# MATHEMATICS 1st YEAR

# UNIT # 14

## TRIGONOMETRIC EQUATIONS

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# Sherazi Mathematics



### اچھی باتیں

1۔ جو کسی کا برا نہیں چاہتے ان کے ساتھ کوئی برا نہیں کر سکتا یہ میرے رب کا وعدہ ہے۔

2۔ برے سلوک کا بہترین جواب اچھا سلوک اور جہالت کا جواب "خاموشی" ہے۔

3۔ کوئی مانے یا نہ مانے لیکن زندگی میں دو ہی اپنے ہوتے ہیں ایک خود اور ایک خدا۔

4۔ جو دو گے وہی لوٹ کے آئے گا عزت ہو یا دھوکہ۔

5۔ جس سے اس کے والدین خوشی سے راضی نہیں اس سے اللہ بھی راضی نہیں۔

# Trigonometric Equations

The equations having at least one trigonometric function are called trigonometric equations. e.g.,

$\sin x = \frac{2}{5}$ , $\sec x = \tan x$ ,

$\sin^2 x - \sec x + 1 = \frac{3}{4}$

**Remember,**
Trigonometric equations have an infinite number of solutions due to the periodicity of trigonometric functions. e.g., if $\sin\theta = 0$ then $\theta = 0, \pm\pi, \pm 2\pi, \ldots$ which can be written as $\theta = n\pi,\ n \in Z$

## Solution of trigonometric Equations

i) first find the solution over the interval whose length is equal to its period.

ii) Find the general solution

iii) when a trigonometric equation contains more than one function then we use trigonometric identities and algebraic formulas to put it in a single function.

**Example 1.** Solve the equation
$\sin x = \frac{1}{2}$

**Solution:-**
$\because \sin x$ is +ive in I and II quadrant.

Reference angle $= x = \frac{\pi}{6}$

$\therefore\ x = \frac{\pi}{6} \longrightarrow$ in I quad

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

$S.S = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\},\ n \in Z$

**Example 2.** Solve the equation $1 + \cos x = 0$

**Solution:-** $1 + \cos x = 0$

$\rightarrow \cos x = -1$

There is only one solution,
$x = \pi$ in $[0, 2\pi]$ $\because$ period of $\cos x$ is $2\pi$ so general value of $x$ is $\pi + 2n\pi,\ n \in Z$ so

$S.S = \{\pi + 2n\pi\},\ n \in Z$

**Example 3.** Solve the equation:
$4\cos^2 x - 3 = 0$

**Solution:-** $4\cos^2 x - 3 = 0$

$\rightarrow 4\cos^2 x = 3$

$\rightarrow \cos^2 x = \frac{3}{4} \rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$ , $\cos x = -\frac{\sqrt{3}}{2}$

$\boxed{\cos x = \frac{\sqrt{3}}{2}}$

$\because \cos x$ is +ive in I and IV quadrants.

Reference angle $= x = \frac{\pi}{6}$

$\therefore\ x = \frac{\pi}{6} \longrightarrow$ in I quad

$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \longrightarrow$ in IV quad

$\therefore$ general values of $x$ are

$\frac{\pi}{6} + 2n\pi$, $\frac{11\pi}{6} + 2n\pi$, $n \in Z$

$\boxed{\cos x = -\frac{\sqrt{3}}{2}}$

$\because \cos x$ is −ive in II and III quadrants.

Reference angle $= x = \frac{\pi}{6}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \longrightarrow$ in III quad

$\therefore$ general values of $x$ are

$\frac{5\pi}{6} + 2n\pi$, $\frac{7\pi}{6} + 2n\pi$, $n \in Z$

$\therefore\ S.S = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{11\pi}{6} + 2n\pi\right\}$
$\cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}$
$,\ n \in Z$

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**Example 1.** Solve $\sin x + \cos x = 0$

**Solution:-** $\sin x + \cos x = 0$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} = 0$$

$$\Rightarrow \tan x + 1 = 0$$

$$\Rightarrow \tan x = -1$$

$\because \tan x$ is -ive in II and IV quadrants.

Reference angle $= x = \frac{\pi}{4}$

$x = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \longrightarrow$ in II quad

$x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \longrightarrow$ not in $[0, \pi]$

so it is not solution.

$\therefore$ general value of $x$ is $\frac{3\pi}{4} + n\pi, n \in Z$

$\therefore$ $S.S = \{\frac{3\pi}{4} + n\pi\}, n \in Z$

**Example 2.** Find the solution set of: $\sin x \cos x = \frac{\sqrt{3}}{4}$

**Solution:-** $\sin x \cos x = \frac{\sqrt{3}}{4}$

$$\Rightarrow \frac{1}{2}(2\sin x \cos x) = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \sin 2x = \frac{\sqrt{3}}{2}$$

$\because \sin 2x$ is +ive in I and II quadrants.

Reference angle $= 2x = \frac{\pi}{3}$

$2x = \frac{\pi}{3} \longrightarrow$ in I quad

$2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \longrightarrow$ in II quad

$\therefore$ general values of $2x$ are

$\frac{\pi}{3} + 2n\pi, \quad \frac{2\pi}{3} + 2n\pi, \quad n \in Z$

$\Rightarrow$ general values of $x$ are

$\frac{\pi}{6} + n\pi, \quad \frac{\pi}{3} + n\pi, \quad n \in Z$

$\therefore$ $S.S = \{\frac{\pi}{6} + n\pi\} \cup \{\frac{\pi}{3} + n\pi\}, \quad n \in Z$

**Example 3.** Solve the equation: $\sin 2x = \cos x$

**Solution:-** $\sin 2x = \cos x$

$$\Rightarrow \sin 2x - \cos x = 0$$

$$\Rightarrow 2\sin x \cos x - \cos x = 0$$

$$\cos x(2\sin x - 1) = 0$$

$\cos x = 0, \quad 2\sin x - 1 = 0$

$\cos x = 0, \quad \sin x = \frac{1}{2}$

$$\boxed{\cos x = 0}$$

$\Rightarrow x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

$\therefore$ general values of $x$ are

$\frac{\pi}{2} + 2n\pi, \quad \frac{3\pi}{2} + 2n\pi, \quad n \in Z$

$$\boxed{\sin x = \frac{1}{2}}$$

$\because \sin x$ is +ive in I and II quad.

Reference angle $= x = \frac{\pi}{6}$

$x = \frac{\pi}{6} \longrightarrow$ in I quad

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

$\therefore$ general values of $x$ are

$\frac{\pi}{6} + 2n\pi, \quad \frac{5\pi}{6} + 2n\pi, \quad n \in Z$

$\therefore$ $S.S = \{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{3\pi}{2} + 2n\pi\} \cup \{\frac{\pi}{6} + 2n\pi\} \cup \{\frac{5\pi}{6} + 2n\pi\}$

**Example 4.** Solve the equation: $\sin^2 x + \cos x = 1$

**Solution:-**

$$\sin^2 x + \cos x = 1$$

$$\Rightarrow 1 - \cos^2 x + \cos x = 1$$

$$\Rightarrow -\cos^2 x + \cos x = 0$$

$$\Rightarrow -\cos x(\cos x - 1) = 0$$

$$\Rightarrow \cos x(\cos x - 1) = 0$$

$\cos x = 0, \quad \cos x - 1 = 0$

or $\cos x = 0, \quad \cos x = 1$

$$\boxed{\cos x = 0}$$

$x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

$\therefore$ general values of $x$ are

$\frac{\pi}{2} + 2n\pi$ , $\frac{3\pi}{2} + 2n\pi$ , $n \in Z$

$\boxed{\cos x = 1}$

$\Rightarrow x = 0, \quad x = 2\pi$

$\therefore$ general values of $x$ are

$0 + 2n\pi$ , $2\pi + 2n\pi$ , $n \in Z$

$\therefore S.S = \{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{3\pi}{2} + 2n\pi\}$

$\cup \{2n\pi\} \cup \{2(n+1)\pi\}$ , $n \in Z$

$\because \{2(n+1)\pi\} \subset \{2n\pi\}$ , $n \in Z$

so $S.S = \{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{3\pi}{2} + 2n\pi\}$

$\cup \{2n\pi\}$, $n \in Z$

**Example 5.** Solve the equation:

$\csc x = \sqrt{3} + \cot x$

**Solution:-**

$\csc x = \sqrt{3} + \cot x$

$\Rightarrow \frac{1}{\sin x} = \sqrt{3} + \frac{\cos x}{\sin x}$

$\Rightarrow 1 = (\sqrt{3} + \frac{\cos x}{\sin x}) \sin x$

$\Rightarrow 1 = \sqrt{3} \sin x + \cos x$

$\Rightarrow 1 - \cos x = \sqrt{3} \sin x$

$\Rightarrow (1 - \cos x)^2 = (\sqrt{3} \sin x)^2$

$1 + \cos^2 x - 2\cos x = 3\sin^2 x$

$1 + \cos^2 x - 2\cos x = 3(1 - \cos^2 x)$

$\cos^2 x - 2\cos x + 1 = 3 - 3\cos^2 x$

$\Rightarrow \cos^2 x + 3\cos^2 x + 2\cos x + 1 - 3 = 0$

$\Rightarrow 4\cos^2 x - 2\cos x - 2 = 0$

or $2\cos^2 x - \cos x - 1 = 0$

$2\cos^2 x - 2\cos x + \cos x - 1 = 0$

$2\cos x(\cos x - 1) + 1(\cos x \;\; 1) = 0$

$(\cos x - 1)(2\cos x + 1) = 0$

$\cos x = 1, \quad \cos x = \frac{-1}{2}$

$\boxed{\cos x = 1}$

$\Rightarrow x = 0, 2\pi$

given equation is not satisfied

$\because \operatorname{cosec} x$ and $\cot x$ becomes undefined for $x = 0$ and $x = 2\pi$.

$\boxed{\cos x = -\frac{1}{2}}$

$\because \cos x$ is -ive in II and III quad.

Ref. angle $= x = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \longrightarrow$ in II quad

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \longrightarrow$ in III quad

but $x = \frac{4\pi}{3}$ does not satisfy given equation. so $\frac{4\pi}{3}$ is extraneous root. so

$S.S = \{\frac{2\pi}{3} + 2n\pi\}$ , $n \in Z$

# Exercise 14

**Q1.** Find the solutions of the following equations which lie in $[0, 2\pi]$

i) $\sin x = -\frac{\sqrt{3}}{2}$

**Solution:-** $\sin x = -\frac{\sqrt{3}}{2}$

$\because \sin x$ is -ive in III and IV quad.

Reference angle $= x = \frac{\pi}{3}$

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ — in III quad

$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ — in IV quad

Principal Solution:-

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

ii) $\operatorname{cosec} \theta = 2$

**Solution:-** $\operatorname{cosec} \theta = 2$

$\Rightarrow \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\because \sin\theta$ is +ive in I and II quad.

Reference angle $= \theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} \longrightarrow$ in I quad

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

Principal solution:-

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

iii) $\sec x = -2$

**Solution:-** $\sec x = -2$

$\rightarrow \frac{1}{\cos x} = -2$

$\rightarrow \cos x = -\frac{1}{2}$

$\because \cos$ is -ive in II and III quad.

Reference angle $= x = \frac{\pi}{3}$

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \longrightarrow$ in II quad

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \longrightarrow$ in III quad

Principal solution:-

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

iv) $\cot\theta = \frac{1}{\sqrt{3}}$

**Solution:-** $\cot\theta = \frac{1}{\sqrt{3}}$

$\rightarrow \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}$

$\rightarrow \tan\theta = \sqrt{3}$

$\because \tan\theta$ is +ive in I and III quad.

Reference angle $= \theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} \longrightarrow$ in I quad

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \longrightarrow$ in III quad

Principal solution:-

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

**Q2-** Solve the trigonometric equations:

i) $\tan^2\theta = \frac{1}{3}$

**Solution:-** $\tan^2\theta = \frac{1}{3}$

$\rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ , $\tan\theta = -\frac{1}{\sqrt{3}}$

$\boxed{\tan\theta = \frac{1}{\sqrt{3}}}$

$\tan\theta$ is +ive in I and III quad.

Ref. angle $= \theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} \longrightarrow$ in I quad

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \longrightarrow$ in III quad

$\therefore$ general values of $\theta$ are

$\frac{\pi}{6} + n\pi, \quad \frac{7\pi}{6} + n\pi, \quad n \in \mathbb{Z}$

$\boxed{\tan\theta = -\frac{1}{\sqrt{3}}}$

$\tan\theta$ is -ive in II and IV quad.

Ref. angle $= \theta = \frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \longrightarrow$ in IV quad

$\therefore$ general values of $\theta$ are

$\frac{5\pi}{6} + n\pi, \quad \frac{11\pi}{6} + n\pi, \quad n \in \mathbb{Z}$

$\therefore S.S = \{\frac{\pi}{6} + n\pi\} \cup \{\frac{7\pi}{6} + n\pi\} \cup$
$\{\frac{5\pi}{6} + n\pi\} \cup \{\frac{11\pi}{6} + n\pi\}, n \in \mathbb{Z}$

ii) $\csc^2\theta = \frac{4}{3}$

**Solution:-** $\csc^2\theta = \frac{4}{3}$

$\rightarrow \frac{1}{\sin^2\theta} = \frac{4}{3}$

$\rightarrow \sin^2\theta = \frac{3}{4}$

$\rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$ , $\sin\theta = -\frac{\sqrt{3}}{2}$

$\boxed{\sin\theta = \frac{\sqrt{3}}{2}}$

$\sin\theta$ is +ive in I and II quad.

Ref. angle $= \theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} \longrightarrow$ in I quad

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \longrightarrow$ in II quad

$\therefore$ general values of $\theta$ are

$\frac{\pi}{3} + 2n\pi, \quad \frac{2\pi}{3} + 2n\pi, \quad n \in Z$

$$\boxed{\sin\theta = -\frac{\sqrt{3}}{2}}$$

$\sin\theta$ is $-$ive in III and IV quad.

Ref. angle $= \theta = \frac{\pi}{3}$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \longrightarrow$ in III quad

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \longrightarrow$ in IV quad

$\therefore$ general values of $\theta$ are

$\frac{4\pi}{3} + 2n\pi, \quad \frac{5\pi}{3} + 2n\pi, \quad n \in Z$

$S.S = \{\frac{\pi}{3} + 2n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\}$
$\cup \{\frac{4\pi}{3} + 2n\pi\} \cup \{\frac{5\pi}{3} + 2n\pi\}, n \in Z$

iii) $\sec^2\theta = \frac{4}{3}$

**Solution:-** $\sec^2\theta = \frac{4}{3}$

$\rightarrow \frac{1}{\cos^2\theta} = \frac{4}{3}$

$\rightarrow \cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}, \quad \cos\theta = -\frac{\sqrt{3}}{2}$

$$\boxed{\cos\theta = \frac{\sqrt{3}}{2}}$$

$\because \cos\theta$ is $+$ive in I and IV quad.

Ref. angle $= \theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} \longrightarrow$ in I quad

$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \longrightarrow$ in IV quad

$\therefore$ general values of $\theta$ are

$\frac{\pi}{6} + 2n\pi, \quad \frac{11\pi}{6} + 2n\pi, \quad n \in Z$

$$\boxed{\cos\theta = -\frac{\sqrt{3}}{2}}$$

$\because \cos\theta$ is $-$ive in II and III quad.

Ref. angle $= \theta = \frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \longrightarrow$ in III quad

$\therefore$ general values of $\theta$ are

$\frac{5\pi}{6} + 2n\pi, \quad \frac{7\pi}{6} + 2n\pi, \quad n \in Z$

$\therefore S.S = \{\frac{\pi}{6} + 2n\pi\} \cup \{\frac{11\pi}{6} + 2n\pi\}$
$\cup \{\frac{5\pi}{6} + 2n\pi\} \cup \{\frac{7\pi}{6} + 2n\pi\}, n \in Z$

iv) $\cot^2\theta = \frac{1}{3}$

**Solution:-** $\cot^2\theta = \frac{1}{3}$

$\rightarrow \frac{1}{\tan^2\theta} = \frac{1}{3}$

$\rightarrow \tan^2\theta = 3$

$\rightarrow \tan\theta = \pm\sqrt{3}$

$\tan\theta = \sqrt{3}, \quad \tan\theta = -\sqrt{3}$

$$\boxed{\tan\theta = \sqrt{3}}$$

$\because \tan\theta$ is $+$ive in I and III quad.

Ref. angle $= \theta = \pi/3$

$\theta = \frac{\pi}{3} \longrightarrow$ in I quad

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \longrightarrow$ in III quad

$\therefore$ general values of $\theta$ are

$\frac{\pi}{3} + n\pi, \quad \frac{2\pi}{3} + n\pi, \quad n \in Z$

$$\boxed{\tan\theta = -\sqrt{3}}$$

$\because \tan\theta$ is $-$ive in II and IV quad.

Ref. angle $= \theta = \frac{\pi}{3}$

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \longrightarrow$ in II quad

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \longrightarrow$ in IV quad

$\because$ general values of $\theta$ are

$\frac{2\pi}{3} + n\pi, \quad \frac{5\pi}{3} + n\pi, \quad n \in Z$

$\because S.S = \{\frac{\pi}{3} + n\pi\} \cup \{\frac{4\pi}{3} + n\pi\} \cup \{\frac{2\pi}{3} + n\pi\}$
$\cup \{\frac{5\pi}{3} + n\pi\}$

Find the values of $\theta$ satisfying the following equations:

**Q3.** $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

**Solution:-** $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

$\rightarrow (\sqrt{3}\tan\theta)^2 + 2\sqrt{3}\tan\theta + (1)^2 = 0$

$\rightarrow (\sqrt{3}\tan\theta + 1)^2 = 0$

$\rightarrow \sqrt{3}\tan\theta + 1 = 0$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\because \tan\theta$ is $-$ive in II and IV quad.

Ref. angle $= \theta = \frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \longrightarrow$ in II quad

$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \longrightarrow$ in IV quad

Required values of $\theta$ are $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

**Q4.** $\tan^2\theta - \sec\theta - 1 = 0$

**Solution:-** $\tan^2\theta - \sec\theta - 1 = 0$

$\rightarrow \sec^2\theta - 1 - \sec\theta - 1 = 0$

$(\sec\theta - 1)(\sec\theta + 1) - 1(\sec\theta + 1) = 0$

$(\sec\theta + 1)(\sec\theta - 1 - 1) = 0$

$\sec\theta + 1 = 0, \quad \sec\theta - 2 = 0$

$\sec\theta = -1, \quad \sec\theta = 2$

$\cos\theta = -1, \quad \cos\theta = \frac{1}{2}$

$\boxed{\cos\theta = -1}$

$\rightarrow \theta = \pi$

$\boxed{\cos\theta = \frac{1}{2}}$

$\because \cos\theta$ is $+$ive in I and IV quad

Ref. angle $= \theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} \longrightarrow$ in I quad

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \longrightarrow$ in IV quad

Required values of $\theta$ are $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

**Q5.** $2\sin\theta + \cos^2\theta - 1 = 0$

**Solution:-** $2\sin\theta + \cos^2\theta - 1 = 0$

$\rightarrow 2\sin\theta + 1 - \sin^2\theta - 1 = 0$

$\rightarrow 2\sin\theta - \sin^2\theta = 0$

$\rightarrow \sin\theta(2 - \sin\theta) = 0$

$\rightarrow \sin\theta = 0, \quad 2 - \sin\theta = 0$

$\rightarrow \sin\theta = 2$

$\boxed{\sin\theta = 0}$, $\boxed{\sin\theta = 2}$ (not possible) $(\because -1 \le \sin\theta \le 1)$

$\rightarrow \theta = 0, \pi$

Required values are $0, \pi$.

**Q6.** $2\sin^2\theta - \sin\theta = 0$

**Solution:-** $2\sin^2\theta - \sin\theta = 0$

$\rightarrow \sin\theta(2\sin\theta - 1) = 0$

$\rightarrow \sin\theta = 0, \quad 2\sin\theta - 1 = 0$

$\sin\theta = 0, \quad \sin\theta = \frac{1}{2}$

$\boxed{\sin\theta = 0}$

$\rightarrow \theta = 0, \pi$

$\boxed{\sin\theta = \frac{1}{2}}$

$\sin\theta$ is $+$ive in I and II quad.

Ref. angle $= \theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{6}$ —— in I quad

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ —— in II quad

Required values of $\theta$ are $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

**Q7.** $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

**Solution:-**

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

($\div$ by $\sin^2\theta$ we get)

$\rightarrow 3\cot^2\theta - 2\sqrt{3}\cot\theta - 3 = 0$

subtract and add $\sqrt{3}\cot\theta$

$3\cot^2\theta - 2\sqrt{3}\cot\theta - \sqrt{3}\cot\theta + \sqrt{3}\cot\theta - 3 = 0$

$3\cot^2\theta - 2\sqrt{3}\cot\theta - \sqrt{3}\cot\theta + \sqrt{3}\cot\theta - \sqrt{3}\sqrt{3} = 0$

$3\cot\theta(\cot\theta-\sqrt{3})+\sqrt{3}(\cot\theta-\sqrt{3})=0$

$(\cot\theta-\sqrt{3})(3\cot\theta+\sqrt{3})=0$

$\cot\theta-\sqrt{3}=0$ or $3\cot\theta+\sqrt{3}=0$

$\cot\theta=\sqrt{3}$ or $\cot\theta=\frac{-\sqrt{3}}{3}$

$\cot\theta=\sqrt{3}$ , $\cot\theta=-\frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\Rightarrow \tan\theta=\frac{1}{\sqrt{3}}$ , $\cot\theta=-\frac{1}{\sqrt{3}}$

$\tan\theta=-\sqrt{3}$

$\boxed{\tan\theta=\frac{1}{\sqrt{3}}}$

$\because \tan\theta$ is +ive in I and III quad.

Ref. angle $=\theta=\frac{\pi}{6}$

$\theta=\frac{\pi}{6}\longrightarrow$ in I quad

$\theta=\pi+\frac{\pi}{6}=\frac{7\pi}{6}\longrightarrow$ in III quad

$\boxed{\tan\theta=-\sqrt{3}}$

$\because \tan\theta$ is −ive in II and IV quad.

Ref angle $=\theta=\frac{\pi}{3}$

$\theta=\pi-\frac{\pi}{3}=\frac{2\pi}{3}\longrightarrow$ in II quad

$\theta=2\pi-\frac{\pi}{3}=\frac{5\pi}{3}\longrightarrow$ in IV quad

Hence

$\theta=\frac{\pi}{6},\frac{7\pi}{6},\frac{2\pi}{3},\frac{5\pi}{3}$

## Q8. $4\sin^2\theta-8\cos\theta+1=0$

**Solution:-**

$4\sin^2\theta-8\cos\theta+1=0$

$\Rightarrow 4(1-\cos^2\theta)-8\cos\theta+1=0$

$\Rightarrow 4-4\cos^2\theta-8\cos\theta+1=0$

$-4\cos^2\theta-8\cos\theta+5=0$

$\Rightarrow 4\cos^2\theta+8\cos\theta-5=0$

$4\cos^2\theta+10\cos\theta-2\cos\theta-5=0$

$2\cos\theta(2\cos\theta+5)-1(2\cos\theta+5)=0$

$(2\cos\theta+5)(2\cos\theta-1)=0$

$2\cos\theta+5=0$, $2\cos\theta-1=0$

$\Rightarrow \cos\theta=-\frac{5}{2}$ (impossible)

$\because -1\le\cos\theta\le 1$

$\cos\theta=\frac{1}{2}$

$\because \cos\theta$ is +ive in I and IV quad.

Ref. angle $=\theta=\frac{\pi}{3}$

$\theta=\frac{\pi}{3}\longrightarrow$ in I quad

$\theta=2\pi-\frac{\pi}{3}=\frac{5\pi}{3}\longrightarrow$ in IV quad.

Hence

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$

Find the solution sets of the following equations:

## Q9. $\sqrt{3}\tan x-\sec x-1=0$

**Solution:-**

$\sqrt{3}\tan x-\sec x-1=0$

$\Rightarrow \sqrt{3}\tan x=\sec x+1$

squaring both sides

$(\sqrt{3}\tan x)^2=(\sec x+1)^2$

$\Rightarrow 3\tan^2 x=\sec^2 x+1+2\sec x$

$\Rightarrow 3(\sec^2 x-1)-\sec^2 x-2\sec x-1=0$

$3\sec^2 x-3-\sec^2 x-2\sec x-1=0$

$\therefore 2\sec^2 x-2\sec x-4=0$

$\sec^2 x-\sec x-2=0$ $(\div$ by $2)$

$\sec^2 x-2\sec x+\sec x-2=0$

$\sec x(\sec x-2)+1(\sec x-2)=0$

$(\sec x+1)(\sec x-2)=0$

$\sec x+1=0$, $\sec x-2=0$

$\sec x=-1$ , $\sec x=2$

$\cos x=-1$ , $\cos x=\frac{1}{2}$

$\boxed{\cos x=-1}$

$x=\cos^{-1}(-1)=\pi$

$\boxed{\cos x=\frac{1}{2}}$

$\because \cos x$ is +ive in I and IV quad.

Ref. angle $= x = \frac{\pi}{3}$

$x = \frac{\pi}{3} \longrightarrow$ in I quad

$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \longrightarrow$ in IV quad

but $x = \frac{5\pi}{3}$ not satisfies given equation. Hence

$S.S = \{\frac{\pi}{3} + 2n\pi\} \cup \{\pi + 2n\pi\}, n \in Z$

## Q10. $\cos 2x = \sin 3x$

**Solution:-** $\cos 2x = \sin 3x$

$\rightarrow 1 - 2\sin^2 x = 3\sin x - 4\sin^3 x$

$\rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

take $\sin x = 1$

| | | | | |
|---|---|---|---|---|
| 1 | 4 | -2 | -3 | 1 |
| | | 4 | 2 | -1 |
| | 4 | 2 | -1 | $\boxed{0}$ |

$4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

$\rightarrow (\sin x - 1)(4\sin^2 x + 2\sin x - 1) = 0$

$\sin x - 1 = 0, \quad 4\sin^2 x + 2\sin x - 1 = 0$

$\rightarrow \sin x = 1$

$x = \frac{\pi}{2}$

$4\sin^2 x + 2\sin x - 1 = 0$

$\rightarrow \sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$

$= \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8}$

$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{2(-1 \pm \sqrt{5})}{8}$

$\sin x = \frac{-1 \pm \sqrt{5}}{4}$

$\rightarrow \sin x = \frac{-1 - \sqrt{5}}{4} = -0.8090$

$\sin x = \frac{-1 + \sqrt{5}}{4} = 0.3090$

$\sin x = -0.8090, \quad \sin x = 0.3090$

$\boxed{\sin x = -0.8090}$

$\because \sin x$ is $-$ive in III and IV quad.

Ref. angle $= x = 54°$

$\rightarrow x = 54° \times \frac{\pi}{180} = \frac{3\pi}{10}$

$x = \pi + \frac{3\pi}{10} = \frac{13\pi}{10} \longrightarrow$ in III quad

$x = 2\pi - \frac{3\pi}{10} = \frac{17\pi}{10} \longrightarrow$ in IV quad.

$\boxed{\sin x = 0.3090}$

$\sin x$ is $+$ive in I and II quad.

Ref. angle $= x = 18°$

$\rightarrow x = 18 \times \frac{\pi}{180} = \frac{\pi}{10}$

$x = \frac{\pi}{10} \longrightarrow$ in I quad

$x = \pi - \frac{\pi}{10} = \frac{9\pi}{10} \longrightarrow$ in II quad

Hence $S.S = \{\frac{\pi}{10} + 2n\pi\} \cup \{\frac{9\pi}{10} + 2n\pi\}$

$\cup \{\frac{13\pi}{10} + 2n\pi\} \cup \{\frac{17\pi}{10} + 2n\pi\}$

$, n \in Z$

## Q11. $\sec 3\theta = \sec\theta$

**Solution:-** $\sec 3\theta = \sec\theta$

$\rightarrow \cos 3\theta = \cos\theta$

$\rightarrow \cos 3\theta - \cos\theta = 0$

$-2\sin\frac{3\theta+\theta}{2}\sin\frac{3\theta-\theta}{2} = 0$

$\rightarrow \sin 2\theta \sin\theta = 0$

$\sin 2\theta = 0$ or $\sin\theta = 0$

$\rightarrow 2\theta = \sin^{-1}(0), \quad \theta = \sin^{-1}(0)$

$\rightarrow 2\theta = n\pi, n \in Z, \theta = n\pi, n \in Z$

$\theta = \frac{n\pi}{2}, n \in Z$ Hence

$S.S = \{\frac{n\pi}{2}\} \cup \{n\pi\}, n \in Z$

## Q12. $\sin 2x + \sin x = 0$

**Solution:-** $\sin 2x + \sin x = 0$

$\rightarrow 2\sin x\cos x + \sin x = 0$

$\rightarrow \sin x\,(2\cos x+1)=0$

$\rightarrow \sin x=0, \quad 2\cos x+1=0$

$x=\sin^{-1}(0) \quad , \quad \cos x=-\frac{1}{2}$

$x=n\pi, n\in Z$

$\boxed{\cos x=-\frac{1}{2}}$

$\because \cos x$ is $-$ive in II and III quad.

Ref. angle $= x=\frac{\pi}{3}$

$x=\pi-\frac{\pi}{3}=2\frac{\pi}{3} \longrightarrow$ in II quad

$x=\pi+\frac{\pi}{3}=\frac{4\pi}{3} \longrightarrow$ in III quad

Hence

$S.S=\{n\pi\}\cup\{\frac{2\pi}{3}+2n\pi\}\cup\{\frac{4\pi}{3}+2n\pi\}, n\in Z$

## Q13. $\tan 2\theta+\cot\theta=0$

**Solution:-** $\tan 2\theta+\cot\theta=0$

$\rightarrow \frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos\theta}{\sin\theta}=0$

$\rightarrow \frac{\sin 2\theta\sin\theta+\cos 2\theta\cos\theta}{\cos 2\theta\sin\theta}=0$

$\rightarrow \cos 2\theta\cos\theta+\sin 2\theta\sin\theta=0$

$\rightarrow \cos(2\theta-\theta)=0$

$\rightarrow \cos\theta=0$

$\rightarrow \theta=\frac{\pi}{2}, \frac{3\pi}{2}$

$S.S=\{\frac{\pi}{2}+2n\pi\}\cup\{\frac{3\pi}{2}+2n\pi\}, n\in Z$

## Q14. $\sin 4x-\sin 2x=\cos 3x$

**Solution:-** $\sin 4x-\sin 2x=\cos 3x$

$\rightarrow 2\cos\frac{4x+2x}{2}\sin\frac{4x-2x}{2}=\cos 3x$

$\rightarrow 2\cos 3x\sin x=\cos 3x$

$\rightarrow 2\cos 3x\sin x-\cos 3x=0$

$\cos 3x\,(2\sin x-1)=0$

$\cos 3x=0, \quad 2\sin x-1=0$

$\sin x=\frac{1}{2}$

$\rightarrow 3x=\frac{\pi}{2}, \frac{3\pi}{2}$

$\rightarrow 3x=\frac{\pi}{2}+2n\pi, \frac{3\pi}{2}+2n\pi, n\in Z$

$\rightarrow x=\frac{\pi}{6}+\frac{2n\pi}{3}, \frac{\pi}{2}+\frac{2n\pi}{3}, n\in Z$

$\boxed{\sin x=\frac{1}{2}}$

$\because \sin x$ is $+$ive in I and II quad.

Ref. angle $= x=\frac{\pi}{6}$

$x=\frac{\pi}{6} \longrightarrow$ in I quad

$x=\pi-\frac{\pi}{6}=\frac{5\pi}{6} \longrightarrow$ in II quad

Hence,

$S.S=\{\frac{\pi}{6}+\frac{2n\pi}{3}\}\cup\{\frac{\pi}{2}+\frac{2n\pi}{3}\}$
$\cup\{\frac{\pi}{6}+2n\pi\}\cup\{\frac{5\pi}{6}+2n\pi\}, n\in Z$

## Q15. $\sin x+\cos 3x=\cos 5x$

**Solution:-** $\sin x+\cos 3x=\cos 5x$

$\rightarrow \sin x+\cos 3x-\cos 5x=0$

or $\cos 5x-\cos 3x-\sin x=0$

$-2\sin\frac{5x+3x}{2}\sin\frac{5x-3x}{2}-\sin x=0$

$-2\sin 4x\sin x-\sin x=0$

$\sin x\,(-2\sin 4x-1)=0$

$\sin x=0, \quad -2\sin 4x-1=0$

$\rightarrow x=0, \pi \quad , \quad \sin 4x=-\frac{1}{2}$

$\boxed{\sin 4x=-\frac{1}{2}}$

$\because \sin x$ is $-$ive in III and IV quad.

Ref. angle $=4x=\frac{\pi}{6}$

$4x=\frac{\pi}{6}$

$4x=\pi+\frac{\pi}{6}=\frac{7\pi}{6} \longrightarrow$ in III quad

$4x=2\pi-\frac{\pi}{6}=\frac{11\pi}{6} \longrightarrow$ in IV quad

$\rightarrow 4x=\frac{7\pi}{6}+2n\pi$

$x=\frac{7\pi}{24}+\frac{n\pi}{2}$

$4x=\frac{11\pi}{6}+2n\pi \quad , n\in Z$

$\rightarrow x=\frac{11\pi}{24}+\frac{n\pi}{2}$

$S.S = \{0+2n\pi\} \cup \{\pi+2n\pi\}$
$\cup \left\{\frac{7\pi}{24}+\frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24}+\frac{n\pi}{2}\right\}$
$, n \in Z$

# Q16. $\sin 3x + \sin 2x + \sin x = 0$

**Solution:-**

$\sin 3x + \sin x + \sin 2x = 0$

$\rightarrow 2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2} + \sin 2x = 0$

$2\sin 2x \cos x + \sin 2x = 0$

$\rightarrow \sin 2x(2\cos x + 1) = 0$

$\sin 2x = 0 \quad 2\cos x + 1 = 0$

$\rightarrow \cos x = -\frac{1}{2}$

$\boxed{\sin 2x = 0,}$

$\rightarrow 2x = \sin^{-1}(0)$

$2x = 0, \pi$

$\rightarrow 2x = 0 + 2n\pi \quad , n \in Z$

and $2x = \pi + 2n\pi$

$x = 0 + n\pi \quad , n \in Z$

and $x = \frac{\pi}{2} + n\pi$

$\boxed{\cos x = -\frac{1}{2}}$

$\because \cos x$ is $-$ive in II and III quad.

Ref. angle $= x = \frac{\pi}{3}$

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \longrightarrow$ in II quad

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \longrightarrow$ in III quad.

$S.S = \{0+n\pi\} \cup \left\{\frac{\pi}{2}+n\pi\right\}$
$\cup \left\{\frac{2\pi}{3}+2n\pi\right\} \cup \left\{\frac{4\pi}{3}+2n\pi\right\}, n \in Z$

# Q17. $\sin 7x - \sin x = \sin 3x$

**Solution:-**

$\sin 7x - \sin x - \sin 3x = 0$

$\rightarrow 2\cos\frac{7x+x}{2}\sin\frac{7x-x}{2} - \sin 3x = 0$

$\rightarrow 2\cos 4x \sin 3x - \sin 3x = 0$

$\rightarrow \sin 3x(2\cos 4x - 1) = 0$

$\sin 3x = 0, \quad 2\cos 4x - 1 = 0$

$\boxed{\sin 3x = 0}$ , $\cos 4x = \frac{1}{2}$

$\rightarrow 3x = \sin^{-1}(0)$

$3x = 0, \pi$

$\rightarrow 3x = 0 + 2n\pi \quad , n \in Z$

and $3x = \pi + 2n\pi$

$\rightarrow x = 0 + \frac{2n\pi}{3} \quad , n \in Z$

and $x = \frac{\pi}{3} + \frac{2n\pi}{3}$

$\boxed{\cos 4x = \frac{1}{2}}$

$\because \cos x$ is $+$ive in I and IV quad.

Ref. angle $= 4x = \frac{\pi}{3}$

$4x = \frac{\pi}{3} \longrightarrow$ in I quad

$4x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \rightarrow$ in IV quad

$4x = \frac{\pi}{3} + 2n\pi, \quad 4x = \frac{5\pi}{3} + 2n\pi$
$, n \in Z$

$x = \frac{\pi}{12} + \frac{n\pi}{2}, \quad x = \frac{5\pi}{12} + \frac{n\pi}{2}, n \in Z$

$S.S = \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3}+\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{12}+\frac{n\pi}{2}\right\}$
$\cup \left\{\frac{5\pi}{12}+\frac{n\pi}{2}\right\}, n \in Z$

# Q18. $\sin x + \sin 3x + \sin 5x = 0$

**Solution:-**

$\sin 5x + \sin x + \sin 3x = 0$

$\rightarrow 2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} + \sin 3x = 0$

$2\sin 3x \cos 2x + \sin 3x = 0$

$\sin 3x(2\cos 2x + 1) = 0$

$\sin 3x = 0, \quad 2\cos 2x + 1 = 0$

$\cos 2x = -\frac{1}{2}$

$\boxed{\sin 3x = 0}$

$\rightarrow 3x = \sin^{-1}(0)$

$3x = 0, \pi$

$3x = 0 + 2n\pi \quad , n \in Z$

and $3x = \pi + 2n\pi$

$x = \frac{2n\pi}{3}, \quad x = \frac{\pi}{3} + \frac{2n\pi}{3}, n \in Z$

$\boxed{\cos 2x = -\frac{1}{2}}$

$\because \cos x$ is $-$ive in II and III quad.

Ref. angle $= 2x = \pi/3$

$2x = \pi - \pi/3 = \frac{2\pi}{3} \rightarrow$ in II quad

$2x = \pi + \pi/3 = 4\pi/3 \rightarrow$ in III quad

$\Rightarrow 2x = \frac{2\pi}{3} + 2n\pi \quad , n \in Z$

and $2x = \frac{4\pi}{3} + 2n\pi$

or $x = \frac{\pi}{3} + n\pi, \quad n \in Z$

$x = \frac{2\pi}{3} + n\pi$

$S.S = \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup$

$\left\{\frac{\pi}{3} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + n\pi\right\}, n \in Z$

**Q19.** $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

**Solution:-**

$\sin 7\theta + \sin\theta + \sin 5\theta + \sin 3\theta = 0$

$2\sin\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2} + 2\sin\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2} = 0$

$\Rightarrow 2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos\theta = 0$

$2\sin 4\theta(\cos 3\theta + \cos\theta) = 0$

$2\sin 4\theta\left[2\cos\frac{3\theta+\theta}{2}\cos\frac{3\theta-\theta}{2}\right] = 0$

$2\sin 4\theta(2\cos 2\theta \cos\theta) = 0$

$\Rightarrow 4\sin 4\theta \cos 2\theta \cos\theta = 0$

$\sin 4\theta = 0, \quad \cos 2\theta = 0$

$\cos\theta = 0$

$\boxed{\sin 4\theta = 0}$

$\Rightarrow 4\theta = \sin^{-1}(0) = 0, \pi$

$4\theta = 0 + 2n\pi \quad , n \in Z$

$4\theta = \pi + 2n\pi$

or $\theta = \frac{n\pi}{2}, \quad \theta = \frac{\pi}{4} + \frac{n\pi}{2}, n \in Z$

$\boxed{\cos 2\theta = 0}$

$\Rightarrow \cos 2\theta = 0$

$2\theta = \cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow 2\theta = \frac{\pi}{2} + 2n\pi, \quad n \in Z$

and $2\theta = \frac{3\pi}{2} + 2n\pi, \quad n \in Z$

$\theta = \frac{\pi}{4} + n\pi, \quad n \in Z$

and $\theta = \frac{3\pi}{4} + n\pi$

$\boxed{\cos\theta = 0}$

$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{2} + 2n\pi \quad , n \in Z$

$\theta = \frac{3\pi}{2} + 2n\pi$

$S.S = \left\{\frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{4} + \frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{4} + n\pi\right\}$

$\cup \left\{\frac{3\pi}{4} + n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\}$

$, n \in Z$

**Q20.** $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

**Solution:-**

$\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

$\cos 7\theta + \cos\theta + \cos 5\theta + \cos 3\theta = 0$

$2\cos\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2} + 2\cos\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2} = 0$

$2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$

$\Rightarrow 2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$

$2\cos 4\theta\left(2\cos\frac{3\theta+\theta}{2}\cos\frac{3\theta-\theta}{2}\right) = 0$

$2\cos 4\theta(2\cos 2\theta \cos\theta) = 0$

$4\cos 4\theta \cos 2\theta \cos\theta = 0$

$\cos 4\theta = 0, \quad \cos 2\theta = 0$

$\cos\theta = 0$

$\boxed{\cos 4\theta = 0}$

$4\theta = \cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$

$4\theta = \frac{\pi}{2} + 2n\pi, \quad n \in Z$

$4\theta = \frac{3\pi}{2} + 2n\pi$

$$\Rightarrow \theta = \frac{\pi}{8} + \frac{n\pi}{2}$$

$$\text{and } \theta = \frac{3\pi}{8} + \frac{n\pi}{2}, \quad n \in Z$$

$$\boxed{\cos 2\theta = 0}$$

$$\Rightarrow 2\theta = \cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2\theta = \frac{\pi}{2} + 2n\pi$$

$$\text{and } 2\theta = \frac{3\pi}{2} + 2n\pi, \quad n \in Z$$

$$\Rightarrow \theta = \frac{\pi}{4} + n\pi,$$

$$\text{and } \theta = \frac{3\pi}{4} + n\pi \quad n \in Z$$

$$\boxed{\cos\theta = 0}$$

$$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = \frac{\pi}{2} + 2n\pi$$

$$\text{and } \theta = \frac{3\pi}{2} + 2n\pi, \quad n \in Z$$

$$S.S = \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{4} + n\pi\right\} \cup \left\{\frac{3\pi}{4} + n\pi\right\}$$
$$\cup \left\{\frac{\pi}{8} + \frac{n\pi}{2}\right\} \cup \left\{\frac{3\pi}{8} + \frac{n\pi}{2}\right\}, \quad n \in Z$$