

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If $f(x) = 3 - \sqrt{x}$ then $f'(1)$ is equal to :	
	(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1	
2	$4 \int_0^{\pi/4} \sin 2x dx = :$	
	(A) $4 - 2\sqrt{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\sqrt{3}$	
3	$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = :$	
	(A) $-\frac{a}{b}$ (B) $-\frac{b}{a}$ (C) $\frac{a}{b}$ (D) $\frac{b}{a}$	
4	$\int \ln x dx = :$	
	(A) $\frac{1}{x} + c$ (B) $x \ln x + c$ (C) $\frac{(\ln x)^2}{2} + c$ (D) $x(\ln x - 1) + c$	
5	Let $f(x) = \sqrt{1 - x^2}$ in R then domain of f is :	
	(A) Real numbers (B) $ x \leq 1$ (C) Negative real numbers (D) Integers	
6	If $\int x e^{x^2} dx = k e^{x^2}$ then $k = :$	
	(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{x}{3}$ (D) $\frac{x}{2}$	
7	If $f(x)$ has second derivative at c such that $f'(c) = 0$ and $f''(c) < 0$ then c is point of :	
	(A) Maxima (B) Minima (C) Point of inflection (D) Origin	
8	If $y = \cot x$, then $\frac{dy}{dx}$ is given by :	
	(A) $\operatorname{cosec}^2 x$ (B) $-\operatorname{cosec}^2 x$ (C) $\tan x$ (D) $-\operatorname{cosec} x \cot x$	
9	$\int \frac{1}{x^2 + a^2} dx = :$	
	(A) $\tan^{-1} \frac{x}{a} + c$ (B) $\frac{1}{a} \tan^{-1} \frac{x}{a} + c$ (C) $\frac{a}{x} \tan^{-1} \frac{x}{a} + c$ (D) $\frac{1}{a} \tan^{-1} \frac{a}{x} + c$	

1-10	For $y = \log_e 5x$, $\frac{dy}{dx} = :$	
	(A) $\frac{1}{x}$ (B) 5 (C) $\frac{1}{5x}$ (D) 1	
11	The straight line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$ if :	
	(A) $c = \frac{a}{m}$ (B) $c = \frac{m}{a}$ (C) $c = \frac{a^2}{m^2}$ (D) $c = am$	
12	y-coordinate of any point on x-axis is :	
	(A) 0 (B) x (C) 1 (D) y	
13	The volume of parallelepiped determined by $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{w} = \underline{i} - 7\underline{j} - 4\underline{k}$ is :	
	(A) 48 (B) 50 (C) 52 (D) 55	
14	The distance between the centres of the circles $x^2 + y^2 + 2x + 2y + 1 = 0$ and $x^2 + y^2 - 4x - 6y - 3 = 0$ is :	
	(A) 1 (B) 4 (C) 5 (D) 15	
15	If $\underline{a} + \underline{b} + \underline{c} = 0$ then which one is correct :	
	(A) $\underline{a} \times \underline{b} \times \underline{c} = 0$ (B) $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ (C) $\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{c} = \underline{c} \cdot \underline{a}$ (D) $\underline{a} = \underline{b} = \underline{c}$	
16	The x-intercept of the line $2x + 3y - 1 = 0$ is :	
	(A) 2 (B) 3 (C) $\frac{1}{3}$ (D) $\frac{1}{2}$	
17	The graph of $2x - 3y \leq 6$ is :	
	(A) On the origin side (B) Not on the origin side (C) Not decided (D) Through the origin	
18	The area of the triangle having $\underline{a}$ and $\underline{b}$ as its two sides is given by :	
	(A) $ \underline{a} \cdot \underline{b} $ (B) $\frac{1}{2} \underline{a} \cdot \underline{b} $ (C) $ \underline{a} \times \underline{b} $ (D) $\frac{1}{2} \underline{a} \times \underline{b} $	
19	Homogeneous equation of second degree $ax^2 + 2hxy + by^2 = 0$ where a, b, h are not all zero, represents two imaginary lines if :	
	(A) $h^2 = ab$ (B) $h^2 > ab$ (C) $h^2 < ab$ (D) $h = ab$	
20	The eccentricity of the ellipse $\frac{x^2}{64} + \frac{y^2}{28} = 1$ is :	
	(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\sqrt{\frac{3}{4}}$ (D) $\sqrt{\frac{4}{3}}$	

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) Prove that $\cos^2 x - \sin^2 x = 1$
- (ii) If $f(x) = \sqrt{x+4}$ then find $f(x-1)$
- (iii) Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- (iv) Evaluate $\lim_{x \rightarrow 0} \frac{1-\cos 2x}{x^2}$
- (v) Differentiate $y = (x^2 + 5)(x^3 + 7)$ with respect to x .
- (vi) Differentiate $\frac{x^2 + 1}{x^2 - 3}$ with respect to x .
- (vii) Find derivative of $(x^3 + 1)^9$ with respect to x .
- (viii) Differentiate $\cos \sqrt{x} + \sqrt{\sin x}$ with respect to the variable involved.
- (ix) $\frac{dy}{dx} = ?$ If $y = e^{x^2+1}$
- (x) Find Maclaurin Series for $\sin x$
- (xi) Determine the interval in which $f(x) = 4 - x^2$, $x \in (-2, 2)$ is increasing or decreasing.
- (xii) Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Using differential to find $\frac{dy}{dx}$ if $xy + x = 4$
- (ii) Evaluate $\int (a-2x)^{\frac{3}{2}} dx$
- (iii) Evaluate $\int \sec x dx$
- (iv) Evaluate $\int x \ln x dx$
- (v) Evaluate $\int_1^2 \frac{x}{x^2+2} dx$
- (vi) Find the area bounded by $\cos$ function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- (vii) Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$
- (viii) Find h such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.
- (ix) The coordinates of a point P are $(3, 2)$. The axes are translated through the point $O'(1, 3)$. Find the coordinates of P referred to new axes.
- (x) Find k so that the line joining $A(7, 3)$; $B(k, -6)$ and the line joining $C(-4, 5)$; $D(-6, 4)$ are parallel.
- (xi) Find the point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- (xii) Find measure of the angle between the lines represented by $9x^2 + 24xy + 16y^2 = 0$

4. Write short answers to any NINE (9) questions :

- (i) Graph the solution set of inequality $3x - 2y \geq 6$
- (ii) Define feasible region.
- (iii) Find the equation of circle whose ends of diameter are $(-3, 2)$ and $(5, -6)$
- (iv) Find the position of the point $(5, 6)$ w.r.t the circle $2x^2 + 2y^2 + 12x - 8y + 1 = 0$
- (v) Find the focus and vertex of parabola $y^2 = -8(x - 3)$
- (vi) Find the eccentricity of ellipse $x^2 + 4y^2 = 16$
- (vii) Find the centre and eccentricity of the conic $\frac{y^2}{4} - x^2 = 1$
- (viii) Identify the conic represented by $4x^2 - 4xy + y^2 - 6 = 0$
- (ix) Find the work done by a constant force $\vec{F} = 2\hat{i} + 4\hat{j}$, if its point of application to a body moves it from $A(1, 1)$ to $B(4, 6)$
- (x) Find the value of ' α ' such that $\alpha\hat{i} + \hat{j}$, $\hat{i} + \hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ are coplanar.
- (xi) If $\vec{u} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{v} = 4\hat{i} + 2\hat{j} - \hat{k}$ find $\vec{u} \times \vec{v}$
- (xii) Find a vector whose magnitude is 4 and is parallel to $2\hat{i} - 3\hat{j} + 6\hat{k}$
- (xiii) If $A(1, -1)$, $B(2, 0)$, $C(-1, 3)$ and $D(-2, 2)$ are given points, find the sum of the vectors $\vec{AB}$ and $\vec{CD}$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Find
- m
- and
- n
- , so that given function
- f
- is continuous at
- $x = 3$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

5

- (b) Prove that
- $y \frac{dy}{dx} + x = 0$
- if
- $x = \frac{1-t^2}{1+t^2}$
- ,
- $y = \frac{2t}{1+t^2}$

5

6. (a) If
- $y = e^{-ax}$
- , then show that
- $\frac{d^3y}{dx^3} + a^3y = 0$

5

- (b) Evaluate the indefinite integral
- $\int \sqrt{x^2 - a^2} dx$

5

7. (a) Solve the differential equation
- $2e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$

5

- (b) Maximize
- $f(x, y) = x + 3y$
- subject to the constraints

$$2x + 5y \leq 30; 5x + 4y \leq 20, \quad x \geq 0, y \geq 0$$

5

8. (a) Find equations of the tangents to the circle
- $x^2 + y^2 = 2$
- perpendicular to the line
- $3x + 2y = 6$

5

- (b) Using vectors, prove that
- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

5

9. (a) Find centre, foci, eccentricity, vertices and equation of directrices of
- $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

5

- (b) Find the equations of altitudes of the triangle whose vertices are

$$A(-3, 2), B(5, 4), C(3, -8)$$

5

Lahore Board-2024

Roll No _____ (To be filled in by the candidate)

(Academic Sessions 2020 – 2022 to 2022 – 2024)

MATHEMATICS

224-1st Annual-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – IL(Objective Type)

GROUP – II

Maximum Marks : 20

PAPER CODE = 8192

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If $f(x) = \frac{1}{x^2}$ when which of the following is equal to $f \circ f(x)$: (A) x^4 (B) x^2 (C) 1 (D) $\frac{1}{x^4}$
2	What is the value of $\lim_{x \rightarrow 0} (x \sin x)$: (A) α (B) -1 (C) 1 (D) 0
3	What is the value of $\sqrt{1-x^2} \frac{d}{dx} (\sin^{-1} x + \cos^{-1} x)$: (A) $\sqrt{1-x^2}$ (B) 0 (C) 2 (D) $\frac{1}{x}$
4	$\frac{d}{dx} (\sinh^{-1} x) = :$ (A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{-1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{-1}{\sqrt{1+x^2}}$
5	Derivative of x^3 w.r.t x^3 is : (A) 0 (B) 1 (C) x^3 (D) $3x^2$
6	If $f(x) = a^x$ then $f'(x) = :$ (A) $a^x \ln a$ (B) $a^x \ln x$ (C) $a^x (\ln a)^2$ (D) $(a^x)^2 \ln a$
7	$\int x^{-1} dx :$ (A) 0 (B) $-x^{-2} + c$ (C) α (D) $\ln x + c$
8	$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = :$ (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$
9	$\int \tan x dx = :$ (A) $\ln \cot x + c$ (B) $\ln \sec x + c$ (C) $\ln \sin x + c$ (D) $\ln \operatorname{cosec} x + c$

(Turn Over)

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(2)

1.10	$\int_0^{\pi} \sin x \, dx = :$ 
	(A) 0 (B) 1 (C) 2 (D) π
11	A linear equation in two variables represents :
	(A) Circle (B) Ellipse (C) Hyperbola (D) Straight line
12	Intercept form of equation of line is :
	(A) $\frac{x}{a} + \frac{y}{b} = 1$ (B) $\frac{x}{a} + \frac{y}{b} = 0$ (C) $\frac{x}{a} - \frac{y}{b} = 1$ (D) $\frac{x}{a} - \frac{y}{b} = 0$
13	Distance of point $(\cos 3x, \sin 3x)$ from origin is :
	(A) 3 (B) 6 (C) 9 (D) 1
14	$(0, 0)$ is one of the solution of inequality :
	(A) $3x + 5y > 4$ (B) $2x + 3y < 4$ (C) $x + 3y > 5$ (D) $2x + 3y > 5$
15	Equation of circle with centre $(3, 0)$ and radius $\sqrt{9}$ is :
	(A) $x^2 + y^2 - 6x = 0$ (B) $x^2 - 6x = 9$ (C) $x^2 + y^2 = 9$ (D) $9x^2 + y^2 = 9$
16	Equation of directrix of parabola $y^2 = -12x$ is :
	(A) $x = -3$ (B) $x = 3$ (C) $y = 3$ (D) $y = -3$
17	Co-vertices of ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1; a > b$ are :
	(A) $(\pm a, 0)$ (B) $(0, \pm a)$ (C) $(0, \pm b)$ (D) $(\pm b, 0)$
18	Which of the following vectors is equal to the vector $\underline{i} \cdot \underline{j} \times \underline{k}$:
	(A) 0 (B) 1 (C) -1 (D) $\underline{i}$
19	For what value of P $[2 \ P \ 5]$ is perpendicular to $[3 \ 1 \ P]$:
	(A) $\frac{2}{3}$ (B) -1 (C) 1 (D) $\sqrt{5}$
20	If $\underline{a}$ and $\underline{b}$ are parallel vectors then $\underline{a} \times \underline{b} = :$
	(A) 0 (B) 1 (C) -1 (D) 2

174-224-II-(Objective Type)- 8500 (8192)

SECTION – I**2. Write short answers to any EIGHT (8) questions :****16**

- (i) Given that $f(x) = \cos x$ find $\frac{f(a+h) - f(a)}{h}$ and simplify.
- (ii) If $f(x) = (-x+9)^3$, find $f^{-1}(x)$
- (iii) By rationalizing, find $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x}$
- (iv) Write down the domain and range of $f(x) = 2x - 5$
- (v) Calculate derivative of $f(x) = x^{2/3}$ at $x = 8$
- (vi) Find derivative of $\frac{1+x}{1-x}$ w.r.t. x
- (vii) If $y = x^4 + 2x^2 + 2$, find $\frac{dy}{dx}$
- (viii) Find $\frac{dy}{dx}$ of implicit function $x^2 - 4xy - 5y = 0$
- (ix) Apply chain rule to find $\frac{dy}{du}$ if $y = x^2 + \frac{1}{x^2}$ and $u = x - \frac{1}{x}$
- (x) Differentiate $\sin^2 x$ w.r.t $\cos^4 x$
- (xi) Find $f'(x)$ if $f(x) = x^3 e^{\frac{1}{x}}$
- (xii) Find y_2 if $y = x^2 \cdot e^{-x}$

**3. Write short answers to any EIGHT (8) questions :****16**

- (i) Using differential to find $\frac{dx}{dy}$ of $x^4 + y^2 = xy^2$
- (ii) Evaluate $\int (2x+3)^{\frac{1}{2}} dx$
- (iii) Evaluate $\int x\sqrt{x-a} dx$
- (iv) Evaluate $\int (\ln x)^2 dx$
- (v) Evaluate $\int_1^2 \left(x + \frac{1}{x}\right)^{\frac{1}{2}} \left(1 - \frac{1}{x^2}\right) dx$
- (vi) Find the area bounded by cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- (vii) Solve $\frac{dy}{dx} + \frac{2xy}{2y+1} = x$
- (viii) Find the mid-points of the line joining the two points A (- 8 , 3) , B (2 , - 1) .
- (ix) Find h such that the points A (- 1 , h) , B (3 , 2) and C (7 , 3) are collinear.
- (x) In the triangle A (8 , 6) , B (- 4 , 2) , C (- 2 , - 6) , find the slope of altitude of triangle.
- (xi) Using slopes, show that the triangle with vertices A (6 , 1) , B (2 , 7) , C (- 6 , - 7) is a right triangle.
- (xii) Find the point of intersection of the lines $x + 4y - 12 = 0$
 $x - 3y + 3 = 0$

4. Write short answers to any NINE (9) questions :

- (i) Define feasible region.
- (ii) Graph the solution set of $5x - 4y \leq 20$
- (iii) Write the standard and general equation of circle.
- (iv) Find centre and radius of $5x^2 + 5y^2 + 24x + 36y + 10 = 0$
- (v) Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$
- (vi) Find the length of the tangent drawn from the point $(-5, 4)$ to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$
- (vii) Find foci and eccentricity of ellipse $x^2 + 4y^2 = 16$
- (viii) Find the points of intersection of $x^2 + y^2 = 8$ and $x^2 - y^2 = 1$
- (ix) If $\underline{u} = 2\underline{i} - 7\underline{j}$, $\underline{v} = \underline{i} - 6\underline{j}$ and $\underline{w} = -\underline{i} + \underline{j}$, find $\frac{1}{2}\underline{u} + \frac{1}{2}\underline{v} + \frac{1}{2}\underline{w}$
- (x) Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$
- (xi) Find α so that the vector $\underline{u}$ and $\underline{v}$ are perpendicular ; $\underline{u} = \alpha\underline{i} + 2\alpha\underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 3\underline{k}$
- (xii) Find the area of parallelogram whose vertices are A $(1, 2, -1)$; B $(4, 2, -3)$; C $(6, -5, 2)$; D $(9, -5, 0)$
- (xiii) Prove that $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v}) = 3\underline{u} \cdot (\underline{v} \times \underline{w})$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Evaluate $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$ 5
- (b) Find the derivative w.r.t. x $\sin \sqrt{\frac{1+2x}{1+x}}$ 5
6. (a) If $y = (\cos^{-1} x)^2$, prove that $(1-x^2)y_2 - xy_1 - 2 = 0$ 5
- (b) Evaluate $\int \frac{2x}{1-\sin x} dx$ 5
7. (a) Find the area between the x-axis and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$ 5
- (b) Maximize $f(x, y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$; $x - y \leq 4$, $x \geq 0$, $y \geq 0$ 5
8. (a) Find equation of the circle passing through the points A $(3, -1)$, B $(0, 1)$ and having centre at $4x - 3y - 3 = 0$ 5
- (b) Use vectors to prove that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ 5
9. (a) Mid-points of sides of triangle are $(1, -1)$, $(-4, -3)$ and $(-1, 1)$. Find coordinates of vertices of triangle. 5
- (b) Show that equation of parabola with focus at $(a \cos \alpha, a \sin \alpha)$ and directrix $x \cos \alpha + y \sin \alpha + a = 0$ is $(x \sin \alpha - y \cos \alpha)^2 = 4a(x \cos \alpha + y \sin \alpha)$ 5

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The perimeter P of a square as a function of its area A is given as : (A) $4A$ (B) $4\sqrt{A}$ (C) $2A$ (D) $2\sqrt{A}$
2	Domain of cosine function $y = \cos x$ is : (A) Real numbers (B) $[-1, 1]$ (C) $(0, \infty)$ (D) $] -1, 1 [$
3	If $y = \tanh^{-1} x$, then $\frac{dy}{dx} =$: (A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $\frac{-1}{1+x^2}$ (D) $\frac{-1}{1-x^2}$
4	$\frac{d}{dx}(a^{\lambda x}) =$: (A) $a^{\lambda x}$ (B) $a^{\lambda x} \ln a$ (C) $\lambda a^{\lambda x} \ln a$ (D) $\frac{a^{\lambda x}}{\lambda \ln a}$
5	$\frac{d}{dx}(\sin \sqrt{x}) =$: (A) $\cos \sqrt{x}$ (B) $\cos \sqrt{x} \cdot \frac{1}{\sqrt{x}}$ (C) $\sqrt{x} \cos \sqrt{x}$ (D) $\cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$
6	If $y = x^2 - 1$, then $dy =$: (A) $x dx + c$ (B) $(x-1) dx$ (C) $2x dx + c$ (D) $2x dx$
7	$\int_0^3 \frac{dx}{x^2+9} =$: (A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) 0 (D) $\frac{\pi}{12}$
8	$\int e^x (\sin x + \cos x) dx =$: (A) $e^x \cos x + c$ (B) $e^x \sin x$ (C) $e^x \sin x + c$ (D) $e^x \cos x$
9	$\int \frac{2}{x+2} dx =$: (A) $\ln x+2 + c$ (B) $\ln x+2 ^2 + c$ (C) $\frac{1}{\ln x+2 } + c$ (D) $2 \ln x + c$
10	$\int \frac{1}{\cos^2 x} dx =$: (A) $\frac{1}{\sin^2 x} + c$ (B) $\tan x + c$ (C) $\sec^2 x + c$ (D) $\operatorname{cosec}^2 x + c$

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(2)

11	The lines represented by $ax^2 + 2hxy + by^2 = 0$ are imaginary if : (A) $h^2 - ab = 0$ (B) $h^2 - ab < 0$ (C) $h^2 - ab > 0$ (D) $h^2 - ab \neq 0$
12	Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if : (A) $a_1a_2 + b_1b_2 = 0$ (B) $a_1a_2 - b_1b_2 = 0$ (C) $a_1b_2 - a_2b_1 = 0$ (D) $a_1b_2 + a_2b_1 = 0$
13	Inclination of the line joining the points (4 , 6) and (4 , 8) is : (A) 90° (B) 45° (C) 30° (D) Undefined
14	A region is said to feasible region which is restricted to : (A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant
15	An angle in a semicircle is of measure : (A) 90° (B) 60° (C) 45° (D) 30°
16	The coordinate of the vertices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is : (A) $(0, \pm b)$ (B) $(\pm b, 0)$ (C) $(0, \pm a)$ (D) $(\pm a, 0)$
17	Focus of the parabola $x^2 - 5y = 0$ is : (A) $(\frac{5}{4}, 0)$ (B) $(0, \frac{5}{4})$ (C) $(0, -\frac{5}{4})$ (D) $(-\frac{5}{4}, 0)$
18	For parabola, value of eccentricity e is : (A) $e = 0$ (B) $e < 1$ (C) $e > 1$ (D) $e = 1$
19	If $\underline{u}, \underline{v}$ and $\underline{w}$ are coterminal edges of a tetrahedron, then its volume is : (A) $[\underline{u} \ \underline{v} \ \underline{w}]$ (B) $\frac{1}{3} [\underline{u} \ \underline{v} \ \underline{w}]$ (C) $\frac{1}{6} [\underline{u} \ \underline{v} \ \underline{w}]$ (D) $\frac{1}{9} [\underline{u} \ \underline{v} \ \underline{w}]$
20	A vector perpendicular to both vectors $\underline{a}$ and $\underline{b}$ is : (A) $\underline{a} \cdot \underline{b}$ (B) $\underline{a} \times \underline{b}$ (C) $\frac{\underline{a} \cdot \underline{b}}{ \underline{a} }$ (D) $\underline{b} \cdot \underline{a}$

173-223-I-(Objective Type)- 7000 (8191)

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) For $f(x) = \frac{2x+1}{x-1}$, find $f^{-1}(x)$
- (ii) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$
- (iii) Discuss the continuity of $f(x)$ at $x = c = 2$, $f(x) = \begin{cases} 2x+5 & \text{if } x \leq 2 \\ 4x+1 & \text{if } x > 2 \end{cases}$
- (iv) Differentiate w.r.t 'x' $(x-5)(3-x)$
- (v) Find $\frac{dy}{dx}$ if $y^2 - xy - x^2 + 4 = 0$
- (vi) Differentiate w.r.t. 'θ' $(\sin 2\theta - \cos 3\theta)^2$
- (vii) Find $\frac{dy}{dx}$ if $y = x^2 \ln \frac{1}{x}$
- (viii) Find y_4 if $y = (2x+5)^{3/2}$
- (ix) Apply Maclaurin series expansion to prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- (x) Find extreme values for $f(x) = x^2 - x - 2$
- (xi) Define feasible region.
- (xii) Graph the inequality $x + 2y \leq 6$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find δy and dy in the case $y = x^2 + 2x$ when x changes from 2 to 1.8
- (ii) Evaluate $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx$, $x > 0$
- (iii) Evaluate $\int a^{x^2} x dx$ ($a > 0, a \neq 1$)
- (iv) Evaluate $\int \sqrt{4-5x^2} dx$
- (v) Evaluate $\int_0^{\frac{\pi}{6}} x \cos x dx$
- (vi) Find area below the curve $y = 3\sqrt{x}$ and above the x-axis between $x=1$ and $x=4$
- (vii) Solve the differential equation $x^2(2y+1)\frac{dy}{dx} - 1 = 0$
- (viii) Find the position vector of the point of division of the line segments joining the following pair of points, in the given ratio, point C with position vector $2\mathbf{i} - 3\mathbf{j}$ and point D with position vector $3\mathbf{i} + 2\mathbf{j}$ in the ratio 4 : 3
- (ix) If $\underline{u} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\underline{v} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\underline{w} = \mathbf{i} + 6\mathbf{j} + z\mathbf{k}$ represent the sides of a triangle, find the value of z .

(Turn Over)

3. (x) Find the angle between the vectors $\underline{u} = 2\hat{i} - \hat{j} + \hat{k}$ and $\underline{v} = -\hat{i} + \hat{j}$
 (xi) If $\underline{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\underline{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, find a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$.
 Also find the sine of angle between the vectors $\underline{a}$ and $\underline{b}$.
 (xii) Find the area of the triangle with vertices A (1, -1, 1), B (2, 1, -1) and C (-1, 1, 2)

4. **Write short answers to any NINE (9) questions :**

18

- (i) Show that the points A (0, 2), B ($\sqrt{3}$, -1) and C (0, -2) are vertices of a right triangle.
 (ii) Find k so that the line joining A (7, 3), B (k, -6) and line joining C (-4, 5), D (-6, 4) are parallel.
 (iii) Find an equation of line if its slope is 2 and y-intercept is 5.
 (iv) Transform the equation $5x - 12y + 39 = 0$ into two-intercept form.
 (v) Find the distance from the points P (6, -1) to the line $6x - 4y + 9 = 0$
 (vi) Find the point of intersection of lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$
 (vii) Find the angle between the lines represented by $x^2 - xy - 6y^2 = 0$
 (viii) Find an equation of circle with centre at ($\sqrt{2}$, $-3\sqrt{3}$) and radius $2\sqrt{2}$
 (ix) Find centre and radius of circle $x^2 + y^2 + 12x - 10y = 0$
 (x) Find vertex and directrix of parabola $x^2 = 16y$
 (xi) Find the focus and vertex of parabola $x^2 = 4(y - 1)$
 (xii) Find centre and foci of $4x^2 + 9y^2 = 36$
 (xiii) Find eccentricity and vertices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$ 5
 (b) If $\frac{y}{x} = \tan^{-1} \frac{x}{y}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$ 5
 6. (a) Evaluate $\int \frac{x}{x^4 + 2x^2 + 5} dx$ 5
 (b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5
 7. (a) Evaluate $\int_0^{\frac{\pi}{4}} (1 + \cos^2 \theta) \tan^2 \theta d\theta$ 5
 (b) Minimize $z = 2x + y$ subject to the constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$ 5
 8. (a) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ 5
 (b) Find equations of the tangents to the circle $x^2 + y^2 = 2$ and parallel to the line $x - 2y + 1 = 0$ 5
 9. (a) Find volume of the tetrahedron with the vertices (0, 1, 2), (3, 2, 1), (1, 2, 1) and (5, 5, 6) 5
 (b) Find the centre, foci, eccentricity and directrices of ellipse $\frac{(2x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$ 5

Roll No _____ **Lahore Board-2023** (To be filled in by the candidate)

(Academic Sessions 2019 – 2021 to 2021 – 2023)

MATHEMATICS

223-1st Annual-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – II

Maximum Marks : 20

PAPER CODE = 8194

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The mid point of the line segment joining the foci of an ellipse is called : (A) Vertex (B) Directrix (C) Centre (D) Minor axis
2	If (3 , 5) is mid point of (5 , a) and (b , 7) then : (A) $a = 4, b = 2$ (B) $a = 3, b = 3$ (C) $a = 7, b = -2$ (D) $a = 3, b = 1$
3	If 2 and 2 are x and y components of a vector, then its angle with x-axis is : (A) 30° (B) 60° (C) 45° (D) 90°
4	(3 , 2) is not a solution of the inequality : (A) $x - y > 1$ (B) $x + y > 2$ (C) $3x + 5y > 8$ (D) $3x - 7y < 3$
5	$\underline{i} \times \underline{j} = :$ (A) $\underline{k}$ (B) $\underline{i}$ (C) $-\underline{k}$ (D) $\underline{j}$
6	Slope of line $3x - 2y + 5 = 0$ is : (A) $-\frac{2}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$
7	Length of the diameter of the circle $(x+5)^2 + (y-8)^2 = 12$: (A) $2\sqrt{3}$ (B) 12 (C) 24 (D) $4\sqrt{3}$
8	Transverse axis of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is : (A) $x = \frac{a}{e}$ (B) $y = 0$ (C) $x = 0$ (D) $y = \frac{a}{e}$
9	Equation of line in slope intercept form is : (A) $y = mx + c$ (B) $\frac{x}{a} + \frac{y}{b} = 1$ (C) $y - y_1 = m(x - x_1)$ (D) $x \cos \alpha + y \sin \alpha = p$
10	The condition for a line $y = mx + c$ to be tangent to the circle $x^2 + y^2 = a^2$ is that : (A) $c = \pm m\sqrt{1+a^2}$ (B) $c = \pm a\sqrt{1+m^2}$ (C) $c = \pm a\sqrt{1-m^2}$ (D) $c = \pm \sqrt{1-m^2}$

(Turn Over)

Lahore Board-2023

(2)

11	$f(x) = f(o) + xf'(o) + \frac{x^2}{2!} f''(o) + \frac{x^3}{3!} f'''(o) + \dots$ is called : (A) Taylor's series (B) Binomial series (C) Maclaurin's series (D) Laurent series
12	$\lim_{x \rightarrow 0} \frac{\sin 7x}{x} = :$ (A) 7 (B) -7 (C) $-\frac{1}{7}$ (D) $\frac{1}{7}$
13	$\int \frac{e^x}{e^x + 1} dx = :$ (A) $\ln(e^x + 1) + c$ (B) $\ln e^x + c$ (C) $e^{-x} + c$ (D) $e^x + c$
14	$\frac{d}{dx} (x^2 + 1)^2 = :$ (A) $2x(x^2 + 1)$ (B) $\frac{(x^2 + 1)^3}{3}$ (C) $2(x^2 + 1)$ (D) $4x(x^2 + 1)$
15	$\int \sin^2 x dx = :$ (A) $\frac{x}{2} - \frac{\sin 2x}{4} + c$ (B) $\frac{x}{2} + \frac{\sin 2x}{4} + c$ (C) $\frac{x}{2} - \frac{\sin 2x}{2} + c$ (D) $\frac{x}{2} + \frac{\sin 2x}{2} + c$
16	$\frac{d}{dx} (\tan x^2) = :$ (A) $\sec^2 x^2$ (B) $2x \sec^2 x^2$ (C) $-\sec^2 x^2$ (D) $-2x \sec x^2$
17	$\int e^x (\cos x - \sin x) dx = :$ (A) $e^x \sin x + c$ (B) $e^x \cos x + c$ (C) $e^x \tan x + c$ (D) $e^x \cot x + c$
18	If $f(x) = \sin x + \cos x$ then $f(x)$ is : (A) Even function (B) Odd function (C) Neither even nor odd (D) Constant function
19	$\int_0^3 \frac{1}{9 + x^2} dx = :$ (A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{12}$ (C) $\frac{-\pi}{4}$ (D) $\frac{\pi}{12}$
20	$\frac{d}{dx} (f(x) \sin x) = :$ (A) $f(x) \cos x + f'(x) \sin x$ (B) $f'(x) \sin x - f(x) \cos x$ (C) $f'(x) \cos x$ (D) $f'(x) \cos x + f(x) \sin x$

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) Find the domain and range of $f(x) = \sqrt{x^2 - 4}$
- (ii) Show that $x = a \sec \theta$, $y = b \tan \theta$ represents the equation of hyperbola.
- (iii) If $f(x) = -2x + 8$, find $f^{-1}(x)$ and $f^{-1}(-1)$
- (iv) Differentiate $(3 - x)(x - 5)$ w.r.t 'x'
- (v) Find derivative of $\sqrt{\frac{1+x}{1-x}}$
- (vi) If $y = x^4 + 2x^2 + 2$, prove $\frac{dy}{dx} = 4x\sqrt{y-1}$
- (vii) Find the derivative of $(x^3 + 1)^9$ w.r.t. 'x'
- (viii) Find $\frac{dy}{dx}$ if $y^3 - 2xy^2 + x^2y + 3x = 0$
- (ix) Differentiate w.r.t. variable involved of $\tan^3 \theta \sec^2 \theta$
- (x) Find $\frac{dy}{dx}$ if $y = a^x$
- (xi) Define feasible region.
- (xii) Graph the feasible region $2x - 3y \leq 6$, $x \geq 0$, $y \geq 0$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Using differentials to find $\frac{dy}{dx}$ if $xy - \ln x = c$
- (ii) Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$
- (iii) Evaluate $\int \frac{e^x}{e^x + 3} dx$
- (iv) Evaluate $\int \ln x dx$
- (v) Evaluate $\int_{-6}^2 \sqrt{3-x} dx$
- (vi) Find the area bounded by cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- (vii) Solve the differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$
- (viii) Find the magnitude of the vector $\underline{u} = \hat{i} + \hat{j}$
- (ix) Find direction cosines of $\vec{v} = 3\hat{i} - \hat{j} + 2\hat{k}$
- (x) Calculate the projection of $\vec{b}$ along $\vec{a}$ if $\vec{a} = \hat{i} - \hat{k}$; $\vec{b} = \hat{j} + \hat{k}$
- (xi) If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, find $\vec{b} \times \vec{a}$
- (xii) Prove that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

4. Write short answers to any NINE (9) questions :

- (i) Find the equation of the straight line whose slope is 2 and y-intercept is 5.
- (ii) Using slopes, show that the triangle with its vertices A (6, 1), B (2, 7) and C (- 6, - 7) is a right triangle.
- (iii) Find an equation of the line through (- 4, 7) and parallel to the line $2x - 7y + 4 = 0$
- (iv) Find h such that A (- 1, h), B (3, 2) and C (7, 3) are collinear.
- (v) Write intercepts form of equation of straight line.
- (vi) Check whether the following lines are concurrent or not
 $3x - 4y - 3 = 0$
 $5x + 12y + 1 = 0$
 $32x + 4y - 17 = 0$
- (vii) Find the slope and inclination of the line joining points (- 2, 4) and (5, 11)
- (viii) Find an equation of circle with centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
- (ix) Define focus and directrix of the parabola.
- (x) Find the centre and foci of the ellipse $x^2 + 4y^2 = 16$
- (xi) Find equation of tangent to $y^2 = 4ax$ at (x_1, y_1)
- (xii) Show that the equation $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ represents a circle. Find its centre.
- (xiii) Find an equation of the ellipse with given data : Foci (0, - 1) and (0, - 5) and major axis of length 6.

SECTION - II

Note : Attempt any THREE questions.

5. (a) If θ is measured in radians then prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ 5
- (b) Find $\frac{dy}{dx}$ if $y = (1 + 2\sqrt{x})^3 \cdot x^{\frac{3}{2}}$ 5
6. (a) Evaluate $\int \ln(x + \sqrt{x^2 + 1}) dx$ 5
- (b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5
7. (a) Solve the differential equation $2e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ 5
- (b) Maximize $f(x, y) = x + 3y$ subject to constraints
 $2x + 5y \leq 30$, $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$ 5
8. (a) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ 5
- (b) Find equations of tangents to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$ 5
9. (a) Show that the equation $9x^2 - 18x + 4y^2 + 8y - 23 = 0$ represents an ellipse. Find its elements and sketch its graph. 5
- (b) Prove that in any triangle ABC $c = a \cos B + b \cos A$ 5

Roll No **Lahore Board-2022** (To be filled in by the candidate)

(Academic Sessions 2018 – 2020 to 2020 – 2022)

MATHEMATICS

222-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – I

Maximum Marks : 20

PAPER CODE = 8193

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	(0 , 0) is the solution of inequality : (A) $3x - 7y < 3$ (B) $x + y > 2$ (C) $x - y > 1$ (D) $3x + 5y > 7$
2	The slope of a line with inclination 90° is : (A) 0 (B) -1 (C) Undefined (D) 1
3	If $\underline{a}$ and $\underline{b}$ are parallel vectors then $\underline{a} \times \underline{b} =$: (A) $\underline{0}$ (B) -1 (C) 1 (D) 2
4	Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if : (A) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2} = -\frac{b_1}{b_2}$ (C) $\frac{b_1}{c_1} = \frac{b_2}{c_2}$ (D) $\frac{a_1}{c_1} = \frac{a_2}{c_2}$
5	The value of $3j \cdot k \times i$ is : (A) -1 (B) -3 (C) 3 (D) 0
6	If a straight line is parallel to x-axis, then its slope is : (A) Undefined (B) -1 (C) 1 (D) 0
7	The centre of the circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ is : (A) $\left(\frac{12}{5}, \frac{18}{5}\right)$ (B) $\left(-\frac{12}{5}, -\frac{18}{5}\right)$ (C) $\left(\frac{12}{5}, -\frac{18}{5}\right)$ (D) $\left(-\frac{12}{5}, \frac{18}{5}\right)$
8	The length of the latus rectum of the parabola $y^2 = 8x$ is : (A) 2 (B) 8 (C) 4 (D) $2\sqrt{8}$
9	The point of intersection of angle bisectors of a triangle is called : (A) Orthocentre (B) Centroid (C) In-centre (D) Circumcentre

(Turn Over)



(2) Lahore Board-2022

10	The coordinates of the vertices of hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$ are : (A) $(0, \pm 7)$ (B) $(\pm 4, 0)$ (C) $(0, \pm 4)$ (D) $(\pm 7, 0)$
11	$\frac{d}{dx}(\sin 2x + \cos 2x) = :$ (A) $(\cos 2x - \sin 2x)$ (B) $(\cos 2x + \sin 2x)$ (C) $(2 \cos 2x + 2 \sin 2x)$ (D) $2(\cos 2x - \sin 2x)$
12	$\lim_{h \rightarrow 0} (1+2h)^{\frac{1}{h}} = :$ (A) e^2 (B) e (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$
13	$\int e^{\sin x} \cos x dx = :$ (A) $e^{\cos x} + c$ (B) $\ln \sin x + c$ (C) $\ln \cos x + c$ (D) $e^{\sin x} + c$
14	$\frac{d}{dx}(\cot^{-1} x) = :$ (A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $-\frac{1}{1+x^2}$ (D) $-\frac{1}{1-x^2}$
15	$\int e^x(\cos x + \sin x) dx = :$ (A) $e^{-x} \sin x + c$ (B) $e^x \sin x + c$ (C) $-e^x \sin x + c$ (D) $e^{-x} \cos x + c$
16	If $y = e^{-ax}$ then $\frac{dy}{dx} = :$ (A) ae^{-ax} (B) e^{-ax} (C) $a^2 e^{-ax}$ (D) $-ae^{-ax}$
17	$\int \frac{1}{1+x^2} dx = :$ (A) $\tan^{-1} x + c$ (B) $-\tan^{-1} x + c$ (C) $\sin^{-1} x + c$ (D) $\cos^{-1} x + c$
18	The range of $f(x) = \sqrt{x^2 - 9}$ is : (A) $(-\infty, 0]$ (B) $[0, +\infty)$ (C) $(0, +\infty)$ (D) $(-\infty, \infty)$
19	$\int \sin x \cos x dx = :$ (A) $\ln \sin x + c$ (B) $\frac{\cos^2 x}{2} + c$ (C) $\frac{\sin^2 x}{2} + c$ (D) $\frac{\sin^2 x \cos^2 x}{2} + c$
20	$\frac{d}{dx}\left(\frac{1}{\operatorname{cosec} x}\right) = :$  (A) $\frac{1}{\sec x}$ (B) $\operatorname{cosec}^2 x$ (C) $\cot x$ (D) $\frac{1}{\operatorname{cosec}^2 x}$

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) Find domain and range of $f(x) = \sqrt{x+1}$
- (ii) Find $f \circ f(x)$ if $f(x) = \sqrt{x+1}$
- (iii) Obtain $f^{-1}(x)$ from $f(x) = 3x^3 + 7$
- (iv) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$
- (v) Express $\lim_{x \rightarrow +\infty} \left(\frac{x}{1+x} \right)^x$ in terms of "e"
- (vi) If $y = \frac{x^2+1}{x^2-3}$, then find $\frac{dy}{dx}$
- (vii) Prove that derivative of $\tan^{-1} x$ w.r.t. "x" is $\frac{1}{1+x^2}$
- (viii) Differentiate $\frac{1}{a} \sin^{-1} \left(\frac{a}{x} \right)$ w.r.t. "x"
- (ix) Find $\frac{dy}{dx}$ if $y = x^2 \ln \sqrt{x}$
- (x) If $y = e^{-x} (x^3 + 2x^2 + 1)$, then find $\frac{dy}{dx}$
- (xi) Apply the Maclaurin's series expansion to prove that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
- (xii) Determine the interval in which $f(x) = \sin x$, $x \in (-\pi, \pi)$ is decreasing.

3. Write short answers to any EIGHT (8) questions :

16

- (i) If $x^2 + 2y^2 = 16$, find $\frac{dy}{dx}$ by using differentials.
- (ii) Evaluate $\int \frac{x}{x+2} dx$
- (iii) Evaluate indefinite integral $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$
- (iv) Evaluate $\int \ln x dx$
- (v) Evaluate the definite integral $\int_{-1}^1 (x^{1/3} + 1) dx$
- (vi) Find the area between the x-axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$
- (vii) Evaluate $\int e^{-x} (\cos x - \sin x) dx$
- (viii) Solve $x dy + y(x-1) dx = 0$
- (ix) Show that the points A (3, 1), B (-2, -3) and C (2, 2) are vertices of an isosceles triangle

3. (x) Find an equation of line having x-intercept : -9 and slope : -4
 (xi) Show that the lines $4x - 3y - 8 = 0$, $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent.
 (xii) What is homogeneous equation?

4. Write short answers to any NINE (9) questions :

18

- (i) Graph the solution set of $2x + 1 \geq 0$
 (ii) Define problem constraint.
 (iii) Find an equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
 (iv) Find slope of tangent to $x^2 + y^2 = 5$ at $(4, 3)$
 (v) Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$
 (vi) Find focus and vertex of $y^2 = 8x$
 (vii) Find equation of ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.
 (viii) Find equation of hyperbola with centre $(0, 0)$, focus $(6, 0)$, vertex $(4, 0)$
 (ix) Find a vector from the point A to the origin where $\vec{AB} = 4\hat{i} - 2\hat{j}$ and B $(-2, 5)$
 (x) Find α so that $|\alpha\hat{i} + (\alpha+1)\hat{j} + 2\hat{k}| = 3$
 (xi) Find the cosine of the angle θ between $\underline{u}$ and $\underline{v}$; $\underline{u} = \hat{i} - 3\hat{j} + 4\hat{k}$; $\underline{v} = 4\hat{i} - \hat{j} + 3\hat{k}$
 (xii) Prove that $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$
 (xiii) A force $\vec{F} = 7\hat{i} + 4\hat{j} - 3\hat{k}$ is applied at P $(1, -2, 3)$. Find its moment about the point Q $(2, 1, 1)$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Discuss the continuity of $f(x)$ at $x = 1$ $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 4x & \text{if } x > 1 \end{cases}$ 5
 (b) Show that $2^{x+h} = 2^x \left\{ 1 + (\ln 2)h + \frac{(\ln 2)^2 h^2}{2!} + \frac{(\ln 2)^3 h^3}{3!} + \dots \right\}$ 5
 6. (a) Evaluate $\int \sqrt{4-5x^2} dx$ 5
 (b) Find the equation of perpendicular bisector of segment joining the points A $(3, 5)$ and B $(9, 8)$ 5
 7. (a) Evaluate the integral $\int_0^{\pi/4} \frac{\cos \theta + \sin \theta}{2 \cos^2 \theta} d\theta$ 5
 (b) Maximize $f(x, y) = x + 3y$ subject to the constraints $2x + 5y \leq 30$; $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$ 5
 8. (a) Find the interior angles whose vertices are A $(-2, 11)$, B $(-6, -3)$, C $(4, -9)$ 5
 (b) Find an equation of the circle passing through the points A $(4, 5)$, B $(-4, -3)$, C $(8, -3)$ 5
 9. (a) Prove angle in a semi circle is right angle. 5
 (b) Find an equation of the tangent to the parabola $y^2 = -6x$ which is parallel to the line $2x + y + 1 = 0$. Also find point of tangency. 5

Roll No **Lahore Board-2022** (To be filled in by the candidate)

(Academic Sessions 2018 – 2020 to 2020 – 2022)

MATHEMATICS

222-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – II

Maximum Marks : 20

PAPER CODE = 8192

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If the degree of a polynomial function is 1, then it is called : (A) Identity function (B) Linear function (C) Constant function (D) Trigonometric function
2	$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - x} = :$ (A) 2 (B) $\frac{1}{2}$ (C) 4 (D) 5
3	If $y = \frac{1}{x^2}$, then $\frac{dy}{dx}$ at $x = -1$ is : (A) 2 (B) 3 (C) $\frac{1}{3}$ (D) 4
4	$\frac{d}{dx}(\cot^{-1} x) = :$ (A) $\frac{1}{1+x^2}$ (B) $\frac{-1}{1+x^2}$ (C) $-\operatorname{cosec}^2 x$ (D) $\sec^2 x$
5	Two positive integer whose sum is 30 and their product will be maximum are : (A) 14, 16 (B) 15, 15 (C) 10, 20 (D) 12, 18
6	$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = :$ (A) $\frac{f(x)g'(x) - f'(x)g(x)}{[g(x)]^2}$ (B) $\frac{f'(x)g(x) - f(x)g'(x)}{[f(x)]^2}$ (C) $\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (D) $\frac{g'(x)f(x) - g(x)f'(x)}{[g(x)]^2}$
7	$\int \sec x \, dx = :$ (A) $\ln(\sec x + \tan x) + c$ (B) $\ln(\operatorname{cosec} x + \cot x) + c$ (C) $\ln(\sin x + \cos x) + c$ (D) $\sec x + \tan x + c$
8	The solution of differential equation $\frac{dy}{dx} = -y$ is : (A) $y = x e^{-x}$ (B) $y = c e^{-x}$ (C) $y = e^x$ (D) $y = c e^x$
9	$\int_{-1}^3 x^3 \, dx = :$ (A) 20 (B) 40 (C) 30 (D) 60

(Turn Over)

Lahore Board-2022

(2)

10	$\int \sin 3x \, dx = :$  (A) $-\frac{\cos 3x}{3} + c$ (B) $\frac{\cos 3x}{3} + c$ (C) $3\cos 3x + c$ (D) $-3\cos 3x + c$
11	An equation of the horizontal line through the point P (7 , - 9) is : (A) $y = -9$ (B) $y = 9$ (C) $x = 7$ (D) $x = -9$
12	The perpendicular distance of line $3x + 4y + 10 = 0$ from the origin is : (A) 0 (B) 1 (C) 2 (D) 3
13	Slope of line perpendicular to line $3x - 4y + 5 = 0$ is : (A) $-\frac{3}{4}$ (B) $-\frac{4}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$
14	Point of intersection of lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$ equals : (A) (1 , 0) (B) (0 , 1) (C) (-1 , 0) (D) (0 , -1)
15	(0 , 0) is the solution of inequality : (A) $7x + 2y > 3$ (B) $x - 3y > 0$ (C) $x + 2y < 6$ (D) $x - 3y < 0$
16	The condition for a line $y = mx + c$ to be the tangent to the circle $x^2 + y^2 = a^2$ is : (A) $c = \pm m\sqrt{1+a^2}$ (B) $c = \pm a\sqrt{1+m^2}$ (C) $c = \pm a\sqrt{1-m^2}$ (D) $c = \pm m\sqrt{1-a^2}$
17	In an ellipse, the foci lie on : (A) Major axis (B) Minor axis (C) Directrix (D) Z-axis
18	The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : (A) $\sqrt{g^2 + f^2 + c}$ (B) $\sqrt{g^2 - f^2 + c}$ (C) $g + f - c$ (D) $\sqrt{g^2 + f^2 - c}$
19	Length of the vector $2\hat{i} - \hat{j} + 2\hat{k}$ is : (A) 6 (B) 4 (C) 3 (D) 5
20	Cosine of the angle between two non-zero vectors $\underline{a}$ and $\underline{b}$ is : (A) $\underline{a} \cdot \underline{b}$ (B) $\frac{ \underline{a} \underline{b} }{\underline{a} \cdot \underline{b}}$ (C) $\frac{\underline{a} \cdot \underline{b}}{ \underline{a} \underline{b} }$ (D) $\frac{\underline{a} \times \underline{b}}{ \underline{a} \underline{b} }$

174-222-II-(Objective Type)- 8750 (8192)

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) Express perimeter “P” of a square as a function of its area “A”
- (ii) Find $f^{-1}(x)$ for $f(x) = -2x + 8$
- (iii) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$
- (iv) Define rational function with example.
- (v) Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x$
- (vi) Find $\frac{dy}{dx}$ from first principle if $y = \sqrt{x+2}$
- (vii) Differentiate w.r.t. “x”; $y = \frac{x^2+1}{x^2-3}$
- (viii) Find $\frac{dy}{dx}$ if $xy + y^2 = 2$
- (ix) Find derivative w.r.t. x if $y = \cot^{-1}\left(\frac{x}{a}\right)$
- (x) Find $\frac{dy}{dx}$ if $y = \log_{10}(ax^2 + bx + c)$
- (xi) Apply the Maclaurin Series to prove that $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$
- (xii) Define increasing function with example.

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find δy and dy in $y = \sqrt{x}$, when x changes from 4 to 4.41
- (ii) Evaluate the integral $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta, \theta > 0$
- (iii) Find $\int \frac{1}{x(\ln x)} dx$
- (iv) Evaluate the integral $\int \frac{x+2}{\sqrt{x+3}} dx$
- (v) Using by part method to evaluate $\int x^2 \ln x dx$
- (vi) Evaluate the definite integral $\int_0^{\frac{\pi}{3}} \cos^2 \theta \sin \theta d\theta$
- (vii) Find the area between the x-axis and the curve $y = \cos \frac{1}{2}x$ from $x = -\pi$ to π
- (viii) Solve the differential equation $\sin y \operatorname{cosec} x \frac{dy}{dx} = 1$
- (ix) Find h such that $A(-1, h), B(3, 2), C(7, 3)$ are collinear.

3. (x) Two points $P(-5, -3)$ and $O'(-2, -6)$ are given in XY-coordinate, find the coordinate of P in xy-coordinate system.

(xi) Find equation of the line having x-intercept -3 and y-intercept 4 .

(xii) Find the distance from the point $P(6, -1)$ to the line $6x - 4y + 9 = 0$



4. Write short answers to any NINE (9) questions :

18

- Define problem constraint.
- Graph the solution set of the linear inequality $3y - 4 \leq 0$
- Find slope of tangent to $x^2 + y^2 = 5$ at $(4, 3)$
- Find α if $\underline{u} = \alpha \underline{i} + 2\alpha \underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha \underline{j} + 3\underline{k}$ are perpendicular to each other.
- Find the direction cosine of the vector $\overline{PQ}$, where $P(2, 1, 5)$ and $Q(1, 3, 1)$
- Find the vector from point A to origin where $\overline{AB} = 4\underline{i} - 2\underline{j}$ and B is the point $(-2, 5)$
- Find cosine of the angle between $\underline{u} = [-3, 5]$ and $\underline{v} = [6, -2]$
- Write standard equation of the hyperbola.
- Find the centre of the ellipse $9x^2 + y^2 = 18$
- Find the equation of the circle with centre $(5, -2)$ and radius is 4 .
- Find the equation of the hyperbola with foci $(\pm 5, 0)$ and vertex $(3, 0)$
- Find centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$
- Find focus and vertex of the parabola $x^2 = 5y$

SECTION - II

Note : Attempt any THREE questions.

- (a) Prove that $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$ 5

(b) If $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ prove that $y \frac{dy}{dx} + x = 0$ 5
- (a) Evaluate $\int \ln(x + \sqrt{x^2 + 1}) dx$ 5

(b) Prove that the linear equation $ax + by + c = 0$ in two variables x and y represents a straight line. 5
- (a) Find the area between the x-axis and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$ 5

(b) Graph the solution region of the system of linear inequalities and find the corner points of $2x - 3y \leq 6$, $2x + 3y \leq 12$, $x \geq 0$ 5
- (a) Find a joint equation of the lines through the origin and perpendicular to the lines represented by $x^2 - 2xy \tan \alpha - y^2 = 0$ 5

(b) Find equations of the tangent lines to the circle $x^2 + y^2 + 4x + 2y = 0$ drawn from $P(-1, 2)$ 5
- (a) Find the centre, foci, eccentricity, vertices and equations of directrices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$ 5

(b) Prove that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ 5

Roll No _____ **Lahore Board-2021** (To be filled in by the candidate)

(Academic Sessions 2017 – 2019 to 2019 – 2021)

MATHEMATICS

221-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – I

Maximum Marks : 20

PAPER CODE = 8195

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	$\int (2x+3)^{\frac{1}{2}} dx = :$  (A) $\frac{(2x+3)^{\frac{3}{2}}}{2} + c$ (B) $\frac{1}{3}(2x+3)^{\frac{3}{2}} + c$ (C) $\frac{1}{2}(2x+3)^{\frac{1}{3}} + c$ (D) $\frac{1}{3}(2x+3)^{\frac{-1}{2}} + c$
2	Distance between A (3 , 1) and B (- 2 , - 4) is : (A) $\sqrt{17}$ (B) $5\sqrt{2}$ (C) $\sqrt{26}$ (D) $2\sqrt{5}$
3	If $f(x) = \frac{x}{x^2 - 4}$ then range of $f(x)$ is : (A) All real number (B) Rational number (C) All negative real number (D) Integer
4	Slope 'm' through A(x_1, y_1) B(x_2, y_2) is : (A) $\frac{x_2 - x_1}{y_2 - y_1}$ (B) $\frac{x_2 + x_1}{y_2 - y_1}$ (C) $\frac{y_2 - y_1}{x_2 - x_1}$ (D) $\frac{y_1 - y_2}{x_1 + x_2}$
5	$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = :$ (A) $\frac{b}{a}$ (B) a (C) $\frac{a}{b}$ (D) $\frac{1}{b}$
6	$\int (a-2x)^{\frac{3}{2}} dx = :$ (A) $\frac{1}{5}(a-2x)^{\frac{3}{2}} + c$ (B) $\frac{1}{5}(a-2x)^{\frac{5}{2}} + c$ (C) $-\frac{1}{5}(a-2x)^{\frac{5}{2}} + c$ (D) $-\frac{3}{5}(a-2x)^{\frac{5}{2}} + c$
7	$\int \sec x dx = :$ (A) $\sec x + \tan x$ (B) $\sec^2 x$ (C) $\ln \sec x - \tan x $ (D) $\ln \sec x + \tan x + c$
8	If $f(x) = \frac{1}{x^m}$ then $f'(x) = :$ (A) $-xm^{-1}$ (B) $-mx^{-m-1}$ (C) $-mx^{-m+1}$ (D) $-m^{-1}x$

(Turn Over)

Lahore Board-2021

(2)

1-9	Midpoint of the line segment joining A (3, 1) and B (- 2 , - 4) is : (A) $\left(\frac{1}{2}, -\frac{3}{2}\right)$ (B) $\left(\frac{5}{2}, \frac{5}{2}\right)$ (C) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (D) $\left(\frac{1}{2}, \frac{5}{2}\right)$
10	The derivative of $\frac{1}{1+x}$ is : (A) x (B) $1+x$ (C) $(1+x)^{-2}$ (D) $-1(1+x)^{-2}$
11	In circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the radius is : (A) $\sqrt{g^2 + f^2 + c}$ (B) $g^2 + f^2 - c$ (C) $\sqrt{g^2 + f^2 - c}$ (D) $g^2 + f^2 + c$
12	$x = 5$ is the solution of inequality : (A) $2x - 3 > 0$ (B) $2x + 3 < 0$ (C) $x + 4 < 0$ (D) $x + 3 < 0$
13	In vectors $\vec{a} \times \vec{b} =$: (A) $\vec{b} \times \vec{a}$ (B) $-\vec{b} \times \vec{a}$ (C) $\vec{a} \times \vec{b}$ (D) $-\vec{a} \times \vec{b}$
14	In equation of circle $x^2 + y^2 = r^2$ the centre of circle is : (A) (x, y) (B) $(0, 0)$ (C) $(1, 0)$ (D) $(0, r)$
15	Magnitude of vector $\vec{u} = 2i - 7j$ is : (A) $\sqrt{53}$ (B) $\sqrt{55}$ (C) $\sqrt{48}$ (D) $\sqrt{52}$
16	$\frac{d}{dx} (\cos^{-1} x) =$: (A) $\frac{1}{\sqrt{1-x^2}}$ (B) $\frac{-1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{1}{1+x^2}$
17	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ is Maclaurin series for : (A) e^x (B) $\sqrt{1+x}$ (C) $\cos x$ (D) $\sin x$
18	The vector $\overrightarrow{PQ}$ through P (0 , 5) and Q (- 1 , - 6) is : (A) $[-1, 11]$ (B) $[-1, -11]$ (C) $[0, 11]$ (D) $[1, 1]$
19	$\frac{d}{dx} \tan^{-1} x =$: (A) $\frac{1}{1-x^2}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{1}{1+x^2}$
20	The focus of parabola $y^2 = 4ax$ is : (A) $(0, a)$ (B) $(-a, 0)$ (C) $(a, 0)$ (D) $(0, -a)$

173-221-I-(Objective Type)- 6875 (8195)

SECTION – I



2. Write short answers to any EIGHT (8) questions :

16

- (i) Find the domain and range of the function g defined by : $g(x) = \sqrt{x^2 - 4}$
- (ii) The real valued functions f and g are given. Find $f \circ g(x)$, if
 $f(x) = 3x^4 - 2x^2$ and $g(x) = \frac{2}{\sqrt{x}}$, $x \neq 0$
- (iii) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$
- (iv) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 2x - 1}{x^3 - x}$
- (v) Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$
- (vi) Differentiate w.r.t. 'x' $\cot^{-1}\left(\frac{x}{a}\right)$
- (vii) Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$
- (viii) Find y_2 if $x^3 - y^3 = a^3$
- (ix) Prove that $\frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{|x| \sqrt{x^2 - 1}}$
- (x) Differentiate $\frac{2x - 1}{\sqrt{x^2 + 1}}$
- (xi) Find the interval in which function is increasing or decreasing :
 $f(x) = \cos x$ $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- (xii) Find y_4 if $y = \sin 3x$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Use differentials to approximate the value of $\sqrt[4]{17}$
- (ii) Solve $\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}$
- (iii) Evaluate $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$
- (iv) Solve $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$
- (v) Solve $\int e^{2x} [-\sin x + 2 \cos x] dx$
- (vi) Evaluate $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$
- (vii) Solve the differential equation $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1 + y^2)$
- (viii) Evaluate $\int x \ln x dx$
- (ix) The points $A(-5, -2)$, $B(5, -4)$ are ends of a diameter of a circle. Find centre and radius of it.

3. (x) Transform the equation $5x - 12y + 39 = 0$ into normal form.
 (xi) Find k so that the lines joining $A(7, 3)$, $B(k, -6)$ and $C(-4, 5)$, $D(-6, 4)$ are parallel.
 (xii) Find the lines represented by $2x^2 + 3xy - 5y^2 = 0$

4. Write short answers to any NINE (9) questions :

18

- (i) Graph the inequality $5x - 4y \leq 20$
 (ii) Find the equation of the circle with ends of diameter at $(-3, 2)$ and $(5, -6)$
 (iii) Find the centre of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$
 (iv) Find the length of the tangent from the point $(-5, 10)$ to the circle $5x^2 + 5y^2 + 14x - 12y - 10 = 0$
 (v) Find the coordinates of the points of intersection of the line $x + 2y = 6$ with the circle $x^2 + y^2 - 2x - 2y - 39 = 0$
 (vi) Find the vertex of the parabola $x^2 = 4(y - 1)$
 (vii) Find the foci of the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$
 (viii) Find a unit vector in the direction of $\underline{v} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$
 (ix) Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$
 (x) If $\underline{v}$ is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$ and $\underline{v} \cdot \underline{k} = 0$, find $\underline{v}$
 (xi) If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
 (xii) Find the volume of parallelepiped for which the vectors $\underline{u} = \underline{i} - 4\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - \underline{j} - 2\underline{k}$ and $\underline{w} = 2\underline{i} - 3\underline{j} + \underline{k}$ are three edges.
 (xiii) Give a force $\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point $A(1, -2, 1)$. Find the moment of $\underline{F}$ about the point $B(2, 0, -2)$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Discuss the continuity of $f(x)$ at $x = c$ $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$, $c = 1$ 5
 (b) Show that $\frac{dy}{dx} = \frac{y}{x}$ if $\frac{y}{x} = \tan^{-1} \frac{x}{y}$ 5
 6. (a) Evaluate $\int x \sin^{-1} x \, dx$ 5
 (b) Find the interior angles of the triangle with vertices $A(6, 1)$, $B(2, 7)$, $C(-6, -7)$ 5
 7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin x} \, dx$ 5
 (b) Minimize $z = 2x + y$ subject to constraints $x + y \geq 3$, $7x + 5y \leq 35$; $x \geq 0$, $y \geq 0$ 5
 8. (a) Prove that in any triangle ABC $b^2 = c^2 + a^2 - 2ca \cos B$. 5
 (b) Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$ 5
 9. (a) If $y = (\cos^{-1} x)^2$ then prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ 5
 (b) Find the points of intersection of the given conic $\frac{x^2}{18} + \frac{y^2}{8} = 1$ and $\frac{x^2}{3} - \frac{y^2}{3} = 1$ 5

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The derivative of $\frac{1}{1+x}$ is : (A) x (B) $1+x$ (C) $(1+x)^{-2}$ (D) $-1(1+x)^{-2}$
2	$\int \cos x \, dx = :$ (A) $1 - \sin^2 x$ (B) $\sqrt{1 - \sin^2 x}$ (C) $\sin x$ (D) $-\sin x$
3	$\int_1^2 (x^2 + 1) \, dx = :$ (A) $\frac{10}{3}$ (B) $\frac{3}{10}$ (C) π (D) $\frac{\pi}{2}$
4	If $y = \cot^{-1} x$, then $\frac{dy}{dx} = :$ (A) $\frac{1}{1-x^2}$ (B) $\frac{-1}{1+x^2}$ (C) $\frac{1}{x^2-1}$ (D) $\frac{1}{x^2+1}$
5	The derivative of $\ln(\tanh x)$ is : (A) $\frac{1}{\tanh x}$ (B) $\frac{\sec^2 x}{\tanh x}$ (C) $\sec^2 x$ (D) $\sec x$
6	$x = at^2$ and $y = 2at$ are parametric equations of : (A) Parabola (B) Ellipse (C) Circle (D) Hyperbola
7	If $y^2 + x^2 = a^2$, then $\frac{dy}{dx} = :$ (A) $-\frac{x}{y}$ (B) $-\frac{y}{x}$ (C) $\frac{x}{y}$ (D) $\frac{y}{x}$
8	The order of $\frac{dy}{dx} = \frac{4}{3}x^3 + x - 3$ is : (A) 1 (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) -3
9	$\int_a^x 3x^2 \, dx = :$ (A) $x^3 + a^3$ (B) $x^3 - a^3$ (C) $3x^3$ (D) x^3

(Turn Over)

1-10	If θ is measured in radian then $\lim_{\theta \rightarrow 0} \frac{\sin 7\theta}{\theta} = :$ (A) 7 (B) $\frac{1}{7}$ (C) $\frac{7\pi}{22}$ (D) $\frac{7\pi}{12}$
11	The measure of the angle between the lines $ax^2 + 2hxy + by^2 = 0$ is given by $\tan \theta = :$ (A) $\frac{\sqrt{h^2 - ab}}{a - b}$ (B) $\frac{2\sqrt{h^2 - ab}}{a + b}$ (C) $\frac{h^2 - ab}{a + b}$ (D) ∞
12	If $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ then $\vec{a} \cdot \vec{b} = :$ (A) 0 (B) 1 (C) -1 (D) $\sqrt{2}$
13	The feasible solution which maximize or minimize the objective function is called : (A) Boundary (B) Half plane (C) Optimal solution (D) Initial values
14	The value of c for $\frac{y^2}{16} - \frac{x^2}{49} = 1$ is : (A) 16 (B) 49 (C) 65 (D) $\sqrt{65}$
15	The equation of a straight line represented by $x \cos \alpha + y \sin \alpha = P$ is called : (A) Normal form (B) Angular form (C) Symmetric form (D) P - form
16	The unit vector in the direction of $\vec{v} = [3, -4]$: (A) $5[3, -4]$ (B) $\frac{1}{5}[3, -4]$ (C) $\hat{i}$ (D) $\hat{j}$
17	The points A (-5, -2), B (5, -4) are ends point of a diameter of the circle. The centre will be : (A) (0, 3) (B) (0, -3) (C) (-5, 2) (D) (-5, 4)
18	$xy = 0$ represents : (A) A pair of lines (B) Hyperbola (C) Parabola (D) Ellipse
19	The projection of $\vec{v}$ along $\vec{u}$ is : (A) $\frac{\vec{u} \cdot \vec{v}}{ \vec{u} }$ (B) $\frac{\vec{u} \cdot \vec{v}}{ \vec{v} }$ (C) $\frac{\vec{u} \cdot \vec{v}}{ \vec{u} \vec{v} }$ (D) $\frac{\vec{u} \cdot \vec{v}}{ \vec{u} + \vec{v} }$
20	An angle inscribed in a semi-circle is : (A) 0 (B) $\frac{\pi}{2}$ (C) π (D) 2π

SECTION – I



2. Write short answers to any EIGHT (8) questions :

16

- (i) Express the area A of a circle as a function of its circumference C .
- (ii) For the real-valued function $f(x) = \frac{2x+1}{2x-1}$, $x > 1$. Find $f^{-1}(x)$
- (iii) Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- (iv) Find the domain and range of $g(x) = |x-3|$
- (v) If $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2$, find $\frac{dy}{dx}$
- (vi) Find $\frac{dy}{dx}$ if $xy + y^2 = 2$
- (vii) Differentiate $\sin x$ w.r.t. $\cot x$
- (viii) Find $\frac{dy}{dx}$ if $y = x^2 \ln \frac{1}{x}$
- (ix) Find y_2 if $y = x^2 \cdot e^{-x}$
- (x) If $y = \ln(\tanh x)$, find $\frac{dy}{dx}$
- (xi) Find $\frac{dy}{dx}$ if $y = (x^2 + 5)(x^3 + 7)$
- (xii) Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Use differential to find $\frac{dy}{dx}$ for $xy + x = 4$
- (ii) Evaluate the integral $\int \frac{3x+2}{\sqrt{x}} dx$
- (iii) Evaluate $\int \frac{x+b}{(x^2+2bx+c)^{1/2}} dx$
- (iv) Evaluate $\int e^x (\cos x + \sin x) dx$
- (v) Evaluate $\int \frac{(a-b)x}{(x-a)(x-b)} dx$
- (vi) Evaluate $\int_{-1}^1 (x^{1/3} + 1) dx$
- (vii) Find the area above the x-axis and under the curve $y = 5 - x^2$ from $x = -1$ to $x = 2$
- (viii) Solve differential equation $ydx + xdy = 0$
- (ix) Find mid-point of line segment joining $A(-8, 3)$; $B(2, -1)$
- (x) Two points 'P' and 'O' given in xy-coordinate system. Find XY-coordinates of 'P' referred to translated axis $O'X$ and $O'Y$ for $P(-2, 6)$; $O'(-3, 2)$
- (xi) Find equation of the line joining $(-5, -3)$ and $(9, -1)$
- (xii) Find equation of vertical line through $(-5, 3)$

4. Write short answers to any NINE (9) questions :

- (i) Graph the solution set of given linear inequality in xy -plane : $2x + y \leq 6$
- (ii) Find the centre and radius of the circle with the given equation
 $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- (iii) Find the focus and vertex of the parabola $x^2 = -16y$
- (iv) Write an equation of parabola with given elements : Focus $(-3, 1)$;
 directrix $x - 2y - 3 = 0$
- (v) Find an equation of directrices of given hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- (vi) Find the centre and eccentricity of given hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$
- (vii) Find the unit vector in the same direction as the vector $\underline{v} = [3, -4]$
- (viii) Find the constant a so that the vectors $\underline{v} = \underline{i} - 3\underline{j} + 4\underline{k}$ and $\underline{w} = a\underline{i} + 9\underline{j} - 12\underline{k}$ are parallel.
- (ix) Find a vector of length 2 in the direction opposite that of $\underline{v} = -\underline{i} + \underline{j} + \underline{k}$
- (x) Find the cosine of the angle θ between $\underline{u}$ and $\underline{v}$ $\underline{u} = [2, -3, 1]$ and $\underline{v} = [2, 4, 1]$
- (xi) Compute $\underline{b} \times \underline{a}$. Check your answer by showing that $\underline{b}$ is perpendicular to $\underline{b} \times \underline{a}$:
 $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$; $\underline{b} = \underline{i} - \underline{j} + \underline{k}$.
- (xii) If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
- (xiii) Give a force $\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point A $(1, -2, 1)$. Find the moment of $\underline{F}$ about the point B $(2, 0, -2)$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Find value of k , if the function $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at $x = 2$ 5
- (b) If $y = \tan(p \tan^{-1} x)$ then show that $(1+x^2)y_1 - p(1+y^2) = 0$ 5
6. (a) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$ 5
- (b) Find an equation of the line through the intersection of the lines $x - y - 4 = 0$ and $7x + y + 20 = 0$ and parallel to the line $6x + y - 14 = 0$ 5
7. (a) Find the area bounded by the curve $y = x^3 - 4x$ and the x -axis. 5
- (b) Maximize $f(x, y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$ 5
8. (a) Write equation of the circle passing through the points A $(-7, 7)$, B $(5, -1)$ and C $(10, 0)$ 5
- (b) Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$ 5
9. (a) Show that $y = \frac{\ell nx}{x}$ has maximum value at $x = e$ 5
- (b) Find focus, vertex and directrix of parabola $x^2 - 4x - 8y + 4 = 0$ 5

Roll No _____ **Lahore Board-2019** (To be filled in by the candidate)

(Academic Sessions 2015 – 2017 to 2017 – 2019)

MATHEMATICS

219-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – I

Maximum Marks : 20

PAPER CODE = 8195

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If $y = e^{2x}$ then $y_2 = :$	(A) e^{2x}	(B) $2e^{2x}$	(C) $4e^{2x}$	(D) $16e^{2x}$
2	$\int a^x dx = :$	(A) $\frac{\ln a}{a^x} + c$	(B) $\frac{a^x}{\ln a} + c$	(C) $\frac{1}{a^x \ln a} + c$	(D) $a^x \ln a + c$
3	$f(x) = ax + b, a \neq 0$ is :	(A) Trigonometric function	(B) Linear function	(C) Cubic function	(D) Quadratic function
4	$\int_0^{\frac{\pi}{2}} \cos x dx = :$	(A) $\frac{\pi}{2}$	(B) 0	(C) -1	(D) 1
5	$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{\frac{n}{2}} = :$	(A) e^{-1}	(B) e^2	(C) $e^{\frac{1}{2}}$	(D) e^3
6	Differential of y is denoted by :	(A) $\frac{dy}{dx}$	(B) dy	(C) dx	(D) dy
7	If $f(x) = \cos x$ then $f'(\pi) = :$	(A) 1	(B) 0	(C) -1	(D) 2
8	The value of $\frac{dy}{dx} = \frac{-2}{x^3}$ at $x = -1$ is :	(A) 4	(B) 5	(C) -2	(D) 2
9	Order of the differential equation $\frac{x d^2 y}{dx^2} + \frac{dy}{dx} - 2x = 0$ is :	(A) 1	(B) 2	(C) 3	(D) 4

(Turn Over)

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) Define explicit function.
- (ii) Determine whether the function $f(x) = x\sqrt{x^2 + 5}$ is even or odd.
- (iii) Prove that $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$
- (iv) If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, find $\frac{dy}{dx}$
- (v) Find $\frac{dy}{dx}$ if $x^2 + y^2 = 4$
- (vi) Prove that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- (vii) Differentiate $\sin^{-1} \sqrt{1-x^2}$ w.r.t. 'x'
- (viii) Differentiate $y = a^{\sqrt{x}}$
- (ix) Prove that $\frac{d}{dx}(\cosh x) = \sinh x$
- (x) Find $\frac{dy}{dx}$ if $y = (x+1)^x$
- (xi) Define decreasing function. Give an example.
- (xii) Determine $f(x) = \cos x$ is increasing or decreasing in the interval $(\frac{\pi}{2}, \pi)$

3. Write short answers to any EIGHT (8) questions :

16

- (i) What is differential coefficient?
- (ii) Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$
- (iii) Integrate by substitution $\int \frac{-2x}{\sqrt{4-x^2}} dx$
- (iv) Find the integral $\int \frac{\cos x}{\sin x \ln(\sin x)} dx$
- (v) Evaluate integral by parts $\int x \cdot \sin x dx$
- (vi) Find indefinite integral $\int a^{ax} \left[a \sec^{-1} x + \frac{1}{x\sqrt{x^2-1}} \right] dx$
- (vii) Evaluate $\int \frac{5x+8}{(x+3)(2x-1)} dx$ using partial fraction.
- (viii) Define definite integral.
- (ix) Calculate the integral $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$
- (x) If $\int_{-2}^1 f(x) dx = 5$, $\int_{-2}^1 g(x) dx = 4$, then evaluate $\int_{-2}^1 [3f(x) - 2g(x)] dx$

(Turn Over)

3. (xi) If a non-vertical line divides a plane into two, then write the name that two planes?
 (xii) Graph the inequality $x + 3y > 6$

4. Write short answers to any NINE (9) questions :

18

- (i) Find coordinates of the point that divide the join of A (- 6 , 3) and B (5 , - 2) in the ratio 2 : 3 internally.
 (ii) Show that the triangle with vertices A (1 , 1) , B (4 , 5) and C (12 , - 5) is right triangle.
 (iii) Find an equation of the line through (- 4 , - 6) and perpendicular to the line having slope $\frac{-3}{2}$.
 (iv) Define trapezium.
 (v) Define parabola.
 (vi) Check the position of the point (5, 6) with respect to the circle $2x^2 + 2y^2 + 12x - 8y + 1 = 0$
 (vii) Find eccentricity of the ellipse $x^2 + 4y^2 = 16$
 (viii) Find an equation of hyperbola if its foci (0 , ± 9) and directrices $y = \pm 4$
 (ix) If $\vec{AB} = \vec{CD}$, find coordinates of point A. If B, C, D are (1 , 2) , (- 2 , 5) , (4 , 11)
 (x) Write direction cosine of $\vec{PQ}$, if P (2 , 1 , 5) Q (1 , 3 , 1)
 (xi) Show that vectors $3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} + 5\vec{k}$ and $2\vec{i} + \vec{j} - 4\vec{k}$ form a right triangle.
 (xii) Find unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ if $\vec{a} = -\vec{i} - \vec{j} - \vec{k}$,
 $\vec{b} = 2\vec{i} - 3\vec{j} + 4\vec{k}$.
 (xiii) A force $\vec{F} = 7\vec{i} + 4\vec{j} - 3\vec{k}$ is applied at P (1 , - 2 , 3). Find its moment about the point Q (2 , 1 , 1)

SECTION - II

Note : Attempt any THREE questions.

5. (a) Find the values of 'm' and 'n' so that $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$

is continuous at $x = 3$

5

- (b) If $y = e^x \cdot \sin x$, then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

5

6. (a) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$

5

- (b) Find an equation of the perpendicular bisector of the segment joining the points A (3 , 5) and B (9 , 8)

5

7. (a) Solve the differential equation $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$

5

- (b) Graph the solution region of the following system of linear inequalities and find the corner points : $x + y \leq 5$, $-2x + y \leq 2$, $y \geq 0$

5

8. (a) Find the lines represented by each of the following and also find measure of the angle between them $x^2 + 2xy \sec \alpha + y^2 = 0$

5

- (b) Find the coordinates of the points of intersection of the line $2x + y + 5 = 0$ and the circle $x^2 + y^2 + 2x - 9 = 0$. Also find the length of intercepted chord.

5

9. (a) Find equation of parabola with elements directrix : $x = -2$, focus (2, 2)

5

- (b) Prove that $\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \sin \beta$ by method of vectors.

5

Roll No _____ **Lahore Board-2019** (To be filled in by the candidate)

(Academic Sessions 2015 – 2017 to 2017 – 2019)

MATHEMATICS

219-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – II

Maximum Marks : 20

PAPER CODE = 8198

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	$\frac{d}{dx}(\sqrt{x}) = :$	
	(A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$ (C) $\frac{1}{2x}$ (D) $\frac{1}{2\sqrt{x}}$	
2	$\int \tan x dx = :$	
	(A) $\ln \sec x + c$ (B) $\ln \csc x + c$ (C) $\ln \sin x + c$ (D) $\ln \cot x + c$	
3	$\int \frac{e^x}{e^x + 3} dx = :$	
	(A) $\ln(e^x + 3) + c$ (B) $e^{2x} + c$ (C) $e^0 + c$ (D) $e^{2x} + 3 + c$	
4	$\frac{d}{dx}(\cos x^2) = :$	
	(A) $2x \sin x^2$ (B) $-2x \sin x^2$ (C) $2 \cos x$ (D) $-2 \sin x$	
5	If $y = \sin^{-1} \frac{x}{a}$, then $\sin y = :$	
	(A) $\cos y$ (B) $\cos x$ (C) $\frac{x}{a}$ (D) $\frac{y}{a}$	
6	The function $y = 27 + x^2$ is a / an :	
	(A) Constant function (B) Even function (C) Implicit function (D) Explicit function	
7	A function $f(x)$ has relative maximum at $x = c$, if $f'(c) = 0$ and :	
	(A) $f''(c) > 0$ (B) $f''(c) < 0$ (C) $f''(c) = 0$ (D) $f'(c) \neq 0$	
8	$\int \sec^2 x dx = :$	
	(A) $\cot x + c$ (B) $\tan x + c$ (C) $2 \sec x + c$ (D) $\frac{1}{\cos^2 x} + c$	
9	$\int_{-\pi}^{\pi} \sin x dx = :$	
	(A) 2π (B) 0 (C) 1 (D) $\cos \pi$	

(Turn Over)

Lahore Board-2019

1-10	If $f(x) = 2x + 1$, then $f^{-1}(x) = ?$: (A) $2x - 1$ (B) $1 - 2x$ (C) $x - \frac{1}{2}$ (D) $\frac{x-1}{2}$
11	y-intercept of the line $2x - y - 4 = 0$ is : (A) 2 (B) -2 (C) 4 (D) -4
12	An angle in the semi circle is of measure : (A) 30° (B) 60° (C) 90° (D) 180°
13	The perpendicular distance of a line $5x + 12y = 7$ from origin is : (A) $\frac{1}{13}$ (B) $\frac{13}{7}$ (C) $\frac{7}{13}$ (D) -7
14	Equation of latus-rectum of parabola $y^2 = 4ax$ is : (A) $x = -a$ (B) $y = -a$ (C) $x = a$ (D) $y = a$
15	The mid point of line segment joining A (-8, 3), B (2, -1) is : (A) (-6, 2) (B) (10, 4) (C) (-3, 1) (D) (-16, -3)
16	The triple scalar product of vectors, calculates the volume of : (A) Triangle (B) Parallelogram (C) Tetrahedron (D) Parallelepiped
17	The equation of line $\frac{x}{b} + \frac{y}{a} = 1$ is in : (A) Normal form (B) Intercept form (C) Point-slope form (D) Two-points form
18	The radius of circle $x^2 + y^2 = 5$ is : (A) 25 (B) $\sqrt{5}$ (C) 5 (D) (0, 0)
19	Non-zero vector $\underline{a}$ and $\underline{b}$ are parallel if $\underline{a} \times \underline{b} =$: (A) 0 (B) 1 (C) -1 (D) (a, b)
20	The solution of the inequality $x + 2y < 6$ is : (A) (1, 1) (B) (1, 3) (C) (1, 4) (D) (1, 5)

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4. Write short answers to any NINE (9) questions :

- (i) Find the coordinates of the point that divides the join of A (- 6 , 3) and B (5 , - 2) internally in ratio 2 : 3.
- (ii) Find the slope and inclination of the line joining the points A (- 2 , 4) and B (5 , 11)
- (iii) By means of slopes show that points A (- 1 , - 3) , B (1 , 5) and C (2 , 9) are collinear.
- (iv) Find equation of the line through (- 4 , 7) and parallel to the line $2x - 7y + 4 = 0$
- (v) Find equation of circle with centre at (5 , - 2) and radius 4.
- (vi) Find focus and vertex of the parabola $y^2 = -8(x - 3)$
- (vii) Find equation of tangent to the parabola $x^2 = 16y$ at the point whose abscissa is 8.
- (viii) Find foci and vertices of the ellipse $25x^2 + 9y^2 = 225$
- (ix) Find the angle between the vectors $\underline{u} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{v} = -\underline{i} + \underline{j}$
- (x) Find scalar α so that the vectors $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular.
- (xi) If $\underline{v}$ is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$, $\underline{v} \cdot \underline{k} = 0$ find $\underline{v}$
- (xii) Prove that $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$
- (xiii) Find the value of α so that $\alpha\underline{i} + \underline{j}$, $\underline{i} + \underline{j} + 3\underline{k}$ and $2\underline{i} + \underline{j} - 2\underline{k}$ are coplanar.

SECTION - II

Note : Attempt any THREE questions.

5. (a) If $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

discuss continuity at $x = 2$ and $x = -2$

5

(b) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

5

6. (a) Integrate $\int \frac{12}{x^3 + 8} dx$

5

(b) Find equations of two parallel lines, perpendicular to $2x - y + 3 = 0$ such that the product of the x- and y-intercepts of each is 3.

5

7. (a) Evaluate the definite integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x (2 + \sin x)} dx$

5

(b) Minimize $z = 2x + y$ subject to the constraints
 $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$

5

8. (a) Find equation of the line through the point (2 , - 9) and intersection of the lines
 $2x + 5y - 8 = 0$
 $3x - 4y - 6 = 0$

5

(b) Show that the circles $x^2 + y^2 + 2x - 2y - 7 = 0$ and $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally.

5

9. (a) Find an equation of the ellipse having foci (± 5 , 0) and passing through the point $\left(\frac{2}{3} , \sqrt{3} \right)$

5

(b) A particle acted upon by constant forces $4\underline{i} + \underline{j} - 3\underline{k}$ and $3\underline{i} - \underline{j} - \underline{k}$ is displaced from A (1 , 2 , 3) to B (5 , 4 , 1). Find the work done.

5

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If $y = \sqrt{1-x^2}$, $0 < x < 1$ then $\frac{dy}{dx} = :$ (A) $\sqrt{x^2-1}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{x}{\sqrt{1-x^2}}$ (D) $\frac{-x}{\sqrt{1-x^2}}$	
2	$\int 3^x dx = :$ (A) $3^x + c$ (B) $3^x \ln 3 + c$ (C) $\frac{3^x}{\ln 3} + c$ (D) $3 \ln 3^x + c$	
3	$\int_0^{\frac{\pi}{2}} \cos x dx = :$ (A) 0 (B) 1 (C) 2 (D) 3	
4	If $f(x)$ has second derivative at "c" such that $f'(c) = 0$ and $f''(c) < 0$ then "c" is a point of : (A) Maxima (B) Minima (C) Zero point (D) Point of inflection	
5	If $y = e^{\sin x}$, then $\frac{dy}{dx} = :$ (A) $e^{\sin x}$ (B) $e^{\sin x} \cos x$ (C) $e^{\sin x} + \cos x$ (D) $-e^{\sin x} \cos x$	
6	$\cosh^2 x - \sinh^2 x = :$ (A) 1 (B) -1 (C) 0 (D) 2	
7	$\frac{d}{dx} \sin^{-1} x = :$ (A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\cos^{-1} x$ (C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{1}{\sqrt{1-x}}$	
8	$\int \frac{1}{f(x)} \times f'(x) dx = :$ (A) $\ln x + c$ (B) $\ln[f'(x) + c]$ (C) $\frac{1}{f(x)} + c$ (D) $\ln f(x) + c$	
9	The order of the differential equation $\frac{d^2 y}{dx^2} - \frac{dy}{dx} + 2x = 0$ is : (A) 2 (B) 1 (C) 0 (D) 3	

1-10	Let $f(x) = x^2 + \cos x$, then $f(x)$ is : (A) Odd function (B) Constant function (C) Even function (D) Neither even nor odd
11	The centroid of a triangle divides each median in ratio : (A) 2 : 1 (B) 1 : 2 (C) 2 : 3 (D) 1 : 1
12	The straight line $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if : (A) $c^2 = a^2 m^2 - b^2$ (B) $c^2 = b^2 m^2 + a^2$ (C) $c^2 = b^2 m^2 - a^2$ (D) $c^2 = a^2 m^2 + b^2$
13	The perpendicular distance of line $3x + 4y - 10 = 0$ from the origin is : (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2
14	Axis of the parabola $x^2 = 4ay$ is : (A) $y = 0$ (B) $x = 0$ (C) $x = y$ (D) $x = 1$
15	If α is the inclination of the line, then $\frac{x - x_1}{\cos \alpha} = \frac{y - y_1}{\sin \alpha} = r$ (say) is called : (A) Point slope form (B) Normal form (C) Symmetric form (D) Intercept form
16	The direction cosines of y-axis are : (A) (0, 1, 0) (B) (1, 0, 0) (C) (0, 0, 1) (D) (0, 0, 0)
17	If α is the inclination of a line " ℓ " then it must be true that : (A) $0 \leq \alpha < \frac{\pi}{2}$ (B) $\frac{\pi}{2} \leq \alpha < \pi$ (C) $0 \leq \alpha < \pi$ (D) $0 \leq \alpha < 2\pi$
18	The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre : (A) $(-g, -f)$ (B) $(-f, +g)$ (C) (f, g) (D) $(0, 0)$
19	Length of the vector $2\hat{i} - \hat{j} - 2\hat{k}$ is : (A) 2 (B) 4 (C) 3 (D) 5
20	The feasible solution which maximizes or minimizes the objective function is called : (A) Exact solution (B) Optimal solution (C) Final solution (D) Objective solution

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) State sandwich theorem.
- (ii) Express the area " A " of a circle as a function of its circumference " C ".
- (iii) If $f(x) = \begin{cases} x+2, & x \leq -1 \\ c+2, & x > -1 \end{cases}$, find " c " so that $\lim_{x \rightarrow -1} f(x)$ exists
- (iv) Define differentiation.
- (v) Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2$ wrt x
- (vi) Find $\frac{dy}{dx}$ if $xy + y^2 = 0$
- (vii) Find $\frac{dy}{dx}$ if $y = x \cos y$
- (viii) Prove that $\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$, $x \in (-1, 1)$
- (ix) Find $\frac{dy}{dx}$ if $y = x e^{\sin x}$
- (x) Define power series.
- (xi) Find extreme values for $f(x) = x^2 - x - 2$
- (xii) Find $\frac{dy}{dx}$ if $y = \sin^{-1} \left(\frac{x}{2} \right)$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find $\frac{dy}{dx}$ using differentials if $xy - \log_e x = c$
- (ii) Evaluate the integral $\int \frac{x}{x+2} \cdot dx$
- (iii) Evaluate the integral $\int \frac{1}{a^2 - x^2} \cdot dx$
- (iv) Evaluate the integral $\int x \sin x \cos x \, dx$
- (v) Evaluate the integral $\int x^2 e^{ax} \cdot dx$
- (vi) Evaluate the integral $\int e^{3x} \left(\frac{3 \sin x - \cos x}{\sin^2 x} \right) dx$
- (vii) Prove that $\int_a^b f(x) \cdot dx = - \int_b^a f(x) \cdot dx$
- (viii) Evaluate the definite integral $\int_0^3 \frac{dx}{x^2 + 9}$
- (ix) Find the area bounded by cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$

3. (x) Solve the differential equation $\sin y \operatorname{cosec} x \frac{dy}{dx} = 1$

(xi) Define optimal solution and feasible solution.

(xii) Graph the region indicated by $4x - 3y \leq 12$, $x \geq -\frac{3}{2}$



4. Write short answers to any NINE (9) questions :

18

(i) Show that the points A (3 , 1) , B (- 2 , - 3) and C (2 , 2) are vertices of an isosceles triangle.

(ii) Find an equation of a line through the points (- 2 , 1) and (6 , - 4)

(iii) Find an equation of the line bisecting the first and third quadrants.

(iv) Find an equation of the line with x-intercept : - 3 and y - intercept : 4

(v) Convert $2x - 4y + 11 = 0$ into slope intercept form.

(vi) Write an equation of the parabola with focus (- 1 , 0) , vertex (- 1 , 2)

(vii) Find the focus and directrix of the parabola $y = 6x^2 - 1$

(viii) Find an equation of the ellipse with centre (0 , 0) , focus (0 , - 3) , vertex (0 , 4)

(ix) Find the eccentricity and directrices of the ellipse whose equation is $25x^2 + 9y^2 = 225$

(x) Define unit vector.

(xi) Find a unit vector in the direction of the vector $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$

(xii) Find a vector whose magnitude is '4' and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$

(xiii) Find a scalar " α " so that the vectors $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular.

SECTION - II

Note : Attempt any THREE questions

5. (a) If $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x^2 - k} , & x \neq 2 \\ k , & x = 2 \end{cases}$

5

Find value of k so that f is continuous at $x = 2$

(b) Show that $y = x^x$ has maximum value at $x = \frac{1}{e}$

5

6. (a) Evaluate $\int e^{2x} \cos 3x dx$

5

(b) The three points A (7 , - 1) , B (- 2 , 2) and C (1 , 4) are consecutive vertices of a parallelogram, find the fourth vertex.

5

7. (a) Find the area bounded by the curve $y = x^3 - 4x$ and x-axis.

5

(b) Minimize $z = 2x + y$ subject to the constraints

5

$$x + y \geq 3, 7x + 5y \leq 35, x \geq 0, y \geq 0$$

8. (a) Find the condition that the line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$ at a single point.

5

(b) Find x so that points A (1 , - 1 , 0) , B (- 2 , 2 , 1) and C (0 , 2 , x) form triangle with right angle at C.

5

9. (a) Find the centre, foci, eccentricity, vertices and equations of directrices of $\frac{y^2}{4} - x^2 = 1$

5

(b) Find volume of the tetrahedron with the vertices A (2 , 1 , 8) , B (3 , 2 , 9) , C (2 , 1 , 4) and D (3 , 3 , 10)

5

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Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

Two or more circles will result in zero mark in that question.



1-1	$\frac{d}{dx} \log_a x = :$	(A) $\frac{1}{x}$	(B) $x \ln x - x$	(C) $\frac{1}{x} \ln a$	(D) $\frac{1}{x \ln a}$
2	$\int \sin x \cos x dx :$	(A) $\frac{1}{2} \cos 2x$	(B) $-\frac{1}{2} \cos 2x$	(C) $\frac{\sin^2 x}{2}$	(D) $\frac{\cos^2 x}{2}$
3	$\int \frac{1}{x\sqrt{x^2-1}} dx :$	(A) $\sin^{-1} x$	(B) $\tan^{-1} x$	(C) $\sec^{-1} x$	(D) $\operatorname{cosec}^{-1} x$
4	If $x = f(\theta), y = g(\theta)$ then $\frac{dy}{dx} :$	(A) $\frac{dy}{d\theta} \frac{d\theta}{dx}$	(B) $\frac{dx}{d\theta} \frac{d\theta}{dy}$	(C) $\frac{d\theta}{dy} \frac{dx}{d\theta}$	(D) $\frac{dy}{d\theta} \frac{dx}{d\theta}$
5	$\frac{d}{dx} \sec hx = :$	(A) $\sec hx \tanh x$	(B) $-\sec hx \tanh x$	(C) $\tan h^2 x$	(D) $\sec h^2 x$
6	If at least one vertical line meets the curve at more than two points then curve is :	(A) A function	(B) Not a function	(C) One – to – one function	(D) Onto function
7	$\frac{d}{dx} \cosh x = :$	(A) $-\sinh x$	(B) $\sec hx$	(C) $-\sec hx$	(D) $\sinh x$
8	$\int \sec^2 x dx :$	(A) $\tan x$	(B) $\frac{\sec^3 x}{3}$	(C) $\tan^2 x$	(D) $\sec x \tan x$
9	Solution of $\frac{dy}{dx} = \frac{-y}{x}$ is :	(A) $\frac{x}{y} = c$	(B) $\frac{y}{x} = c$	(C) $y = cx$	(D) $xy = c$

1-10	Domain of $f(x) = x^2 + 1$: (A) $\mathbb{R}$ (B) $\mathbb{R} - \{1\}$ (C) $\mathbb{R} - \{-1\}$ (D) $[1, \infty)$
11	Equation of line bisecting II and IV quadrant : (A) $y = x$ (B) $y = -x$ (C) $y = \frac{1}{x}$ (D) $x + y = 1$
12	Set of all points equidistant from a fixed point form : (A) Ellipse (B) Parabola (C) Hyperbola (D) Circle
13	Joint equation of two lines is $ax^2 + 2hxy + by^2 = 0$, if θ is angle between them, then $\tan \theta =$: (A) $\frac{2\sqrt{h^2 + ab}}{a + b}$ (B) $\frac{2\sqrt{h^2 - ab}}{a + b}$ (C) $\frac{\sqrt{h^2 + ab}}{a + b}$ (D) $\frac{\sqrt{h^2 - ab}}{a + b}$
14	Focal chord perpendicular to axis of parabola is called : (A) Latus Rectum (B) Eccentricity (C) Vertex (D) Axis
15	Horizontal line through $(7, -9)$ is : (A) $x = 7$ (B) $x = -9$ (C) $y = 7$ (D) $y = -9$
16	Projection of vector $\vec{u}$ on vector $\vec{v}$ is : (A) $\frac{\vec{u} \cdot \vec{v}}{ \vec{v} }$ (B) $\frac{\vec{u} \cdot \vec{v}}{ \vec{u} }$ (C) $\frac{\vec{u} \times \vec{v}}{ \vec{v} }$ (D) $\frac{\vec{u} \times \vec{v}}{ \vec{u} }$
17	Distance of (x_1, y_1) from line $ax + by + c = 0$ is : (A) $\frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$ (B) $\frac{ ax_1 + by_1 - c }{\sqrt{a^2 + b^2}}$ (C) $\frac{ ax_1 + by_1 + c }{\sqrt{a + b}}$ (D) $\frac{ ax_1 + by_1 - c }{\sqrt{a + b}}$
18	For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$ then eccentricity $e =$: (A) $\frac{\sqrt{a^2 - b^2}}{a}$ (B) $\frac{\sqrt{a^2 + b^2}}{a}$ (C) $\frac{\sqrt{b^2 - a^2}}{a}$ (D) $\frac{\sqrt{b^2 - a^2}}{b}$
19	If $\vec{v}$ is any vector then vector of magnitude 5 opposite to $\vec{v}$ is : (A) $5\vec{v}$ (B) $-5\vec{v}$ (C) $5\frac{\vec{v}}{ \vec{v} }$ (D) $-5\frac{\vec{v}}{ \vec{v} }$
20	System of linear inequalities involved in the problem is called : (A) Coefficients (B) Solution (C) Problem constraints (D) Boundaries

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

- (i) Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$
- (ii) Determine whether function $f(x) = \frac{x^3 - x}{x^2 + 1}$ is even or odd.
- (iii) Evaluate $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$
- (iv) Find $\frac{dy}{dx}$ if $y = \frac{a+x}{a-x}$
- (v) Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$
- (vi) Differentiate $x^2 - \frac{1}{x^2}$ w.r.t x^4
- (vii) Differentiate $\sin^{-1} \sqrt{1-x^2}$ w.r.t x
- (viii) Find $\frac{dy}{dx}$ if $y = \ln(x + \sqrt{x^2 + 1})$
- (ix) Find $\frac{dy}{dx}$ if $y = e^{-2x} \sin 2x$
- (x) Find $\frac{d^2 y}{dx^2}$ if $y^3 + 3ax^2 + x^3 = 0$
- (xi) Find y_2 if $y = \cos^3 x$
- (xii) Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{x^2 - 1}{x^2 + 1}\right)^{\frac{1}{2}}$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find δy and dy : $y = \sqrt{x}$, when x changes from 4 to 4.41
- (ii) Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$
- (iii) Evaluate $\int (a - 2x)^{\frac{3}{2}} dx$
- (iv) Evaluate $\int \frac{x+b}{(x^2 + 2bx + c)^{\frac{1}{2}}} dx$
- (v) Evaluate $\int x e^x dx$
- (vi) Evaluate $\int e^x \left(\frac{1}{x} + \ln x \right) dx$
- (vii) Evaluate $\int_{-1}^3 (x^3 + 3x^2) dx$
- (viii) Evaluate $\int_{\pi/3}^{\pi/2} \cos^2 \theta \sin \theta d\theta$

3. (ix) Find the area between the x-axis and the curve $y = 4x - x^2$ from $x = 0$ to $x = 4$
 (x) Define differential equation.
 (xi) Solve $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$
 (xii) Solve $\frac{dy}{dx} = 2x$

4. Write short answers to any NINE (9) questions :

18

- (i) Write down equation of straight line with x-intercept (2, 0) and y-intercept (0, -4)
 (ii) Find an equation of a line bisecting 2nd and 4th quadrants.
 (iii) Find an equation of a line with x-intercept : -9 and slope : -4.
 (iv) Prove that if the lines are perpendicular, then product of their slopes = -1
 (v) Find the measure of angle between the lines represented by $x^2 - xy - 6y^2 = 0$
 (vi) Find focus and vertex of the parabola $y = 6x^2 - 1$
 (vii) Find equation of latus rectum of parabola $y^2 = -8(x - 3)$
 (viii) Find an equation of an ellipse with foci ($\pm 3, 0$) and minor axis of length 10.
 (ix) Find the foci and length of the latus rectum of the ellipse $9x^2 + y^2 = 18$
 (x) Define direction angles and direction cosines of a vector.
 (xi) Find the projection of vector $\underline{a}$ along vector $\underline{b}$ and projection of vector $\underline{b}$ along $\underline{a}$
 when $\underline{a} = \hat{i} - \hat{k}$, $\underline{b} = \hat{j} + \hat{k}$
 (xii) Find a vector perpendicular to each of the vectors $\underline{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\underline{b} = 4\hat{i} + 2\hat{j} - \hat{k}$
 (xiii) Convert $2x - 4y + 11 = 0$ into slope intercept form.

SECTION - II

Note : Attempt any THREE questions.

5. (a) Prove that $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$ 5
 (b) Prove that $y \frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ 5
 6. (a) Show that $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c$ 5
 (b) The points A (-1, 2), B (6, 3) and C (2, -4) are vertices of a triangle, then show that the line joining the mid-point "D" of $\overline{AB}$ and mid-point "E" of $\overline{AC}$ is parallel to $\overline{BC}$ and $\overline{DE} = \frac{1}{2} \overline{BC}$. 5
 7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t \, dt$ 5
 (b) Graph the feasible region of system of linear inequalities and find the corner points $2x + 3y \leq 18, x + 4y \leq 12, 3x + y \leq 12, x \geq 0, y \geq 0$ 5
 8. (a) Find an equation of parabola having its focus at the origin and directrix parallel to y-axis. 5
 (b) Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long. 5
 9. (a) Find the centre, foci, eccentricity, vertices and equations of directrices of $\frac{y^2}{4} - x^2 = 1$ 5
 (b) Find the value of α , in the coplanar vectors $\alpha \hat{i} + \hat{j}, \hat{i} + \hat{j} + 3\hat{k}, 2\hat{i} + \hat{j} - 2\hat{k}$ 5