UNIT NO. 7:-

Permutation, Combination and Probability

Factorial:-

The product of natural number is called factorial. It is denoted by !

For example:-

$$1! = 1$$

$$2! = 2 \cdot 1$$

Exercise 7.1

Q.NO. 1:-

Evaluate each of the following.

i) $4!$

$= 4 \cdot 3 \cdot 2 \cdot 1$

$= 24$

ii) $6!$

$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 720$

iii) $\dfrac{8!}{7!}$

$= \dfrac{8 \cdot \cancel{7!}}{\cancel{7!}}$

$= 8$

(iv) $\dfrac{10!}{7!}$

$= \dfrac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}}$

$= 720$

(v) $\dfrac{11!}{4!\ 7!}$

$\dfrac{11 \cdot \cancel{9}^{3} \cdot \cancel{10} \cdot 8 \cdot \cancel{7!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2}^{1} \cdot 1 \cdot \cancel{7!}}$

$= 330$

(vi) $\dfrac{6!}{3!\ 3!}$

$= \dfrac{\cancel{6}^{2} \cdot 5 \cdot \cancel{4}^{2} \cdot \cancel{3!}}{\cancel{3}^{1} \cdot \cancel{2}^{1} \cdot 1 \cdot \cancel{3!}}$

$= 20$

(vii) $\dfrac{8!}{4!\ 2!}$

$= \dfrac{8 \cdot 7 \cdot \cancel{6}^{3} \cdot 5 \cdot \cancel{4!}}{\cancel{4!} \cdot \cancel{2}^{1} \cdot 1}$

$= 840$

(viii) $\dfrac{11!}{2!\ 4!\ 5!}$

$$= \frac{11\cdot10\cdot9\cdot\cancel{8}\cdot7\cdot\cancel{6}\cdot\cancel{5!}}{\cancel{2}\cdot1\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1\cdot\cancel{5!}}$$

$$= 6930$$

(ix) $\frac{9!}{2!\,(9-2)!}$

$$= \frac{9!}{2!\;7!}$$

$$= \frac{9\cdot\overset{4}{\cancel{8}}\cdot\cancel{7!}}{\cancel{2}\cdot1\cdot\cancel{7!}}$$

$$= 36$$

(x) $\frac{15!}{15!\,(15-15)!}$

$$= \frac{\cancel{15!}}{\cancel{15!}\;0!}$$

$$= \frac{1}{1}$$

$$= 1$$

(xi) $\frac{3!}{0!}$

$$= \frac{3!}{1}$$

$$= 3\cdot2\cdot1$$

$$= 6$$

(xii) $4! \cdot 0!, 1!$

$= 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1$

$= 24$

Q.NO.2:-

Write each of the fallowing in factorial form.

(i) $6 \cdot 5 \cdot 4$

$= \dfrac{6 \cdot 5 \cdot 4 \cdot 3!}{3!}$

$= \dfrac{6!}{3!}$

(ii) $12 \cdot 11 \cdot 10$

$= \dfrac{12 \cdot 11 \cdot 10 \cdot 9!}{9!}$

$= \dfrac{12!}{9!}$

(iii) $20 \cdot 19 \cdot 18 \cdot 17$

$= \dfrac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16!}{16!}$

$= \dfrac{20!}{16!}$

(iv) $\dfrac{10 \cdot 9}{2 \cdot 1}$

$$= \frac{10\cdot 9\cdot 8!}{2!\,8!}$$

$$= \frac{10!}{2!\,8!}$$

(v) $\frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}$

$$= \frac{8\cdot 7\cdot 6\cdot 5!}{3!\;5!}$$

$$= \frac{8!}{3!\,5!}$$

(vi) $\frac{52\cdot 51\cdot 50\cdot 49}{4\cdot 3\cdot 2\cdot 1}$

$$= \frac{52\cdot 51\cdot 50\cdot 49\cdot 48!}{4!\,48!}$$

$$= \frac{52!}{4!\,8!}$$

(ii) $n(n-1)(n-2)$

$$= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$$

$$= \frac{n}{(n-3)!}$$

(iii) $(n+2)(n+1)(n)$

$$= \frac{(n+2)(n+1)(n)(n-1)!}{(n-1)!}$$

$$= \frac{(n+2)!}{(n-1)!}$$

(ix) $\dfrac{(n+1)(n)(n-1)}{3\cdot 2\cdot 1}$

$$= \frac{(n+1)(n)(n-1)(n-2)!}{3!\,(n-2)!}$$

$$= \frac{(n+1)!}{3!\,(n-2)!}$$

(x) $n(n-1)(n-2)\cdots(n-r+1)$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

$\therefore (n-r+1)(n-r+\cancel{r}-\cancel{1})$

$\therefore (n-r+1)(n-r)$

Definitions

Permutation:-

An ordering (arrangement) of n objects is called a permutation of the objects.

A permutation of n different objects is an ordering (arrangement) of the objects such that one object is first, one is second, one is third and so on.

Fundamental Principle of Counting: imp.

Suppose A and B are two events. The first event A can occur in p different ways. After A has occurred, B can occur in q different ways. The number of ways that the two events can occur is the product p.q.

Permutation imp.

A permutation of n different objects taken $r (\leq n)$ at a time is an arrangement of the r objects. Generally it is denoted by ${}^nP_r$ or $P(n,r)$.

Prove that:

$${}^nP_r = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

→ If $r = n$ then

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$$

$$= \frac{n!}{1}$$

$$= n!$$

Exercise 7.2

Q.NO. 1:-

Evaluate the following.

(i) $^{20}P_3$

$$^{n}P_r = \frac{n!}{(n-r)!}$$

$$^{20}P_3 = \frac{20!}{(20-3)!}$$

$$^{20}P_3 = \frac{20!}{17!}$$

$$^{20}P_3 = \frac{20.19.18.17!}{17!}$$

$$^{20}P_3 = 6840$$

(ii) $^{16}P_4$

$$^{n}P_r = \frac{n!}{(n-r)!}$$

$$^{16}P_4 = \frac{16!}{(16-4)!}$$

$$^{16}P_4 = \frac{16}{12!}$$

$$= \frac{16.15.14.13.\cancel{12!}}{\cancel{12!}}$$

$$= 43680$$

(iii) $^{12}P_5$

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^{12}P_5 = \frac{12!}{(12-5)!}$$

$$= \frac{12!}{7!}$$

$$= \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}}$$

$$= 95040$$

(iv) $^{10}P_7$

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^{10}P_7 = \frac{10!}{(10-7)!}$$

$$= \frac{10!}{3!}$$

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}}$$

$$= 604800$$

(v) $^9P_8$

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^9P_8 = \frac{9!}{(9-8)!}$$

$$= \frac{9!}{1!}$$

$$= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 362,880$$

Q.NO.2:-

Find the value of n when

(i) $^nP_2 = 30$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$$

$$n(n-1) = 30$$

$$n(n-1) = 6 \cdot 5$$

$$\boxed{n = 6}$$

(ii) $^{11}P_n = 11 \cdot 10 \cdot 9$

$$^{11}P_n = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8!}$$

$$= \frac{11!}{8!}$$

$$^{11}P_n = \frac{11!}{(11-3)!}$$

$^{11}P_n = {}^{11}P_3$

$\boxed{n=3}$

(iii) ${}^nP_4 : {}^{n-1}P_3 = 9:1$

$$\frac{{}^nP_4}{{}^{n-1}P_3} = \frac{9}{1}$$

$$\frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = 9$$

$$\frac{\frac{n!}{\cancel{(n-4)!}}}{\frac{(n-1)!}{\cancel{(n-4)!}}} = 9$$

$$\frac{n!}{(n-1)!} = 9$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = 9$$

$\boxed{n=9}$

Q.NO.3:-

Prove from the first principle that:

(i) ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$

R.H.S

$= n \cdot {}^{n-1}P_{r-1}$

$$= \frac{n \cdot (n-1)!}{(n-1-r+1)!}$$

$$= \frac{n \cdot (n-1)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^{n}P_{r}$$

$$L.H.S = R.H.S$$

(ii) $\quad {}^{n}P_{r} = {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$

R.H.S

$$= {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$$

$$= \frac{(n-1)!}{(n-1-r)!} + r \cdot \frac{(n-1)!}{(n-\cancel{1}-r+\cancel{1})!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r\,(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[1 + \frac{r}{n-r}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[\frac{n-\cancel{r}+\cancel{r}}{n-r}\right]$$

$$= \frac{n\,(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^nP_r$$

L.H.S = R.H.S

Q.NO.4:-

How many signals can be given by 5 flags of different colours when any number of flags can be used at a time?

Solution:-

$n = 5$, $r = 3$

NO. of signals = ?

$${}^nP_r = {}^5P_3$$

$${}^5P_3 = \frac{5!}{(5-3)!}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}}$$

$$= 60$$

Q.NO.5:-

How many signals can be given by 6 flags of different colours when any number of flags can be used at a time?

Solution:-

$n = 6$

no. of signals of 1 flag $= {}^6P_1$

$= 6$

no. of signals of 2 flags $= {}^6P_2$

$= 30$

no. of signals of 3 flags $= {}^6P_3$

$= 120$

no. of signals of 4 flags $= {}^6P_4$

$= 360$

no. of signals of 5 flags $= {}^6P_5$

$= 720$

no. of signals of 6 flags $= {}^6P_6$

$= 720$

Q.NO.6:-

How many words can be formed from the letters of the following words using all letters when no letter is to be repeated.

(i) PLANE

$n = 5, \; r = 5$

${}^nP_r = {}^5P_5$

$= 120$

(ii) OBJECT

$n = 6$, $r = 6$

$^nP_r = {}^6P_6$

$= 720$

(iii) FASTING

$n = 7$, $r = 7$

$^nP_r = {}^7P_7$

$= 5040$

Q.NO.7:-

How many 3-digit numbers can be formed by using each one of the digits 2, 3, 5, 7, 9 only once?

Solution:-

$n = 5$, $r = 3$

$^nP_r = {}^5P_3$

no. of digits $= 60$

Q.NO.8:-

Find the numbers greater than 23000 that can be formed from the digits 1, 2, 3, 5, 6 without repeating any digit.

Solution:-

$$n = 5 \quad , \quad r = 5$$

$$\text{Total} = {}^5P_5 = 120$$

Less:- Case - I

1 is fixed at extreme left

$\boxed{1}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

$$n = 4 \quad , \quad r = 4$$

$$\text{number} = {}^4P_4$$
$$= 24$$

Case - II:-

$\boxed{2}\ \boxed{1}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

$$n = 3 \quad , \quad r = 3$$

Less:-

$$= 24 + 6$$
$$= 30$$

Greater:- $= \text{Total} - \text{less}$

$$= 120 - 30$$
$$= 90$$

Q.NO.9:-

Find the number of 5-digit numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated) but:

(i) the digit 2 and 8 are next to each other.

$n = 5 \quad , \; r = 3$

5-digit numbers $= {}^5P_5$

$= 120$

(i) Next to each other:-

Case - I

$\boxed{28}\ \square\square\square$

$n = 4 \quad , \; r = 4$

numbers $= {}^4P_4$

$= 24$

Case - II

$\boxed{82}\ \square\square\square$

$n = 4 \; , \; r = 4$

numbers $= {}^4P_4$

$= 24$

Next to each other $= 24 + 24$

$= 48$

(ii) Not next to each other:-

= Total - next to each other

$= 120 - 48$

$= 72$

Q. NO. 11:-

How many 5-digit multiples of 5 can be formed from the digits 2, 3, 5, 7, 9 when no digit is repeated.

Solution:-

$$n = 4 \quad , r = 4$$

no. of 5-digits x of 5 $= {}^nP_r$

$= {}^4P_4$

$= 24$

Q.NO. 12:-

In how many ways can 8 books including 2 on English be arranged on a shelf in such a way that the English books are never together.

Solution:-

Let E_1 and E_2 be English books & $B_1, B_2, B_3, B_4, B_5, B_6$ be the other books.

$$n = 8$$

$$r = 8$$

Total arrangement $= {}^8P_8$

$= 40320$

Together:- **Case - I**

$\boxed{E_1 E_2}\square\square\square\square\square\square$

$n = 7$

$r = 7$

Arrangements $= {}^7P_7$

$= 5040$

Case - II

$\boxed{E_2 E_1}\square\square\square\square\square\square$

$n = 7$

$r = 7$

Arrangements $= {}^7P_7$

$= 5040$

Together $= 5040 + 5040$

$= 10080$

Never Together:-

$=$ Total $-$ Together

$= 40320 - 10080$

$= 30240$

Q.NO.10:-

How many 6-digit numbers can be formed, without repeating any digit from the digits 0, 1, 2, 3, 4, 5? In how many of them will zero be at the tens place?

Solutions:-

$n=6$, $r=6$

${}^nP_r = {}^6P_6 = 720$

(i) If 0 is fixed at extreme left

$\boxed{0}\ \square\ \square\ \square\ \square\ \square$

$n=5$, $r=5$

${}^5P_5 = 120$

6-digit numbers = Total − 5-digit

$= 720 - 120$

$= 600$

(ii) If 0 is fixed at extreme right.

$\square\ \square\ \square\ \square\ \square\ \boxed{0}$

$n=5$, $r=5$

${}^5P_5 = 120$

*Exercise 7.3

Q.NO. 12:-

How many necklaces can be made from 6 beads of different colours?

Solution:-

$n = 6$

$$\text{No. of ways} = \frac{(n-1)!}{2}$$

$$= \frac{(6-1)!}{2}$$

$$= \frac{5!}{2}$$

$$= \frac{\cancel{120}\,60}{\cancel{2}\,1}$$

$$= 60$$

Q.NO. 11:-

In how many ways can 4 keys be arranged on a circular key ring?

~~Answer:-~~ Solution:-

$n = 4$

$$\text{No. of ways} = \frac{(n-1)!}{2}$$

$$= \frac{(4-1)!}{2}$$

$$= \frac{3!}{2} = \frac{\cancel{6}\,3}{\cancel{2}\,1}$$

$$= 2$$

Q.NO.7:-

The D.C.Os of 11 districts meet to discuss the law and order situation in their districts. In how many ways can they be seated at a round table, when two particular D.C.Os insist on sitting together?

Solution:-

NO. of D.C.Os = 11

2 D.C.Os insist to sit together

$n = 10$

no. of ways $= (n-1)!$

$= (10-1)!$

$= 9!$

$= 362880 \times 2$

$= 725760$ ways

Q.NO.8:-

The Governor of the Punjab calls a meeting of 12 officers. In how many ways can they be seated at a round table?

Solution:-

$n = 12$ officers

No. of ways $= (n-1)!$

$= (12-1)!$

$= 11!$

$= 39916800$

Q.No.9:-

Fatima invites 14 people to a dinner. There are 9 males and 5 females who are seated at two different tables so that guests of one sex at a round table and the guests of the other sex at the second table. Find the number of ways in which all guests are seated?

Solution:-

Men = 9 , Women = 5

No. of ways $= (n-1)! \times (n-1)!$

$= (9-1)! \times (5-1)!$

$= 8! \times 4!$

$= 967680$

Q.NO. 10:-

Find the number of ways in which 5 men and 5 women can be seated at a round table in such a way that not two persons of the same sex sit together.

Solution:-

men = 5 , women = 5

$n = 10$

$$\text{No. of ways} = (n-1)!$$
$$= (10-1)!$$
$$= 9!$$
$$= 362880$$

Q.NO. 1:-

How many arrangements of the letters of the following words, taken all together, can be made:

(i) PAKPATTAN

P is repeated = 2

A is repeated = 3

K is repeated = 1

T is repeated = 2

N is repeated = 1

$$\text{No. of ways} = \frac{n!}{n_1!\,n_2!\,n_3!\,n_4!\,n_5!}$$

$$= \frac{9!}{2!\,3!\,2!\,1!\,1!}$$

$$= \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3!}{2\cdot 1\cdot 3!\cdot 2\cdot 1\cdot 1\cdot 1}$$

$$= \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}!}{\cancel{2}\cdot 1\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{2}!\cdot 1\cdot 1}$$

$$= 15120$$

(iii) PAKISTAN

P is repeated = 1

A is repeated = 2

K is repeated = 1

I is repeated = 1

S is repeated = 1

T is repeated = 1

N is repeated = 1

$$\text{No. of ways} = \frac{n!}{n_1!\,n_2!\,n_3!\,n_4!\,n_5!\,n_6!\,n_7!}$$

$$= \frac{8!}{1!\,2!\,1!\,1!\,1!\,1!\,1!}$$

$$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot \cancel{2}!}{\cancel{2}!\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1}$$

$= 20160$

(iii) MATHEMATICS

M is repeated = 2

A is repeated = 2

T is repeated = 2

H is repeated = 1

E is repeated = 1

I is repeated = 1

C is repeated = 1

S is repeated = 1

$$\text{No. of ways} = \frac{n!}{n_1!\,n_2!\,n_3!\,n_4!\,n_5!\,n_6!\,n_7!\,n_8!}$$

$$= \frac{11!}{2!\cdot 2!\cdot 2!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!}$$

$$= \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot \cancel{4}\cdot 3\cdot \cancel{2!}}{\cancel{2}\cdot 1\cdot \cancel{2}\cdot 1\cdot \cancel{2!}}$$

$$= 4989600$$

(iv) ASSASSINATION

A is repeated = 3

S is repeated = 4

I is repeated = 2

N is repeated = 2

T is repeated = 1

O is repeated = 1

$$\text{No. of ways} = \frac{n!}{n_1!\, n_2!\, n_3!\, n_4!\, n_5!\, n_6!}$$

$$= \frac{13!}{3! \cdot 4! \cdot 2! \cdot 2! \cdot 1! \cdot 1!}$$

$$= \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot 4 \cdot 3 \cdot \cancel{2!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{2} \cdot 1 \cdot \cancel{2!}}$$

$$= 10810800$$

Q.NO.2:-

How many permutation of the letters of the word PANAMA can be made, if P is to be the first letter in each arrangement?

Solution:-

$\boxed{P}\,\Box\,\Box\,\Box\,\Box\,\Box$

A is repeated = 3

N is repeated = 1

M is repeated = 1

$$\text{No. of ways} = \frac{n!}{n_1!\, n_2!\, n_3!}$$

$$= \frac{5!}{3! \cdot 1! \cdot 1!}$$

$$= \frac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!} \cdot 1 \cdot 1}$$

$$= 20$$

Q.NO.3:-

How many arrangements of the letters of the words ATTACKED can be made if each arrangement begins with C and ends with K?

Solution:-

$\boxed{C}\square\square\square\square\square\square\boxed{K}$

$n = 6$

A is repeated = 2

T is repeated = 2

E is repeated = 1

D is repeated = 1

$$\text{No. of ways} = \frac{n!}{n_1!\,n_2!\,n_3!\,n_4!}$$

$$= \frac{6!}{2!\cdot 2!\cdot 1!\cdot 1!}$$

$$= \frac{6\cdot 5\cdot \overset{2}{\cancel{4}}\cdot 3\cdot \cancel{2!}}{\cancel{2}\cdot 1\cdot \cancel{2!}}$$

$$= 180$$

Q.NO.4:-

How many numbers greater than 1000,000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4?

Solutions:-

$n = 7$

0 is repeated = 1

2 is repeated = 3

3 is repeated = 1

4 is repeated = 2

No. of ways = $\dfrac{7!}{3!\cdot 2!\cdot 1!\cdot 1!}$

$= \dfrac{7\cdot \cancel{6}\cdot 5\cdot 4\cdot 3\cdot \cancel{2!}}{\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{2!}\cdot 1\cdot 1}$

$= 420$

If 0 is fixed at extreme left

$\boxed{0}\ \Box\ \Box\ \Box\ \Box\ \Box\ \Box$

$n = 6$

No. of ways = $\dfrac{6!}{3!\cdot 2!}$

$= \dfrac{\cancel{6}\cdot 5\cdot 4\cdot 3\cdot \cancel{2!}}{\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{2!}}$

$= 60$

Greater than 200,000 = Total - Less

$= 420 - 60$

$= 360$

Q.NO.5:-

How many 6-digits numbers can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them will lie between 400,000 and 430,000?

Solution:-

$n = 6$

2 is repeated = 2

3 is repeated = 2

4 is repeated = 2

$$\text{No. of ways} = \frac{6!}{2!\cdot 2!\cdot 2!}$$

$$= \frac{6\cdot 5\cdot 4\cdot 3\cdot \cancel{2!}}{2\cdot 1\cdot 2\cdot 1\cdot \cancel{2!}}$$

$$= 90$$

Condition:-

between 400,000 and 430,000

$\boxed{4}\ \boxed{2}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

$n = 4$

2 is repeated = 1

4 is repeated = 1

3 is repeated = 2

No. of ways b/w 400,000 and 430,000 $= \frac{4!}{2!}$

$$= \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}}$$

$$= 12$$

Q.NO. 6:-

11 members of a club form 4 committees of 3, 4, 2, 2 members so that no member is a member of more than one committee. Find the number of committes?

Solution:-

$$n = 11$$

$$\text{No. of ways} = \frac{11!}{3! \cdot 4! \cdot 2! \cdot 2!}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2!}$$

$$= 69300$$

Definitions

Combinations:-

While counting the number of possible permutation of a set of objects, the order is important.

The number of combinations of n different objects taken r at a time is denoted by ${}^nC_r$, or $C(n,r)$ or $\binom{n}{r}$ and is given by

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

Proof:-

$$ {}^nC_r \times r! = {}^nP_r $$

$$ {}^nC_r \times r! = \frac{n!}{(n-r)!} $$

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

Complementary Combination:-

$$ {}^nC_r = {}^nC_{n-r} $$

Proof:-

$$ {}^nC_{n-r} = \frac{n!}{(n-r)!(n-\cancel{r}+\cancel{r})!} $$

$$ {}^nC_{n-r} = \frac{n!}{(n-r)!\,r!} $$

$$ {}^nC_{n-r} = \frac{n!}{r!(n-r)!} $$

$$ {}^nC_{n-r} = {}^nC_r $$

* Exercise 7.4

Q. NO. 1:-

Evaluate the following:

(i) $^{12}C_3$

$$^{12}C_3 = \frac{12!}{3!(12-3)!}$$

$$= \frac{\cancel{12}^{2}\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{9!}}$$

$$= 220$$

(ii) $^{20}C_{17}$

$$^{20}C_{17} = \frac{20!}{17!(20-17)!}$$

$$= \frac{20\cdot 19\cdot 18\cdot\cancel{17!}}{\cancel{17!}\ 3!}$$

$$= \frac{\cancel{6840}\ 1140}{\cancel{6}}$$

$$= 1140$$

(iii) $^{n}C_4$

$$^{n}C_4 = \frac{n!}{4!(n-4)!}$$

$$= \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{4!\ \cancel{(n-4)!}}$$

$$= \frac{n(n-1)(n-2)(n-3)}{24}$$

Q.NO.2:-

Find the value of n, when

(i) $^nC_5 = {}^nC_4$

$^nC_r = {}^nC_{n-r}$

$^nC_5 = {}^nC_{n-5}$

$^nC_{n-5} = {}^nC_4$

$n-5 = 4$

$n = 4+5$

$\boxed{n = 9}$

imp. S.Q

(ii) $^nC_{10} = \dfrac{12 \times 11}{2!}$

$^nC_{10} = \dfrac{12 \times 11 \times 10!}{2!\ 10!}$

$^nC_{10} = \dfrac{12!}{2!\ 10!}$

$^nC_{10} = \dfrac{12!}{10!(12-10)!}$

$^nC_{10} = {}^{12}C_{10}$

$\boxed{n = 12}$

(iii) $^nC_{12} = {}^nC_6$

$^nC_r = {}^nC_{n-r}$

$^nC_{12} = {}^nC_{n-12}$

$^nC_{n-12} = {}^nC_6$

$$n - 12 = 6$$

$$n = 6 + 12$$

$$\boxed{n = 18}$$

Q.NO.3:-

Find the value of n and r, when.

i) ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$

$$r! \times {}^{n}C_r = {}^{n}P_r$$

$$r! \times 35 = 210$$

$$r! = \frac{\cancel{210}\ 6}{\cancel{35}\ 1}$$

$$r! = 6$$

$$r! = 3 \cdot 2 \cdot 1$$

$$r! = 3!$$

$$\boxed{r = 3}$$

$${}^{n}P_3 = 210$$

$$\frac{n!}{(n-3)!} = 210$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 210$$

$$n(n-1)(n-2) = 7 \cdot 6 \cdot 5$$

$$\boxed{n = 7}$$

Q.NO. 18:-

Show that:

$$^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$$

L.H.S:-

$$= {}^{16}C_{11} + {}^{16}C_{10}$$

$$= \frac{16!}{11!(16-11)!} + \frac{16!}{10!(16-10)!}$$

$$= \frac{16!}{11!\,5!} + \frac{16!}{10!\,6!}$$

$$= \frac{16!}{11\cdot 10!\cdot 5!} + \frac{16!}{10!\,6\cdot 5!}$$

$$= \frac{16!}{10!\cdot 5!}\left[\frac{1}{11} + \frac{1}{6}\right]$$

$$= \frac{16!}{10!\cdot 5!}\left[\frac{6+11}{11\cdot 6}\right]$$

$$= \frac{17\cdot 16!}{11\cdot 10!\cdot 6\cdot 5!}$$

$$= \frac{17!}{11!\cdot 6!}$$

$$= \frac{17!}{11!(17-11)!}$$

$$= {}^{17}C_{11}$$

So L.H.S = R.H.S

Q.NO. 10:-

Prove that:

$${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$$

L.H.S:-

$$= {}^{n}C_r + {}^{n}C_{r-1}$$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-(r-1))!}$$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} \qquad \because (n-r+1)!(n-r+1-1) \quad (n-r)!$$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right]$$

$$= \frac{(n+1)n!}{r(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!}$$

$$= \frac{(n+1)!}{r!(n+1-r)!} = {}^{n+1}C_r$$

Q.NO. 4:-

How many (a) diagonals and (b) triangles can be formed by joining the vertices of the polygon having:

(i) 5 sides

No. of diagnals = $^nC_2$

$^5C_2 = 10$

No. of triangles = $^nC_3 - n$

$^5C_3 - 5 = 5$

(ii) 8 sides

No. of diagonals = $^nC_2$

$^8C_2 = 28$

No. of triangle = $^nC_3 - n$

$^8C_3 - 8 = 48$

(iii) 12 sides

No. of diagonals = $^nC_2$

$^{12}C_2 = 66$

No. of triangles = $^nC_3 - n$

$^{12}C_3 - 12 = 208$

Q.NO. 5:-

The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution:-

No. of boys = 12 , Members = 3 boys

No. of total girls = 8 ,

Committee members = 2 girls

$$\text{No. of ways} = {}^{n}C_r \cdot {}^{n}C_r$$

$n = 12$, $r = 3$ | $n = 8$, $r = 2$

$$= {}^{12}C_3 \cdot {}^{8}C_2$$

$$= 220 \cdot 28$$

$$= 6160$$

Q.NO.6:-

How many committees of 5 members can be chosen from a group of 8 persons when each committee must include 2 particular persons?

Solution:-

Total Persons = 8

Committee members = 5

$n = 6$, $r = 3$

$$\text{No. of ways} = {}^{n}C_r = ?$$

$$\text{No. of ways} = {}^{6}C_3$$

$$= 20$$

Q.NO.7:-

In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

Solution:-

Total players = 15

Players in team = 11

$n = 14, \quad r = 10$

$^{n}C_{r} = {}^{14}C_{10} = 1001$

Q.NO.8:-

Show that:

$^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

L.H.S

$$\frac{16!}{11!(16-11)!} + \frac{16!}{10!(16-10)!} = {}^{17}C_{11}$$

$$\frac{16!}{11!\ 5!} + \frac{16!}{10!\ 6!} = {}^{17}C_{11}$$

$$\frac{16!}{11 \cdot 10! \cdot 5!} + \frac{16!}{10!\ 6 \cdot 5!} = {}^{17}C_{11}$$

$$\frac{16!}{10! \cdot 5!}\left(\frac{1}{11} + \frac{1}{6}\right) = {}^{17}C_{11}$$

$$\frac{16!}{10!\,5!}\left(\frac{6+11}{6\cdot 11}\right) = {}^{17}C_{11}$$

$$\frac{17\cdot 16!}{6\cdot 5!\cdot 11\cdot 10!} = {}^{17}C_{11}$$

$$\frac{17!}{11!\cdot 6!} = {}^{17}C_{11}$$

$$\frac{17!}{11!(17-11)!} = {}^{17}C_{11}$$

$${}^{17}C_{11} = {}^{17}C_{11}$$

Q.NO.9:-

There are 8 men and 10 women members of a club. How many committees of seven members can be formed?

Solution:-

Men = 8

Women = 10

Committee members = 7

(i) 4 women

NO. of committees $= {}^{8}C_3 \times {}^{10}C_4$

$= 11760$

(ii) at the most 4 women

$= {}^{8}C_7 \times {}^{10}C_0 + {}^{8}C_6 \times {}^{10}C_1 + {}^{8}C_5 \times {}^{10}C_2 + {}^{8}C_4 \times {}^{10}C_3$
$+ {}^{8}C_3 \times {}^{10}C_4$

(iii) At least 4-women

$$= {}^{8}C_3 \times {}^{10}C_4 + {}^{8}C_2 \times {}^{10}C_5 + {}^{8}C_1 \times {}^{10}C_6 + {}^{8}C_0 \times {}^{10}C_7$$

$$= 20616$$

Definitions

Probability:-

Probability is the nummerical evaluation of a chance that a particular event would occur.

Sample Space:-

The set S consisting of all possible out comes of a given experiment is called the sample space.

Event:-

A particular outcome is called an event and usually denoted by E

* Exercise 7.5

For the following experiments, find the probability in each case:

1- Experiment:

From a box containing orange-flavoured sweets, Bilal takes out one sweet with looking.

Event Happening:

i) the sweet is orange-flavoured

$n(S) = 1$

A represents 'orange flavoured'

$P(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{1}$$

$= 1$

ii) the sweet is lemon-flavoured

$n(B) = 0$

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{1}$$

$= 0$

Q.NO.2:- Experiment

Pakistan and India play a cricket match. The result is:

Events Happenings:

i) Pakistan wins

A represents 'Pakistan win'

$n(S) = 3$

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{1}{3}$$

(ii) India does not lose

B represents 'india does'nt lose'

$n(B) = 2$

$$P(B) = \frac{2}{3}$$

Q.NO.3:- Experiment:

There are 5 green and 3 red balls in a box, one ball is taken out,

Event Happenings:

(i) the ball is green

$n(S) = 8$

A represents 'ball is green'

$n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{5}{8}$$

(ii) the ball is red

B represents 'ball is red.'

$$n(B) = 3$$

$$P(B) = \frac{n(B)}{P(B)}$$

$$= \frac{3}{8}$$

Q.NO.6:- Experiment:

From a box containing slips numbered 1,2,3,4,5 one slip is picked up.

Events Happening:

i) the number on the slip is prime number.

$$n(S) = 5$$

A represents 'prime numbers'

$$A = \{2,3,5\}$$

$$n(A) = 3$$

$$P(A) = \frac{3}{5}$$

ii) the number on the slip is multiple of 3.

$$n(B) = 1$$

$$P(B) = \frac{1}{5}$$

Q.NO.4:- Experiment:

A fair coin is tossed three times. It shows

Events Happening:

(i) One tail

$S = \{HHH, HTT, THT, TTH, THH, HTH, HHT, TTT\}$

$n(S) = 8$

A represents 'one tail'

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

(ii) at least one head

B represents 'at least one head'

$n(B) = 7$

$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

Q.NO.5:- Experiment:

A die is rolled. The top shows

Events Happening:

(i) 3 or 4 dots

$n(S) = 6$

A represents '3 or 4 dots'

$n(A) = 2$

$P(A) = \frac{\cancel{2}\,1}{\cancel{6}\,3}$

$= \frac{1}{3}$

(ii) dots less than 5

B represents 'dot less than 5'

$n(B) = 4$

$P(B) = \frac{\cancel{4}\,2}{\cancel{6}\,3}$

$= \frac{2}{3}$

Q.NO.7:- Experiment:

Two dice, one red and the other but blue, are rolled simultaneously. The numbers of dots on the tops are added. The total of two scores is:

Events Happening:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

(i) 5

A represents 'total score is 5'

$$n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{39}$$

$$= \frac{1}{9}$$

(ii) 7

B represents 'total score is 7'

$$n(B) = 6$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{6}\,1}{\cancel{36}\,6} = \frac{1}{6}$$

(iii) 11

C represents 'total score is 7'

$$n(C) = 2$$

$$P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{\cancel{2}\,1}{\cancel{36}\,18} = \frac{1}{18}$$

Q.NO.8:- Experiment:

A bag contains 40 balls out of which 5 are green, 15 are black and the remaining are yellow. A ball is taken out of the bag.

Events Happening:

(i) The ball is black

Green balls = 5 , Black ball = 15

Yellow balls = 20

$n(S) = 40$

A represents 'Black balls'

$n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{15}^{3}}{\cancel{40}^{8}}$$

$$= \frac{3}{8}$$

(ii) The ball is green

B represents 'Green balls'

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{5}^{1}}{\cancel{40}_{8}}$$

$$= \frac{1}{8}$$

(iii) The ball is not green

C represents 'ball is not green'

$n(C) = 15 + 20$

$= 35$

$$P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{\cancel{35}^{7}}{\cancel{40}_{8}}$$

Q.NO.9:- Experiment:

One chit out of 30 containing the names of 30 students of a class of 18 boys and 12 girls is taken out at random, for nomination as the monitor of the class.

$n(S) = 30$

Events Happening:

(i) the monitor is a boy

A represents 'the monitor is boy'

$n(A) = 18$

$$P = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{18}\,3}{\cancel{30}\,5}$$

$$= \frac{3}{5}$$

(ii) the monitor is a girl

B represents 'the monitor is girl.'

$n(B) = 12$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{12}\,2}{\cancel{30}\,5}$$

Q.NO. 10:- Experiment:

A coin is tossed four times. The tops show.

Events Happening:

(i) all heads

S = {HHHH, HHHT, HHTH, HTHH, THHH, TTTT, TTTH, TTHT, THTT, HTTT, HHTT, TTHH, HTHT, THTH, THHT, HTTH}

$n(S) = 16$

A represents 'all heads'

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{1}{16}$$

(ii) 2 heads and two tails

B represents '2 heads and 2 tails'

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{6}\,3}{\cancel{16}\,8}$$

$$= \frac{3}{8}$$

Exercise 7.7

Q.NO.1:-

If sample spaces = {1,2,3 ...,9}
Event A = {2,4,6,8} and Event
B = {1,3,5}, find P(PUB).

Solution:-

$S = \{1,2,3,\ldots 9\}$

$n(S) = 9$

$A = \{2, 4, 6, 8\}$

$n(A) = 4$

$B = \{1, 3, 5\}$

$n(B) = 3$

Using formula.

$$P(A \cup B) = P(A) + P(B)$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{9}$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{3}{9}$$

$$P(A \cup B) = \frac{4}{9} + \frac{3}{9}$$

$$= \frac{4+3}{9}$$

$$= \frac{7}{9}$$

Q.NO.2:-

A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

Solution:-

$$n(S) = 60$$

A represents 'red marbles'

$$n(A) = 10$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{10}{60}$$

B represents 'white marble'.

$$n(B) = 30$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{30}{60}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{10}{60} + \frac{30}{60}$$

$$= \frac{10+30}{60}$$

$$= \frac{\cancel{40}^{2}}{\cancel{60}^{3}}$$

$$= \frac{2}{3}$$

Q.NO.3:-

A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen numbers is a multiple of 3 or of 5?

Solution:-

$$S = \{1, 2, 3, \dots, 50\}$$

$$n(S) = 50$$

A represents 'multiple of 3'.

$$A = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48\}$$

$$n(A) = 16$$

B represents 'multiple of 5'.

$$B = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$$

$$n(B) = 10$$

$$A \cap B = \{15, 30, 45\} \Rightarrow n(A \cap B) = 3$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{16}{50}$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{10}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$= \frac{3}{50}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{16}{50} + \frac{10}{50} - \frac{3}{50}$$

$$= \frac{16 + 10 - 3}{50}$$

$$= \frac{23}{50}$$

Q.NO.5:-

A die thrown twice. What is the probability that the sum of the number of dots shown is 3 or 11?

Solution:-

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$

$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)$

A represents 'the sum is 3'

$A = \{(1,2), (2,1)\}$

$n(A) = 2$

B represents 'the sum is 11'

$B = \{(6,5), (5,6)\}$

$n(B) = 2$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{2}{36}$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{2}{36}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{2}{36} + \frac{2}{36}$$

$$= \frac{2+2}{36}$$

$$= \frac{\cancel{4}}{\cancel{36}\,9}$$

$$= \frac{1}{9}$$

Q.NO.6:-

Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6.

Solution:

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$n(S) = 36$$

A represents "the sum is 4."

$$A = \{(1,3),(2,2),(3,1)\}$$

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{3}{36}$$

B represents 'the sum is 6'

$B = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{5}{36}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{3}{36} + \frac{5}{36}$$

$$= \frac{3+5}{36}$$

$$= \frac{\cancel{8}^{2}}{\cancel{36}_{9}}$$

$$= \frac{2}{9}$$

Q.NO.8:-

There are 10 girls and 20 girls in a class. Half of the boys and half of the girls have blue eyes. Find the probability that one student chosen as monitor is either a girl or has blue eyes.

Solutions:-

Girls = 10

Boys = 20

Blue eyes = 15

$n(S) = 30$

A represents 'monitor is a girl'

$n(A) = 10$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{10}{30}$$

B represents 'monitor is blue eyes'

$n(B) = 15$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{15}{30}$$

$n(A \cap B) = 5$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$= \frac{5}{30}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{10}{30} + \frac{15}{30} - \frac{5}{30}$$

$$= \frac{10+15-5}{30}$$

$$= \frac{\cancel{20}\,2}{\cancel{30}\,3}$$

$$= \frac{2}{3}$$

Q.NO.7:-

Two dice are thrown simultaneously. If the event A is that the sum of the number of dots shown is an odd number and the event B is that the number of dots shown on atleast one die is 3. Find (AUB).

Solution:-

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) = 36$

A represents 'the sum is odd number'

$= \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),$
$(3,2),(3,4),(3,6),(4,1),(4,3),$
$(4,5),(5,4),(5,6),(6,1),(6,3),(6,5)\}$

$n(A) = 18$

$P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{18}{36}$

B represents 'atleast one dice is 3'

$= \{(3,1),(2,3),(3,3),(1,3),(4,3),$
$(3,4),(5,3),(3,6),(3,5),(3,2),$
$(6,3)\}$

$n(B) = 11$

$P(B) = \dfrac{n(B)}{n(S)}$

$= \dfrac{11}{36}$

$A \cap B = \{(3,2),(2,3),(3,4),(3,6),(4,3),$
$(6,3)\}$

$n(A \cap B) = 6$

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)}$$

$$= \frac{6}{36}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{18}{36} + \frac{11}{36} - \frac{6}{36}$$

$$= \frac{18+11-6}{36}$$

$$= \frac{23}{36}$$

Q.NO. 4:-

A card is drawn from a deck of 52 playing cards. What is the probability that it is a diamond card or an ace?

Solution:-

$$n(S) = 52$$

A represents 'diamond card'

$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{13}{52}$$

B represents 'Ace cards'

$$n(B) = 4$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{4}{52}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$= \frac{1}{52}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{13 + 4 - 1}{52}$$

$$= \frac{\cancel{16}\ 4}{\cancel{52}\ 13}$$

$$= \frac{4}{13}$$

Exercise 7.8

Q.NO. 1:-

The probability that a person A will be alive 15 year hence is $\frac{5}{7}$ and the probability that another person B will be alive 15 years hence is $\frac{7}{9}$. Find the probability that both will be alive 15 year hence.

Solution:-

$$P(A) = \frac{5}{7}$$

$$P(B) = \frac{7}{9}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{5}{\cancel{7}} \times \frac{\cancel{7}}{9}$$

$$= \frac{5}{9}$$

Q.NO. 2:-

A die is rolled twice:

Event E_1 is the apperance of even number of dots and event

E_2 is the appearance of more than 4 dots. Prove that:

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

$E_1 = \{2, 4, 6\}$

$n(E_1) = 3$

$$P(E_1) = \frac{n(E_1)}{n(S)}$$

$$= \frac{3}{6}$$

$E_2 = \{5, 6\}$

$n(E_2) = 2$

$$P(E_2) = \frac{2}{6}$$

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$E_1 \cap E_2 = 6$ $\quad = \frac{3}{6} \cdot \frac{2}{6}$

$n(E_1 \cap E_2) = 1$

$$P(E_1 \cap E_2) = \frac{1}{6} = \frac{\cancel{6}}{\cancel{6} \cdot 6}$$

$$\frac{1}{6} = \frac{1}{6}$$

So L.H.S = R.H.S

Q.NO.3:-

Determine the probability of getting 2 heads in two successive tosses of a balanced coin.

Solution:-

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$

A represents '2 heads'

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{1}{4}$$

Q.NO.4:-

Two coins are tossed twice each. Find the probability that the head appears on the first toss and same faces appear in the two tosses.

Solution:-

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, TTHH, THTH, HHTT, TTTT, HTTT, TTHT, TTTH, THHT, HTTH, HTHT, THTT\}$

$$n(S) = 16$$

A represents 'head appears on 1st toss'

$$n(A) = 8$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{8}^{1}}{\cancel{16}_{2}}$$

$$= \frac{1}{2}$$

Similarly,

$$n(B) = 8$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{8}^{1}}{\cancel{16}_{2}}$$

$$= \frac{1}{2}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{4}$$

Q. NO. 7:-

Two dice are thrown twice. Find the probability that ~~a prime number~~ the sum of the dots shown in the first throw is 7 and that of the second throw is 11?

Solution:-

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(S) = 36$

A represents 'sum is 7'

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),$
$(6,1)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{6}\,^{1}}{\cancel{36}\,_{6}}$$

$$= \frac{1}{6}$$

B represents 'sum is 11'

$B = \{(5,6), (6,5)\}$

$n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{2}^{1}}{\cancel{36}_{18}}$$

$$= \frac{1}{18}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{6} \cdot \frac{1}{18}$$

$$= \frac{1}{108}$$

Q.NO.8:-

Find the probability that the sum of dots appearing in two successive throws of two dice is every time 7?

Solution:-

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(S) = 36$

A represents 'sum is 7.'

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{6}^{1}}{\cancel{36}_{6}}$$

$$= \frac{1}{6}$$

Similarly,

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{6}^{1}}{\cancel{36}_{6}}$$

$$= \frac{1}{6}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{6} \cdot \frac{1}{6}$$

$$= \frac{1}{36}$$

Q.NO.9:-

A fair die is throw twice. Find the probability that a prime number of dots appears in the first throw and the number of dots in the second throw is less than 5.

Solution:-

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6$$

A represents 'prime numbers'

$$A = \{2, 3, 5\}$$

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$$

$$= \frac{1}{2}$$

B represents 'less than 5'

$B = \{1, 2, 3, 4\}$

$n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{4}^{2}}{\cancel{6}_{3}}$$

$$= \frac{2}{3}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}$$

$$= \frac{1}{3}$$

Question NO. 10:-

A bag contain 8 red, 5 white and 7 black balls. 3 balls are drawn from the bag. What is the probability that the first ball is red, the second ball is white and the third ball is black, when every time the ball is replaced?

Solution:-

$n(S) = 20$

A represents 'red balls'

$n(A) = 8$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{8}^{2}}{\cancel{20}_{5}}$$

$$= \frac{2}{5}$$

B represents 'white balls'

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{5}^{1}}{\cancel{20}_{4}}$$

$$= \frac{1}{4}$$

C represents 'black balls'

$n(C) = 7$

$$P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{7}{20}$$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$= \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{7}{20}$$

$$= \frac{\cancel{14}\,7}{\cancel{400}\,200}$$

$$= \frac{7}{200}$$

Q. NO. 5:-

Two cards are drawn from a deck of 52 playing cards. If one card is drawn and replaced before drawing the second card, find the probability that both the cards are aces.

Solution:-

$$n(S) = 52$$

A represents 'Ace card'

$$n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{4}\,1}{\cancel{52}\,13}$$

$$= \frac{1}{13}$$

Similarly,

$$P(B) = \frac{1}{13}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{13} \cdot \frac{1}{13}$$

$$= \frac{1}{169}$$

Q.NO.6:-

Two cards from a deck of 52 playing cards are drawn in such a way that the card is replaced after the first draw. Find the probabilities in the following cases.

(i) first card is king and the second is a queen.

$$n(S) = 52$$

A represents 'king cards'

$$n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{4}\,1}{\cancel{52}\,13}$$

$$= \frac{1}{13}$$

B represents 'Queen cards'

$$n(B) = 4$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{4}^{1}}{\cancel{52}_{13}}$$

$$= \frac{1}{13}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{13} \cdot \frac{1}{13}$$

$$= \frac{1}{169}$$

ii) both the cards are faced cards

ie. king, queen, jack

$$n(S) = 52$$

A represents 'face cards'

$$n(A) = 12$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{\cancel{12}^{3}}{\cancel{52}_{13}}$$

$$= \frac{3}{13}$$

Similarly,

$$n(B) = 12$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\cancel{12}\,3}{\cancel{52}\,13}$$

$$= \frac{3}{13}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{3}{13} \cdot \frac{3}{13}$$

$$= \frac{9}{169}$$