B}{y+c^2} + \frac{C}{y+d^2} \rightarrow (i)$$

Multiplying $(y+b^2)(y+c^2)(y+d^2)$ on b.s of (i)

$$y+a^2 = A(y+c^2)(y+d^2)+B(y+b^2)(y+d^2)+C(y+b^2)(y+c^2) \rightarrow (ii)$$

Put $y = -b^2$ in eq (ii)

$$-b^2+a^2 = A(-b^2+c^2)(-b^2+d^2)+0+0$$

$$\boxed{\frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)} = A}$$

Put $y = -c^2$ in eq (ii)

$$-c^2+a^2 = 0+B(-c^2+b^2)(-c^2+d^2)+0$$

$$\frac{a^2-c^2}{(d^2-c^2)(b^2-c^2)} = B$$

Put $y=-d^2$ in eq (ii)

$$-d^2+a^2 = 0+0+C(-d^2+b^2)(-d^2+c^2)$$

$$\boxed{\frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)} = C}$$

Put A, B and C in eq (i)

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)} = \frac{a^2-b^2}{(y+b^2)(c^2-b^2)(d^2-b^2)} + \frac{a^2-c^2}{(y+c^2)(d^2-c^2)(b^2-c^2)} + \frac{a^2-d^2}{(y+d^2)(b^2-d^2)(c^2-d^2)}$$

Now put $y=x^2$

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)} = \frac{a^2-b^2}{(x^2+b^2)(c^2-b^2)(d^2-b^2)} + \frac{a^2-c^2}{(x^2+c^2)(d^2-c^2)(b^2-c^2)} + \frac{a^2-d^2}{(x^2+d^2)(b^2-d^2)(c^2-d^2)}$$

Exercise 3.2

Resolve into Partial Fractions.

Q. NO. 1:-

$$\frac{2x^2-3x+4}{(x-1)^3}$$

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} \rightarrow (i)$$

Multiplying $(x-1)^3$ on b.S of (i)

$2x^2-3x+4 = A(x-1)^2+B(x-1)+C \rightarrow$ (ii)

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$2(1)^2-3(1)+4 = 0+0+C$

$2-3+4 = C$

$\boxed{3 = C}$

By compairing coefficient method

$2x^2-3x+4 = A(x^2-2x+2)+Bx-B+C$

$2x^2-3x+4 = Ax^2-2Ax+2A+Bx-B+C$

$2x^2-3x+4 = Ax^2+(-2A+B)x-2A-B+C$

$\boxed{2 = A}$

$-3 = -2A+B$

$-3 = -2(2)+B$

$-3 = -4+B$

$-3+4 = B$

$\boxed{1 = B}$

Put A, B and C in eq (i)

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{3}{(x-1)^3}$$

Q.NO.2:-

$$\frac{5x^2-2x+3}{(x+2)^3}$$

$$\frac{5x^2-2x+3}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} \rightarrow (i)$$

Multiplying $(x+2)^3$ on b.S of (i)

$5x^2-2x+3 = A(x+2)^2 + B(x+2) + C \rightarrow$ (ii)

Put $x+2=0 \Rightarrow x=-2$ in eq (ii)

$5(-2)^2-2(-2)+3 = 0+0+C$

$5(4)+4+3 = C$

$20+4+C = C$

$\boxed{27 = C}$

By compairing coefficient method

$5x^2-2x+3 = A(x^2+4x+4)+Bx+2B+C$

$5x^2-2x+3 = Ax^2+4Ax+4A+Bx+2B+C$

$5x^2-2x+3 = Ax^2+(4A+B)x+4A+2B+C$

$\boxed{5 = A}$

Put A, B and C in eq (i)

$$\frac{5x^2-2x+3}{(x+2)^3} = \frac{5}{x+2} - \frac{22}{(x+2)^2} + \frac{27}{(x+2)^3}$$

Q. NO. 3:-

$$\frac{4x}{(x+1)^2(x-1)}$$

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} \rightarrow \text{(i)}$$

Multiplying $(x+1)^2(x-1)$ on b.S of (i)

$4x = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \rightarrow$ (ii)

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$4(-1) = 0 + B(-1-1) + 0$

$$-4 = -2B$$

$$\frac{\cancel{-4}\,2}{\cancel{-2}\,1} = B$$

$$\boxed{2 = B}$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$4(1) = 0+0+C(1+1)^2$$

$$4 = 4C$$

$$\frac{\cancel{4}\,1}{\cancel{4}\,1} = C$$

$$\boxed{1 = C}$$

By compairing coefficient method

$$4x = A(x+1)(x-1) + B(x-1) + C(x+1)^2$$

$$4x = A(x^2 \cancel{-x} \cancel{+x} - 1) + Bx - B + C(x^2 + 2x + 1)$$

$$4x = Ax^2 - A + Bx - B + Cx^2 + 2Cx + C$$

$$4x = Ax^2 + Cx^2 + Bx + 2Cx - A - B + C$$

$$4x = (A+C)x^2 + (B+2C)x - A - B + C$$

$$0 = A + C$$

$$0 = A + 1$$

$$\boxed{-1 = A}$$

$$4 = B + 2C$$

$$4 = B + 2(1)$$

$$4 - 2 = B$$

$$\boxed{2 = B}$$

Put A, B and C in eq (i)

$$\frac{4x}{(x+1)^2(x-1)} = \frac{-1}{x+1} + \frac{2}{(x+1)^2} + \frac{1}{x-1}$$

Q.NO.4:-

$$\frac{9}{(x+2)^2(x-1)}$$

$$\frac{9}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} \rightarrow (i)$$

Multiplying $(x+2)^2(x-1)$ on b.s of (i)

$$9 = A(x+2)(x-1) + B(x-1) + C(x+2)^2 \rightarrow (ii)$$

Put $x+2=0 \Rightarrow x=-2$ in eq (ii)

$$9 = 0 + B(-2-1) + 0$$

$$9 = -3B$$

$$-\frac{\cancel{9}^{3}}{\cancel{3}_{1}} = B$$

$$\boxed{-3 = B}$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$9 = 0+0 + C(1+2)^2$$

$$9 = C(3)^2$$

$$9 = 9C$$

$$\frac{\cancel{9}^{1}}{\cancel{9}_{1}} = C$$

$$\boxed{1 = C}$$

By compairing coefficient method

$9 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$

$9 = A(x^2 - x + 2x - 2) + Bx + B + C(x^2 + 4x + 4)$

$9 = Ax^2 - Ax - 2A + Bx - B + Cx^2 + 4Cx + 4C$

$9 = Ax^2 + Cx^2 + Ax + Bx + 4Cx - 2A - B + 4C$

$0 = A + C$

$0 = A + 1$

$\boxed{-1 = A}$

Put A, B and C in eq (i)

$$\frac{9}{(x+2)^2(x-1)} = \frac{-1}{x+2} - \frac{3}{(x+2)^2} + \frac{1}{x-1}$$

Q.NO.5:-

$$\frac{1}{(x-3)^2(x+1)}$$

$$\frac{1}{(x-3)^2(x+1)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+1} \rightarrow (i)$$

Multiplying $(x-3)^2(x+1)$ on b.s of (i)

$1 = A(x-3)(x+1) + B(x+1) + C(x-3)^2 \rightarrow (ii)$

Put $x-3=0 \Rightarrow x=3$ in eq (ii)

$1 = 0 + B(3+1) + 0$

$1 = 4B$

$\boxed{\frac{1}{4} = B}$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$1 = 0 + 0 + C(-1-3)^2$

$1 = C(-4)^2$

$1 = 16C$

$\boxed{\frac{1}{16} = C}$

By compairing coefficient method

$1 = A(x-3)(x+1) + B(x+1) + C(x-3)^2$

$1 = A(x^2 + x - 3x - 3) + Bx + B + C(x^2 - 6x + 9)$

$1 = Ax^2 - 2Ax - 3A + Bx + B + Cx^2 - 6Cx + 9C$

$1 = Ax^2 + Cx^2 - 2Ax + Bx - 6Cx - 3A + B + 9C$

$0 = A + C$

$0 = A + \frac{1}{16}$

$\boxed{\frac{-1}{16} = A}$

Put A, B and C in eq (i)

$$\frac{1}{(x-3)^2(x+1)} = \frac{-1}{16(x-3)} + \frac{1}{4(x+1)} + \frac{1}{16(x+1)}$$

Q.NO.6:-

$$\frac{x^2}{(x-2)(x-1)^2}$$

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \rightarrow (i)$$

Multiplying $(x-2)(x-1)^2$ on b.S of (i)

$x^2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2) \rightarrow (ii)$

Put $x-2=0 \Rightarrow x=2$ in eq (ii)

$(2)^2 = A(2-1)^2 + 0 + 0$

$4 = A(1)^2$

$\boxed{4 = A}$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$1 = 0 + 0 + C(1-2)$

$1 = -C$

$\boxed{-1 = C}$

By compairing coefficient method

$x^2 = A(x^2-2x+1) + B(x^2-x-2x+1) + C(x-2) \rightarrow (ii)$

$x^2 = Ax^2 - 2Ax + A + Bx^2 - 3Bx + B + Cx - 2C$

$x^2 = Ax^2 + Bx^2 - 2Ax - 3Bx + Cx + A + B - 2C$

$1 = A + B$

$1 = 4 + B$

$1 - 4 = B$

$\boxed{-3 = B}$

Put A, B and C ieq (i)

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{4}{x-2} - \frac{3}{x-1} - \frac{1}{(x-1)^2}$$

Q. NO. 7:-

$$\frac{1}{(x-1)^2(x+1)}$$

$$\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \rightarrow (i)$$

Multiplying $(x-1)^2(x+1)$ on b.s of (i)

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$1 = 0 + B(1+1) + 0$$

$$1 = 2B$$

$$\boxed{\frac{1}{2} = B}$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$1 = 0 + 0 + C(-1-1)^2$$

$$1 = C(-2)^2$$

$$1 = 4C$$

$$\boxed{\frac{1}{4} = C}$$

By compairing coefficient method

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$1 = A(x^2 \cancel{+x} \cancel{-x} - 1) + Bx + B + C(x^2 - 2x + 1)$$

$$1 = Ax^2 + Cx^2 + Bx - 2Cx - A + B + C$$

$$0 = A + C$$

$$0 = \frac{A+1}{4}$$

$$\boxed{\frac{-1}{4} = A}$$

Put A, B and C in eq (i)

$$\frac{1}{(x-1)^2(x+1)} = \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}$$

Q. NO. 8:-

$$\frac{x^2}{(x-1)^3(x+1)}$$

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1} \rightarrow (i)$$

Multiplying $(x-1)^3(x+1)$ on b.s of (i)

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3 \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$(1)^2 = 0 + 0 + C(1+1) + 0$$

$$1 = 2C$$

$$\boxed{\frac{1}{2} = C}$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$(-1)^2 = 0 + 0 + 0 + D(-1-1)^3$$

$$1 = D(-2)^3$$

$$1 = -8D$$

$$\boxed{\frac{-1}{8} = D}$$

By compairing coefficient method

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3$$

$$x^2 = A(x^2-2x+1)(x+1) + B(x^2-1) + Cx + C + D(x^3-1-3x^2+3x)$$

$$x^2 = A(x^3+x^2-2x^2-2x+x+1) + Bx^2 - B + Cx + C + Dx^3 - D - 3Dx^2 + 3Dx$$

$$x^2 = Ax^3 - Ax^2 - Ax + A + Bx^2 - B + Cx + C + Dx^3 - D - 3Dx^2 + 3Dx$$

$$x^2 = Ax^3 + Dx^3 + Bx^2 - 3Dx^2 - Ax^2 - Ax + Cx + 3Dx + A - B + C - D$$

$$x^2 = (A+D)x^3 + (B-3D-A)x^2 + (-A+C+3D)x + A - B + C - D$$

$$0 = A + D$$

$$0 = A - \frac{1}{8}$$

$$\boxed{\frac{1}{8} = A}$$

$$1 = B - 3D - A$$

$$1 = B - 3\left(\frac{-1}{8}\right) - \left(\frac{1}{8}\right)$$

$$1 = B + \frac{3}{8} - \frac{1}{8}$$

$$1 = B + \frac{3-1}{8}$$

$$1 = B + \frac{2}{8}$$

$$\frac{1-2}{8} = B$$

$$\frac{8-2}{8} = B$$

$$\frac{\cancel{6}^{3}}{\cancel{8}_{4}} = B$$

$$\boxed{\frac{3}{4} = B}$$

Put A, B, C and D in eq (i)

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{1}{8(x+1)}$$

Q. NO. 9:-

$$\frac{x-1}{(x-2)(x+1)^3}$$

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \rightarrow (i)$$

Multiplying $(x-2)(x+1)^3$ on b.s of (i)

$$x-1 = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2) \rightarrow (ii)$$

Put $x-2=0 \Rightarrow x=2$ in eq (ii)

$$2-1 = A(2+1)^3 + 0 + 0 + 0$$

$$1 = A(3)^3$$

$$1 = A(27$$

$$\boxed{\frac{1}{27} = A}$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$-1-1 = 0+0+0+D(-1-2)$$

$$-2 = -3D$$

$$\frac{+2}{+3} = D$$

$$\boxed{\frac{2}{3} = D}$$

By compairing coefficient method

$$x-1 = A(x^3+1+3x^2+3)+B(x-2)(x+1)^2+C(x-2)(x+1)+D(x-2)$$

$$x-1 = A(x^3+1+3x^2+3)+B(x-2)(x^2+2x+1)+C(x^2+x-2x-2)+Dx-2D$$

$$x-1 = Ax^3+A+3Ax^2+3A+B(x^3+\cancel{2x^2}+x-\cancel{2x^2}-4x-2)+Cx^2-Cx-2C+Dx-2D$$

$$x-1 = Ax^3+A+3Ax^2+3A+Bx^3-3Bx-2B+Cx^2-Cx-2C+Dx-2D$$

$$x-1 = (A+B)x^3+(3A+C)x^2+(-3B-C+D)+A-2B-2C-2D$$

$$0 = A+B$$

$$0 = \frac{1}{27}+B$$

$$\frac{-1}{27} = B$$

$$0 = 3A+C$$

$$0 = 3\left(\frac{1}{27}\right)+C$$

$$0 = \frac{3}{27}+C$$

$$\frac{-\cancel{3}^{1}}{\cancel{27}_{9}} = C$$

$$\boxed{\frac{-1}{9} = C}$$

Put A, B and C in eq (i)

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{1}{27(x-2)} - \frac{1}{27(x+1)} - \frac{1}{9(x+1)^2} + \frac{2}{3(x+1)^3}$$

Q. NO. 10:-

$$\frac{4x^3}{(x^2-1)(x+1)^2}$$

$$= \frac{4x^3}{(x+1)(x-1)(x+1)^2}$$

$$\frac{4x^3}{(x+1)(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \rightarrow (i)$$

Multiplying $(x-1)(x+1)^3$ on b.s of (i)

$$4x^3 = A(x+1)^3 + B(x-1)(x+1)^2 + C(x-1)(x+1) + D(x-1) \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x = 1$ in eq (ii)

$$4(1)^3 = A(1+1)^3 + 0 + 0 + 0$$

$$4 = A(2)^3$$

$$4 = 8A$$

$$\frac{\cancel{4}^{1}}{\cancel{8}_{2}} = A$$

$$\boxed{\frac{1}{2} = A}$$

Put $x+1=0 \Rightarrow x=-1$ in eq (iii)

$$4(-1)^3 = 0 + 0 + 0 + D(-1-1)$$

$$-4 = -2D$$

$$\frac{\cancel{-4}^{\,2}}{\cancel{-2}_{\,1}} = D$$

$$\boxed{2 = D}$$

By compairing coefficient method

$$4x^3 = A(x+1)^3 + B(x-1)(x+1)^2 + C(x+1)(x-1) + D(x-1)$$

$$4x^3 = A(x^3+1+3x^2+3) + B(x-1)(x^2+2x+1) + C(x^2-1) + D(x-1)$$

$$4x^3 = Ax^3 + A + 3Ax^2 + 3A + B(x^3+2x^2+\cancel{x}-x-2x-1) + Cx^2 - C + Dx - D$$

$$4x^3 = Ax^3 + A + 3Ax^2 + 3A + Bx^3 + 2Bx^2 - 2Bx - B + Cx^2 - C + Dx - D$$

$$4x^3 = (A+B)x^3 + (3A+2B+C)x^2 + (-2B+D)x + A - B - C - D$$

$$4 = A + B$$

$$4 = \frac{1}{2} + B$$

$$4 - \frac{1}{2} = B$$

$$\frac{8-1}{2} = B$$

$$\boxed{\frac{7}{2} = B}$$

$$0 = 3A + 2B + C$$

$$0 = 3\left(\frac{1}{2}\right) + 2\left(\frac{7}{2}\right) + C$$

$$\frac{-3}{2} - \frac{7}{2} = C$$

$$\frac{-3-7}{2} = C$$

$$\frac{\cancel{-10}\,5}{\cancel{2}\,1} = C$$

$$\boxed{-5 = C}$$

Put A, B and C in eq (i)

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{1}{2(x-1)} + \frac{7}{2(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3}$$

Q. NO. 11:-

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2}$$

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2} \rightarrow (i)$$

Multiplying $(x+3)(x-1)(x+2)^2$ on b.s of (i) $\rightarrow$ (ii)

$$2x+1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1)$$

Put $x+3=0 \Rightarrow x=-3$ in eq (ii)

$$2(-3)+1 = A(-3-1)(-3+2)^2 + 0 + 0 + 0$$

$$-6+1 = A(-4)(-1)^2$$

$$-5 = -4A$$

$$\frac{+5}{+4} = A$$

$$\boxed{\frac{5}{4} = A}$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$2(1)+1 = 0 + B(1+3)(1+2)^2 + 0 + 0$$

$$2+1 = B(4)(3)^2$$

$$3 = 36B \Rightarrow \frac{\cancel{3}^{1}}{\cancel{36}_{12}} = B$$

$$\boxed{\frac{1}{12} = B}$$

Put $x+2=0 \Rightarrow x=-2$ in eq (ii)

$$2(-2)+1 = 0 + 0 + 0 + D(-2+3)(-2-1)$$

$$-4+1 = D(1)(-3)$$

$$-3 = -3D$$

$$\frac{\cancel{+3}^{1}}{\cancel{+3}_{1}} = D$$

$$\boxed{1 = D}$$

By compairing coefficient method

$$2x+1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1)$$

$$2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4) + C(x+3)(x^2+2x-x-2) + D(x^2-x+3x-3)$$

$$2x+1 = A(x^3+4x^2+4x-x^2-4x-4) + B(x^3+4x^2+4x+3x^2+12x+12) + C(x+3)(x^2+x-2) + D(x^2+2x-3)$$

$$2x+1 = A(x^3-3x^2-4) + B(x^3+7x^2+16x+12) + C(x^3+x^2-2x+3x^2+3x-6) + D(x^2+2x-3)$$

$$2x+1 = Ax^3-3Ax^2-4A+Bx^3+7Bx^2+16Bx+12B+Cx^3+4Cx^2+Cx-6C+Dx^2+2Dx-3D$$

$$2x+1 = (A+B+C)x^3 + (-3A+7B+4C+D)x^2 + (16B+C+2D)x - 4A + 12B - 6C - 3D$$

$$0 = A+B+C$$

$$0 = \frac{5}{4} - \frac{1}{2} + C$$

$$\frac{-5}{4} + \frac{1}{2} = C$$

$$\frac{-5+2}{4} = C$$

$$\boxed{\frac{-3}{4} = C}$$

Put A, B, C and D in eq (i)

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{3}{4(x+2)} + \frac{1}{(x+2)^2}$$

Q. NO. 12:-

$$\frac{2x^4}{(x-3)(x+2)^2}$$

$$\frac{2x^4}{(x-3)(x+2)^2} = \frac{2x^4}{(x^2+4x+4)(x-3)}$$

$$\frac{2x^4}{(x^3+4x^2+4x-3x^2-12x-12)} = \frac{2x^4}{x^3+x^2-8x-12}$$

$$\begin{array}{r|l} & 2x-2 \\ \hline x^3+x^2-8x-12 & 2x^4 \\ & \cancel{\pm 2x^4} \pm 2x^3 \mp 16x^2 \mp 24x \\ \hline & \cancel{-2x^3} + 16x^2 + 24x \\ & \cancel{\mp 2x^3} \mp 2x^2 \pm 16x \pm 24 \\ \hline & 18x^2 + 8x - 24 \end{array}$$

$$\frac{2x^4}{x^3+x^2-8x-12} = 2x-2+\frac{18x^2+8x-24}{x^3+x^2-8x-12} \rightarrow (i)$$

Taking

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3}+\frac{B}{x+2}+\frac{C}{(x+2)^2} \rightarrow (ii)$$

Multiplying $(x-3)(x+2)^2$ on b.s of (ii)

$$18x^2+8x-24 = A(x+2)^2+B(x-3)(x+2)+C(x-3) \rightarrow (iii)$$

Put $x-3=0 \Rightarrow x=3$ in eq (iii)

$$18(3)^2+8(3)-24 = A(3+2)^2+0+0$$

$$18(9)+\cancel{24}-\cancel{24} = A(25)$$

$$162 = 25A$$

$$\boxed{\frac{162}{25} = A}$$

Put $x+2=0 \Rightarrow x=-2$ in eq (iii)

$$18(-2)^2+8(-2)-24 = 0+0+C(-2-3)$$

$$18(4)-16-24 = -5C$$

$$72-16-24 = -5C$$

$$-32 = -5C$$

$$\boxed{\frac{-32}{5} = C}$$

By compairing coefficient method

$$18x^2+8x-24 = A(x+2)^2+B(x-3)(x+2)+C(x-3)$$

$$18x^2+8x-24 = A(x^2+4x+4)+B(x^2+2x-3x-6)+C(x-3)$$

$$18x^2+8x-24 = Ax^2+4Ax+4A+Bx^2-Bx-6B+Cx-3C$$

$$18x^2+8x-24=(A+B)x^2+(4A-B+C)x+4A-6B-3C$$

$$18 = A+B$$

$$18 = \frac{162}{25}+B$$

$$18-\frac{162}{25}=B$$

$$\frac{450-162}{25}=B$$

$$\boxed{\frac{288}{25}=B}$$

Put A, B and C in eq (ii)

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2}=\frac{162}{25(x-3)}+\frac{288}{25(x+2)}-\frac{32}{5(x+2)^2}$$

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2}=\frac{162}{25(x-3)}+\frac{288}{25(x+2)}-\frac{32}{5(x+2)^2}$$

* Exercise 5.3

Q.NO.1:-

$$\frac{9x-7}{(x^2+1)(x+3)}$$

$$\frac{9x-7}{(x^2+1)(x+3)}=\frac{9x-7}{(x+3)(x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{x^2+1}\rightarrow(i)$$

Multiplying $(x+3)(x^2+1)$ on b.S of (i)

$9x-7 = A(x^2+1)+(Bx+C)(x+3) \rightarrow (ii)$

Put $x+3=0 \Rightarrow x=-3$ in eq (ii)

$9(-3)-7 = A[(-3)^2+1]+0$

$-27-7 = A(9+1)$

$\frac{-34}{10}^{-17} = A$

$\boxed{\frac{-17}{5} = A}$

By compairing coefficient method

$9x-7 = A(x^2+1)+(Bx+C)(x+3)$

$9x-7 = Ax^2+A+Bx^2+3Bx+Cx+3C$

$9x-7 = (A+B)x^2+(3B+C)+A+3C$

$0 = A+B$

$0 = \frac{-17}{5}+B$

$\boxed{\frac{17}{5} = B}$

$9 = 3B+C$

$9 = 3\left(\frac{17}{5}\right)+C$

$9 = \frac{51}{5}+C$

$9-\frac{51}{5} = C$

$$\frac{45-51}{5} = C$$

$$\boxed{\frac{-6}{5} = C}$$

Put A, B and C in eq (i)

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{\left(\frac{17}{5}\right)x + \frac{-6}{5}}{x^2+1}$$

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$$

Q. NO. 2:-

$$\frac{1}{(x^2+1)(x+1)}$$

$$\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} \rightarrow (i)$$

Multiplying $(x^2+1)(x+1)$ on b.s of (i)

$$1 = (Ax+B)(x+1) + C(x^2+1) \rightarrow (ii)$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$1 = 0 + C[(-1)^2+1]$$

$$1 = C(1+1)$$

$$1 = 2C$$

$$\boxed{\frac{1}{2} = C}$$

By compairing coefficient method

$$1 = (Ax+B)(x+1) + C(x^2+1)$$

$$1 = Ax^2 + Ax + Bx + B + Cx^2 + C$$

$$1 = (A+C)x^2 + (A+B)x + B+C$$

$$0 = A+C$$

$$0 = A + \frac{1}{2}$$

$$\boxed{\frac{-1}{2} = A}$$

$$0 = A+B$$

$$0 = \frac{-1}{2} + B$$

$$\boxed{\frac{1}{2} = B}$$

Put A, B and C in eq (i)

$$\frac{1}{(x^2+1)(x+1)} = \frac{\left(\frac{-1}{2}\right)x + \frac{1}{2}}{x^2+1} + \frac{1}{2(x+1)}$$

$$\frac{1}{(x^2+1)(x+1)} = \frac{-x+1}{2(x^2+1)} + \frac{1}{2(x+1)}$$

Q. NO. 3:-

$$\frac{3x+7}{(x^2+4)(x+3)}$$

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+3} \rightarrow \text{(i)}$$

Multiplying $(x^2+4)(x+3)$ on b.s of (ii)

$3x+7 = (Ax+B)(x+3) + C(x^2+4)$

Put $x+3=0 \Rightarrow x=-3$ in eq (ii)

$3(-3)+7 = 0 + C[(-3)^2+4]$

$-9+7 = C(9+4)$

$-2 = 13C$

$$\boxed{\frac{-2}{13} = C}$$

By compairing coefficient method

$3x+7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + 4C$

$3x+7 = (A+C)x^2 + (3A+B)x + 3B + 4C$

$0 = A+C$

$0 = A - \frac{2}{13}$

$$\boxed{\frac{2}{13} = A}$$

$3 = 3A+B$

$3 = 3\left(\frac{2}{13}\right)+B$

$3 = \frac{6}{13} + B$

$3 - \frac{6}{13} = B$

$$\frac{39-6}{13} = B$$

$$\boxed{\frac{33}{13} = B}$$

Put A, B and C in eq (i)

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{\left(\frac{2}{13}\right)x+\frac{33}{13}}{x^2+4} - \frac{2}{13(x+3)}$$

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{2x+33}{13(x^2+4)} - \frac{2}{13(x+3)}$$

Q. NO. 4:-

$$\frac{x^2+15}{(x^2+2x+5)(x-1)}$$

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{Ax+B}{x^2+2x+5} + \frac{C}{x-1} \rightarrow \text{(i)}$$

Multiplying $(x^2+2x+5)(x-1)$ on b.s of (i)

$$x^2+15 = (Ax+B)(x-1)+C(x^2+2x+5) \rightarrow \text{(ii)}$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$(1)^2+15 = 0+C[(1)^2+2(1)+5]$$

$$1+15 = C(1+2+5)$$

$$16 = 8C$$

$$\frac{\cancel{16}^{2}}{\cancel{8}_{1}} = C$$

$$\boxed{2 = C}$$

By compairing coefficient method

$x^2+15 = (Ax+B)(x-1) + C(x^2+2x+5)$

$x^2+15 = Ax^2 - Ax + Bx - B + Cx^2 + 2Cx + 5C$

$x^2+15 = (A+C)x^2 + (-A+B+2C)x - B + 5C$

$1 = A+C$

$1 = A+2$

$1-2 = A$

$\boxed{-1 = A}$

$0 = -A+B+2C$

$0 = -(-1)+B+2(2)$

$0 = 1+4+B$

$0 = 5+B$

$-5 = B$

$\boxed{-5 = B}$

Put A, B and C in eq (i)

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{-1x+10}{x^2+2x+5} + \frac{2}{x-1}$$

Q.NO.5:-

$$\frac{x^2}{(x^2+4)(x+2)}$$

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+2} \rightarrow (i)$$

Multiplying $(x^2+4)(x+2)$ on b.S of (i)

$x^2 = (Ax+B)(x+2) + C(x^2+4) \rightarrow (ii)$

$(-2)^2 = 0 + C[(-2)^2+4]$

$4 = C(4+4)$

$4 = 8C$

$\frac{\cancel{4}^{1}}{\cancel{8}_{2}} = C$

$\boxed{\frac{1}{2} = C}$

By compairing coefficient method.

$x^2 = (Ax+B)(x+2) + C(x^2+4)$

$x^2 = Ax^2+2Ax+Bx+2B + Cx^2+4C$

$x^2 = (A+C)x^2+(2A+B)x + 2B+4C$

$1 = A+C$

$1 = A + \frac{1}{2}$

$1 - \frac{1}{2} = A$

$\frac{2-1}{2} = A$

$\boxed{\frac{1}{2} = A}$

$0 = 2A + B$

$0 = \cancel{2}\left(\frac{1}{\cancel{2}}\right) + B$

$$0 = 1 + B$$
$$\boxed{-1 = B}$$

Put A, B and C in eq (i)

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{\left(\frac{1}{2}\right)x - 1}{x^2+4} + \frac{1}{2(x+2)}$$

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{1x - 2}{2(x^2+4)} + \frac{1}{2(x+2)}$$

Q.NO.6:-

$$\frac{x^2+1}{x^3+1}$$

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \rightarrow (i)$$

Multiplying $(x+1)(x^2-x+1)$ on b.s of

$$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1) \rightarrow (ii)$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$(-1)^2+1 = A\left[(-1)^2-(-1)+1\right]+0$$
$$1+1 = A(1+1+1)$$
$$2 = 3A$$
$$\boxed{\frac{2}{3} = A}$$

By compairing coefficient method

$$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$$
$$x^2+1 = Ax^2-Ax+A + Bx^2+Bx+Cx+C$$

$$x^2+1 = (A+B)x^2 + (-A+B+C)x + A+C$$

$$1 = A+B$$

$$1 = \frac{2}{3} + B$$

$$1 - \frac{2}{3} = B$$

$$\frac{3-2}{3} = B$$

$$\boxed{\frac{1}{3} = B}$$

$$1 = A+C$$

$$1 = \frac{2}{3} + C$$

$$1 - \frac{2}{3} = C$$

$$\frac{3-2}{3} = C$$

$$\boxed{\frac{1}{3} = C}$$

Put A, B and C in eq (i)

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{\left(\frac{1}{3}\right)x + \frac{1}{3}}{x^2-x+1}$$

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$

Q.NO.7:-

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$$

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)} = \frac{Ax+B}{x^2+3} + \frac{C}{x+1} + \frac{D}{x-1} \rightarrow (i)$$

Multiplying $(x^2+3)(x+1)(x-1)$ on b.s of (i)

$$x^2+2x+2 = (Ax+B)(x+1)(x-1)+C(x^2+3)(x-1)+D(x^2+3)(x+1) \rightarrow (ii)$$

Put $x+1=0 \Rightarrow x=-1$ in eq (ii)

$$(-1)^2+2(-1)+2 = 0+C[(-1)^2+3][-1-1]$$

$$1-\cancel{2}+\cancel{2} = 0+C(1+3)(-2)$$

$$1 = -8C$$

$$\boxed{\frac{1}{-8} = C}$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$(1)^2+2(1)+2 = 0+0+D[(1)^2+3][1+1]$$

$$1+2+2 = D(4)(2)$$

$$5 = 8D$$

$$\boxed{\frac{5}{8} = D}$$

By compairing coefficient method

$$x^2+2x+2 = (Ax+B)(x+1)(x-1)+C(x^2+3)(x-1)+D(x^2+3)(x+1)$$

$$x^2+2x+2 = (Ax+B)(x^2-1)+C(x^3-x^2+3x-3)+D(x^3+x^2+3x+3)$$

$$x^2+2x+2 = Ax^3-Ax+Bx^2-B+Cx^3-Cx^2+3Cx-3C+Dx^3+Dx^2+3Dx+3D$$

$$x^2+2x+2 = (A+C+D)x^3+(B-C+D)x^2+(-A+3C+3D)x-B-3C+3D$$

$0 = A + C + D$

$0 = A - \frac{1}{8} + \frac{5}{8}$

$0 = -\frac{1}{8} + \frac{5}{8} = A$

$\frac{-1+5}{8} = A \Rightarrow 0 = A + \frac{4}{5}$

$\boxed{\frac{-4}{5} = A}$

$1 = B - C + D$

$1 = B + \frac{1}{8} + \frac{5}{8} \Rightarrow 1 = B + \frac{1+5}{8}$

$1 = B + \frac{6}{8}$

$1 - \frac{6}{8} = B \Rightarrow B = \frac{8-6}{8}$

$\boxed{\frac{2}{8} = B}$

Put A, B and C in eq (i)

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)} = \frac{\frac{-x}{2}+\frac{1}{4}}{x^2+3} - \frac{1}{4(x+1)} + \frac{5}{8(x-1)}$$

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)} = \frac{-2x+1}{4(x^2+3)} - \frac{1}{4(x+1)} + \frac{5}{8(x-1)}$$

Q. NO. 8:-

$$\frac{1}{(x-1)^2(x^2+2)}$$

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2} \rightarrow (i)$$

Multiplying $(x-1)^2(x^2+2)$ on b.S of (i)

$$1 = A(x-1)(x^2+2)+B(x^2+2)+(Cx+D)(x-1)^2 \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$1 = 0+B[(1)^2+2]+0$$

$$1 = 3B$$

$$\boxed{\frac{1}{3} = B}$$

By Equating Coefficient Method

$$1 = A(x-1)(x^2+2)+B(x^2+2)+(Cx+D)(x-1)^2$$

$$1 = (Ax-A)(x^2+2)+Bx^2+2B+(Cx+D)(x^2-2x+1)$$

$$1 = Ax^3+2Ax-Ax^2-2A+Bx^2+2B+Cx^3-2Cx^2+Cx+Dx^2-2Dx+D$$

$$1 = (A+C)x^3+(-A+B-2C+D)x^2+(2A+C-2D)x-2A+2B+D$$

$$0 = A+C$$

$$A = -C$$

$$\boxed{A = -\frac{2}{9}}$$

$$0 = -A+B-2C+D$$

$$0 = C+\frac{1}{3}-2C+D$$

$$0 = -C + \frac{1}{3} + D$$

$$C = D + \frac{1}{3}$$

$$\boxed{C - \frac{1}{3} = D}$$

$$\frac{2}{9} - \frac{1}{3} = D$$

$$\frac{2-3}{9} = D$$

$$\boxed{\frac{-1}{9} = D}$$

$$0 = 2A + C - 2D$$

$$0 = -2C + C - 2\left(C - \frac{1}{3}\right)$$

$$0 = -C - 2C + \frac{2}{3}$$

$$0 = -3C + \frac{2}{3}$$

$$3C = \frac{2}{3}$$

$$\boxed{C = \frac{2}{9}}$$

Put A, B, C and D in eq (i)

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{-2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{\left(\frac{2}{9}\right)x-\left(\frac{1}{9}\right)}{x^2+2}$$

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{-2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{2x-1}{9(x^2+2)}$$

Q. NO. 9:-

$$\frac{x^4}{1-x^4}$$

$$= \frac{x^4}{-x^4+1}$$

$$\begin{array}{r|l} & -1 \\ -x^4+1 & x^4 \\ & \pm x^4 \mp 1 \\ \hline & 1 \end{array}$$

$$\frac{x^4}{1-x^4} = -1 + \frac{1}{1-x^4} \rightarrow (i)$$

Taking

$$= \frac{1}{1-x^4}$$

$$= \frac{1}{(1-x^2)(1+x^2)}$$

$$= \frac{1}{(1-x)(1+x)(1+x^2)}$$

$$\frac{1}{(1-x)(1+x)(1+x^2)} = \frac{A}{1-x} + \frac{B}{1+x} + \frac{Cx+D}{1+x^2} \rightarrow (i)$$

Multiplying $(1-x)(1+x)(1+x^2)$ on b.s of (i)

$$1 = A(1+x)(1+x^2) + B(1-x)(1+x^2) + (Cx+D)(1-x^2) \rightarrow (iii)$$

Put $1-x=0 \Rightarrow x=1$ in (iii)

$$1 = A(1+1)(1+1) + 0 + 0$$

$$1 = A(2)(2)$$

$$1 = 4A$$

$$\boxed{\frac{1}{4} = A}$$

Put $x=-1$ in (iii)

$$1 = 0 + B(1+1)[1+(-1)^2]$$

$$1 = B(2)(2)$$

$$1 = 4B$$

$$\boxed{\frac{1}{4} = B}$$

$$(A+Ax)(1+x^2) + (B-Bx)(1+x^2) + Cx - Cx^3 + D - Dx^2$$

$$A + Ax^2 + Ax + Ax^3 + B + Bx^2 - Bx - Bx^3 + Cx - Cx^3 + D - Dx^2$$

$$(A-B-C)x^3 + (A+B-D)x^2 + (A-B+C)x + A+B+D$$

$$0 = A-B-C$$

$$C = \frac{1}{4} - \frac{1}{4}$$

$$\boxed{C=0}$$

$$0 = A + B - D$$

$$D = \frac{1}{4} + \frac{1}{4}$$

$$D = \frac{1+1}{4}$$

$$D = \frac{\cancel{2}\,1}{\cancel{4}\,2}$$

$$\boxed{D = \frac{1}{2}}$$

Put A, B, C and D in eq (ii)

$$\frac{1}{(1-x)(1+x)(1+x^2)} = \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{(0)x + \frac{1}{2}}{1+x^2}$$

$$\frac{1}{(1-x)(1+x)(1+x^2)} = \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{1}{2(1+x^2)}$$

$$\frac{x^4}{1-x^4} = -1 + \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{1}{2(1+x^2)}$$

Q.NO.10:-

$$\frac{x^2 - 2x + 3}{x^4 + x^2 + 1}$$

$$= (x^2 - x + 1)(x^2 + x + 1)$$

$$= x^4 + \cancel{x^3} + \cancel{x^2} - \cancel{x^3} - \cancel{x^2} - \cancel{x} + x^2 + \cancel{x} + 1$$

$$= x^4 + x^2 + 1$$

$$\frac{x^2 - 2x + 3}{(x^2 - x + 1)(x^2 + x + 1)} = \frac{Ax + B}{x^2 - x + 1} + \frac{Cx + D}{x^2 + x + 1} \rightarrow (i)$$

Multiplying $(x^2-x+1)(x^2+x+1)$ on b.s of (i)

$x^2-2x+3 = (Ax+B)(x^2+x+1)+(Cx+D)(x^2-x+1)$

$x^2-2x+3 = Ax^3+Ax^2+Ax+Bx^2+Bx+B+Cx^3-Cx^2+Cx+Dx^2-Dx+D$

$x^2-2x+3 = (A+C)x^3+(A-B-C+D)x^2+(A+B+C-D)x+B+D$

$0 = A+C$

$A = -C$

$\boxed{A = -1}$

$1 = A+B-C+D$

$1 = -C+3-\cancel{D}-C+\cancel{D}$

$1-3 = -2C$

$-2 = -2C$

$\frac{\cancel{+2}}{\cancel{+2}} = C$

$\boxed{1 = C}$

$-2 = A+B+C-D$

$-2 = \cancel{-1}+3-D+\cancel{1}-D$

$-2-3 = -2D$

$-5 = -2D$

$\frac{+5}{+2} = D$

$\boxed{\frac{5}{2} = D}$

$3 = B+D$

$$3-D=B$$

$$3-\frac{5}{2}=B$$

$$\frac{6-5}{2}=B$$

$$\boxed{\frac{1}{2}=B}$$

Put A, B, C and D in eq (i)

$$\frac{x^2-2x+3}{(x^2-x+1)(x^2+x+1)}=\frac{-x+\frac{1}{2}}{x^2-x+1}+\frac{x+\frac{5}{2}}{x^2+x+1}$$

$$\frac{x^2-2x+3}{(x^2-x+1)(x^2+x+1)}=\frac{-2x+1}{2(x^2-x+1)}+\frac{2x+5}{2(x^2+x+1)}$$

* Exercise 5.4

Q.NO.1:-

$$\frac{x^3+2x+2}{(x^2+x+1)^2}$$

$$\frac{x^3+2x+2}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}\rightarrow(i)$$

Multiplying $(x^2+x+1)^2$ on b.S of (i)

$$x^3+2x+2=(Ax+B)(x^2+x+1)+(Cx+D)\rightarrow(ii)$$

By compairing coefficient method

$$x^3+2x+2=(Ax+B)(x^2+x+1)+(Cx+D)$$

$x^3+2x+2 = Ax^3+Ax^2+Ax+Bx^2+Bx+B+Cx+D$

$\boxed{1 = A}$

$0 = A+B$

$0 = 1+B$

$\boxed{-1 = B}$

$2 = A+B+C$

$2 = \cancel{1}-\cancel{1}+C$

$\boxed{2 = C}$

$2 = B+D$

$2 = -1+D$

$2+1 = D$

$\boxed{3 = D}$

Put A, B, C and D in eq (i)

$$\frac{x^3+2x+2}{(x^2+x+1)^2} = \frac{x-1}{x^2+x+1} + \frac{2x+3}{(x^2+x+1)^2}$$

Q.NO.2:-

$$\frac{x^2}{(x^2+1)(x-1)}$$

$$\frac{x^2}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} \rightarrow (i)$$

Multiplying $(x^2+1)(x-1)$ on b.s of (i)

$$x^2 = (Ax+B)(x-1) + C(x^2+1) \rightarrow (ii)$$

Put $x-1=0 \Rightarrow x=1$ in eq (ii)

$$(1)^2 = [A(1)+B][1-1] + C[(1)^2+1]$$

$$1 = 0 + C(2)$$

$$\boxed{\frac{1}{2} = C}$$

By compairing coefficient method

$$x^2 = (Ax+B)(x-1) + C(x^2+1)$$

$$x^2 = Ax^2 - Ax + Bx - B + Cx^2 + C$$

$$x^2 = (A+C)x^2 + (-A+B)x - B + C$$

$$1 = A + C$$

$$1 = A + \frac{1}{2}$$

$$\boxed{1 - \frac{1}{2} = A}$$

$$\frac{2-1}{2} = A$$

$$\boxed{\frac{1}{2} = A}$$

$$0 = -A+B$$

$$0 = -\frac{1}{2}+B$$

$$\boxed{\frac{1}{2} = B}$$

Put A, B and C in eq

$$\frac{x^2}{(x^2+1)(x-1)} = \frac{\left(\frac{1}{2}\right)x+\frac{1}{2}}{x^2+1} + \frac{1}{2(x-1)}$$

$$\frac{x^2}{(x^2+1)(x-1)} = \frac{x+1}{2(x^2+1)} + \frac{1}{2(x-1)}$$

Q.NO.3:-

$$\frac{2x-5}{(x^2+2)^2(x-2)}$$

$$\frac{2x-5}{(x^2+2)^2(x-2)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} + \frac{E}{x-2} \rightarrow (i)$$

Multiplying $(x^2+2)^2(x-2)$ on b.S of (i)

$$2x-5 = (Ax+B)(x^2+2)(x-2)+(Cx+D)(x-2)+E(x^2+2)^2 \rightarrow (ii)$$

Put $x-2=0 \Rightarrow x=2$ in eq. (ii)

$$2(2)-5 = 0+0+E\left[(2)^2+2\right]^2$$

$4-5 = E\,(4+2)^2$

$-1 = 36E$

$$\boxed{\frac{-1}{36} = E}$$

By compairing coefficient method

$2x-5 = (Ax+B)(x^2+2)(x-2) + (Cx+D)(x-2) + E(x^2+2)^2$

$2x-5 = (Ax+B)(x^3-2x^2+2x-4) + Cx^2-2Cx+Dx-2D+$
$E(x^4+4x^2+4)$

$2x-5 = Ax^4-2Ax^3+2Ax^2-4Ax+Bx^3-2Bx^2+2Bx-4B$
$+Cx^2-2Cx+Dx-2D+Ex^4+4Ex^2+4E$

$2x-5 = (A+E)x^4+(-2A+B)x^3+(2A-2B+C+4E)x^2$
$+(-4A+2B-2C+D)x-4B-2D+4E$

$0 = A+E$

$0 = A - \frac{1}{36}$

$$\boxed{\frac{1}{36} = A}$$

$0 = -2A+B$

$0 = -2\left(\frac{1}{36}\right)+B$

$0 = -\frac{2}{36}+B$

$$\boxed{\frac{2}{36} = B}$$

$$0 = 2A - 2B + C + 4E$$

$$0 = 2\left(\frac{1}{36}\right) - 2\left(\frac{2}{36}\right) + C + 4\left(\frac{-1}{36}\right)$$

$$0 = \frac{2}{36} - \frac{4}{36} + C - \frac{4}{36}$$

$$0 = C + \frac{2 - 4 - 4}{36}$$

$$0 = C + \left(\frac{-6}{36}\right)$$

$$\frac{\cancel{6}\,1}{\cancel{36}\,6} = C$$

$$\boxed{\frac{1}{6} = C}$$

$$-5 = -4B - 2D + 4E$$

$$-5 = -4\left(\frac{2}{36}\right) - 2D + 4\left(\frac{-1}{36}\right)$$

$$-5 = \frac{-8}{36} - 2D - \frac{4}{36}$$

$$2D = \frac{-8}{36} - \frac{4}{36} + 5$$

$$2D = \frac{-8 - 4 + 180}{36}$$

$$2D = \frac{168}{36}$$

$D = \frac{\cancel{-168}^{\cancel{-84}\ \cancel{42}\ \cancel{21}\ 7}}{\cancel{36}\times\cancel{2}\ 1}$
$_{18\ 9\ 3}$

$$\boxed{D = \frac{7}{3}}$$

Put A, B, C, D and E in eq (i)

$$\frac{2x-5}{(x^2+2)^2(x-2)} = \frac{\left(\frac{1}{36}\right)x + \frac{2}{36}}{x^2+2} + \frac{\left(\frac{1}{6}\right)x + \frac{7}{3}}{(x^2+2)^2} - \frac{1}{36(x-2)}$$

$$\frac{2x-5}{(x^2+2)^2(x-5)} = \frac{x+2}{36(x^2+2)} + \frac{x+14}{6(x^2+2)^2} - \frac{1}{36(x-2)}$$

Q.NO.4:-

$$\frac{8x^2}{(x^2+1)^2(1-x^2)}$$

$$\frac{8x^2}{(x^2+1)^2(1-x)(1+x)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{1-x} + \frac{F}{1+x} \rightarrow (i)$$

Multiplying $(x^2+1)^2(1-x)(1+x)$ on b.s of (i)

$$8x^2 = (Ax+B)(x^2+1)(1-x)(1+x) + (Cx+D)(1-x)(1+x) + E(x^2+1)^2(1+x) + F(x^2+1)^2(1-x) \rightarrow (ii)$$

Put $1-x=0 \Rightarrow x=1$ in eq (ii)

$$8(1)^2 = E[(1)^2+1]^2[1+1]$$

$$8 = E(2)^2(2)$$

$$8 = E(4)(2)$$

$$8 = 8E$$

$$\frac{\cancel{8}^{1}}{\cancel{8}} = E$$

$$\boxed{E = 1}$$

Put $1+x=0 \Rightarrow x=-1$ in eq (ii)

$$8(-1)^2 = F[(-1)^2+1]^2(1+1)$$

$$8 = F(1+1)^2(+2)$$

$$8 = F(2)^2(+2)$$

$$8 = F(4)(+2)$$

$$8 = +8F$$

$$\frac{+\cancel{8}^{1}}{\cancel{8}_{1}} = F$$

$$\boxed{+1 = F}$$

By compairing coefficient method

$$8x^2 = (Ax+B)(x^2+1)(1-x^2)+(Cx+D)(1-x^2)+$$
$$E(x^4+2x^2+1)(1+x)+F(x^4+2x^2+1)(1-x)$$

$$8x^2 = (Ax+B)(\cancel{x^2}-x^4+1-\cancel{x^2})+Cx-Cx^3+D-Dx^2+$$
$$E(x^4+x^5+2x^2+2x^3+1+x)+F(x^4-x^5+2x^2-2x^3+1-x)$$

$$8x^2 = (Ax+B)(1-x^4)+Cx-Cx^3+D-Dx^2+Ex^4+$$
$$Ex^5+2Ex^2+2Ex^3+2E+2Ex+Fx^4-Fx^5+2Fx^2-2Fx^3+F-Fx$$

$$8x^2 = Ax-Ax^5+B-Bx^4+Cx-Cx^3+D-Dx^2+Ex^4+Ex^5+$$
$$2Ex^2+2Ex^3+2E+2Ex+Fx^4-Fx^5+2Fx^2-2Fx^3+F-Fx$$

$$8x^2 = (-A+E-F)x^5+(-B+E+F)x^4+(-C+2E-2F)x^3+$$
$$(-D+2E+2F)x^2+(A+C+2E-F)x+(B+D+2E+F)$$

$$0 = -A+E-F$$

$$A = E-F$$

$A = 1-(+1)$

$A = 1-1$

$\boxed{A = 0}$

$0 = -B+E+F$

$B = E+F$

$B = 1+1$

$\boxed{B = 2}$

$0 = -C+2E-2F$

$C = 2(1)-2(+1)$

$C = 2-2$

$\boxed{C = 0}$

$8 = -D+2E+2F$

$8 = -D+2(1)+2(+1)$

$8 = -D+2+2 \Rightarrow 8 = -D+4$

$\boxed{-4 = D}$ $\quad -D = 8-4$

$-D = 4$

Put A, B, C, D and E in eq (i)

$$\frac{8x^2}{(x^2+1)^2(1-x)(1+x)} = \frac{2}{x^2+1} - \frac{4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x}$$