Math Sci 9: Test		Total No. 40

Name:____________________ Roll No. :_______

Date:_____-_____-20 Teacher's Signature: ________

Q.1: Tick (✓) the correct answer. سوال نمبر1۔ درست جواب پر (✓) کا نشان لگائیں۔

The symbol of ratio is: 1۔ نسبت کو علامتی طور پر ظاہر کیا جاتا ہے

(A) : (B) : : (C) || (D) =

Notation || means: 2۔ علامت || کا مطلب ہے:

(A) Congruent/مماثل ہے (B) Unequal/برابر نہیں ہے (C) Parallel/متوازی ہے (D) Equal/برابر

Ratio has ______unit. 3۔ نسبت کی اکائی______ہوتی ہے:

(A) m^{-1} / میٹرفی سیکنڈ (B) Kg / کلوگرام (C) J / جول (D) No any / کوئی نہیں

In right triangle, there can be _____ right angles: 4۔ قائمۃ الزاویہ مثلث میں_____قائمہ زاویے ہوسکتے ہیں:

(A) 4 (B) 1 (C) 2 (D) 3

In a right angled triangle the greatest angle is of _____: 5۔ قائمۃ الزاویہ مثلث میں سب سے بڑا زاویہ ہوتا ہے:

(A) 30^o (B) 45^o (C) 90^o (D) 60^o

A triangle has _________ types with respect of angels: 6۔ مثلثوں کی زاویوں کے لحاظ سے اقسام ہیں:

(A) Five/پانچ (B) Four/چار (C) Two/دو (D) Three/تین

The unit of area is: 7۔ رقبہ کی اکائی ہے:

(A) m (B) m^2 (C) m^3 (D) ms^{-1}

A triangular _____ is the union of a triangle and its interior: 8۔ کسی مستطیل اوراس کے اندرونہ کے یونین کو مستطیلی_______کہتے ہیں:

(A) Region/علاقہ (B) Interior/اندرونہ (C) Area/رقبہ (D) Exterior/بیرونہ

If three altitudes of a triangle are congruent, then triangle is: 9۔ اگرایک مثلث کے تینوں عمود متماثل ہوں تو وہ مثلث____ہوگی:

(A) Isosceles / مساوی الاضلاع (B) Equilateral / قائمۃ الزاویہ (C) Right angled / متساوی الساقین (D) Acute angled / حادۃ الزاویہ

Median of a triangle divide it into ____ triangle of equal area: 10۔ مثلث کا ہر ایک وسطانیہ اسے برابر رقبے والی__ مثلثوں میں تقسیم کرتا ہے

(A) 1 (B) 2 (C) 3 (D) 4

$10 \times 2 = 20$ **Write short answers to any ten (10) questions.** سوال نمبر2۔ کوئی سے 10 سوالات کے جوابات تحریر کیجیے۔

i۔ What is the importance of knowledge of ratios and proportions? نسبت تناسب کا علم کیا اہمیت رکھتا ہے؟

ii۔ Write two practicle applications of similar triangles in daily life. روزمرہ زندگی میں متشابہ مثلثوں کے دو عملی استعمال لکھیے۔

iii۔ What is meant by proportion? تناسب سے کیا مراد ہے؟

iv۔ Who was pythagoras and what did he discover? فیثا غورث کون تھا اور اس نے کیا دریافت کیا؟

v۔ What is meant by converse of theorem? مسئلہ کے عکس سے کیا مراد ہے

vi۔ Verify that this triangel is right angled: $a = 5cm,\ b = 12cm, c = 13cm$ تصدیق کیجیے کہ یہ مثلث قائمۃ الزاویہ ہے:

vii۔ Write the axiom of congruent trianlge. متماثل رقبوں کا اصول متعارفہ کی تعریف کریں۔

viii۔ When are two triangles considered to be between two parallels? دو مثلثیں کب دو متوازی خطوط کے درمیان سمجھی جاتی ہیں؟

ix۔ Define the altitude of triangle. مثلث کا ارتفاع کی تعریف کیجیے۔

x۔ What is meant by concurrent lines? نقطہ اتصال سے کیا مراد ہے؟

xi۔ Define circumcentre. مثلث کے محاصرہ مرکز کی تعریف کیجیے۔

xii۔ Define point of concurrency of the lines. نقطہ تثلیث کی تعریف کیجیے۔

$1 \times 10 = 10$ **Write answer to any One (1) question.** نوٹ: کوئی سے ایک سوال کا جواب لکھیے۔

سوال نمبر3۔ Prove that triangles on equal bases and equal altitudes are equal in area. ثابت کریں کہ ایسی مثلثیں جن کے قاعدے اور ارتفاع برابر ہوں وہ رقبہ میں برابر ہوں گی۔

سوال نمبر4۔ ثابت کیجیے کہ برابر قاعدوں پر واقع اور برابر ارتفاع والی متوازی الاضلاع اشکال رقبہ میں برابر ہوتی ہیں۔
Prove that the parallelograms on equal basis and having the same or equal altitude are equal in area.

Exercise 11.1

Q.1 One angle of a parallelogram in 130°. Find the measures of its remaining angles.

In parallelogram

$m\angle B = 130°$

$\angle D = \angle B$ (Opposite angles of a parallelogram)

$m\angle D = m\angle B = 130°$

We know that

$\angle A + \angle B = 180$

$\angle A + 130 = 180$ (sum of int. $\angle$s on same side of a parallelogram is 180°)

$\angle A = 180 - 130$

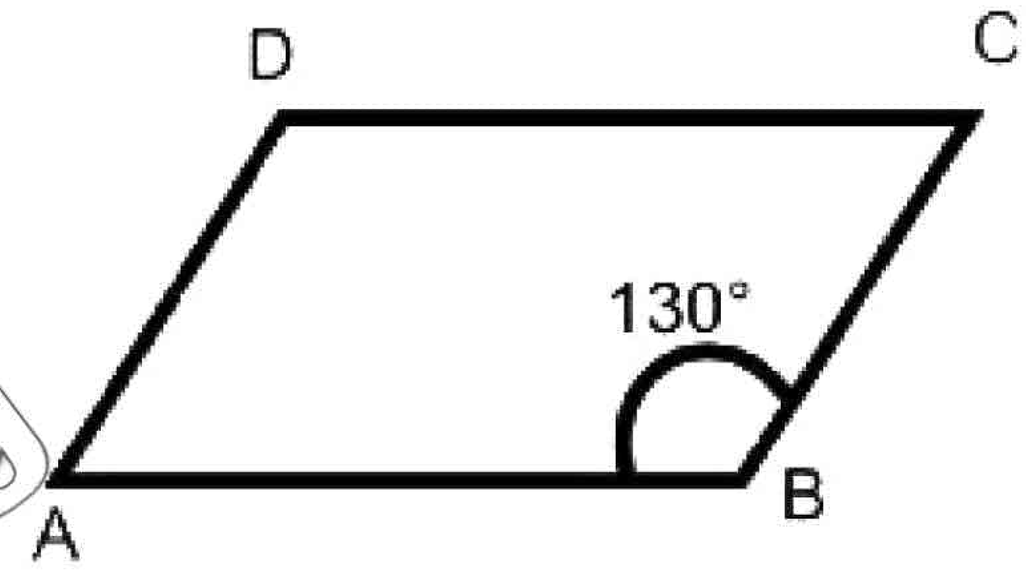
$\angle A = 50°$

If $\angle D = \angle B$

Then

$\angle C = \angle A$

$\angle C = 50°$



Q.2 One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles.

$ABCD$ is a parallelogram. $\overline{BA}$ is produced towards A.

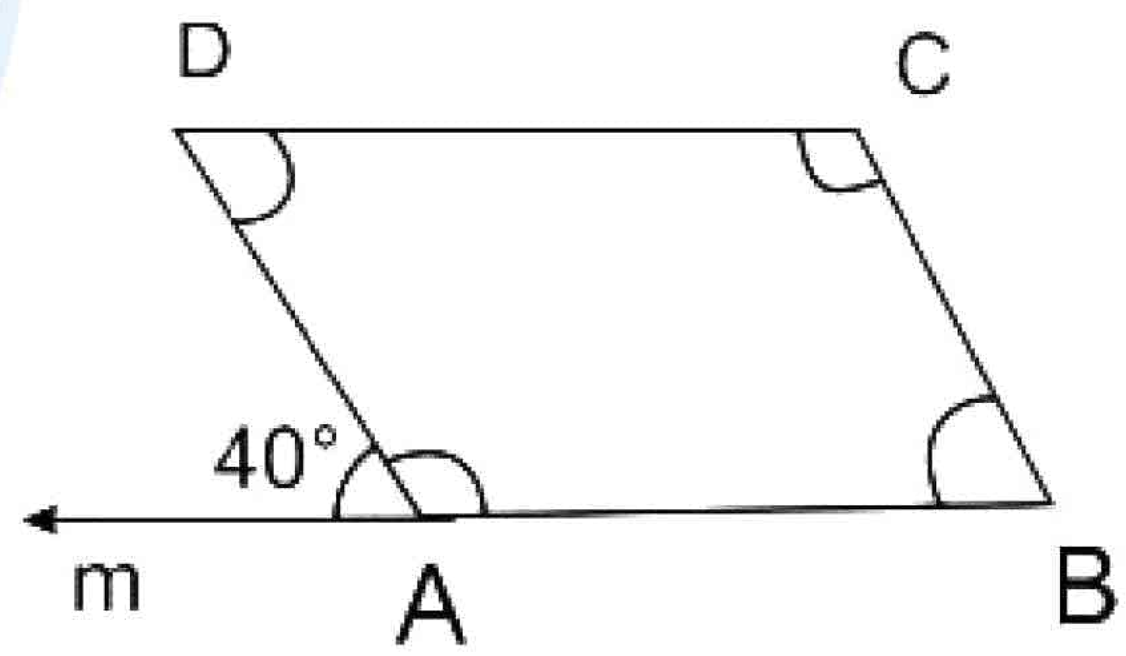
$m\angle DAM = 40°$

$m\angle DAB = ?$

$m\angle D = ?$

$m\angle B = ?$

$m\angle C = ?$



$\angle DAM + \angle DAB = 180°$

$40° + \angle DAB = 180°$

$\angle DAB = 180° - 40°$

$\angle DAB = 140°$

$\angle DAB + \angle B = 180°$

$140° + \angle B = 180°$

$\angle B = 180° - 140°$

$\angle B = 40°$

$\angle D = \angle B = 40°$

$\angle D = 40°$

$\angle C = \angle DAB$

$\angle C = 140°$

Theorem 11.1.2

Statement: If two opposite sides of quadrilateral are congruent and parallel, it is a parallelogram

Given

In quadrilateral $ABCD$,

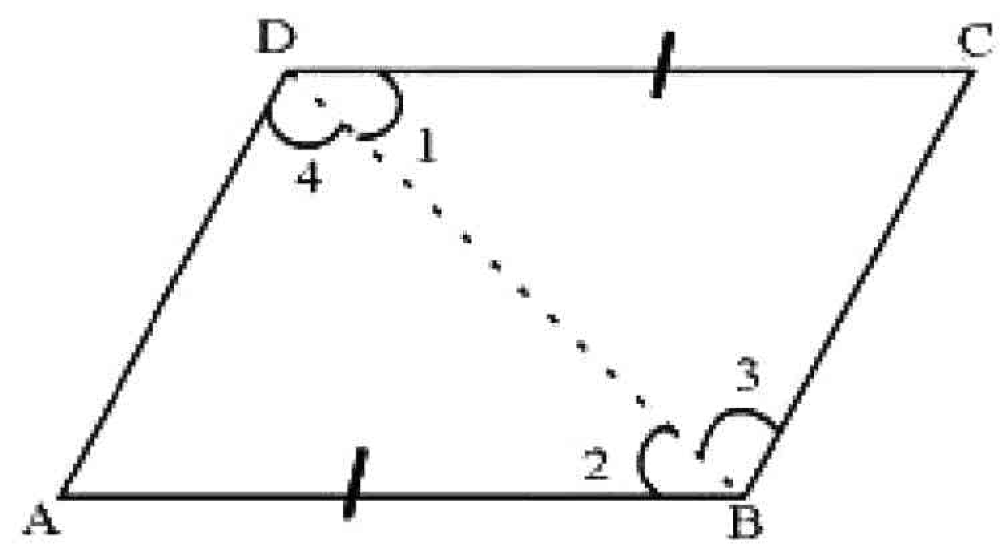
$\overline{AB} \cong \overline{DC}$ *and* $\overline{AB} \parallel \overline{DC}$

To prove

ABCD is a parallelogram

Construction

Join the point B to D and in the figure name the angles as



Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	SAS postulate
Now $\angle 4 \cong \angle 3$............(i)	(Corresponding angles of congruent triangles)
$\therefore \overline{AD} \parallel \overline{BC}$...............(ii)	from (i)
and $\overline{AD} = \overline{BC}$...........(iii)	corresponding of sides of congruent triangles
Also $\overline{AB} \parallel \overline{DC}$...........(iv)	Given
Hence ABCD is a parallelogram	From (ii)-(iv)

Exercise 11.2

Q.1 Prove that a quadrilateral is a parallelogram if its
(a) Opposite angles are congruent
(b) Diagonals bisects each other

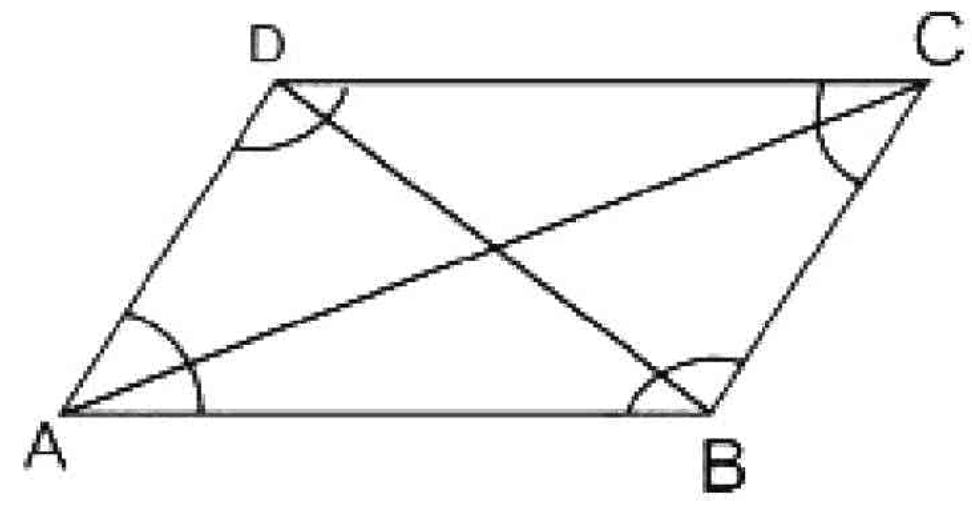
(a) Given

In quadrilateral ABCD

$m\angle A = m\angle C, m\angle B = m\angle D$

To Prove

$ABCD$ is a parallelogram



Statements	Reasons
$m\angle A = m\angle C \ldots(\text{i})$	Given
$m\angle B = m\angle D \ldots(\text{ii})$	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360°$	Angles of quadrilateral
$m\angle A + m\angle B = 180°$	
$m\angle C + m\angle D = 180°$	
$\overline{AD} \parallel \overline{BC}$	
Similarity it can be proved that $\overline{AB} \parallel \overline{DC}$	
Hence ABCD is a parallelogram	

(b) Given

In quadrilateral ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

i.e. $\overline{OA} = \overline{OC}, \overline{OB} = \overline{OD}$

To prove ABCD is a parallelogram

Proof

Statements	Reasons
In $\Delta ABO \leftrightarrow \Delta CDO$	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Vertical opposite angles
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\Delta ABO \cong \angle CDO$	$S.A.S \cong S.A.S$
Hence, $\overline{AB} \parallel \overline{CD} \ldots(\text{i})$	$\angle 1 \cong \angle 2$
By taking BOC and is ΔAOD it can be prove that	
$\overline{AD} \parallel \overline{BC} \ldots(\text{ii})$	From (i) and (ii)
Hence ABCD is a parallelogram	

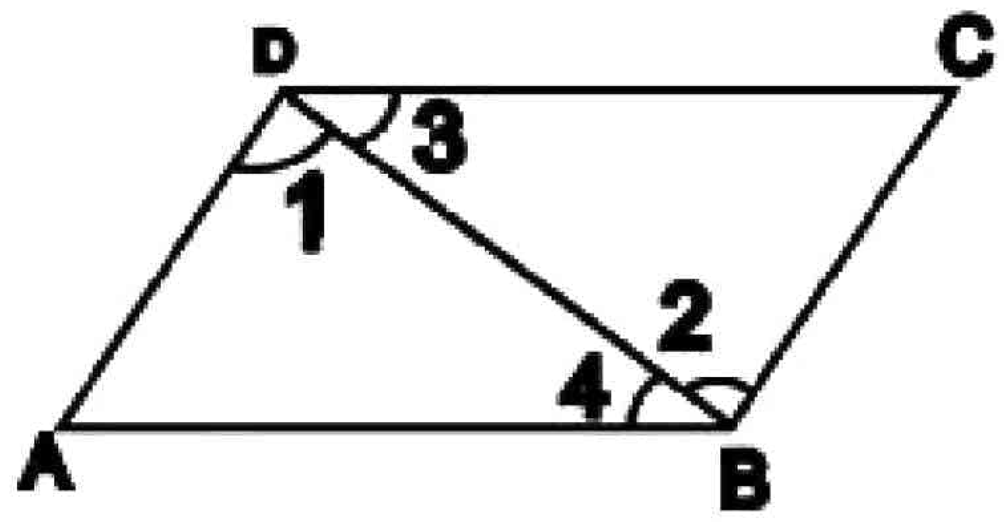
Q.2 Prove that a quadrilateral is a parallelogram if its opposite sides are congruent

Given

In quadrilateral $ABCD$

(i) $\overline{AB} \cong \overline{DC}$

(ii) $\overline{AD} \cong \overline{BC}$



To prove

$ABCD$ is a parallelogram i.e. $\overline{AD} \parallel \overline{BC}$

Prove

Statements	Reasons
$\Delta CDB \leftrightarrow \Delta ABD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Thus, $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 4 \cong \angle 3$	Corresponding angles of congruent triangles
(i) $\overline{AD} \parallel \overline{BC}$	Alternate angles are congruent
$\overline{AB} \parallel \overline{DC}$	Alternate angles are congruent
$\therefore$ ABCD is a parallelogram	

Example

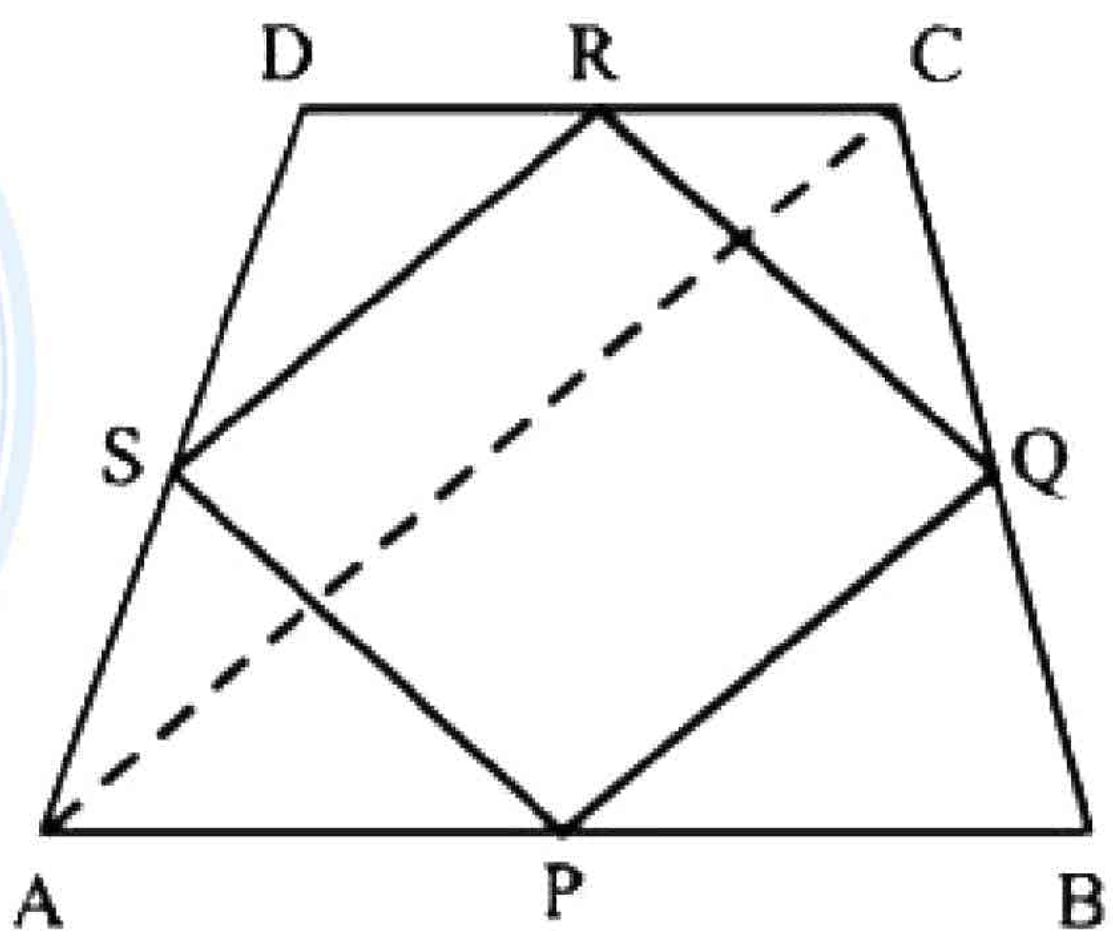
The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.

Given

A quadrilateral $ABCD$, in which P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$ R is the mid-point of $\overline{CD}$ S is the mid-point of $\overline{DA}$

P is joined to Q, Q is joined to R.

R is joined to S and S is joined to P.



To prove

PQRS is a parallelogram.

Construction

Join A to C.

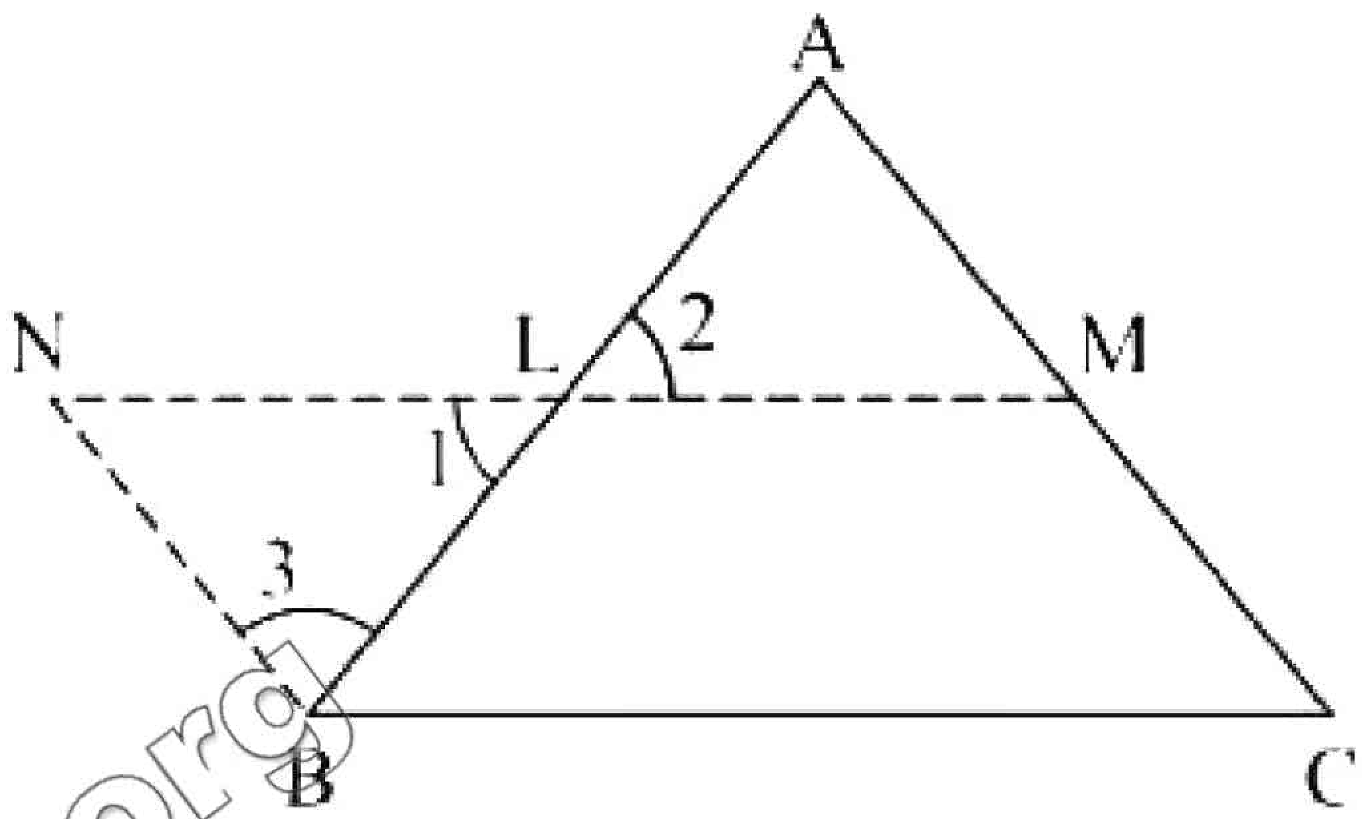
Proof

Statements	Reasons
In ΔDAC,	
$\overline{SR} \parallel \overline{AC}$	S is the midpoint of $\overline{DA}$
$m\overline{SR} = \frac{1}{2} m\overline{AC}$	R is the midpoint of $\overline{CD}$

In ΔBAC,	
$\overline{PQ} \parallel \overline{AC}$	P is the midpoint of $\overline{AB}$
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$	Q is the midpoint of $\overline{BC}$
$\overline{SR} \parallel \overline{PQ}$	Each $\parallel \overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each$=\frac{1}{2}\overline{AC}$
Thus $PQRS$ is a parallelogram	$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)

Theorem 11.1.3

The line segment, joining the midpoint of two sides of triangle, is parallel to the third side and is equal to one half of its length.



Given

In ΔABC, the mid-point of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure, name the angles $\angle 1$, $\angle 2$ and $\angle 3$ as shown.

Proof

Statements	Reasons
In Δ BLN ↔ Δ ALM	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
∴ ΔBLN ≅ ΔALM	S.A.S postulate
$\therefore \angle A \cong \angle 3$...(i)	(Corresponding angles of congruent triangles)
And $\overline{NB} \cong \overline{AM}$...(ii)	(Corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	from (i), alternative <s
Thus	
$\overline{NB} \parallel \overline{MC}$ ....(iii)	(M is a point of $\overline{AC}$)

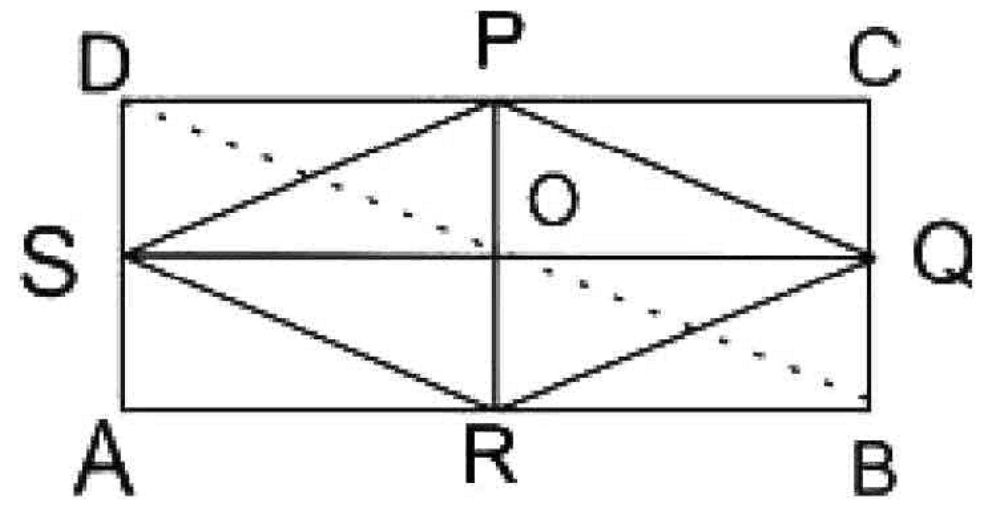
$\overline{MC} \cong AM$ ....(iv)	Given
$\overline{NB} \cong \overline{MC}$ ....(v)	from (ii) and (iv)
$BC\,MN$ is a parallelogram	From (iii) and (v)
$\therefore \overline{BC} \parallel \overline{LM}$ *or* $\overline{BC} \parallel \overline{NL}$	(Opposite sides of a parallelogram BCMN) (Opposite sides of a parallelogram)
$\overline{BC} \cong \overline{NM}$ ....(vi)	Construction.
$m\overline{LM} = \frac{1}{2} m\overline{NM}$ ....(vii)	
Hence, $m\overline{LM} = \frac{1}{2} m\overline{BC}$	from (vi) and (vii)

Exercise 11.3

Q.1 Prove that the line segments joining the midpoint of the opposite side of a quadrilateral bisect each other.

Given

$ABCD$ is quadrilaterals point $QRSP$ are the mid point of the sides $\overline{RP}$ and $\overline{SQ}$ are joined they meet at O.



$\overline{OP} \cong \overline{OR}$ $\overline{OQ} \cong \overline{OS}$

Construction

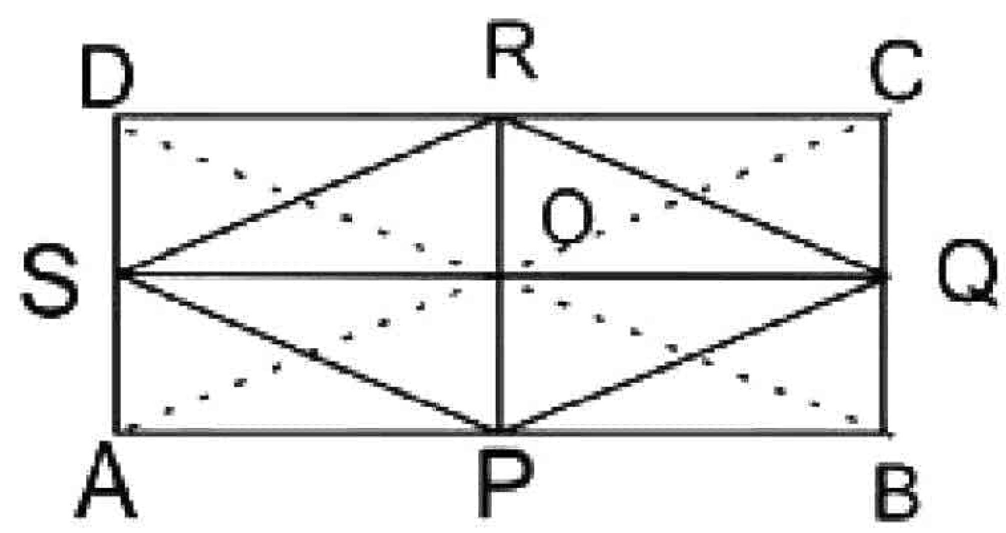
Join P, Q, R and S in order join C to A or A to C

Proof

Statements	Reasons
$SP \parallel AC$ ... (i)	In $\Delta ADC, S, P$ are mid point of AD, DC
$m\overline{SP} = \frac{1}{2} m\overline{AC}$...(ii)	
$\overline{AC} \parallel \overline{RQ}$...(iii)	In $\Delta ABC, Q, R$ are midpoint of $\overline{BC}$, $\overline{AB}$
$m\overline{RQ} = \frac{1}{2}\overline{AC}$...(iv)	
$m\overline{SP} \parallel \overline{RQ}$...(v)	
and $\overline{RQ} = \overline{SP}$...(vi)	From (ii) and (iv)
Now $\overline{RP}$ *and* $\overline{QS}$ diagonals of parallelogram PQRS intersect at O.	
$\therefore$ $\overline{OP} \cong \overline{OR}$ $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram bisects each other.

Q.2 Prove that the line segments joining the midpoint of the opposite sides of a rectangle are the right bisectors of each other.
[Hint: Diagonals of a rectangle are congruent]

Given



(i) $ABCD$ is a rectangle

(ii) P, Q, R, S are the midpoints of $\overline{AB}$, $\overline{CD}$ and $\overline{DA}$

(iii) $\overline{SQ}$ and $\overline{RP}$ cut each other at point O

$\overline{OS} \cong \overline{OQ}$

$\overline{OP} \cong \overline{OR}$

Construction

Join P to Q and Q to R and R to S and S to P

Join A to C and B to D

Proof

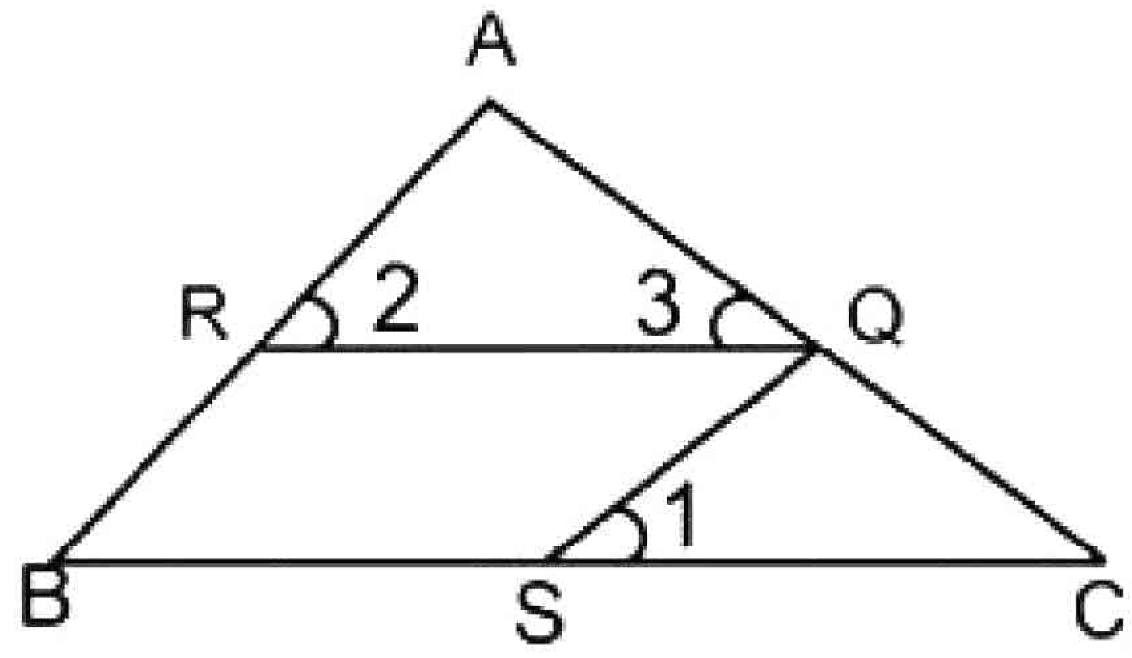
Statements	Reasons
Midpoint of $\overline{BC}$ is Q	Given
Midpoint of $\overline{AB}$ is P	Given
$\therefore \overline{AC} \parallel \overline{PQ}$................(i)	
$\frac{1}{2}\overline{AC} = \overline{PQ}$............(ii)	
In ΔADC	
$\overline{AC} \parallel \overline{SR}$...............(iii)	
$\frac{1}{2}\overline{AC} = \overline{SR}$............(iv)	
$\overline{PQ} = \overline{SR}$	From equation (i) and (ii) each are parallel to
$\overline{SP} = \overline{RQ}$	$\overline{AC}$ each are half of $\overline{DB}$
By joined B to D we can prove	
$\overline{RQ} \parallel \overline{SP}$	
$m\overline{SR} \parallel m\overline{PQ}$	Each of them = $\frac{1}{2}\overline{AC}$
$m\overline{AC} \parallel m\overline{BD}$	
PQRS is a parallelogram all it sides are equal	
$\overline{OP} \cong \overline{OR}$	
$\overline{OS} \cong \overline{OQ}$	
$\Delta OQR \leftrightarrow \Delta OQP$	
$\overline{OR} \cong \overline{OP}$	Proved
$\overline{OQ} \cong \overline{OQ}$	Common
$\overline{RQ} \cong \overline{PQ}$	Adjacent
$\therefore \Delta OQR \cong \Delta OQP$	
$\angle ROQ \cong \angle POQ$..............(vii)	
$\angle ROQ + \angle POQ = 180$........(viii)	Supplementary angle
$\angle ROQ = \angle POQ = 90°$	From (vii) and (viii)
Thus $\overline{PR} \perp \overline{QS}$	

Q.3 Prove that line segment passing the midpoint of one side and parallel to other side of a triangle also bisects the third side.

Given

In ΔABC, R is the midpoint of $\overline{AB}$, $\overline{RQ} \parallel \overline{BC}$

$\overline{RQ} \parallel \overline{BS}$



To prove

$\overline{AQ} = \overline{QC}$

Construction

$\overline{QS} \parallel \overline{AB}$

Proof

Statements	Reasons
$\overline{RQ} \parallel \overline{BS}$	Given
$\overline{QS} \parallel \overline{BR}$	Construction
$RBSQ$ is a Parallelogram	
$\overline{QS} \cong \overline{BR}$...(i)	Opposite side
$\overline{AR} \cong \overline{RB}$...(ii)	Given
$\overline{QS} \cong \overline{AR}$...(iii)	From (i) and (ii)
$\angle 1 \cong \angle B$ and	
$\angle 1 \cong \angle 2$...(iv)	
$\Delta ARQ \leftrightarrow \Delta QSC$	
$\angle 2 \cong \angle 1$	From (iv)
$\angle 3 \cong \angle C$	
$\overline{AR} \cong \overline{SQ}$	From (iii)
Hence, $\Delta ARQ \cong \Delta QSC$	$A.A.S \cong A.A.S$
$\overline{AQ} \cong \overline{QC}$	Corresponding sides

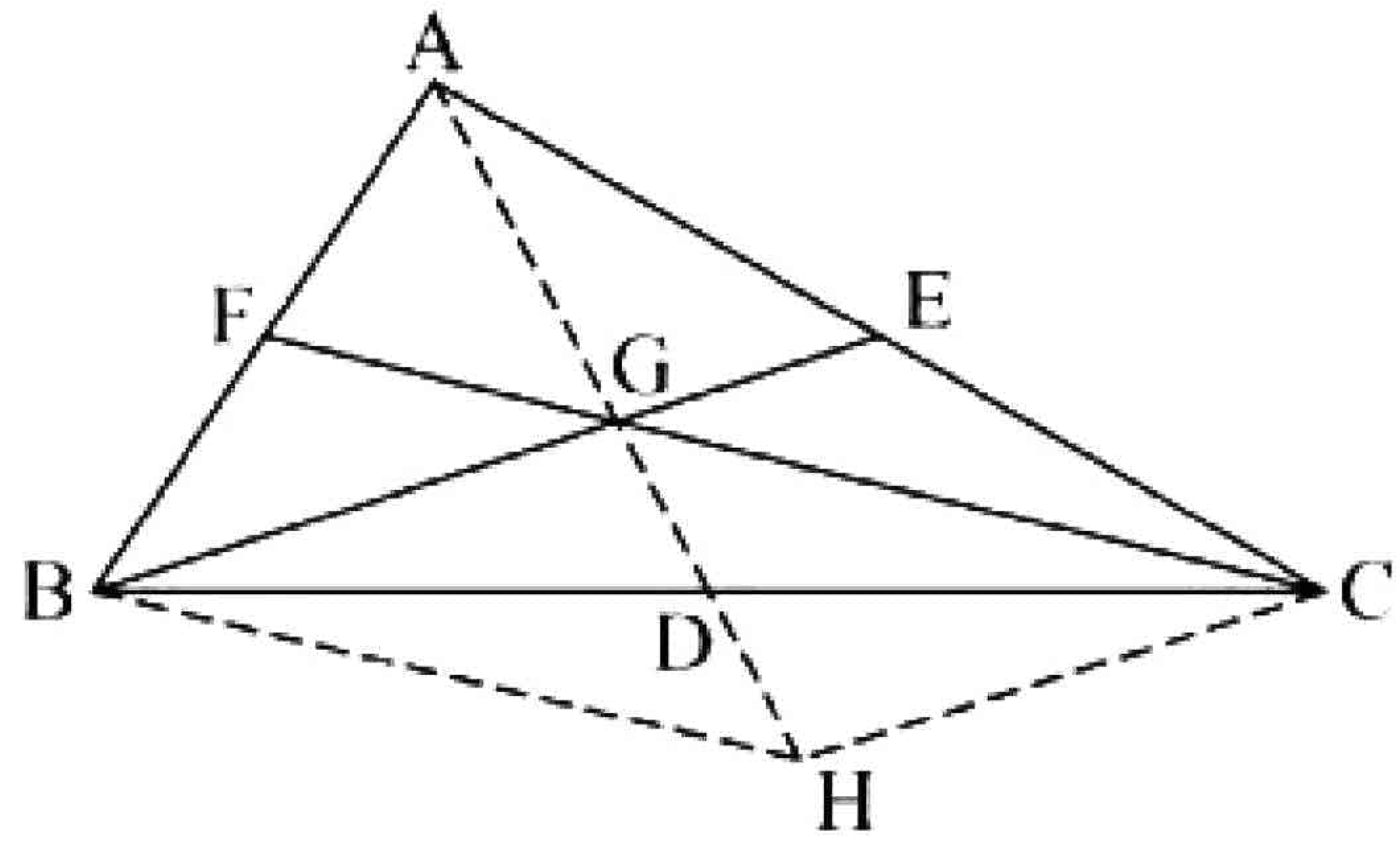
Theorem: 11.1.4

Statement: The median of triangle are concurrent and their point of concurrency is the point of trisection of each median.

Given ΔABC

To prove

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median



Construction

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to the point H such that $AG \simeq \overline{GH}$ Join H to the points B and C $\overline{AH}$ Intersects $\overline{BC}$ at the point D.

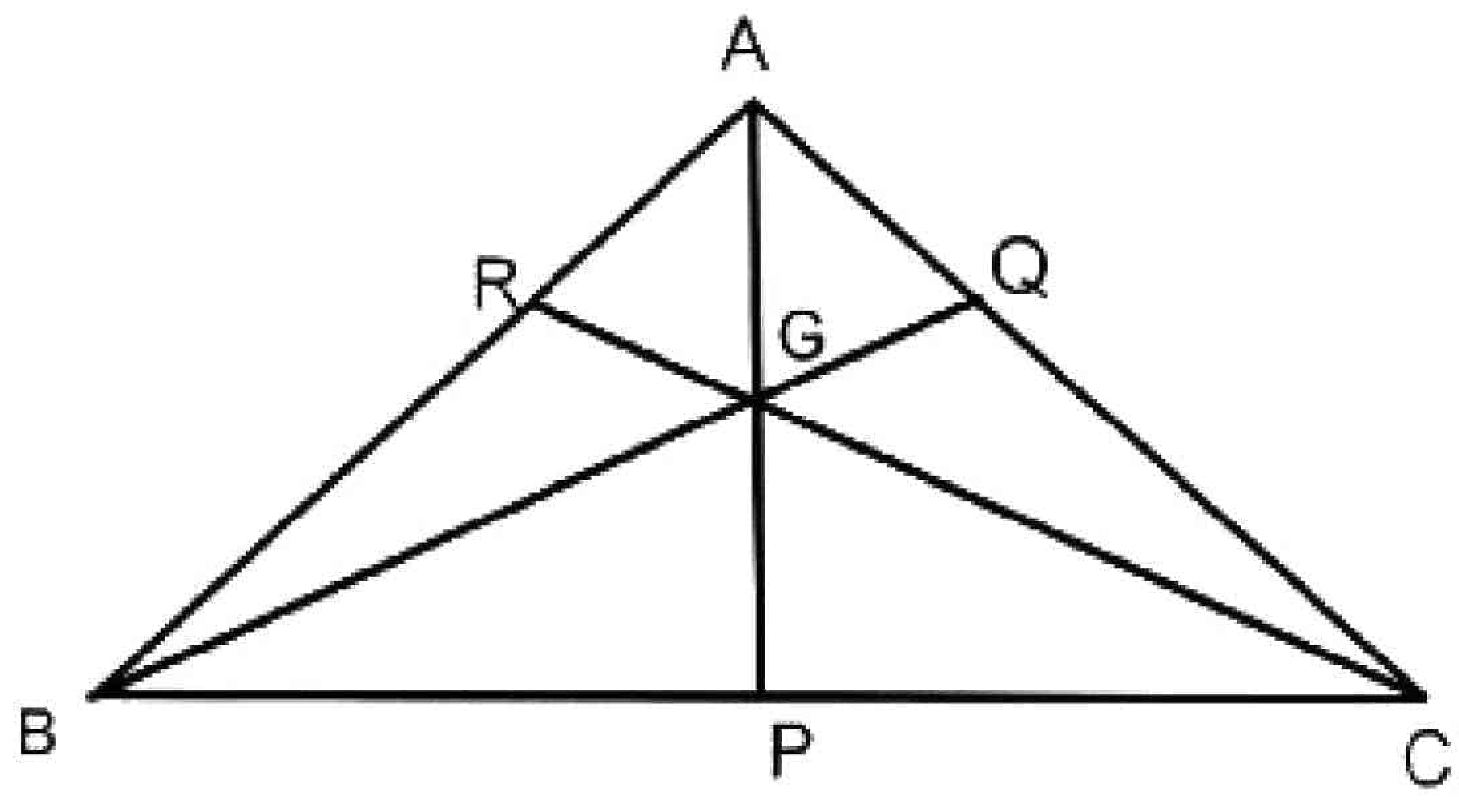
Proof

Statements	Reasons
In Δ ACH,	
$\overline{GE} \parallel \overline{HC}$	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
Or $\overline{BE} \parallel \overline{HC}$........(i)	G is point of $\overline{BE}$ diagonals $\overline{BC}$
Similarly $\overline{CF} \parallel \overline{HB}$...(ii)	
$\therefore$ BHCG is a parallelogram	From (i) and (ii)
And	
$m\overline{GD} = \frac{1}{2} m\overline{GH}$...(iii)	Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram $BHCG$ intersect each other at point D.
$\overline{BD} = \overline{CD}$	
$\overline{AD}$ is a median of ΔABC medians $\overline{AD}, \overline{BE}$ and $\overline{CF}$ pass through the point G	G is the interesting point of $\overline{BE}$, $\overline{CF}$ and $\overline{AD}$ pass through it.
Now $\overline{GH} \cong \overline{AG}$...(iv)	Construction
$m\overline{GD} = \frac{1}{2} m\overline{AG}$	From (iii) and (iv)
and G is the point of trisection of $\overline{AD}$...(v)	
similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$	

Exercise 11.4

Q.1 The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm. 1.4 cm and 1.6 cm. Find the length of its medians.

Let ΔABC with the point of concurrency of medians at G



$\overline{AG}=1.2cm,\ \overline{BG}=1.4cm\ and\ \overline{CG}=1.6cm$

$\overline{AP}=\frac{3}{2}\overline{AG}=\frac{3}{2}\times1.2=1.8cm$

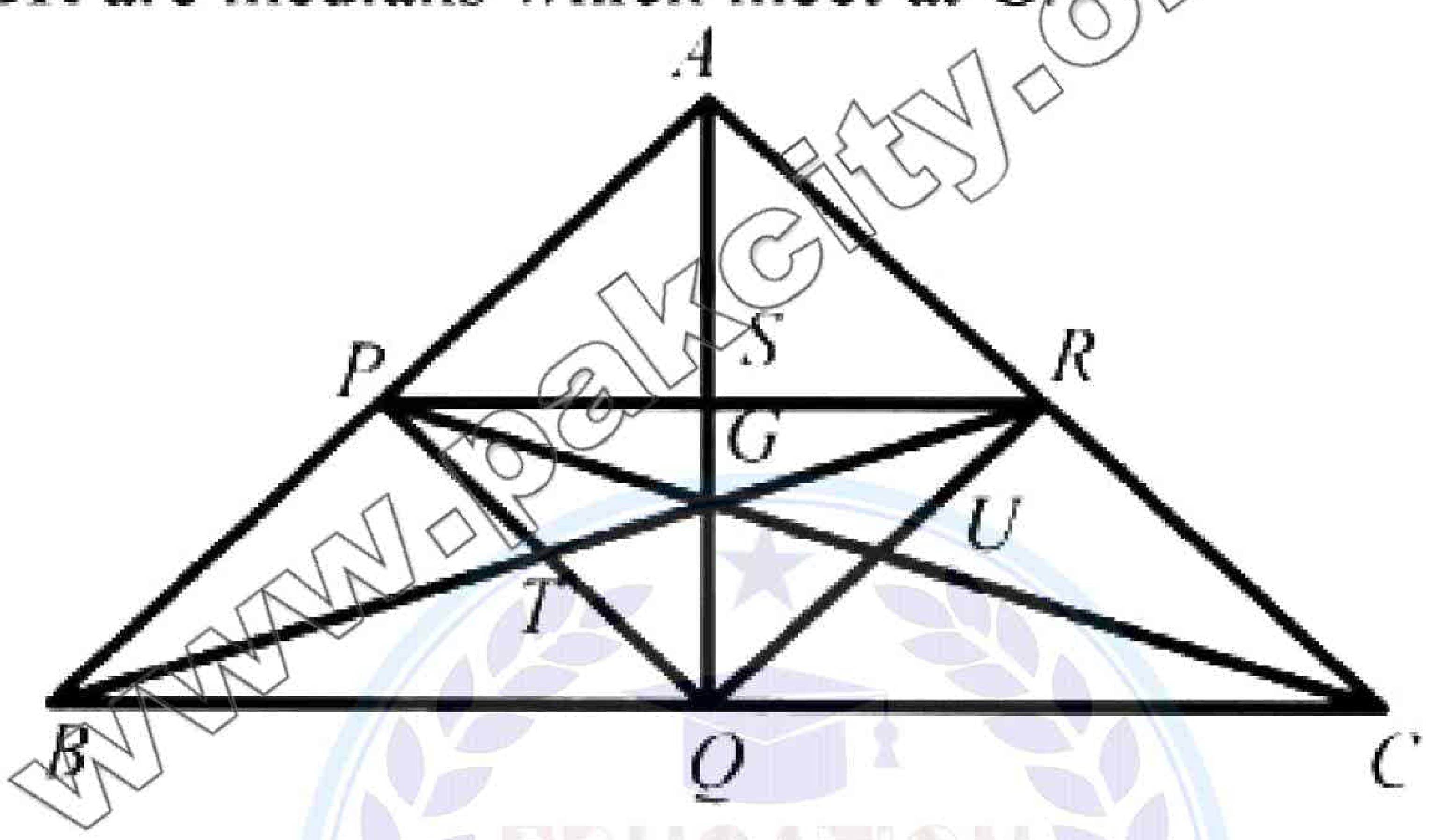
$\overline{BQ}=\frac{3}{2}\overline{BG}=\frac{3}{2}\times1.4=2.1cm$

$\overline{CR}=\frac{3}{2}\overline{CG}=\frac{3}{2}\times1.6=2.4cm$

Q.2 Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the midpoint of its sides to the same.

Given

In ΔABC, AQ, CP, BR are medians which meet at G.



To prove

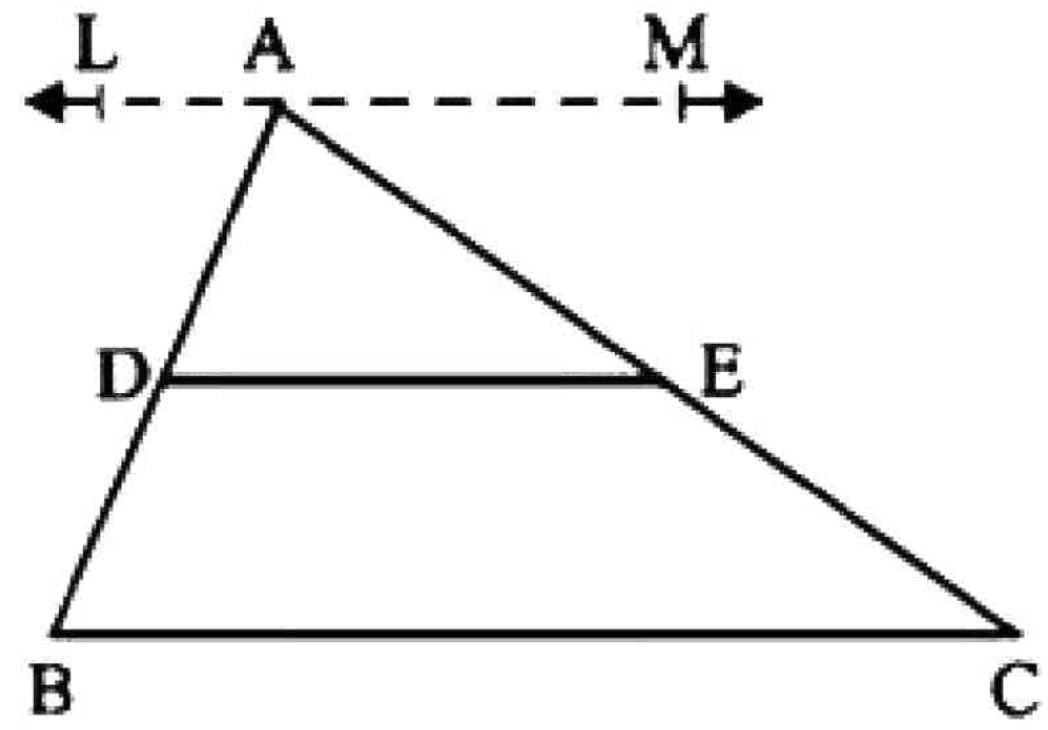
G is the point of concurrency of the medians of ΔABC and ΔPQR

Proof

Statements	Reasons
$\overline{PR}\parallel\overline{BC}$	P, R are midpoint of $\overline{AB},\overline{AC}$
$\overline{BQ}\parallel\overline{PR}$	
Similarly $\overline{QR}\parallel\overline{BP}$	
$\therefore PBQR$ is a parallelogram it diagonals $\overline{BR}$ and $\overline{PQ}$ bisector each other at T	
Similarly U is the midpoint of QR and S is midpoint of $\overline{PR}$	
$\therefore \overline{PU},\overline{QS},\overline{RT}$ are medians of ΔPQR	
(i) $\overline{AQ}$ and $\overline{SQ}$ pass through G	
(ii) $\overline{BR}$ and $\overline{TR}$ pass through G	
(iii) $\overline{UP}$ and $\overline{CP}$ pass through G	
Hence G is point of concurrency of medians of ΔPQR and ΔABC	

Example

A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.



Given

In ΔABC, D is the mid-point of $\overline{AB}$.

$\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{BC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}, \overline{DE}, \overline{BC}$ on $\overline{AC}$ are congruent. i.e., $\overline{AE} \cong \overline{EC}$.	Intercepts cut by parallels $\overleftrightarrow{LM}, \overline{DE}$. $\overline{BC}$ on $\overline{AB}$ are congruent (given)

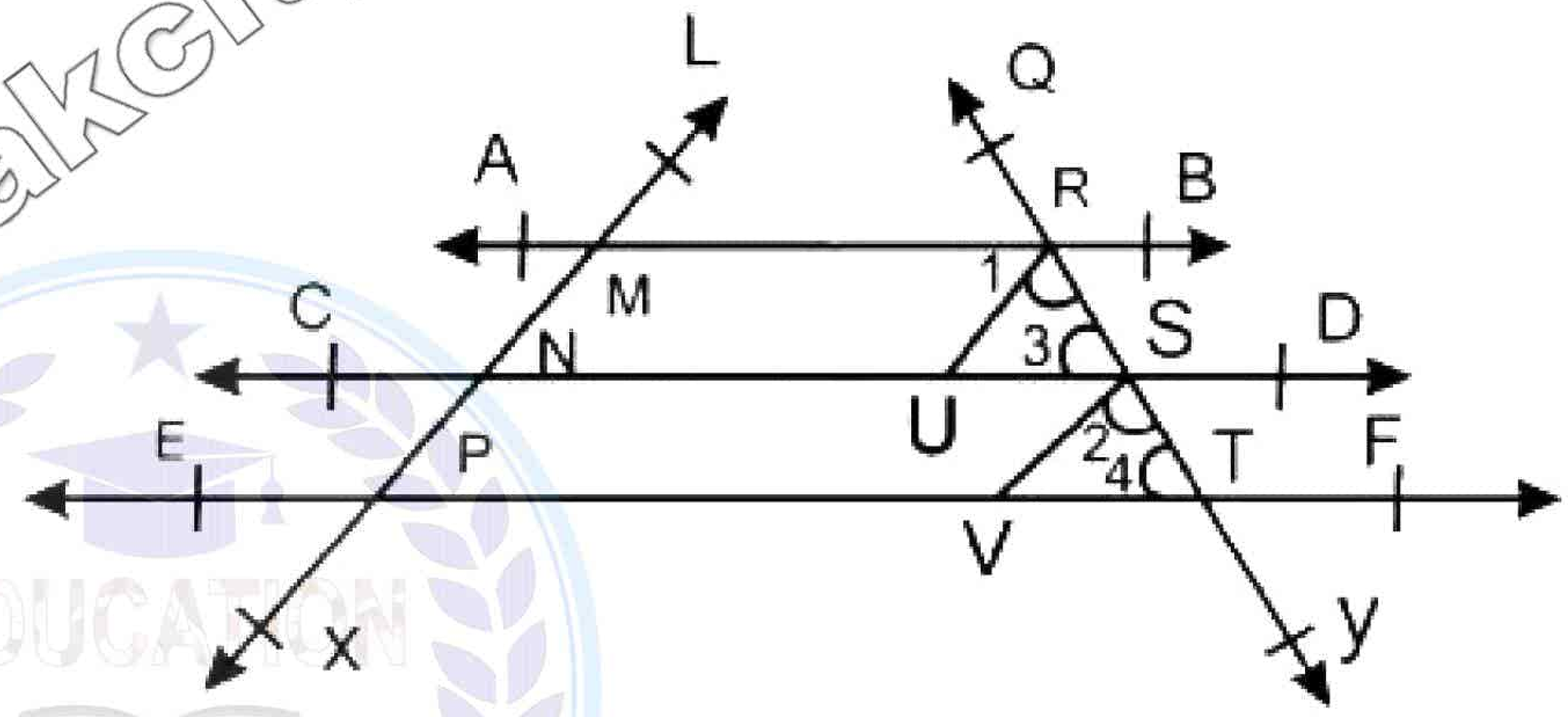
Theorem 11.1.5

Statement: In three or more parallel lines make congruent segments on a traversal they also intercept congruent segments on any other line that cuts them.

Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at point R, S and T respectively.



Prove

$\overline{RS} \cong \overline{ST}$

Construction

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U, from S draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Proof

Statements	Reasons
$MNUR$ is parallelogram	$\overline{RU} \parallel \overline{LX}$ (Construction) $\overline{AB} \parallel \overline{CO}$ (given)
$\therefore \overline{MN} \cong \overline{RU}$ (i)	(Opposite side of parallelogram).
Similarly.	
$\overline{NP} \cong \overline{SV}$ (ii)	
But $\overline{MN} \cong \overline{NP}$ (iii)	Given
$\therefore \overline{RU} \cong \overline{SV}$	{from (i) (ii) and (iii)} each is $\parallel$ $\overline{LX}$ (construction)

Also $\overline{RU} \parallel \overline{SV}$	
$\therefore \angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore \Delta RUS \cong \Delta SVT$	$S.A.A \cong S.A.A$
Hence $\overline{RS} \cong \overline{ST}$	(Corresponding sides of congruent triangles)

Exercise 11.5

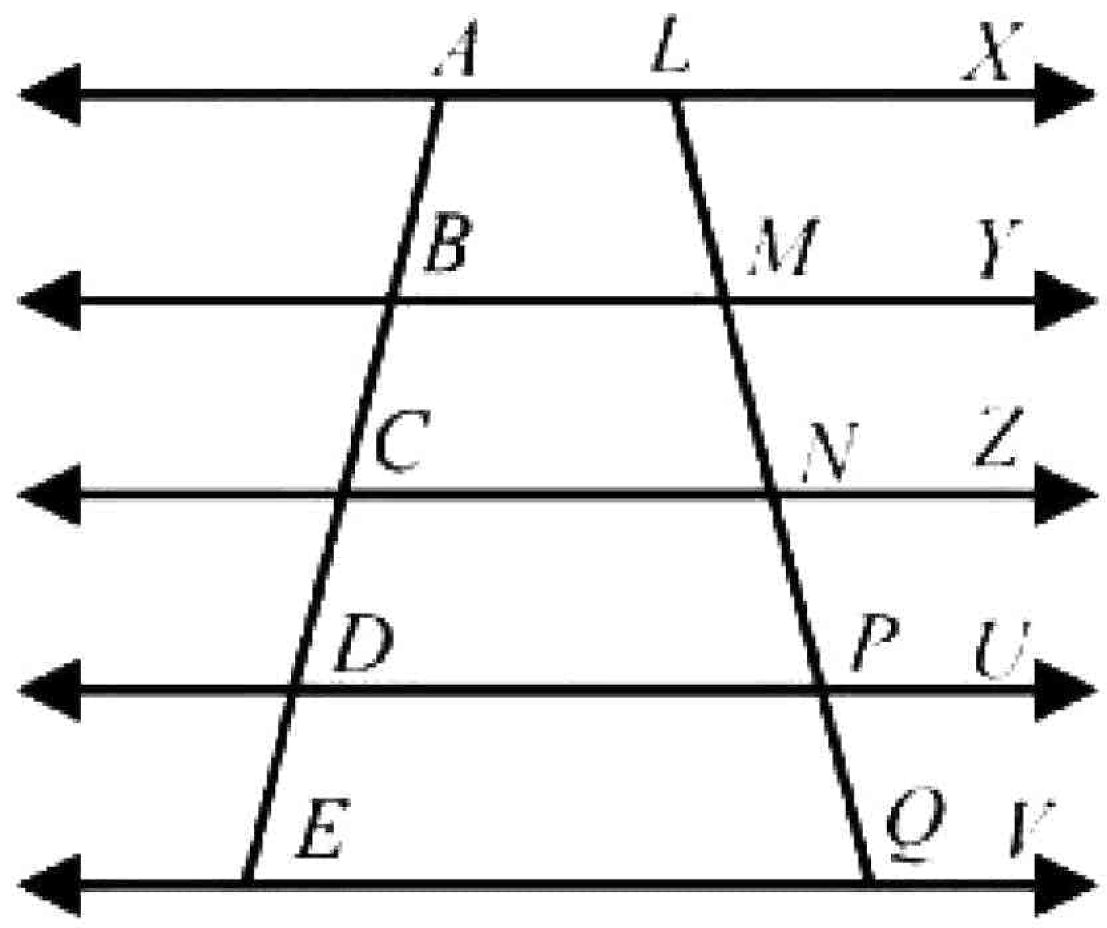
Q.1 In the given figure

$\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

If $\overline{MN} = 1cm$ then find the length of $\overline{LN}$ and $\overline{LQ}$

$\therefore \overline{PQ} \cong \overline{NP} \cong \overline{MN} \cong \overline{LM}$

$MN = 1cm$



Given

$\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

Therefore, $\overline{LN} = \overline{LM} + \overline{MN}$

$\overline{LM} = \overline{MN}$

so, $\overline{LN} = \overline{MN} + \overline{MN}$

$\overline{LN} = 1 + 1$

$\overline{LN} = 2cm$

$\overline{LM} = \overline{NP} = \overline{PQ} = \overline{MN} = 1cm$

So, $\overline{LM} = 1cm, \overline{NP} = 1cm, \overline{PQ} = 1cm$

$LQ = \overline{LM} + \overline{MN} + \overline{NP} + \overline{PQ}$

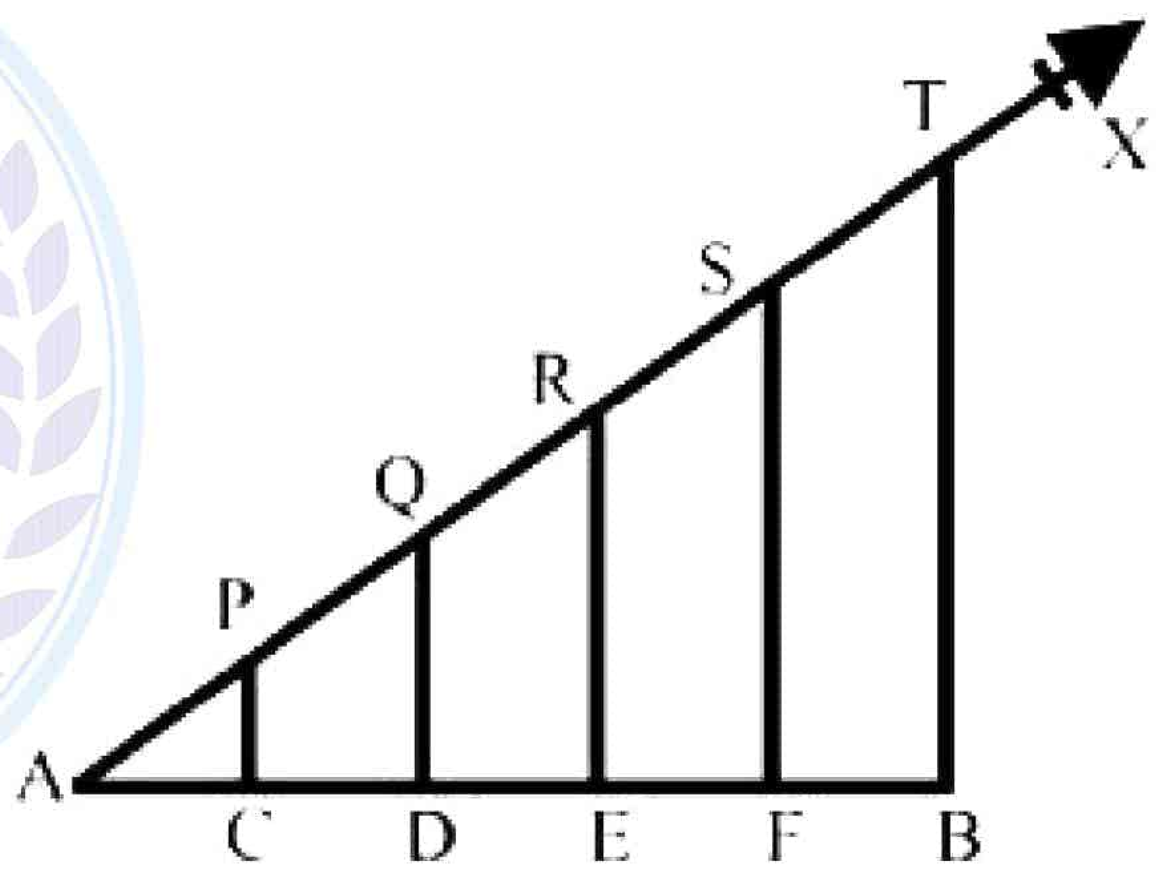
$LQ = 1 + 1 + 1 + 1$

$LQ = 4cm$

Q.2 Take a line segment of length $5.5cm$ and divide it into five congruent parts [Hint: draw an acute angle $\angle B$ AX. On $\overline{AX}$ take $\overline{AP} \cong \overline{PQ} \cong \overline{RS} \cong \overline{ST}$ join T to B draw lines parallel to $\overline{TB}$ from the point P, Q R and S.

Proof

Construction:



(i) Take a line segment $AB = 5.5cm$

(ii) Draw any acute angle $\angle BAX$

(iii) Draw arcs on $\overrightarrow{AX}$ which are $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

(iv) Join T to B

(v) Draw lines $\overline{SF}, \overline{RE}, \overline{QD}, \& \overline{PC}$ Parallel to $\overline{TB}$.

Result line segment $\overline{AB}$ is divided into congruent line segments $\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FB}$.

Review Exercise 11

Q.1 Fill in the blanks

(i) In a parallelogram opposite side are ……

Ans: Congruent

(ii) In a parallelogram opposite angles are …….

Ans: Congruent

(iii) Diagonals of a parallelogram …….. each other at a point.

Ans: Bisects

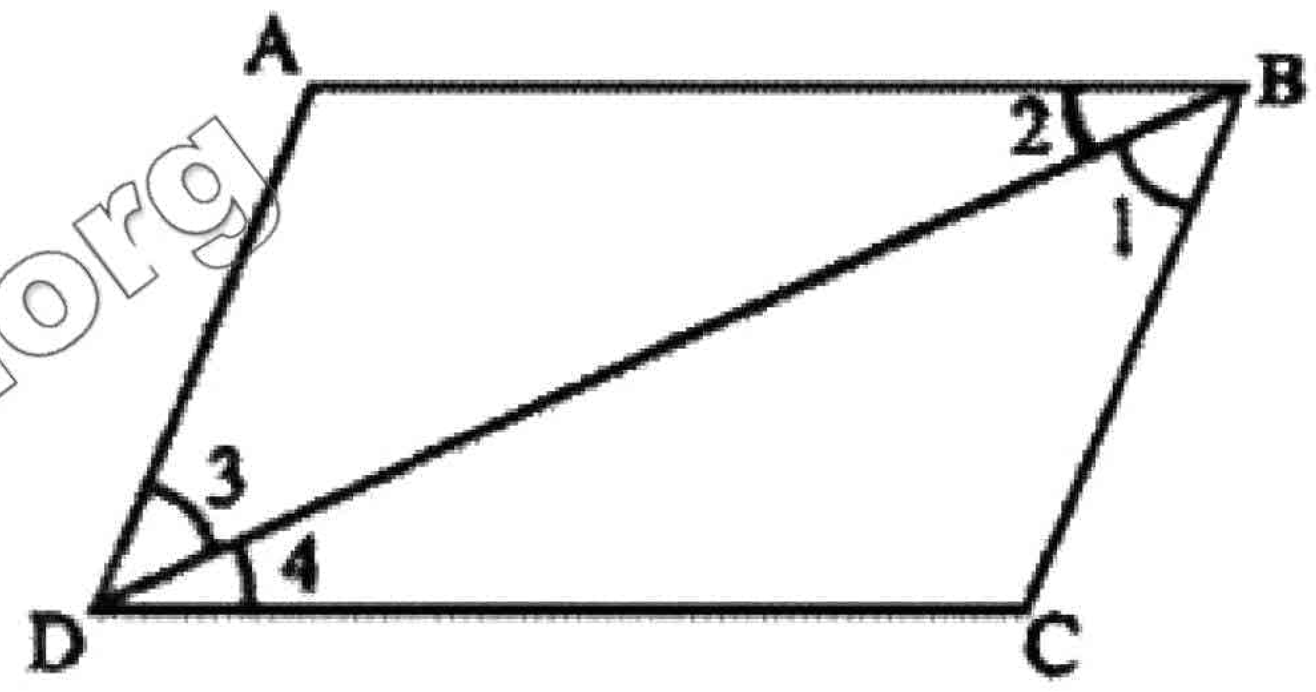
(iv) Medians of a triangle are ……….

Ans: Concurrent

(v) Diagonals of a parallelogram divide the parallelogram into two ……. Triangles

Ans: Congruent

Q.2 In parallelogram *ABCD*



(i) $m\overline{AB} = \ldots\ldots\ldots\ldots$

Ans: $m\overline{AB} = m\overline{DC}$

(ii) $m\overline{BC} \ldots\ldots\ldots\ldots$

Ans: $m\overline{BC} = m\overline{AD}$

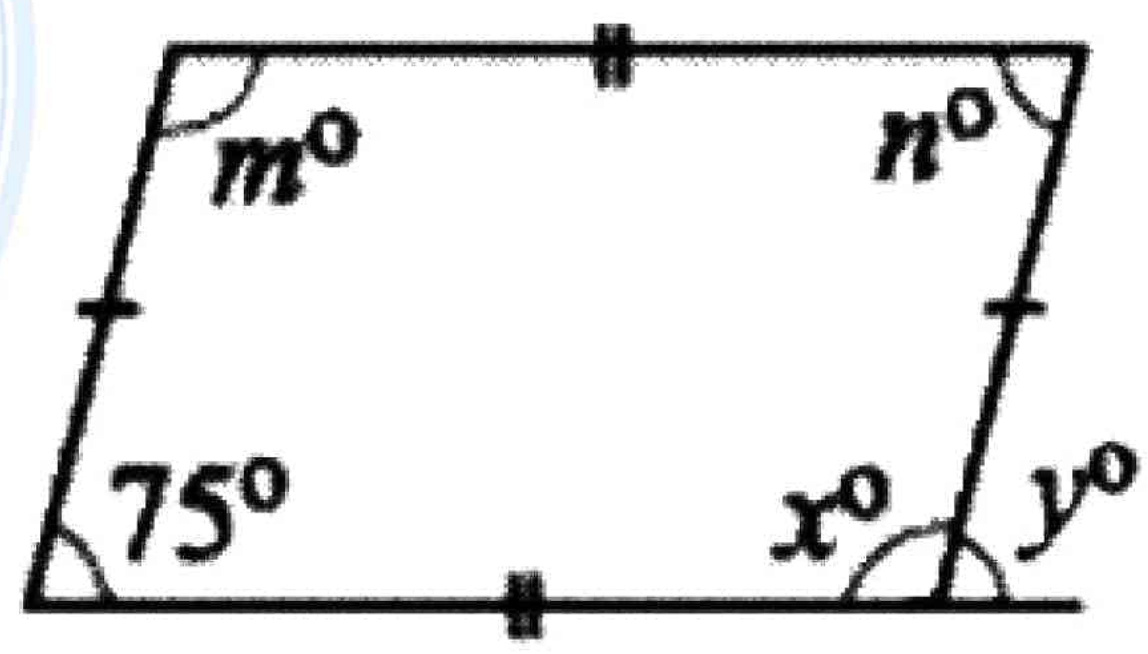
(iii) $m\angle 1 \cong \ldots\ldots\ldots\ldots$

Ans: $m\angle 1 = m\angle 3$

(iv) $m\angle 2 = \ldots\ldots\ldots\ldots$

Ans: $m\angle 2 = m\angle 4$

Q.3 Find the unknown in the figure given



Solution

$n° = 75$

$y° = n°$

Substituting the value of $n°$

$y° = 75°$

$x° + 75 = 180$ Adjacent and supplementary

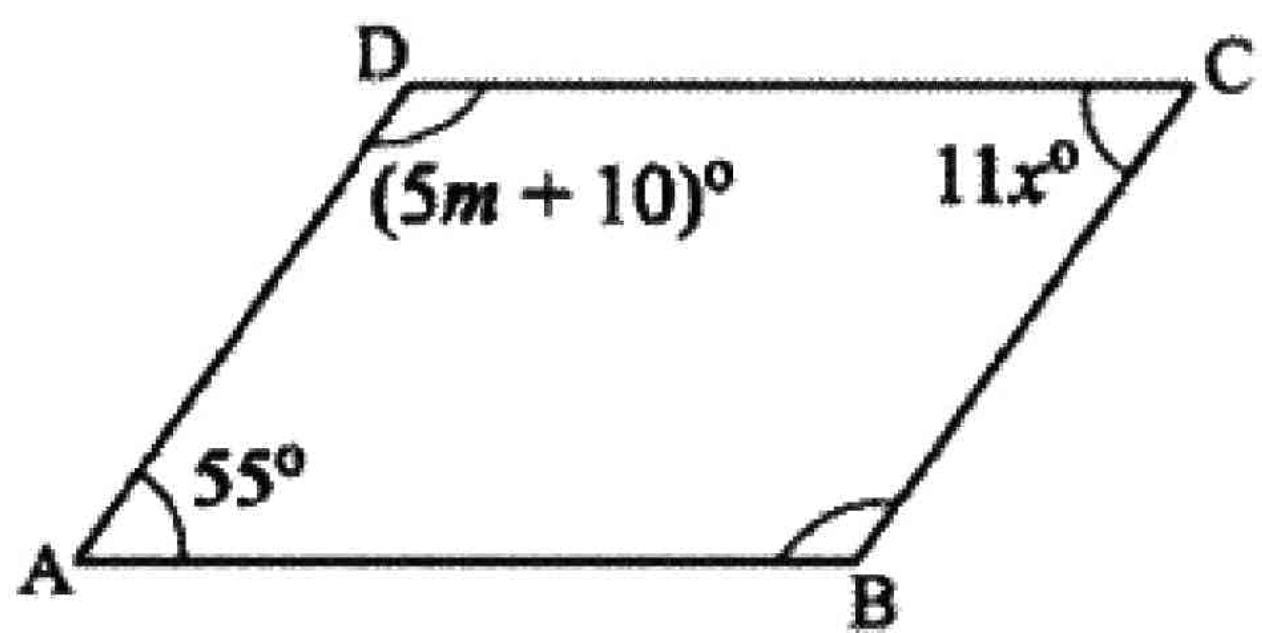
$x° = 180 - 75$

$x° = 105°$

$m° = x°$

$m° = 105°$

Q.4 If the given figure ABCD is a parallelogram then find *x*, *m*



$11x° = 55°$

$x° = \frac{\cancel{55}^{5}}{\cancel{11}}$

$x° = 5°$

$\angle A + \angle B = 180°$

$\angle B = 180° - \angle A$

$\angle B = 180° - 55 = 125°$

$\angle B = 130°$

$\angle D + \angle C = 180°$

$5m + 10° + 55° = 180°$

$5m + 65° = 180°$

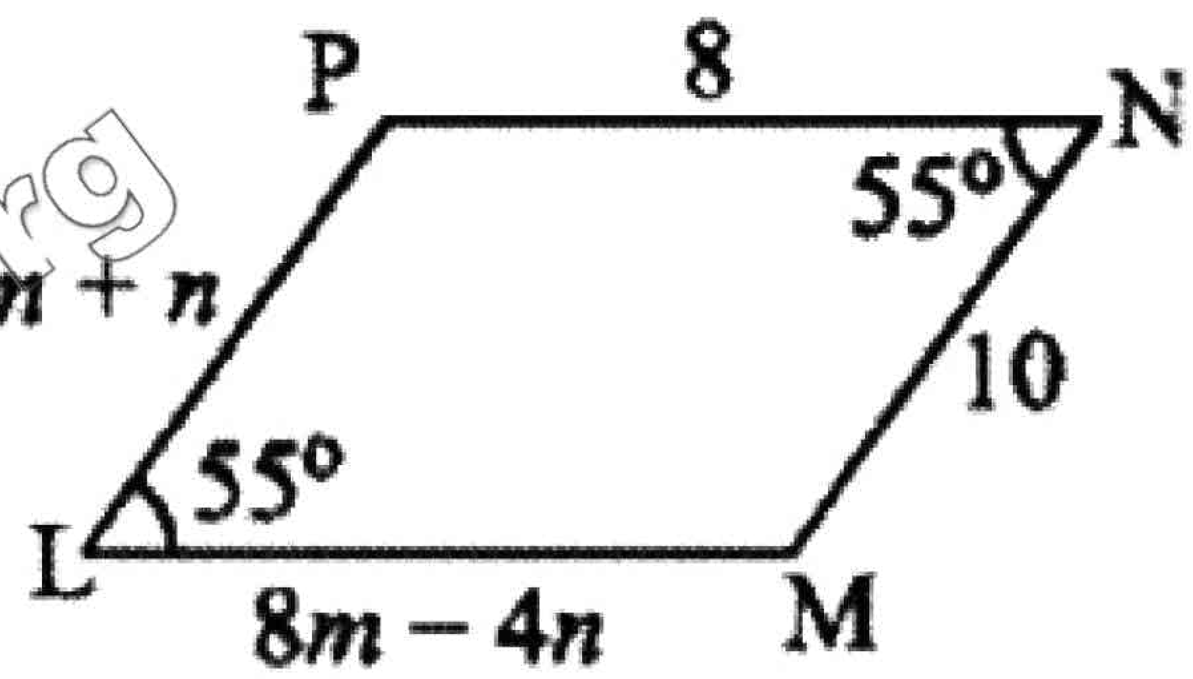
$5m = 180° - 65°$

$5m = 115°$

$m = \frac{115°}{5°}$

$m = 23°$

Q.5 The given figure ∠MNP is a parallelogram finds the value of m, n



$4m + n = 10$.......... (i)

In parallelogram opposite sides are congruent $8m - 4n = 8$ … (ii)

Multiply 4 with equation

$4(4m + n) = 4 \times 10$

$16m + 4n = 40$… (iii)

Adding equation (ii) and (iv)

$8m - \cancel{4n} = 8$

$16m + \cancel{4n} = 40$

$24m = 48$

$m = \frac{48}{24}$

$m = 2$

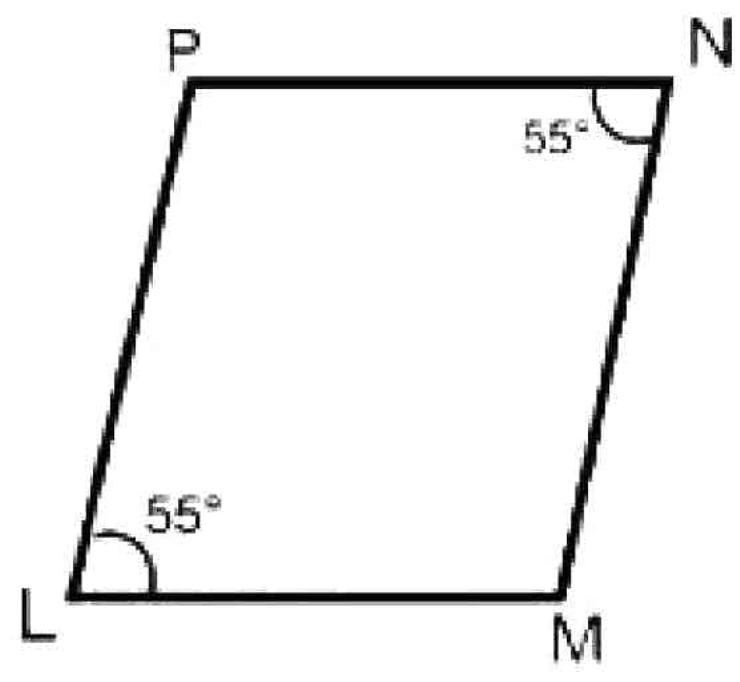
Putting the value of m in equation (i) $4(2) + n = 10$

$8 + n = 10$

$n = 10 - 8$

$n = 2$

Q.6 In the equation 5, sum of the opposite angles of the parallelogram in 110°



$\angle L + \angle M = 180$

$55° + \angle M = 180°$

$\angle M = 180° - 55°$

$\angle M = 125°$

$\angle P = \angle M$ opposite angles are congruent in parallelogram

$\angle P = 125°$

Unit 11: Parallelograms and Triangles

Overview

Parallelogram:

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.

Medians

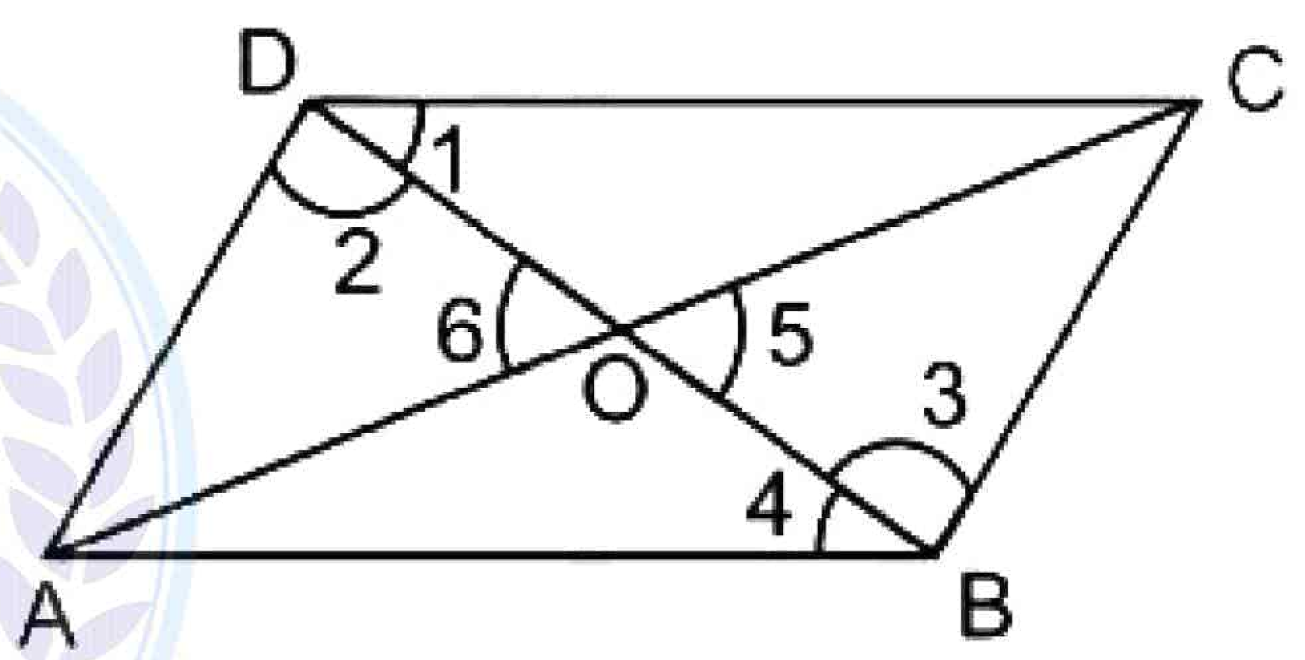
A line segment joining a vertex of a triangle to the mid-point of the opposite side is called median of the triangle.

Trisection

The process to divide a line segment into three equal parts.

Theorem 11.11

In a parallelograms
(i) Opposite sides are congruent
(ii) Opposite angles are congruent
(iii) The diagonals bisect each other



Given

In a quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}$, $\overline{BC} \parallel \overline{AD}$ and the diagonals $\overline{AC}, \overline{BD}$ meet each other at point O.

To Prove

(i) $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

(ii) $\angle ADC \cong \angle ABC, \ \angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC}, \ \overline{OB} \cong \overline{OD}$

Construction

In the figure as shown, we label the angles as $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

Proof

Statements	Reasons
(i) In $\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 4 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 2 \cong \angle 3$	Alternate angles
$\therefore \Delta ABD \cong \Delta CDB$	$A.S.A \cong A.S.A$
So, $\overline{AB}, \overline{DC}, \overline{AD} \cong \overline{BC}$	(Corresponding sides of congruent triangles)
and $\angle A \cong \angle C$	(Corresponding angles of congruent triangles)
(ii) Since	
and $\angle 1 \cong \angle 4$................(a)	Proved

$\angle 2 \cong \angle 3$................(b)	Proved
$\therefore m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$ or m$\angle$ADC = m$\angle$ABC or $\angle ADC \cong \angle ABC$	From (a) and (b)
and $\angle BAD \cong m\angle BCD$	Proved in (i)
(iii) In ΔBOC $\leftrightarrow$ ΔDOA	
$\overline{BC} \cong \overline{AD}$	Proved in (i)
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\because \Delta BOC \cong \Delta DOA$	(A.A.S $\cong$ A.A.S)
Hence $\overline{OC} \cong \overline{OA}, \overline{OB} \cong \overline{OD}$	(Corresponding sides of congruent triangles)

Example

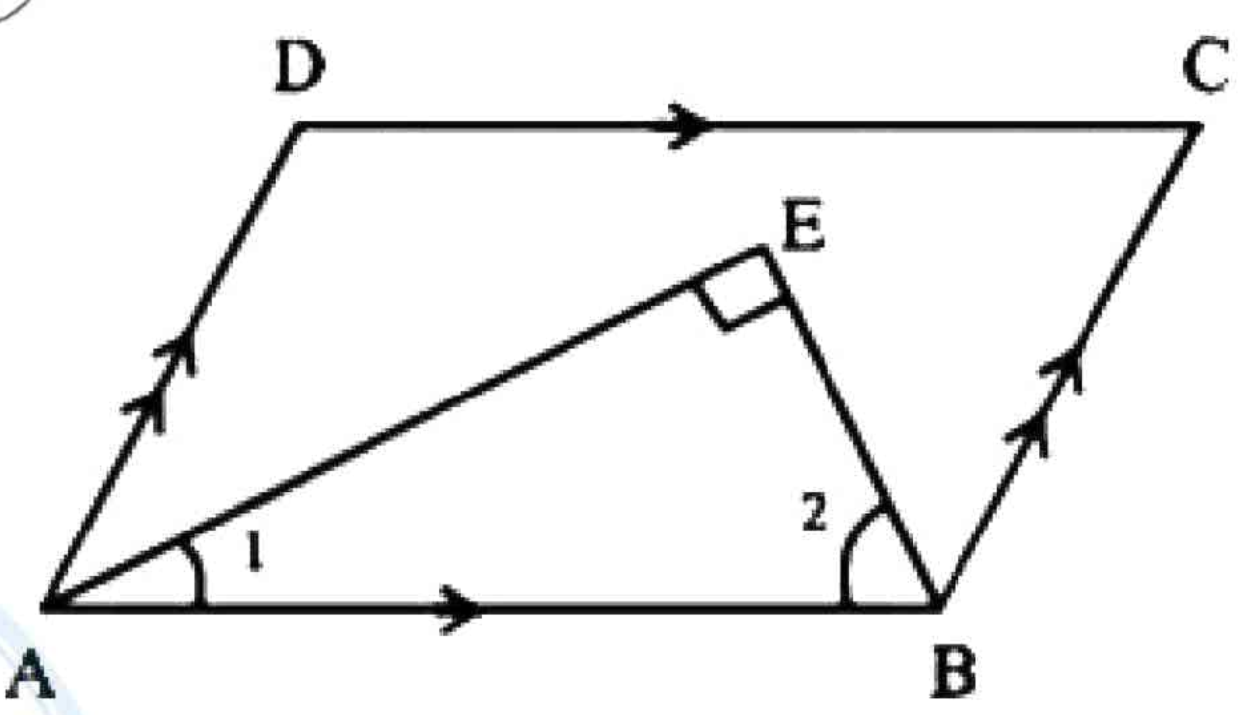
The bisectors of two angles on the same side of a parallelogram cut each other at right angles.

Given

A parallelogram ABCD, in which

$\overline{AB} \parallel \overline{DC}, \overline{AD} \parallel \overline{BC}$

The bisectors of EA and EB cut each other at E.



To Prove

m$\angle$E=90°

Construction:

Name the angles $\angle 1$ and $\angle 2$ as shown in the figure.

Proof

Statements	Reasons
$m\angle 1 + m\angle 2$ $= \frac{1}{2}(m\angle BAD + m\angle ABC)$	$m\angle 1 = \frac{1}{2} m\angle BAD$ $m\angle 2 = \frac{1}{2} m\angle ABC$
$= \frac{1}{2}(180°)$ $= 90°$	int. angles on the same side of $\overline{AB}$ which cuts $\parallel$ segments $\overline{AD}$ and $\overline{BC}$ are supplementary.
Hence in $\Delta ABE, m\angle E = 90°$	$m\angle 1 + m\angle 2 = 90°$ (proved)