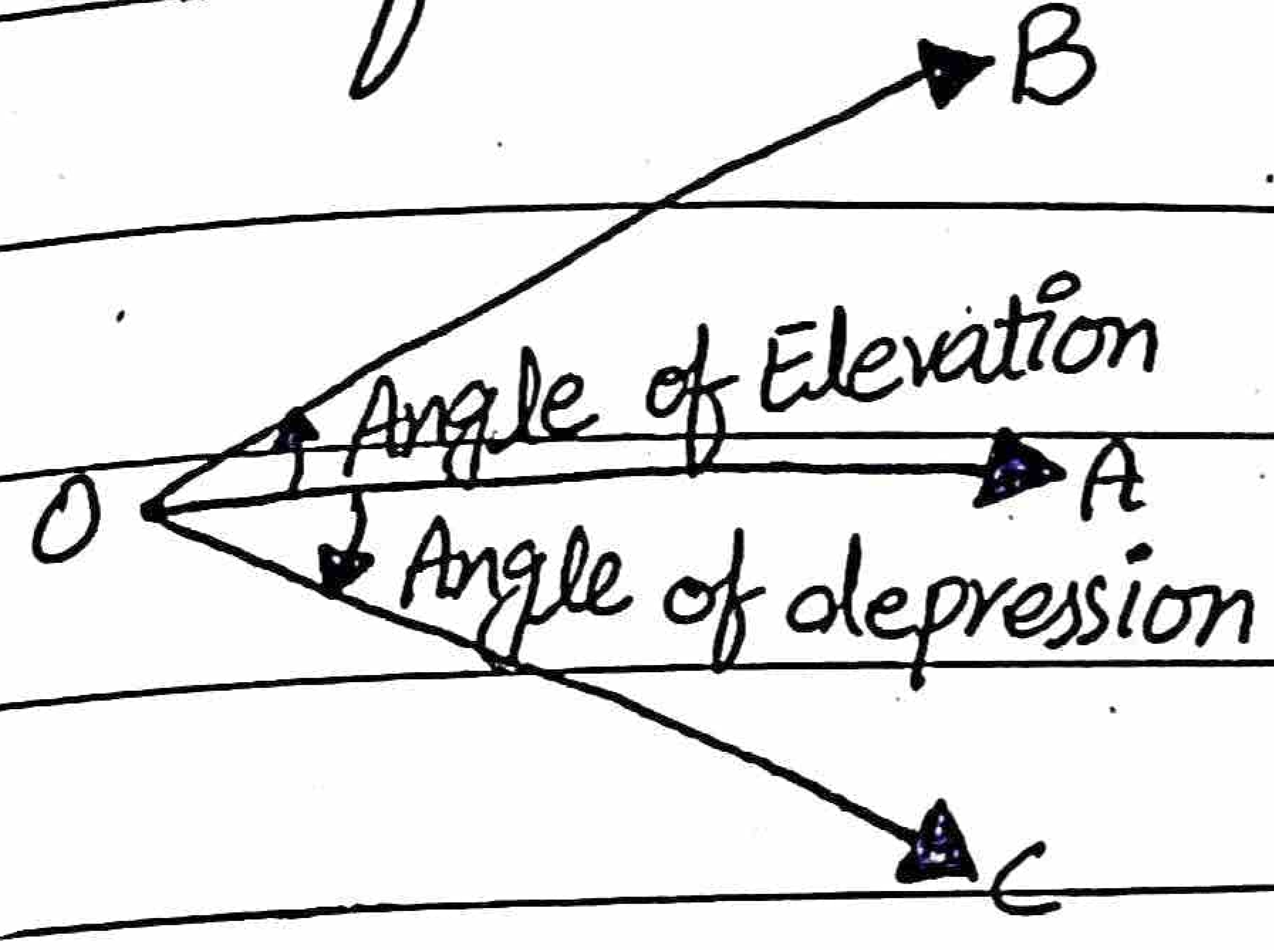
Chapter NO. 12:-

Application of Trigonometry

Definition



Angle of Elevation:-

For looking at B above the horizontal ray, we have to raise our eyes, and $\angle AOB$ is called the **Angle of Elevation**.

Angle of Depression:-

For looking at C below the horizontal ray we have to lower our eyes, and $\angle AOC$ is called the **Angle of Depression**.

Exercise 12.1

Q.NO.1:-

Find the values of:

(i) $\sin 53^\circ 40'$

$= \sin 53^\circ 40'$

$= 0.8055$

(ii) $\cos 36^\circ 20'$

$= \cos 36^\circ 20'$

$= 0.8055$

(iii) $\tan 19^\circ 30'$

$= \tan 19^\circ 30'$

$= 0.354$

(iv) $\cot 33^\circ 50'$

$= \cot 33^\circ 50'$

$= 1.4920$

(v) $\cos 42^\circ 38'$

$= \cos 42^\circ 38'$

$= 0.7357$

(vi) $\tan 25^\circ 34'$

$= \tan 25^\circ 34'$

$= 0.4784$

(vii) $\sin 18^\circ 31'$

$= \sin 18^\circ 31'$

$= 0.3175$

(viii) $\cos 52^\circ 13'$

$= \cos 52^\circ 13'$

$= 0.6126$

(ix) $\cot 89°9'$

$= \cot 89°9'$

$= 0.0148$

Q.NO.26-

Find θ, if

(i) $\sin\theta = 0.5791$

$\theta = \sin^{-1} 0.5791$

$= 35.38$

$= 35°23'$

(ii) $\cos\theta = 0.9316$

$\theta = \cos^{-1} 0.9316$

$= 21.31$

$= 21°18'$

(iii) $\cos\theta = 0.5257$

$\theta = \cos^{-1} 0.5257$

$= 58.28$

$= 58°17'$

(iv) $\tan\theta = 1.705$

$\theta = \tan^{-1} 1.705$

$= 59.60$

$= 59°36'$

(v) $\tan\theta = 21.943$

$\theta = \tan^{-1} 21.943$

$= 87.39$

$= 87^\circ 23'$

(vi) $\sin\theta = 0.5186$

$\theta = \sin^{-1} 0.5186$

$= 31.23$

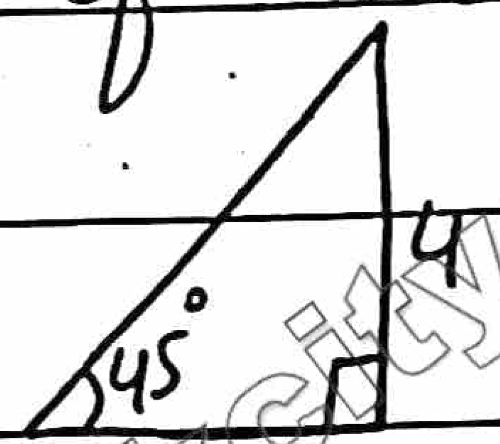
$= 31^\circ 14'$

Exercise 12.2

Q.No.1:-

Find the unknown angles and sides of the following triangles:

(i)



$\sin\theta = \dfrac{P}{H}$

$\sin 45 = \dfrac{4}{H}$

$H = \dfrac{4}{\sin 45}$

$\boxed{H = 5.65}$

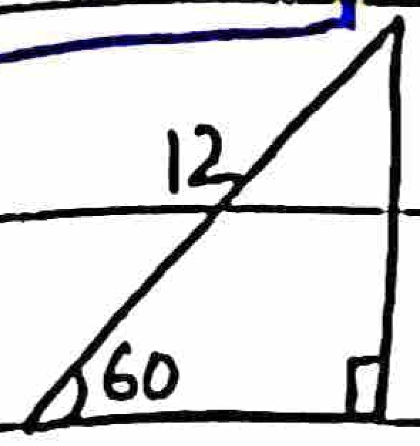
$\tan\theta = \dfrac{P}{B}$

$\tan 45^\circ = \dfrac{4}{B}$

$$B = \frac{4}{\tan 45^\circ}$$

$$\boxed{B = 4}$$

(ii)



$$\alpha + \beta + \gamma = 180^\circ$$

$$60 + \beta + 90^\circ = 180^\circ$$

$$\beta = 180 - 150$$

$$\boxed{\beta = 30^\circ}$$

$$\sin \theta = \frac{P}{H}$$

$$\sin 60 \times 12 = P$$

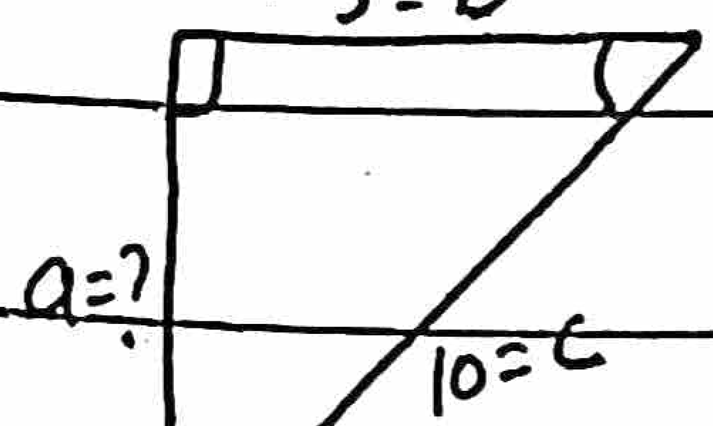
$$\boxed{10.39 = P}$$

$$\cos \theta = \frac{B}{H}$$

$$\cos 80^\circ \times 12 = B$$

$$\boxed{6 = B}$$

(iii)



$\gamma = 90^\circ$, $b = 5$, $c = 10$

By pythagoras theorem

$$c^2 = b^2 + a^2$$

$$(10)^2 - (5)^2 = a^2$$

$$\sqrt{75} = \sqrt{a^2}$$

$$\boxed{8.66 = a}$$

$$\cos \alpha = \frac{B}{H}$$

$$\alpha = \cos^{-1} \frac{B}{H}$$

$$\alpha = \cos^{-1}\left(\frac{5}{10}\right)$$

$$\boxed{\alpha = 60^\circ}$$

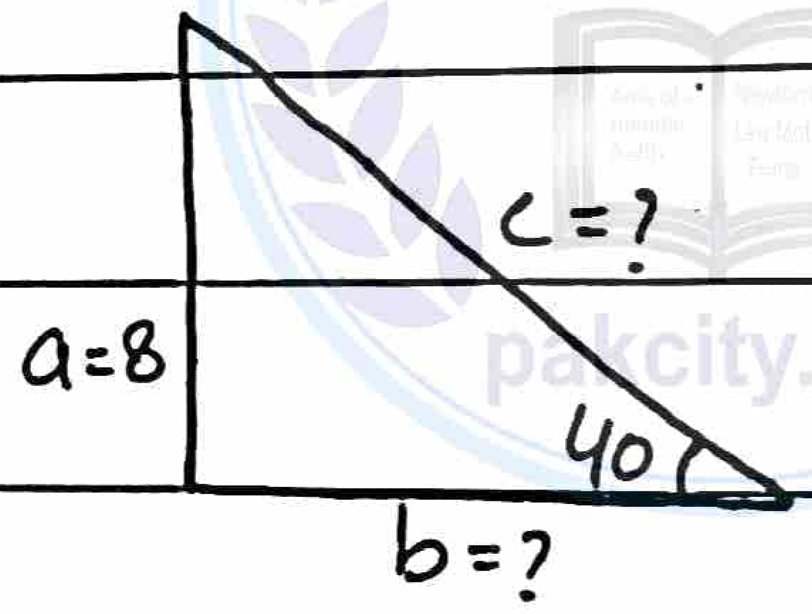
$$\alpha + \beta + \gamma = 180^\circ$$

$$60 + \beta + 90 = 180$$

$$B = 180^\circ - 150^\circ$$

$$\boxed{B = 30^\circ}$$

(iv)



$$a = 8, \quad \alpha = 40^\circ, \quad \gamma = 90^\circ$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$40^\circ + \beta + 90^\circ = 180^\circ$$

$$= 180 - 130$$

$$\boxed{\beta = 50^\circ}$$

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$$\sin\alpha = \frac{P}{H} = \frac{8}{H}$$

$$\sin 40 = \frac{8}{H}$$

$$H = \frac{8}{\sin 40}$$

$$\boxed{H = 12.44}$$

By pythagoras theorem

$$H^2 = B^2 + P^2$$

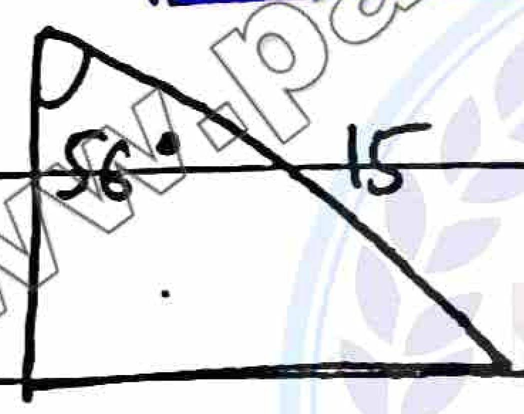
$$(12.44)^2 = B^2 + (8)^2$$

$$154.75 - 64 = B^2$$

$$\sqrt{90.75} = \sqrt{B^2}$$

$$\boxed{9.52 = B}$$

(v)



$$c = 5, \quad \alpha = 56^\circ, \quad \gamma = 90^\circ$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$56 + \beta + 90^\circ = 180$$

$$\beta = 180 - 146$$

$$\boxed{\beta = 34}$$

$$\sin\alpha = \frac{P}{H}$$

$$\sin 56 = \frac{P}{15}$$

$\sin 56 \times 15 = P$

$\boxed{12.4 = P}$

By pythagoras theorem

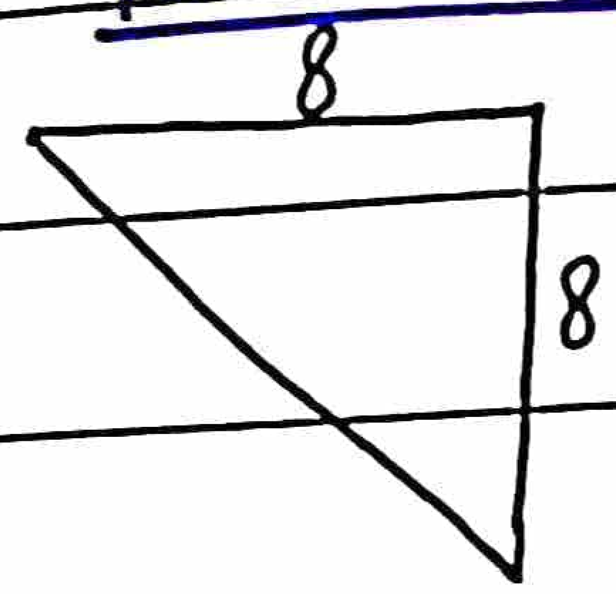
$(15)^2 = B^2 + (12.4)^2$

$225 - 153.76 = B^2$

$\sqrt{72} = \sqrt{B^2}$

$\boxed{8.4 = B}$

(vi)



$a = 8$, $b = 8$, $\gamma = 90°$

By pythagoras theorem

$H^2 = B^2 + P^2$

$H^2 = (8)^2 + (8)^2$

$H^2 = 64 + 64$

$\sqrt{H^2} = \sqrt{128}$

$\boxed{H = 11.3137}$

$\tan \alpha = \frac{P}{B}$

$\alpha = \tan^{-1} \frac{P}{B}$

$\alpha = \tan^{-1} \left(\frac{8}{8}\right)$

$\alpha = \tan^{-1}(1)$

$\boxed{\alpha = 45^\circ}$

$\alpha + \beta + \gamma = 180^\circ$

$45 + \beta + 90 = 180$

$\beta = 180 - 135$

$\boxed{\beta = 45}$

Solve the right triangle ABC, in which $\gamma = 90^\circ$

Q·NO·26-

$\alpha = 37^\circ 20'$, $a = 243$

$\gamma = 90^\circ$

$\alpha + \beta + \gamma = 180^\circ$

$37^\circ 20' + \beta + 90^\circ = 180^\circ$

$\beta = 180 - 127.33$

$\boxed{\beta = 52^\circ 40'}$

$\sin \alpha = \dfrac{P}{H}$

$\sin 37^\circ 20' = \dfrac{243}{H} \Rightarrow H = \dfrac{243}{\sin 37^\circ 20'}$

$\boxed{H = 400}$

By pythagoras theorem

$H^2 = B^2 + P^2$

$(400)^2 = B^2 + (243)^2$

$(400)^2 - (243)^2 = B^2$

$\sqrt{100951} = \sqrt{B^2}$

$\boxed{318 = B}$

Q.NO.3:-

$\alpha = 62^\circ 40'$, $b = 796$

$\gamma = 90^\circ$

$\alpha + \beta + \gamma = 180^\circ$

$62^\circ 40' + \beta + 90^\circ = 180^\circ$

$\beta = 180 - 152.66$

$\beta = 27.33$

$\boxed{\beta = 27^\circ 20'}$

$\cos \alpha = \frac{B}{H}$

$\cos 62^\circ 40' = \frac{796}{H}$

$H \cancel{\cos} = \frac{796}{\cos 62^\circ 40'}$

$\boxed{H = 1734}$

By pythagoras theorem

$H^2 = B^2 + P^2$

$(1734)^2 = (796)^2 + P^2$

$\sqrt{2373140} = \sqrt{P^2}$

$\boxed{1540 = P}$

Q.NO.4:-

$a = 3.28$, $b = 5.74$

$\gamma = 90^\circ$

By pythagoras theorem

$c^2 = b^2 + a^2$

$c^2 = (5.74)^2 + (3.28)^2$

$\sqrt{c^2} = \sqrt{43.706}$

$\boxed{c = 6.61}$

$\sin\alpha = \frac{P}{H} = \frac{a}{c}$

$\sin\alpha = \frac{3.28}{6.61}$

$\alpha = \sin^{-1}(0.496217)$

$= 29.75$

$\boxed{\alpha = 29^\circ 45'}$

$\alpha + \beta + \gamma = 180^\circ$

$29^\circ 45' + \beta + 90 = 180^\circ$

$\beta = 180 - 119.75$

$\beta = 60.25$

$\boxed{\beta = 60^\circ 15'}$

Q.NO.5:-

$b = 68.4$, $c = 96.2$

$\gamma = 90^\circ$

By pythagoras theorem

$$c^2 = b^2 + a^2$$

$$(96.2)^2 = (68.4)^2 + a^2$$

$$(96.2)^2 - (68.4)^2 = a^2$$

$$\sqrt{4575.88} = \sqrt{a^2}$$

$$\boxed{67.7 = a}$$

$$\tan\alpha = \frac{P}{B} = \frac{a}{b}$$

$$\tan\alpha = \frac{67.7}{68.4}$$

$$\alpha = \tan^{-1}\left(\frac{67.7}{68.4}\right)$$

$$\boxed{\alpha = 44.683}$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$44^\circ 40' + \beta + 90^\circ = 180^\circ$$

$$\beta = 180^\circ - 134.66$$

$$\beta = 45.34$$

$$\boxed{\beta = 45^\circ 20'}$$

Q.NO.6:-

$$a = 5429, \quad c = 6294$$

By pythagoras theorem

$$c^2 = b^2 + a^2$$

$$(6294)^2 = b^2 + (5429)^2$$

$(6294)^2 - (5429)^2 = b^2$

$\sqrt{10140395} = \sqrt{b^2}$

$\boxed{3184 = b}$

$\tan \alpha = \frac{P}{B}$

$\tan \alpha = \frac{5429}{3184}$

$\alpha = \tan^{-1} \frac{5429}{3184}$

$= \tan^{-1}(1.705)$

$= 59.60$

$\boxed{\alpha = 59^\circ 36'}$

$\alpha + \beta + \gamma = 180^\circ$

$59^\circ 36' + \beta + 90^\circ = 180^\circ$

$\beta = 180^\circ - 149.6$

$\beta = 30.4$

$\boxed{\beta = 30^\circ 24'}$

Q.NO.7:-

$\beta = 50^\circ 10' \quad , \quad c = 0.832$

$\gamma = 90^\circ$

$\alpha + \beta + \gamma = 180^\circ$

$\alpha + 50^\circ 10' + 90^\circ = 180^\circ$

$\alpha = 180^\circ - 140.16$

$\alpha = 39.84$

$\boxed{\alpha = 39°50'}$

$\sin\alpha = \frac{P}{H} = \frac{a}{c}$

$\sin 39°50' = \frac{a}{0.832}$

$\sin 39°50' \times 0.832 = a$

$0.6405 \times 0.832 = a$

$\boxed{a = 0.533}$

By pythagoras theorem

$H^2 = B^2 + P^2$

$(0.832)^2 = B^2 + (0.533)^2$

$(0.832)^2 - (0.533)^2 = B^2$

$\sqrt{0.408135} = \sqrt{B^2}$

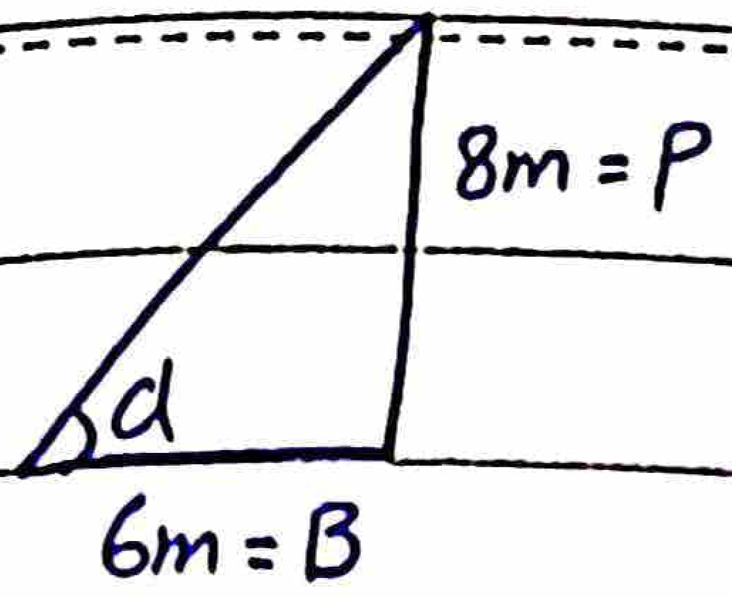
$0.6388 = B$

$\boxed{B = 0.639}$

Exercise 12.3

Q.No.1:-

A vertical pole is 8m high and the length of its shadow is 6m. What is the angle of elevation of the sun at that moment?



$$\tan \alpha = \frac{P}{B}$$

$$\tan \alpha = \frac{8}{6}$$

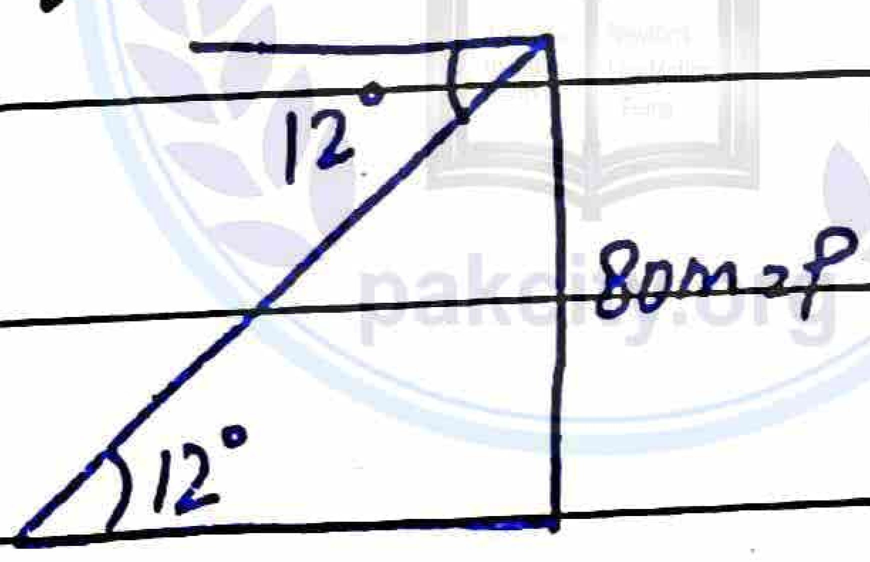
$$\alpha = \tan^{-1} 1.33$$

$$= 53.13$$

Angle of elevation $\boxed{\alpha = 53^\circ 7^\circ}$

Q.NO.36-

At the top of a cliff 80m high, the angle of depression of a boat is 12°. How far is the boat from the cliff?



$$\tan \alpha = \frac{P}{B}$$

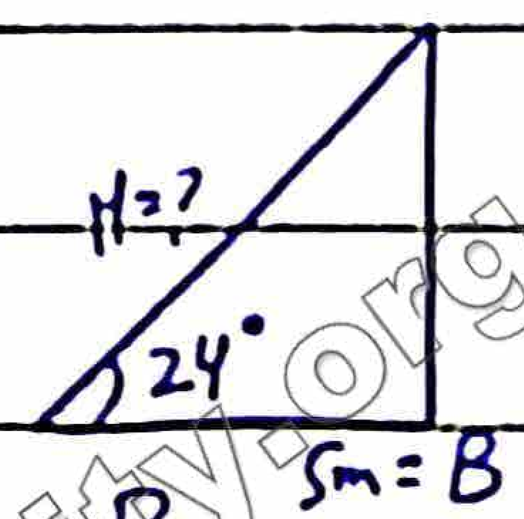
$$\tan 12 = \frac{80}{B}$$

$$B = \frac{80}{\tan 12}$$

$\boxed{\text{Distance of boat} = B = 376.31m \text{ from cliff}}$

Q.NO. 4:-

A ladder leaning against a vertical wall makes an angle of 24° with the wall. Its foot is 5m from the wall. Find its length?



$$\cos\theta = \frac{B}{H}$$

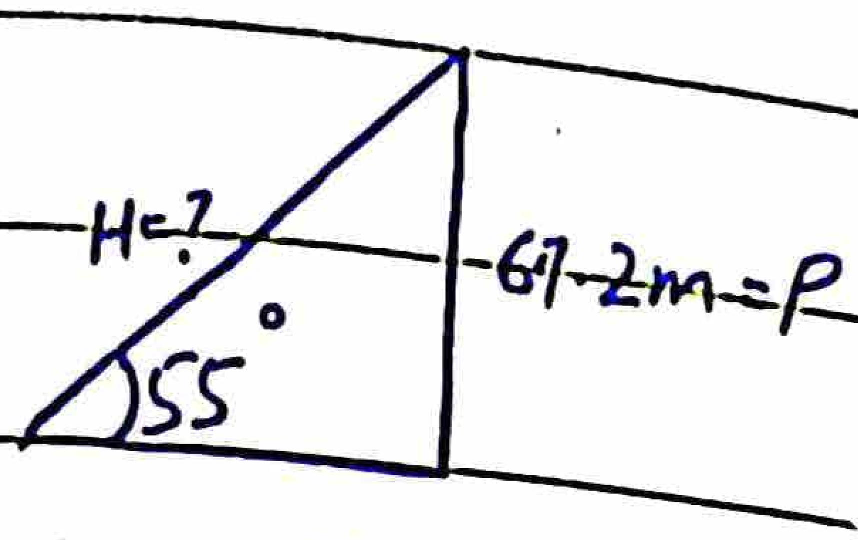
$$\cos 24 = \frac{5}{H}$$

$$H = \frac{5}{\cos 24}$$

$\boxed{\text{Length of ladder} = H = 5.47m}$

Q.NO.5:-

A kite flying at a height of 67.2m is attached to a fully streched string inclined at an angle of 55° to the horizontal. Find the length of the string?



$$\sin\theta = \frac{P}{H}$$

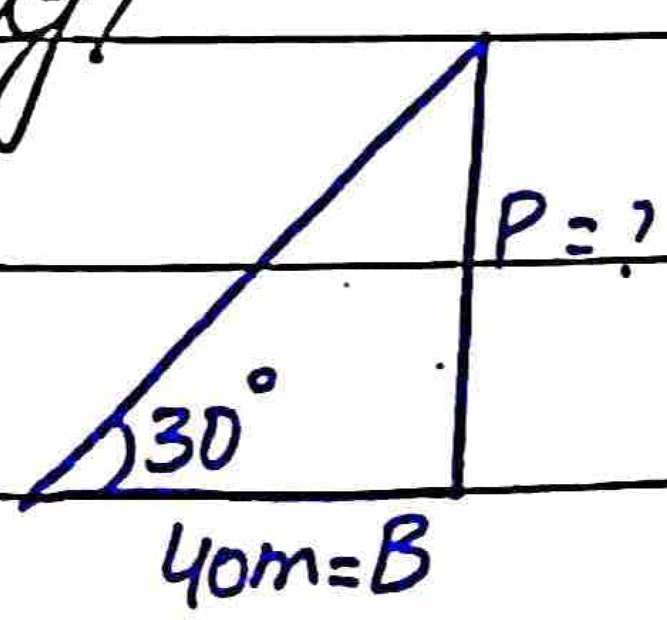
$$\sin 55 = \frac{67.2}{H}$$

$$H = \frac{67.2}{\sin 55}$$

$$\boxed{\text{Length of string} = H = 82.036\,m}$$

Q.NO.66-

When the angle between the ground and the sun is 30°, flag pole casts a shadow of 40m long. Find the height of the top of the flag?



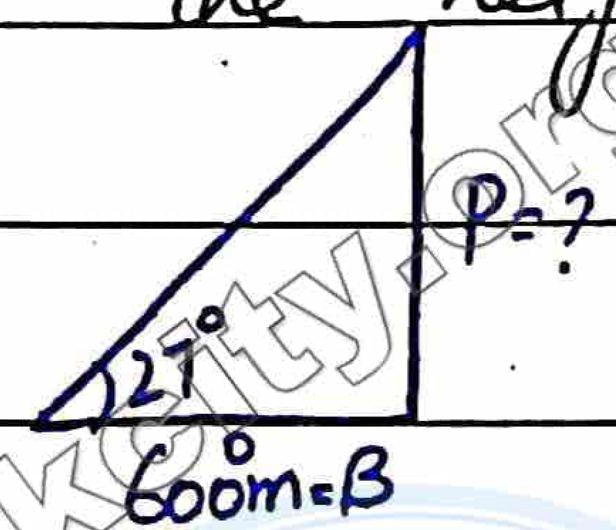
$$\tan\theta = \frac{P}{B}$$

$$\tan 30^\circ = \frac{P}{40} \Rightarrow P = \tan 30^\circ \times 40$$

$$\boxed{\text{Height of flag pole} = P = 23.09\text{m}}$$

Q.NO.7:-

A plane flying directly above a post 6000m away from an anti-aircraft gun observes the gun at an angle of depression of 27°. Find the height of the plane?



$$\tan \theta = \frac{P}{B}$$

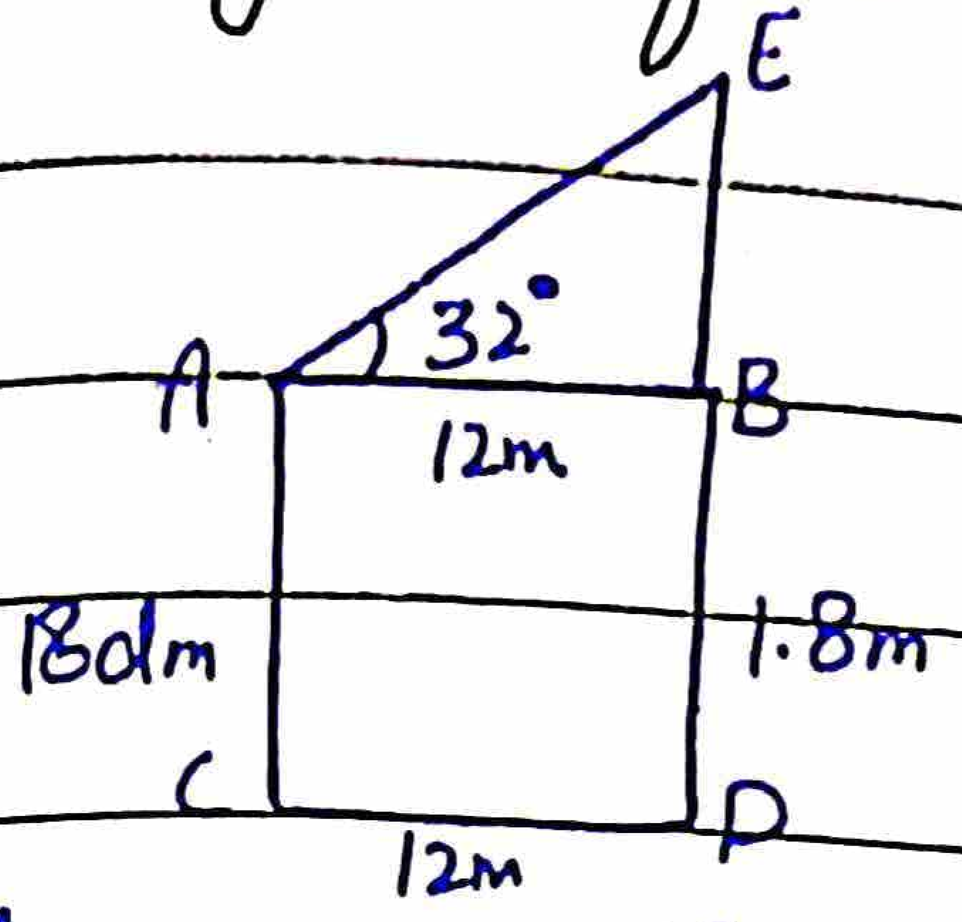
$$\tan 27 = \frac{P}{6000}$$

$$P = \tan 27 \times 6000$$

$$\boxed{\text{Height of plane} = P = 3057.15\text{m}}$$

Q.NO.26-

A man 18dm tall observes that the angle of elevation of the top of a tree at a distance of 12m from him is 32°. What is the height of the tree?



Height of man = 18 dm

$= \frac{18}{10}$

$= 1.8m$

$\tan\theta = \frac{P}{B}$

$\tan 32 = \frac{P}{12}$

$P = \tan 32 \times 12$

$P = 7.49$

$$\boxed{\text{Height of tree} = 7.49 + 1.8 = 9.3m}$$

*Exercise 12.4

Solve the triangle ABC, if

Q.NO.1:-

$\beta = 60°$, $\gamma = 15°$, $b = \sqrt{6}$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\alpha + 60+15 = 180$$

$$\alpha = 180-75$$

$$\boxed{\alpha = 105}$$

$$\therefore \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

By law of sin

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$a = \frac{b\sin\alpha}{\sin\beta}$$

$$a = \frac{\sqrt{6}\sin 105}{\sin 60}$$

$$\boxed{a = 2.732}$$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$c = \frac{b\sin\gamma}{\sin\beta}$$

$$c = \frac{\sqrt{6}\sin 15}{\sin 60}$$

$$\boxed{c = 0.732}$$

Q.NO.2:-

$\beta = 52^\circ$, $\gamma = 89^\circ 35'$, $\alpha = 89^\circ 35$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + 52^\circ + 89^\circ 35' = 180^\circ$$

$$\alpha = 180 - 141.58$$

$$\alpha = 38.416$$

$$\boxed{\alpha = 38^\circ 25'}$$

$$\therefore \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

By law of sin

$$\frac{a}{\sin \alpha} = \frac{b}{\sin\beta}$$

$$b = \frac{a \sin\beta}{\sin \alpha}$$

$$b = \frac{89.35 \sin 52}{\sin 38.416}$$

$$\boxed{b = 113.31}$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$c = \frac{a \sin\gamma}{\sin\alpha}$$

$$c = \frac{89.35 \sin 89^\circ 35'}{\sin 38.416}$$

$$\boxed{c = 143.792}$$

Q.NO.38-

$b = 125, \quad \gamma = 53^\circ, \quad \alpha = 47^\circ$

$\alpha + \beta + \gamma = 180^\circ$

$47 + \beta + 53 = 180$

$\beta = 180 - 100$

$\boxed{\beta = 80}$

$\therefore \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

By laws of sin

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$a = \dfrac{b\sin\alpha}{\sin\beta}$

$a = \dfrac{125\sin 47}{\sin 80}$

$\boxed{a = 93}$

$\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$c = \dfrac{b\sin\gamma}{\sin\beta}$

$c = \dfrac{125\sin 53}{\sin 80}$

$\boxed{c = 101.369}$

Q.NO.4:-

$c = 16.1, \ \alpha = 42^\circ 45', \ \gamma = 74^\circ 32'$

$\alpha + \beta + \gamma = 180^\circ$

$42^\circ 45' + \beta + 74^\circ 32' = 180$

$\beta = 180 - 117^\circ 17'$

$\beta = 62.716$

$\boxed{\beta = 62^\circ 43'}$

$\because \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

By laws of sin

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$a = \dfrac{c \sin\alpha}{\sin\gamma}$

$a = \dfrac{16.1 \sin 42^\circ 45'}{\sin 74^\circ 32'}$

$\boxed{a = 11.339}$

$\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$b = \dfrac{c \sin\beta}{\sin\gamma}$

$b = \dfrac{16.1 \sin 62.716}{\sin 74^\circ 32'}$

$\boxed{b = 14.846}$

Q.NO.5:-

$a = 53, \beta = 88°36', \gamma = 31°54'$

$\alpha + \beta + \gamma = 180°$

$\alpha + 88°36' + 31°54' = 180°$

$\alpha + 120°30' = 180$

$\alpha = 180 - 120°30'$

$\boxed{\alpha = 59°30'}$

$\therefore \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

By laws of sin

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$b = \frac{a \sin\beta}{\sin\alpha}$

$b = \frac{53 \sin 88°36'}{\sin 59.5}$

$\boxed{b = 61.493}$

$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$c = \frac{a \sin\gamma}{\sin\alpha}$

$c = \frac{53 \sin 31°54'}{\sin 59.5}$

$\boxed{c = 32.5049}$

Exercise 12.6

Solve the following triangles, in which:

Q.NO.3:-

$a = 28.3$, $b = 31.7$, $c = 42.8$

By law of cosine

$$\cos \alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(31.7)^2+(42.8)^2-(28.3)^2}{2(31.7)(42.8)}$$

$$\cos\alpha = 0.7502$$

$$\alpha = \cos^{-1}(0.7502)$$

$$= 41.38$$

$$\boxed{\alpha = 41^\circ 23'}$$

$$\cos \beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(28.3)^2+(42.8)^2-(31.7)^2}{2(28.3)(42.8)}$$

$$\cos \beta = 0.6719$$

$$\beta = \cos^{-1}(0.6719)$$

$$\beta = 47.789$$

$$\boxed{\beta = 47^\circ 46'}$$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\gamma = 180^\circ - 41^\circ 23' - 47^\circ 46'$$

$$\boxed{\gamma = 90^\circ 51'}$$

imp

Q.NO.6:-

Find the smallest angle of the triangle ABC, when

$a = 37.34$, $b = 3.24$, $c = 35.06$

As b is smallest side, then β is smallest angle.

By law of cosine

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{(37.34)^2 + (35.06)^2 - (3.24)^2}{2(37.34)(35.06)}$$

$$\cos\beta = 0.9979$$

$$\beta = \cos^{-1}(0.9979)$$

$$\beta = 3.646$$

$$\boxed{\beta = 3^\circ 38'}$$

imp

Q.NO.7:-

Find the measure of the greatest angle, if sides of the triangle are 16, 20, 33.

As c is greatest side, then γ is greatest angle.

By law of cosine

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{(16)^2+(20)^2-(33)^2}{2(16)(20)}$$

$$\cos\gamma = -0.676$$

$$\gamma = \cos^{-1}(-0.676)$$

$$= 132.5$$

$$\boxed{\gamma = 132^\circ 34'}$$

imp

Q.NO.8:-

Three sides of a triangle are x^2+x+1, $2x+1$ and x^2-1. Prove that the greatest angle of the triangle is 120°.

$a = x^2+x+1$, $b = 2x+1$, $c = x^2-1$

As a is greatest side than α is greater angle.

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(2x+1)^2+(x^2-1)^2-(x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2+1+4x+x^4+1-2x^2-(x^4+x^2+1+2x^3+2x+2x^2)}{2(2x^3-2x+x^2-1)}$$

$$= \frac{\cancel{4x^2} + 1 + 4x + \cancel{x^4} + \cancel{1} - 2\cancel{x^2} - \cancel{x^4} - x^2 - \cancel{1} - 2x^3 - 2x - \cancel{2x^2}}{2(2x^3 - 2x + x^2 - 1)}$$

$$= \frac{-2x^3 + 2x + 1 - x^2}{2(2x^3 - 2x + x^2 - 1)}$$

$$= \frac{-\cancel{(2x^3 - 2x - 1 + x^2)}}{2\cancel{(2x^3 - 2x - 1 + x^2)}}$$

$$\cos\alpha = \frac{-1}{2}$$

$$\alpha = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$\boxed{\alpha = 120^\circ}$$

Q.NO.16-

$a = 7,\ b = 7,\ c = 9$

By law of cosine

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(7)^2 + (9)^2 - (7)^2}{2(7)(9)}$$

$$= \frac{81}{126}$$

$$\boxed{\cos\alpha = 0.642}$$

$$\alpha = \cos^{-1}(0.642)$$

$$= 49.9$$

$$\boxed{\alpha = 49^\circ 59'}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(7)^2+(9)^2-(7)^2}{2(7)(9)}$$

$$= \frac{81}{126}$$

$$\boxed{\cos\beta = 0.642}$$

$$\beta = \cos^{-1}(0.642)$$

$$\beta = 49^\circ 59'$$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\gamma = 180 - 49^\circ 59' - 49^\circ 59'$$

$$\boxed{\gamma = 80^\circ 2'}$$

Q.NO.2:-

$a = 32$, $b = 40$, $c = 66$

By law of cosine

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(40)^2+(66)^2-(32)^2}{2(40)(66)}$$

$$\cos\alpha = 0.9340$$

$$\alpha = \cos^{-1}(0.9340)$$

$$= 20.91$$

$$\boxed{\alpha = 20^\circ 55'}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(32)^2+(66)^2-(40)^2}{2(32)(66)}$$

$$\boxed{\cos\beta = 0.8948}$$

$$\beta = \cos^{-1}(0.8948)$$

$$\beta = 26.50$$

$$\beta = 26^\circ 30'$$

$$\alpha+\beta+\gamma = 180^\circ$$

$$20^\circ 55' + 26^\circ 30' + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 20^\circ 55' - 26^\circ 30'$$

$$\boxed{\gamma = 132^\circ 35'}$$

Q.NO.4:-

$a = 31.9$, $b = 56.31$, $c = 40.27$

By law of cosine

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(56.31)^2+(40.27)^2-(31.9)^2}{2(56.31)(40.27)}$$

$$\cos\alpha = 0.70100$$

$$\alpha = \cos^{-1}(0.70100)$$

$$\boxed{\alpha = 45^\circ 29'}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(31.9)^2 + (56.3)^2 - (40.27)^2}{2(31.9)(56.31)}$$

$$\cos\beta = 0.71446$$

$$\beta = \cos^{-1}(0.71446)$$

$$\boxed{\beta = 44^\circ 24'}$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$45^\circ 29' + 44^\circ 24' + \gamma = 180^\circ$$

$$\gamma = 180 - 89^\circ 53'$$

$$\boxed{\gamma = 90^\circ 7'}$$

Q.NO.5:-

$a = 4584$, $b = 5740$, $c = 3624$

By law of cosine

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{(5140)^2 + (3624)^2 - (4584)^2}{2(5140)(3624)}$$

$$\cos\alpha = 0.498$$

$$\alpha = \cos^{-1}(0.498)$$

$$\boxed{\alpha = 60^\circ 7'}$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{(4584)^2 + (3624)^2 - (5140)^2}{2(4584)(3624)}$$

$$\cos\beta = 0.233$$

$$\beta = \cos^{-1}(0.233)$$

$$\boxed{\beta = 76^\circ 31'}$$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\gamma = 180^\circ - 136^\circ 38'$$

$$\boxed{\gamma = 43^\circ 22'}$$

Q.NO.9:-

The measures of side of a triangular plot are 413, 214 and 375 meters. Find the measures of the corner angles of the plot?

By law of cosine

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\alpha = \frac{(214)^2+(375)^2-(413)^2}{2(214)(375)}$$

$$\cos\alpha = 0.0987$$

$$\alpha = \cos^{-1}(0.0987)$$

$$\boxed{\alpha = 84^\circ 20'}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos\beta = \frac{(375)^2+(413)^2-(214)^2}{2(375)(413)}$$

$\cos\beta = 0.8568$

$\beta = \cos^{-1}(0.8568)$

$\boxed{\beta = 31^\circ 1'}$

$\alpha + \beta + \gamma = 180^\circ$

$84^\circ 20' + 31^\circ 1' + \gamma = 180^\circ$

$\gamma = 180^\circ - 115^\circ 21'$

$\boxed{\gamma = 64^\circ 39'}$

Q.NO.10:-

Three villagers A, B and C are connected by straight roads 6km, 9km, 13km. What angles these roads make with each other?

By law of cosine

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(9)^2+(13)^2-(6)^2}{2(9)(13)}$$

$\cos\alpha = 0.915$

$\alpha = \cos^{-1}(0.915)$

$\boxed{\alpha = 23^\circ 47'}$

$$\cos\beta = \frac{c^2+a^2-b^2}{2ac}$$

$$= \frac{(13)^2+(6)^2-(9)^2}{2(13)(6)}$$

$\cos\beta = 0.795$

$\beta = \cos^{-1}(0.795)$

$\boxed{\beta = 37^\circ 20'}$

$\alpha + \beta + \gamma = 180^\circ$

$23^\circ 47' + 37^\circ 20' + \gamma = 180$

$\gamma = 180 - 61^\circ 7'$

$\boxed{\gamma = 118^\circ 53'}$

*Exercise 12.5

Solve the triangle, in which

Q.NO.1:-

$b = 95, \; c = 34, \; \alpha = 52^\circ$

By law of cosine

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = (95)^2 + (34)^2 - 2(95)(34)\cos 52^\circ$

$\sqrt{a^2} = \sqrt{6203.82}$

$\boxed{a = 78.7}$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{(78.7)^2 + (34)^2 - (95)^2}{2(78.7)(34)}$$

$$\cos\beta = \frac{-1675.31}{5351.6}$$

$\cos\beta = -0.313$

$\beta = \cos^{-1}(-0.313)$

$\beta = 108.24$

$\boxed{\beta = 108^\circ 14'}$

$\alpha + \beta + \gamma = 180^\circ$

$78.7 + 108.24 + \gamma = 180$

$\gamma = 180 - 160^\circ 7'$

$\boxed{\gamma = 19^\circ 53'}$

Q. NO. 3:-

$a = \sqrt{3} - 1 \quad , \; b = \sqrt{3} + 1 , \; \gamma = 60^\circ$

By laws of cosine

$c^2 = a^2 + b^2 - 2ab \cos\gamma$

$c^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 - 2(\sqrt{3}-1)(\sqrt{3}+1)\cos 60^\circ$

$= (\sqrt{3})^2 + 1 - \cancel{2\sqrt{3}} + (\sqrt{3})^2 + 1 + \cancel{2\sqrt{3}} - 2[(\sqrt{3})^2 - 1]\cos 60^\circ$

$= 3 + 1 + 3 + 1 - \cancel{2}(3-1)\frac{1}{\cancel{2}}$

$= 4 + 4 - 2$

$\sqrt{c^2} = \sqrt{6}$

$\boxed{c = \sqrt{6}}$

$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{(\sqrt{3}+1)^2 + (\sqrt{6})^2 - (\sqrt{3}-1)}{2(\sqrt{3}+1)(\sqrt{6})}$

$\cos\alpha = 0.9659$

$\alpha = \cos^{-1}(0.9659)$

$\boxed{\alpha = 15^\circ}$

$\alpha + \beta + \gamma = 180^\circ$

$\beta = 180^\circ - 15^\circ - 60^\circ$

$\boxed{\beta = 105^\circ}$

Q.NO.6B

$a = 36.21$, $b = 42.09$, $\gamma = 42^\circ 29'$

$\alpha + \beta + \gamma = 180^\circ$

$\alpha + \beta + 44^\circ 29' = 180^\circ$

$\alpha + \beta = 135^\circ 31' \rightarrow (i)$

By law of Tangent

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

$$\frac{36.21-42.09}{36.21+42.09} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{135^\circ 31'}{2}\right)}$$

$$-0.075 = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{2.44}$$

$$-0.075 \times 2.44 = \tan\left(\frac{\alpha-\beta}{2}\right)$$

$$-0.18 = \tan\left(\frac{\alpha-\beta}{2}\right)$$

$$\tan^{-1} -0.18 = \frac{\alpha-\beta}{2}$$

$$-10.392 = \frac{\alpha-\beta}{2}$$

$$-20.80 = \alpha-\beta \longrightarrow \text{(ii)}$$

Add eq (i) and (ii)

$$\alpha+\beta = 135^\circ 31'$$
$$\underline{\alpha-\beta = -20^\circ 48'}$$
$$2\alpha = 114.71$$
$$\alpha = 57.35$$
$$\boxed{\alpha = 57^\circ 21'}$$

Put $\alpha = 57^\circ 21'$ in eq (i)

$$\alpha+\beta = 135^\circ 31'$$
$$57^\circ 21' + \beta = 135^\circ 31'$$
$$\beta = 135^\circ 31' - 57^\circ 21'$$
$$\boxed{\beta = 78^\circ 9'}$$

By law of sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$c = \frac{a\sin\gamma}{\sin\alpha}$$

$$c = \frac{36.21 \sin 44^\circ 29'}{\sin 57^\circ 21'}$$

$$\boxed{c = 30.134}$$

Q.NO.2b-

$b = 12.5,\ c = 23,\ \alpha = 38^\circ 20'$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = (12.5)^2 + (23)^2 - 2(12.5)(23)\cos 38^\circ 20'$

$a^2 = 234.211$

$\sqrt{a^2} = \sqrt{234.211}$

$\boxed{a = 15.3}$

By law of sin

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$\sin\beta = \dfrac{b\sin\alpha}{a}$

$\beta = \sin^{-1}\left(\dfrac{12.5}{15.3}\sin 38^\circ 20'\right)$

$\beta = \sin^{-1}(0.5067)$

$\boxed{\beta = 30^\circ 26'}$

$\alpha + \beta + \gamma = 180^\circ$

$\gamma = 180^\circ - 38^\circ 20' - 30^\circ 26'$

$\boxed{\gamma = 111^\circ 14'}$

Q.NO.4b-

$a = 3,\ c = 6,\ \beta = 36^\circ 20'$

By law of cosine

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = (3)^2 + (6)^2 - 2(3)(6)\cos 36^\circ 20'$

$\sqrt{b^2} = \sqrt{15.99}$

$\boxed{b = 3.998}$

$\therefore \cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos\alpha = \dfrac{(3.998)^2 + (6)^2 - (3)^2}{2(3.998)(6)}$

$\cos\alpha = 0.896$

$\alpha = \cos^{-1}(0.896)$

$\boxed{\alpha = 26^\circ 21'}$

$\alpha + \beta + \gamma = 180^\circ$

$26^\circ 21' + 36^\circ 20' + \gamma = 180^\circ$

$\gamma = 180 - 26^\circ 21' - 36^\circ 20'$

$\boxed{\gamma = 117^\circ 19'}$

Q.NO.5:-

$a = 7$, $b = 3$, $\gamma = 38^\circ 13'$

By law of cosine

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = (7)^2 + (3)^2 - 2(7)(3)\cos 38^\circ 13'$

$\sqrt{c^2} = \sqrt{25}$

$\boxed{c = 5}$

$\therefore \cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$$\cos \alpha = \frac{(3)^2+(5)^2-(7)^2}{2(3)(5}$$

$$\alpha = \cos^{-1}(-0.5)$$

$$\boxed{\alpha = 120^\circ}$$

$$\therefore \cos \beta = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos \beta = \frac{(7)^2+(5)^2-(3)^2}{2(7)(5)}$$

$$\beta = \cos^{-1}(0.928)$$

$$\boxed{\beta = 21^\circ 52'}$$

Q.NO.76-

$$a = 93, \quad c = 101, \quad \beta = 80^\circ$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + \gamma = 180 - 80$$

$$\alpha + \gamma = 100^\circ \longrightarrow (i)$$

By law of tangent

$$\frac{c-a}{c+a} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$$

$$\frac{101-93}{101+93} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{100}{2}\right)}$$

$$0.0412 \times 1.1917 = \tan\left(\frac{\gamma-\alpha}{2}\right)$$

$$\tan^{-1}(0.0491) = \frac{\gamma-\alpha}{2}$$

$2.81096 \times 2 = \gamma - \alpha$

$5.62192 = \gamma - \alpha$

$5^\circ 37' = \gamma - \alpha \rightarrow (ii)$

Add (i) + (ii)

$\gamma + \cancel{\alpha} = 100^\circ$

$\underline{\gamma - \cancel{\alpha} = 5^\circ 37'}$

$2\gamma = 105.62$

$\gamma = \frac{105.62}{2}$

$\boxed{\gamma = 52^\circ 48'}$

Put γ in eq (i)

$\alpha + \gamma = 100$

$\alpha = 52^\circ 48' - 100^\circ$

$\boxed{\alpha = 47^\circ 12'}$

By law of sin

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$b = \frac{a \sin\beta}{\sin\alpha}$

$b = \frac{93 \sin 60^\circ}{\sin 47^\circ 12'}$

$\boxed{b = 125}$

Q.NO.8:-

$b = 14.8, \quad c = 16.1, \quad \alpha = 42°45'$

$\alpha + \beta + \gamma = 180°$

$42°45' + \beta + \gamma = 180°$

$\beta + \gamma = 180° - 42°45'$

$\beta + \gamma = 137°15' \longrightarrow (i)$

By law of tangent

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$$

$$\frac{14.8 - 16.1}{14.8 + 16.1} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{137°15'}{2}\right)}$$

$$-0.0421 = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{-0.4622}$$

$$-0.0421 \times -0.4622 = \tan\left(\frac{\beta-\gamma}{2}\right)$$

$$\tan^{-1}(0.0195) = \frac{\beta-\gamma}{2}$$

$$1.1171 \times 2 = \beta - \gamma$$

$$2°14' = \beta - \gamma \longrightarrow (ii)$$

Add (i) + (ii)

$$\beta + \gamma = 137°15'$$
$$\beta - \gamma = 2°14'$$
$$2\beta = 139°29'$$

$$\beta = \frac{139°29'}{2}$$

$$\boxed{\beta = 69°44'}$$

Put β in eq (i)

$$\beta + \gamma = 137°15'$$

$$\gamma = 137°15' - 69°44'$$

$$\boxed{\gamma = 67°31'}$$

By law of sin

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$a = \frac{b \sin \alpha}{\sin \beta}$$

$$a = \frac{14.8 \sin 42°45'}{\sin 69°44'}$$

$$\boxed{a = 10.709}$$

Q.NO.9:-

$a = 319$, $b = 168$, $\gamma = 110°22'$

$$\alpha + \beta + \gamma = 180°$$

$$\alpha + \beta + 110°22' = 180°$$

$$\alpha + \beta = 180 - 110°22'$$

$$\alpha + \beta = 69°38' \rightarrow (i)$$

By law of tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

$$\frac{319-168}{319+168} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{69^\circ 38'}{2}\right)}$$

$$0.31 = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{1.3468}$$

$$0.31 \times 1.3468 = \tan\left(\frac{\alpha-\beta}{2}\right)$$

$$\tan^{-1}(0.4175) = \frac{\alpha-\beta}{2}$$

$$2 \times 22.66 = \alpha-\beta$$

$$\alpha-\beta = 45^\circ 19' \rightarrow \text{(ii)}$$

Add (i) and (ii)

$$\alpha+\cancel{\beta} = 69^\circ 38'$$

$$\underline{\alpha-\cancel{\beta} = 45^\circ 19'}$$

$$2\alpha = 114^\circ 57'$$

$$\boxed{\alpha = 57^\circ 28'}$$

Put α in eq (i)

$$\alpha+\beta = 69^\circ 38'$$

$$57^\circ 28' + \beta = 69^\circ 38'$$

$$\beta = 69^\circ 38' - 57^\circ 28'$$

$$\boxed{\beta = 12^\circ 10'}$$

By law of sin

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$c = \frac{a \sin\gamma}{\sin\alpha}$$

$$c = \frac{319 \sin 110^\circ 22'}{\sin 57^\circ 28'}$$

$$\boxed{c = 354.72}$$

Q.NO.10 B-

$b = 61, \quad a = 32, \quad \alpha = 59^\circ 30'$

$\alpha + \beta + \gamma = 180^\circ$

$59^\circ 30' + \beta + \gamma = 180^\circ$

$\beta + \gamma = 180^\circ - 59^\circ 30'$

$\beta + \gamma = 120^\circ 30' \longrightarrow (i)$

By law of tangent

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$$

$$\frac{61-32}{61+32} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{120^\circ 30'}{2}\right)}$$

$$0.3118 \times -0.8488 = \tan\left(\frac{\beta-\gamma}{2}\right)$$

$$\tan^{-1}(-0.2647) = \frac{\beta-\gamma}{2}$$

$-14.826 \times 2 = \beta - \gamma$

$-29^\circ 39' = \beta - \gamma \longrightarrow (ii)$

Add (i) + (ii)

$$\beta + \gamma = 120^\circ 30'$$
$$\beta - \gamma = -29^\circ 39'$$
$$2\beta = 90^\circ 51'$$
$$\beta = \frac{90^\circ 51'}{2}$$
$$\boxed{\beta = 45^\circ 25'}$$

Put β in eq (i)

$$\beta + \gamma = 120^\circ 30'$$
$$\gamma = 120^\circ 30' - 45^\circ 25'$$
$$\boxed{\gamma = 75^\circ 5'}$$

By law of sin

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$a = \frac{b \sin\alpha}{\sin\beta}$$
$$a = \frac{61 \sin 59^\circ 31'}{\sin 45^\circ 25'}$$
$$\boxed{a = 73.79}$$

Exercise 12.7

Q.NO.1:-

Find the area of the triangle ABC, given two sides and their included angle.

(i) $a = 200$, $b = 120$, $\gamma = 150^\circ$

$= \frac{1}{2} ab \sin \gamma$

$= \frac{1}{2}(200)(120) \sin 150^\circ$

$= 6000$ sq. unit

(ii) $b = 37$, $c = 45$, $\alpha = 30^\circ 50'$

Area $= \frac{1}{2} bc \sin \alpha$

$= \frac{1}{2}(37)(45) \sin 30^\circ 50'$

$= 426.69$ sq. unit

(iii) $a = 4.33$, $b = 9.25$, $\gamma = 56^\circ 44'$

Area $= \frac{1}{2} ab \sin \gamma$

$= \frac{1}{2}(4.33)(9.25) \sin 56^\circ 44'$

$= 16.744$ sq. unit

Q.NO.2:-

Find the area of the triangle ABC, given one side and two angles:

(i) $b = 25.4, \quad \gamma = 36°41', \quad \alpha = 45°17'$

$$\alpha + \beta + \gamma = 180°$$

$$45°17' + \beta + 36°41' = 180°$$

$$\beta = 180 - 81°58'$$

$$\beta = 98°2$$

$$\Delta = \frac{b^2 \sin\gamma \sin\alpha}{2 \sin\beta}$$

$$= \frac{(25.4)^2 \sin 36°41' \sin 45°17'}{2 \sin 98°2'}$$

$$= 138.29 \text{ sq. units}$$

(ii) $c = 32, \quad \alpha = 47°24', \quad \beta = 70°16'$

$$\alpha + \beta + \gamma = 180°$$

$$47°24' + 70°16' + \gamma = 180°$$

$$\gamma = 180° - 117°40'$$

$$\gamma = 62°20'$$

$$\Delta = \frac{c^2 \sin\alpha \sin\beta}{2 \sin\gamma}$$

$$= \frac{(32)^2 \sin 47°24' \sin 70°16'}{2 \sin 62°20'}$$

$$= 400.54 \text{ sq. units}$$

(iii) $a = 4.8, \quad \beta = 83°42', \quad \gamma = 37°12'$

$$\alpha + \beta + \gamma = 180°$$

$$\alpha + 83°42' + 37°12' = 180°$$

$$\alpha = 180^\circ - 120^\circ 54'$$

$$\alpha = 59^\circ 6'$$

$$\Delta = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$$

$$\Delta = \frac{(4.8)^2 \sin 83^\circ 42' \sin 37^\circ 12'}{2\sin 59^\circ 6'}$$

$$\Delta = 8.068 \text{ sq. unit}$$

Q.NO.3:-

Find the area of the triangle ABC, given three sides:

(i) $a = 18$, $b = 24$, $c = 30$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

$$= \frac{18+24+30}{2}$$

$$s = 36$$

$$\Delta = \sqrt{36(36-18)(36-24)(36-30)}$$

$$= 216 \text{ sq. unit}$$

(ii) $a = 524$, $b = 276$, $c = 315$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

$$s = \frac{524+276+315}{2}$$

$$s = 557.5$$

$$\Delta = \sqrt{557.5(557.5-524)(557.5-276)(557.5-315)}$$

$$\Delta = 35705.89 \text{ sq. unit}$$

(iii) $a = 32.65,\ b = 42.81,\ c = 64.92$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

$$s = \frac{32.65+42.81+64.92}{2}$$

$$s = 70.19$$

$$\Delta = \sqrt{70.19(70.19-32.65)(70.19-42.81)(70.19-64.92)}$$

$$\Delta = 616.60 \text{ sq. units}$$

Q.NO.4:-

The area of the triangle is 2437. If $a = 79$ and $c = 97$, then find β.

$$\text{Area} = \frac{1}{2} ca \sin\beta$$

$$2437 = \frac{1}{2}(97)(79)\sin\beta$$

$$2437 = \frac{7663}{2}\sin\beta$$

$2(2437) = 7663 \sin\beta$

$\frac{4874}{7663} = \sin\beta$

$\sin\beta = 0.6360$

$\beta = \sin^{-1}(0.6360)$

$\beta = 39^\circ 29'$

Q.NO.5:-

The area of the triangle is 121.34. If $\alpha = 32^\circ 15'$, $\beta = 65^\circ 37'$, then find c and angle γ:

$\alpha + \beta + \gamma = 180^\circ$

$32^\circ 15' + 65^\circ 37' + \gamma = 180^\circ$

$\gamma = 180^\circ - 97^\circ 52'$

$\gamma = 82^\circ 8'$

Area $a = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$

$121.34 = \frac{c^2 \sin 32^\circ 15' \sin 65^\circ 37'}{2\sin 82^\circ 8'}$

$121.34 = \frac{c^2 (0.4860)}{1.9811}$

$(121.34)(1.9811) = c^2(0.4860)$

$\frac{240.39}{0.4860} = c^2$

$$\sqrt{494.62} = \sqrt{c^2}$$

$$c = 22.24$$

Q.NO.6:-

One side of a triangular garden is 30m. If its two corner angles are $22^\circ \frac{1}{2}$ and $112^\circ \frac{1}{2}$, find the cost of planting the grass at the rate of Rs.5 per square meter.

$a = 30m$, $\beta = 22.5^\circ$, $\gamma = 112.5^\circ$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + 22.5^\circ + 112.5^\circ = 180^\circ$$

$$\alpha = 180^\circ - 135^\circ$$

$$\alpha = 45^\circ$$

$$\Delta = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$$

$$= \frac{(30)^2 \sin 22.5^\circ \sin 112.5^\circ}{2\sin 45^\circ}$$

$$\Delta = 225 \text{ sq. meter}$$

Cost per sq. meter = 5 Rs.

Cost of 225 sq. meter = 5 X 225

= 1125 Rs.

Exercise 12.8

Q.NO.7:-

Show that:

(i) $r = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$

R.H.S:-

$= 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$

$= 4\left(\frac{abc}{4\Delta}\right)\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}$

$= \frac{abc}{\Delta}\sqrt{\frac{(s-a)^2(s-b)^2(s-c)^2}{a^2b^2c^2}}$ $\therefore \Delta^2 = s($

$= \frac{\cancel{abc}}{\Delta}\left[\frac{(s-a)(s-b)(s-c)}{\cancel{abc}}\right]$

$= \frac{s(s-a)(s-b)(s-c)}{s\Delta}$

$= \frac{\Delta^2}{s\cancel{\Delta}}$

$= \frac{\Delta}{s} = r = L.H.S$

(ii) $s = 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

R.H.S:-

$= 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

$= 4\left(\frac{abc}{4\Delta}\right)\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$

$= \frac{abc}{\Delta}\sqrt{\frac{s^2 \cdot s(s-a)(s-b)(s-c)}{a^2b^2c^2}}$

$= \left(\frac{abc}{\Delta}\right)\left(s\sqrt{\frac{s(s-a)(s-b)(s-c)}{abc}}\right)$

$= \frac{s\Delta}{\Delta} = s = L.H.S$

Q.NO.2:-

Show that:

(i) $r = a \sin\frac{\beta}{2} \sin\frac{\gamma}{2} \sec\frac{\alpha}{2}$

$= b \sin\frac{\gamma}{2} \sin\frac{\alpha}{2} \sec\frac{\beta}{2}$

$= c \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sec\frac{\gamma}{2}$

$r = a \sin\frac{\beta}{2} \sin\frac{\gamma}{2} \sec\frac{\alpha}{2}$

R.H.S:-

$$= a \sin\frac{\beta}{2} \sin\frac{\gamma}{2} \sec\frac{\alpha}{2}$$

$$= a\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}\sqrt{\frac{bc}{s(s-a)}}$$

$$= a\sqrt{\frac{(s-a)^2(s-b)(s-c)\cancel{bc}}{a^2\cdot \cancel{bc}\cdot s\cancel{(s-a)}}}$$

$$= \frac{\cancel{a}}{\cancel{a}}\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$= \frac{\Delta}{s} = r = L.H.S$$

R.H.S:- $= b \sin\frac{\gamma}{2} \sin\frac{\alpha}{2} \sec\frac{\beta}{2}$

$$= b\sqrt{\frac{(s-a)(s-b)}{ab}}\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{ac}{s(s-b)}}$$

$$= b\sqrt{\frac{(s-a)(s-b)^2(s-c)\cancel{ac}}{b^2\cdot \cancel{ac}\cdot s\cancel{(s-b)}}}$$

$$= \frac{\cancel{b}}{\cancel{b}}\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$= \frac{\Delta}{s} = r = L.H.S$$

R.H.S:- $= c \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sec\frac{\gamma}{2}$

$$= c\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{ab}{s(s-c)}}$$

$$= c\sqrt{\frac{(s-a)(s-b)(s-c)^2 \cdot \cancel{ab}}{\cancel{ab} \cdot c^2 \cdot s\cancel{(s-c)}}}$$

$$= \frac{\cancel{c}}{\cancel{c}}\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$= \frac{\Delta}{s} = r = L.H.S$$

Q.NO.3:-

Show that:

(i) $r_1 = 4R \sin\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$

R.H.S:-

$$= 4R \sin\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$$

$$= \cancel{4}\left(\frac{abc}{\cancel{4}\Delta}\right)\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \left(\frac{abc}{\Delta}\right)\sqrt{\frac{s^2(s-b)^2(s-c)^2}{a^2b^2c^2}}$$

$$= \left(\frac{\cancel{abc}}{\Delta}\right)\left(\frac{s(s-b)(s-c)}{\cancel{abc}}\right)$$

$$= \frac{s(s-a)(s-b)(s-c)}{\Delta(s-a)}$$

$$= \frac{\Delta^2}{\cancel{\Delta}(s-a)} = \frac{\Delta}{s-a} = r_1 = L.H.S$$

(ii) $r_2 = 4R \cos\frac{\alpha}{2} \sin\frac{\beta}{2} \cos\frac{\gamma}{2}$

R.H.S:-

$$= 4R \cos\frac{\alpha}{2} \sin\frac{\beta}{2} \cdot \cos\frac{\gamma}{2}$$

$$= 4\left(\frac{abc}{4\Delta}\right)\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \frac{abc}{\Delta}\sqrt{\frac{s^2(s-a)^2(s-c)^2}{a^2b^2c^2}}$$

$$= \left(\frac{abc}{\Delta}\right)\left(\frac{s(s-a)(s-c)}{abc}\right)$$

$$= \frac{s(s-a)(s-b)(s-c)}{\Delta(s-b)}$$

$$= \frac{\Delta^2}{\cancel{\Delta}(s-b)} = \frac{\Delta}{s-b} = r_2 = L.H.S$$

(iii) $r_3 = 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \sin\frac{\gamma}{2}$

R.H.S:-

$$= 4R \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \sin\frac{\gamma}{2}$$

$$= 4\left(\frac{abc}{4\Delta}\right)\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$= \frac{abc}{\Delta}\sqrt{\frac{s^2(s-a)^2(s-b)^2}{a^2b^2c^2}}$$

$$= \left(\frac{abc}{\Delta}\right)\left(\frac{s(s-a)(s-b)}{abc}\right)$$

$$= \frac{s(s-a)(s-b)(s-c)}{\Delta(s-c)}$$

$$= \frac{\Delta^2}{\cancel{\Delta}(s-c)} = \frac{\Delta}{s-c} = r_3 = L.H.S$$

Q.NO.46:-

Show that:

(i) $r_1 = s \tan \frac{\alpha}{2}$

R.H.S:- $= s \tan \frac{\alpha}{2}$

$$= s\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= s\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-a)^2}}$$

$$= \cancel{s}\cdot\frac{\Delta}{\cancel{s}(s-a)} = \frac{\Delta}{s-a} = r_1 = L.H.S$$

(ii) $r_2 = s \tan \frac{\beta}{2}$

R.H.S:- $= s \tan \frac{\beta}{2}$

$$= s\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= s\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-b)^2}}$$

$$= \cancel{s}\cdot\frac{\Delta}{\cancel{s}(s-b)} = \frac{\Delta}{s-b} = r_2 = L.H.S$$

(iii) $r_3 = s\tan\frac{\gamma}{2}$

R.H.S:- $= s\tan\frac{\gamma}{2}$

$$= s\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= s\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-c)^2}}$$

$$= \cancel{s}\cdot\frac{\Delta}{\cancel{s}(s-c)} = \frac{\Delta}{s-c} = r_3 = L.H.S$$

Q.NO.5:-

Prove that:

(i) $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

L.H.S:

$$=\left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right)+\left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)+\left(\frac{\Delta}{s-c}\right)\left(\frac{\Delta}{s-a}\right)$$

$$= \Delta^2\left[\frac{1}{(s-a)(s-b)} + \frac{1}{(s-b)(s-c)} + \frac{1}{(s-c)(s-a)}\right]$$

$$= \Delta^2 \left[\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)} \right]$$

$$= s\Delta^2 \left[\frac{3s-(a+b+c)}{s(s-a)(s-b)(s-c)} \right] \quad \because 2s = a+b+c$$

$$= s\cancel{\Delta^2} \left[\frac{3s-2s}{\cancel{\Delta^2}} \right]$$

$$= s(s) = s^2 = R.H.S$$

(ii) $r r_1 r_2 r_3 = \Delta^2$

L.H.S:-

$$= \left(\frac{\Delta}{s}\right)\left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)$$

$$= \frac{\Delta^2 \cdot \Delta^2}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta^2 \cdot \cancel{\Delta^2}}{\cancel{\Delta^2}}$$

$$= \Delta^2 = R.H.S$$

(iii) $r_1 + r_2 + r_3 - r = 4R$

L.H.S:-

$$= \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s}$$

$$= \Delta \left[\left(\frac{1}{s-a} + \frac{1}{s-b} \right) + \left(\frac{1}{s-c} - \frac{1}{s} \right) \right]$$

$$= \Delta \left[\frac{s-b+s-a}{(s-a)(s-b)} + \frac{\cancel{s}-\cancel{s}+c}{s(s-c)} \right]$$

$$= \Delta\left[\frac{2s-a-b}{(s-a)(s-b)} + \frac{c}{s(s-c)}\right]$$

$$= \Delta\left[\frac{a+b+c-a-b}{(s-a)(s-b)} + \frac{c}{s(s-c)}\right]$$

$$= \Delta c\left[\frac{1}{(s-a)(s-b)} + \frac{1}{s(s-c)}\right]$$

$$= \Delta c\left[\frac{s(s-c)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right]$$

$$= \cancel{\Delta} c\left[\frac{s^2-sc+s^2-sb-sa+ab}{\Delta^{\cancel{2}}}\right]$$

$$= c\left[\frac{2s^2-s(a+b+c)+ab}{\Delta}\right]$$

$$= c\left[\frac{\cancel{2s^2}-\cancel{s(2s)}+ab}{\Delta}\right]$$

$$= \frac{4abc}{4\Delta} = 4R = R.H.S$$

(iv) $r_1\, r_2\, r_3 = rs^2$

L.H.S:-

$$= \left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)$$

$$= \frac{s\,\Delta^2\cdot\Delta}{s(s-a)(s-b)(s-c)} \qquad \because r = \frac{\Delta}{s}, \quad \Delta = rs$$

$$= \frac{s\cancel{\Delta^2}\cdot\Delta}{\cancel{\Delta^2}} = s(rs) = rs^2 = R.H.S$$

Q.NO.6:-

Find R, r, r_1, r_2 and r_3, if measures of sides of the triangle ABC are

(i) $a = 13$, $b = 14$, $c = 15$

$$S = \frac{a+b+c}{2}$$

$$= \frac{13+14+15}{2} = 21$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$\Delta = \sqrt{7056}$$

$$\Delta = 84$$

$$r = \frac{\Delta}{s} = \frac{84}{21} = 4$$

$$r_1 = \frac{\Delta}{s-a} = \frac{84}{21-13} = 10.5$$

$$r_2 = \frac{\Delta}{s-b} = \frac{84}{21-14} = 12$$

$$r_3 = \frac{\Delta}{s-c} = \frac{84}{21-15} = 14$$

$$R = \frac{abc}{4\Delta} = \frac{(13)(14)(15)}{4(84)} = 8125$$

(iii) $a = 34$, $b = 20$, $c = 42$

$$s = \frac{a+b+c}{2}$$

$$= \frac{34+20+42}{2} = 48$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta = \sqrt{48(48-34)(48-20)(48-42)}$$

$$\Delta = 336$$

$$r = \frac{\Delta}{s} = \frac{336}{48} = 7$$

$$r_1 = \frac{\Delta}{s-a} = \frac{336}{48-34} = 24$$

$$r_2 = \frac{\Delta}{s-b} = \frac{336}{48-20} = 12$$

$$r_3 = \frac{\Delta}{s-c} = \frac{336}{48-42} = 56$$

$$R = \frac{abc}{4\Delta} = \frac{(34)(20)(42)}{4(336)} = 21.25$$

Q.NO.7:-

Prove that in an equilateral triangle.

(i) $r : R : r_1 = 1 : 2 : 3$

$a = b = c$

$$s = \frac{a+b+c}{2} = \frac{a+a+a}{2}$$

$$\boxed{s = \frac{3a}{2}}$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{s(s-a)(s-a)(s-a)}$$

$$= \sqrt{s(s-a)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a-2a}{2}\right)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)^3}$$

$$= \sqrt{\frac{3a^4}{16}}$$

$$\boxed{\Delta = \frac{\sqrt{3}\,a^2}{4}}$$

$$\Rightarrow \quad r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}\,a^2}{4}}{\frac{3a}{2}}$$

$$= \frac{\sqrt{3}\,a^{\cancel{2}}}{\cancel{4}_2} \times \frac{\cancel{2}}{3\cancel{a}}$$

$$= \frac{\cancel{\sqrt{3}}\,a}{2\cancel{\sqrt{3}} \times \sqrt{3}}$$

$$\boxed{r = \frac{a}{2\sqrt{3}}}$$

$$R = \frac{abc}{4\Delta} = \frac{(a)(a)(a)}{4\left(\frac{\sqrt{3}a^2}{4}\right)}$$

$$\boxed{R = \frac{a}{\sqrt{3}}}$$

(ii) $r : R : r_1 = 1 : 2 : 3$

L.H.S = $r : R : r_1$

$$R = \frac{abc}{4\Delta} = \frac{(a)(a)(a)}{4\left(\frac{\sqrt{3}a^2}{4}\right)}$$

$$R = \frac{a}{\sqrt{3}}$$

$$r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}a^2}{4_2}}{\frac{3a}{2}} = \frac{\sqrt{3}a}{2\sqrt{3}\sqrt{3}}$$

$$r = \frac{a}{2\sqrt{3}}$$

$$r_1 = \frac{\Delta}{s-a}$$

$$= \frac{\Delta}{\frac{3a}{2} - a}$$

$$= \frac{\Delta}{\frac{3a-2a}{2}}$$

$$= \frac{\frac{\sqrt{3}a^2}{4_2}}{\frac{a}{2}} = \frac{\sqrt{3}a}{2}$$

$$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}a}{2}$$

Multiply by $\frac{2\sqrt{3}}{a}$.

$$= \frac{a}{2\sqrt{3}} \times \frac{2\sqrt{3}}{a} : \frac{a}{\sqrt{3}} \times \frac{2\sqrt{3}}{a} : \frac{\sqrt{3}a}{2} \times \frac{2\sqrt{3}}{a}$$

$$= 1 : 2 : (\sqrt{3})^2$$

$$= 1 : 2 : 3 = R.H.S$$

(ii) $r : R : r_1 : r_2 : r_3 = 1 : 2 : 3 : 3 : 3$

In equilateral triangle

$$a = b = c$$

$$S = \frac{a+b+c}{2} = \frac{a+a+a}{2}$$

$$S = \frac{3a}{2}$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{S(S-a)(S-a)(S-a)}$$

$$= \sqrt{S\,(S-a)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a-2a}{2}\right)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)^3}$$

$$= \sqrt{\frac{3a^4}{16}} \Rightarrow \Delta = \frac{\sqrt{3}\,a^2}{4}$$

$$r = \frac{\Delta}{s}$$

$$r = \frac{\frac{\sqrt{3}\,a^2}{\cancel{4}\,2}}{\frac{3a}{\cancel{2}}}$$

$$r = \frac{\cancel{\sqrt{3}}\,a}{2\cancel{\sqrt{3}}\sqrt{3}}$$

$$r = \frac{a}{2\sqrt{3}}$$

$$r_1 = \frac{\Delta}{s-a}$$

$$= \frac{\Delta}{\frac{3a}{2}-a}$$

$$= \frac{\Delta}{\frac{3a-2a}{2}}$$

$$= \frac{\frac{\sqrt{3}\,a^2}{\cancel{4}\,2}}{\frac{a}{\cancel{2}}}$$

$$r_1 = \frac{\sqrt{3}a}{2}$$

Similarly,

$$r_2 = \frac{\sqrt{3}\,a}{2}, \quad r_3 = \frac{\sqrt{3}\,a}{2}$$

$$R = \frac{abc}{4\Delta}$$

$$= \frac{a \cdot a^2}{4\left(\frac{\sqrt{3}a^2}{4}\right)}$$

$$R = \frac{a}{\sqrt{3}}$$

L.H.S:- $r : R : r_1 : r_2 : r_3$

$$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}a}{2} : \frac{\sqrt{3}a}{2} : \frac{\sqrt{3}a}{2}$$

Multiply by $\frac{2\sqrt{3}}{a}$

$$= \frac{a}{2\sqrt{3}} \times \frac{2\sqrt{3}}{a} : \frac{a}{\sqrt{3}} \times \frac{2\sqrt{3}}{a} : \frac{\sqrt{3}a}{2} \times \frac{2\sqrt{3}}{a} : \frac{\sqrt{3}a}{2} \times \frac{2\sqrt{3}}{a} : \frac{\sqrt{3}a}{2} \times \frac{2\sqrt{3}}{a}$$

$$= 1 : 2 : (\sqrt{3})^2 : (\sqrt{3})^2 : (\sqrt{3})^2$$

$$= 1 : 2 : 3 : 3 : 3 = \text{R.H.S}$$

Q.NO.8:-

Prove that:

(i) $\Delta = r^2 \cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

R.H.S:- $= r^2 \cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

$$= \left(\frac{\Delta}{s}\right)^2 \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \frac{\Delta^2}{s^2}\sqrt{\frac{s^{\cancel{2}}\cdot s(s-a)(s-b)(s-c)}{(s-a)^2(s-b)^2(s-c)^2}}$$

$$= \frac{\Delta^2}{s^{\cancel{2}}}\cdot\frac{s\Delta}{(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta^2\cdot\Delta}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\cancel{\Delta^2}\cdot\Delta}{\cancel{\Delta^2}} = \Delta = L.H.S$$

(iii) $r = s\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

R.H.S:-

$$= s\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= s\sqrt{\frac{(s-a)^2(s-b)^2(s-c)^2}{s^2\cdot s(s-a)(s-b)(s-c)}}$$

$$= \frac{\cancel{s}}{\cancel{s}}\cdot(s-a)(s-b)(s-c)\sqrt{\frac{1}{s(s-a)(s-b)(s-c)}}$$

$$= (s-a)(s-b)(s-c)\cdot\frac{1}{\Delta}$$

$$= \frac{s(s-a)(s-b)(s-c)}{\Delta s}$$

$$= \frac{\Delta^2}{\Delta s} = \frac{\Delta}{s} = r = L.H.S$$

(iii) $\Delta = 4Rr\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

R.H.S:-

$$= 4\left(\frac{abc}{4\Delta}\right)\left(\frac{\Delta}{s}\right)\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \frac{abc}{s}\sqrt{\frac{s^2 \cdot s(s-a)(s-b)(s-c)}{a^2b^2c^2}}$$

$$= \frac{\cancel{abc}}{\cancel{s}} \cdot \frac{\cancel{s}}{\cancel{abc}} \cdot \Delta = \Delta = L.H.S$$

Q.NO.9:-

Show that:

(i) $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}$

$$L.H.S = \frac{1}{2rR}$$

$$= \frac{1}{2\left(\frac{\Delta}{s}\right)\left(\frac{abc}{4\Delta}\right)}$$

$$= \frac{2s}{abc}$$

$$= \frac{a+b+c}{abc}$$

$$= \frac{\cancel{a}}{\cancel{a}bc} + \frac{\cancel{b}}{a\cancel{b}c} + \frac{\cancel{c}}{ab\cancel{c}}$$

$$= \frac{1}{bc} + \frac{1}{ac} + \frac{1}{ab} = R.H.S$$

(ii) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

R.H.S:- $= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$$= \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$

$$= \frac{s-a+s-b+s-c}{\Delta}$$

$$= \frac{3s-(a+b+c)}{\Delta}$$

$$= \frac{3s-2s}{\Delta} = \frac{s}{\Delta} = \frac{1}{r} = L.H.S$$

Q.NO.10:-

Prove that:

(i) $r = \frac{a \sin\frac{\beta}{2} \sin\frac{\gamma}{2}}{\cos\frac{\alpha}{2}}$

$$= \frac{a\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{\cancel{(s-a)}(s-b)}{a\cancel{b}}}}{\sqrt{\frac{s\cancel{(s-a)}}{b\cancel{c}}}}$$

$$= \frac{a\sqrt{\frac{(s-a)(s-b)(s-c)}{a^2}}}{\sqrt{s}}$$

$$= \frac{\cancel{a}}{\cancel{a}}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$= \sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$= \frac{\Delta}{s} = r = L.H.S$$

(ii) $r = \dfrac{b \sin\frac{\alpha}{2} \cdot \sin\frac{\gamma}{2}}{\cos\frac{\beta}{2}}$

R.H.S :-

$$= \frac{b\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-b)}{ab}}}{\sqrt{\frac{s(s-b)}{ac}}}$$

$$= b\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-b)}{ab}}\sqrt{\frac{ac}{s(s-b)}}$$

$$= b\sqrt{\frac{(s-a)\cancel{(s-b)}(s-b)(s-c)\cancel{(ac)}}{b^2 \cdot \cancel{ac} \cdot s\cancel{(s-b)}}}$$

$$= \frac{\cancel{b}}{\cancel{b}}\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$= \frac{\Delta}{s} = r = L.H.S$$

(iii) $r = \dfrac{c \sin\frac{\alpha}{2} \sin\frac{\beta}{2}}{\cos\frac{\gamma}{2}}$

R.H.S:-

$$c\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-c)(s-a)}{ac}}$$

$$=\sqrt{\frac{s(s-c)}{ab}}$$

$$=c\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-c)(s-a)}{ac}}\sqrt{\frac{ab}{s(s-c)}}$$

$$=c\sqrt{\frac{(s-a)(s-b)(s-c)^2\cancel{(ab)}}{\cancel{ab}\cdot c^2\cdot s\cancel{(s-c)}}}$$

$$=\frac{\cancel{c}}{\cancel{c}}\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$=\frac{\Delta}{s}=r=L.H.S$$

Q.NO. 11:-

Prove that:

$$abc(\sin\alpha+\sin\beta+\sin\gamma)=4\Delta s$$

L.H.S:-

$$R=\frac{a}{2\sin\alpha}=\frac{b}{2\sin\beta}=\frac{c}{2\sin\gamma}$$

$$R=\frac{a}{2\sin\alpha},\quad \sin\alpha=\frac{a}{2R}$$

$$R=\frac{b}{2\sin\beta},\quad \sin\beta=\frac{b}{2R}$$

$R = \frac{c}{2\sin\gamma}$, $\sin\gamma = \frac{c}{2R}$

$= abc\left(\frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}\right)$

$= abc\left(\frac{a+b+c}{2R}\right)$

$= \cancel{abc}\left(\frac{\cancel{2}s}{\cancel{2}\left(\frac{\cancel{abc}}{4\Delta}\right)}\right)$

$= 4\Delta s = R.H.S$

Q.NO.12:-

Prove that:

(i) $(r_1 + r_2)\tan\frac{\gamma}{2} = c$

L.H.S:-

$= \left(\frac{\Delta}{s-a} + \frac{\Delta}{s-b}\right)\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$= \Delta\left(\frac{1}{s-a} + \frac{1}{s-b}\right)\sqrt{\frac{(s-a)^2(s-b)^2}{s(s-a)(s-b)(s-c)}}$

$= \cancel{\Delta}\left[\frac{s-b+s-a}{\cancel{(s-a)}\cancel{(s-b)}}\right]\left[\frac{\cancel{(s-a)}\cancel{(s-b)}}{\cancel{\Delta}}\right]$

$= 2s - a - b$

$= \cancel{2s}\ \cancel{a} + \cancel{b} + c - \cancel{a} - \cancel{b}$

$= c$

$= R.H.S$

(iii) $(r_3 - r_4)\cot\dfrac{\gamma}{2} = c$

L.H.S:-

$$= \left(\frac{\Delta}{s-c} - \frac{\Delta}{s}\right)\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \Delta\left(\frac{1}{s-c} - \frac{1}{s}\right)\sqrt{\frac{s^2(s-c)^2}{s(s-a)(s-b)(s-c)}}$$

$$= \cancel{\Delta}\left[\frac{s-(s-c)}{\cancel{s}\cancel{(s-c)}}\right]\left[\frac{\cancel{s}\cancel{(s-c)}}{\cancel{\Delta}}\right]$$

$$= \cancel{s} - \cancel{s} + c$$

$$= c = R.H.S$$