Exercise 15

Q.1 Verify that the Δs having the following measures of sides are right-angled to verify weather the Δs are right angled or not we use Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

(i) $a = 5cm$
$b = 12cm$
$c = 13cm$
$a^2 = 25cm^2$
$b^2 = 144cm^2$
$c = 169cm^2$
Larger Size is Hypotenuse So
$169 = 25 + 144$
$169 = 169$
L.H.S = R.H.S
So it is right angled triangle

(ii) $a = 1.5cm$
$b = 2cm$
$c = 2.5cm$
$a^2 = 2.25cm^2$
$b^2 = 4cm^2$
$c^2 = 6.25$
$6.25 = 2.25 + 4$
$6.25 = 6.25$
L.H.S = R.H.S
So it is right-angled triangle

(iii) $a = 9cm$
$b = 12cm$
$c = 15cm$
$a^2 = 81cm^2$
$b^2 = 144cm^2$
$c = 225cm^2$
$225cm^2 = 8.1cm + 144cm$
$225cm^2 = 225cm^2$
L.H.S = R.H.S
So it is right angled triangle

(iv) $a = 16cm$
$b = 30cm$
$c = 34cm$
$a^2 = 256cm^2$
$b^2 = 900cm$
$c^2 = 1156cm^2$
$1156 = 256 + 900$
$1156 = 1156$
L.H.S = R.H.S
It is right angled triangle

Q.2 Verify that a^2+b^2, a^2-b^2 and $2ab$ are the measures of the sides of a right angled Triangle where a and b are any two real numbers (a >b)

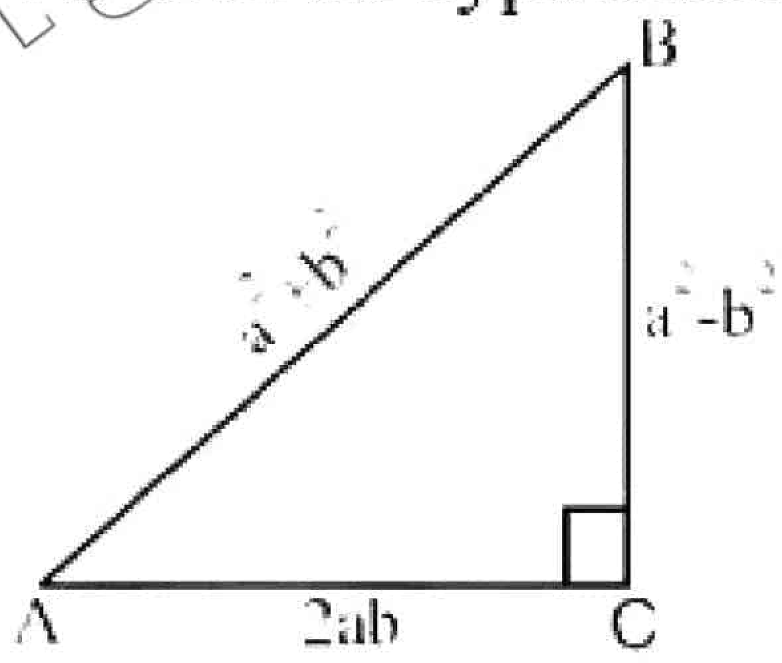
Let $a = z$ and $b = 1$

$a^2 + b^2 = (2)^2 + (1)^2 = 4 + 1 = 5$

$a^2 - b^2 = (2)^2 - (1)^2 = 4 - 1 = 3$

$2ab = 2(2)(1) = 4$

Since $a^2 + b^2$ is the largest side so $a^2 + b^2$ will be hypotenuse



So

$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$

$(a^2+b^2)^2 = (2ab)^2 + (a^2-b^2)^2$

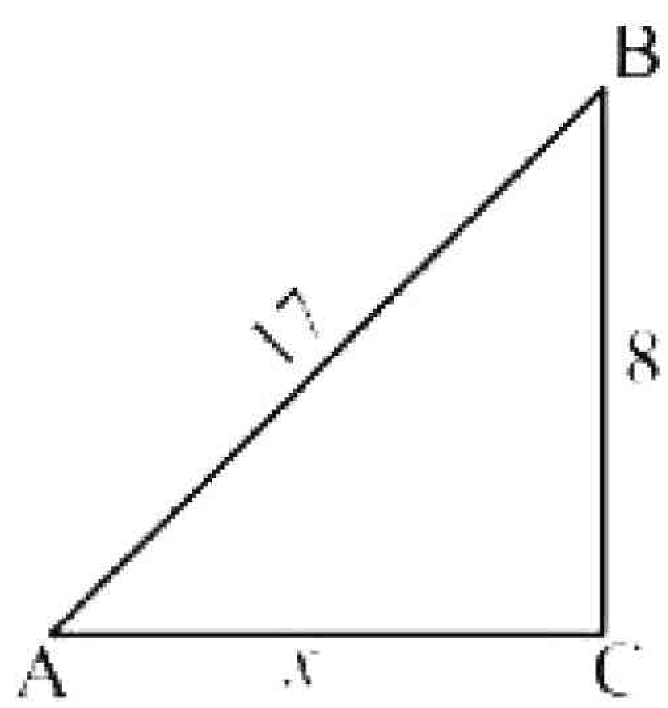
$a^4 + b^4 + 2a^2b^2 = 4a^2b^2 + a^4 + b^4 - 2a^2b^2$

$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 + 2a^2b^2$

L.H.S = R.H.S

It is proved that it is a right angled triangle

Q.3 The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of right angled triangle by Pythagoras theorem

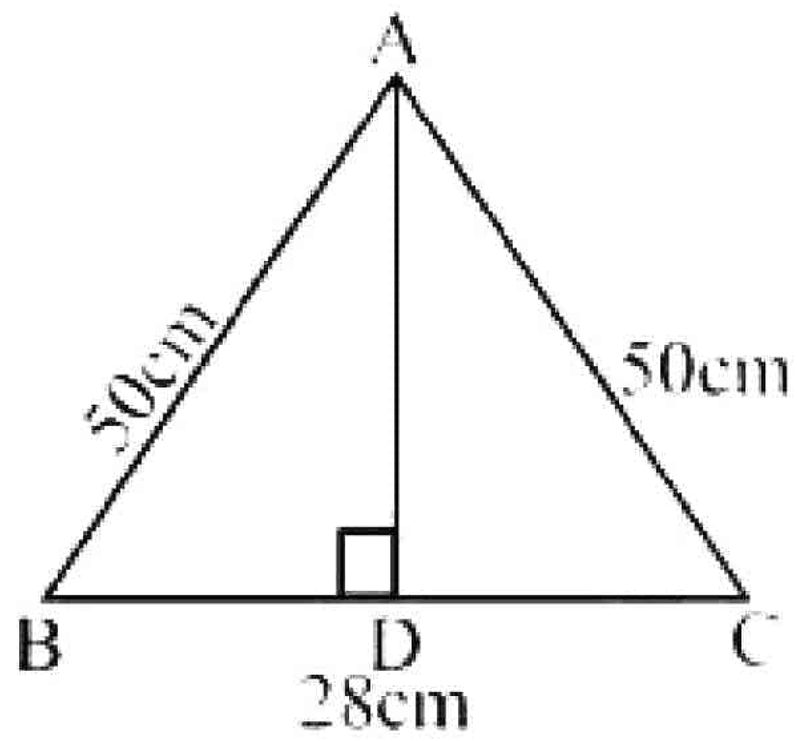


$$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$$
$$(17)^2 = (x)^2 + (8)^2$$
$$289 = x^2 + 64$$
$$289 - 64 = x^2$$
$$x^2 = 225$$
Taking square root both side
$$\sqrt{x^2} = \sqrt{225}$$
$$x = 15$$

Q.4 In an isosceles Δ the base $\overline{BC} = 28\text{cm}$ and

$\overline{AB} = \overline{AC} = 50\text{cm}$

If $\overline{AD} \perp \overline{BC}$ then find



(i) Length of $\overline{AD}$

Solution:

$\overline{AD} \perp \overline{BC}$

So $\overline{BD} = \overline{CD}$

$$\frac{1}{2}\overline{BC} = \frac{1}{2}(28)$$
$$\frac{1}{2}\overline{BC} = 14$$

So

$$\overline{BD} = \overline{CD} = 14$$
$$(\overline{AB})^2 = (\overline{BD})^2 + (\overline{AD})^2$$
$$2500 = (14)^2 + (\overline{AD})^2$$
$$2500 = 196 + (\overline{AD})^2$$
\ $$2500 - 196 = (\overline{AD})^2$$
$$(\overline{AD})^2 = 2304$$

Taking square root on both side
$$\sqrt{(\overline{AD})^2} = \sqrt{2304}$$
$$\overline{AD} = 48\text{cm}$$

(ii) Area of Δ ABC

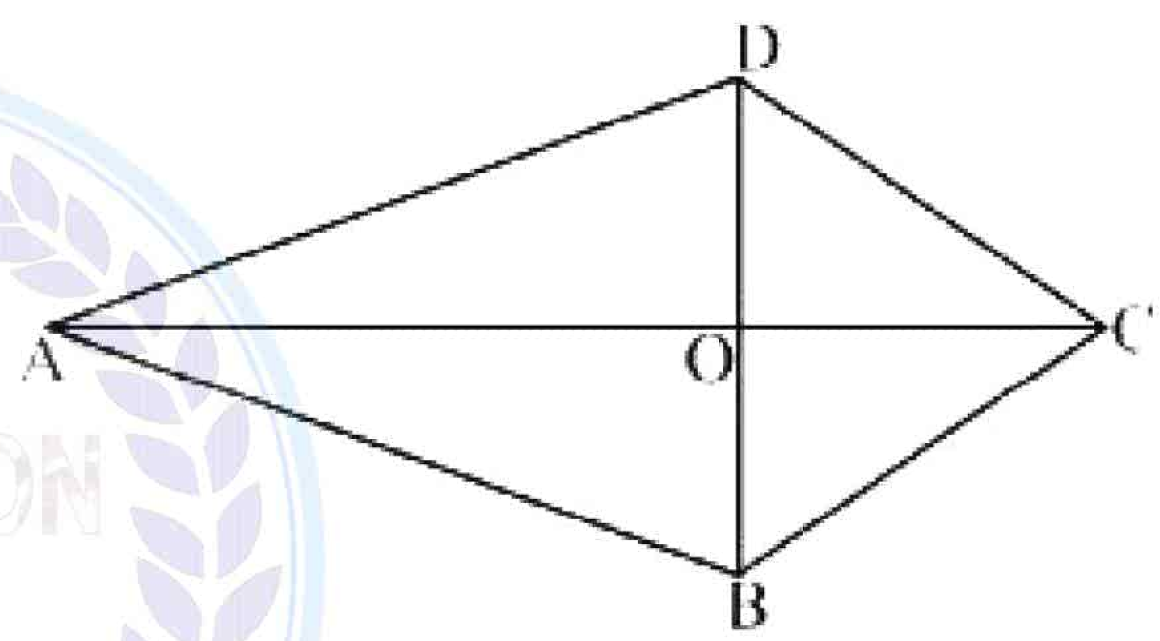
Area of Δ ABC $= \frac{1}{2}$ (base) (height)

$$= \frac{1}{2}(28)(48)$$
$$= (14)(48)$$
$$= 672\ \text{cm}^2$$

Q.5 In a quadrilateral ABCD the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. Prove that

$$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{AD})^2 + (\overline{BC})^2$$

ΔAOB



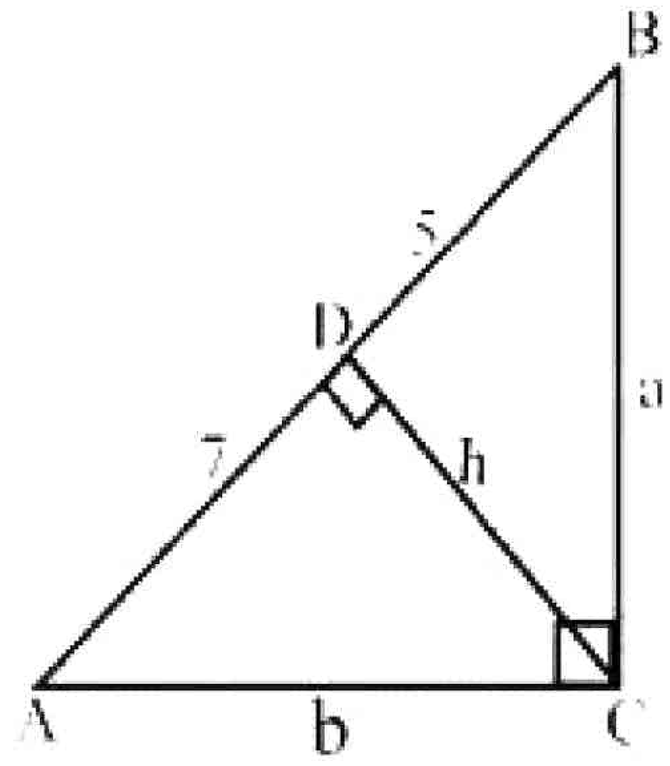
$$(\overline{AB})^2 = (\overline{OB})^2 + (\overline{OA})^2 \longrightarrow \text{(i)}$$
ΔBOC
$$(\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2 \longrightarrow \text{(ii)}$$
ΔCOD
$$(\overline{CD})^2 = (\overline{OD})^2 + (\overline{OC})^2 \longrightarrow \text{(iii)}$$
ΔDOA
$$(\overline{AD})^2 = (\overline{OA})^2 + (\overline{OD})^2 \longrightarrow \text{(iv)}$$
By adding (i) and (iii)
$$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{OB})^2 + (\overline{OA})^2 + (\overline{OD})^2 + (\overline{OC})^2 \rightarrow \text{(v)}$$
By adding (ii) and (iv)
$$(\overline{AD})^2 + (\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2 + (\overline{OA})^2 + (\overline{OD})^2 \rightarrow \text{(vi)}$$
By comparing v and vi

$$\left(\overline{AB}\right)^2+\left(\overline{CD}\right)^2=\left(\overline{AD}\right)^2+\left(\overline{BC}\right)^2$$

Hence proved

Q.6 the ΔABC as shown in the figure m∠ACB = 90° and $\overline{CD}\perp\overline{AB}$ find the length a, h and b if $m\overline{BD}=5$ units and $m\overline{AD}=7$ units

(i)



ΔACB

$$(7+5)^2=(b)^2+(a)^2$$

$$a^2+b^2=(12)^2$$

$$a^2+b^2=144 \_\_\_\_\_\_ (i)$$

ΔADC

$$(b)^2=(7)^2+(h)^2$$

$$b^2-h^2=49 \_\_\_\_\_\_ (ii)$$

ΔCDB

$$a^2=(5)^2+(h)^2$$

$$a^2-h^2=25 \_\_\_\_\_\_ (iii)$$

Subtracting ii from iii

$$a^2-\cancel{h^2}=25$$

$$\pm b^2\mp\cancel{h^2}=\pm49$$

$$a^2-b^2=-24$$

$$a^2-b^2=-24 \_\_\_\_\_\_ (iv)$$

Adding equation I and IV

$$a^2+\cancel{b^2}=144$$

$$a^2-\cancel{b^2}=-24$$

$$2a^2=120$$

$$2a^2=120$$

$$a^2=\frac{\cancel{120}^{60}}{\cancel{2}}$$

$$a^2=60$$

Prime factor

$2\times2\times15$

4×15

2	60
2	30
	15

$$a^2=4\times15$$

Taking square root both side

$$\sqrt{a^2}=\sqrt{4\times15}$$

$$a=2\sqrt{15}$$

Putting the value of a in equation (i)

$$\left(2\sqrt{15}\right)^2+b^2=144$$

Prime factor

$$4\times15+b^2=144$$

$$60+b^2=144$$

$$b^2=144-60$$

$$b^2=84$$

$$b^2=4\times21$$

$$2\times2\times21$$

$$4\times21$$

2	84
2	42
21	21
	1

Taking square root both side

$$b^2=\sqrt{4\times21}$$

$$b=2\sqrt{21}$$

Putting the value of b in equation (ii)

$$\left(2\sqrt{21}\right)^2-h^2=49$$

$$4\times21-49=h^2$$

$$h^2=84-49$$

$$h^2=35$$

Taking square root both side

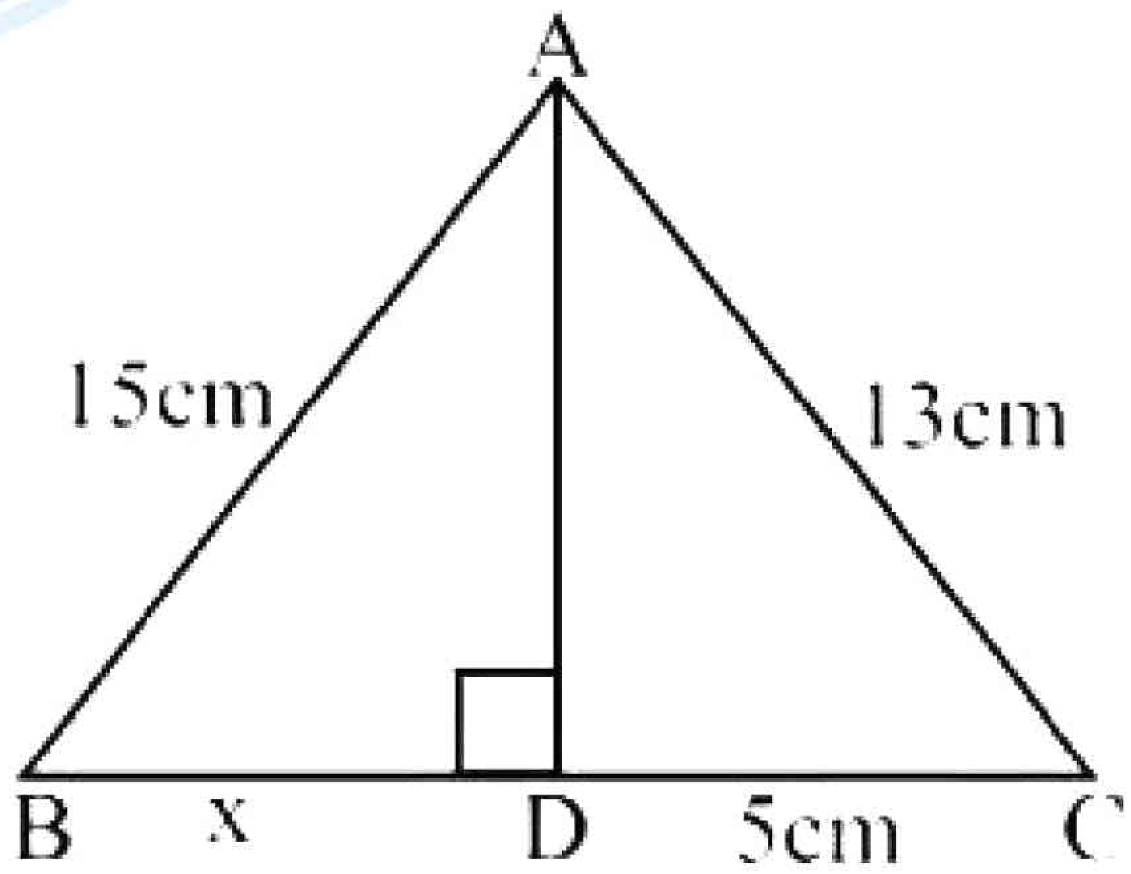
$$\sqrt{h^2}=\sqrt{35}$$

$$h=\sqrt{35}$$

(ii) Find the value of *x* in the shown figure

From ΔADC

$$\left(\overline{AC}\right)^2=\left(\overline{DC}\right)^2+\left(\overline{AD}\right)^2$$



$$(13)^2=(5)^2+\left(\overline{AD}\right)^2$$

$$169=25+\left(\overline{AD}\right)^2$$

$$169-25=\left(\overline{AD}\right)^2$$

$$\left(\overline{AD}\right)^2=144$$

Taking square root both side

$$\sqrt{\left(\overline{AD}\right)^2} = \sqrt{(144)}$$

$$\overline{AD} = 12$$

From Δ ADB

$$\left(\overline{AB}\right)^2 = \left(BD\right)^2 + \left(\overline{AD}\right)^2$$

$$(15)^2 = x^2 + (12)^2$$

$$225 = x^2 + 144$$

$$225 - 144 = x^2$$

$$x^2 = 81$$

Taking square on both side

$$\sqrt{x^2} = \sqrt{81}$$

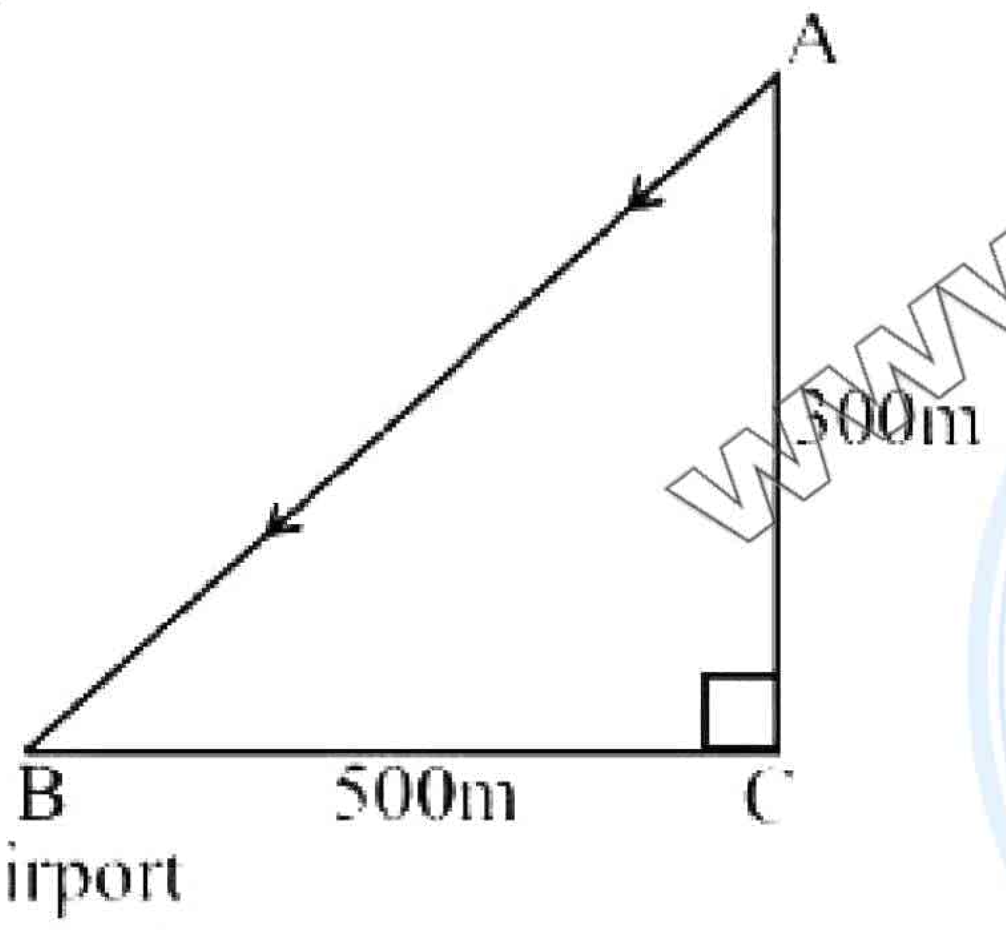
$$x = 9$$

Q.7 A plane is at a height of 300m and is 500m away from the airport as shown in the figure How much distance will it travel to land at the airport?

ΔABC is right angle triangle

$$\left(\overline{AB}\right)^2 = \left(\overline{BC}\right)^2 + \left(\overline{AC}\right)^2$$

$$\left(\overline{AB}\right)^2 = (500)^2 + (300)^2$$



$$\left(\overline{AB}\right)^2 = 250000 + 90000$$

$$\left(\overline{AB}\right)^2 = 340000$$

$$\left(\overline{AB}\right)^2 = 10000 \times 34$$

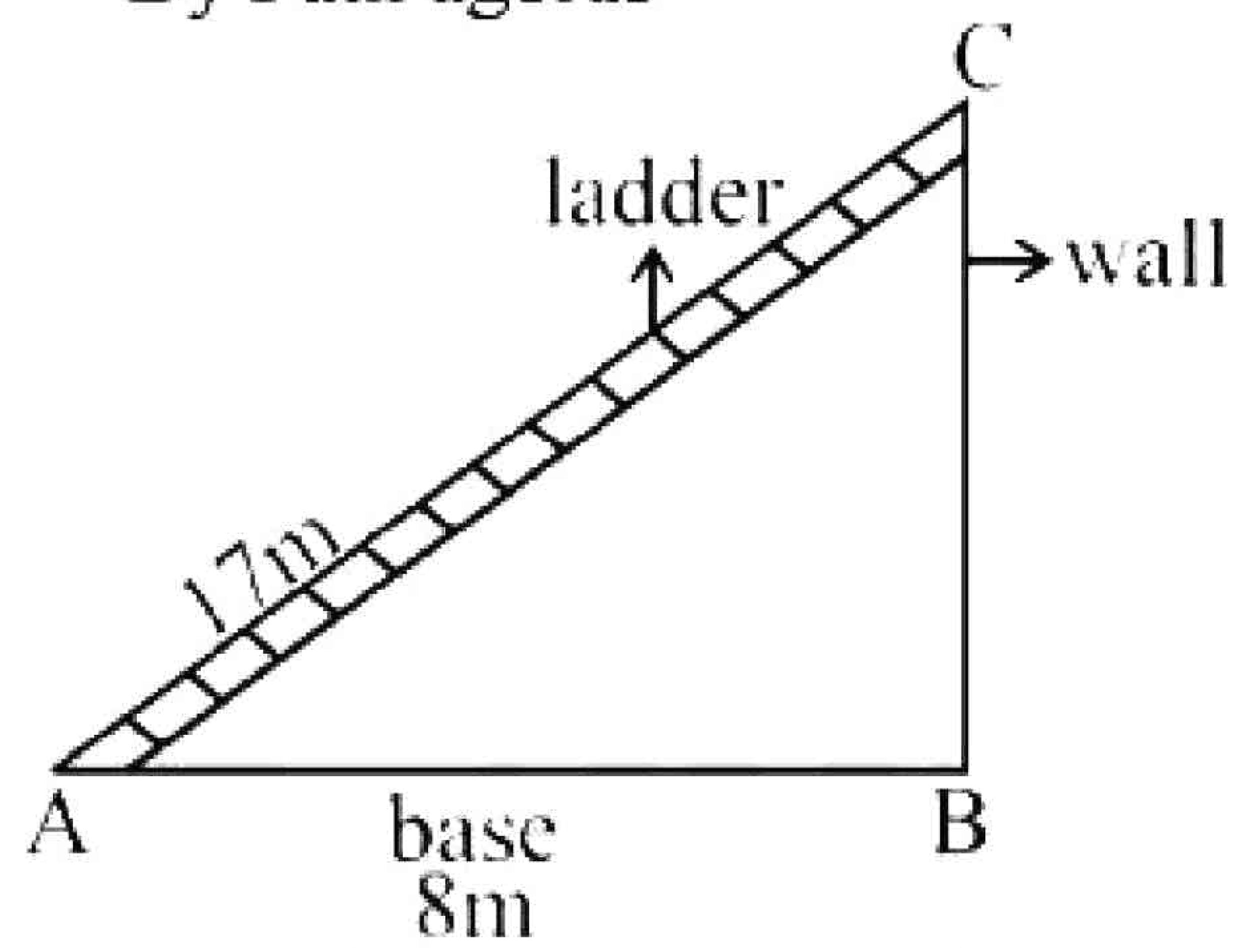
Taking square root on both side

$$\sqrt{\left(\overline{AB}\right)^2} = \sqrt{10000 \times 34}$$

$$AB = 100\sqrt{34}m$$

Q.8 A ladder 17m long rests against a vertical wall. The foot of the ladder is 8m away from the base of thewall. How high up the wall will the ladder reach?

By Path agoras



$$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$$

$$(17)^2 = (8)^2 + \left(\overline{BC}\right)^2$$

$$289 = 64 + \left(\overline{BC}\right)^2$$

$$284 - 64 = \left(\overline{BC}\right)^2$$

$$\left(\overline{BC}\right)^2 = 225$$

Taking square root on both side

$$\sqrt{\left(\overline{BC}\right)^2} = \sqrt{225}$$

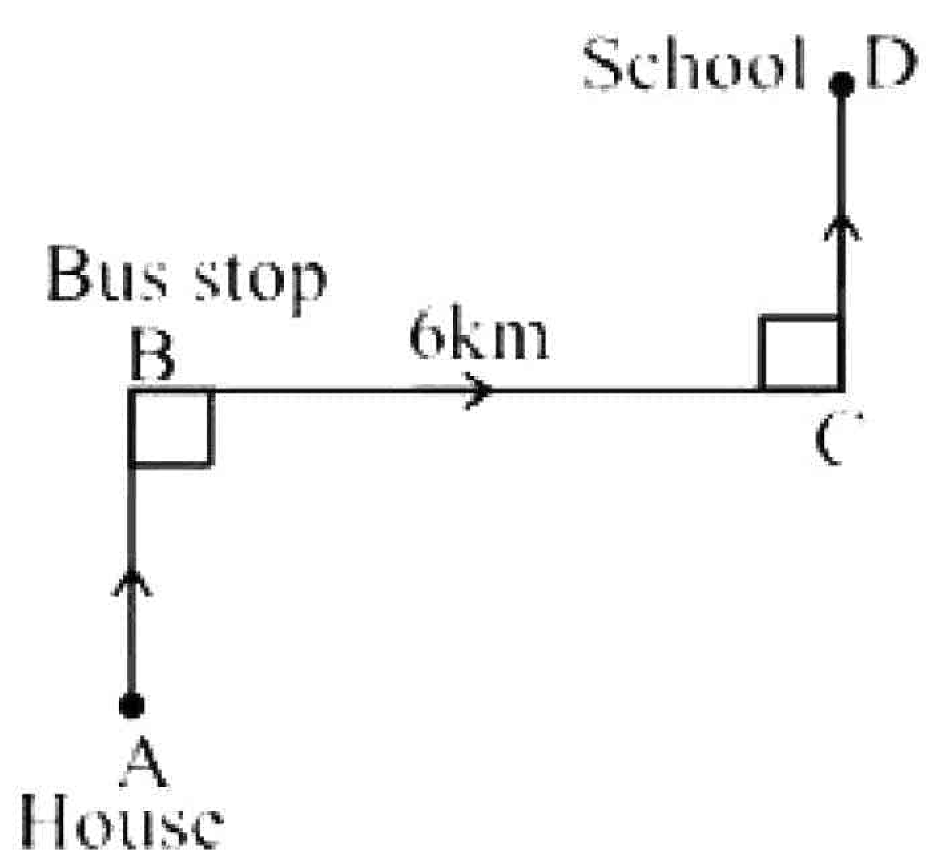
$$\overline{BC} = 15m$$

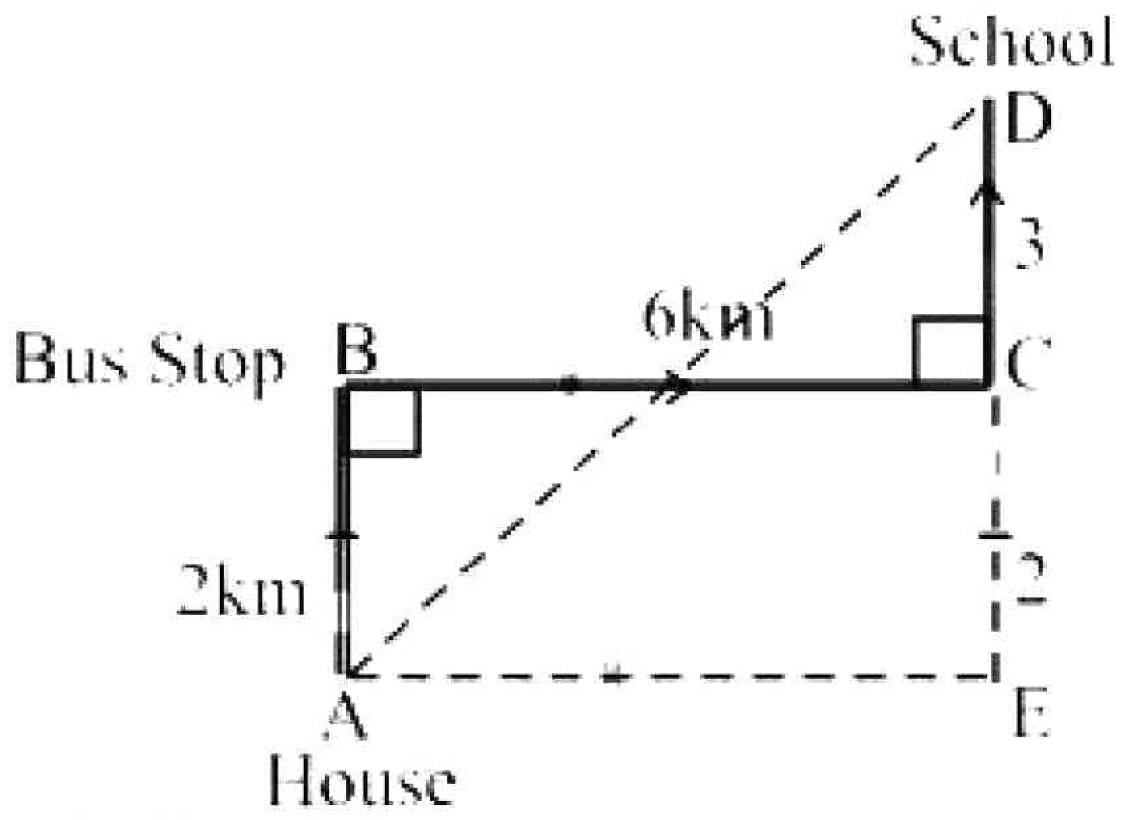
The height of wall $= \overline{BC} = 15m$

Q.9 A student travels to his school by the route as shown in the figure. Find m$\overline{AD}$, the direct distance from his house to school.

Solution:

As we know that in rectangular opposite sides are equal so





$\overline{AB} = \overline{CE} = 2km$

$\overline{BC} = \overline{AE} = 6km$

$\overline{DE} = \overline{DC} + \overline{CE}$

$\therefore$ We get triangle

Δ ADF which is right angled triangle

$$\left(\overline{AD}\right)^2 = \left(\overline{AE}\right)^2 + \left(\overline{ED}\right)^2$$

$$\left(\overline{AD}\right)^2 = (6)^2 + (3+2)^2$$

$$\left(\overline{AD}\right)^2 = 36 + (5)^2$$

$$\left(\overline{AD}\right)^2 = 36 + 25$$

$$\left(\overline{AD}\right)^2 = 61$$

Taking square root on both side

$$\sqrt{\left(\overline{AD}\right)^2} = \sqrt{61}$$

$$\overline{AD} = \sqrt{61}km$$

Review Exercise 15

Q.1 Which of the following are true and which are false

(i) In a right angled triangle greater angle is of 90° (True)

(ii) In a right angled triangle right angle is of 60° (False)

(iii) In a right triangle hypotenuse is a side opposite to right angle (True)

(iv) If a,b,c are sides of right angled triangle with c as longer side then c

$c^2 = a^2 + b^2$ (True)

(v) If 3cm and 4cm are two sides of a right angled triangle, the hypotenuse is 5cm (True)

(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2cı (False)

Q.2 Find the unknown value in each of the following figures.

(i) By Path agoras theorem

(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

$(x)^2 = (3)^2 + (4)^2$

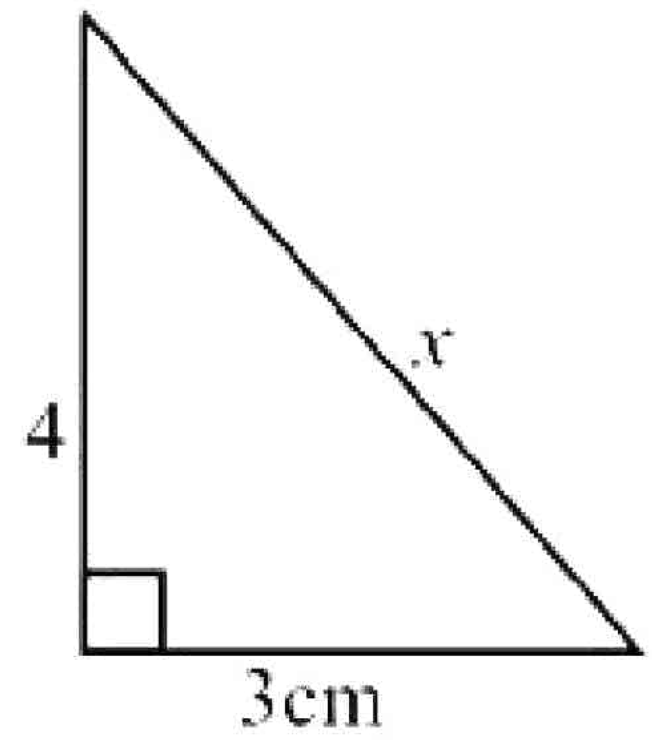
$x^2 = 9 + 16$

$x^2 = 25$

Taking square root on both side

$\sqrt{x^2} = \sqrt{25}$

$x = 5cm$



(ii) By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$(10)^2 = (x)^2 + (6)^2$

$100 = x^2 + 36$

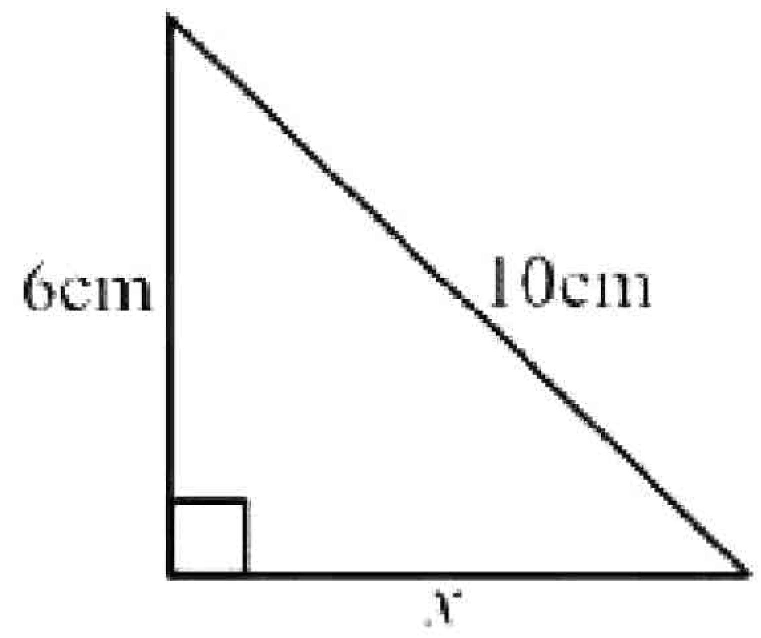
$100 - 36 = x^2$

$x^2 = 64$

Taking square root on both side

$\sqrt{x^2} = \sqrt{64}$

$x = 8cm$



(iii) By Pythagoras theorem

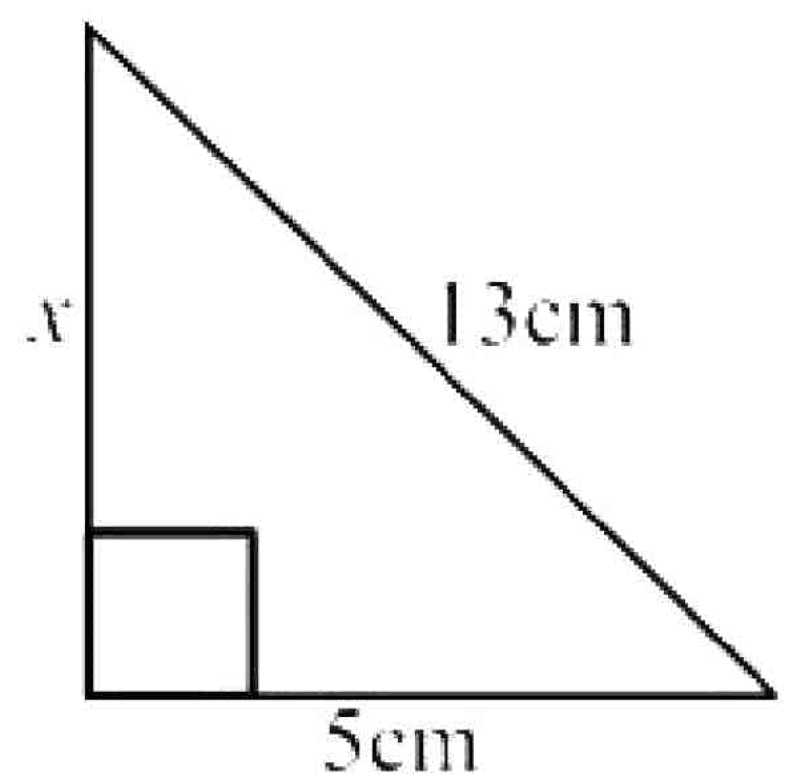
$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$(13)^2 = (5)^2 + (x)^2$

$169 = 25 + x^2$

$169 - 25 = x^2$

$x^2 = 144$

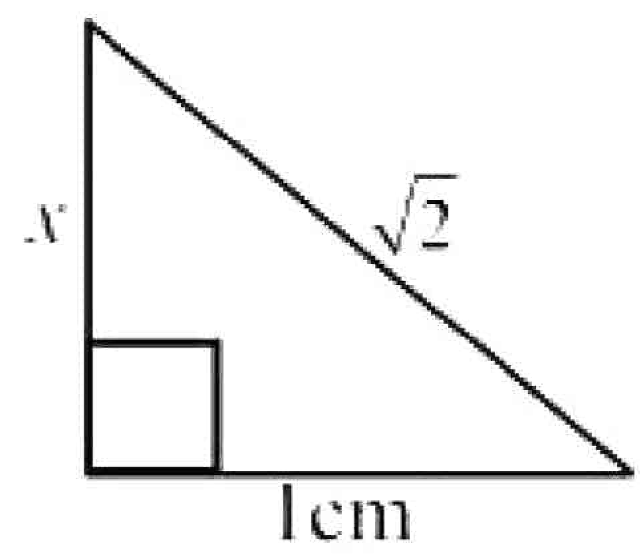


Taking square root on both side

$\sqrt{x^2} = \sqrt{144}$

$x = 12\ cm$

(iv) By Path agoras theorem



$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

$(\sqrt{2})^2 = (1)^2 + (x)^2$

$2 = 1 + x^2$

$2 - 1 = x^2$

$x^2 = 1$

Taking square root on both side

$\sqrt{x^2} = \sqrt{1}$

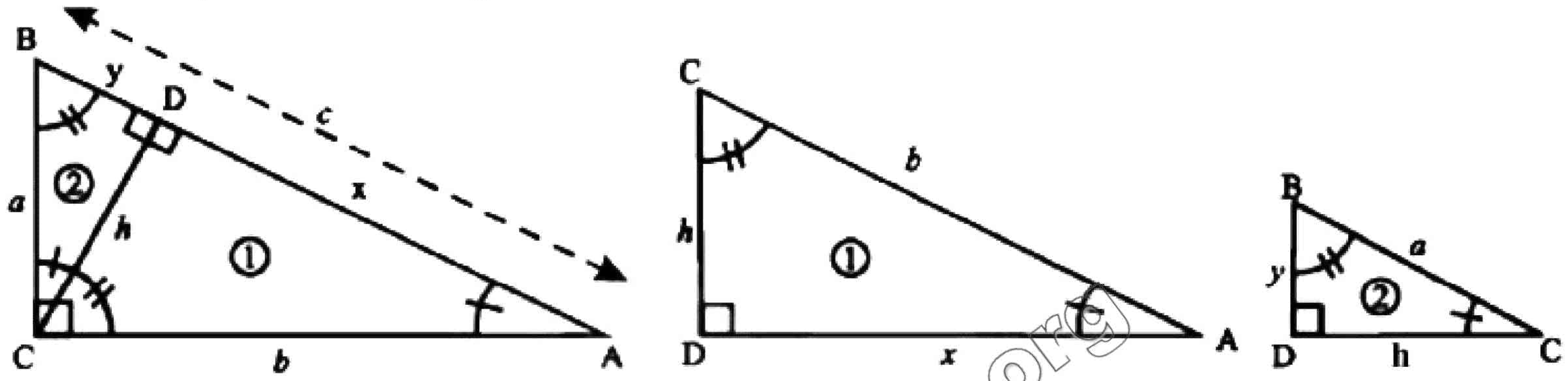
$x = 1cm$

Unit 15: Pythagoras Theorem

Overview

Theorem 15.1.1

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides



(i) (ii)-a (ii)-b

Given

Δ ACB is a right angled triangle in which m∠C = 90° and m$\overline{BC}$ = a, m$\overline{AC}$ = b and m$\overline{AB}$ = c

To prove

$c^2 = a^2 + b^2$

Construction

Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$

Let m$\overline{CD}$ = h, m$\overline{AD}$ = x and m$\overline{BD}$ = y. Line segment CD splits ΔABC into two Δs ADC and BDC which are separately shown in the figures (ii) –a and (ii) –b respectively

Proof (using similar Δs)

Statements	Reasons
In Δ ADC ↔ Δ ACB	Refer to figure (ii)-a and (i)
∠A ≅ ∠A	Common – Self Congruent
∠ADC ≅ ∠ACB	Construction- given each angle = 90°
∠C ≅ ∠B	∠C and ∠B complements of ∠A

$\therefore \Delta ADC \sim \Delta ACB$	Congruency of three angles
$\therefore \frac{x}{b} = \frac{b}{c}$	(Measures of corresponding sides of similar triangles are proportional)
or $x = \frac{b^2}{c}$ ________ (i)	
Again in $\Delta BDC \leftrightarrow \Delta BCA$	Refer to figure (ii)-b and (i)
$\angle B \cong \angle B$	Common – self Congruent
$\angle BDC \cong \angle BCA$	Construction – given each angle = 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$ complements of $\angle B$
$\therefore \Delta BDC \sim \Delta BCA$	Congruency of three angles
$\therefore \frac{y}{a} = \frac{a}{c}$	(Corresponding sides of similar triangles are proportional)
or $y = \frac{a^2}{c}$ ________ (ii)	
But $y + x = c$	Supposition
$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	Multiplying both side by c
i-e $c^2 = a^2 + b^2$	

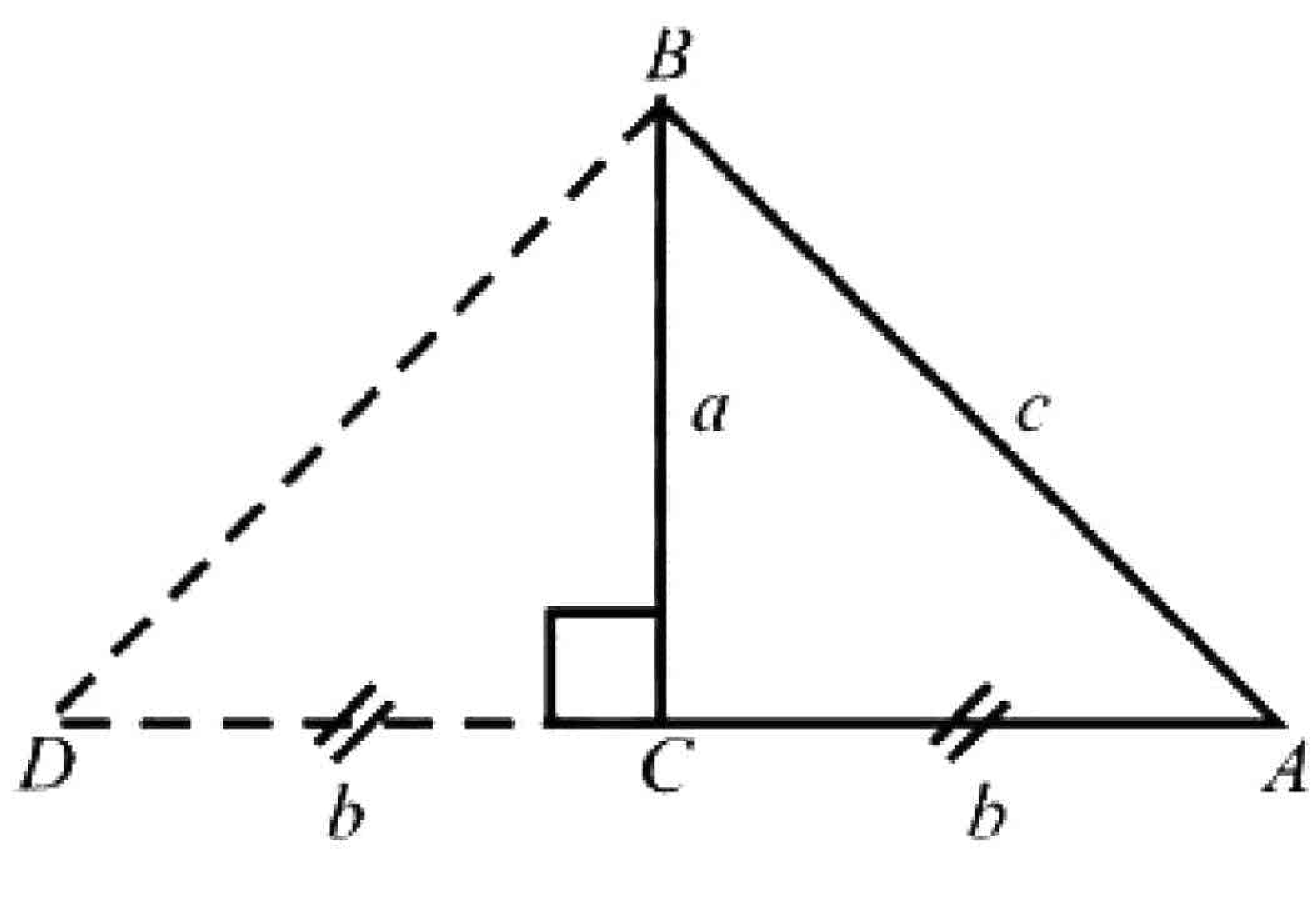
Theorem 15.1.2 Converse of Pythagoras Theorem 15.1.1

If the Square of one side of a triangle is equal to the sum of the square of the other two sides then the triangle is a right angled triangle

Given

In a ΔABC , $m\overline{AB} = c, m\overline{BC} = a, m\overline{AC} = b$

Such that $a^2 + b^2 = c^2$



To prove

ΔACB is a right angled triangle

Construction

Draw $\overline{CD}$ perpendicular to $\overline{BC}$ Such that

$\overline{CD} \cong \overline{CA}$. Join the points B and D

Proof

Statements	Reasons
ΔDCB is a right angled triangle	Construction
$\therefore \left(m\overline{BD}\right)^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore \left(m\overline{BD}\right)^2 = c^2$	
or $m\overline{BD} = c$	Taking Square root on both sides
Now in $\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each side = c
$\therefore \Delta DCB \cong \Delta ACB$	S.S.S ≅ S.S.S
$\therefore \Delta DCB \cong \angle ACB$	(Corresponding angles of congruent triangle)
But $m\angle DCB = 90°$	Construction
$\therefore m\angle ACB = 90°$	
Hence the ΔACB is a Right angled triangle	