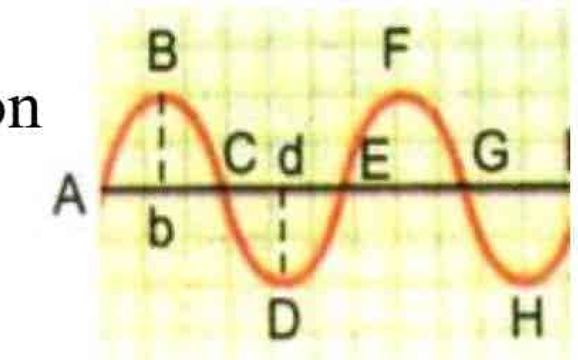
Chapter = 07

Oscillations

Oscillatory motion/vibratory motion: To and fro motion of a body about a mean position is called vibratory or oscillatory motion. For example motion of mass suspended from a spring and motion of bob of simple pendulum.
Periodic motion: The vibratory motion that repeats itself in equal interval of time is called periodic motion.
Restoring force: The force which brings the system back to its equilibrium position is called restoring force. F=-Kx.
Requirements for oscillations: There are two requirements for oscillations

(1) Oscillating system must have restoring force (2) Oscillating system has inertia

How oscillations are produced: A body is pulled away on one side from its equilibrium position and then released the body starts to oscillate due to restoring force.
Hook's law: Within elastic limit, the applied force is directly proportional to the displacement. F=Kx.
Spring constant: Force per unit extension is called spring constant. K=F/x its SI unit is N/m and dimension $[MT^{-2}]$
Simple harmonic motion: A type of motion in which acceleration is directly proportional to displacement from mean position and directed towards mean position is called SHM. $a \propto -x$
Conditions for SHM: The system must have inertia, restoring force and frictionless for SHM.
Waveform of SHM: The curve which shows the variations of displacement with time is called wave form. Wave form of SHM is sine wave.
Characteristics of wave form of SHM:
Instantaneous displacement: The displacement of vibrating body at any instant of time
Amplitude: the maximum displacement of vibrating body on either position from its mean position
Vibration: one complete round trip of vibrating body about is mean position
Time period: The time required to complete one vibration. It is shown by T and its unit is second
Frequency: the number of vibrations completed in one second f=1/T.



The product of frequency and time period is equal to 1, fT=1
Angular frequency: If time period is T of a body executing SHM, its angular frequency $\omega = 2\pi / T = 2\pi f$. Basically the angular frequency is the property of circular motion.

Prove that $a \propto -x$?

Consider a mass m attached to one end of spring which can move freely on horizontal surface by applying force
According to Hook's law F = Kx and opposing force brings the mass towards mean position which is

F = -Kx - - - - - - - - - - - - - - - - - -(1)

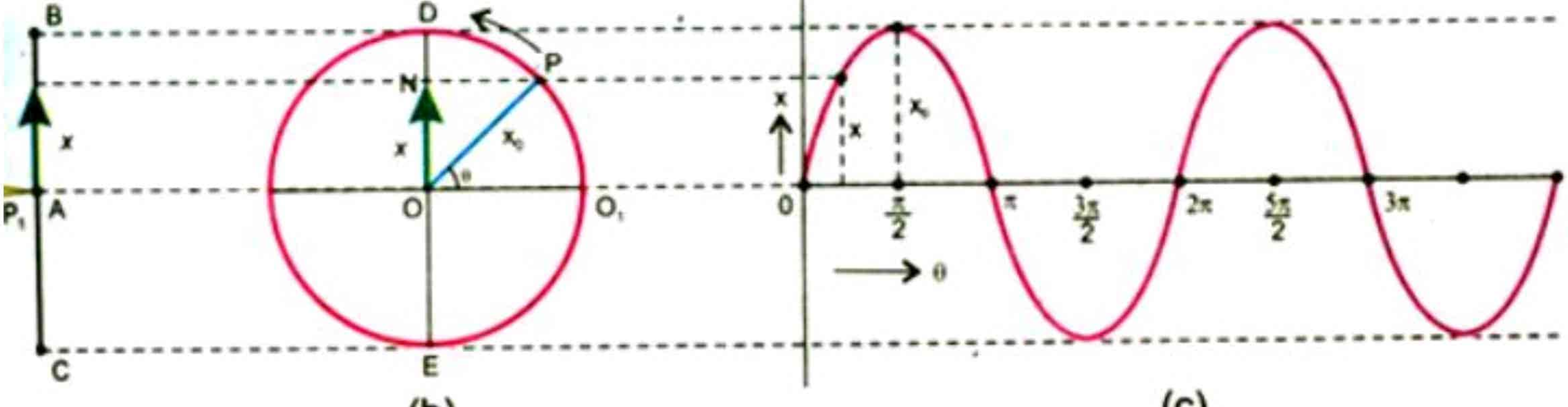
According to Newton 2nd law acceleration is produced by force F = ma - - - - - - - -(2)

comparing (1) and (2) - Kx = ma

$$a = \frac{-K}{m}x \Rightarrow a \propto -x \text{ which is req result as } \frac{K}{m} = \text{constant}$$

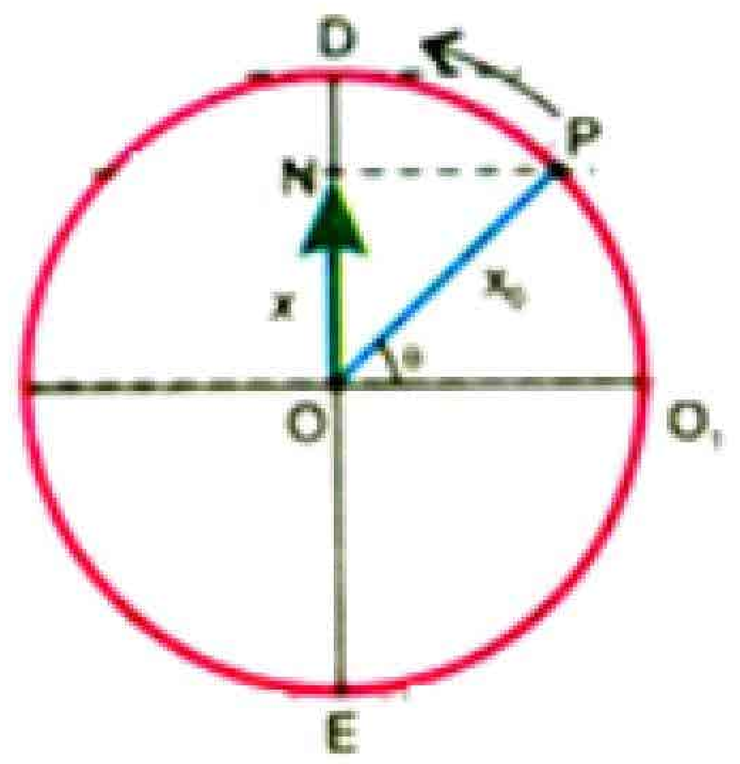
EXPLAIN SHM AND UNIFORM CIRCULAR MOTION.

Consider a mass m attached with end of vertically suspended spring. It vibrates simple harmonically with period T, frequency f and amplitude x_o. The motion of mass is displayed by the pointer P_1 on the line BC with A as mean position and "B" and "C" as extreme position as shown in fig.
At t=0 pointer is at position A then at position B, A, C and back to A at instant T/4, T/2, 3T/4 and T respectively.
In circular motion pointer P is moving in a circle of radius xo with uniform angular frequency ω. Let us consider the motion of point N, the projection of P on diameter DE, the levels of D and E are similar to points B and C, hence the comparison of motion of point N and P1 shows that motion of projection of particle p moving in a circle is SHM.



(b) (c)

Instantaneous Displacement: let N be the projection of a particle P moving in a circle of Angular frequency ω and angle subtended is $\Theta=\omega t$ in radius of circle x_o.



From right angle triangle (OPN)

$$\sin\theta = \frac{ON}{OP} \Rightarrow ON = OP\sin\theta$$

$$x = x_o \sin\theta$$

$$x = x_o \sin\omega t$$

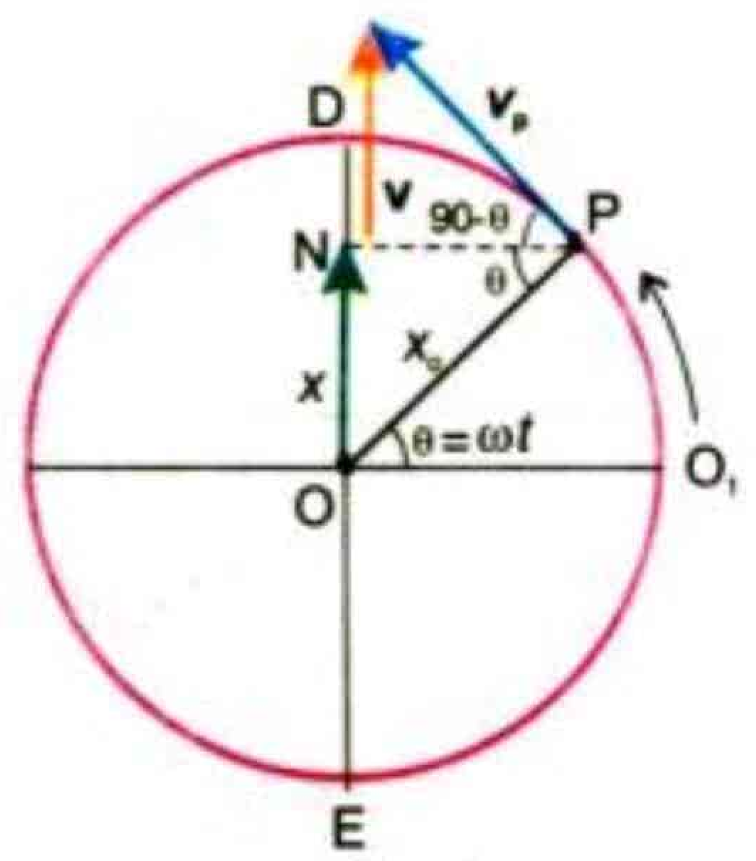
$$at\ \theta = \omega t = 0^o \Rightarrow x = x_o \sin 0^o = 0 \qquad at\ \ \theta = \omega t = 90^o \Rightarrow x = x_o \sin 90^o = x_o$$

$$at\ \ \theta = \omega t = 180^o \Rightarrow x = x_o \sin 180^o = 0\ \ at\ \ \theta = \omega t = 270^o \Rightarrow x = x_o \sin 270^o = -x_o$$

$$at\ \ \theta = \omega t = 360^o \Rightarrow x = x_o \sin 360^o = 0 \quad \theta = \omega t \text{ is the phase of vibration}$$

Instantaneous velocity: The velocity of point P at the instant t, will directed along the tangent to the circle at P and its magnitude is $V_p=x_o\omega$. The velocity of N is actually the vertical component of velocity Vp in direction parallel to DE.

The component of velocity parallel to DE is



$$V = V_p \sin(90^o - \theta)$$

$$V = V_p \cos\theta$$

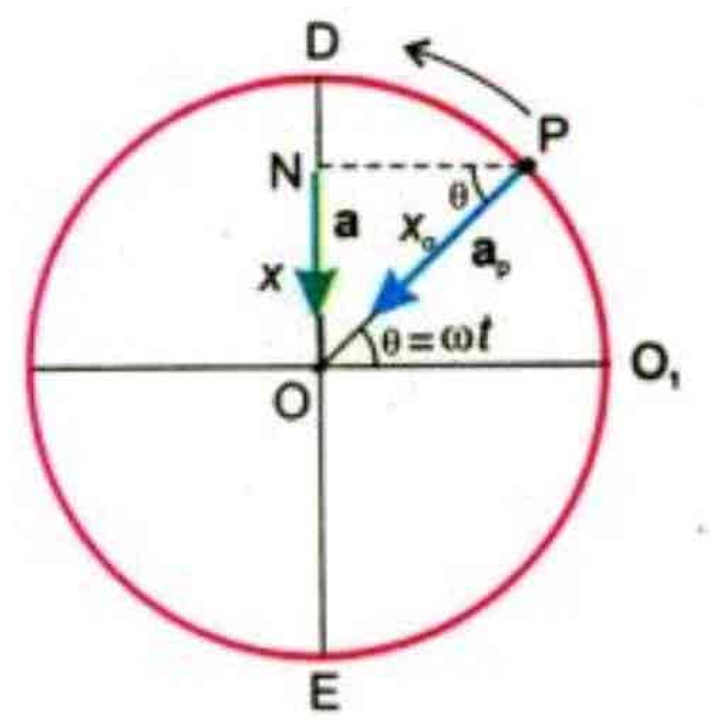
$$V = x_o\omega \cos\theta \text{ - (1)}$$

$$\text{From fig } \cos\theta = \frac{NP}{OP} = \frac{\sqrt{x_o^2 - x^2}}{x_o}, \text{ putting the value in eq(1)}$$

$$V = x_o\omega \frac{\sqrt{x_o^2 - x^2}}{x_o} = \omega\sqrt{x_o^2 - x^2}$$

At mean position velocity is maximum and at extreme position velocity is minimum

Instantaneous Acceleration (a): The acceleration at any point P along the circle is $a_p=x_o\omega^2$ which is directed towards the center O. the acceleration of point N will be the component of acceleration ap along the diameter DE as $a=a_p\sin\Theta$



$$a = x_o\omega^2\cos(90+\theta)$$

$$a = -x_o\omega^2 Sin\,\theta$$

$$\text{as } \sin\theta = \frac{x}{x_o}, \text{ putting in equation (1)}$$

$$a = -x_o\omega^2(\frac{x}{x_o}) =$$

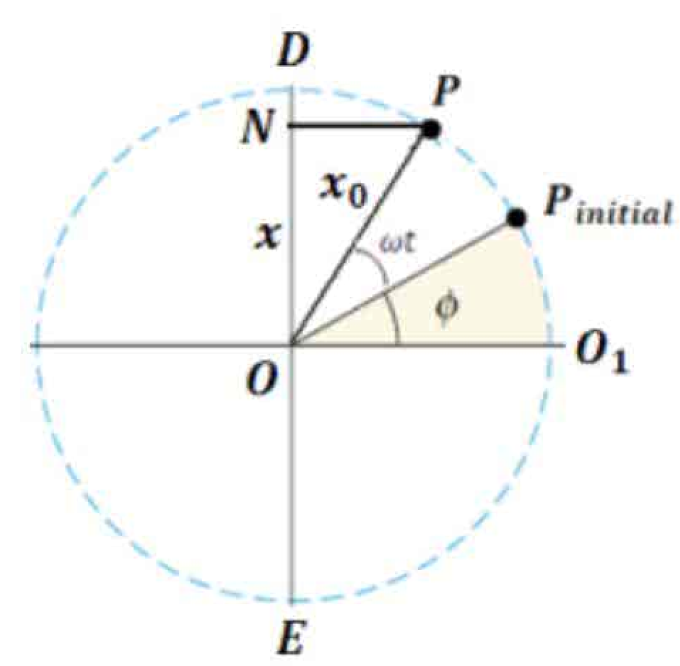
$$a = -\omega^2 x$$

$$a = -\text{constant } x$$

$$a \propto -x, \text{ This shows that acceleration is directly proportional to displacement and directed towards mean position}$$

What is Phase? Give its two cases.

The angle $\theta = \omega t$ which gives the displacement as well as the direction of motion executing SHM is called phase. This angle is obtained when SHM is related with circular motion. Phase determine the state of motion of vibrating body.



Case 01: When motion starts from mean position its phase would at this point be 0.
Let at t=0 the angle made by rotating radius OP with reference line OO1=ϕ, after a time t, the radius rotate through angle=ωt and angle made by rotating radius OP is (ωt+ϕ) and displacement is $x=x_o \sin(\omega t+\phi)$. This is shown in fig 1

Case 02: When motion starts at the extreme position, its phase would be π/2.
In this case we take initial phase as 90° or π/2 as shown in fig 2 then displacement is $x=x_o\sin(\omega t+90)= x_o\cos\omega t$

Write a note on characteristics of Horizontal mass spring system.

Let us consider a mass which is attached with a spring at one side and at the other side
Spring is fixed with rigid support

Instantaneous Acceleration of spring system: let the Restoring force produces acceleration then

$F = ma$ ----------(1) $F = -Kx$ ------(1) comparing both eqs

$ma = -Kx$

$a = \frac{-Kx}{m}$ This is the formula for acceleration of mass spring system

$a = -\text{constant}\ x$ as $\frac{K}{m} = \text{constant}$

$a \propto -x$, This shows that acceleration is directly proportional to displacment and directed towards mean position

Angular frequency: as we know that acceleration for simple harmonic motion is

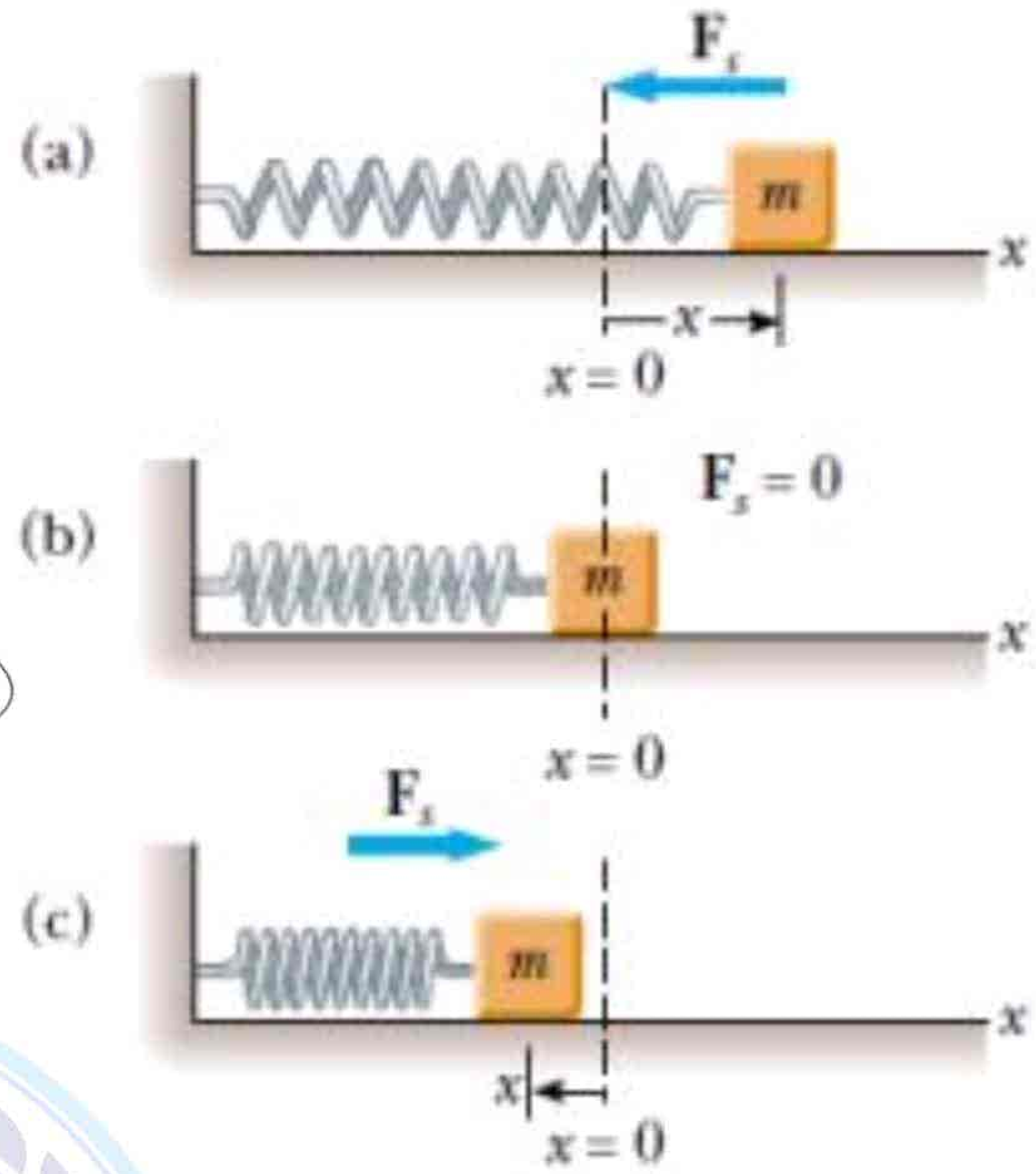
$a = -\omega^2 x$ ---------(1)

$a = \frac{-K}{m}x$ ---------(2) comparing both eqs

$-\omega^2 x = \frac{-K}{m}x$

$\omega^2 = \frac{K}{m}$

$\omega = \sqrt{\frac{K}{m}}$, This is the formula for angular frequency



Time period and frequency: Time period and frequency of mass spring system having SHM are

$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\sqrt{\frac{K}{m}}}$

$T = 2\pi\sqrt{\frac{m}{K}}$, this is the formula for time period

$f = 1/T = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$ This is the formula for frequency of mass spring system

Instantaneous displacement: The displacement at any instant of time is calculate as $x = x_o \sin\omega t = x_o \sin\sqrt{K/m}\,t$.

Instantaneous velocity: The velocity at any instant of time is calculated as

$v = \omega\sqrt{x_o^2 - x^2}$

$v = \sqrt{K/m}\sqrt{x_o^2 - x^2} = \sqrt{K/m\,(x_o^2 - x^2)}$

$v = x_o\sqrt{K/m\,(1 - x^2/x_o^2)}$

Max velocity at mean position at $x = 0$ $v = x_o\sqrt{K/m}$

Min velocity at extreme position $x = x_o$, $v = 0$

Relation b/w maximum velocity and instantaneous velocity: $v = v_o\sqrt{1-\frac{x^2}{x_o^2}}$.

What is Simple pendulum? Prove that motion of simple pendulum is SHM. Also derive the relation for time period of simple pendulum.

Definition: A small heavy mass suspended by a weightless and inextensible string with frictionless support is called simple pendulum.

Motion of simple pendulum: Let us consider an object of mass m attached with the end of a light weight string whose length is l

When the pendulum is displaced from mean position through a small angle Θ and released then it start to oscillate to and fro motion about mean position. Two forces acting on it

Equation: Weight mg of bob acting vertically down and Tension T of string acting upward

The weight has two components mgsinΘ and mgcosΘ, the only component mgsinΘ is responsible for motion of pendulum which brings the bob back towards the mean position

Restoring force = F = $-mg\sin\theta$ ----------(1) -ive sign shows that force is directed towards mean position

F = ma -------------------(2) comparing both equation

$ma = -mg\sin\theta$ where angle is very small so for small angle $\sin\theta \approx \theta$

$ma = -mg\theta$

$a = -g\theta$ as we know from fig $\theta = x/l$

$a = -g\frac{x}{l} \Rightarrow \frac{g}{l} = \text{constant}$

a = −constant x

$a \propto -x$ This relation shows that acceleration is directly proportional to displacment and directed towards mean position

Angular frequency

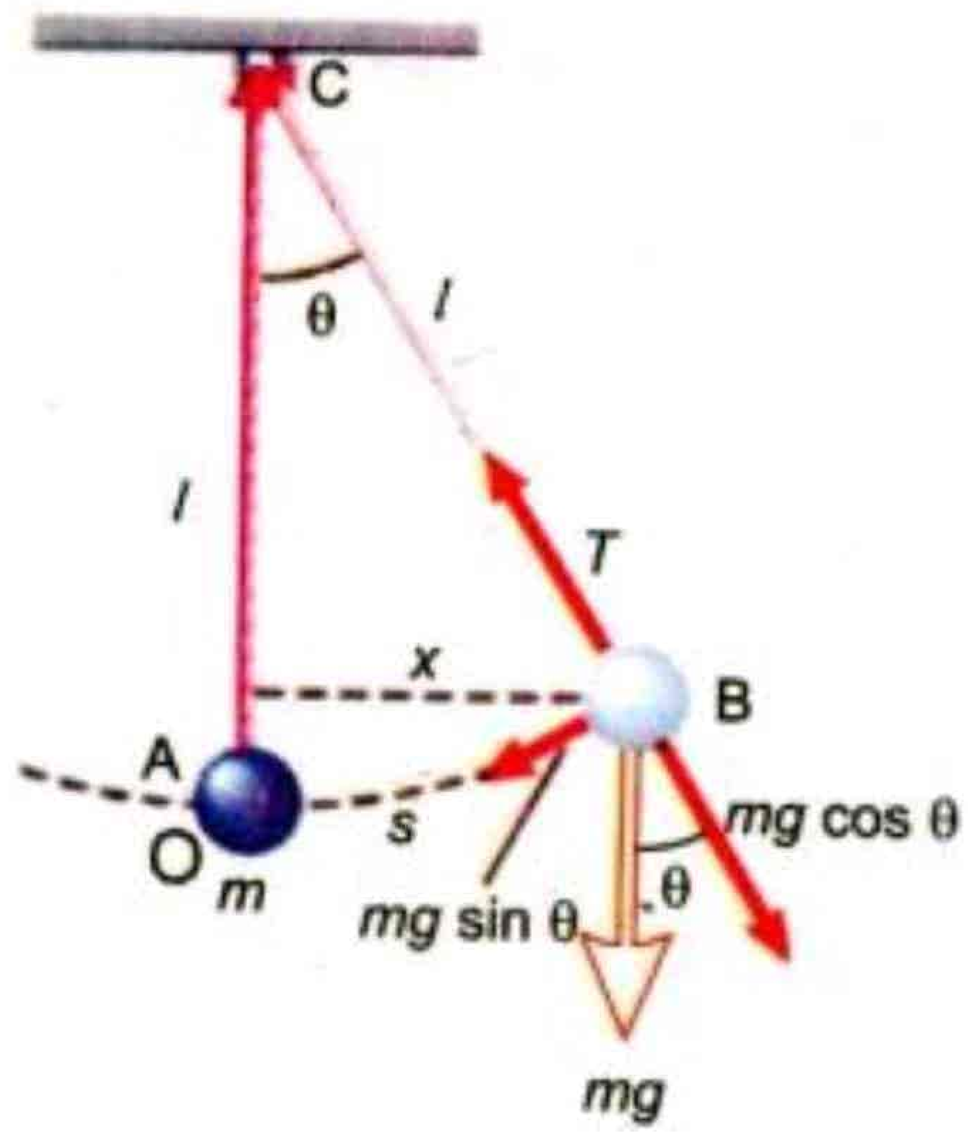
$a = -\omega^2 x$ ---------(1)

$a = \frac{-g}{l}x$ ---------(2) comparing both eqs

$-\omega^2 x = \frac{-g}{l}x$

$\omega^2 = \frac{g}{l}$

$\omega = \sqrt{\frac{g}{l}}$, This is the formula for angular frequency



$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\sqrt{\frac{g}{l}}}$

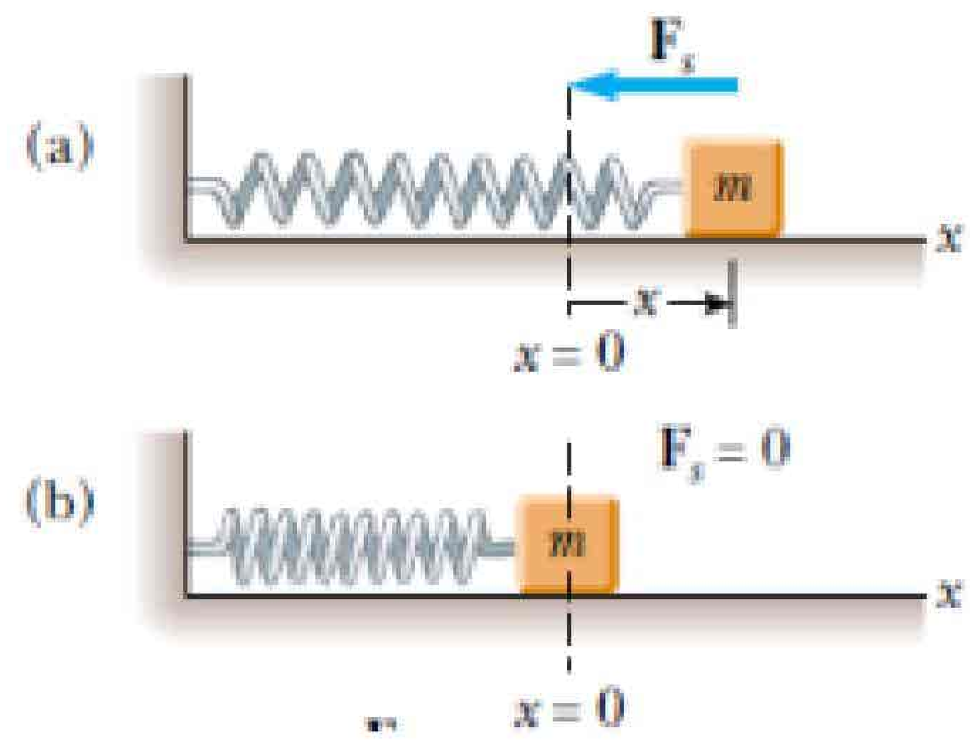
$T = 2\pi\sqrt{\frac{l}{g}}$, this is the formula for time period

$f = 1/T = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ This is the formula for frequency of simple pendulum

Double pendulum: A simple pendulum whose time period is 2 second. Its frequency is 0.5 Hz and length is 99.2 cm.

Explain Energy conservation in SHM.

Let us consider a mass spring system. when the mass is pulled and stretched the spring by distance xo along horizontal frictionless surface. Instantaneous potential energy can be calculated



According to Hook's law $F = Kx$ when displacment is x at extreme position

$F = 0$ when displacment is zero at mean position

average force is $F = \frac{0+kx}{2} = \frac{kx}{2}$,

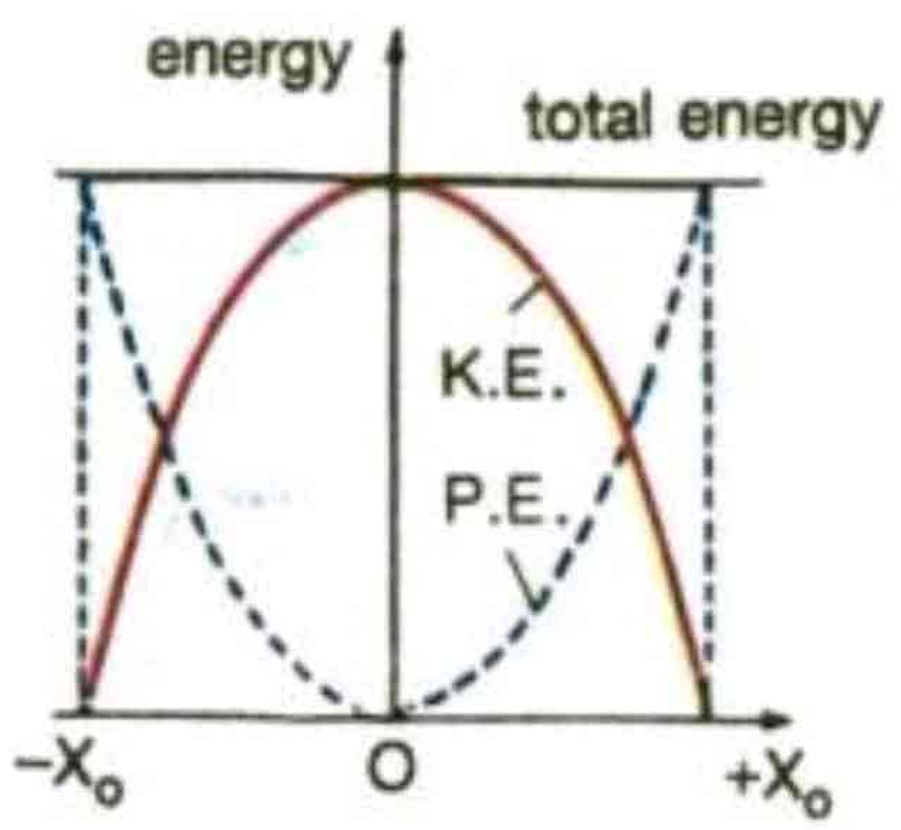
work done is equal to P.E in this case so

$$W = Fd = Fx = (\frac{kx}{2})x = \frac{1}{2}kx^2$$

$P.E = \frac{1}{2}kx^2$, if displacment is at maximum value xo at extrement position then

$P.E = \frac{1}{2}kx_o^2$ this is maximum P.E and at mean position $x = 0$ so $P.E = 0$ min

Instantaneous kinetic energy can be calcualted by using the formula $K.E = \frac{1}{2}mv^2$



$$v = x_o\sqrt{K/m(1 - x^2/x_o^2)}$$

$$K.E = \frac{1}{2}m(x_o\sqrt{K/m(1 - x^2/x_o^2)})^2$$

$$K.E = \frac{1}{2}mx_o^2\frac{K}{m}(1 - x^2/x_o^2)$$

$$K.E = \frac{1}{2}Kx_o^2(1 - x^2/x_o^2)$$

at mean position $x = 0$, kinetic energy will be maximum

$$K.E = \frac{1}{2}Kx_o^2(1 - 0^2/x_o^2) \Rightarrow K.E = \frac{1}{2}Kx_o^2 --------(2)$$

Kinetic energy at extreme position will be minimum at at $x = x_o \Rightarrow K.E = 0$

Total Energy $= P.E + K.E$

$$E = \frac{1}{2}Kx^2 + \frac{1}{2}Kx_o^2(1 - x^2/x_o^2)$$

$$E = \frac{1}{2}Kx^2 + \frac{1}{2}Kx_o^2 - \frac{1}{2}Kx^2$$

$$E = \frac{1}{2}Kx_o^2, -----------------(3)$$

Equation (1), (2) and (3) show that total energy remains constant in SHM.

What are Free and forced oscillations? Give example of each.

Free oscillations: If a body executes oscillations with its natural frequency without the interference of external force, then these oscillations are called free oscillations. For example a simple pendulum vibrates freely with its natural frequency.

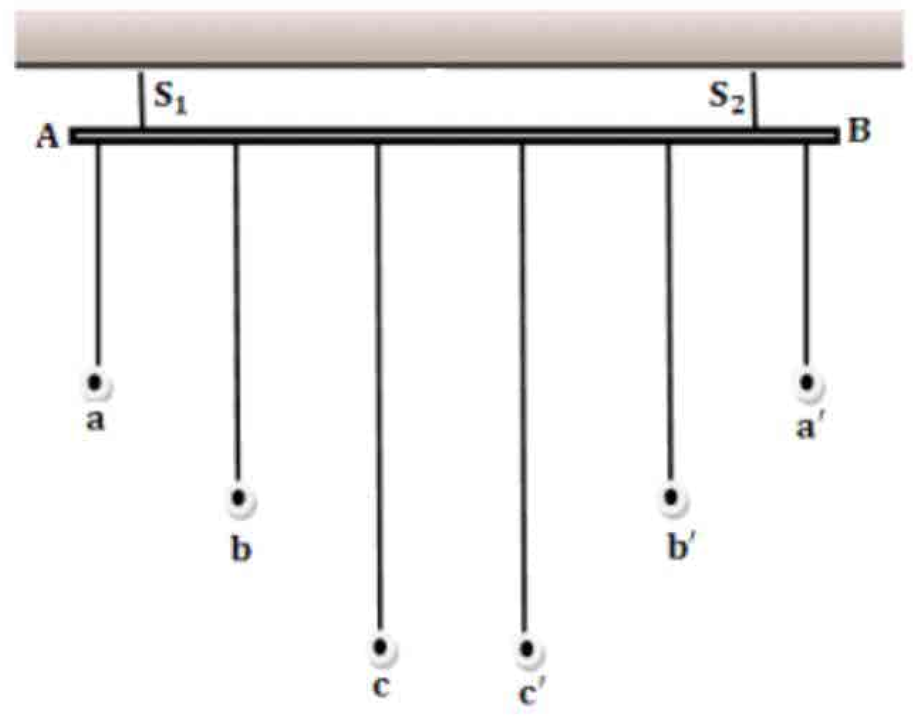
Free oscillations: A body is said to be executing forced vibrations if it oscillate under the action of an external force. for example if mass of simple pendulum is struck repeatedly then forced vibrations are produced.

Driven harmonic oscillator: The physical system that undergoes forced vibrations is called driven harmonic oscillator.

What is Resonance? Describe Barton pendulum experiment. And examples of resonance.

Resonance: The phenomenon in which the amplitude of vibrating body increase when the frequency of applied force is equal to natural frequency of harmonic oscillator is called resonance.
For example, motion of swing, tuning of a radio, microwave oven etc.

Barton pendulum experiment: Consider a horizontal rod AB is supported by two strings
Three pairs of pendulums, and are suspended to this rod.
If one of these pendulums, say c, is displaced
from its mean position, then its resultant oscillatory motion causes
Slight disturbance motion in rod AB.
This causes the pendulum to oscillate back with steadily increasing amplitude.
However, the amplitude of the other pendulums remains small.
The increase of the amplitude of pendulum is due to effect of resonance,
Because the periods as well as the natural frequencies of pendulum and are equal.



Examples of Resonance

Mechanical Resonance for the case of swing: A swing is the good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

March of soldiers on bridge: The column of soldiers, while marching on a bridge of long span is advised to break their steps. Their rhythmic march might set up oscillation of dangerously large amplitude in the bridge structure.

Electrical Resonance in Tuning of a Radio: Tuning of a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of electrical circuit of receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

Cooking of a Food in Microwave Oven: Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven. The waves produced in this type of oven have a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food.

Why microwave oven use 2450 MHz?

Microwave oven use 2450M Hz frequency because at this frequency the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

What is undamped and damped oscillations? Also define damping.

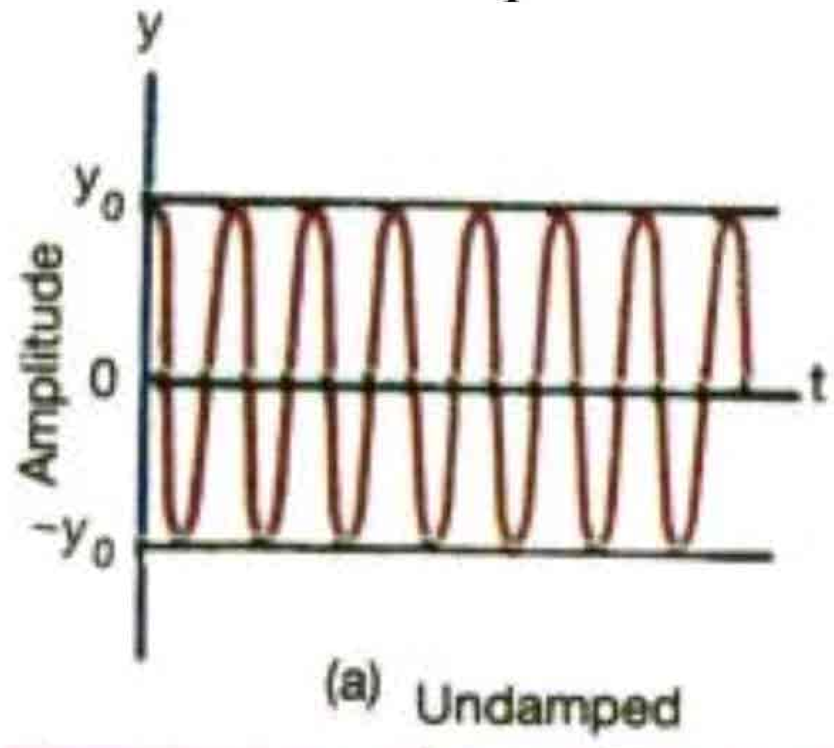
The oscillations in which amplitude remains same with time are called undamped oscillations. For example oscillations of an ideal simple pendulum.
The oscillations in which amplitude decrease with time due to energy dissipation are called damped oscillations. Its example is the shock absorber of a car
Damping is the process whereby energy is dissipated from the oscillating system.

What is Effect of damping on vibration of a body?

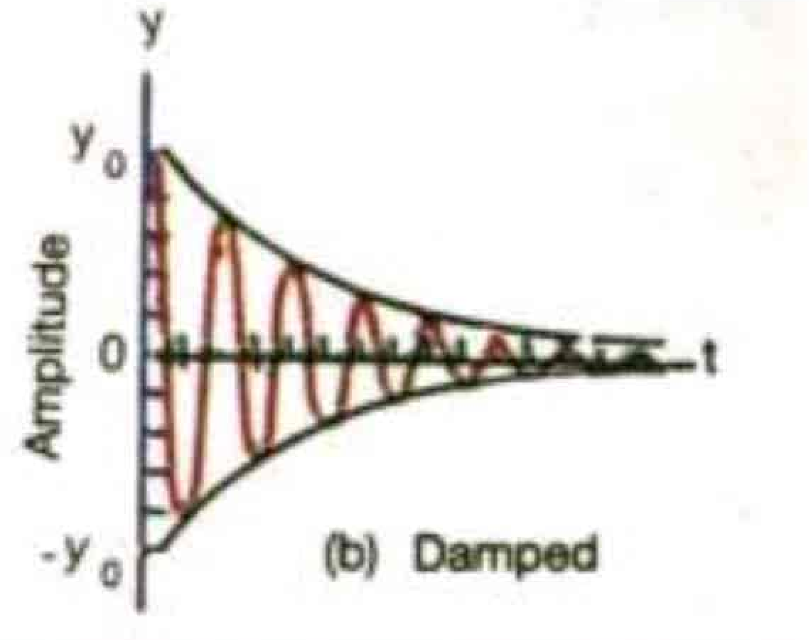
The amplitude of vibration of a body increases when the damping is small and amplitude decrease when damping is large. A heavily damped system has fairly flat resonance curve.
Smaller the damping, greater will be the amplitude and more sharp will be the resonance

What is Sharpness of resonance?

Such a type of resonance in which amplitude of vibration of a body increase at maximum value when damping is very small is called sharpness of resonance.



Graph between amplitude and time



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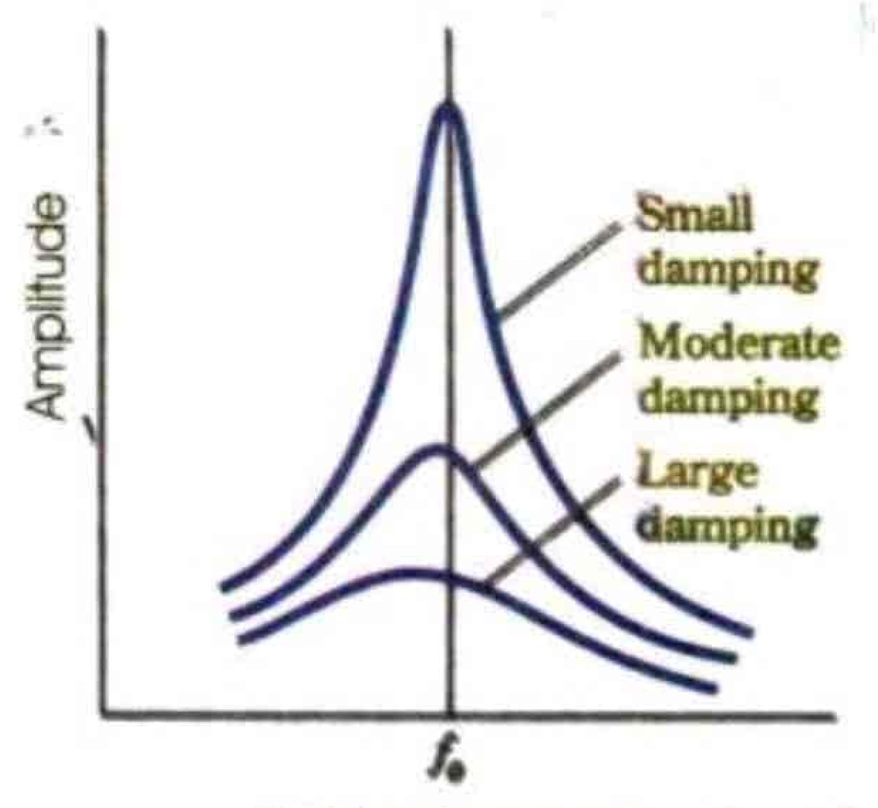

Chapter = 07

Exercise Short Questions

Q.1 ***Name two characteristics of simple harmonic motion.***

i) $a \propto -x$ Acceleration is directly proportional to the displacement and directed towards mean position

ii) Total Energy remains constant K.E+P.E=constant

Q.2 ***Does frequency depends on amplitude for harmonic oscillators?***

No. Frequency of harmonic oscillator is independent of amplitude. Because for simple pendulum $f = \frac{1}{2\pi}\sqrt{g/l}$ it depends upon length and g where for mass spring system $f = \frac{1}{2\pi}\sqrt{K/m}$ it depends upon mass and spring constant K.

Q.3 ***Can we realize an ideal simple pendulum?***

No. Due to friction and weight of the string. For an ideal simple pendulum, the string should be massless, inextensible and suspended from frictionless support and these condition are difficult to achieve.

Q.4 ***What is the total distance traveled by an object moving with SHM in a time equal to its period, if its amplitude is A?***

Total distance traveled will be 4A. Time period is time during which vibrating body completes one round trip and in one round trip total distance is A+A+A+A=4A.

Q.5 ***What happens to the period of a simple pendulum if its length is doubled? What happens if the suspended mass is doubled?***

As we know that simple pendulum, $T = 2\pi\sqrt{l/g}$ for doubling the length $T = 2\pi\sqrt{2l/g} = \sqrt{2} \times 2\pi\sqrt{l/g} = \sqrt{2}\,T$ So the time period increases by $\sqrt{2}$ (=1.414) times, as length is doubled. ii) There will be no change, when suspended mass is doubled. Since time period, T, is independent of mass, m.

Q.6 ***Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.***

No. Acceleration depends upon x, $a = -\omega^2 x$ the acceleration is zero at mean position (x = 0) and it becomes maximum at extreme position ($x = x_o$) so the acceleration of simple harmonic oscillator does not remain constant during its motion

Q.7 ***What is meant by phase angle? Does it define angle between maximum displacement and the driving force?***

i) Phase angle (or phase): "The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM". It indicates the state and direction of motion of a vibrating particle.

ii) No, It does not define angle between maximum displacement and the driving force.

Q.8 ***Under what conditions does the addition of two simple harmonic motions produce a resultant, which is also simple harmonic?***

The addition of two simple harmonic motion produce a resultant, which is also simple harmonic when

i. They have same frequency
ii. Same phase
iii. They are parallel

Q.9 ***Show that in SHM the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest.***

We have for SHM; $v = \omega\sqrt{x_o^2 - x^2}$ & $a = -\omega^2 x$ At mean position, from the above equations, X = 0 then a = 0 & v = ωx_o—maximum value, i.e. acceleration is zero and velocity is greatest. &

at extreme positions; $x = x_o$ then v = 0 & $a = -\omega^2 x_o$—maximum value. i. e. velocity is zero when acceleration is greatest.

Q.10 ***In relation to SHM, explain the equations;***

(i) $y = A\sin(\omega t + \phi)$

(ii) $a = -\omega^2 x$

$y = A\sin(\omega t + \phi)$ initial phase

Instantaneous displacement y and A is Amplitude angle subtended in time t this equation shows that displacement of SHM as a function of amplitude and phase angle depending upon time.

$a = -\omega^2 x$ where a = acceleration of a particle executing SHM ω = constant angular frequency x = instantaneous displacement from the mean position.

Q.11 ***Describe some common phenomena in which resonance plays an important role.***

1) **Tuning radio/TV** we change the frequency with knob. When it becomes equal to a particular transmitted station, resonance occurs. Then we receive amplified audio/video signals.
2) Microwave oven Microwaves (of frequency 2450 MHz) with $\lambda = 12$ cm, are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.
3) Musical instruments In some instruments (e.g. drums) air columns resonate in the wooden box. In string instruments (e.g. sitar) strings resonate with their frequencies and loud music is heard.

Q.12 ***If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?***
Due to friction and air resistance mass-spring oscillating system eventually stops. When it oscillates, due to frictional forces energy is dissipated into heat and finally it stops.

Numerical problems

No.7.1: A 100.0 g body hung on a spring elongates the spring by 4.0cm. When a certain object is hung on the spring and set vibrating, its period is 0.568s. What is the mass of the object pulling the spring?

Given Data : $m = 100g = 10/1000kg = 0.1kg$, $x = 4cm = 4/100m = 0.04m$, $T = 0.568sec$, mass of object $= m' = ?$

As $F = Kx \Rightarrow mg = Kx \Rightarrow K = \frac{mg}{x} = \frac{0.1*9.8}{0.04} = 24.5Nm^{-1}$, Now using the formula for time period of mass spring

$$T = 2\pi\sqrt{\frac{m'}{K}} \Rightarrow m' = \frac{T^2K}{4\pi^2} = \frac{(0.568)^2*24.5}{4(3.14)^2} = 0.2Kg$$

7.2: A load of 15.0g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

Given Data : $m = 15g = 15/1000 = 0.015kg$, $x = 2cm = 0.02m$, $m' = 294g = 0.294kg$, $x_o = 0.1m$, $T = ?, K = ?, V_o = ?$

$$F = Kx, \Rightarrow mg = Kx \Rightarrow K = \frac{mg}{x} = \frac{0.015*9.8}{0.02} = 7.35Nm^{-1}$$

$$T = 2\pi\sqrt{\frac{m}{K}} = 2*3.14\sqrt{\frac{0.294}{7.35}} = 1.26\sec$$

$$v_o = x_o\sqrt{\frac{K}{m+m'}} = 0.1\sqrt{\frac{7.35}{0.015+0.29}} = 0.49ms^{-1}$$

7.3: An 8.0kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find (i) Period (ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12m.

Given Data : $m = 8kg$, $xo = 0.3m$, $F = 60$ N, $x = 0.3m$, $T = ?, a = ?, v = ?$ k.E = ? P.E = ?(when $x = 0.12m$)

$$F = Kx, \Rightarrow K = \frac{F}{x} = \frac{60}{0.3} = 200Nm^{-1} \Rightarrow T = 2\pi\sqrt{\frac{m}{K}} = 2*3.14\sqrt{\frac{8}{200}} = 1.3\sec$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{200}{8}} = 4.82Hz \Rightarrow a = \omega^2 x = (4.82)^2*0.12 = 3ms^{-2}$$

$$v = \omega\sqrt{x_o^2 - x^2} = 4.82\sqrt{(0.3)^2-(0.12)^2} = 1.33ms^{-1}$$

$$K.E = \frac{1}{2}Kx_o^2(1-\frac{x^2}{x_o^2}) = \frac{1}{2}*200*(0.3)^2(1-\frac{(0.12)^2}{(0.3)^2}) = 7.6J$$

$$P.E = \frac{1}{2}Kx^2 = \frac{1}{2}(200)(0.12)^2 = 1.44J$$

7.4: A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant k = 1960 Nm^{-1}, Find the maximum distance through which the spring will be compressed.

$$\text{Given Data}: m = 4Kg, h = 0.8m, K = 1960\ Nm^{-1}, x_o = ?$$

$$P.E = mgh \text{ also } \frac{1}{2}Kx_o^2 = mgh \Rightarrow x_o^2 = \frac{2mgh}{K} \Rightarrow x_o = \sqrt{\frac{2mgh}{K}} = \sqrt{\frac{2*4*9.8*0.8}{1960}} = 0.18m$$

7.5: A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where

g = 9.8 ms^{-2}?

$$\text{Given Data}: l = 50cm = 50/100m = 0.5m, g = 9.8ms^{-2}, f = ?$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2*3.14}\sqrt{\frac{9.8}{0.5}} = 0.7Hz$$

7.6: A block of mass 1.6 kg is attached to a spring with spring constant 1000 Nm^{-1}, as shown in Fig.7.14. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x=0, if the surface is frictionless.

$$\text{Given data}: m = 1.6kg, K = 1000\ N/m, x_o = 2cm = 0.02m, v = ?$$

$$v = x_o\sqrt{\frac{K}{m}} = 0.02\sqrt{\frac{1000}{1.6}} = 0.5ms^{-1}$$

7.7: A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

$$\text{Given Data}: m_1 = 1300kg, m_2 = 160\ kg, m = m_1 + m_2 = 1300 + 160 = 1460kg$$

$$\text{for one spring, } K = 20{,}000N/m, \text{ for 4 spring} = 4*20000 = 80000\ N/m, f = ?$$

$$\text{using } f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2*3.14}\sqrt{\frac{80000}{1460}} = 1.18Hz$$

7.8: Find the amplitude, frequency and period of an object vibrating at the end of spring, if the equation for its position, as a function of time, is $x = 0.25\cos\left(\frac{\pi}{8}\right)t$ what is the displacement of the object after 2.0s?

$$\text{Given Data}: t = 20\ sec, \text{amplitude} = x_o = ?, f = ?, T = ?, \text{ x when } t = 2\ sec = ?$$

$$\text{comparing given eq } x = 0.25\cos\left(\frac{\pi}{8}\right)t \text{ with } x = x_o\cos(\omega)t \Rightarrow x_o = 0.25m$$

$$\omega = \frac{\pi}{8} \Rightarrow 2\pi\pi = \frac{\pi}{8} \Rightarrow f = \frac{1}{16}Hz \qquad T = \frac{1}{f} = \frac{1}{1/16} = 16\sec$$

$$x = 0.25\cos\left(\frac{\pi}{8}\right)*2 = 0.25\cos\left(\frac{\pi}{4}\right) = 0.18\ m$$

Chapter = 07

Previous all Punjab Board Exams Solved MCQs

	Questions	Option A	Option B	Option C	Option D
1)	Distance covered by oscillating body during one vibration of amplitude A	2A	**4A**	0	A^2
2)	Which of these quantity has unit kgs^{-2}	Surface tension	**Spring constant**	Force	Momentum
	$K=F/x=N/m=Kgms^{-2}/m=Kgs^{-2}$				
3)	The product of frequency and time period	**1**	0	3.14	2
	As f=1/T, fT=1				
4)	The frequency of second pendulum is	2 Hz	**0.5 Hz**	1 Hz	0.25 Hz
	Time period=2sec, f=1/T=1/2=0.5Hz				
5)	Angle of projection of projectile moving around a circle is given by relation a =?	-gx/l	**$-w^2x$**	-x	-gsinΘ
6)	In mass spring system mass 'm' is attached with spring of spring constant 'k' with time period 'T_1'.. Then the mass is replaced by '2m' with same spring, what is the time period 'T_2'	$T_2 = T_1$	**$T_2 = \sqrt{2}\, T_1$** Time period is directly proportional to sq root of mass so	$T_2 = 2T_1$	$T_2 = T_1/\sqrt{2}$
7)	Mass attached to spring is pulled slowly from mean position to x0 then work done will be ?	½ Kx_0	**½ Kx_0^2**	Kx_0	W^2x_0
8)	Angular displacement of a point moving in a circle 10cm when displacement of projection of this point along vertical diameter of circle is 8.66cm will	30°	**60°** Sinx=x/xo =10/8.66 Sin-(0.866) 60°	45°	75°
9)	A body performing SHM with displacement $x=x_o \sin(wt+fi)$, when t=0, $x=x_o$.. Then what is the phase angle fi??	π	π/4	**π/2** at this angle $x=x_o$	–π
10)	If distance covered by wave is 20 cm then what is the amplitude is	10cm	**5cm**	15cm	20cm
	Distance=4*amplitude, 20=4A. A=5cm				
11)	The motion of simple pendulum is SHM is only if	Amplitude is large	Mass is small	**Amplitude is small**	Length is small
12)	The expression for instantaneous displacement of particle executing SHM is given as	$a=-w^2x$	**$x=x_0 \sin\omega t$**	$F=kx$	All of these
13)	If a load of 15g is elongated a spring by 2cm, then K	7.5 N/m	7 N/m	**7.35 N/m**	0.75 N/m
14)	Instantaneous potential energy of spring mass system is given by	$\frac{1}{2}Kx^2o$	**$\frac{1}{2}Kx^2$**	Mgh	None of these

15)	At which place the motion of simple pendulum is slowest	Karachi	**K-2**	Murree	Lahore
As we know that frequency and speed is directly to g so the value of g is minimum in these places at K-2 so at that point motion of simple pendulum is slowest					
16)	A simple pendulum 50cm long its frequency of vibration at place where $g=9.8\ m/s^2$	**0.70 Hz** To see n#7.5	7 Hz	70 Hz	10 Hz
17)	A simple pendulum is moved from Earth to moon, how does it change the period of oscillation(g at moon=1.6)	Remains same	**Increased by factor $\sqrt{6}$**	Increased by factor four	Decreased by factor $\sqrt{6}$
As we know that T is inversely proporitional to sq.rt of g so at moon the value of g is g/6 so it increased T by $\sqrt{6}$ times					
18)	The frequency of waves produced in microwave oven is	2450 Hz	**2450 MHz**	2450 KHz	12 Hz
19)	Tunning of radio is an example of	Mechanical resonance	Light wave resonance	**Electrical resonance**	Physical resonance
20)	A spring of spring constant 10 N/m amplitude 2m. the maximum P.E	10 J	**20 J**	30 J	40 J
$P.E=1/2\ Kx^2=1/2*10*(2)^2=20$ J					
21)	A particle takes 0.2 sec to complete one revolution, its frequency is	10Hz	**5Hz** f=1/T=1/0.2=5 Hz	50Hz	None of these
22)	In SHM the restoring force is directly proportional to	Velocity	Acceleration	**Displacement**	Time period
23)	The waveform of simple harmonic motion is	**Sine wave**	Square wave	Pulsed wave	Cosine wave
24)	If f is frequency of body executing SHM, its angular frequency w is	**2πf**	1/f	4πf	2π/f
25)	If mass of pendulum is doubled then its time period will be	Double	Half	Four times	**Remains same**
26)	SI unit of frequency is	Radian	m/s	**Hertz**	Meter
27)	If F=0.08N and x=4cm then K=?	6 N/m	4 N/m	8N/m	**2 N/m**
K=F/x=0.08/4cm=0.08/4*10-2=2 N/m					
28)	The phase angle $\theta = wt$ of a body performing SHM indicates	Only direction of amplitude	Only magnitude of displacement	**Both A&B**	None of these
29)	The process in which energy is dissipated in oscillating system is called	Resonance	Forced oscillations	**Damping**	None of these
30)	If the frequency of oscillator is 5Hz then time period will be	0.1 Sec	**0.2 Sec**	0.4 Sec	0.5 Sec
Time period =1/f=1/5=0.2 sec					
31)	Angular frequency is basically a property of	**Circular motion**	Linear motion	Vibratory motion	Elliptical motion
32)	If the mass attached with a spring becomes four times, the time period of vibration becomes:	One fourth	Half	**Double**	3/4
33)	The displacement of projection is given as	Xo	W	t	**Wt**

	$x = x_o \sin wt$, which quantity represents phase				
34)	In a simple pendulum, the tension of the string is	g cos θ	mg cos θ	**mg sin θ**	mg
35)	An oscillating body is at mean position at t = 0. At t = T/4 it will be at	**Extreme position**	Between extreme and mean position	Mean position	Beyond extreme position
36)	At mean position during SHM	P.E is Max and K.E min	**P.E is min and K.E max**	Both K.E and P.E maximum	Both K.E and P.E min
37)	What is kinetic of body executing SHM when displacement form mean position is half of its displacement	½	**¼**	3/2	¾
Energy is directly proportional to square of amplitude, so when half then its square is ¼					
38)	Which expression is correct for the time period of simple pendulum	$T \alpha l$	**$T \alpha \sqrt{l}$**	$T \alpha m$	None of these
39)	If time period of simple pendulum is 2sec its frequency will be	1 Hz	2 Hz	**0.5 Hz**	4 Hz
Frequency=1/T=1/2=0.5Hz					
40)	SI unit of spring constant are	m^{-1}	**Nm^{-1}**	Nm^{-2}	Nm^{2}
41)	Time period of simple pendulum only depends on	Mass of bob	**Length of pendulum**	Amplitude of vibration	Size of bob
42)	A simple harmonic oscillator has a time period of 10 seconds. Which equation rotates its acceleration 'a' and displacement 'x'?	$a = -2x$	$a = -(20\pi)x$	$a = -(20\pi)^2 x$	**$a = -(2\pi/10)^2 x$** by applying $a = -w^2 x$ $a = -(2\pi/T)^2 x$
43)	The oscillation in which amplitude decreased steadily with time are called	Natural oscillations	Free oscillations	**Damped oscillations**	Forced oscillations
44)	When the length of a simple pendulum is doubled, find the ratio of the new frequency to the old frequency?	1/4	√2	1/2	**1/√2**
45)	In SHM the velocity of particle is maximum at	Extreme position	**Mean position**	Between extreme and mean position	None
46)	What is the period of mass spring system during SHM if the ratio of mass to spring constant is ¼?	**π** apply time period formula to get result	1/π	2 π	½ π
47)	Acceleration of mass spring system is	Uniform	Variable due to change in direction	Variable due to change in magnitude	**Both B&C**
48)	The unit used for factor $\sqrt{\frac{l}{g}}$ may be	Meter	**Second**	Kilogram	Radian
As $T = 2\pi\sqrt{\frac{l}{g}}$ in this formula 2π has no dimension, so $\sqrt{\frac{l}{g}}$ has unit of time also $\left(\frac{m}{ms^{-2}}\right)^{1/2} = \left(s^{-2}\right)^{1/2} = s$					
49)	The acceleration of body performing SHM depends upon its	Mass	Time period	Amplitude	**Displacement**

50	If the time period of simple pendulum is doubled its amplitude becomes	**Remains same**	Doubled	Half	1.41 as large
Time period is independent of amplitude of simple pendulum					
51)	The wave length used in micro wave oven is	**12cm**	10cm	24 cm	2470 cm
52)	10cm extension is produced in a spring due to a force of 20N. the spring constant is	2 N/m	20 N/m	**200 N/m**	2000 N/m
As extension x=10cm=10/100=0.1m, F=20N, K=F/x= 20/0.1=200 N/m					
53)	If length of simple pendulum becomes double then time period	Increase double	**Increase 1.41 times**	Increase 4 times	Decrease 1.41 times
As time period is directly proportional to sq.rt of length					
54)	One complete round trip of body in motion is called	Frequency	Amplitude	**Vibration**	Time period
55)	The expression for restoring force is	F=Kx	F=ma	F=dp/dt	**F= -Kx**
56)	A quantity which indicates the state and direction of vibrating body is called	Time period	Amplitude	**Phase**	Frequency
57)	For vibrating mass-spring system, the expression of kinetic energy at any displacement 'x' is given by:	$\frac{1}{2}kxo^2(1-\frac{x^2}{x_o{}^2})$	$\frac{1}{2}m\omega(1-\frac{x^2}{x_o{}^2})$	$\frac{1}{2}kx_o{}^2$	$\frac{1}{2}mx_o w^2$
58)	When soldiers cross a bridge, they are advised to march out of step due to	**Resonance**	High frequency	Noise produced	Fact that bridge is weak
59)	Which of the following quantity can be expressed in Kgs^{-2}	**Spring constant**	Density	Momentum	Force
60)	The wavelength of transverse wave travelling with speed v having frequency f is equal to	f/v	Vf	**V/f**	f/V^2
61)	When a particle is moving along a circular path, its projection along the diameter executes	Linear motion	Vibratory motion	Rotatory motion	**SHM**
62)	A simple pendulum complete 20 vibrations in 5 sec, frequency will be	2 Hz	3 Hz	5Hz	**4Hz**
63)	the dimension of spring constant are	$[MT^{-1}]$	**$[MT^{-2}]$**	$[MT^{-3}]$	[MT]
64)	Oscillations of shock absorber of car is an example of	SHM	Forced oscillations	**Damped oscillations**	Undamped oscillations
65)	Potential energy at mean position in SHM	Maximum	Equal to K.E	**Zero**	Negligible
66)	The maximum velocity in SHM	$x_o\omega$	$x_o\omega^2$	$x\omega$	$x\omega^2$
67)	Food being cooked in microwave oven is an example	Beats	**Resonance**	Overtones	Stationary waves

68)	What should be the ration of kinetic energy to total energy for simple harmonic oscillator? Hint take ratio of both to get the result	$1-\frac{X^2}{X_o^2}$	$(Xo^2 - X^2)$	1	$\frac{1}{2}X^2$
69)	Resonance occurs when the driving frequency is:	Greater than natural frequency	Less than natural frequency	Unequal the natural frequency	**Equal to the natural frequency**
70)	What should be the length of simple pendulum whose period is 6.28 second at a place where g = 10 ms-2.	0.28 m	6.28 m	10 m	10.8 m
$\text{apply formula of time period}, 6.28 = 2\pi\sqrt{\frac{1}{10}} = l = 10m$ as T=6.28m and 2pi has 6.28 value					
71)	A body performs simple harmonic motion with a period of 0.063 s. The maximum speed of 3.0 ms-1. What are the values of the amplitude 'xo (m) and angular frequency 'ω (rads-1)	$x_o = 5.3, \omega = 16$	**$x_o = 0.03, \omega = 100$**	$x_o = 0.19, \omega = 16$	$x_o = 3.3, \omega = 100$
$\omega = \frac{2\pi}{T} = \frac{2*3.14}{0.063} = 100, x_o = \frac{v}{w} = \frac{3}{100} = 0.03m$					
72)	Frequency of simple pendulum of length 9.8 m will be	2 π Hertz	**1/2π Hertz**	π/2 Hertz	π/4 Hertz
Apply formula for frequency of pendulum to put l=9.8m and g=9.8ms^{-2}, to get the result					