F.Sc Mathematics Part-I

Ch#9 (Fundamentals of Trigonometry)

Exercise#9.3

QNO#5: Find the Values of the trigonometric functions of the following quadrantal Angles.

(i) $-\pi$

نوٹ: (i) ہم اپنے دیئے ہوئے Angle کو π کے even multiples میں تبدیل کر کے لکھ لیا کریں گے۔

(ii) اگر ہم even multiples کا کوئی لے تو پھر اسے Ignore کر دیں گے۔

(i) $-\pi = \pi - 2\pi$

As we Know that

$\boxed{\theta + 2k\pi = \theta}$, $k \in Z$

θ میں 2π کو جمع کرنے سے جواب θ آتا ہے کیونکہ یہ coterminal Angle ہے اس لیے ہمارے سوال میں یہی concept کا استعمال ہوگا۔

$\sin(-\pi) = \sin\pi = 0$

$\cos(-\pi) = \cos\pi = -1$

$\tan(-\pi) = +\tan\pi = 0$

$\operatorname{cosec}(-\pi) = \dfrac{1}{\sin\pi} = \dfrac{1}{0} = \infty$

$\sec(-\pi) = \dfrac{1}{\cos\pi} = \dfrac{1}{-1} = -1$

$\cot(-\pi) = \dfrac{1}{\tan\pi} = \dfrac{1}{0} = \infty$

Key Points:

Quadrantal Angle:

If the terminal side of an angle falls on x-axis or y-axis, it is called a Quadrantal Angle.

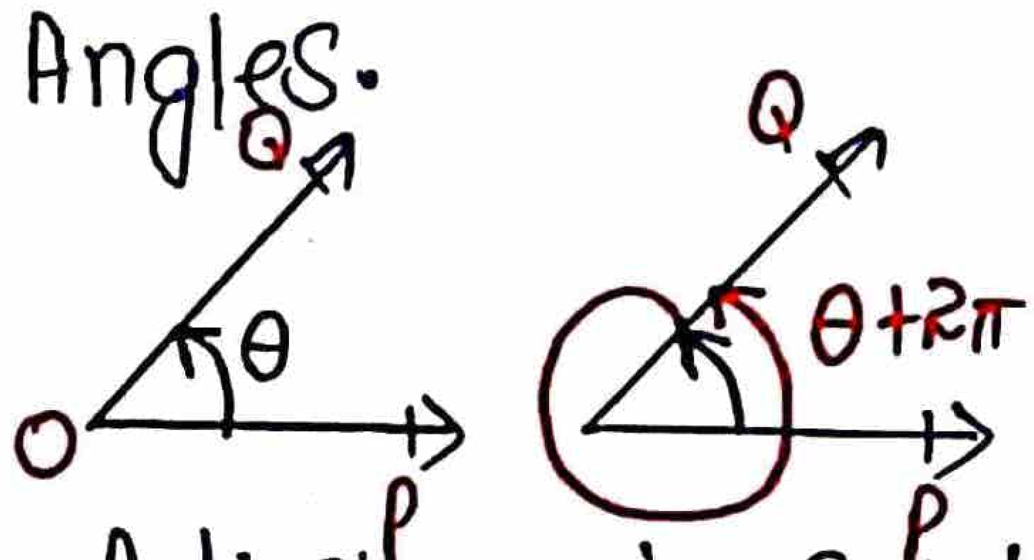
i-e (Id est) مثال کے طور پر یا دوسرے الفاظ میں $90^\circ, 180^\circ, 270^\circ$ and 360° are Quadrantal Angles.

e.g (Exempli Gratia)

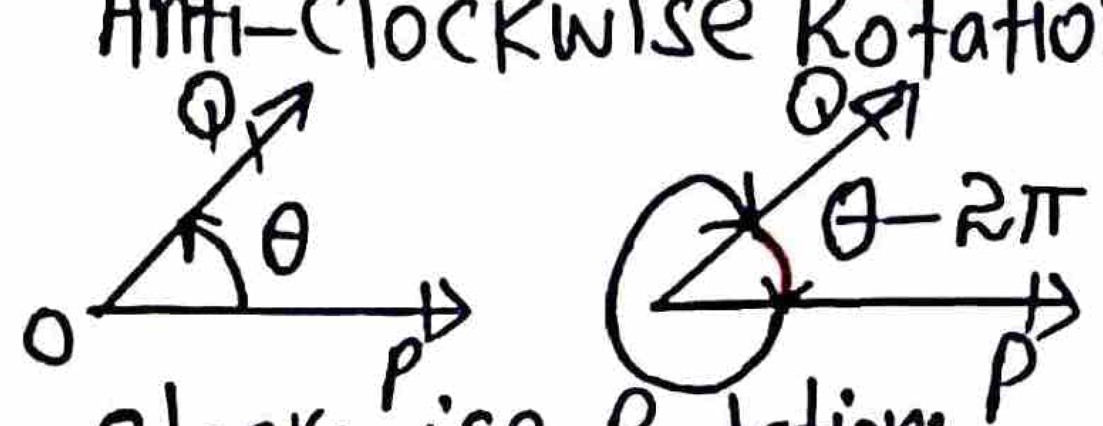
i-e and e.g both are Latin abbreviations.

Coterminal Angles:

There can be many angles with the same initial and terminal sides. These are called Coterminal Angles.



Anti-clockwise Rotation



Clockwise Rotation:

General Angle: The Angle $\theta + 2k\pi$, $k \in Z$ is called General Angle.

(ii) -3π

$$-3\pi = \pi - 4\pi$$

As we know that

$$\theta + 2k\pi = \theta, \quad k \in \mathbb{Z}$$

$$\sin(-3\pi) = \sin\pi = 0$$

$$\cos(-3\pi) = \cos\pi = -1$$

$$\tan(-3\pi) = \tan\pi = 0$$

$$\operatorname{cosec}(-3\pi) = \operatorname{cosec}\pi = \frac{1}{\sin\pi} = \frac{1}{0} = \infty$$

$$\sec(-3\pi) = \sec\pi = \frac{1}{\cos\pi} = \frac{1}{-1} = -1$$

$$\cot(-3\pi) = \cot\pi = \frac{1}{\tan\pi} = \frac{1}{0} = \infty$$

(iii) $\frac{5}{2}\pi$

$$\frac{5}{2}\pi = \frac{\pi}{2} + 2\pi$$

As we know that

$$\theta + 2k\pi = \theta, \quad k \in \mathbb{Z}$$

$$\sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos\left(\frac{5\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\tan\left(\frac{5\pi}{2}\right) = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$\operatorname{cosec}\left(\frac{5\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = \frac{1}{1} = 1$$

$$\sec\left(\frac{5\pi}{2}\right) = \frac{1}{\cos\frac{\pi}{2}} = \frac{1}{0} = \infty$$

$$\cot\left(\frac{5\pi}{2}\right) = \frac{1}{\tan\frac{\pi}{2}} = \frac{1}{\infty} = 0$$

Key Point:

(i) اگر سوال میں fraction آجائے تو Simple Division کے طریقے سے اسے even multiple میں تبدیل کر لیں۔

$$\begin{array}{r|l} & 2\pi \\ 2 & 5\pi \\ & -4\pi \\ \hline & 1\cdot\pi \end{array}$$

$$\frac{5\pi}{2} = \frac{\pi}{2} + 2\pi$$

(iv) $-\frac{9}{2}\pi$

$$-\frac{9}{2}\pi = \frac{3\pi}{2} - 6\pi$$

As we Know that

$$\theta + 2K\pi = \theta \,,\ K \in Z$$

$$\sin\left(-\frac{9}{2}\pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1$$

$$\cos\left(-\frac{9}{2}\pi\right) = \cos\left(\frac{3\pi}{2}\right) = 0$$

$$\tan\left(-\frac{9}{2}\pi\right) = \tan\left(\frac{3\pi}{2}\right) = \infty$$

$$\operatorname{cosec}\left(-\frac{9}{2}\pi\right) = \operatorname{cosec}\left(\frac{3\pi}{2}\right) = \frac{1}{\sin\frac{3\pi}{2}} = -1$$

$$\sec\left(-\frac{9}{2}\pi\right) = \sec\left(\frac{3\pi}{2}\right) = \frac{1}{\cos\frac{3\pi}{2}} = \frac{1}{0} = \infty$$

$$\cot\left(-\frac{9}{2}\pi\right) = \cot\left(\frac{3\pi}{2}\right) = \frac{1}{\tan\frac{3\pi}{2}} = \frac{1}{\infty} = 0$$

Key Point:

$$-\frac{9}{2}\pi = \frac{3\pi}{2} - 6\pi$$

$$\begin{array}{r} -6\pi \\ 2\overline{)-9\pi} \\ \mp 12\pi \\ \hline 3\pi \end{array}$$

اگر ہم 4 سے ضرب کرتے ہیں

$$\begin{array}{r} -4\pi \\ 2\overline{)-9\pi} \\ \mp 8\pi \\ \hline -\pi \end{array}$$

تو جواب -ve آجاتا ہے

ہم نے +ve رکھنا ہے

کیونکہ

$$\theta + 2K\pi = \theta$$

(v) -15π

$$-15\pi = \pi - 16\pi$$

As we Know that

$$\theta + 2K\pi = \theta \,,\ K \in Z$$

$$\sin(-15\pi) = \sin\pi = 0$$

$$\cos(-15\pi) = \cos\pi = -1$$

$$\tan(-15\pi) = \tan\pi = 0$$

$$\operatorname{cosec}(-15\pi) = \operatorname{cosec}\pi = \frac{1}{\sin\pi} = \frac{1}{0} = \infty$$

$$\sec(-15\pi) = \sec\pi = \frac{1}{\cos\pi} = \frac{1}{-1} = -1$$

$$\cot(-15\pi) = \cot\pi = \frac{1}{\tan\pi} = \frac{1}{0} = \infty$$

(vi) 1530°

$$1530^\circ = 90^\circ + 4(360^\circ)$$

As we Know that

$$\theta + 2K\pi = \theta, \quad K \in Z$$

$$\sin(1530^\circ) = \sin(90^\circ) = 1$$

$$\cos(1530^\circ) = \cos(90^\circ) = 0$$

$$\tan(1530^\circ) = \tan(90^\circ) = \infty$$

$$\operatorname{cosec}(1530^\circ) = \operatorname{cosec} 90^\circ = \frac{1}{\sin 90^\circ} = 1$$

$$\sec(1530^\circ) = \sec 90^\circ = \frac{1}{\cos 90^\circ} = \frac{1}{0} = \infty$$

$$\cot(1530^\circ) = \cot 90^\circ = \frac{1}{\tan 90^\circ} = \frac{1}{\infty} = 0$$

Key Point:

$$1530^\circ = 90^\circ + 4(360^\circ)$$

(vii) -2430°

$$-2430^\circ = -2430 \times \frac{\pi}{180}$$

$$-2430^\circ = -\frac{27}{2}\pi$$

As we Know that

$$2\pi K + \theta = \theta$$

$$2\pi(-7) + \frac{\pi}{2} = \frac{\pi}{2}$$

$$-14\pi + \frac{\pi}{2} = \frac{\pi}{2}$$

So

$$-\frac{27}{2}\pi = \frac{\pi}{2}$$

$$\sin\left(-\frac{27}{2}\pi\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos\left(-\frac{27}{2}\pi\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\tan\left(-\frac{27}{2}\pi\right) = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$\operatorname{cosec}\left(-\frac{27}{2}\pi\right) = \operatorname{cosec}\frac{\pi}{2} = \frac{1}{\sin\pi/2} = 1$$

$$\sec\left(-\frac{27}{2}\pi\right) = \sec(\pi/2) = \frac{1}{\cos\pi/2} = \frac{1}{0} = \infty$$

$$\cot\left(-\frac{27}{2}\pi\right) = \cot(\pi/2) = 1/\tan\pi/2 = \frac{1}{\infty} = 0$$

Key Point:

$$\frac{27}{2} \times \frac{180}{360}$$

$$\frac{27}{2} \times \frac{1}{2}$$

$$= \frac{27}{4}$$

$$= 6.75$$

(اگر سوال میں .75 ہو تو
سو کو ایک بڑھا کر
لکھیں گے)

QNO#5:

Type 1: (i) 1530°

Type 2: (ii) $-\pi, -3\pi, -15\pi$

Type 3: (iii) $\frac{407}{2}\pi, \frac{235}{2}\pi, \frac{5}{2}\pi$

Type 4: (iv) $-\frac{9}{2}\pi, -\frac{17}{3}\pi$

$$\boxed{2\pi K + \theta = \theta}$$

Formula

(viii) $\frac{235}{2}\pi$

$$\frac{235\pi}{2}=\frac{3\pi}{2}+\boxed{116\pi}$$

As we Know that

$$\theta+2K\pi=\theta \;,\; K\in Z$$

$$\sin\left(\frac{235}{2}\pi\right)=\sin\left(\frac{3\pi}{2}\right)=-1$$

$$\cos\left(\frac{235}{2}\pi\right)=\cos\left(\frac{3\pi}{2}\right)=0$$

$$\tan\left(\frac{235}{2}\pi\right)=\tan\left(\frac{3\pi}{2}\right)=\infty$$

$$\operatorname{cosec}\left(\frac{235}{2}\pi\right)=\operatorname{cosec}\left(\frac{3\pi}{2}\right)=\frac{1}{\sin\left(\frac{3\pi}{2}\right)}=\frac{1}{-1}=-1$$

$$\sec\left(\frac{235\pi}{2}\right)=\sec\left(\frac{3\pi}{2}\right)=\frac{1}{\cos\left(\frac{3\pi}{2}\right)}=\frac{1}{0}=\infty$$

$$\cot\left(\frac{235\pi}{2}\right)=\cot\left(\frac{3\pi}{2}\right)=\frac{1}{\tan\left(\frac{3\pi}{2}\right)}=\frac{1}{\infty}=0$$

Key Point:

$$\begin{array}{r|l} & 116\pi \\ 2 & 235\pi \\ & -232\pi \\ \hline & 3\pi \end{array}$$

$$\frac{235\pi}{2}=\frac{3\pi}{2}+116\pi$$

(ix) $\frac{407\pi}{2}$

$$\frac{407\pi}{2}=\frac{3\pi}{2}+202\pi$$

As we Know that

$$\theta+2K\pi=\theta \;,\; K\in Z$$

$$\sin\left(\frac{407}{2}\pi\right)=\sin\left(\frac{3\pi}{2}\right)=1$$

$$\cos\left(\frac{407}{2}\pi\right)=\cos\left(\frac{3\pi}{2}\right)=0$$

$$\tan\left(\frac{407}{2}\pi\right)=\tan\left(\frac{3\pi}{2}\right)=\infty$$

$$\operatorname{cosec}\left(\frac{407}{2}\pi\right)=\operatorname{cosec}\left(\frac{3\pi}{2}\right)=\frac{1}{\sin\left(\frac{3\pi}{2}\right)}=\frac{1}{-1}=-1$$

$$\sec\left(\frac{407}{2}\pi\right)=\sec\left(\frac{3\pi}{2}\right)=\frac{1}{\cos\left(\frac{3\pi}{2}\right)}=\frac{1}{0}=\infty$$

$$\cot\left(\frac{407\pi}{2}\right)=\cot\left(\frac{3\pi}{2}\right)=\frac{1}{\tan\left(\frac{3\pi}{2}\right)}=\frac{1}{\infty}=0$$

Key Point:

$$\begin{array}{r|l} & 202\pi \\ 2 & 407\pi \\ & -404\pi \\ \hline & 3\pi \end{array}$$

$$\frac{407\pi}{2}=\frac{3\pi}{2}+202\pi$$

QNO6: Find the values of the trigonometric functions of the following angles.

(i) 390°

As we know that

$\theta + 2\pi K = \theta$, $K \in Z$

So $390^\circ = 30^\circ + 1(360^\circ)$, $K = 1$

$\sin(390^\circ) = \sin(30^\circ) = \frac{1}{2}$

$\cos(390^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$

$\tan(390^\circ) = \tan(30^\circ) = \frac{1}{\sqrt{3}}$

$\operatorname{cosec}(390^\circ) = \operatorname{cosec}(30^\circ) = \frac{1}{\sin 30^\circ} = \frac{1}{1/2} = 2$

$\sec(390^\circ) = \sec(30^\circ) = \frac{1}{\cos 30^\circ} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\cot(390^\circ) = \cot(30^\circ) = \frac{1}{\tan 30^\circ} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$

(ii) -330°

As we know that

$\theta + 2\pi K = \theta$, $K \in Z$

So $-330^\circ = 30^\circ - 1(360^\circ)$

$-330^\circ = 30^\circ + (360^\circ)(-1)$, $K = -1$

$\sin(-330^\circ) = \sin(30^\circ) = \frac{1}{2}$

$\cos(-330^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$

$\tan(-330^\circ) = \tan(30^\circ) = \frac{1}{\sqrt{3}}$

$\operatorname{cosec}(-330^\circ) = \operatorname{cosec}(30^\circ) = \frac{1}{\sin(30^\circ)} = 2$

$\sec(-330^\circ) = \sec(30^\circ) = \frac{1}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$

$\cot(-330^\circ) = \cot(30^\circ) = \frac{1}{\tan 30^\circ} = \sqrt{3}$

(iii) $765°$

As we Know that

$$\theta + 2\pi K = \theta \ , \ K \in Z$$

So $765° = 45° + 2(360°) \ , K = 2$

$$\sin(765°) = \sin 45° = \frac{1}{\sqrt{2}}$$
$$\cos(765°) = \cos 45° = \frac{1}{\sqrt{2}}$$
$$\tan(765°) = \tan 45° = 1$$
$$\operatorname{cosec}(765°) = \operatorname{cosec} 45° = \sqrt{2}$$
$$\sec(765°) = \sec 45° = \sqrt{2}$$
$$\cot(765°) = \cot 45° = 1$$

(iv) $-675°$

As we Know that

$$\theta + 2\pi K = \theta \ , \ K \in Z$$

So

$$-675° = 45° - 2(360°) \ , K = -2$$
$$-675° = 45° + (-2)(360°)$$
$$\sin(-675°) = \sin(45°) = \frac{1}{\sqrt{2}}$$
$$\cos(-675°) = \cos(45°) = \frac{1}{\sqrt{2}}$$
$$\tan(-675°) = \tan(45°) = 1$$
$$\operatorname{cosec}(-675°) = \operatorname{cosec}(45°) = \sqrt{2}$$
$$\sec(-675°) = \sec(45°) = \sqrt{2}$$
$$\cot(-675°) = \cot(45°) = 1$$

(V) $-\frac{17\pi}{3}$

As we Know that

$$\theta + 2\pi K = \theta, \quad K \in Z$$

So

$$-\frac{17\pi}{3} = -\frac{17\cancel{\pi}}{3} \times \frac{180^\circ}{\cancel{\pi}}$$

$$= -\frac{17}{\cancel{3}} \times \cancel{180^\circ}\,60^\circ$$

$$-\frac{17}{3}\pi = -1020^\circ$$

$$= 60^\circ - 3(360^\circ)$$

$$-\frac{17}{3}\pi = 60^\circ + 360^\circ(-3), \quad \boxed{K=-3}$$

$$\sin\left(-\frac{17}{3}\pi\right) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos\left(-\frac{17}{3}\pi\right) = \cos 60^\circ = \frac{1}{2}$$

$$\sec\left(-\frac{17}{3}\pi\right) = \sec 60^\circ = 2$$

$$\tan\left(-\frac{17}{3}\pi\right) = \tan 60^\circ = \sqrt{3}$$

$$\operatorname{cosec}\left(-\frac{17}{3}\pi\right) = \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$

$$\cot\left(-\frac{17}{3}\pi\right) = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

(VI) $\frac{13}{3}\pi$ (LHR-2016) <u>Long Question</u>

As we Know that

$$\theta + 2\pi K = \theta, \quad K \in Z$$

So

$$\frac{13}{3}\pi = \frac{13\cancel{\pi}}{\cancel{3}} \times \frac{\cancel{180^\circ}\,60^\circ}{\cancel{\pi}}$$

$$\frac{13}{3}\pi = 780^\circ$$

$$\frac{13}{3}\pi = 60^\circ + 2(360^\circ)$$

$$\sin\left(\frac{13}{2}\pi\right) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{13}{2}\pi\right) = \cos 60^\circ = \frac{1}{2}$$

$$\tan\left(\frac{13}{2}\pi\right) = \tan 60^\circ = \sqrt{3}$$

$$\operatorname{cosec}\left(\frac{13}{2}\pi\right) = \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$

$$\sec\left(\frac{13}{2}\pi\right) = \sec 60^\circ = 2$$

$$\cot\left(\frac{13}{2}\pi\right) = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

(vii) $\frac{25}{6}\pi$

$$\frac{25}{6}\pi = \frac{25}{\cancel{6}}\cancel{\pi} \times \frac{\overset{30^\circ}{\cancel{180^\circ}}}{\cancel{\pi}}$$

$$\frac{25}{6}\pi = 750^\circ$$

$$\frac{25}{6}\pi = 30^\circ + 2(360^\circ), \quad K = 2$$

$$\sin\left(\frac{25}{6}\pi\right) = \sin(30^\circ) = \frac{1}{2}$$

$$\cos\left(\frac{25}{6}\pi\right) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{25}{6}\pi\right) = \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec}\left(\frac{25}{6}\pi\right) = \operatorname{cosec}(30^\circ) = 2$$

$$\sec\left(\frac{25}{6}\pi\right) = \sec(30^\circ) = \frac{2}{\sqrt{3}}$$

$$\cot\left(\frac{25}{6}\pi\right) = \cot(30^\circ) = \sqrt{3}$$

(viii) $-\frac{71}{6}\pi$

$$-\frac{71}{6}\pi = -\frac{71\cancel{\pi}}{\cancel{6}} \times \frac{\overset{30^\circ}{\cancel{180^\circ}}}{\cancel{\pi}}$$

$$-\frac{71}{6}\pi = -2130^\circ$$

$$-\frac{71}{6}\pi = 30^\circ - 6(360^\circ)$$

$$-\frac{71}{6}\pi = 30^\circ + (360^\circ)(-6), \; K = -6$$

As we know that

$$\theta + 2\pi K = \theta, \; K \in Z$$

$$\sin\left(-\frac{71}{6}\pi\right) = \sin(30^\circ) = \frac{1}{2}$$

$$\cos\left(-\frac{71}{6}\pi\right) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{71}{6}\pi\right) = \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec}\left(-\frac{71}{6}\pi\right) = \operatorname{cosec}(30^\circ) = 2$$

$$\sec\left(-\frac{71}{6}\pi\right) = \sec(30^\circ) = \frac{2}{\sqrt{3}}$$

$$\cot\left(-\frac{71}{6}\pi\right) = \cot(30^\circ) = \sqrt{3}$$

(ix) -1035°

$$-1035^\circ = 45^\circ - 3(360^\circ)$$

$$-1035^\circ = 45^\circ + (360^\circ)(-3), \quad \boxed{K=-3}$$

As we know that

$$\sin(-1035^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos(-1035^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sec(-1035^\circ) = \sec 45^\circ = \sqrt{2}$$

$$\tan(-1035^\circ) = \tan 45^\circ = 1$$

$$\operatorname{cosec}(-1035^\circ) = \operatorname{cosec} 45^\circ = \sqrt{2}$$

$$\cot(-1035^\circ) = \cot 45^\circ = 1$$