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|------------------------------|----------------------------|---------------------|
| Math Sci 9: Test | | Total No. 40 |
| Name: _____ Roll No. : _____ | | |
| Date: _____ - _____ -20 | Teacher's Signature: _____ | |

Q.1: Tick (✓) the correct answer.

Factors of $3x^2 - 75y^2$ is:

$3(x - 25y)$ (D) $3(x + 5y)(x - 5y)$ (C) $3(x + 25y)(x - 25y)$ (B) $(3x + 75y)(3x - 75y)$ (A)

Factors of $x^2 - 11x - 42$ is _____.

$(x + 14)(x - 3)$ (D) $(x - 14)(x + 3)$ (C) $(x - 14)(x - 3)$ (B) $(x + 14)(x + 3)$ (A)

What will be added to complete the square of $9a^2 - 12ab$?

$-4b^2$ (D) $4b^2$ (C) $16b^2$ (B) $-16b^2$ (A)

Find m so that $x^2 + 4x + m$ is a complete square.

16 (D) 4 (C) -8 (B) 8 (A)

Factors of $5x^2 - 17xy - 12y^2$ are:

$(5x - 4y), (x + 3y)$ (D) $(x - 4y), (5x + 3y)$ (C) $(x - 4y), (5x - 3y)$ (B) $(x + 4y), (5x + 3y)$ (A)

The factors of $x^2 - 5x + 6$ are:

$x + 2, x + 3$ (D) $x + 6, x - 1$ (C) $x - 2, x - 3$ (B) $x + 1, x - 6$ (A)

Factors of $3x^2 - x - 2$ are:

$(x - 1)(3x + 2)$ (D) $(x - 1), (3x - 2)$ (C) $(x + 1), (3x + 2)$ (B) $(x + 1), (3x - 2)$ (A)

Factors of $8x^3 + 27y^3$ are:

$(2x - 3y), (4x^2 + 6xy + 9y^2)$ (D) $(2x + 3y), (4x^2 - 6xy + 9y^2)$ (C) $(2x - 3y), (4x^2 - 9y^2)$ (B) $(2x + 3y), (4x^2 + 9y^2)$ (A)

Factors of $a^4 - 4b^4$ are:

$(a - 2b), (a^2 + 2b^2)$ (D) $(a - b), (a + b), (a^2 - 4b^2)$ (C) $(a^2 - 2b^2), (a^2 + 2b^2)$ (B) $(a - b), (a + b), (a^2 + 4b^2)$ (A)

Factorize $3x^2 - 75y^2$.

$3(x - 5y)$ (D) $(x - 5y)(x + 5y)$ (C) $x(3 - 5y)(3 + 5y)$ (B) $3(x - 5y)(x + 5y)$ (A)

10 × 2 = 20 Write short answers to any ten (10) questions.

What is meant by factor?

Factorize $25x^2 + 16 + 40x$.

Factorize $128am^2 - 242an^2$.

Factorize

$$x^2 + x - 132$$

Factorize

$$x^2 - 7x + 12$$

What is meant by remainder theorem?

Factorize $4x^2 - 16y^2$.

Define factorization.

Factorize $9xy - 12x^2y + 18y^2$.

Factorize $x^2 - a^2 + 2a - 1$.

Factorize $x^2 + 5x - 36$.

Define Factor Theorem.

1 × 10 = 10 Write answer to any One (1) question.

Factorize $1 - 12pq + 36p^2q^2$.

Factorize the cubic polynomial by factor theorem:

$$x^3 - 2x^2 - x + 2$$

سوال نمبر 1- درست جواب پر (✓) کا نشان لگائیں۔

1- $3x^2 - 75y^2$ کی اجزائے ضربی ہے:

$3(x - 25y)$ (D) $3(x + 5y)(x - 5y)$ (C) $3(x + 25y)(x - 25y)$ (B) $(3x + 75y)(3x - 75y)$ (A)

2- $x^2 - 11x - 42$ کی اجزائے ضربی ہے:

$(x + 14)(x - 3)$ (D) $(x - 14)(x + 3)$ (C) $(x - 14)(x - 3)$ (B) $(x + 14)(x + 3)$ (A)

3- $9a^2 - 12ab$ کو کامل مربع بنانے کے لیے اس میں کیا جمع کریں گے؟

$-4b^2$ (D) $4b^2$ (C) $16b^2$ (B) $-16b^2$ (A)

4- m کی کس قیمت کے لیے $x^2 + 4x + m$ کامل مربع بن جائے گا؟

16 (D) 4 (C) -8 (B) 8 (A)

5- $5x^2 - 17xy - 12y^2$ کے اجزائے ضربی ہیں:

$(5x - 4y), (x + 3y)$ (D) $(x - 4y), (5x + 3y)$ (C) $(x - 4y), (5x - 3y)$ (B) $(x + 4y), (5x + 3y)$ (A)

6- $x^2 - 5x + 6$ کے اجزائے ضربی ہیں:

$x + 2, x + 3$ (D) $x + 6, x - 1$ (C) $x - 2, x - 3$ (B) $x + 1, x - 6$ (A)

7- $3x^2 - x - 2$ کے اجزائے ضربی ہیں:

$(x - 1)(3x + 2)$ (D) $(x - 1), (3x - 2)$ (C) $(x + 1), (3x + 2)$ (B) $(x + 1), (3x - 2)$ (A)

8- $8x^3 + 27y^3$ کے اجزائے ضربی ہیں:

$(2x - 3y), (4x^2 + 6xy + 9y^2)$ (D) $(2x + 3y), (4x^2 - 6xy + 9y^2)$ (C) $(2x - 3y), (4x^2 - 9y^2)$ (B) $(2x + 3y), (4x^2 + 9y^2)$ (A)

9- $a^4 - 4b^4$ کے اجزائے ضربی ہیں:

$(a - 2b), (a^2 + 2b^2)$ (D) $(a - b), (a + b), (a^2 - 4b^2)$ (C) $(a^2 - 2b^2), (a^2 + 2b^2)$ (B) $(a - b), (a + b), (a^2 + 4b^2)$ (A)

10- $3x^2 - 75y^2$ کی تجزی کیجیے:

$3(x - 5y)$ (D) $(x - 5y)(x + 5y)$ (C) $x(3 - 5y)(3 + 5y)$ (B) $3(x - 5y)(x + 5y)$ (A)

سوال نمبر 2- کوئی سے 10 سوالات کے جوابات تحریر کیجیے۔

What is meant by factor?

i- اجزائے ضربی سے کیا مراد ہے؟

Factorize $25x^2 + 16 + 40x$.

ii- $25x^2 + 16 + 40x$ کی تجزی کیجیے۔

Factorize $128am^2 - 242an^2$.

iii- $128am^2 - 242an^2$ تجزی کریں۔

Factorize

$$x^2 + x - 132$$

iv- تجزی کریں:

Factorize

$$x^2 - 7x + 12$$

v- تجزی کریں۔

What is meant by remainder theorem?

vi- مسئلہ باقی سے کیا مراد ہے؟

Factorize $4x^2 - 16y^2$.

vii- $4x^2 - 16y^2$ تجزی کیجیے۔

Define factorization.

viii- تجزی کی تعریف کیجیے؟

Factorize $9xy - 12x^2y + 18y^2$.

ix- $9xy - 12x^2y + 18y^2$ تجزی کیجیے۔

Factorize $x^2 - a^2 + 2a - 1$.

x- $x^2 - a^2 + 2a - 1$ تجزی کریں۔

Factorize $x^2 + 5x - 36$.

xi- $x^2 + 5x - 36$ تجزی کریں۔

Define Factor Theorem.

xii- مسئلہ تجزی کی تعریف کیجیے۔

1 × 10 = 10 Write answer to any One (1) question.

نوٹ: کوئی سے ایک سوال کا جواب لکھیے۔

Factorize $1 - 12pq + 36p^2q^2$.

سوال نمبر 3- $1 - 12pq + 36p^2q^2$ کی تجزی کیجیے۔

Factorize the cubic polynomial by factor theorem:

$$x^3 - 2x^2 - x + 2$$

سوال نمبر 4- مسئلہ تجزی کی مدد سے کثیررتی جملے کی تجزی کیجیے:

Exercise 5.1

Q.1 Factorize

(i) $2abc - 4abx + 2abd$

Solution: $2abc - 4abx + 2abd$
 $= 2ab(c - 2x + d)$

(ii) $9xy - 12x^2y + 18y^2$

Solution: $9xy - 12x^2y + 18y^2$
 $= 3y(3x - 4x^2 + 6y)$

(iii) $-3x^2y - 3x + 9xy^2$

Solution: $-3x^2y - 3x + 9xy^2$
 $= -3x(xy + 1 - 3y^2)$

(iv) $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

Solution: $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$
 $= 5abc(bc^2 - 2ab^2 - 4a^2c)$

(v) $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

Solution: $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$
 $= (x - 3y)(3x^3y - 7x^2y^2)$
 $= (x - 3y)x^2y(3x - 7y)$
 $= x^2y(x - 3y)(3x - 7y)$

(vi) $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$

Solution: $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$
 $= (x^2 + 5)(2xy^3 + 8xy^2)$
 $= (x^2 + 5)2xy^2(y + 4)$
 $= 2xy^2(x^2 + 5)(y + 4)$

Q.2 Factorize

(i) $5ax - 3ay - 5bx + 3by$

Solution: $5ax - 3ay - 5bx + 3by$
 $= 5ax - 5bx - 3ay + 3by$
 $= 5x(a - b) - 3y(a - b)$
 $= (a - b)(5x - 3y)$

(ii) $3xy + 2y - 12x - 8$

Solution: $3xy + 2y - 12x - 8$
 $= 3xy - 12x + 2y - 8$
 $= 3x(y - 4) + 2(y - 4)$
 $= (y - 4)(3x + 2)$

(iii) $x^3 + 3xy^2 - 2x^2y - 6y^3$

Solution: $x^3 + 3xy^2 - 2x^2y - 6y^3$
 By cyclic order
 $= x^3 - 2x^2y + 3xy^2 - 6y^3$
 $= x^2(x - 2y) + 3y^2(x - 2y)$
 $= (x - 2y)(x^2 + 3y^2)$

(iv) $(x^2 - y^2)z + (y^2 - z^2)x$

Solution: $(x^2 - y^2)z + (y^2 - z^2)x$
 $= x^2z - y^2z + xy^2 - xz^2$
 Arrange in cyclic order
 $x^2z + xy^2 - xz^2 - y^2z$
 $= x^2z + xy^2 - y^2z - xz^2$
 $= x(xz + y^2) - z(xz + y^2)$
 $= (xz + y^2)(x - z)$

Q.3 Factorize

(i) $144a^2 + 24a + 1$

Solution: $144a^2 + 24a + 1$

By using formula

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$= (12a)^2 + 2(12a)(1) + (1)^2$$

$$= (12a + 1)^2$$

$$(ii) \quad \frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$$

$$\text{Solution: } \frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$$

$$\text{Formula } a^2 - 2ab + b^2 = (a - b)^2$$

$$= \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$$

$$= \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

$$(iii) \quad (x + y)^2 - 14z(x + y) + 49z^2$$

$$\text{Solution: } (x + y)^2 - 14z(x + y) + 49z^2$$

$$\text{Formula } a^2 - 2ab + b^2 = (a - b)^2$$

$$= (x + y)^2 - 2(x + y)(7z) + (7z)^2$$

$$= (x + y - 7z)^2$$

$$(iv) \quad 12x^2 - 36x + 27$$

$$\text{Solution: } 12x^2 - 36x + 27$$

$$= 3(4x^2 - 12x + 9)$$

$$\text{Formula } a^2 - 2ab + b^2 = (a - b)^2$$

$$= 3[(2x)^2 - 2(2x)(3) + (3)^2]$$

$$= 3(2x - 3)^2$$

Q.4 Factorize

$$(i) \quad 3x^2 - 75y^2$$

$$\text{Solution: } 3x^2 - 75y^2$$

$$= 3(x^2 - 25y^2)$$

$$\text{Formula } a^2 - b^2 = (a + b)(a - b)$$

$$= 3[(x)^2 - (5y)^2]$$

$$= 3(x + 5y)(x - 5y)$$

$$(ii) \quad x(x - 1) - y(y - 1)$$

$$\text{Solution: } x(x - 1) - y(y - 1)$$

$$= x^2 - x - y^2 + y$$

Arranging in cyclic order

$$= x^2 - y^2 - x + y$$

Taking common

$$= (x^2 - y^2) - (x - y)$$

$$= [(x + y)(x - y)] - (x - y)$$

$$= (x - y)(x + y - 1)$$

$$(iii) \quad 128am^2 - 242an^2$$

$$\text{Solution: } 128am^2 - 242an^2$$

$$= 2a(64m^2 - 121n^2)$$

$$= 2a[(8m)^2 - (11n)^2]$$

$$= 2a(8m + 11n)(8m - 11n)$$

$$(iv) \quad 3x - 243x^3$$

$$\text{Solution: } 3x - 243x^3$$

$$= 3x(1 - 81x^2)$$

$$= 3x[(1)^2 - (9x)^2]$$

$$= 3x(1 + 9x)(1 - 9x)$$

Q.5 Factorize

$$(i) \quad x^2 - y^2 - 6y - 9$$

$$\text{Solution: } x^2 - y^2 - 6y - 9$$

$$= x^2 - [y^2 + 6y + 9]$$

$$= x^2 - [(y)^2 + 2(y)(3) + (3)^2]$$

$$= x^2 - (y + 3)^2$$

$$= (x)^2 - (y + 3)^2$$

$$= (x + y + 3)[x - (y + 3)]$$

$$= (x + y + 3)(x - y - 3)$$

$$(ii) \quad x^2 - a^2 + 2a - 1$$

$$\text{Solution: } x^2 - a^2 + 2a - 1$$

$$= x^2 - [a^2 - 2a + 1]$$

$$= x^2 - (a - 1)^2$$

$$= [x + (a - 1)][x - (a - 1)]$$

$$= (x + a - 1)(x - a + 1)$$

(iii) $4x^2 - y^2 - 2y - 1$

Solution: $4x^2 - y^2 - 2y - 1$
 $= 4x^2 - (y^2 + 2y + 1)$
 $= 4x^2 - [(y)^2 + 2(y)(1) + (1)^2]$
 $= 4x^2 - (y + 1)^2$
 $= (2x)^2 - (y + 1)^2$
 $= [2x + (y + 1)][2x - (y + 1)]$
 $= (2x + y + 1)(2x - y - 1)$

(iv) $x^2 - y^2 - 4x - 2y + 3$

Solution: $x^2 - y^2 - 4x - 2y + 3$
 $= x^2 - 4x + 4 - y^2 - 2y - 1$
 $= (x^2 - 4x + 4) - (y^2 + 2y + 1)$
 $= [(x)^2 - 2(x)(2) + (2)^2]$
 $- [(y)^2 + 2(y)(1) + (1)^2]$
 $= (x - 2)^2 - (y + 1)^2$
 $= (x - 2 + y + 1)[x - 2 - (y + 1)]$
 $= (x - 2 + y + 1)(x - 2 - y - 1)$
 $= (x + y - 2 + 1)(x - y - 2 - 1)$
 $= (x + y - 1)(x - y - 3)$

(v) $25x^2 - 10x + 1 - 36z^2$

Solution: $25x^2 - 10x + 1 - 36z^2$
 $= (5x)^2 - 2(5x)(1) + (1)^2 - 36z^2$
 $= (5x - 1)^2 - (6z)^2$
 $= [(5x - 1) + 6z][(5x - 1) - 6z]$
 $= (5x - 1 + 6z)(5x - 1 - 6z)$

(vi) $x^2 - y^2 - 4xz + 4z^2$

Solution: $x^2 - y^2 - 4xz + 4z^2$
 $= x^2 - 4xz + 4z^2 - y^2$
 $= [(x)^2 - 2(x)(2z) + (2z)^2] - y^2$
 $= (x - 2z)^2 - (y)^2$
 $= (x - 2z + y)(x - 2z - y)$



Exercise 5.2

Q.1 Factorize

(i) $x^4 + \frac{1}{x^4} - 3$

Solution: $x^4 + \frac{1}{x^4} - 3$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 3$$

By adding and subtracting by 2

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 - 2 - 3$$

$$= \left[(x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2 \right] + 2 - 3$$

$$= \left[(x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2 \right] - 1$$

$$= \left(x^2 - \frac{1}{x^2} \right)^2 - (1)^2$$

$$= \left(x^2 - \frac{1}{x^2} + 1 \right) \left(x^2 - \frac{1}{x^2} - 1 \right)$$

(ii) $3x^4 + 12y^4$

Solution: $3x^4 + 12y^4$

$$= 3(x^4 + 4y^4)$$

By adding and subtracting by $2(x^2)(2y^2)$

$$= 3 \left[(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - 2(x^2)(2y^2) \right]$$

$$= 3 \left[(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - 2(x^2)(2y^2) \right]$$

$$= 3 \left[(x^2 + 2y^2)^2 - 4x^2y^2 \right]$$

$$= 3 \left[(x^2 + 2y^2)^2 - (2xy)^2 \right]$$

$$= 3 \left[(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy) \right]$$

$$= 3 \left[(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2) \right]$$

(iii) $a^4 + 3a^2b^2 + 4b^4$

Solution: $a^4 + 3a^2b^2 + 4b^4$

$$= (a^4 + 4b^4) + 3a^2b^2$$

$$= (a^2)^2 + (2b^2)^2 + 3a^2b^2$$

By adding and subtracting by $2(a^2)(2b^2)$

$$= (a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) - 2(a^2)(2b^2) + 3a^2b^2$$

$$= \left[(a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) \right] - 2(a^2)(2b^2) + 3a^2b^2$$

$$= (a^2 + 2b^2)^2 - a^2b^2$$

$$= (a^2 + 2b^2)^2 - (ab)^2$$

$$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$$

(iv) $4x^4 + 81$

Solution: $4x^4 + 81$

$$= (2x^2)^2 + (9)^2$$

By adding and subtracting by $2(2x^2)(9)$

$$= \left[(2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9) \right]$$

$$= \left[(2x^2)^2 + (9)^2 + 2(2x^2)(9) \right] - 2(2x^2)(9)$$

$$= (2x^2 + 9)^2 - 36x^2$$

$$= (2x^2 + 9)^2 - (6x)^2$$

$$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$$

$$= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$$

(v) $x^4 + x^2 + 25$

Solution: $x^4 + x^2 + 25$

$$= (x^4 + 25) + x^2$$

$$= \left[(x^2)^2 + (5)^2 \right] + x^2$$

By adding and subtracting by $2(x^2)(5)$

$$= \left[(x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5) \right] + x^2$$

$$= \left[(x^2)^2 + (5)^2 + 2(x^2)(5) \right] - 2(x^2)(5) + x^2$$

$$= (x^2 + 5)^2 - 10x^2 + x^2$$

$$= (x^2 + 5)^2 - 9x^2$$

$$\begin{aligned}
 &= (x^2 + 5)^2 - (3x)^2 \\
 &= (x^2 + 5 + 3x)(x^2 + 5 - 3x) \\
 &= (x^2 + 3x + 5)(x^2 - 3x + 5)
 \end{aligned}$$

(vi) $x^4 + 4x^2 + 16$

Solution: $x^4 + 4x^2 + 16$

$$\begin{aligned}
 &= (x^2)^2 + 16 + 4x^2 \\
 &= (x^2)^2 + (4)^2 + 4x^2
 \end{aligned}$$

By adding and subtracting by $2(x^2)(4)$

$$\begin{aligned}
 &= (x^2)^2 + (4)^2 + 2(x^2)(4) - 2(x^2)(4) + 4x^2 \\
 &= (x^2)^2 + (4)^2 + 2(x^2)(4) - 2(x^2)(4) + 4x^2 \\
 &= (x^2 + 4)^2 - 8x^2 + 4x^2 \\
 &= (x^2 + 4)^2 - 4x^2 \\
 &= (x^2 + 4)^2 - (2x)^2 \\
 &= (x^2 + 4 + 2x)(x^2 + 4 - 2x) \\
 &= (x^2 + 2x + 4)(x^2 - 2x + 4)
 \end{aligned}$$

Q.2 Factorize

(i) $x^2 + 14x + 48$

Solution: $x^2 + 14x + 48$

$$\begin{aligned}
 &= x^2 + 8x + 6x + 48 \\
 &= x(x + 8) + 6(x + 8) \\
 &= (x + 8)(x + 6)
 \end{aligned}$$

(ii) $x^2 - 21x + 108$

Solution: $x^2 - 21x + 108$

$$\begin{aligned}
 &= x^2 - 12x - 9x + 108 \\
 &= x(x - 12) - 9(x - 12) \\
 &= (x - 9)(x - 12)
 \end{aligned}$$

(iii) $x^2 - 11x - 42$

Solution: $x^2 - 11x - 42$

$$\begin{aligned}
 &= x^2 - 14x + 3x - 42 \\
 &= x(x - 14) + 3(x - 14) \\
 &= (x + 3)(x - 14)
 \end{aligned}$$

(iv) $x^2 + x - 132$

Solution: $x^2 + x - 132$

$$\begin{aligned}
 &= x^2 + 12x - 11x - 132
 \end{aligned}$$

$$\begin{aligned}
 &= x(x + 12) - 11(x + 12)
 \end{aligned}$$

$$\begin{aligned}
 &= (x - 11)(x + 12)
 \end{aligned}$$

Q.3 Factorize

(i) $4x^2 + 12x + 5$

Solution: $4x^2 + 12x + 5$

$$\begin{aligned}
 &= 4x^2 + 2x + 10x + 5 \\
 &= 2x(2x + 1) + 5(2x + 1) \\
 &= (2x + 5)(2x + 1)
 \end{aligned}$$

(ii) $30x^2 + 7x - 15$

Solution: $30x^2 + 7x - 15$

$$\begin{aligned}
 &= 30x^2 + 25x - 18x - 15 \\
 &= 5x(6x + 5) - 3(6x + 5) \\
 &= (5x - 3)(6x + 5)
 \end{aligned}$$

(iii) $24x^2 - 65x + 21$

Solution: $24x^2 - 65x + 21$

$$\begin{aligned}
 &= 24x^2 - 56x - 9x + 21 \\
 &= 8x(3x - 7) - 3(3x - 7) \\
 &= (8x - 3)(3x - 7)
 \end{aligned}$$

(iv) $5x^2 - 16x - 21$

Solution: $5x^2 - 16x - 21$

$$\begin{aligned}
 &= 5x^2 + 5x - 21x - 21 \\
 &= 5x(x + 1) - 21(x + 1) \\
 &= (5x - 21)(x + 1)
 \end{aligned}$$

(v) $4x^2 - 17xy + 4y^2$

Solution: $4x^2 - 17xy + 4y^2$

$$\begin{aligned}
 &= 4x^2 - 16xy - xy + 4y^2 \\
 &= 4x(x - 4y) - y(x - 4y) \\
 &= (4x - y)(x - 4y)
 \end{aligned}$$

(vi) $3x^2 - 38xy - 13y^2$

Solution: $3x^2 - 38xy - 13y^2$

$$\begin{aligned}
 &= 3x^2 - 39xy + xy - 13y^2 \\
 &= 3x(x - 13y) + y(x - 13y) \\
 &= (3x + y)(x - 13y)
 \end{aligned}$$

(vii) $5x^2 + 33xy - 14y^2$

Solution: $5x^2 + 33xy - 14y^2$

$$\begin{aligned}
 &= 5x^2 + 35xy - 2xy - 14y^2 \\
 &= 5x(x + 7y) - 2y(x + 7y) \\
 &= (5x - 2y)(x + 7y)
 \end{aligned}$$

(viii) $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0$

Solution: $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0$

$$\begin{aligned}
 &= \left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right)(2) + (2)^2 \\
 &= \left(5x - \frac{1}{x} + 2\right)^2 \\
 &= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)
 \end{aligned}$$

Q.4

(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Solution: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Suppose that

$$x^2 + 5x = y$$

So,

$$\begin{aligned}
 &(x^2 + 5x + 4)(x^2 + 5x + 6) - 3 \\
 &= (y + 4)(y + 6) - 3 \\
 &= [y(y + 6) + 4(y + 6)] - 3 \\
 &= (y^2 + 6y + 4y + 24) - 3 \\
 &= (y^2 + 10y + 24) - 3 \\
 &= y^2 + 10y + 24 - 3 \\
 &= y^2 + 10y + 21 \\
 &= y^2 + 7y + 3y + 21 \\
 &= y(y + 7) + 3(y + 7) \\
 &= (y + 3)(y + 7)
 \end{aligned}$$

We know that $y = x^2 + 5x$

$$= (x^2 + 5x + 3)(x^2 + 5x + 7)$$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Solution: $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Suppose that

$$x^2 - 4x = y$$

So,

$$\begin{aligned}
 &= (y)(y - 1) - 20 \\
 &= (y^2 - y) - 20 \\
 &= y^2 - y - 20 \\
 &= y^2 - 5y + 4y - 20 \\
 &= y(y - 5) + 4(y - 5) \\
 &= (y + 4)(y - 5) \\
 &\text{We know that } a = x^2 - 4x \\
 &= (x^2 - 4x + 4)(x^2 - 4x - 5) \\
 &= [(x)^2 - 2(x)(2) + (2)^2][x^2 - 5x + x - 5] \\
 &= (x - 2)^2 [x(x - 5) + 1(x - 5)] \\
 &= (x - 2)^2 (x - 5)(x + 1) \\
 &= (x - 5)(x + 1)(x - 2)^2
 \end{aligned}$$

(iii) $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Solution: $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

$$\begin{aligned}
 &= [(x + 2)(x + 5)][(x + 3)(x + 4)] - 15 \\
 &= [x(x + 5) + 2(x + 5)][x(x + 4) + 3(x + 4)] - 15 \\
 &= [x^2 + 5x + 2x + 10][x^2 + 4x + 3x + 12] - 15 \\
 &= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15
 \end{aligned}$$

Suppose that

$$x^2 + 7x = y$$

So,

$$\begin{aligned}
 &(x^2 + 7x + 10)(x^2 + 7x + 12) - 15 \\
 &= (y + 10)(y + 12) - 15 \\
 &= [y(y + 12) + 10(y + 12)] - 15 \\
 &= (y^2 + 12y + 10y + 120) - 15 \\
 &= (y^2 + 22y + 120) - 15 \\
 &= y^2 + 22y + 120 - 15 \\
 &= y^2 + 22y + 105 \\
 &= y^2 + 15y + 7y + 105 \\
 &= y(y + 15) + 7(y + 15) \\
 &= (y + 7)(y + 15)
 \end{aligned}$$

We know that $y = x^2 + 7x$

$$= (x^2 + 7x + 7)(x^2 + 7x + 15)$$

(iv) $(x + 4)(x - 5)(x + 6)(x - 7) - 504$

$$\begin{aligned} \text{Solution: } & (x+4)(x-5)(x+6)(x-7) - 504 \\ & = [(x+4)(x-5)][(x+6)(x-7)] - 504 \\ & = [x(x-5) + 4(x-5)][x(x-7) + 6(x-7)] - 504 \\ & = (x^2 - 5x + 4x - 20)(x^2 - 7x + 6x - 42) - 504 \\ & = (x^2 - x - 20)(x^2 - x - 42) - 504 \end{aligned}$$

Suppose that

$$x^2 - x = y$$

So,

$$\begin{aligned} & = (y-20)(y-42) - 504 \\ & = [y(y-42) - 20(y-42)] - 504 \\ & = (y^2 - 42y - 20y + 840) - 504 \\ & = y^2 - 62y + 840 - 504 \\ & = y^2 - 62y + 336 \\ & = y^2 - 56y - 6y + 336 \\ & = y(y-56) - 6(y-56) \\ & = (y-6)(y-56) \end{aligned}$$

We know that $a = x^2 - x$

$$\begin{aligned} & = (x^2 - x - 6)(x^2 - x - 56) \\ & = (x^2 - 3x + 2x - 6)(x^2 - 8x + 7x - 56) \\ & = [x(x-3) + 2(x-3)][x(x-8) + 7(x-8)] \\ & = (x+2)(x-3)(x+7)(x-8) \end{aligned}$$

$$(v) \quad (x+1)(x+2)(x+3)(x+6) - 3x^2$$

$$\begin{aligned} \text{Solution: } & (x+1)(x+2)(x+3)(x+6) - 3x^2 \\ & = [(x+1)(x+6)][(x+2)(x+3)] - 3x^2 \\ & = [x(x+6) + 1(x+6)][x(x+3) + 2(x+3)] - 3x^2 \\ & = (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2 \\ & = (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2 \end{aligned}$$

Suppose that

$$x^2 + 6 = y$$

So,

$$\begin{aligned} & = (y+7x)(y+5x) - 3x^2 \\ & = [y(y+5x) + 7x(y+5x)] - 3x^2 \\ & = (y^2 + 5xy + 7xy + 35x^2 - 3x^2) \\ & = y^2 + 12xy + 32x^2 \\ & = y^2 + 8xy + 4xy + 32x^2 \\ & = y(y+8x) + 4x(y+8x) \\ & = (y+4x)(y+8) \end{aligned}$$

We know that $y = x^2 + 6$

$$\begin{aligned} & = (x^2 + 6 + 4x)(x^2 + 6 + 8x) \\ & = (x^2 + 4x + 6)(x^2 + 8x + 6) \end{aligned}$$

Q.5

$$(i) \quad x^3 + 48x - 12x^2 - 64$$

$$\begin{aligned} \text{Solution: } & x^3 + 48x - 12x^2 - 64 \\ & = x^3 - 12x^2 + 48x - 64 \\ & \quad a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3 \\ & = (x)^3 - 3(x)^2(4) + 3(x)(4)^2 - (4)^3 \\ & = (x-4)^3 \end{aligned}$$

$$(ii) \quad 8x^3 + 60x^2 + 150x + 125$$

$$\begin{aligned} \text{Solution: } & 8x^3 + 60x^2 + 150x + 125 \\ & \quad a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3 \\ & = (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3 \\ & = (2x+5)^3 \end{aligned}$$

$$(iii) \quad x^3 - 18x^2 + 108x - 216$$

$$\begin{aligned} \text{Solution: } & x^3 - 18x^2 + 108x - 216 \\ & \quad a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3 \\ & = (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3 \\ & = (x-6)^3 \end{aligned}$$

$$(iv) \quad 8x^3 - 125y^3 - 60x^2y + 150xy^2$$

$$\begin{aligned} \text{Solution: } & 8x^3 - 125y^3 - 60x^2y + 150xy^2 \\ & = 8x^3 - 60x^2y + 150xy^2 - 125y^3 \\ & \quad a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3 \\ & = (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3 \\ & = (2x-5y)^3 \end{aligned}$$

Q.6

$$(i) \quad 27 + 8x^3$$

$$\begin{aligned} \text{Solution: } & 27 + 8x^3 \\ & = (3)^3 + (2x)^3 \\ & = (3+2x)[(3)^2 - (3)(2x) + (2x)^2] \\ & = (3+2x)(9 - 6x + 4x^2) \end{aligned}$$

$$(ii) \quad 125x^3 - 216y^3$$

$$\begin{aligned} \text{Solution: } & 125x^3 - 216y^3 \\ & = (5x)^3 - (6y)^3 \\ & \quad (a-b)(a^2 + ab + b^2) = a^3 - b^3 \\ & = (5x-6y)[(5x)^2 + (5x)(6y) + (6y)^2] \end{aligned}$$

$$= (5x - 6y)(25x^2 + 30xy + 36y^2)$$

(iii) $64x^3 + 27y^3$

Solution: $64x^3 + 27y^3$

$$= (4x)^3 + (3y)^3$$

$$(a + b)(a^2 + ab + b^2) = a^3 + b^3$$

$$= (4x + 3y) \left[(4x)^2 - (4x)(3y) + (3y)^2 \right]$$

$$= (4x + 3y)(16x^2 - 12xy + 9y^2)$$

(iv) $(2x)^3 + (5y)^3$

Solution: $(2x)^3 + (5y)^3$

$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$= (2x + 5y) \left[(2x)^2 - (2x)(5y) + (5y)^2 \right]$$

$$= (2x + 5y)(4x^2 - 10xy + 25y^2)$$



Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3 - 10x^2 + 13x - 6$ is divided by $(x - 2)$.

Solution:

$$P(x) = 3x^3 - 10x^2 + 13x - 6$$

Since $P(x)$ is divided by $(x - 2)$,

$$\therefore P(2) = R$$

$$R = 3(2)^3 - 10(2)^2 + 13(2) - 6$$

$$= 3(2)^3 - 10(2)^2 + 13(2) - 6$$

$$= 24 - 40 + 26 - 6$$

$$R = 4$$

Hence 4 is the remainder

(ii) $4x^3 - 4x + 3$ is divided by $(2x - 1)$

Solution:

$$P(x) = 4x^3 - 4x + 3$$

Since $P(x)$ is divided by $(2x - 1)$

$$\therefore R = P\left(\frac{1}{2}\right)$$

$$= 4\left[\frac{1}{2}\right]^3 - \cancel{A}^2 \times \frac{1}{\cancel{2}} + 3$$

$$= \cancel{A} \times \frac{1}{\cancel{2}^2} - 2 + 3$$

$$= \frac{1}{2} - 2 + 3$$

$$= \frac{1 - 4 + 6}{2} = \frac{3}{2}$$

$$R = \frac{3}{2}$$

Hence $\frac{3}{2}$ is the remainder

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(iii) $6x^4 + 2x^3 - x + 2$ is divided by $(x + 2)$ from $x + 2 = 0$

Solution: Given that

$$P(x) = 6x^4 + 2x^3 - x + 2$$

Since $P(x)$ is divided by $(x + 2)$

$$\therefore R = P(-2)$$

$$= 6(-2)^4 + 2(-2)^3 - (-2) + 2$$

$$= 96 - 16 + 2 + 2$$

$$R = 84$$

Hence 84 is the remainder

(iv) $(2x - 1)^3 + 6(3 + 4x)^2 - 10$ is divided by $2x + 1$ from $2x + 1 = 0$

$$x = -\frac{1}{2}$$

Solution: Given that

$$P(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10$$

Since $P(x)$ is divided by $2x + 1$

$$\therefore R = P\left(-\frac{1}{2}\right)$$

$$= \left[2\left(-\frac{1}{2}\right) - 1\right]^3 + 6\left[3 + 4\left(-\frac{1}{2}\right)\right]^2 - 10$$

$$= [-1 - 1]^3 + 6[3 - 2]^2 - 10$$

$$= [-2]^3 + 6 - 10 = -8 + 6 - 10$$

$$R = -12$$

Hence -12 is the remainder

(v) $x^3 - 3x^2 + 4x - 14$ is divided by $(x + 2)$ from $x + 2 = 0, x = -2$

Solution: Given that

$$P(x) = x^3 - 3x^2 + 4x - 14$$

Since $P(x)$ is divided by $(x + 2)$

$$\therefore R = P(-2)$$

$$= (-2)^3 - 3(-2)^2 + 4(-2) - 14$$

$$= -8 - 12 - 8 - 14$$

$$R = -42$$

Hence -42 is the remainder

Q.2

(i) If $(x+2)$ is a factor of $3x^2 - 4kx - 4k^2$ then find the values of k $x+2=0$ $x=-2$

Solution: Given that

$$P(x) = 3x^2 - 4kx - 4k^2$$

$$P(-2) = 3(-2)^2 - 4k(-2) - 4k^2$$

$$P(-2) = 12 + 8k - 4k^2$$

If $(x+2)$ is the factor then remainder is equal to zero

$$P(-2) = 0$$

$$12 + 8k - 4k^2 = 0$$

$$4(3 + 2k - k^2) = 0$$

$$-k^2 + 2k + 3 = \frac{0}{4}$$

$$-k^2 + 3k - k + 3 = 0$$

$$-k(k-3) - 1(k-3) = 0$$

$$(k-3)(-k-1) = 0$$

$$k-3=0 \quad -k-1=0$$

$$k=3 \quad -1=k$$

$$k=-1$$

(ii) If $(x-1)$ is a factor of $x^3 - kx^2 + 11x - 6$ then find the value of k from $x-1=0$ $x=1$

Solution: Given that

$$P(x) = x^3 - kx^2 + 11x - 6$$

$$P(1) = (1)^3 - k(1)^2 + 11(1) - 6$$

$$P(1) = 1 - k + 11 - 6$$

$$P(1) = 6 - k$$

If $(x-1)$ is the factor then remainder is equal to zero

$$P(1) = 0$$

$$6 - k = 0$$

$$k = 6$$

Q.3 Without long division determine whether

(i) $(x-2)$ and $(x-3)$ are factor of $P(x) = x^3 - 12x^2 + 44x - 48$ from $x-2=0$ $x=2$

Solution: Given that

$$P(x) = x^3 - 12x^2 + 44x - 48$$

If $(x-2)$ is the factor then remainder is equal to zero

$$P(2) = (2)^3 - 12(2)^2 + 44(2) - 48 = 8 - 48 + 88 - 48 = 0$$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$$R = P(3)$$

$$= (3)^3 - 12(3)^2 + 44(3) - 48$$

$$= (3)^3 - 12(3)^2 + 44(3) - 48$$

$$= 27 - 108 + 132 - 48$$

$$= 159 - 156$$

$$R = 3$$

3 is remainder hence $x-3$ is not factor of $P(x)$

$P(3)$ is not equal to zero then $x-3$ is not factor of $P(x) = x^3 - 12x^2 + 44x - 48$

(ii) $(x-2)$, $(x+3)$ and $(x-4)$ are factor of $q(x) = x^3 + 2x^2 - 5x - 6$ from $x-2=0$, $x=2$

Solution: Given that

$$q(x) = x^3 + 2x^2 - 5x - 6$$

For $(x-2)$, put $x-2=0$

$$x = 2$$

$$R = q(2)$$

$$= (2)^3 + 2(2)^2 - 5(2) - 6$$

$$R = 8 + 8 - 10 - 6$$

$$R = 16 - 16$$

$$R = 0$$

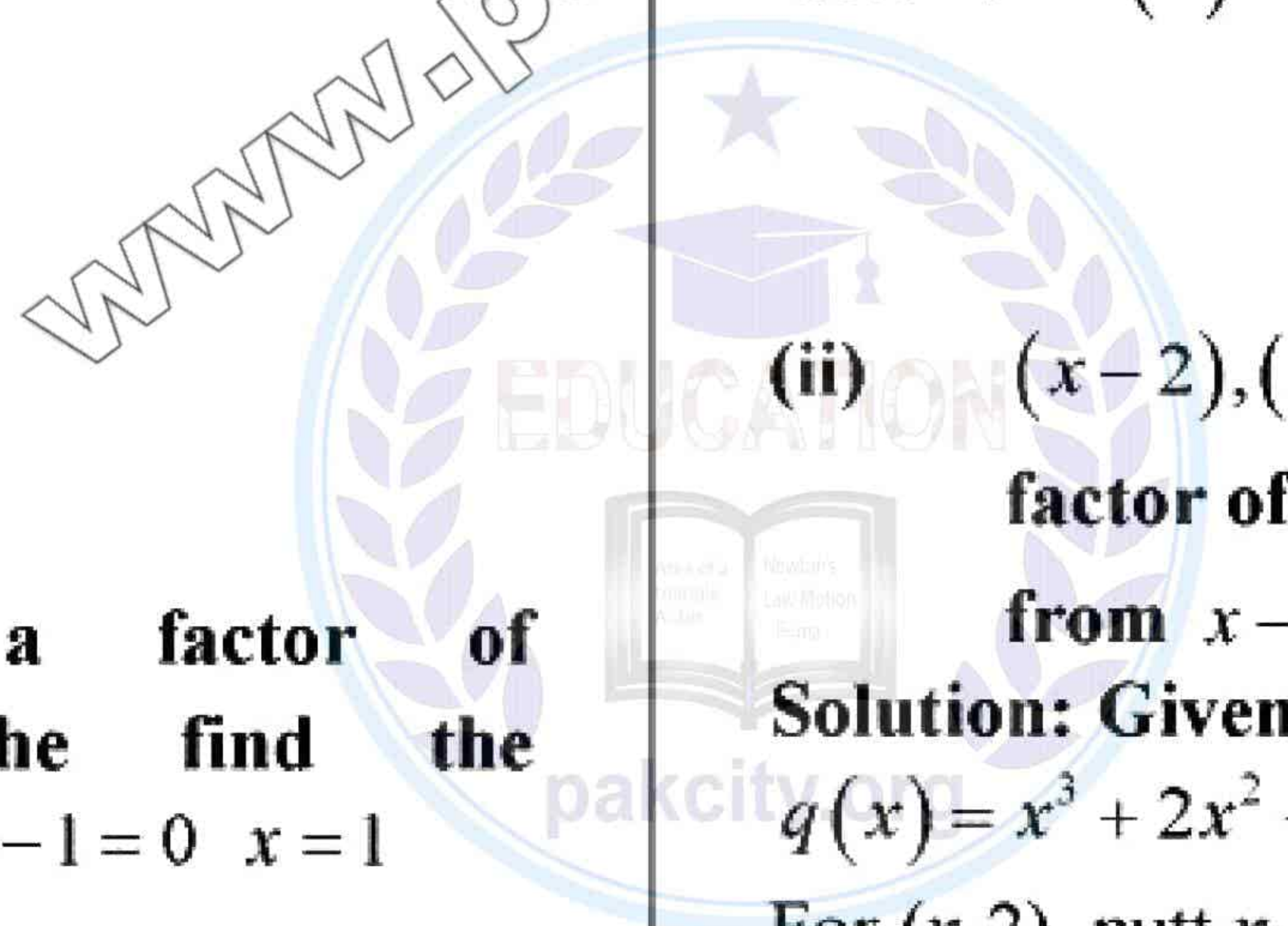
Hence $x-2$ is factor of

$$q(x) = x^3 + 2x^2 - 5x - 6$$

For $(x+3)$, put $x+3=0$

$$x = -3$$

$$R = q(-3)$$



$$= (-3)^3 + 2(-3)^2 - 5(-3) - 6$$

$$= -27 + 18 + 15 - 6$$

$$R = 0$$

Hence $x-2$ is factor of

$$q(x) = x^3 + 2x^2 - 5x - 6$$

For $x-4$, $x-4=0$

$$x=4$$

$$R = q(4)$$

$$= (4)^3 + 2(4)^2 - 5(4) - 6$$

$$= 64 + 32 - 20 - 6$$

$$R=70$$

Hence $x-4$ is not a factor of

$$q(x) = x^3 + 2x^2 - 5x - 6$$

Q.4 For what value of m is the polynomial $P(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $x+2$?

Solution:

$$P(x) = 4x^3 - 7x^2 + 6x - 3m$$

From $x+2=0$, $x=-2$

$$P(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$$

$$P(-2) = -32 - 28 - 12 - 3m = -72 - 3m$$

If $(x+2)$ is the factor then remainder is equal to zero

$$P(-2) = 0$$

$$-72 - 3m = 0$$

$$-72 = 3m$$

$$m = -\frac{72}{3}$$

$$m = -24$$

Q.5 Determine the value of k if $P(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$$q(x) = x^3 - 4x + k$$

from $x-3=0$ $x=3$

$$R_1 = q(3)$$

$$= (3)^3 - 4(3) + k$$

$$= 27 - 12 + k$$

$$= 15 + k$$

$$R_1 = 15 + k \quad \dots(i)$$

$$R_2 = P(3)$$

$$= k(3)^3 + 4(3)^2 + 3(3) - 4$$

$$= 27k + 36 + 9 - 4$$

$$R_2 = 27k + 41 \quad \dots(ii)$$

Since it leaves the same remainder.

$$\text{Hence } R_1 = R_2$$

$$15 + k = 27k + 41$$

$$15 - 41 = 27k - k$$

$$-26 = 26k$$

$$k = \frac{-26}{26}$$

$$k = -1$$

Q.6 The remainder after dividing the polynomial $P(x) = x^3 + ax^2 + 7$ by $(x+1)$ is $2b$ calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$$P(x) = x^3 + ax^2 + 7$$

Since $P(x)$ is divided by $(x+1)$

$$\text{Put } x+1=0 \quad x=-1$$

$$R = P(-1)$$

$$= (-1)^3 + a(-1)^2 + 7$$

$$= -1 + a + 7$$

$$R = a + 6$$

According to first condition remainder is $2b$

$$2b = a + 6 \quad \dots(i)$$

Since $P(x)$ is divided by $(x-2)$

$$\text{Put } x-2=0$$

$$x=2$$

$$P(2) = (2)^3 + a(2)^2 + 7$$

$$= 8 + 4a + 7$$

$$R = 15 + 4a$$

According to second condition remainder is $(b+5)$

$$15 + 4a = b + 5$$

$$4a - b = 5 - 15$$

$$4a - b = -10 \quad \dots(ii)$$

Solving equations (i) and (ii)

From equation (ii) $b=10+4a$ putting the value of b in equation (i)

$$a+6=2(10+4a)$$

$$a=20+8a-6$$

$$-8a+a=14$$

$$-7a=14$$

$$a = \frac{14}{-7}$$

$$a=-2$$

Putting the value of a in equation (ii)

$$4a - b = -10$$

$$4(-2) - b = -10$$

$$-8 - b = -10$$

$$-8 + 10 = b$$

$$2 = b$$

$$b = 2$$

Q.7 The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x + 4)$ and it leaves a remainder of 36 when divided by $(x - 2)$

Find the values of l and m .

Solution:

Let

$$P(x) = x^3 + lx^2 + mx + 24$$

$$\text{From } x + 4 = 0 \quad x = -4$$

$$P(-4) = (-4)^3 + l(-4)^2 + m(-4) + 24$$

$$P(-4) = -64 + 16l - 4m + 24$$

$$P(-4) = 16l - 4m - 40$$

According to condition $(x+4)$ is the factor then

$$16l - 4m - 40 = 0$$

$$4[4l - m - 10] = 0$$

$$4l - m - 10 = 0 \quad (i)$$

$$\text{from } x - 2 = 0 \quad x = 2$$

$$\text{Now } P(2) = (2)^3 + l(2)^2 + m(2) + 24$$

$$P(2) = 8 + 4l + 2m + 24$$

$$P(2) = 4l + 2m + 32$$

According the condition

$$4l + 2m + 32 = 36$$

$$4l + 2m = 36 - 32$$

$$4l + 2m = 4$$

$$4l + 2m - 4 = 0 \quad (ii)$$

Subtracting (i) from (ii)

$$\cancel{4l} + 2m - 4 = 0$$

$$\pm \cancel{4l} \mp m \mp 10 = 0$$

$$3m + 6 = 0$$

$$3m + 6 = 0$$

$$3m = -6$$

$$m = \frac{-6}{3}$$

$$m = -2$$

Putting the value of m in equation (i)

$$4l - (-2) - 10 = 0$$

$$4l + 2 - 10 = 0$$

$$4l - 8 = 0$$

$$4l = 8$$

$$l = \frac{8}{4}$$

$$l = 2$$

Q.8 The expression $lx^3 + mx^2 - 7$ leaves remainder of -3 and 12 when divided by $(x - 1)$ and $(x + 2)$ respectively. Calculate the value of l and m .

Solution:

$$P(x) = lx^3 + mx^2 - 7$$

$$\text{from } x - 1 = 0 \quad x = 1$$

$$P(1) = l(1)^3 + m(1)^2 - 7$$

$$P(1) = l + m - 7$$

According to conditions $l + m - 7 = -3$

$$l + m = 7 - 3$$

$$l + m = 4 \quad (i)$$

$$\text{From } x + 2 = 0 \quad x = -2$$

$$P(-2) = l(-2)^3 + m(-2)^2 - 7$$

$$P(-2) = -8l + 4m - 7$$

According to condition

$$-8l + 4m - 7 = 12$$

Putting the value of l in the equation

$$-8[1-m] + 4m = 16$$

$$-8 + 8m + 4m = 16$$

$$12m = 16 + 8$$

$$12m = 24$$

$$m = \frac{24}{12}$$

$$m = 2$$

Putting the value of m in equation (i)

$$l = 1 - 2$$

$$l = -1$$

$$m = 2$$

$$l = -1$$

$$11a + 2[27 - 10a] - 36 = 0 \quad \text{www.pakcity.org}$$

$$11a + 54 - 20a - 36 = 0$$

$$-9a + 18 = 0$$

$$+18 = 9a$$

$$a = \frac{+18}{9}$$

$$a = +2$$

Putting the value of a in equation (iii)

$$b = 27 - 10(+2)$$

$$b = 27 - 20$$

$$b = 7$$

$$a = 2$$

Q.9 The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$. Find the value of a and b .

Solution: Given that

$$P(x) = ax^3 - 9x^2 + bx + 3a$$

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$

$$= x[x - 2] - 3[x - 2]$$

$$= [x - 2][x - 3]$$

$(x - 2)(x - 3)$ divides the expression $ax^3 -$

$9x^2 + bx + 3a$ from $x - 2 = 0, x = 2$

$$P(2) = a(2)^3 - 9(2)^2 + b(2) + 3a$$

$$P(2) = 8a - 36 + 2b + 3a$$

$$P(2) = 11a + 2b - 36$$

According to condition $(x - 2)$ is the factor so

$$11a + 2b - 36 = 0 \quad \text{(i)}$$

From $x - 3 = 0, x = 3$

$$P(3) = a(3)^3 - 9(3)^2 + b(3) + 3a$$

$$P(3) = 27a - 81 + 3b + 3a$$

$$P(3) = 30a + 3b - 81$$

According to condition $(x - 3)$ is the factor so

$$30a + 3b - 81 = 0 \quad \text{(ii)}$$

$$3(10a + b - 27) = 0$$

$$10a + b - 27 = \frac{0}{3}$$

$$b = 27 - 10a \quad \text{(iii)}$$

Putting the value of b in equation (i)

Exercise 5.4

Q.1 $x^3 - 2x^2 - x + 2$

Solution: Given that

$$P(x) = x^3 - 2x^2 - x + 2$$

$P=2$ and possible factor of 2 are $\pm 1, \pm 2$.

Here $q=1$ and possible factor of 1 are ± 1 .

So possible factor of $P(x)$ will be form $\frac{P}{q} = \pm 1, \pm 2$

$$P(x) = x^3 - 2x^2 - x + 2$$

Put $x=1$

$$P(1) = (1)^3 - 2(1)^2 - 1 + 2 = 1 - 2 - 1 + 2 = 0 \text{ Remainder is equal to zero so } (x-1) \text{ is factor}$$

Put $x=-1$

$$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2 = -1 - 2 + 1 + 2 = 0 \text{ Remainder is equal to zero so } (x+1) \text{ is factor}$$

Put $x=2$

$$P(2) = (2)^3 - 2(2)^2 - (2) + 2 = 8 - 8 - 2 + 2 = 0 \text{ Remainder is equal to zero so } (x-2) \text{ is factor}$$

$$x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2)$$

Q.2 $x^3 - x^2 - 22x + 40$

Solution: Given that

$$P(x) = x^3 - x^2 - 22x + 40$$

$P=40$

possible factor of 40 = $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

Here $q=1$ and possible factor of 1 are ± 1

So possible factor of $P(x)$ will be from

$$\frac{P}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$$

$$P(x) = x^3 - x^2 - 22x + 40$$

Put $x = 2$

$$P(2) = (2)^3 - (2)^2 - 22(2) + 40$$

$$= 8 - 4 - 44 + 40 = 0$$

Remainder is equal to zero so $(x-2)$ is a factor

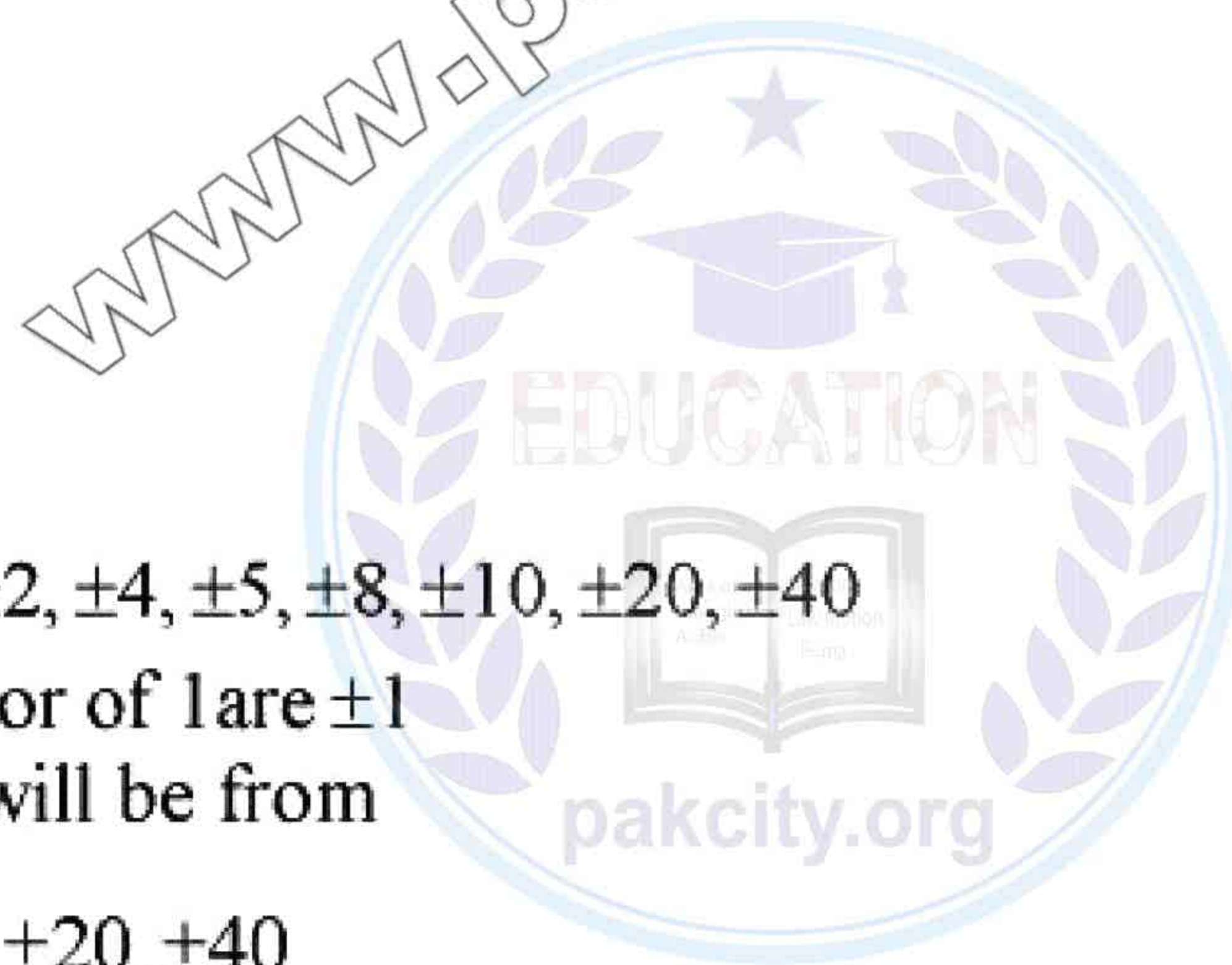
Put $x=4$

$$P(4) = (4)^3 - (4)^2 - 22(4) + 40$$

$$= 64 - 16 - 88 + 40 = 0$$

Remainder is not equal to zero so $(x-4)$ is a factor

Put $x=-5$



$$\begin{aligned}
 P(-5) &= (-5)^3 - (-5)^2 - 22(-5) + 40 \\
 &= -125 - 25 + 110 + 40 \\
 &= -150 + 150 \\
 &= 0
 \end{aligned}$$

Remainder is equal to zero so $(x+5)$ is a factor

$$\text{Hence } x^3 - x^2 - 22x + 40 = (x-2)(x-4)(x+5)$$

Q.3 $x^3 - 6x^2 + 3x + 10$

Solution: Given that

$$P(x) = x^3 - 6x^2 + 3x + 10$$

$$P=10$$

So possible factor of 10 are $\pm 1, \pm 2, \pm 5, \pm 10$

Here $q=1$ So, possible factor of 1 are ± 1 .

So possible of factor of $P(x)$ can be found from $\frac{P}{q} = \pm 1, \pm 2, \pm 5, \pm 10$

$$P(x) = x^3 - 6x^2 + 3x + 10$$

Put $x=-1$

$$P(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10 = -1 - 6 - 3 + 10 = 0$$

Remainder is equal to zero so $(x+1)$ is a factor

Put $x=2$

$$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x=5$

$$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0 \text{ Remainder is equal to zero so } (x-5) \text{ is a factor}$$

$$\text{Hence } x^3 - 6x^2 + 3x + 10 = (x+1)(x-2)(x-5)$$

Q.4 $x^3 + x^2 - 10x + 8$

Solution: Given that

$$P(x) = x^3 + x^2 - 10x + 8$$

$P=8$ So possible factor of 8

are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8$.

Here $q=1$ So possible factor can be found from $\frac{P}{q} = \pm 1, \pm 2, \pm 4, \pm 8$

$$P(x) = x^3 + x^2 - 10x + 8$$

Put $x=1$

$$P(1) = (1)^3 + (1)^2 - 10(1) + 8 = 1 + 1 - 10 + 8 = 0$$

Remainder is equal to zero so $(x-1)$ is a factor

Put $x=2$

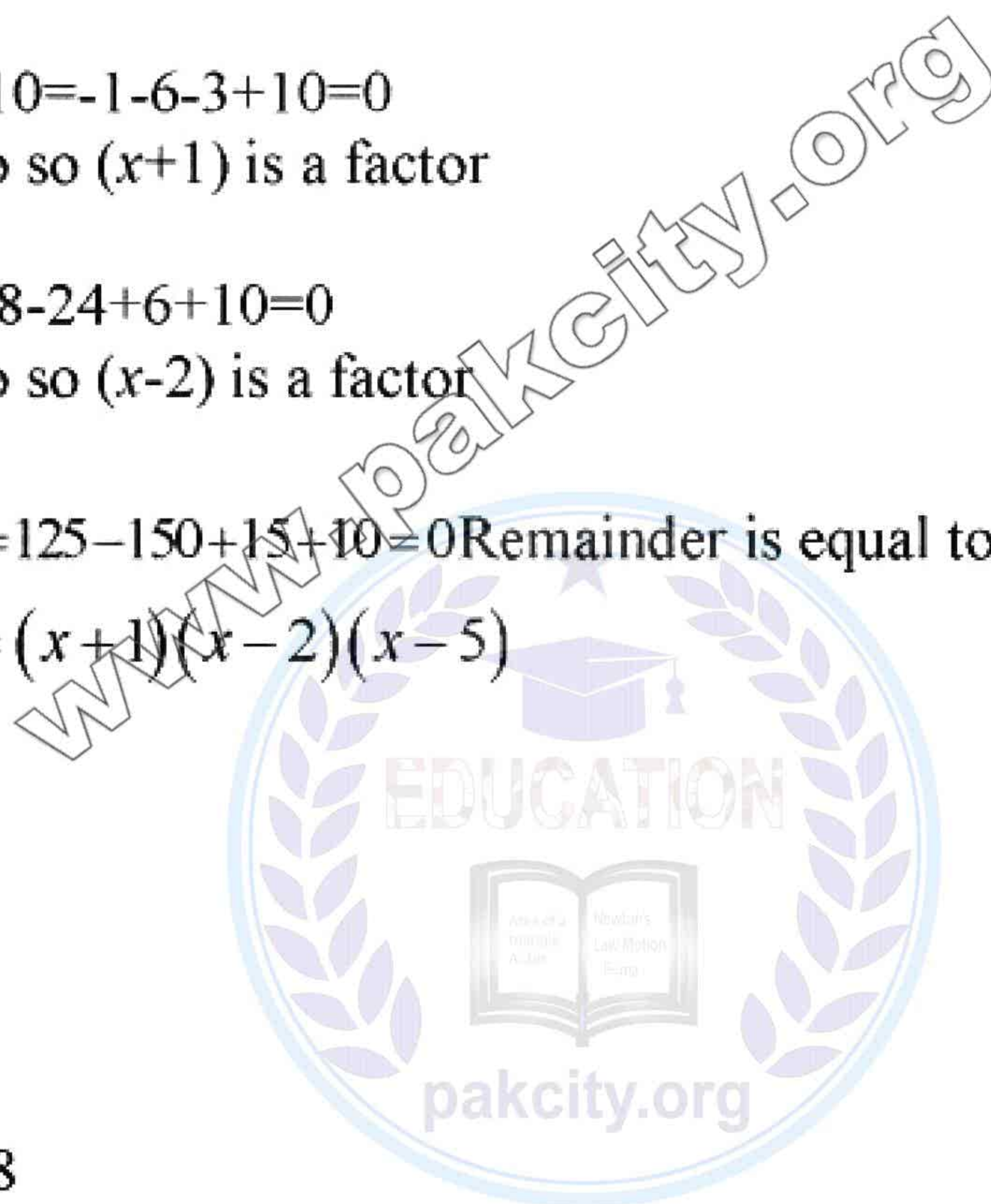
$$P(2) = 2^3 + 2^2 - 10(2) + 8$$

$$= 8 + 4 - 20 + 8$$

$$= 20 - 20$$

$$= 0$$

Remainder is equal to zero so $(x-2)$ is a factor



Put $x=-4$

$$P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8$$

$$= -64 + 16 + 40 + 8$$

$$= -64 + 64$$

$$= 0$$

Remainder is equal to zero so $(x+4)$ is a factor

$$\text{Hence } x^3 + x^2 - 10x + 8 = (x-1)(x-2)(x+4)$$

Q.5 $x^3 - 2x^2 - 5x + 6$

Solution: Given that

$$P(x) = x^3 - 2x^2 - 5x + 6$$

$$P = 6 \text{ So factors of 3 are } \pm 1, \pm 2, \pm 3, \pm 6$$

Here $q=1$ So factors of 1 are ± 1 .

So possible factors of $P(x)$ can be found from $\frac{P}{q} = \pm 1, \pm 2, \pm 3, \pm 6$

$$P(x) = x^3 - 2x^2 - 5x + 6$$

Put $x=1$

$$P(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$

$$= 1 - 2 - 5 + 6$$

$$= -7 + 7$$

$$= 0$$

Remainder is equal to zero so $(x-1)$ is a factor

Put $x=-2$

$$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$$

$$= -8 - 8 + 10 + 6$$

$$= -16 + 16$$

$$= 0$$

Remainder is equal to zero so $(x+2)$ is a factor

Put $x=3$

$$P(3) = (3)^3 - 2(3)^2 - 5(3) + 6$$

$$= 27 - 6 - 15 + 6$$

$$27 - 27$$

$$= 0$$

Remainder is equal to zero so $(x-3)$ is a factor

$$\text{Hence } x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$$

Q.6 $x^3 + 5x^2 - 2x - 24$

Solution: Given that

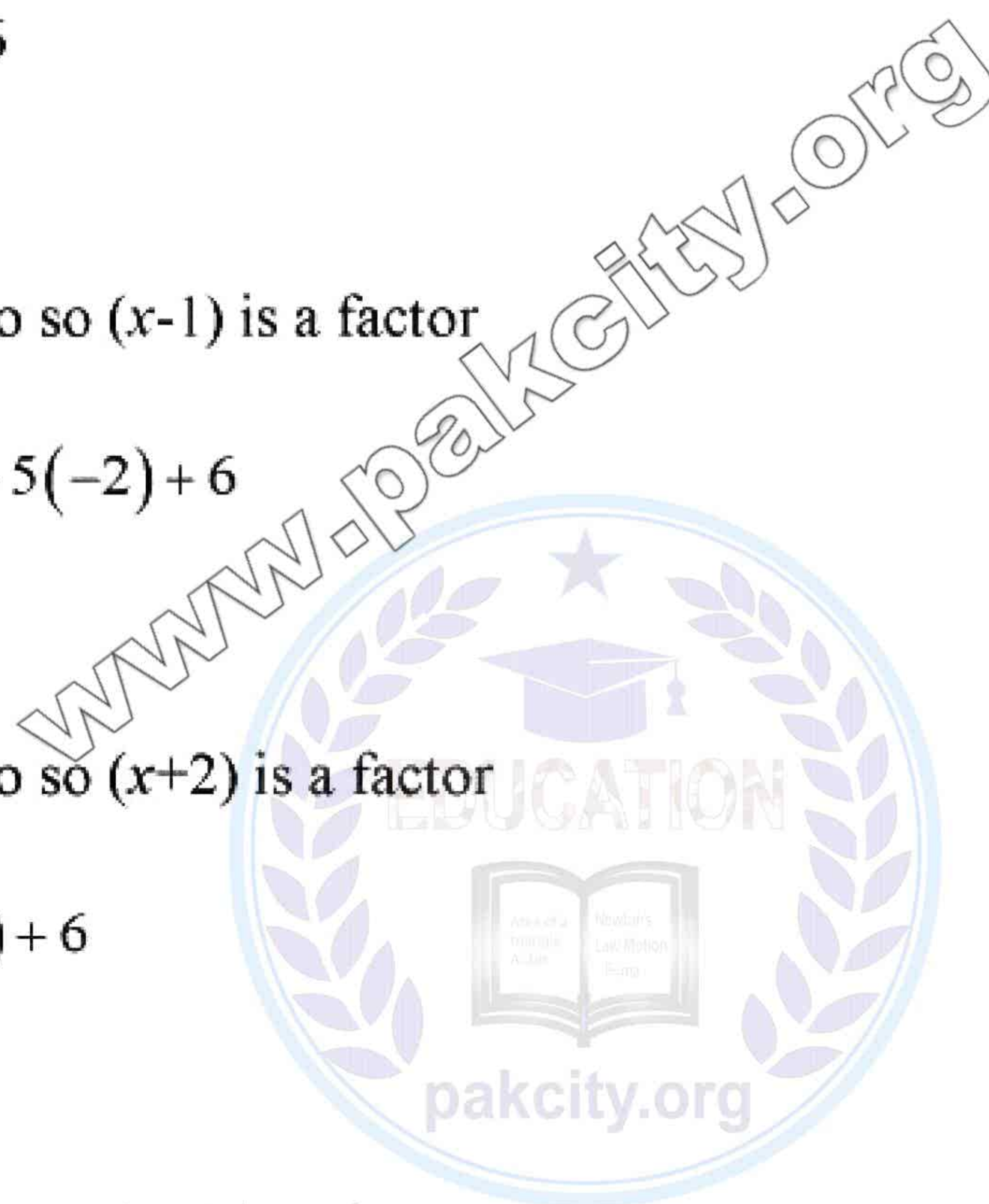
$$P(x) = x^3 + 5x^2 - 2x - 24$$

$$P = -24 \text{ So possible factors of 24 are } \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$

Here $q=1$. So possible factors of 1 are ± 1 .

So possible factors of $P(x)$ will be found from

$$\frac{P}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$



$$P(x) = x^3 + 5x^2 - 2x - 24$$

Put $x=2$

$$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$$

$$= 8 + 20 - 4 - 24$$

$$= 28 - 28$$

$$= 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x=-3$

$$P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24$$

$$= -27 + 45 + 6 - 24$$

$$= -51 + 51$$

$$= 0$$

Remainder is equal to zero so $(x+3)$ is a factor

Put $x=-4$

$$P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$$

$$= -64 + 80 + 8 - 24$$

$$= -88 + 88$$

$$= 0$$

Remainder is equal to zero so $(x+4)$ is a factor

$$\text{Hence } x^3 + 5x^2 - 2x - 24 = (x-2)(x+3)(x+4)$$

Q.7 $3x^3 - x^2 - 12x + 4$

Solution: Given that

$$P(x) = 3x^3 - x^2 - 12x + 4$$

$P=4$ So possible factors of 4 are $\pm 1, \pm 2, \pm 4$.

Here $q=3$ So possible factors of 3 are $\pm 1, \pm 3$.

So possible factors of $P(x)$ can be found from

$$\frac{P}{q} = \pm 1, \pm 2, \pm 4, \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$$

Put $x=2$

$$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$$

$$= -24 - 4 + 24 + 4$$

$$= 28 - 28$$

$$= 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x=-2$

$$P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$$

$$= -24 - 4 + 24 + 4$$

$$= 28 - 28$$

$$= 0$$

Remainder is equal to zero so $(x+2)$ is a factor

Put $x = \frac{1}{3}$

$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 4$$



$$= \cancel{3} \left(\frac{1}{\cancel{27}9} \right) - \frac{1}{9} - \cancel{A} + \cancel{A}$$

$$P\left(\frac{1}{3}\right) = \frac{1}{9} - \frac{1}{9}$$

$$= 0$$

$$x = \frac{1}{3} \Rightarrow 3x = 1$$

$$3x - 1$$

Remainder is equal to zero so $(3x-1)$ is a factor

$$\text{Hence } 3x^3 - x^2 - 10x + 4 = (x-2)(x+2)(3x-1)$$

Q.8 $2x^3 + x^2 - 2x - 1$

Solution: Given that

$$P(x) = 2x^3 + x^2 - 2x - 1$$

$P = -1$ So possible factors of -1 are $\pm 1, \pm 2$.

Here $q=1$. So possible factors of $P(x)$ will be found from $\frac{p}{q}$

$$\frac{P}{q} = \pm 1, \pm 2, \pm \frac{1}{2}$$

$$P(x) = 2x^3 + x^2 - 2x - 1$$

Put $x=1$

$$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$$

$$= 2 + 1 - 2 - 1$$

$$= 3 - 3$$

$$= 0$$

Remainder is equal to zero $(x-1)$ is a factor

Put $x=-1$

$$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

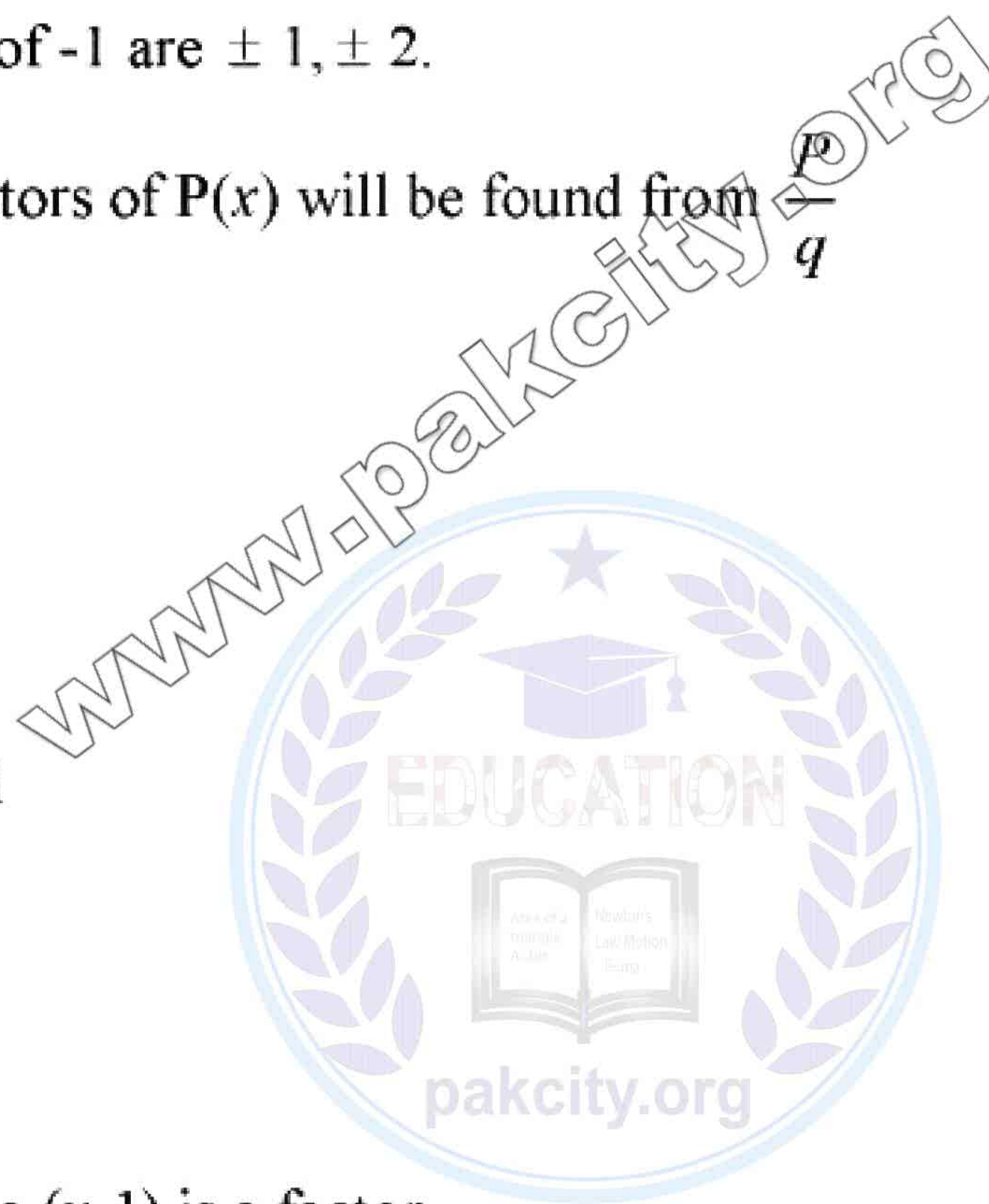
$$= 2 + 1 - 2 - 1$$

$$= 3 - 3$$

$$= 0$$

Remainder is equal to zero $(x+1)$ is a factor

$$\text{Put } x = \frac{-1}{2}$$



$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{2}\right]^3 + \left[\frac{-1}{2}\right]^2 - \cancel{2}\left[\frac{-1}{\cancel{2}}\right] - 1$$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{\cancel{2}4}\right] + \frac{1}{4} + \cancel{1} - \cancel{1}$$

$$P\left(\frac{-1}{2}\right) = -\frac{1}{\cancel{2}4} + \frac{1}{4}$$
$$= 0$$

$$x = -\frac{1}{2}$$

$$2x = -1$$

$$2x + 1 = 0$$

Remainder is equal to zero so $(2x+1)$ is a factor

$$\text{Hence } 2x^3 + x^2 - 2x - 1 = (x-1)(x+1)(2x+1)$$



Review Exercise 5

Q.1 Filling the blanks:

1. The factor of $x^2 - 5x + 6$ are _____.

| | |
|--------------------|--------------------|
| (a) $x + 1, x - 6$ | (b) $x - 2, x - 3$ |
| (c) $x + 6, x - 1$ | (d) $x + 2, x + 3$ |

2. Factors of $8x^3 + 27y^3$ are _____.

| | |
|--------------------------------------|--------------------------------------|
| (a) $(2x - 3y), (4x^2 + 9y^2)$ | (b) $(2x - 3y), (4x^2 - 9y^2)$ |
| (c) $(2x + 3y), (4x^2 - 6xy + 9y^2)$ | (d) $(2x - 3y), (4x^2 + 6xy + 9y^2)$ |

3. Factors of $3x^2 - x - 2$ are _____.

| | |
|-------------------------|-------------------------|
| (a) $(x + 1), (3x - 2)$ | (b) $(x + 1), (3x + 2)$ |
| (c) $(x - 1), (3x - 2)$ | (d) $(x - 1), (3x + 2)$ |

4. Factors of $a^4 - 4b^4$ are _____.

| | |
|--------------------------------------|----------------------------------|
| (a) $(a - b), (a + b), (a^2 + 4b^2)$ | (b) $(a^2 - 2b^2), (a^2 + 2b^2)$ |
| (c) $(a - b), (a + b)(a^2 + 4b^2)$ | (d) $(a - 2b), (a^2 + 2b^2)$ |

5. What will be added to complete the square of $9a^2 - 12ab$?.....

| | | | |
|--------------|-------------|------------|-------------|
| (a) $-16b^2$ | (b) $16b^2$ | (c) $4b^2$ | (d) $-4b^2$ |
|--------------|-------------|------------|-------------|

6. Find m so that $x^2 + 4x + m$ is a complete square

| | |
|-------|--------|
| (a) 8 | (b) -8 |
| (c) 4 | (d) 16 |

7. Factors of $5x^2 - 17xy - 12y^2$ are _____.

| | |
|---------------------------|---------------------------|
| (a) $(x + 4y), (5x + 3y)$ | (b) $(x - 4y), (5x - 3y)$ |
| (c) $(x - 4y), (5x + 3y)$ | (d) $(5x - 4y), (x + 3y)$ |

8. Factors of $27x^3 - \frac{1}{x^3}$ are

| | |
|--|--|
| (a) $\left(3x - \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$ | (b) $\left(3x + \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$ |
| (c) $\left(3x - \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$ | (d) $\left(\frac{3x + 1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$ |

ANSWERS KEYS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| b | c | d | b | c | c | c | a |

Q.2 Completion items

- (i) $x^2 + 5x + 6 =$ _____
- (ii) $4a^2 - 16 =$ _____
- (iii) $4a^2 + 4ab + (\text{_____})$ is a complete square.
- (iv) $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} =$ _____
- (v) $(x + y)(x^2 - xy + y^2) =$ _____
- (vi) Factored form of $x^4 - 16$ is _____
- (vii) If $x-2$ is factor of $P(x) = x^2 + 2kx + 8$ then = _____

ANSWER KEYS

- (i) $(x + 3)(x + 2)$
- (ii) $(2a + 4)(2a - 4) = 4(a + 2)(a - 2)$
- (iii) $(b)^2$
- (iv) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$
- (v) $x^3 + y^3$
- (vi) $(x + 2)(x - 2)(x^2 + 4)$
- (vii) -3

Q.3 Factorize the following

- (i) $x^2 + 8x + 16 - 4y^2$
Solution: $x^2 + 8x + 16 - 4y^2$
 $= [x^2 + 8x + 16] - 4y^2$
 $= [(x)^2 + 2(x)(4) + (4)^2] - (2y)^2$
 $= (x + 4)^2 - (2y)^2$
 Now arrange them
 $= (x + 4 + 2y)(x + 4 - 2y)$
 $= (x + 2y + 4)(x - 2y + 4)$

- (ii) $4x^2 - 16y^2$
Solution: $4x^2 - 16y^2$
 $= 4[x^2 - 4y^2]$
 $= 4[(x)^2 - (2y)^2]$
 $= 4(x - 2y)(x + 2y)$

- (iii) $9x^2 + 24x + 3x + 8$
Solution: $= 9x^2 + 24x + 3x + 8$
 $= 3x(3x + 8) + 1(3x + 8)$
 $= (3x + 8)(3x + 1)$

- (iv) $1 - 64z^3$
Solution: $1 - 64z^3$
 $= (1)^3 - (4z)^3$
 $= (1 - 4z)[(1)^2 + (1)(4z) + (4z)^2]$
 $= (1 - 4z)(1 + 4z + 16z^2)$

- (v) $8x^3 - \left(\frac{1}{3y}\right)^3$
 $= (2x)^3 - \left(\frac{1}{3y}\right)^3$
 $= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

(vi) $2y^2 + 5y - 3$

Solution: $= 2y^2 + 6y - y - 3$
 $= 2y(y+3) - 1(y+3)$
 $= (2y-1)(y+3)$

(vii) $x^3 + x^2 - 4x - 4$

Solution: $x^3 + x^2 - 4x - 4$
 $= x^2(x+1) - 4(x+1)$
 $= (x+1)(x^2 - 4)$
 $= (x+1)(x-2)(x+2)$

(viii) $25m^2n^2 + 10mn + 1$

Solution: $25m^2n^2 + 10mn + 1$
 $= (5mn)^2 + 2(5mn)(1) + (1)^2$
 $= (5mn+1)^2$

(ix) $1 - 12pq + 36p^2q^2$

Solution: $1 - 12pq + 36p^2q^2$
 $\therefore (a)^2 - 2ab + (b)^2$
 $= (1)^2 - 2(1)(6pq) + (6pq)^2$
 $= (1 - 6pq)^2$



Unit 5: Factorization

Overview

Factorization:

The process of expressing an algebraic expression in terms of its factors is called factorization.

Remainder Theorem:

If a polynomial $p(x)$ is divided by a linear divisor $(x-a)$, then the remainder is $p(a)$.

Zero of a Polynomial:

If a specific number $x = a$ is substituted for the variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x = a$ is called a zero of the polynomial $p(x)$.

Factor Theorem:

The polynomial $(x-a)$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$.

Rational Root Theorem:

Let $a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0$, $a_0 \neq 0$ be a polynomial equation of degree n with integral coefficients. If $\frac{p}{q}$ is a rational root (expressed in lowest terms) of the equation, then p is a factor of the constant term a_n and q is a factor of the leading coefficient a_0 .