Chapter : 3

Integration

EXERCISE 3.6

Evaluate the following definite Integrals.

(1)

$$\int_1^2 (x^2+1)\,dx$$

Solution:

$$\int_1^2 (x^2+1)\,dx$$

$$= \left[\frac{x^3}{3}+x\right]_1^2$$

$$= \left[\frac{(2)^3}{3}+2\right]-\left[\frac{(1)^3}{3}+1\right]$$

$$= \frac{8}{3}+2-\frac{1}{3}-1$$

$$= \frac{8+6}{3}-\frac{1+3}{3}$$

A. Good work
19/01/2024

Day: Date:

$= \frac{14-4}{3}$

$= \frac{10}{3}$ **Answer**

(2)

$\int_{-1}^{1} (x^{1/3} + 1)\, dx$

Solution:

$$\int_{-1}^{1} (x^{1/3} + 1)\, dx$$

$$= \int_{-1}^{1} x^{1/3}\, dx + \int_{-1}^{1} 1\, dx$$

$$= \left[\frac{x^{1/3+1}}{1/3+1}\right]_{-1}^{1} + [x]_{-1}^{1}$$

$$= \left[\frac{x^{4/3}}{4/3}\right]_{-1}^{1} + [x]_{-1}^{1}$$

$$= \frac{3}{4}[x^{4/3}]_{-1}^{1} + [x]_{-1}^{1}$$

$$= \frac{3}{4}[x]_{-1}^{1}{}^{4/3} + [x]_{-1}^{1}$$

$$= \frac{3}{4}[(1)^{4/3} - (-1)^{4/3}] + [1-(-1)]$$

$$= \frac{3}{4}[1-(-1)^{\cancel{3}\times 4/\cancel{3}}] + [1+1]$$

$$= \frac{3}{4}[1-1] + 2$$

$= 0 + 2 \Rightarrow 2$ **Answer**

(3)

Solution: $\int_{-2}^{0} \frac{1}{(2x-1)^2}\,dx$

$$\int_{-2}^{0} \frac{1}{(2x-1)^2}\,dx$$

$$= \int_{-2}^{0} (2x-1)^{-2}\,dx$$

$$= \frac{1}{2}\int_{-2}^{0} (2x-1)^{-2}\,dx$$

$$= \frac{1}{2}\left[\frac{(2x-1)^{-2+1}}{-2+1}\right]_{-2}^{0}$$

$$= \frac{1}{2}\left[\frac{(2x-1)^{-1}}{-1}\right]_{-2}^{0}$$

$$= \frac{1}{2}\left[\frac{1}{(2x-1)}\right]_{-2}^{0}$$

$$= -\frac{1}{2}\left[\frac{1}{2(0)-1}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{2(-2)-1}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{0-1} - \frac{1}{-4-1}\right]$$

$$= -\frac{1}{2}\left[-1-\left(-\frac{1}{5}\right)\right]$$

$$= -\frac{1}{2}\left[-1+\frac{1}{5}\right]$$

$$= -\frac{1}{2}\left[\frac{-5+1}{5}\right]$$

$$= \frac{1}{2}\left[\frac{-4}{5}\right] \Rightarrow \frac{2}{5}$$ **Answer.**

(4)

$$\int_{-6}^{2} \sqrt{3-x}\, dx$$

Solution:

$$\int_{-6}^{2} \sqrt{3-x}\, dx$$

$$= \int_{-6}^{2} (3-x)^{1/2}\, dx$$

$$= -\int_{-6}^{2} (3-x)^{1/2}\, dx$$

$$= -\left[\frac{(3-x)^{1/2+1}}{1/2+1}\right]_{-6}^{2}$$

$$= -\left[\frac{(3-x)^{3/2}}{3/2}\right]_{-6}^{2}$$

$$= -\frac{2}{3}\left[(3-2)^{3/2} - (3-(-6))^{3/2}\right]$$

$$= -\frac{2}{3}\left[(1)^{3/2} - (3+6)^{3/2}\right]$$

$$= -\frac{2}{3}\left[1 - (9)^{3/2}\right]$$

$$= -\frac{2}{3}\left[1 - (3^{2})^{3/2}\right.$$

$= -\frac{2}{3}[1-3^3]$

$= -\frac{2}{3}[1-9]$

$= -\frac{2}{3}[-8]$

$= \frac{16}{3}$ **Answer.**

(5)

$$\int_1^{\sqrt{5}} \sqrt{(2t-1)^3}\,dt$$

Solution:

$$\int_1^{\sqrt{5}} \sqrt{(2t-1)^3}\,dt$$

Let

$$I = \int_1^{\sqrt{5}} (2t-1)^{3/2}\,dt$$

÷ and × by (2)

$$= \frac{1}{2}\int_1^{\sqrt{5}} (2t-1)^{3/2} \cdot (2)\,dt$$

$$= \frac{1}{2}\left[\frac{(2t-1)^{3/2+1}}{3/2+1}\right]_1^{\sqrt{5}}$$

$$= \frac{1}{2}\left[\frac{(2t-1)^{5/2}}{5/2}\right]_1^{\sqrt{5}}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5}\left[(2t-1)^{5/2}\right]_1^{\sqrt{5}}$$

$$= \frac{1}{5}\left[(2\sqrt{5}-1)^{5/2} - (2(1)-1)^{5/2}\right]$$

Day:

$$= \frac{1}{5}\left[(2\sqrt{5}-1)^{5/2} - (1)^{5/2}\right]$$

$$= \frac{1}{5}\left[(2\sqrt{5}-1)^{5/2} - 1\right]$$ **Answer.**

(6)

$$\int_{2}^{\sqrt{5}} x\sqrt{x^2-1}\,dx$$

Solution:

$$\int_{2}^{\sqrt{5}} x\sqrt{x^2-1}\,dx$$

Let

$$I = \int_{2}^{\sqrt{5}} x(x^2-1)^{1/2}\,dx$$

÷ and × by (2)

$$= \frac{1}{2}\int_{2}^{\sqrt{5}} (x^2-1)^{1/2}\, 2x\,dx$$

$$= \frac{1}{2}\left[\frac{(x^2-1)^{1/2+1}}{1/2+1}\right]_{2}^{\sqrt{5}}$$

$$= \frac{1}{2}\left[\frac{(x^2-1)^{3/2}}{3/2}\right]_{2}^{\sqrt{5}}$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\left[(x^2-1)^{3/2}\right]_{2}^{\sqrt{5}}$$

$$= \frac{1}{3}\left[((\sqrt{5})^2-1)^{3/2} - ((2)^2-1)^{3/2}\right]$$

$$= \frac{1}{3}\left[(5-1)^{3/2} - (4-1)^{3/2}\right]$$

$= \frac{1}{3}\left[(4)^{3/2} - (3)^{3/2}\right]$

$= \frac{1}{3}\left[(2)^{2\times 3/2} - 3 \cdot 3^{1/2}\right]$

$= \frac{1}{3}\left[8 - 3\sqrt{3}\right]$

$= \frac{1}{3}\left[8 - 3\sqrt{3}\right]$ **Answer.**

(7)

Solution: $\int_1^2 \frac{x}{x^2+2}\,dx$

$\int_1^2 \frac{x}{x^2+2}\,dx$

Let $\div$ **and** $\times$ **by (2)**

$I = \frac{1}{2}\int_1^2 \frac{2x}{x^2+2}\,dx$

$= \frac{1}{2}\left[\ln|x^2+2|\right]_1^2$

$= \frac{1}{2}\left[\ln|(2)^2+2| - \ln|(1)^2+2|\right]$

$= \frac{1}{2}\left[\ln|4+2| - \ln|1+2|\right]$

$= \frac{1}{2}\left[\ln 6 - \ln 3\right]$

$\because \ln a - \ln b = \ln\frac{a}{b}$

$= \frac{1}{2}\left[\ln\frac{6}{3}\right]$

$= \frac{1}{2}\ln 2$ **Answer.**

$$\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$$

Solution:

$$\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$$

$$= \int_2^3 \left[x^2 + \frac{1}{x^2} - 2x\frac{1}{x}\right] dx$$

$$= \int_2^3 x^2 dx + \int_2^3 x^{-2} dx - 2\int_2^3 1\, dx$$

$$= \left[\frac{x^3}{3}\right]_2^3 + \left[\frac{x^{-2+1}}{-2+1}\right]_2^3 - 2\left[x\right]_2^3$$

$$= \frac{1}{3}\left[(3)^3 - (2)^3\right] + \left[\frac{(x^{-1})}{-1}\right]_2^3 - 2\left[3 - 2\right]$$

$$= \frac{1}{3}\left[27 - 8\right] - \left[\frac{1}{x}\right]_2^3 - 2(1)$$

$$= \frac{1}{3}(19) - \left[\frac{1}{3} - \frac{1}{2}\right] - 2$$

$$= \frac{19}{3} - \left[\frac{2-3}{6}\right] - 2$$

$$= \frac{19}{3} - \left[\frac{-1}{6}\right] - 2$$

$$= \frac{19}{3} + \frac{1}{6} - 2$$

$$= \frac{38 + 1 - 12}{6}$$

$$= \frac{27}{6} \Rightarrow \frac{9}{2}$$ **Answer.**

Day:

(9)

Solution: $\int_{-1}^{1}\left(x+\frac{1}{2}\right)\sqrt{x^2+x+1}\,dx$

$$\int_{-1}^{1}\left(x+\frac{1}{2}\right)\sqrt{x^2+x+1}\,dx$$

$$=\int_{-1}^{1}(x^2+x+1)^{1/2}\cdot\left(x+\frac{1}{2}\right)dx$$

$$=\int_{-1}^{1}(x^2+x+1)^{1/2}\cdot(2x+1)\,dx$$

$$=\frac{1}{2}\int_{-1}^{1}(x^2+x+1)^{1/2}\cdot(2x+1)\,dx$$

$$=\frac{1}{2}\left[\frac{(x^2+x+1)^{1/2+1}}{1/2+1}\right]_{-1}^{1}$$

$$=\frac{1}{2}\left[\frac{(x^2+x+1)^{3/2}}{3/2}\right]_{-1}^{1}$$

$$=\frac{1}{2}\cdot\frac{2}{3}\left[(x^2+x+1)^{3/2}\right]_{-1}^{1}$$

$$=\frac{1}{3}\left[(1+1+1)^{3/2}-((-1)^2+(-1)+1)^{3/2}\right]$$

$$=\frac{1}{3}\left[(3)^{3/2}-(1-1+1)^{3/2}\right]$$

$$=\frac{1}{3}\left[3^1\cdot3^{1/3}-(1)^{3/2}\right]$$

$$=\frac{1}{3}\left[3\sqrt{3}-1\right]$$ **Answer.**

(10)

MCQ + ShQ

$$\int_{0}^{3}\frac{dx}{x^2+9}$$

Solution:

Let $\int_0^3 \frac{dx}{x^2+9}$

$$I = \int_0^3 \frac{dx}{x^2+9}$$

$$I = \int_0^3 \frac{1}{(x)^2+(3)^2}\,dx$$

Using $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$= \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^3$$

$$= \frac{1}{3}\left[\left(\tan^{-1}\frac{3}{3}\right)-\left(\tan^{-1}\frac{0}{3}\right)\right]$$

$$= \frac{1}{3}\left[\tan^{-1}(1)-\tan^{-1}(0)\right]$$

$$= \frac{1}{3}\left[\frac{\pi}{4}\right]$$

$$= \frac{\pi}{12}$$ **Answer.**

(11) Short Q

Solution: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t\,dt$

Let $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t\,dt$

$$I = [\sin t]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \left[\sin\frac{\pi}{3} - \sin\frac{\pi}{6}\right]$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2} \quad \text{Answer.}$$

(12)

Solution: $\int_1^2 \left[x+\frac{1}{x}\right]^{\frac{1}{2}}\left[1-\frac{1}{x^2}\right]dx$

$$\int_1^2 \left[x+\frac{1}{x}\right]^{\frac{1}{2}}\left[1-\frac{1}{x^2}\right]dx$$

Using $\int [f(x)]^n . f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

$$= \left[\frac{(x+\frac{1}{x})^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^2$$

$$= \frac{2}{3}\left[(x+\frac{1}{x})^{3/2}\right]_1^2$$

$$= \frac{2}{3}\left[(2+\frac{1}{2})^{3/2} - (1+\frac{1}{1})^{3/2}\right]$$

$$= \frac{2}{3}\left[(\frac{5}{2})^{3/2} - (2)^{3/2}\right]$$

$$= \frac{2}{3}\left[\frac{5}{2}^{1}.(\frac{5}{2})^{1/2} - 2^{1}.(2)^{1/2}\right]$$

$$= \frac{2}{3}\left[\frac{5.\sqrt{5}}{2\sqrt{2}} - 2\sqrt{2}\right]$$

$$= \frac{2}{3} . \frac{5\sqrt{5}}{2\sqrt{2}} - \frac{2.\sqrt{2}}{3}$$

$$= \frac{5\sqrt{5}}{3\sqrt{2}} - \frac{4\sqrt{2}}{3}$$

$$= \frac{5\sqrt{5} - 4(2)}{3\sqrt{2}}$$ **Answer.**

(13)

$$\int_1^2 \ln x \, dx$$

Solution:

$$\int_1^2 \ln x \, dx$$

Consider

$$\int_1^2 \ln x dx = \int_1^2 \ln x \cdot 1 dx$$

Integrating by parts, we get

$$= \ln x \int_1^2 2 dx - \int_1^2 \left(\frac{d}{dx} \ln x \cdot \int_1^2 1 dx\right) dx$$

$$= \ln x \cdot x - \int_1^2 \left(\frac{1}{x} \cdot x\right) dx$$

$$= x \cdot \ln x - \int_1^2 1 dx$$

$$\int_1^2 \ln x dx = x \ln x - x + c$$

$$\int_1^2 \ln x \, dx = \left[x \ln x - x\right]_1^2$$

$$= [(2\ln 2 - 2) - (1\ln 1 - 1)]$$

$$= [2\ln 2 - 2 - \ln 1 + 1]$$

$$= 2\ln 2 - 1$$ **Answer.**

(14)

Solution: $\int_0^2 \left[e^{\frac{x}{2}} - e^{-\frac{x}{2}}\right]dx$

$$\int_0^2 \left[e^{\frac{x}{2}} - e^{-\frac{x}{2}}\right]dx$$

$$= \int_0^2 e^{x/2}dx - \int_0^2 e^{-x/2}dx$$

$$= \left[\frac{e^{x/2}}{1/2}\right]_0^2 - \left[\frac{e^{-x/2}}{-1/2}\right]_0^2$$

$$= 2\left[e^{x/2}\right]_0^2 + 2\left[e^{-x/2}\right]_0^2$$

$$= 2\left[e^{2/2} - e^{0/2}\right] + 2\left[e^{-2/2} - e^{-0/2}\right]$$

$$= 2\left[(e-1)\right] + 2\left[\frac{1}{e} - 1\right]$$

$$= 2e - 2 + \frac{2}{e} - 2$$

$$= 2e - \frac{2}{e} - 2 - 2$$

$$= 2e - \frac{2}{e} - 4$$

$$= \left[e - \frac{1}{e}\right] - 4 \quad \left[\frac{e^2 - 1}{e}\right] - 4$$ **Answer.**

(15)

Solution: $\int_0^{\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{2\cos^2\theta}\, d\theta$

$$\int_0^{\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{2\cos^2\theta}\, d\theta$$

Let

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\frac{\cos\theta + \sin\theta}{\cos^2\theta}\right) d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\frac{\cos\theta}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} \right) d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos\theta} + \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} \right) d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} (\sec\theta + \sec\theta\tan\theta)\, d\theta$$

$$= \frac{1}{2}\left[\ln(\sec\theta + \tan\theta) + \sec\theta\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left[\ln\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) + \sec\frac{\pi}{4}\right] - (\ln(\sec(0) + \tan(0)) + \sec(0))]$$

$$= \frac{1}{2}\left[(\ln(\sqrt{2}+1) + \sqrt{2}) - (\ln(1+0) + 1)\right]$$

$$= \frac{1}{2}\left[\ln(\sqrt{2}+1) + \sqrt{2} - \ln 1 - 1\right]$$

$$= \frac{1}{2}\left[\ln|\sqrt{2}+1| + \sqrt{2} - 1\right]$$ **Answer.**

(16)

$$\int_0^{\frac{\pi}{6}} \cos^3\theta\, d\theta$$

Solution:

$$\int_0^{\frac{\pi}{6}} \cos^3\theta\, d\theta$$

Let

$$I = \int_0^{\frac{\pi}{6}} \cos\theta \cdot \cos^2\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \cos\theta \cdot (1 - \sin^2\theta)\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} (\cos\theta - (\sin\theta)^2 \cos\theta\, d\theta$$

$$I = \left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{6}}$$

$$I = \left[\left(\sin\frac{\pi}{6} - \frac{\sin\left(\frac{\pi}{6}\right)^3}{3}\right) - \left(\sin(0) - \frac{\sin^3(0)}{3}\right)\right]$$

$$I = \left[\left(\frac{1}{2} - \frac{1}{24}\right) - (0 - 0)\right]$$

$$I = \left[\frac{12-1}{24}\right]$$

$$I = \frac{11}{24}$$ **Answer.**

(17)

Solution: $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2\theta \cot^2\theta \, d\theta$

Let $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2\theta \cot^2\theta \, d\theta$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2\theta . (\csc^2\theta - 1) \, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos^2\theta . \csc^2\theta - \cos^2\theta) \, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\cos^2\theta . \frac{1}{\sin^2\theta} - \cos^2\theta\right) d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cot^2\theta - \cos^2\theta) \, d\theta$$

$\because 1 + \cot^2\theta = \csc^2\theta$

$\because \cos 2\theta = 2\cos^2\theta - 1$

$\frac{1+\cos^2\theta}{2} = \cos^2\theta$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} [(\text{cosec}^2\theta - (\frac{1+\cos 2\theta}{2})] d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} [(\text{cosec}^2\theta - 1 - \frac{1}{2}(1+\cos 2\theta)] d\theta$$

$$= [-\cot\theta - \theta - \frac{1}{2}\theta - \frac{1}{2}\frac{\sin 2\theta}{2}]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= [-\cot\theta - \frac{3\theta}{2} - \frac{1}{4}\sin 2\theta]_{\pi/6}^{\pi/4}$$

$$= [(\cot\frac{\pi}{4} - \frac{3}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\sin 2(\frac{\pi}{4})) - (-\cot\frac{\pi}{6} - \frac{3}{2}\cdot\frac{\pi}{6} - \frac{1}{4}\sin 2(\frac{\pi}{6})]$$

$$= [(-1 - \frac{3\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2}) - (-\sqrt{3} - \frac{3\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3})]$$

$$= [-1 - \frac{3\pi}{8} - \frac{1}{4} + \sqrt{3} + \frac{3\pi}{12\,4} + \frac{\sqrt{3}}{8}]$$

$$= [-1 - \frac{1}{4} - \sqrt{3} + \frac{3\pi}{8} + \frac{\pi}{4} + \frac{\sqrt{3}}{8}]$$

$$= [\frac{-5}{4} + \sqrt{3} - \frac{3\pi}{8} + \frac{\pi}{4} + \frac{\sqrt{3}}{8}]$$

$$= [\frac{-10 + 8\sqrt{3} - 3\pi + 2\pi + \sqrt{3}}{8}]$$

$$= [\frac{8\sqrt{3} + \sqrt{3} - \pi - 10}{8}]$$

$$= [\frac{9\sqrt{3} - \pi - 10}{8}]$$ **Answer.**

(18)

$$\int_0^{\frac{\pi}{4}} \cos^4 t\, dt$$

Solution:

$$\int_0^{\frac{\pi}{4}} \cos^4 t\, dt$$

$\because \cos 2t = 2\cos^2 t - 1$

$1 + \cos 2t = 2\cos^2 t$

$\frac{1+\cos 2t}{2} = \cos^2 t$

Let

$$I = \int_0^{\frac{\pi}{4}} (\cos^2 t)^2\, dt$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1+\cos 2t}{2}\right)^2$$

$\because \cos^2 t = \frac{1+\cos 2t}{2}$

$\because \cos^2 2t = \frac{1+\cos 4t}{2}$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} (1+\cos^2 2t + 2\cos 2t)\, dt$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} (1+2\cos 2t + \cos^2 2t)\, dt$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \left(1+2\cos 2t + \frac{1+\cos 4t}{2}\right) dt$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \frac{2+4\cos 2t + 1+\cos 4t}{2}\, dt$$

$$= \frac{1}{4 \cdot 2}\int_0^{\frac{\pi}{4}} (3+4\cos 2t + \cos 4t)\, dt$$

$$= \frac{1}{8}\left[3t + 4 \cdot \frac{\sin 2t}{2} + \frac{\sin 4t}{4}\right]_0^{\pi/4}$$

$$= \frac{1}{8}\left[3t + 2\sin 2t + \frac{\sin 4t}{4}\right]_0^{\pi/4}$$

$$= \frac{1}{8}\left[\left(3\frac{\pi}{4} + 2\sin 2\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\right) - \left(3(0) + 2\sin 2(0) + \sin(0)\right)\right]$$

$$= \frac{1}{8}\left[\frac{3\pi}{4} + 2\sin\frac{\pi}{2} + \sin\frac{\pi}{4}\right] - (0)$$

$$= \frac{1}{8}\left[\frac{3\pi}{4} + 2 + \frac{0}{4}\right]$$

$$= \frac{1}{8}\left[\frac{3\pi + 8}{4}\right]$$

$$= \frac{3\pi + 8}{32}$$ **Answer.**

(19)

$$\int_0^{\frac{\pi}{3}} \cos^2\theta \sin\theta \, d\theta$$

Solution:

Let $\int_0^{\frac{\pi}{3}} \cos^2\theta \sin\theta \, d\theta$

$$I = \int_0^{\frac{\pi}{3}} \cos^2\theta \sin\theta \, d\theta$$

÷ and × by (−)

$$= -\int_0^{\frac{\pi}{3}} (\cos\theta)^2 \cdot (-\sin\theta) \, d\theta$$

$$= \left[\frac{(\cos\theta)^{2+1}}{2+1}\right]_0^{\pi/3}$$

$$= \left[\frac{\cos\theta^3}{3}\right]_0^{\pi/3}$$

$$= \frac{1}{3}\left[\left(\cos\frac{\pi}{3}\right)^3 - (\cos(0))^3\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2}\right)^3 - (1)^3\right]$$

$$= \frac{1}{3}\left[\frac{1}{8} - 1\right]$$

$$= \frac{1}{3}\left[\frac{1-8}{8}\right]$$

$$= \frac{7}{24}$$ **Answer.**

(20)

$$\int_0^{\frac{\pi}{4}} (1+\cos^2\theta)\tan^2\theta \, d\theta$$

Solution:

$$\int_0^{\frac{\pi}{4}} (1+\cos^2\theta)\tan^2\theta \, d\theta$$

Let

$$I = \int_0^{\frac{\pi}{4}} (1+\cos^2\theta) \cdot \tan^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} (\tan^2\theta + \cos^2\theta\tan^2\theta)\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left((\sec^2\theta - 1) + \cos^2\theta \cdot \frac{\sin^2\theta}{\cos^2\theta}\right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} (\sec^2\theta - 1 + \sin^2\theta)\, d\theta$$

$$\because \cos 2\theta = 1 - 2\sin^2\theta$$

$$= \int_0^{\pi/4} \left(\sec^2\theta - 1 + \left(\frac{1-\cos 2\theta}{2}\right)\right) d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \int_0^{\frac{\pi}{4}} \left(\sec^2\theta - 1 + \left(\frac{1-\cos 2\theta}{2}\right)\right) d\theta$$

$$= \left[\tan\theta - \theta + \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)\right]_0^{\pi/4}$$

$$= \left[\tan\theta - \theta + \frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\pi/4}$$

$$= \left[\tan\theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/4}$$

$$= \left[\tan\theta - \theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\pi/4}$$

$$= \left[\tan\theta - \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\pi/4}$$

$$= \left[\tan\frac{\pi}{4} - \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\sin 2\left(\frac{\pi}{4}\right) - (0-0-0)\right]$$

$$= 1 - \frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2} - 0$$

$$= 1 - \frac{\pi}{8} - \frac{1}{4}(1)$$

$$= \frac{8-\pi-2}{8} = \frac{6-\pi}{8}$$ **Answer.**

(21)

Solution: $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sec\theta}{\sin\theta + \cos\theta}\, d\theta$

Let $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sec\theta}{\sin\theta + \cos\theta}\, d\theta$

$$I = \int_0^{\frac{\pi}{4}} \frac{\sec\theta}{\cos\left(\frac{\sin\theta}{\cos\theta}\right)+1}\, d\theta$$

$$I = \int_0^{\frac{\pi}{4}} \frac{\sec\theta}{\cos\left(\frac{\sin\theta}{\cos\theta}+1\right)}\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{\sec\theta \cdot \sec\theta}{\tan\theta + 1} \right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{\sec^2\theta}{\tan\theta + 1} \right) d\theta$$

$$= \left[\ln(\tan\theta + 1) \right]_0^{\pi/4}$$

$$= \left[\ln\left(\tan\frac{\pi}{4} + 1\right) - \ln(\tan(0) + 1) \right]$$

$$= \left[\ln(1+1) - \ln 1 \right]$$

$$= \ln 2 - 0 \Rightarrow \ln 2$$ **Answer.**

(22) Short Q

Solution: $\int_{-1}^{5} |x-3|\,dx$

$$\int_{-1}^{5} |x-3|\,dx$$

$$|x| = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|x-3| = \begin{cases} +(x-3), & x-3 \geq 0, \; x \geq 3 \\ -(x-3), & x-3 < 0. \; x < 3 \end{cases}$$

$$I = \int_{-1}^{3} -(x-3) + \int_{3}^{5} (x-3)\,dx$$

$$= -\int_{-1}^{3} (x-3)'.\, 1 + \int_{3}^{5} (x-3)'(1)\,dx$$

$$= -\left[\frac{(x-3)^2}{2} \right]_{-1}^{3} + \left[\frac{(x-3)}{2} \right]_{3}^{5}$$

$$= \left[\frac{(3-3)^2}{2} - \frac{(-1-3)^2}{2}\right] + \left[\frac{(5-3)^2}{2} - \frac{(3-3)^2}{2}\right]$$

$$= \left[\frac{0}{2} - \frac{(-4)^2}{2}\right] + \left[\frac{(2)^2}{2} - \frac{0}{2}\right]$$

$$= \frac{0}{2} - \frac{16}{2} + \frac{4}{2} - \frac{0}{2}$$

$$= 0 - \frac{\cancel{16}\,8}{\cancel{2}} + \frac{\cancel{4}\,2}{\cancel{2}} - 0$$

$= 8 + 2 \Rightarrow 10$ **Answer.**

(23)

$$\int_{1/8}^{1} \frac{\left[x^{\frac{1}{3}}+2\right]^2}{x^{\frac{2}{3}}}\, dx$$

Solution:

$$\int_{1/8}^{1} \frac{\left[x^{\frac{1}{3}}+2\right]^2}{x^{\frac{2}{3}}}\, dx$$

Let

$$I = \int_{1/8}^{1} (x^{\frac{1}{3}}+2)^2 \cdot x^{-\frac{2}{3}}\, dx$$

÷ and × by (3)

$$= 3\int_{1/8}^{1} (x^{1/3}+2)^2 \cdot \left(\frac{1}{3}x^{-\frac{2}{3}}\right) dx$$

$$= 3\left[\frac{(x^{\frac{1}{3}}+2)^{2+1}}{2+1}\right]_{1/8}^{1}$$

$$= \frac{3}{3}\left[(x^{\frac{1}{3}}+2)\right]_{1/8}^{1}$$

Day:

$= [((1)^{\frac{1}{3}}+2)^3 - ((1/8)^{\frac{1}{3}}+2)^3]$

$= [3^3 - (\frac{1}{2^3})^{\frac{1}{3}}+2^3]$

$= [27 - (\frac{1}{2}+2)^3]$

$= [27 - [\frac{5}{2}]^3]$

$= 27 - \frac{125}{8}$

$= \frac{216-125}{8} \Rightarrow \frac{91}{8}$ **Answer.**

(24) Short Q

$\int_1^3 \frac{x^2-2}{x+1}\,dx$

Solution:

Let $\int_1^3 \frac{x^2-2}{x+1}\,dx$

$I = \int_1^3 \frac{x^2-2}{x+1}\,dx$

Improper

$$
\begin{array}{r|l}
 & x-1 \\
x+1 & x^2-2 \\
 & \pm x^2 \pm x \\
\hline
 & -x-2 \\
 & \pm x \pm 1 \\
\hline
 & -1
\end{array}
$$

$= \int_1^3 (Q+\frac{R}{D})\,dx$

$= \int_1^3 (x-1+\frac{-1}{x+1})\,dx$

$= \left[\frac{x^2}{2} - x - \ln(x+1)\right]_1^3$

$= \left[(\frac{(3)^2}{2} - 3 - \ln(3+1)) - (\frac{(1)^2}{2} + 1 + \ln(1+1)\right]$

$= \frac{9}{2} - 3 - \ln 4 - \frac{1}{2} + 1 + \ln 2$

$= \frac{9}{2} - \frac{1}{2} - 3 + 1 - \ln 4 + \ln 2$

$= \frac{9}{2} - \frac{1}{2} - 2 - \ln 2^2 + \ln 2$

$= \frac{9}{2} - \frac{1}{2} - 2 - 2\ln 2 + \ln 2$

$= \frac{9-1-4}{2} - 2\ln 2 + \ln 2$

$= \frac{4}{\cancel{4}} - \ln 2 \Rightarrow 2 - \ln 2$ **Answer.**

Q(25)b

$$\int_2^3 \frac{3x^2 - 2x + 1}{(x-1)(x^2+1)}\, dx$$

Solution:

Let

$$I = \int_2^3 \frac{3x^2 - 2x + 1}{(x^3 - x^2 + x - 1)}\, dx$$

Using $\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$

$\because \ln a - \ln b = \ln \frac{a}{b}$

$= [\ln(x^3 - x^2 + x - 1)]_2^3$

$= [\ln(3^3 - 3^2 + 3 - 1) - \ln(2^3 - 2^2 + 2 - 1)]$

$= [\ln(27 - 9 + 3 - 1) - \ln(8 - 4 + 2 - 1)]$

$= [\ln(20) - \ln(5)]$

$= \ln \frac{20}{5} \Rightarrow \ln 4$ **Answer.**

Solution: $\int_0^{\frac{\pi}{4}} \frac{\sin x - 1}{\cos^2 x} dx$

Let

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin x - 1}{\cos^2 x} dx$$

$$I = \int_0^{\frac{\pi}{4}} \left[\frac{\sin x}{\cos^2 x} - \frac{1}{\cos^2 x}\right] dx$$

$$= \int_0^{\frac{\pi}{4}} \left[\frac{\sin x}{\cos x \cdot \cos x} - \frac{1}{\cos^2 x}\right] dx$$

$$= \int_0^{\frac{\pi}{4}} \left[\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} - \frac{1}{\cos^2 x}\right] dx$$

$$= \int_0^{\frac{\pi}{4}} (\sec x \tan x - \sec^2 x) dx$$

$$= \int_0^{\frac{\pi}{4}} (\sec x - \tan x) dx$$

$$= [\sec x - \tan x]_0^{\pi/4}$$

$$= \left[\sec\frac{\pi}{4} - \tan\frac{\pi}{4}\right] - [\sec(0) - \tan(0)]$$

$$= (\sqrt{2} - 1) - (1 - 0)$$

$$= \sqrt{2} - 1 - 1$$

$$= \sqrt{2} - 2$$ **Answer.**

(27)

Solution: $\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$$

$$I=\int_0^{\frac{\pi}{4}}\left(\frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\right)dx$$

$$I=\int_0^{\frac{\pi}{4}}\left(\frac{1-\sin x}{1-\sin x}\right)dx$$

$\because \cos^2 x = 1-\sin^2 x$

$$I=\int_0^{\frac{\pi}{4}}\left(\frac{1-\sin x}{\cos x}\right)dx$$

$$=\int_0^{\frac{\pi}{4}}\left(\frac{1}{\cos^2 x}-\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\right)dx$$

$$=\int_0^{\frac{\pi}{4}}(\sec^2 x-\sec x\tan x)\,dx$$

$$=\int_0^{\frac{\pi}{4}}(\tan x-\sec x)\,dx$$

$$=\left[\tan x-\sec x\right]_0^{\pi/4}$$

$$=\left[\tan\frac{\pi}{4}-\sec\frac{\pi}{4}\right]-\left[\tan(0)-\sec(0)\right]$$

$$=(1-\sqrt{2})-(0-1)$$

$$=1-\sqrt{2}+1$$

$$=2-\sqrt{2}$$ **Answer**

Solution: $\int_0^1 \frac{3x}{\sqrt{4-3x}}dx$

Let

$$I = \int_0^1 \frac{3x}{\sqrt{4-3x}}dx$$

$$I = -\int_0^1 \frac{-3x}{\sqrt{4-3x}}dx$$

$$I = -\int_0^1 \frac{(4-3x)-4}{\sqrt{4-3x}}dx$$

$$I = -\int_0^1 \left(\frac{4-3x}{\sqrt{4-3x}} - \frac{4}{\sqrt{4-3x}}\right)dx$$

$$= -\int_0^1 \left(\sqrt{4-3x} - 4(4-3x)^{-\frac{1}{2}}\right) dx$$

$$= -\int_0^1 (4-3x)^{1/2} \cdot (-3)\,dx - \frac{4}{3}\int_0^1 (4-3x)^{-\frac{1}{2}} \cdot (-3)\,dx$$

$$= \frac{1}{3}\int_0^1 (4-3x)^{\frac{1}{2}} \cdot (-3)\,dx - \frac{4}{3}\int_0^1 (4-3x)^{-\frac{1}{2}} \cdot (-3)\,dx$$

$$= \frac{1}{3}\left[\frac{(4-3x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^1 - \frac{4}{3}\left[\frac{(4-3x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_0^1$$

$$= \frac{1}{3}\left[\frac{(4-3x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - \frac{4}{3}\left[\frac{(4-3x)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1$$

$$= \frac{1}{3}\cdot\frac{2}{3}\left[(4-3x)^{\frac{3}{2}}\right]_0^1 - \frac{4}{3}\cdot\frac{2}{1}\left[(4-3x)^{\frac{1}{2}}\right]_0^1$$

$$= \frac{2}{9}\left[(4-3(1))^{3/2} - (4-3(0))^{3/2}\right] - \frac{8}{3}\left[(4-3(1))^{\frac{1}{2}} - (4-3(0))^{\frac{1}{2}}\right]$$

$$= \frac{2}{9}\left[(4-3)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right] - \frac{8}{3}\left[(4-3)^{\frac{1}{2}} - (4)^{\frac{1}{2}}\right]$$

$$= \frac{2}{9}\left[(1)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right] - \frac{8}{3}\left[(1)^{\frac{1}{2}} - (4)^{\frac{1}{2}}\right]$$

$$= \frac{2}{9}\left[1 - (2)^{2\times\frac{3}{2}}\right] - \frac{8}{3}\left[1 - (2^{2})^{\frac{1}{2}}\right]$$

$$= \frac{2}{9}[1-8] - \frac{8}{3}[1-2]$$

$$= \frac{2}{9}(-7) - \frac{8}{3}(-1)$$

$$= \frac{-14}{9} + \frac{8}{3} \Rightarrow \frac{-14+24}{9}$$

$$= \frac{10}{9}$$ **Answer.**

(29)

Solution:

let

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x(2+\sin x)}\, dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{\sin x(2+\sin x)}\right) \cdot \cos x\, dx$$

$\div$ **and $\times$ by (2)**

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{2}{\sin x\cdot(2+\sin x)}\right) \cos x\, dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{(2+\sin x) - \sin x}{\sin x\cdot(2+\sin x)}\right) \cos x\, dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{(2+\sin x)}{\sin x \cdot (2+\sin x)} - \frac{\sin x}{\sin x(2+\sin x)} \right) \cos x \, dx$$

$$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{\sin x} - \frac{1}{2+\sin x} \right) \cos x \, dx$$

$$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{\cos x}{\sin x} - \frac{\cos x}{2+\sin x} \right) dx$$

$$= \frac{1}{2} \left[\ln \sin x - \ln(2+\sin x) \right]_{\pi/6}^{\pi/2}$$

$$= \frac{1}{2} \left[\left(\ln \sin\left(\frac{\pi}{2}\right) - \ln\left(2+\sin\left(\frac{\pi}{2}\right)\right) \right) - \left(\ln \sin\frac{\pi}{6} - \ln\left(2+\sin\left(\frac{\pi}{6}\right)\right) \right) \right]$$

$$= \frac{1}{2} \left[(\ln 1 - \ln 3) - \left(\ln \frac{1}{2} - \ln\left(2+\frac{1}{2}\right) \right) \right]$$

$$= \frac{1}{2} \left[\ln 1 - \ln 3 - \ln \frac{1}{2} - \ln \frac{5}{2} \right]$$

$$= \frac{1}{2} \left[0 - \ln 3 - \ln \frac{1}{2} + \ln \frac{5}{2} \right]$$

$$= \frac{1}{2} \left[\ln \frac{5/2}{3 \times \frac{1}{2}} \right]$$

$$= \frac{1}{2} \ln \frac{5}{3}$$ **Answer.**

(30)

$$\int_{0}^{\frac{\pi}{2}} \frac{\sin x}{(1+\cos x)(2+\cos x)} dx$$

Solution:

$$\int_{0}^{\pi/2} \frac{\sin x}{(1+\cos)(2+\cos x)} dx$$

$$I = \int_0^{\frac{\pi}{2}} \left(\frac{1}{(1+\cos x)(2+\cos x)} \sin x dx \right.$$

$$I = \int_0^{\frac{\pi}{2}} \left(\frac{(2+\cos x) - (1-\cos x)}{(1+\cos x)(2+\cos x)} \right) \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{(2+\cos x)}{(1+\cos x)\cdot(2+\cos x)} - \frac{(1+\cos x)}{(1+\cos x)(2+\cos x)} \right) \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sin x}{1+\cos x} - \frac{\sin x}{2+\cos x} \right) dx$$

$$= -\int_0^{\frac{\pi}{2}} \left(-\frac{\sin x}{1+\cos x} - \frac{(-\sin x)}{2+\cos x} \right) dx$$

$$= -\left[\ln(1+\cos x) - \ln(2+\cos x)\right]_0^{\pi/2}$$

$$= -\left[\ln\left(1+\cos\frac{\pi}{2}\right) - \ln\left(2+\cos\frac{\pi}{2}\right) - \ln(1+\cos(0)) - \ln(2+\cos(0)\right.$$

$$= -\left[\ln(1+0) - \ln(2+0) - \ln(1+1) - \ln(2+1)\right]$$

$$= -\left[\ln(1) - \ln(2) - \ln(2) - \ln(3)\right]$$

$$= -\left[0 - \ln 2 - \ln 2 - \ln 3\right]$$

$$= \ln \frac{2\times 2}{3}$$

$$= \ln \frac{4}{3}$$ **Answer.**

Good work