

Note:- You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of the question number in your answer book. Use marker or pen to fill the circles. Cutting or filling up two or more circles will result no mark.

SECTION-A

Q.1	Questions	A	B	C	D
1.	$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} =$	$f'(x)$	$f'(0)$	$f'(x-a)$	$f'(a)$
2.	The range of $f(x) = 2 + \sqrt{x+1}$ is:	$[-1, \infty[$	$[0, \infty[$	$[2, \infty[$	$[-2, \infty[$
3.	If $f(x) = \tan x$ then $f'\left(\frac{\pi}{3}\right) =$	4	2	1	0
4.	If $f(x) = x^3 + 2x + 9$ then $f'''(0)$ is:	0	2	3	6
5.	Maclaurin series for $\frac{1}{1+x}$ is:	$1 - x + x^2 - x^3 + \dots$	$1 - x - x^2 - x^3 - x^4 \dots$	$1 + x + x^2 + x^3 + \dots$	$-1 - x - x^2 - x^3 - \dots$
6.	A function $f(x)$ is increasing in the interval (a, b) if $f(x_2) > f(x_1)$ whenever:	$x_2 > x_1$	$x_2 < x_1$	$x_2 = x_1$	$x_1 = 0, x_2 =$
7.	$\int \frac{\sin 2x}{\sin x} dx =$	$\sin 2x + c$	$2\sin 2x + c$	$\frac{1}{2}\sin x + c$	$2\sin x + c$
8.	Solution of differential equation $\frac{dy}{dx} = -y$ is:	$y = c e^{-x}$	$y = ce^x$	$y = e^{cx}$	$y = x e^{-x}$
9.	$\int \frac{e^x}{e^x - 2} dx =$	$\ln(e^x + 2) + c$	$\ln(e^x + 3) + c$	$\ln(e^x - 2) + c$	$\ln(e^x - 3) +$
10.	If $\int f(x) dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$ then $f(x) =$	$\frac{1}{\sqrt{x^2 - a^2}}$	$\frac{1}{x\sqrt{x^2 - a^2}}$	$\frac{1}{x\sqrt{x^2 + a^2}}$	$\frac{1}{x\sqrt{a^2 - x^2}}$

	Questions	A	B	C	D
11.	Equation of a line passing through $(-2, 5)$ having slope 0 is:	$y = -5$	$y = 5$	$x = -2$	$x = 2$
12.	If the distance of the point $(5, x)$ from x -axis is 3 then $x =$	7	5	3	-5
13.	The slope of the line with inclination 60° is:	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
14.	$(3, 2)$ is not in the solution of inequality:	$x + y > 2$	$x - y > 1$	$3x + 5y > 7$	$3x - 7y < 3$
15.	The vertex of the parabola $(x-1)^2 = 8(y+2)$ is:	$(1, -2)$	$(0, 1)$	$(2, 0)$	$(0, 0)$
16.	The end points of the major axis of the ellipse are called its:	Foci	Vertices	Covertices	Directrix
17.	Directrix of parabola $x^2 = 16y$ is:	$x + 4 = 0$	$x - 4 = 0$	$y - 4 = 0$	$y + 4 = 0$
18.	For any two vectors \underline{a} and \underline{b} projection of \underline{a} on \underline{b} is:	$\frac{\underline{a} \cdot \underline{b}}{ \underline{a} }$	$\frac{\underline{a} \cdot \underline{b}}{ \underline{b} }$	$\frac{\underline{a} \cdot \underline{b}}{ \underline{a} \underline{b} }$	$\underline{a} \cdot \underline{b}$
19.	The unit vector of $2\underline{i} + \underline{j}$ is:	$2\underline{i} - \underline{j}$	$\frac{2\underline{i} + \underline{j}}{5}$	$\frac{2\underline{i} + \underline{j}}{3}$	$\frac{2\underline{i} + \underline{j}}{\sqrt{5}}$
20.	If $\frac{\underline{U} \cdot \underline{V}}{ \underline{U} \underline{V} } = \frac{1}{2}$, then the angle between \underline{U} and \underline{V} is:	30°	60°	300°	90°

SECTION – B

Note: - Section B is compulsory.

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(8 x 2 = 16)

2. Write short answers to any EIGHT parts.

- i. Express the volume V of a cube as a function of area A of its base.
- ii. Prove the identity $\sec^2 x = 1 + \tan^2 x$.
- iii. Determine whether $f(x) = x^{\frac{2}{3}} + 6$ is even or odd.
- iv. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$.
- v. Differentiate $\frac{2x-1}{\sqrt{x^2+1}}$ w.r.t 'x'.
- vi. Find $\frac{dy}{dx}$ if $y^2 + x^2 - 4x = 5$.
- vii. Find $\frac{dy}{dx}$ by making suitable substitution if $y = (3x^2 - 2x + 7)^6$.
- viii. Differentiate $\sin x$ w.r.t $\cot x$.
- ix. Find $\frac{dy}{dx}$ if $y = \ln(x + \sqrt{x^2 + 1})$.
- x. Find y_2 if $y = \sqrt{x} + \frac{1}{\sqrt{x}}$.
- xi. Find the extreme value of $f(x) = x^2 - x - 2$.
- xii. Divide 20 into two parts so that the sum of their squares will be minimum.

(8 x 2 = 16)

3. Write short answers to any EIGHT parts.

- i. Using differential find $\frac{dx}{dy}$ if $xy - \ln x = c$.
- ii. Evaluate $\int (2x-3)^{\frac{1}{2}} dx$
- iii. Evaluate $\int \frac{e^x}{e^x + 3} dx$.
- iv. Find $\int x \cos x dx$.
- v. Evaluate $\int_2^{\sqrt{5}} x \sqrt{x^2 - 1} dx$.
- vi. Find the area between the x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$.
- vii. Solve the differential equation $ydx + xdy = 0$.
- viii. Find the distance between the points $A(-8, 3)$; $B(2, -1)$. Find the mid-point of the line-segment joining the given points also.
- ix. Find the point three-fifth of the way along the line-segment from $A(-5, 8)$ to $B(5, 3)$.
- x. Find the slope and inclination of the line joining the points $(-2, 4)$; $(5, 11)$.
- xi. Determine the value of p such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.
- xii. Find an equation of each of the lines represented by $20x^2 + 17xy - 24y^2 = 0$.

4. Write short answers to any NINE parts.

(9 x 2 = 18)

- i. Graph the solution set of inequality $5x - 4y \leq 20$.
- ii. Define a linear inequality.
- iii. Show that the equation $2x^2 - xy + 5x - 2y + 2 = 0$ represents a pair of lines.
- iv. If centre is $(0, 0)$, focus is $(6, 0)$ and vertex is $(4, 0)$, find the equation of hyperbola.
- v. Find the length of latus rectum of the ellipse $9x^2 + y^2 = 18$.
- vi. Find the focus and vertex of parabola $y^2 = 8x$.
- vii. Find the equation of tangent to the circle $x^2 + y^2 = 25$ at the point $(4, 3)$.
- viii. Find the centre and radius of the circle $x^2 + y^2 + 12x - 10y = 0$.
- ix. If $\vec{u} = 2\hat{i} - 7\hat{j}$, $\vec{v} = \hat{i} - 6\hat{j}$ and $\vec{w} = -\hat{i} + \hat{j}$, find $2\vec{u} - 3\vec{v} + 4\vec{w}$.
- x. Find a vector of length 5 in the direction opposite to $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$.
- xi. Find the projection of \vec{a} along \vec{b} and \vec{b} along \vec{a} when $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} - \hat{j} + \hat{k}$.
- xii. Prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$.
- xiii. A force $\vec{F} = 4\hat{i} - 3\hat{k}$ passes through the point $A(2, -2, 5)$. Find the moment of \vec{F} about the point $B(1, -3, 1)$.

SECTION-C

Note: Attempt any THREE questions. Each question carries (5+5=10) marks.

5. (a) Prove that $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$. (b) Differentiate with respect to 'x' $\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$.
6. (a) If $y = (\cos^{-1} x)^2$, prove that $(1-x^2)y_2 - xy_1 - 2 = 0$. (b) Evaluate the indefinite integral $\int \sqrt{a^2 + x^2} dx$.
7. (a) Solve the differential equation $\left(y - x \frac{dy}{dx}\right) = 2\left(y^2 + \frac{dy}{dx}\right)$. (b) Maximize $f(x, y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$; $x - y \leq 4$, $x \geq 0$; $y \geq 0$.
8. (a) Find the equation of the tangent to the circle $x^2 + y^2 = 2$ parallel to the line $x - 2y + 1 = 0$. (b) Show that mid-point of hypotenuse a right triangle is equidistant from its vertices (use vectors).
9. (a) Prove that the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$. (b) The points $(4, -2)$, $(-2, 4)$ and $(5, 5)$ are vertices of a triangle. Find in-centre of the triangle.

Note: – You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of that question number in your answer book. Use marker or pen to fill the circles. Cutting or filling up two or more circles will result no mark.

SECTION-A

Q.1	Questions	A	B	C	D
1.	Focus of parabola $x^2=4ay$ is	(a,0)	(0,a)	(-b,0)	(0,-a)
2.	Two lines l_1 and l_2 with slope m_1 and m_2 are perpendicular if:	$m_1 = m_2$	$m_1 m_2 = 1$	$m_1 m_2 + 1 = 0$	$m_1 = \frac{2}{m_2}$
3.	The lines represented by $ax^2+2hxy+by^2=0$ are parallel if:	$h^2 - ab = 0$	$h^2 - ab > 0$	$h^2 - ab < 0$	$h^2 = a + b$
4.	A solution of $x + 2y < 6$ is:	(8,0)	(0,8)	(5,1)	(1,2)
5.	The line $y = mx + c$ intersect the circle $x^2 + y^2 = a^2$ at most _____ point/s	One	Two	three	Infinite
6.	Equation of point circle is:	$x^2 + y^2 = 1$	$x^2 + y^2 = -1$	$x^2 + y^2 = 0$	$x^2 - y^2 = 0$
7.	Equation of horizontal line through (a,b) is	$y = a$	$x = a$	$x = b$	$y = b$
8.	For hyperbola value of eccentricity is:	$e = 0$	$e < 1$	$e = 1$	$e > 1$
9.	If $f(x) = e^{\sqrt{x}-1}$, then $f'(x) = \dots\dots$	$\frac{1}{2\sqrt{x}} e^{\sqrt{x}-1}$	$e^{\sqrt{x}-1}$	$\frac{1}{2x}$	$\frac{e^{\sqrt{x}-1}}{\sqrt{x}}$
10.	$\int \frac{x+2}{x+2} dx = \dots\dots$	$1+c$	$x+c$	$-x+c$	$2x$
11.	$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = \dots\dots$	1	0	-1	∞
12.	$\lim_{n \rightarrow \infty} (1 - \frac{1}{n})^n = \dots\dots$	e^{-1}	e	e^2	$\frac{1}{e^2}$
13.	If $x = at^2, y = 2at$, then $\frac{dy}{dx} = \dots\dots$	$\frac{2}{y}$	$\frac{2a}{y}$	$2ay$	$2a$
14.	$\underline{i} \cdot (\underline{i} \times \underline{k}) = \dots\dots$	\underline{i}	$-\underline{j}$	0	1
15.	If $f(x) = \cos x$, then $f'(\sin^{-1} x) = \dots\dots$	$-\sin x$	$-x$	1	$\frac{1}{\sqrt{1-x^2}}$
16.	If $y = e^{2x}$, then $y_4 = \dots\dots$	$16e^{2x}$	$8e^{2x}$	$4e^{2x}$	$-16e^{2x}$
17.	The projection of $\vec{a} = \underline{i} - \underline{j}$ along $\vec{b} = \underline{j} + \underline{k}$ is:	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	1	-2
18.	The order of differential equation $y \left(\frac{dy}{dx} \right)^2 + 2x = 0$ is:	1	-1	2	-2
19.	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos \theta d\theta = \dots\dots$	$\frac{\sqrt{3}-1}{2}$	$\frac{\sqrt{3}}{2}$	$1 + \frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} - 1$
20.	$\int \ln x dx = \dots\dots$	$(\ln x)^2 + c$	$x \ln x + c$	$x \ln x - x + c$	$-x \ln x + c$

Mathematics

H.S.S.C (12th) 1st Annual 2023

Time : 2:30 Hours

Paper : II

Subjective

Marks : 80

Note: - Section B is compulsory. Attempt any three questions from section C.

SECTION - B

2. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. Express the volume V of a cube as a function of the area of its base
- ii. For any real valued function, $g(x) = \frac{1}{\sqrt{x^2}}$; $x \neq 0$, find $g \circ g(x)$
- iii. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$
- iv. Differentiate $\frac{2x-3}{2x+1}$ w.r.t. x
- v. Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$
- vi. Differentiate $(\sin 2\theta - \cos 3\theta)^2$ w.r.t. θ
- vii. Find $f'(x)$ if $f(x) = \ln(e^x + e^{-x})$
- viii. Find y_2 if $y = x^2 e^{-x}$
- ix. Apply Maclaurin series expansion to prove that, $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots$
- x. Find the extreme values for the function, $f(x) = x^2 - x - 2$
- xi. Define objective function.
- xii. Graph the solution set of the inequality, $3x - 2y \geq 6$

3. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. Evaluate $\int \frac{3x+2}{\sqrt{x}} dx$, ($x > 0$)
- ii. Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$
- iii. Evaluate $\int \sec n \, dx$
- iv. Evaluate $\int \frac{x}{\sqrt{4+x^2}} dx$
- v. Evaluate $\int a^x x \, dx$ ($a > 0, a \neq 1$)
- vi. Evaluate $\int \frac{x+b}{(x^2+2bx+c)^{\frac{1}{2}}} dx$
- vii. Evaluate definite integral $\int_1^2 (x^2+1) dx$
- viii. Find the volume of the parallelepiped determined by, $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{w} = \underline{i} - 7\underline{j} - 4\underline{k}$
- ix. Find the value of $[\underline{i} \quad \underline{j} \quad \underline{k}]$
- x. Find the constant α such that the given vectors are coplanar. $\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 2\underline{j} - 3\underline{k}$ and $3\underline{i} - \alpha \underline{j} + 5\underline{k}$
- xi. Write the formula to find the volume of tetrahedron.
- xii. Write two properties of cross product.

(Turn Over)

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4. Write short answers to any Nine parts.

(9 x 2 = 18)

- i. Find distance between A and B, midpoint of AB, where A(3,1), B(-2,-4)
- ii. Find slope and inclination of line joining A(3,-2) and B(2,7)
- iii. By means of slope, show that (-1,-3), (1,5) and (2,9) are collinear.
- iv. Find equation of line through A(-6,5) having slope 7
- v. Find point of intersection of $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- vi. Find the lines represented by $2x^2 + 3xy - 5y^2 = 0$
- vii. Find the distance from P(6,-1) to the line $6x - 4y + 9 = 0$
- viii. Find the equation of circle with centre (5,-2) and radius 4
- ix. Write down equation of tangent and normal to $x^2 + y^2 = 25$ at (4,3)
- x. Write equation of parabola with Focus (1,2); vertex (3,2)
- xi. Find foci and eccentricity of the ellipse $x^2 + 4y^2 = 16$
- xii. Find equation of hyperbola with centre (0,0) Focus (6,0) and vertex (4,0)
- xiii. Find centre and radius of circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$



SECTION - C

Note: Attempt any Three questions. Each question carries 10 marks

5. (a) Evaluate the following limit

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$$

5

(b) Show that $\frac{dy}{dx} = \frac{y}{x}$ if $y = \tan^{-1} \frac{x}{y}$

5

6. (a) Evaluate $\int \frac{e^x(1 + \sin x)}{1 + \cos x} dx$

5

(b) Find an equation of the perpendicular bisector of the segment joining the points A(5,3) and B(9,8)

5

7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx$

5

(b) Minimize $z = 2x + y$, subject to the constraints $x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$

5

8. (a) Divide 20 into two parts so that the sum of their squares will be minimum.

5

(b) Find equations of the tangents to the circle $x^2 + y^2 = 2$ parallel to the $x - 2y + 1 = 0$

5

9. (a) Find the centre, foci, eccentricity, vertices and directrices of ellipse $x^2 + 16x + 4y^2 - 16y + 76 = 0$

5


(b) Prove that by vector method $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

5

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Note: – You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of that question number in your answer book. Use marker or pen to fill the circles. Cutting or filling up two or more circles will result no mark.

SECTION-A

Q.1	Questions	A	B	C	D
1.	$\int 2^x dx =$ 	$2^x + c$	$2^x \cdot \ln 2 + c$	$\frac{\ln 2}{2^x} + c$	$\frac{1}{\ln 2} \cdot 2^x + c$
2.	$\int_0^3 \frac{1}{9+x^2} dx =$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{\pi}{12}$	$\frac{\pi}{8}$
3.	Slope of vertical line is:	0	∞	1	-1
4.	Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, are perpendicular if:	$a_1a_2 + b_1b_2 = 0$	$a_1a_2 - b_1b_2 = 0$	$a_1b_2 + a_2b_1 = 0$	$a_1b_2 - a_2b_1 = 0$
5.	The distance of the line $4x - 3y - 25 = 0$ from the origin is:	1	5	25	2
6.	Normal form of the equation of straight line is:	$y = mx + c$	$y - y_1 = m(x - x_1)$	$\frac{x}{a} + \frac{y}{b} = 1$	$x \cos \alpha + y \sin \alpha = p$
7.	$2x + y < 6$ is satisfied by which point?	(3,1)	(1,3)	(0,7)	(4,0)
8.	Equation of the tangent to the circle $x^2 + y^2 = 4$ at (1,3) is:	$x + 3y = 4$	$x - 3y = 4$	$3x + y = 4$	$3x - y = 4$
9.	$\sinh^{-1} x =$	$\frac{e^x + e^{-x}}{2}$	$\frac{e^x - e^{-x}}{2}$	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	$\frac{e^x + e^{-x}}{e^x - e^{-x}}$
10.	If $g(x) = \frac{1}{x^2}$, $x \neq 0$, then $g \circ g(x) =$	x^2	$\frac{1}{x^2}$	x^4	$\frac{1}{x^4}$
11.	Derivative of $(x^3 + 1)^9$ w.r.t. x^3 equals:	$9(x^3 + 1)^8$	$27x^2(x^3 + 1)^8$	$3x(x^3 + 1)^8$	$27(x^3 + 1)^8$
12.	If $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ exists, then it is equal to:	$f'(x)$	$f'(a)$	zero	∞
13.	Length of latus-rectum of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is:	$\frac{2a^2}{b}$	$\frac{2b^2}{a}$	$\frac{b^2}{2a}$	$\frac{b}{2a^2}$
14.	Centre of the Hyperbola $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{4} = 1$, is:	(1,2)	(2,1)	(-1,2)	(1,-2)
15.	Unit vector perpendicular to \underline{a} and \underline{b} is:	$\frac{\underline{a} \times \underline{b}}{ \underline{a} \underline{b} }$	$\frac{\underline{a} \cdot \underline{b}}{ \underline{a} \underline{b} }$	$\frac{\underline{a} \cdot \underline{b}}{\underline{a} \times \underline{b}}$	$\frac{\underline{a} \times \underline{b}}{ \underline{a} \times \underline{b} }$
16.	$2\hat{i} \cdot \hat{j} \times \hat{k} =$	2	Zero	$\frac{1}{2}$	∞
17.	$\int e^{\tan^{-1} x} \cdot \frac{1}{1+x^2} dx =$	$e^{\tan^{-1} x} + c$	$\frac{1}{1+x^2} + c$	$e^{\cos^{-1} x} + c$	$e^{\sec^2 x} + c$
18.	$\int e^x (1+x) dx =$	$e^x + c$	$x e^x + c$	$x^2 + c$	$\frac{1}{2} x^2 e^x + c$
19.	If $y = \ln(x^2)$, then $\frac{dy}{dx} =$	$\frac{1}{x^2}$	$\frac{1}{2x^2}$	$\frac{2}{x}$	$\frac{2}{x^2}$
20.	$\frac{d}{dx}(\cos x^2) =$	$-\sin x^2$	$-2x \sin x^2$	$2x \sin x^2$	$-x \sin x^2$

Note: - Section B is compulsory. Attempt any three questions from section C.

SECTION - B

2. Write short answers to any Eight parts.

(8 x 2 = 16)

- Determine whether the function $f(x) = \sin x + \cos x$ is even or odd.
- Find the composition function $f \circ f(x)$ if $f(x) = \frac{1}{\sqrt{x-1}}$
- Express $\lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^{2n}$ in term of "e".
- What is the implicit function?
- Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$
- Differentiate $\frac{2x-3}{2x+1}$ w.r.t. x
- Find $\frac{dy}{dx}$ if $y^2 - xy - x^2 + 4 = 0$
- Find derivative of $(\sin 2\theta - \cos 3\theta)^2$ w.r.t θ
- If $y = x^2 \ln\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$
- Find y_2 if $y = x^2 e^{-x}$
- Find the intervals in which $f(x) = 4 - x^2; x \in (-2, 2)$ is increasing or decreasing.
- Differentiate $\log_{10}(ax^2 + bx + c)$ w.r.t. x

3. Write short answers to any Eight parts.

(8 x 2 = 16)

- Using differentials find $\frac{dy}{dx}$ in the equation $xy + x = 4$
- Evaluate $\int \sin^2 x \, dx$
- Evaluate $\int \frac{\sec^2 x}{\sqrt{\tan x}} \, dx$
- Evaluate $\int \tan^{-1} x \, dx$
- Evaluate $\int e^{-x} (\cos x - \sin x) \, dx$
- Evaluate $\int \frac{3x+1}{x^2 - x + 6} \, dx$
- Evaluate $\int_1^2 \ln x \, dx$
- Find the area between the x -axis and the curve $y = 4x - x^2$.
- Show that the points $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.
- Find an equation of line passing through $A(-5, -3)$ and $B(9, -1)$.
- Find an equation of the line through $(-4, 7)$ and parallel to the line $2x - 7y + 4 = 0$
- Find lines represented by $3x^2 + 7xy + 2y^2 = 0$

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4. Write short answers to any Nine parts.

(9 x 2 = 18)



- i. How would you obtain the optimal solution from feasible region?
- ii. Indicate the solution region of $3x - 2y \geq 6$
- iii. Find an equation of the circle passing through $A(1,4), B(-1,8)$ and tangent to the line $x + 3y - 3 = 0$
- iv. Write down the equation of normal to circle $3x^2 + 3y^2 + 5x - 13y + 2 = 0$ at $\left(1, \frac{10}{3}\right)$.
- v. Investigate vertex and directrix of $x + 8 - y^2 + 2y = 0$
- vi. Form the equation of ellipse from that data, foci $(\pm 3, 0)$ and minor axis of length 10.
- vii. Find foci and eccentricity of $\frac{y^2}{16} - \frac{x^2}{9} = 1$
- viii. Use vectors, to prove that the diagonals of a parallelogram bisect each other.
- ix. For what values of a and b from the given parallel vectors $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $a\mathbf{i} + b\mathbf{j} - 2\mathbf{k}$
- x. Prove that in any triangle ABC: $a = b \cos C + c \cos B$
- xi. Find area of parallelogram, where vertices are $A(-1,1,1), B(-1,2,2), C(-3,4,-5)$ and $D(-3,5,-4)$.
- xii. Find α from the given coplanar vectors $\mathbf{i} - 2\alpha\mathbf{j} - \mathbf{k}$, $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\alpha\mathbf{i} - \mathbf{j} + \mathbf{k}$
- xiii. A force $\mathbf{F} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ is applied at the point $(1, -1, 2)$. Find the moment of the force about the point $(2, -1, 3)$

SECTION - C

(Each question carries 10 marks)

5. (a) If $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$ 5
- Discuss the continuity at $x = 2$ and $x = -2$
- (b) Differentiate $\frac{x^2 + 1}{x^2 - 1}$ w.r.t $\frac{x - 1}{x + 1}$ 5
6. (a) Evaluate $\int \frac{dx}{\sqrt{7 - 6x - x^2}}$ 5
- (b) Find h such that the points $A(h, 1), B(2, 7)$ and $C(-6, -7)$ are vertices of a right triangle. 5
7. (a) Find area between the x -axis and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$ 5
- (b) Graph the feasible region of the following system of linear inequality and find corner points
 $2x + 3y \leq 18, 2x + y \leq 10, x + 4y \leq 12, x \geq 0, y \geq 0$ 5
8. (a) Find a joint equation of the straight lines through the origin and perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$ 5
- (b) Find an equation of the parabola whose focus is $F(-3, 4)$ and directrix is $3x - 4y + 5 = 0$ 5
9. (a) Find centre, foci, eccentricity and vertices of hyperbola $4x^2 - 8x - y^2 - 2y - 1 = 0$ 5
- (b) Prove that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ 5

Note: – You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of that question number in your answer book. Use marker or pen to fill the circles. Cutting or filling up two or more circles will result no mark.

Q.1	Questions	A	B	C	D
1.	The conic is called circle if:	$e = 1$	$e < 1$	$e = 0$	$e > 1$
2.	The direction cosines of Z – axis are:	$(1, 0, 0)$	$(0, 1, 0)$	$(0, 0, 1)$	$(0, 0, 0)$
3.	Angle between nonzero vectors \underline{a} and $\underline{a} \times \underline{b}$ is:	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
4.	Distance between $(-1, 2)$ and $(7, 5)$ is:	± 7	$\sqrt{73}$	73	$\sqrt[3]{73}$
5.	Opening of parabola $x^2 = -16y$ is:	downward	upward	left	right
6.	If \underline{u} and \underline{v} are parallel vectors having same direction then $\underline{u} \cdot \underline{v}$ is equal to:	$-uv$	uv	$uv \sin \theta$	$uv \tan \theta$
7.	If $f(x) = \sqrt{x+4}$ then $f(x-1)$ is equal to:	$\sqrt{x+4}$	$\sqrt{x+3}$	$\sqrt{x+2}$	$\sqrt{x+1}$
8.	The distance of point $(1, -2)$ from Y – axis is:	2	3	4	1
9.	Vertices of $\frac{y^2}{16} - \frac{x^2}{49} = 1$ are:	$(\pm 4, 0)$	$(0, \pm 4)$	$(0, \pm 7)$	$(\pm 7, 0)$
10.	Inclination of line perpendicular to Y – axis is:	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	zero
11.	$\frac{d}{dx}(\sin^2 x + \cos^2 x)$ is equal to:	zero	1	2	3
12.	$\int e^x (x+1) dx$ is equal to:	$e^x + c$	$x e^x + c$	$x^2 e^x + c$	$\frac{x e^x}{2} + c$
13.	$\int_0^{\pi} \cos x dx$ is equal to:	2	-1	zero	2
14.	If $V = x^3$ then differential of V is:	$3x^2$	$3x^2 dv$	$x^3 dv$	$3x^2 dx$
15.	If $f'(c) = 0$, then f has relative minima at C if $f''(c)$ is:	negative	zero	any value	positive
16.	$\frac{d}{dx}(\sin^{-1} x)$ is equal to:	$\frac{-1}{\sqrt{1-x^2}}$	$\frac{1}{\sqrt{x^2-1}}$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{-1}{\sqrt{x^2-1}}$
17.	The general solution of $\frac{1}{x} \frac{dy}{dx} - 2y = 0$ is:	$y = c e^x$	$c e^x$	$c e^{-x}$	$c e^{\frac{y}{x}}$
18.	$\frac{d}{dx} \left[\sin \left(\frac{1}{x} \right) \right]$ is equal to:	$x \cos \frac{1}{x}$	$\frac{-1}{x^2} \cos \frac{1}{x}$	$\frac{1}{x^2} \cos \frac{1}{x}$	$\frac{-1}{x} \cos \frac{1}{x}$
19.	$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{3n} \right)^n$ is equal to:	$e^{\frac{1}{3}}$	e	e^2	e^3
20.	$x = 0$ is solution of inequality:	$2x - 1 < 0$	$2x + 1 < 0$	$x < 0$	$x < -1$

Note: - Section I is compulsory. Attempt any three questions from section II.

Section - I

2. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. Given $f(x) = x^3 - ax^2 + bx + 1$. if $f(2) = -3$ and $f(-1) = 0$. Find the value of a and b .
- ii. Find $f^{-1}(x)$ if $f(x) = (-x+9)^3$
- iii. Express the $\lim_{n \rightarrow +\infty} \left(1 + \frac{3}{n}\right)^{2n}$ in terms of "e".
- iv. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- v. Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t "x"
- vi. Find $\frac{dy}{dx}$ if $x = at^2$ and $y = 2at$
- vii. Prove that $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
- viii. Find $\frac{dy}{dx}$ if $y = x \cos y$
- ix. Find $\frac{dy}{dx}$ if $y = e^{-x}(x^3 + 2x^2 + 1)$
- x. Find $\frac{dy}{dx}$ if $y = e^{-2x} \cdot \sin 2x$
- xi. Find y_2 if $y = 2x^5 - 3x^4 + 4x^3 + x - 2$
- xii. Apply the Maclaurin series expansion to prove that $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$

3. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. Find dy if $y = x^2 - 1$, when x changes from 3 to 3.02
- ii. Evaluate $\int \frac{3x+2}{\sqrt{x}} dx$ ($x > 0$)
- iii. Evaluate $\int \frac{1-x^2}{1+x^2} dx$
- iv. Evaluate $\int \frac{1}{x \ln x} dx$
- v. Evaluate $\int x^4 \ln x dx$
- vi. Evaluate $\int_{-1}^1 (x^{1/5} + 1) dx$
- vii. Find the area above the x -axis and under the curve $y = 5 - x^2$ from $x = -1$ to $x = 2$
- viii. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$
- ix. Find an equation of line through $A(-6, 5)$ having slope 7.
- x. Find the lines represented by $x^2 - 2xy \sec \alpha + y^2 = 0$
- xi. The points $A(-5, -2)$ and $B(5, -4)$ are ends of diameter of a circle. Find the centre of that circle.
- xii. Check whether the point $(-7, 6)$ lies above or below the line $4x + 3y - 9 = 0$

Sahiwal Board-2021

4. Write short answers to any Nine parts.

(9 x 2 = 18)

- i. Graph the solution set of the linear inequality in xy -plane given by $2x + y \leq 6$.
- ii. Find the equation of the circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$.
- iii. Show that the circles $x^2 + y^2 + 2x - 2y - 7 = 0$ and $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally.
- iv. Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$.
- v. Write the equation of a parabola with focus $(-3, 1)$ and directrix $x = 3$.
- vi. Find the centre and foci of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
- vii. Find a unit vector in the direction of the vector $\underline{v} = [-2, 4]$.
- viii. Find equation of normal to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a \cos \theta, b \sin \theta)$.
- ix. Find α so that $|\alpha \underline{i} + (\alpha + 1)\underline{j} + 2\underline{k}| = 3$.
- x. Find the cosine of the angle between $\underline{u} = [2, -3, 1]$ and $\underline{v} = [2, 4, 1]$.
- xi. Compute the product $\underline{a} \times \underline{b}$, $\underline{a} = -4\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} + \underline{k}$.
- xii. Find α so that the vectors $\alpha \underline{i} + \underline{j}$, $\underline{i} + \underline{j} + 3\underline{k}$ and $2\underline{i} + \underline{j} - 2\underline{k}$ are coplanar.
- xiii. Find equation of tangent to $x^2 - 2y^2 = 2$ through $(1, -2)$.

Section - II

(Each question carries 10 marks)

5. (a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$ 5
- (b) If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$, show that $a \frac{dy}{dx} + b \tan \theta = 0$ 5
6. (a) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$ 5
- (b) Find an equation of the line through the intersection of the lines $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercepts on the axes. 5
7. (a) Solve the differential equation $\sec x + \tan y \frac{dy}{dx} = 0$ 5
- (b) Minimize $z = 3x + y$; subject to the constraints:
 $3x + 5y \geq 15$; $x + 6y \geq 9$ $x \geq 0$; $y \geq 0$ 5
8. (a) Write equation of circle that passes through the points $A(5, 6)$, $B(-3, 2)$ and $C(3, -4)$. 5
- (b) Prove that: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ 5
9. (a) If $y = e^x \sin x$, then show that $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$ 5
- (b) Find centre, foci, eccentricity and vertices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$ 5

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Roll No. _____ Annual 2019

Mathematics
Paper : II

(INTER PART II CLASS 12th) - (I)
OBJECTIVE

Time :30 Minutes
Marks : 20

Code : 8191

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number with marker or pen. Cutting or filling two or more circles will result in zero mark in that question.

1. $\lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\theta} =$

- (A) 1 (B) ∞ (C) $\frac{0}{0}$ (D) $\frac{0}{0}$

2. Parametric equations $x = at$, $y = 2at$ represent

- (A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $y^2 = 4ax$ (D) $x^2 + y^2 = a^2$

3. If $y = x^3$ then dy is

- (A) $2x$ (B) $2x dx$ (C) $3x^2 dx$ (D) $2x^2$

4. If $f'(c) = 0$, then $f(x)$ is minimum at $x = c$ if

- (A) $f''(c) > 0$ (B) $f''(c) < 0$ (C) $f''(c) = 0$ (D) $f''(c) < -1$

5. $\frac{d}{dx} (\cos x^3) =$

- (A) $\sin x^3$ (B) $-\sin x^3$ (C) $2x \sin x^2$ (D) $-2x \sin x^2$

6. $\frac{d}{dx} \left(\frac{a}{x} \right) =$

- (A) a (B) $\frac{1}{x}$ (C) $\frac{a}{x^2}$ (D) $-\frac{a}{x^2}$

7. The order of differential equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + 2 = 0$ is

- (A) 1 (B) 2 (C) 3 (D) 4

8. $\int \ln x dx =$

- (A) $x \ln x - x$ (B) $x \ln x + x$ (C) $x - x \ln x$ (D) $-x \ln x - x$

9. If $\int_{-1}^3 f(x) dx = 5$, then $\int_5^{-1} f(x) dx =$

- (A) $\frac{1}{5}$ (B) $-\frac{1}{5}$ (C) -5 (D) 5

(Turn Over)

Sahiwal Board-2019

(2)



10. $\int 3 \sin 3x dx =$
 (A) $\cos 3x$ (B) $-\cos 3x$ (C) $a \sin 3x$ (D) $9 \cos 3x$
11. Slope – intercept form of line is
 (A) $y - y_1 = m(x - x_1)$ (B) $\frac{x}{a} + \frac{y}{b} = 1$ (C) $y = mx + c$ (D) $x \cos \alpha + y \sin \alpha = p$
12. Slope of line AB, A(1,2), B(1,4)
 (A) 0 (B) 1 (C) 2 (D) undefined
13. Distance of A (1, 1) from origin is
 (A) 2 (B) $\sqrt{2}$ (C) 0 (D) 1
14. Equation of vertical line is
 (A) $y = c$ (B) $y = -c$ (C) $x = c$ (D) $y = x$
15. (1, 1) is solution of
 (A) $x + y < 1$ (B) $2x + y < 1$ (C) $2x - y < 1$ (D) $x - y < 1$
16. Radius of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
 (A) $\sqrt{g^2 + f^2 - c}$ (B) $\sqrt{g^2 + f^2 + c}$ (C) $\sqrt{g^2 + f^2 - c^2}$ (D) $\sqrt{g^2 + f^2 + c^2}$
17. Axis of parabola $y^2 = 4ax$ is
 (A) $y = 0$ (B) $x = 0$ (C) $y = a$ (D) $x = a$
18. Eccentricity e of circle is
 (A) $e < 1$ (B) $e = 1$ (C) $e > 1$ (D) $e = 0$
19. Two vectors \underline{u} and \underline{v} are perpendicular if
 (A) $\underline{u} \cdot \underline{v} = 0$ (B) $\underline{u} \times \underline{v} = 0$ (C) $\underline{u} = t\underline{v}$ (D) $\underline{u} + \underline{v} = 0$
20. $2\hat{i} \times 2\hat{j} =$
 (A) $4\hat{i}$ (B) 4 (C) $4\hat{k}$ (D) 0

309- 419 – 14500 *

Note :- Section I is compulsory. Attempt any three Questions from section II.

Section = I

2. Write short answers to any Eight parts. (8 x 2 = 16)

- Express the perimeter p of a square as a function of its area A .
- For the function $f(x) = -2x + 8$, find $f^{-1}(x)$.
- Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$.
- Differentiate $\frac{2x-3}{2x+1}$ w.r.t x .
- Differentiate $\sin^2 x$ w.r.t $\cos^4 x$.
- Differentiate $(\ln x)^x$ w.r.t x .
- Find $f'(x)$ if $f(x) = \frac{e^x}{e^{-x}+1}$.
- Find $\frac{dy}{dx}$ if $y = x^2 \ln \sqrt{x}$.
- Find y_2 if $y = \ln \left(\frac{2x+3}{3x+2} \right)$.
- Find $\frac{dy}{dx}$ if $y = \ln \sqrt{\frac{x^2-1}{x^2+1}}$.
- Find $\frac{dy}{dx}$ if $y = \cosh^{-1}(\sec x)$ $0 \leq x \leq \pi/2$.
- Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$.

3. Write short answers to any Eight parts. (8 x 2 = 16)

- Evaluate $\int \frac{\sqrt{y(y+1)}}{y} dy$.
- Evaluate $\int \frac{e^x}{e^x+3} dx$.
- Evaluate $\int x \ln x dx$.
- Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t dt$.
- Give the definition of differential equation and write an example.
- Evaluate $\int (a-2x)^{\frac{2}{3}} dx$.
- Evaluate $\int e^x \left(\frac{1}{x} + \ln x \right) dx$.
- Find area bounded by $y = x^2 + 1$ and x -axis, from $x=1$ to $x=2$.
- Write any two properties of definite integrals.
- Solve the differential equation $\frac{dy}{dx} = -y$.
- Define objective function.
- Show that the ordered pair $(1, 1)$ is a solution of the inequality $x + 2y < 6$.

(Turn Over)

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(2)

4. Write short answers to any Nine parts. (9 x 2 = 18)

- i. The points A (-5, -2) and B (5, -4) are ends of a diameter of a circle. Find the centre and radius of the circle.
- ii. Define latus rectum of parabola.
- iii. Find point of intersection of lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$.
- iv. Find whether the given point lies above or below the line (5, 8) ; $2x - 3y + 6 = 0$.
- v. Find the centre and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- vi. Find length of tangent drawn from the point (-5, 4) to circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$
- vii. Find focus and vertex of parabola $x^2 = 5y$
- viii. Find an equation of ellipse with foci (0, -1) and (0, -5) and major axis of length 6.
- ix. Compute the cross product $\underline{a} \times \underline{b}$ where $\underline{a} = 3\underline{i} - 2\underline{j} + \underline{k}$, $\underline{b} = \underline{i} + \underline{j}$
- x. Define cross product of two vectors.
- xi. Prove that $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v}) = 3\underline{u} \cdot (\underline{v} \times \underline{w})$.
- xii. Find direction cosines of vector \overline{PQ} where P(2,1,5) and Q(6,3,1)
- xiii. Find volume of parallelepiped for which given vectors are three edges.
 $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{v} = 2\underline{i} - \underline{j} - \underline{k}$, $\underline{w} = \underline{j} + \underline{k}$

Section = II

Note : Attempt any three questions

(10 x 3 = 30)

5. (a) If θ is measured in radian, then prove $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$
- (b) Show that $y = x^x$ has minimum value at $x = \frac{1}{e}$.
6. (a) Evaluate $\int \sqrt{1 + \sin x} \, dx$, $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$
- (b) Find distance between parallel lines $x + 2y - 5 = 0$, $2x + 4y = 1$ also find equation of parallel line which is lying mid way between them.
7. (a) Find the area between the x -axis and the curve $y = \sqrt{2ax - x^2}$; when $a > 0$.
- (b) Minimize $f(x, y) = 2x + y$; subject to the constraints $x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$
8. (a) Find a joint equation of the straight lines through the origin perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$
- (b) Show that the circles $x^2 + y^2 + 2x - 8 = 0$ and $x^2 + y^2 - 6x + 6y - 46 = 0$ touch internally.
9. (a) Find the centre, foci, eccentricity vertices and directrices of the ellipse whose equation is given $25x^2 + 9y^2 = 225$
- (b) The position vectors of the points A, B, C and D are $2\underline{i} - \underline{j} + \underline{k}$, $3\underline{i} + \underline{j}$, $2\underline{i} + 4\underline{j} - 2\underline{k}$ and $-\underline{i} - 2\underline{j} + \underline{k}$ respectively. Show that \overline{AB} is parallel to \overline{CD}




Code : 8195

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number with marker or pen. Cutting or filling two or more circles will result in zero mark in that question.

- i. If $\int_2^k 2x \cdot dx = 12$ then $K =$
 (A) 2, -2 (B) 2, 6 (C) 4, -4 (D) 4, 2
- ii. Distance of the point $P(x, y)$ from y -axis is
 (A) $|x|$ (B) $|y|$ (C) x (D) y
- iii. Y - co-ordinate of centroid of the triangle with vertices A(-2, 3) B(-4, 1) C(3, 5) is
 (A) 9 (B) 3 (C) $9/2$ (D) $3/2$
- iv. The line $ax + by + c = 0$ is parallel to x -axis if
 (A) $a = 0$ (B) $b = 0$ (C) $c = 0$ (D) $b = c$
- v. Equation of a line passing through (5, -7) having slope undefined is
 (A) $y = -7$ (B) $x = 5$ (C) $x = -5$ (D) $y = 7$
- vi. Length of the diameter of the circle $(x+5)^2 + (y-8)^2 = 12$ is
 (A) $4\sqrt{3}$ (B) $2\sqrt{3}$ (C) 12 (D) 24
- vii. (3, 2) is not in the solution of inequality
 (A) $x + y > 2$ (B) $x - y > 1$ (C) $3x + 5y > 7$ (D) $3x - 7y < 3$
- viii. The length of latus rectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ is
 (A) $\frac{25}{6}$ (B) $\frac{25}{3}$ (C) $\frac{25}{36}$ (D) $\frac{3}{25}$
- ix. Length of the major and minor axes of the ellipse $x^2 + 16y^2 = 16$ is
 (A) 4, 1 (B) 10, 5 (C) 8, 2 (D) 16, 2
- x. Projection of a vector \underline{v} along vector \underline{u} is
 (A) $\frac{\underline{u} \times \underline{v}}{|\underline{v}|}$ (B) $\frac{\underline{u} \cdot \underline{v}}{|\underline{u}|}$ (C) $\frac{\underline{u} \cdot \underline{v}}{\underline{u}}$ (D) $\frac{\underline{u} \cdot \underline{v}}{|\underline{v}|}$
- xi. $[\hat{i} \hat{j} \hat{k}] =$
 (A) 1 (B) 2 (C) 0 (D) -1
- xii. $\int \frac{\sec^2 x}{\sqrt{\tan x}} \cdot dx =$
 (A) $\log_e \tan x + c$ (B) $\log_e \sqrt{\tan x} + c$ (C) $2\sqrt{\tan x} + c$ (D) $2\sqrt{\sec x} + c$

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xiii.

If $\int f(x).dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$ then $f(x) =$ 

- (A) $\frac{1}{\sqrt{x^2 - a^2}}$ (B) $\frac{1}{x\sqrt{x^2 - a^2}}$ (C) $\frac{1}{x\sqrt{x^2 + a^2}}$ (D) $\frac{1}{x\sqrt{a^2 - x^2}}$

xiv. $\frac{d}{dx} \left(\frac{1}{g(x)} \right) =$

- (A) $(g(x))^{-2} \cdot g'(x)$ (B) $-1(g'(x))^{-2} \cdot g(x)$ (C) $(-1(g'(x))^{-2} \cdot g'(x)$ (D) $(-1)(g(x))^{-2} \cdot g'(x)$

xv. If $f'(a - \varepsilon) < 0$ and $f'(a + \varepsilon) > 0$ then at $x = a$ there is

- (A) relative maxima (B) relative minima (C) point of inflection (D) critical point

xvi. $1 + 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots =$

- (A) $\sin x$ (B) $\cos x$ (C) e^x (D) e

xvii. $\ln \left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|} \right) =$; $x \neq 0$

- (A) $\coth^{-1} x$ (B) $\tanh^{-1} x$ (C) $\operatorname{sech}^{-1} x$ (D) $\operatorname{cosech}^{-1} x$

xviii. The range of $f(x) = 2 + \sqrt{x-1}$ is

- (A) $[-1, \infty)$ (B) $[0, \infty)$ (C) $[2, \infty)$ (D) $(-2, \infty)$

xix. If $\frac{1}{\sqrt{x^2 - 1}} = f'(x)$ then $f(x) =$

- (A) $\cos^{-1} x$ (B) $\sinh^{-1} x$ (C) $\cosh^{-1} x$ (D) $\operatorname{cosech}^{-1} x$

xx. $\int \cot^{-1} \sqrt{\frac{1+\cos x}{1-\cos x}} . dx$

- (A) $\frac{x^2}{2} + c$ (B) $\frac{x^2}{4} + c$ (C) $\frac{x^4}{2} + c$ (D) $\frac{x}{4} + c$

Note :- Section I is compulsory. Attempt any three Questions from section II.

Section = I

2. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. $f(x) = \frac{x}{x^2 - 4}$, find the domain and range of $f(x)$.
- ii. Prove the identities $\cosh^2 x - \sinh^2 x = 1$
- iii. Find $f \circ g(x)$ if $f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{x^2}$, $x \neq 1$
- iv. Define derivative of a function.
- v. If $y = \sqrt{x+2}$ find dy/dx from first principle.
- vi. Differentiate $\frac{x^2+1}{x^2-3}$ w.r to "x".
- vii. Differentiate w. r. to "x" $(x-5)(3-x)$
- viii. Find dy/dx if $x = at^2$ and $y = 2at$.
- ix. Find dy/dx if $3x + 4y + 7 = 0$
- x. Prove that $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, $x \in (-1, 1)$.
- xi. Differentiate $\sin^3 x$ w.r.to $\cos^2 x$
- xii. Find $f'(x)$ if $f(x) = e^{\sqrt{x-1}}$

3. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. Find δy and dy if $y = x^2 + 2x$ when x changes from 2 to 1.8.
- ii. Evaluate $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$, $x > 0$
- iii. Evaluate $\int \frac{ax+b}{ax^2+2bx+c} dx$.
- iv. Evaluate $\int \frac{x^2}{4+x^2} dx$
- v. Evaluate $\int \frac{1}{(1+x^2) \tan^{-1} x} dx$
- vi. Evaluate $\int x \ln x dx$
- vii. Evaluate $\int \frac{xe^x}{(1+x)^2} dx$
- viii. Evaluate $\int_0^{\pi/4} \sec x (\sec x + \tan x) dx$
- ix. Find the area bounded by the curve $y = x^3 + 2x^2$ and x -axis.
- x. Solve $ydx + xdy = 0$
- xi. Define a corner point or vertex of solution region.
- xii. Graph the inequality $x + 2y < 6$.

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(2)

4. Write short answers to any Nine parts. (9 x 2 = 18)

- i. Find h such that the points $A(h, 1)$, $B(2, 7)$ and $C(-6, -7)$ are vertices of a right triangle with right angle at vertex A .
- ii. Find the point three-fifth of the way along the line segment from $A(-5, 8)$ to $B(5, 3)$.
- iii. Find the equation of the line through $(-4, -6)$ and perpendicular to a line having slope $\frac{-3}{2}$.
- iv. Find the area of the region bounded by the triangle with vertices $(a, b+c)$, $(a, b-c)$ and $(-a, c)$.
- v. Show that lines $4x-3y-8=0$, $3x-4y-6=0$ and $x-y-2=0$ are concurrent.
- vi. Find the direction cosines of vector $\underline{v} = 4\underline{i} - 5\underline{j}$
- vii. Calculate the projection of the vector $\underline{a} = \underline{i} - \underline{k}$ along vector $\underline{b} = \underline{j} + \underline{k}$.
- viii. Find area of parallelogram whose vertices are $P(0, 0, 0)$, $Q(-1, 2, 4)$, $R(2, -1, 4)$, $S(1, 1, 8)$.
- ix. Find value of " α " so that $\alpha\underline{i} + \underline{j}$, $\underline{i} + \underline{j} + 3\underline{k}$ and $2\underline{i} + \underline{j} - 2\underline{k}$ are coplaner.
- x. Find vertex and directrix of the parabola, $x^2 = 4(y-1)$.
- xi. Find equation of the parabola with focus $(2, 2)$ and directrix $x = -2$.
- xii. Find equation of ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.
- xiii. Find the foci and vertices of the ellipse $25x^2 + 9y^2 = 225$.

Section = II

Note : Attempt any three questions

(10 x 3 = 30)

5. (a) Find m and n so that the given function " f " is continuous at $x = 3$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$
- (b) If $y = (\cos^{-1} x)^2$, prove that $(1-x^2) y_2 - x y_1 - 2 = 0$
6. (a) Evaluate the indefinite integral using partial fraction $\int \frac{2x^3 - 3x^2 - x - 7}{2x^2 - 3x - 2} dx$
- (b) Find a joint equation of the lines through the origin and perpendicular to the lines represented by $x^2 - 2xy \tan \alpha - y^2 = 0$
7. (a) Find the area between the x -axis and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$.
- (b) Minimize $z = 3x + y$; subject to the constraints $3x + 5y \geq 15$; $x + 6y \geq 9$, $x \geq 0$, $y \geq 0$.
8. (a) Write an equation of the circle that passes through the given points $A(-7, 7)$, $B(5, -1)$, $C(10, 0)$
- (b) Prove that in any triangle $\triangle ABC$, $C = a \cos B + b \cos A$.
9. (a) Find the centre, foci, eccentricity vertices and equations of directrices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$
- (b) Find a unit vector perpendicular to the plane containing \underline{a} and \underline{b} . Also find the "sine" of the angle between them.