Chapter NO. 10:-

Trigonometric Identities

Fundamental Law of trigonometry

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Examples related to Exercise 10.2

Example 3:-

Prove ~~tat~~ that $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$

R.H.S:-

$$= \tan 56^\circ$$

$$= \tan(45^\circ + 11^\circ)$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ}$$

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{1 + \frac{\sin 11^\circ}{\cos 11^\circ}}{1 - \frac{\sin 11^\circ}{\cos 11^\circ}}$$

$$= \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

$$= \text{R.H.S}$$

Example 5:

Express $3 \sin \theta + 4 \cos \theta$ in the form $r \sin(\theta + \phi)$, where the terminal side of the angle of measure ϕ is in the I quadrant

Exercise 10.2

Q.NO.1:-

Prove that:

$\sin(180^\circ + \theta) = -\sin\theta$

L.H.S:-

$= \sin(180^\circ + \theta)$

$\because \sin(\alpha + \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= (\sin 180)\cdot(\cos\theta) + (\sin\theta)(\cos 180)$

$= (0)\cos\theta + \sin\theta(-1)$

$= -\sin\theta \quad = R.H.S$

(ii) $\cos(180°+\theta) = -\cos\theta$

L.H.S:-

$= \cos(180°+\theta)$

$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= (\cos 180)(\cos\theta) - (\sin 180)(\sin\theta)$

$= (-1)\cos\theta - (0)\sin\theta$

$= -\cos\theta$

$=$ R.H.S

(iii) $\tan 270° - \theta = \cot\theta$

L.H.S:-

$= \tan 270° - \theta$

$$= \frac{\sin(270°-\theta)}{\cos(270°-\theta)}$$

$$= \frac{(\sin 270)(\cos\theta) - (\sin\theta)(\cos 270)}{(\cos 270)(\cos\theta) + (\sin 270)(\sin\theta)}$$

$$= \frac{(-1)\cos\theta - \sin\theta(0)}{(0)\cos\theta + (-1)(0)}$$

$$= \frac{+\cos\theta}{+\sin\theta}$$

$= \cot\theta$

$=$ R.H.S

(vii) $\tan(180°+\theta) = \tan\theta$

L.H.S:-

$= (\tan 180^\circ + \theta)$

$\because \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \dfrac{\tan 180^\circ + \tan\theta}{1 - (\tan 180^\circ)(\tan\theta)}$

$= \dfrac{\tan\theta}{1}$

$= \tan\theta$

$=$ R.H.S

(iv) $\cos(\theta - 180^\circ) = -\cos\theta$

L.H.S:-

$= \cos(\theta - 180^\circ)$

$\because \cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$= \cos\theta \cos 180^\circ + \sin\theta \sin 180^\circ$

$= \cos(-1) + \sin\theta(0)$

$= -\cos\theta + 0$

$= -\cos\theta$

$=$ R.H.S

(v) $\cos(270^\circ + \theta) = \sin\theta$

L.H.S:-

$= \cos(270^\circ + \theta)$

$\because \cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$= \cos 270^\circ \cos\theta - \sin 270^\circ \sin\theta$

$= (0)\cos\theta - (-1)\sin\theta$

$= 0 + \sin\theta$

$= \sin\theta$

$= R.H.S$

(vi) $\sin(\theta + 270°) = -\cos\theta$

L.H.S:-

$= \sin(\theta + 270°)$

$\therefore (\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \sin\theta\cos 270° + \cos\theta\sin 270°$

$= \sin\theta(0) + \cos\theta(-1)$

$= 0 - \cos\theta$

$= -\cos\theta$

$= R.H.S$

viii) $\cos(360° - \theta) = \cos\theta$

L.H.S:-

$= \cos(360° - \theta)$

$\therefore \cos(\alpha \mp \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$

$= \cos 360 \cos\theta + \sin 360 \sin\theta$

$= (1)\cos\theta + (0)\sin\theta$

$= \cos\theta$

$= R.H.S$

Q.NO.2:-

Find the values of the

following.

(i) $\sin 15^\circ$

$$= \sin (45^\circ - 30^\circ)$$

$\therefore \sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

$$= \sin 45^\circ \sin 30^\circ - \cos 45^\circ \cos 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{3}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

(ii) $\cos 15^\circ$

$$= \cos (45^\circ - 30^\circ)$$

$\therefore \cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

(iii) $\tan 15^\circ$

$$= \tan (45^\circ - 30^\circ)$$

$\therefore \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$= \dfrac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ}$

$= \dfrac{1-\frac{1}{\sqrt{3}}}{1+(1)\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

(iv) $\sin 105^\circ$

$= \sin(60^\circ+45^\circ)$

$\therefore \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

(v) $\cos 105^\circ$

$= \cos(60^\circ+45^\circ)$

$\therefore \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

(vi) $\tan 105^\circ$

$= \tan(60^\circ + 45^\circ)$

$\because \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}(1)}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$

Q.NO.3:-

Prove that:

$\sin(45^\circ + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$

L.H.S:-

$= \sin(45^\circ + \alpha)$

$\because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \sin 45 \cos\alpha + \cos 45 \sin\alpha$

$= \left(\frac{1}{\sqrt{2}}\right)\cos\alpha + \left(\frac{1}{\sqrt{2}}\right)\sin\alpha$

$= \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha) = R.H.S$

(ii) $\cos(\alpha + 45^\circ) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

L.H.S:-

$= \cos(\alpha + 45^\circ)$

$\because \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos\alpha\cos 45 - \sin\alpha\sin 45$

$= \cos\alpha\left(\frac{1}{\sqrt{2}}\right) - \sin\alpha\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

$= R.H.S$

Q.NO. 4:-

Prove that:

(i) $\tan(45^\circ + A)\tan(45^\circ - 1) = 1$

L.H.S:-

$$= \frac{\tan(45+A)+\tan(45-A)}{1-\tan(45+A)\tan(45-A)}$$

$$= \left(\frac{\tan(45+A)}{1-\tan 45 \tan A}\right)\left(\frac{\tan(45-A)}{1+\tan 45 \tan A}\right)$$

$$= 1$$

$$= R.H.S$$

(ii) $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)=0$

L.H.S:-

$$= \frac{\tan\left(\frac{\pi}{4}\right)-\tan\theta}{1+\tan\left(\frac{\pi}{4}\right)\tan\theta} + \frac{\tan\left(\frac{3\pi}{4}\right)+\tan\theta}{1-\tan\left(\frac{3\pi}{4}\right)\tan\theta}$$

$$= \frac{1-\tan\theta}{1+\tan\theta} + \frac{-1+\tan\theta}{1+\tan\theta}$$

$$= \frac{\cancel{1}-\cancel{\tan\theta}-\cancel{1}+\cancel{\tan\theta}}{1+\tan\theta}$$

$$= \frac{0}{1+\tan\theta}$$

$$= 0$$

$$= R.H.S.$$

$\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$

L.H.S:-

$$= \sin\theta \cos\frac{\pi}{6} + \cos\theta \sin\frac{\pi}{6} + \cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}$$

$$= \sin\theta\left(\cancel{\frac{\sqrt{3}}{2}}\right) + \cos\theta\left(\frac{1}{2}\right) + \cos\theta\left(\frac{1}{2}\right) - \sin\theta\left(\cancel{\frac{\sqrt{3}}{2}}\right)$$

$$= \cos\theta\left(\frac{1}{2} + \frac{1}{2}\right)$$

$$= \cos\theta\left(\frac{1+1}{2}\right)$$

$$= \cos\theta\left(\frac{\cancel{2}}{\cancel{2}}\right)$$

$$= \cos\theta(1)$$

$$= 1\cos\theta$$

$$= R.H.S$$

(iv) $$\frac{\sin\theta - \cos\theta \tan\frac{\theta}{2}}{\cos\theta - \sin\theta\tan\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

L.H.S:-

$$= \frac{\sin\theta - \cos\theta\cdot\left[\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\right]}{\cos\theta - \sin\theta\left[\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\right]}$$

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$$= \frac{\frac{(\sin\theta)\left(\cos\frac{\theta}{2}\right) - (\cos\theta)\left(\sin\frac{\theta}{2}\right)}{\cancel{\cos\frac{\theta}{2}}}}{\frac{(\cos\theta)\left(\cos\frac{\theta}{2}\right) + (\sin\theta)\left(\sin\frac{\theta}{2}\right)}{\cancel{\cos\frac{\theta}{2}}}}$$

$$= \frac{\sin\left(\theta - \frac{\theta}{2}\right)}{\cos\left(\theta - \frac{\theta}{2}\right)}$$

$$= \tan\left(\theta - \frac{\theta}{2}\right)$$

$$= \tan\frac{2\theta - \theta}{2}$$

$$= \tan\frac{\theta}{2}$$

$= R.H.S$

$$\frac{1 - \tan\theta\tan\phi}{1 + \tan\theta\tan\phi} = \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$$

L.H.S:-

$$= \frac{1 - \tan\theta\tan\phi}{1 + \tan\theta\tan\phi}$$

$$= \frac{1 - \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\phi}{\cos\phi}\right)}{1 + \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\phi}{\cos\phi}\right)}$$

$$= \frac{\frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi}}{\frac{\cos\theta\cos\phi + \sin\theta\sin\phi}{\cos\theta\cos\phi}}$$

$$= \frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi + \sin\theta\sin\phi}$$

$$= \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$$

$= R.H.S$

Q.NO.5:-

Show that:

$$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

L.H.S:-

$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta \longrightarrow (i)$

$= \cos^2\alpha(1-\sin^2\beta) - (1-\cos^2\alpha)\sin^2\beta$

$= \cos^2\alpha - \cancel{\cos^2\alpha\sin^2\beta} - \sin^2\beta + \cancel{\cos^2\alpha\sin^2\beta}$

$= \cos^2\alpha - \sin^2\beta$

From eq (i)

$= (1-\sin^2\alpha)\cos^2\beta - \sin^2\alpha(1-\cos^2\beta)$

$= \cos^2\beta - \cancel{\cos^2\beta\sin^2\alpha} - \sin^2\alpha + \cancel{\cos^2\beta\sin^2\alpha}$

$= \cos^2\beta - \sin^2\alpha$

$= R.H.S$

Q.NO.6:-

Show that:

$$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \tan\alpha$$

L.H.S:-

$$= \frac{\sin\alpha\cos\beta+\cancel{\cos\alpha\sin\beta}+\sin\alpha\cos\beta-\cancel{\sin\beta\cos\alpha}}{\cos\alpha\cos\beta-\cancel{\sin\alpha\sin\beta}+\cos\alpha\cos\beta+\cancel{\sin\alpha\sin\beta}}$$

$$= \frac{\cancel{2}\sin\alpha\cancel{\cos\beta}}{\cancel{2}\cos\alpha\cancel{\cos\beta}}$$

$$= \tan\alpha$$

$$= R.H.S$$

Q.NO.11:-

Prove that:

$$\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$$

R.H.S:-

$$= \tan 37^\circ$$

$$= \tan(45^\circ - 8^\circ)$$

$$= \frac{\sin(45-8)}{\cos(45-8)}$$

$$= \frac{\sin 45\cos 8 - \cos 45\sin 8}{\cos 45\cos 8 + \sin 45\sin 8}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)\cos 8 - \left(\frac{1}{\sqrt{2}}\right)\sin 8}{\left(\frac{1}{\sqrt{2}}\right)\cos 8 + \left(\frac{1}{\sqrt{2}}\right)\sin 8}$$

$$= \frac{\cancel{\frac{1}{\sqrt{2}}}(\cos 8 - \sin 8)}{\cancel{\frac{1}{\sqrt{2}}}(\cos 8 + \sin 8)}$$

$$= \frac{\cos 8 - \sin 8}{\cos 8 + \sin 8}$$

$$= R.H.S$$

Q.NO.7:-

Show that:

(i) $\cot(\alpha+\beta) = \dfrac{\cot\alpha \cot\beta - 1}{\cot\alpha + \cot\beta}$

R.H.S:-

$$= \frac{\cot\alpha \cot\beta - 1}{\cot\alpha + \cot\beta}$$

$$= \frac{\left[\left(\frac{\cos\alpha}{\sin\alpha}\right)\left(\frac{\cos\beta}{\sin\beta}\right) - 1\right]}{\frac{\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta}}$$

$$= \frac{\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cancel{\sin\alpha\sin\beta}}}{\frac{\cos\alpha\sin\beta + \cos\beta\sin\alpha}{\cancel{\sin\alpha\sin\beta}}}$$

$$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\sin\beta + \cos\beta\sin\alpha}$$

$$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}$$

$$= \cot(\alpha+\beta)$$

$$= L.H.S$$

ii) $\cot(\alpha-\beta) = \dfrac{\cot\alpha \cot\beta + 1}{\cot\beta - \cot\alpha}$

R.H.S:-

$$= \frac{\cot\alpha \cot\beta + 1}{\cot\beta - \cot\alpha}$$

$$= \frac{\left[\left(\frac{\cos\alpha}{\sin\alpha}\right)\left(\frac{\cos\beta}{\sin\beta}\right) + 1\right]}{\left(\frac{\cos\beta}{\sin\beta}\right) - \left(\frac{\cos\alpha}{\sin\alpha}\right)}$$

$$= \frac{\dfrac{\cos\alpha \cos\beta + \sin\alpha \sin\beta}{\cancel{\sin\alpha \sin\beta}}}{\dfrac{\cos\alpha \sin\beta - \cos\beta \sin\alpha}{\cancel{\sin\alpha \sin\beta}}}$$

$$= \frac{\cos\alpha \cos\beta + \sin\alpha \sin\beta}{\cos\alpha \sin\beta - \cos\beta \sin\alpha}$$

$$= \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)}$$

$$= \cot(\alpha-\beta)$$

$$= L.H.S$$

(iii) $\dfrac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

L.H.S:-

$$= \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}$$

$$= \frac{\dfrac{\sin\alpha}{\cos\beta} + \dfrac{\sin\beta}{\cos\beta}}{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{\sin\beta}{\cos\beta}}$$

$$= \frac{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cancel{\cos\alpha\cos\beta}}}{\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cancel{\cos\alpha\cos\beta}}}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

$$= \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$$

$= R.H.S$

Q.NO. 12:-

If α, β, γ are the angles of a triangle ABC, show that:

$$\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\alpha+\beta = 180-\gamma$$

$$\frac{\alpha+\beta}{2} = \frac{180-\gamma}{2}$$

$$\frac{\alpha}{2}+\frac{\beta}{2} = \frac{\cancel{180^\circ}^{90}}{\cancel{2}} - \frac{\gamma}{2}$$

$$\frac{\alpha}{2}+\frac{\beta}{2} = 90^\circ - \frac{\gamma}{2}$$

$$\cot\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \cot\left(90^\circ - \frac{\gamma}{2}\right)$$

$$\cot\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \tan\frac{\gamma}{2}$$

$$\frac{\left(\cot\frac{\alpha}{2}\right)\left(\cot\frac{\beta}{2}\right)-1}{\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}} = \frac{1}{\cot\frac{\gamma}{2}}$$

By cross multiplication

$$\cot\frac{\gamma}{2}\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1\right) = \cot\frac{\alpha}{2}+\cot\frac{\beta}{2}$$

$$\cot\frac{\gamma}{2}\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}+\cot\frac{\beta}{2}$$

$$\cot\frac{\gamma}{2}\cot\frac{\alpha}{2}\cot\frac{\beta}{2} = \cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2}$$

Hence proved

Q.NO.13:-

If $\alpha+\beta+\gamma = 180^\circ$, show that
$\cot\alpha \cot\beta + \cot\beta \cot\gamma + \cot\gamma \cot\alpha = 1$

$$\alpha+\beta+\gamma = 180^\circ$$
$$\alpha+\beta = 180 - \gamma$$

Taking cot on both sides

$$\cot(\alpha+\beta) = \cot(180^\circ - \gamma)$$
$$\frac{\cot\alpha \cot\beta - 1}{\cot\alpha + \cot\beta} = -\cot\gamma$$
$$\cot\alpha \cot\beta - 1 = -\cot\gamma(\cot\alpha + \cot\beta)$$
$$\cot\alpha \cot\beta - 1 = -\cot\alpha \cot\gamma - \cot\gamma \cot\beta$$
$$\cot\alpha \cot\beta + \cot\beta \cot\gamma + \cot\gamma \cot\alpha = 1$$

Hence proved

Q.NO.8:-

If ~~sict~~ $\sin\alpha = \frac{5}{4}$ and $\cos\beta = \frac{40}{41}$, where $0 < \alpha < \frac{\pi}{2}$ and $0 < \beta < \frac{\pi}{2}$.

show that $(\alpha - \beta) = \frac{133}{205}$.

$$\sin\alpha = \frac{4}{5} = \frac{P}{H}$$

By pythagoras theorem

$$H^2 = B^2 + P^2$$

$$(5)^2 = B^2 + (4)^2$$

$$25 - 16 = B^2$$

$$\sqrt{9} = \sqrt{B^2}$$

$$3 = B$$

$$\cos\alpha = \frac{B}{H} = \frac{3}{5}$$

$$\cos\beta = \frac{40}{41} = \frac{B}{H}$$

By pythagoras theorem

$$H^2 = B^2 + P^2$$

$$(41)^2 = (40)^2 + P^2$$

$$1681 - 1600 = P^2$$

$$\sqrt{81} = \sqrt{P^2}$$

$$9 = P$$

$$\sin\beta = \frac{P}{H} = \frac{9}{41}$$

L.H.S:-

$$= \sin(\alpha - \beta)$$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \left(\frac{4}{5}\right)\left(\frac{40}{41}\right) - \left(\frac{3}{5}\right)\left(\frac{9}{41}\right)$$

$$= \frac{160}{205} - \frac{27}{205}$$

$$= \frac{133}{205}$$

$$= R.H.S$$

Q.NO.9B-

If $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$ where $\frac{\pi}{2} < \alpha < \pi$ and $\frac{\pi}{2} < \beta < \pi$. Find

(i) $\sin^2\alpha + \cos^2\alpha = 1$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$= 1 - \left(\frac{4}{5}\right)^2$$

$$= 1 - \frac{16}{25}$$

$$= \frac{25-16}{25}$$

$$\sqrt{\cos^2\alpha} = \sqrt{\frac{9}{25}}$$

$$\cos\alpha = \frac{3}{5}$$

$$\cos^2\beta = 1 - \sin^2\beta$$

$$= 1 - \left(\frac{12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$= \frac{169-144}{169}$$

$$\sqrt{\cos^2\beta} = \sqrt{\frac{25}{169}}$$

$$\cos\beta = -\frac{5}{13}$$

$$\tan\alpha = -\frac{4}{3}$$

$$\tan\beta = -\frac{12}{5}$$

$\sin(\alpha+\beta)$

$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-20}{65} - \frac{36}{65}$$

$$= \frac{-20-36}{65}$$

$$= \frac{-56}{65}$$

$\cos(\alpha+\beta)$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= \frac{15-48}{65}$$

$$= \frac{-33}{65}$$

(iii) $\tan(\alpha+\beta)$

$$= \frac{\alpha+\beta}{1-\alpha\beta}$$

$$= \frac{\left(\frac{-4}{3}\right)+\left(\frac{-12}{5}\right)}{1-\left(\frac{-4}{3}\right)\left(\frac{-12}{5}\right)}$$

$$= \frac{\frac{-20-36}{\cancel{15}}}{\frac{15-48}{\cancel{15}}}$$

$$= \frac{-56}{+33}$$

$$= \frac{56}{33}$$

~~Q.NO 14:~~

(iv) $\sin(\alpha-\beta)$

$$= \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

$$= \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-20}{65} + \frac{36}{65}$$

$= \dfrac{-20+36}{65}$

$= \dfrac{16}{65}$

ii) $\cos(\alpha-\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\dfrac{-3}{5}\right)\left(\dfrac{-5}{13}\right) + \left(\dfrac{4}{5}\right)\left(\dfrac{12}{13}\right)$

$= \dfrac{15}{65} + \dfrac{48}{65}$

$= \dfrac{15+48}{65}$

$= \dfrac{63}{65}$

iii) $\tan(\alpha-\beta)$

$= \dfrac{\alpha-\beta}{1+\alpha\beta}$

$= \dfrac{\left(\frac{-4}{3}\right) - \left(\frac{-12}{5}\right)}{1+\left(\frac{-4}{3}\right)\left(\frac{-12}{5}\right)}$

$= \dfrac{\frac{-20+36}{15}}{\frac{15+48}{15}}$

$= \dfrac{16}{63}$

Q.NO. 14B-

Express the following in the form $r\sin(\theta+\phi)$ or $r\sin(\theta-\phi)$, where terminal sides of the angles of measures θ and ϕ are in first quadrant.

i) $12\sin\theta + 5\cos\theta$

$$12 = r\cos\theta \longrightarrow (i)$$

$$5 = r\sin\theta \longrightarrow (ii)$$

$(i)^2 + (ii)^2$

$$(12)^2 + (5)^2 = r^2\cos^2\theta + r^2\sin^2\theta$$

$$144+25 = r^2(\cos^2\theta + \sin^2\theta)$$

$$169 = r^2(1)$$

$$\sqrt{169} = \sqrt{r^2}$$

$$13 = r$$

$(ii) \div (i)$

$$\frac{\sin\phi}{\cos\phi} = \frac{5}{12}$$

$$\tan\phi = \frac{5}{12} \Rightarrow \tan^{-1}\left(\frac{5}{12}\right) = \phi$$

ii) $3\sin\theta - 4\cos\theta$

$$3 = r\cos\theta \rightarrow (i)$$

$$4 = r\sin\theta \rightarrow (ii)$$

$(i)^2 + (ii)^2$

$$(3)^2 + (4)^2 = r^2\sin^2\theta + r^2\cos^2\theta$$

$$9+16 = r^2(\sin^2\theta + \cos^2\theta)$$

$$25 = r^2(1)$$

$$\sqrt{25} = \sqrt{r^2}$$

$$5 = r$$

$(ii) \div (i)$

$$\frac{4}{3} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}$$

$$\frac{4}{3} = \tan\theta$$

$$\theta = \tan^{-1}\frac{4}{3}$$

NOW;

$$3\sin\theta - 4\cos\theta = r\cos\phi\sin\theta - r\sin\phi\cos\theta$$

$$= r\sin(\theta - \phi)$$

ii) $\sin\theta - \cos\theta$

let $1 = r\cos\phi \rightarrow (i)$

$1 = r\sin\phi \rightarrow (ii)$

$(i)^2 + (ii)^2$

$$(1)^2 + (1)^2 = r^2\cos^2\phi + r^2\sin^2\phi$$

$1+1 = r^2\cos^2\phi + r^2\sin^2\phi$

$2 = r^2(\cos^2\phi + \sin^2\phi)$

$\sqrt{2} = \sqrt{r^{\cancel{2}}}$

$\sqrt{2} = r$

(ii) $\div$ (i)

$1 = \dfrac{r\sin\phi}{r\cos\phi}$

$1 = \tan\phi$

$\phi = \tan^{-1} 1$

$\phi = 45$

(iv) $5\sin\theta - 4\cos\theta$

$5 = r\cos\phi \rightarrow$ (i)

$4 = r\sin\phi \rightarrow$ (ii)

$(i)^2 + (ii)^2$

$(5)^2 + (4)^2 = r^2\cos^2\phi + r^2\sin^2\phi$

$25+16 = r^2(\cos^2\phi + \sin^2\phi)$

$41 = r^2(1)$

$\sqrt{41} = \sqrt{r^{\cancel{2}}}$

$\sqrt{41} = r$

(ii) $\div$ (i)

$\dfrac{4}{5} = \dfrac{r\sin\phi}{r\cos\phi}$

$\dfrac{4}{5} = \tan\phi$

$$\phi = \tan^{-1}\left(\frac{4}{5}\right)$$

Now;

$$5\sin\theta - 4\cos\theta = r\cos\phi\,\sin\theta - r\sin\phi\,\cos\theta$$
$$= r(\cos\phi\,\sin\theta - \sin\phi\,\cos\theta)$$
$$= r\sin(\theta - \phi)$$

$\sin\theta + \cos\theta$

$$1 = r\cos\phi \longrightarrow (i)$$
$$1 = r\sin\phi \longrightarrow (ii)$$

$(i)^2 + (ii)^2$

$$1 + 1 = r^2\cos^2\theta + r^2\sin^2\theta$$
$$2 = r^2(\cos^2\theta + \sin^2\theta)$$
$$2 = r^2(1)$$
$$\sqrt{2} = \sqrt{r^2}$$
$$\sqrt{2} = r$$

$(ii) \div (i)$

$$\frac{1}{1} = \frac{\cancel{r}\sin\phi}{\cancel{r}\cos\phi}$$

$$1 = \tan\phi$$
$$\phi = \tan^{-1} 1$$

$$\sin\theta + \cos\theta = r\cos\phi\,\sin\theta + r\cos\theta\,\sin\phi$$
$$= r\sin(\theta + \phi)$$

(vi) $3\sin\theta - 5\cos\theta$

$3 = r\cos\theta \longrightarrow$ (i)

$5 = r\sin\theta \longrightarrow$ (ii)

$(i)^2 + (ii)^2$

$(3)^2 + (5)^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$9 + 25 = r^2(\cos^2\theta + \sin^2\theta)$

$34 = r^2(1)$

$\sqrt{34} = \sqrt{r^2}$

$\sqrt{34} = r$

(ii) $\div$ (i)

$\frac{5}{3} = \frac{\cancel{r}\sin\phi}{\cancel{r}\cdot\cos\phi}$

$\frac{5}{3} = \tan\phi$

$\phi = \tan^{-1}\left(\frac{5}{3}\right)$

Q.NO.10:-

Find $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, given than.

(i) $\tan\alpha = \frac{3}{4}$, $\cos\beta = \frac{5}{13}$ and neither the terminal side of the angle of measure α nor that β is in the I quadrant.

$\tan \alpha = \frac{3}{4} = \frac{P}{B}$

By pythagoras theorem

$H^2 = B^2 + P^2$

$H^2 = (4)^2 + (3)^2$

$H^2 = 16 + 9$

$\sqrt{H^2} = \sqrt{25}$

$H = 5$

$\sin \alpha = \frac{P}{H} = -\frac{3}{5}$

$\cos \alpha = \frac{B}{H} = -\frac{4}{5}$

$\cos \beta = \frac{5}{13} = \frac{B}{H}$

By pythagoras theorem

$H^2 = B^2 + P^2$

$(13)^2 = (5)^2 + P^2$

$169 - 25 = P^2$

$\sqrt{144} = \sqrt{P^2}$

$12 = P$

$\sin \beta = \frac{P}{H} = -\frac{12}{13}$

$\tan \beta = \frac{P}{B} = -\frac{12}{5}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{-15}{65} + \frac{48}{65}$$

$$= \frac{-15+48}{65}$$

$$= \frac{33}{65}$$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{-20}{65} - \frac{36}{65}$$

$$= \frac{-20-36}{65}$$

$$= \frac{-56}{65}$$

(iii) $\tan\alpha = \frac{-15}{8}$ and $\sin\beta = \frac{-7}{25}$ and neither the terminal side of the angle of measure α nor that of β is in the IV quadrant.

$\tan\alpha = \frac{-15}{8} = \frac{P}{B}$

θ - II

By pythagoras theorem

$H^2 = B^2 + P^2$

$H^2 = (8)^2 + (15)^2$

$H^2 = 64 + 225$

$\sqrt{H^2} = \sqrt{289}$

$H = 17$

$\sin\alpha = \frac{P}{H} = \frac{15}{17}$

$\cos\alpha = \frac{B}{H} = \frac{-8}{17}$

$\sin\beta = \frac{-7}{25} = \frac{P}{H}$

θ - III

By pythagoras theorem

$H^2 = B^2 + P^2$

$(25)^2 = B^2 + (-7)^2$

$625 - 49 = B^2$

$\sqrt{576} = \sqrt{B^2}$

$24 = B$

$\cos\beta = \frac{B}{H} = \frac{-24}{25}$

$$\tan\beta = \frac{P}{B} = \frac{7}{24}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(\frac{15}{17}\right)\left(\frac{-24}{25}\right) + \left(\frac{-8}{17}\right)\left(\frac{-7}{25}\right)$$

$$= \frac{-360}{425} + \frac{56}{425}$$

$$= \frac{-360+56}{425}$$

$$= \frac{-304}{425}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \left(\frac{-8}{17}\right)\left(\frac{-24}{25}\right) - \left(\frac{15}{17}\right)\left(\frac{-7}{25}\right)$$

$$= \frac{192}{425} + \frac{105}{425}$$

$$= \frac{192+105}{425}$$

$$= \frac{297}{425}$$

*Exercise 10.3

Q.NO.2:-

$$\cot\alpha - \tan\alpha = 2\cot 2\alpha$$

L.H.S:-

$$= \frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\cos\alpha \sin\alpha}$$

$$\because \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \frac{\cos 2\alpha}{\cos\alpha \sin\alpha}$$

$$= \frac{2\cos 2\alpha}{2\cos\alpha \sin\alpha}$$

$$\therefore \sin 2\alpha = 2\cos\alpha \sin\alpha$$

$$= \frac{2\cos 2\alpha}{\sin 2\alpha}$$

$$= 2\cot 2\alpha$$

$$= R.H.S$$

Q.NO.3:-

$$\frac{\sin 2\alpha}{1+\cos 2\alpha} = \tan\alpha$$

L.H.S:-

$$= \frac{\sin 2\alpha}{1+\cos\alpha}$$

$$= \frac{\sin 2\alpha}{1+2\cos^2\alpha-1} \quad \because \cos 2\alpha = 2\cos^2\alpha - 1$$

$$= \frac{\cancel{2}\sin\alpha \cancel{\cos\alpha}}{\cancel{2}\cos^{\cancel{2}}\alpha} \quad \because \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$$= R.H.S$$

Q.NO.4:-

$$\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$$

L.H.S:-

$$\because \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\because \cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2}$$

$$2\sin^2\frac{\alpha}{2} = 1-\cos\alpha$$

$$= \frac{\cancel{2}\sin^{\cancel{2}}\frac{\alpha}{2}}{\cancel{2}\cancel{\sin\frac{\alpha}{2}}\cos\frac{\alpha}{2}}$$

$$= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$

$$= \frac{\tan \alpha}{2}$$

$$= R.H.S$$

Q.NO.5:-

$$\frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} = \sec 2\alpha - \tan 2\alpha$$

L.H.S:-

$$= \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha}$$

$$= \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} \times \frac{\cos\alpha - \sin\alpha}{\cos\alpha - \sin\alpha}$$

$$= \frac{(\cos\alpha - \sin\alpha)^2}{\cos^2\alpha - \sin^2\alpha}$$

$$= \frac{\cos^2\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha}{\cos 2\alpha} \quad \because \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \frac{1 - 2\sin\alpha\cos\alpha}{\cos 2\alpha}$$

$$= \frac{1 - \sin 2\alpha}{\cos 2\alpha} \quad \because \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= \frac{1}{\cos 2\alpha} - \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \sec 2\alpha - \tan 2\alpha$$

$$= R.H.S$$

Q.NO.8:-

$$1 + \tan\alpha \tan 2\alpha = \sec 2\alpha$$

L.H.S:-

$$1 + \left(\frac{\sin\alpha}{\cos\alpha}\right) \cdot \left(\frac{\sin 2\alpha}{\cos 2\alpha}\right) = \sec 2\alpha$$

$$= \frac{\cos 2\alpha \cos\alpha + \sin\alpha \sin 2\alpha}{\cos\alpha \cos 2\alpha}$$

$$= \frac{\cos(2\alpha - \alpha)}{\cos\alpha \cos 2\alpha}$$

$$= \frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha} \cos 2\alpha}$$

$$= \frac{1}{\cos 2\alpha}$$

$$= \sec 2\alpha$$

$= R.H.S$

Q.NO.13:-

$$\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$$

L.H.S:-

$$= \frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$$

$$= \frac{\sin 3\theta \sin\theta + \cos 3\theta \cos\theta}{\cos\theta \sin\theta}$$

$$= \frac{\cos(3\theta - \theta)}{\cos\theta \sin\theta}$$

$$= 2\left(\frac{\cos 2\theta}{2\cos\theta \sin\theta}\right)$$

$$= \frac{2 \cos 2\theta}{\sin 2\theta}$$

$$= 2 \cot 2\theta$$

$= R.H.S$

Q.NO.10:-

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$$

L.H.S:-

$$= \frac{\sin 3\theta \cos\theta - \cos 3\theta \sin\theta}{\sin\theta \cos\theta}$$

$$= \frac{\sin(3\theta - \theta)}{\sin\theta \cos\theta}$$

$$= \frac{\sin 2\theta}{\cos\theta \sin\theta}$$

$$= 2\left(\frac{\sin 2\theta}{2\cos\theta \sin\theta}\right)$$

$$= 2\left(\frac{\cancel{\sin 2\theta}}{\cancel{\sin 2\theta}}\right)$$

$= 2$

$= R.H.S$

Q.NO.11:-

$$\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$$

L.H.S:-

$$= \frac{\cos 3\theta \sin\theta + \sin 3\theta \cos\theta}{\cos\theta \sin\theta}$$

$$= \frac{\sin(\theta + 3\theta)}{\cos\theta \sin\theta}$$

$$= \frac{\sin 4\theta}{\cos\theta \sin\theta}$$

$$= \frac{2\sin 2\theta \cos 2\theta}{\cos\theta \sin\theta}$$

$$= \frac{2(2\cancel{\sin\theta \cos\theta})\cos 2\theta}{\cancel{\cos\theta \sin\theta}}$$

$$= 4\cos 2\theta$$

$$= R.H.S$$

Q.NO.12:-

$$\frac{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \sec\theta$$

L.H.S:-

$$= \frac{\dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}{\dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}$$

$$\frac{\dfrac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\cancel{\cos\frac{\theta}{2}\sin\frac{\theta}{2}}}}{\dfrac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\cancel{\cos\frac{\theta}{2}\sin\frac{\theta}{2}}}}$$

$$= \frac{1}{\cos\theta} \qquad \because \sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}=1 \qquad \because \cos 2\theta = \cos^2\theta-\sin^2\theta$$

$$= \sec\theta$$

Q.NO.7:-

$$\frac{\operatorname{cosec}\theta + 2\operatorname{cosec}2\theta}{\sec\theta} = \cot\frac{\theta}{2}$$

L.H.S:-

$$= \frac{\dfrac{1}{\sin\theta} + 2\left(\dfrac{1}{\sin 2\theta}\right)}{\sec\theta}$$

$$= \frac{\dfrac{1}{\sin\theta} + \cancel{2}\left(\dfrac{1}{\cancel{2}\sin\theta\cos\theta}\right)}{\sec\theta}$$

$$= \frac{\dfrac{1}{\sin\theta} + \dfrac{1}{\sin\theta\cos\theta}}{\sec\theta}$$

$$= \frac{\dfrac{\cos\theta+1}{\sin\theta\cos\theta}}{\sec\theta}$$

$$= \frac{\dfrac{\cos\theta+1}{\sin\theta\cancel{\cos\theta}}}{\dfrac{1}{\cancel{\cos\theta}}}$$

$$= \frac{\cos\theta+1}{\sin\theta}$$

$$\because \cos 2\theta = 2\cos^2\theta - 1$$

$$= \cos\theta = \frac{2\cos^2\frac{\theta}{2} - 1}{}$$

$$= \frac{\cancel{2}\cos^2\frac{\theta}{2} \cancel{-1} \cancel{+1}}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \qquad \therefore \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2}$$

$$= R.H.S$$

Q.NO.6:-

$$\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}$$

L.H.S:-

$$= \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$$

$$\therefore \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} = 1$$

$$\therefore \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$= \sqrt{\frac{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$$

$$= \sqrt{\frac{\left(\cos\frac{a}{2} + \sin\frac{a}{2}\right)^2}{\left(-\cos\frac{a}{2} + \sin\frac{a}{2}\right)^2}}$$

$$= \frac{\sin\frac{a}{2} + \cos\frac{a}{2}}{\sin\frac{a}{2} - \cos\frac{a}{2}}$$

$= R.H.S$

Q.NO.14:-

$$\sin^4\theta$$

$$= (\sin^2\theta)^2$$

$$\because \cos 2\theta = 1 - \sin^2\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$\sin\theta = \frac{1-\cos 2\theta}{2}$$

$$= \left(\frac{1-\cos 2\theta}{2}\right)^2$$

$$= \frac{1 - 2\cos 2\theta + \cos^2 2\theta}{4}$$

$$= \frac{1}{4}\left[1 - 2\cos 2\theta + \cos^2 2\theta\right]$$

$$\because \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 4\theta = 2\cos^2 2\theta - 1$$

$$\frac{1+\cos 4\theta}{2} = \cos^2 2\theta$$

$$= \frac{1}{4}\left[1 - 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right]$$

$$= \frac{1}{4}\left[\frac{2 - 4\cos 2\theta + 1 + \cos 4\theta}{2}\right]$$

$$= \frac{1}{8}\left[3 - 4\cos 2\theta + \cos 4\theta\right]$$

Q.NO.18-

Find the values of $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$, when:

(ii) $\cos\alpha = \frac{3}{5}$

$$\cos\alpha = \frac{B}{H}$$

By pythagoras theorem

$$H^2 = B^2 + P^2$$

$$(5)^2 = (3)^2 + P^2$$

$$25 - 9 = P^2$$

$$\sqrt{16} = \sqrt{P^2}$$

$$4 = P$$

$$\sin\alpha = \frac{4}{5}$$

$$\tan \alpha = \frac{3}{4}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$$

$$= \frac{9}{25} - \frac{16}{25}$$

$$= \frac{9 - 16}{25}$$

$$= \frac{-7}{25}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2\alpha}$$

$$= \frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{6}{4}}{\frac{16-9}{16}}$$

$$= \frac{6}{\cancel{4}_1} \times \frac{\cancel{16}^4}{7}$$

$$= \frac{24}{7}$$

(ii) $\sin \alpha = \frac{12}{13}$

$$\sin \alpha = \frac{P}{H}$$

By pythagoras theorem

$$H^2 = B^2 + P^2$$

$$(13)^2 = B^2 + (12)^2$$

$$169 - 144 = B^2$$

$$\sqrt{25} = \sqrt{B^2}$$

$$5 = B$$

$$\cos \alpha = \frac{B}{H} = \frac{5}{13}$$

$$\tan \alpha = \frac{P}{B} = \frac{12}{5}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)$$

$$= \frac{120}{169}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$= \frac{25}{169} - \frac{144}{169}$$

$$= \frac{25-144}{169}$$

$$= \frac{-119}{169}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2\left(\frac{12}{5}\right)}{1-\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{24}{5}}{\frac{25-144}{25}}$$

$$= \frac{24}{\cancel{5}_1} \times -\frac{\cancel{25}^{5}}{119}$$

$$= -\frac{120}{119}$$

Exercise 10.4

Q.NO.1:-

Express the following products as sums or difference:

(i) $2\sin 3\theta \cos\theta$

$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$= \sin(3\theta+\theta) + \sin(3\theta-\theta)$

$= \sin 4\theta + \sin 2\theta$

(ii) $2\cos 5\theta \sin 3\theta$

$2\cos\alpha \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

$= \sin(5\theta+3\theta) - \sin(5\theta-3\theta)$

$= \sin 8\theta - \sin 2\theta$

(iii) $\sin 5\theta \cos 2\theta$

$\frac{1}{2}\left[2\sin 5\theta \cos 2\theta\right]$

$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$= \frac{1}{2}\left[\sin(5\theta+2\theta) + \sin(5\theta-2\theta)\right]$

$= \frac{1}{2}\left[\sin 7\theta + \sin 3\theta\right]$

(iv) $2\sin 7\theta \sin 2\theta$

$= -\left[-2\sin 7\theta \sin 2\theta\right]$

$-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= -[\cos(7\theta+2\theta) - \cos(7\theta-2\theta)]$

$= -[\cos 9\theta - \cos 5\theta]$

$= -\cos 9\theta + \cos 5\theta$

(v) $\cos(x+y)\sin(x-y)$

$= \frac{1}{2}[2\cos(x+y)\sin(x-y)]$

$\therefore 2\cos\alpha \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

$= \frac{1}{2}[\sin(x+y+x-\cancel{y}) - \sin(\cancel{x}+y-\cancel{x}+y)]$

$= \frac{1}{2}(\sin 2x - \sin 2y)$

(vi) $\cos(2x+30^\circ)\cos(2x-30^\circ)$

$= \frac{1}{2}[2\cos(2x+30)\cos(2x-30)]$

$2\cos\alpha \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$= \frac{1}{2}[\cos(2x+30+2x-30) + \cos(2\cancel{x}+30-2\cancel{x}+30)]$

$= \frac{1}{2}[\cos 4x + \cos 60]$

(vii) $\sin 12^\circ \sin 46^\circ$

$= \frac{-1}{2}[-2\sin 12 \sin 46]$

$\therefore -2\sin\alpha \sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= \frac{-1}{2}[\cos(12+46) - \cos(12-46)]$

$= \frac{-1}{2}[\cos 58 - \cos(-34)]$

$= \frac{-1}{2}[\cos 58 + \cos 34]$

(viii) $\sin(x+45^\circ)\sin(x-45^\circ)$

$= \frac{-1}{2}[-2\sin(x+45)\sin(x-45)]$

$-2\sin\alpha\cos\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= \frac{-1}{2}[\cos(x+45+x-45) - \cos(x+45-x+45)]$

$= \frac{-1}{2}[\cos(2x) - \cos(90)]$

$= \frac{-1}{2}[\cos 2x - \cos 90]$

Prove that:

(i) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

L.H.S:-

$= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$= \cos 20^\circ \cos 40^\circ \left(\frac{1}{2}\right) \cos 80^\circ$

$= \frac{1}{2}(\cos 20^\circ \cos 40^\circ) \cos 80^\circ$

$$= \frac{1}{2\cdot 2}\,(2\cos 40^\circ \cos 20^\circ)\cos 80^\circ$$

$\therefore\ 2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$$= \frac{1}{4}\left[\cos(40+20) + \cos(40-20)\right]\cos 80^\circ$$

$$= \frac{1}{4}\left[\cos 60 + \cos 20\right]\cos 80$$

$$= \frac{1}{4}\left[\frac{1}{2} + \cos 20\right]\cos 80$$

$$= \frac{1}{4}\left[\frac{1 + 2\cos 20}{2}\right]\cos 80$$

$$= \frac{1}{8}\left[\cos 80 + 2\cos 80\cos 20\right]$$

$\therefore\ 2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$$= \frac{1}{8}\left[\cos 80 + (\cos(80+20) + \cos(80-20))\right]$$

$$= \frac{1}{8}\left[\cos 80 + \cos 100 + \cos 60\right]$$

$$= \frac{1}{8}\left[\cos 80 + \cos(180-80) + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[\cos 80 - \cos 80 + \frac{1}{2}\right]$$

$$= \frac{1}{16} = R.H.S$$

ii) $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$

L.H.S:-

$= \sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9}$

$= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \sin 20^\circ \sin 40^\circ \left(\frac{\sqrt{3}}{2}\right) \sin 80^\circ$

$= \frac{\sqrt{3}}{2}(\sin 40 \sin 20) \sin 80$

$= -\frac{\sqrt{3}}{2\cdot 2}(-2\sin 40 \sin 20) \sin 80$

$= \frac{-\sqrt{3}}{4}[\cos(40+20) - \cos(40-20)] \sin 80$

$= \frac{-\sqrt{3}}{4}[\cos 60^\circ - \cos 20^\circ] \sin 80$

$= \frac{-\sqrt{3}}{4}\left(\frac{1}{2} - \cos 20^\circ\right) \sin 80$

$= \frac{-\sqrt{3}}{4}\left[\frac{1 - 2\cos 20}{2}\right] \sin 80$

$= \frac{-\sqrt{3}}{8}(1 - 2\cos 20) \sin 80$

$= \frac{-\sqrt{3}}{8}(\sin 80 - 2\sin 80 \cos 20)$

$\therefore 2\sin\alpha\cos\beta = \sin(\alpha+\beta)+\sin(\alpha-\beta)$

$$= \frac{-\sqrt{3}}{8}\left[\sin 80^\circ - (\sin(80+20)+\sin(80-20))\right]$$

$$= \frac{-\sqrt{3}}{8}\left[\sin 80 - [\sin 100 + \sin 60]\right]$$

$$= \frac{-\sqrt{3}}{8}\left[\sin 80 - \sin(180-80) - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{-\sqrt{3}}{8}\left[\cancel{\sin 80} - \cancel{\sin 80} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{(\sqrt{3})^2}{16}$$

$$= \frac{3}{16}$$

$= R.H.S$

$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

L.H.S:-

$= \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

$$= \sin 10\left(\frac{1}{2}\right)\sin 50 \sin 70$$

$$= \frac{1}{2}(\sin 10 \sin 50)\sin 70$$

$$= \frac{1}{2}(\sin 50 \sin 10)\sin 70$$

$$= \frac{-1}{2\cdot 2}(-2\sin 50\ \sin 10)\sin 70$$

$$\therefore -2\sin\alpha\sin\beta = \cos(\alpha+\beta)-\cos(\alpha-\beta)$$

$$= \frac{-1}{4}\left[\cos(50+10)-\cos(50-10)\right]\sin 70$$

$$= \frac{-1}{4}\left[\cos 60-\cos 40\right]\sin 70$$

$$= \frac{-1}{4}\left[\frac{1}{2}-\cos 40\right]\sin 70$$

$$= \frac{-1}{4}\left[\frac{1-2\cos 40}{2}\right]\sin 70$$

$$= \frac{-1}{8}\left[1-2\cos 40\right]\sin 70$$

$$= \frac{-1}{8}\left[\sin 70-2\sin 70\cos 40\right]$$

$$\therefore 2\sin\alpha\cos\beta = \sin(\alpha+\beta)-\sin(\alpha-\beta)$$

$$= \frac{-1}{8}\left[\sin 70-\left[\sin(40+70)-\sin(70-40)\right]\right]$$

$$= \frac{-1}{8}\left[\sin 70^\circ-\sin 110^\circ+\sin 30^\circ\right]$$

$$= \frac{-1}{8}\left[\sin 70-\sin(70+40)-\sin 30\right]$$

$$= \frac{-1}{8}\left[\cancel{\sin 70}-\cancel{\sin 70}-\sin 30\right]$$

$= \frac{+1}{8}\left(\frac{+1}{2}\right)$

$= \frac{1}{16}$

$= R.H.S$

Q.NO.4:-

Prove that:

i) $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$

L.H.S:-

$= 2\cos\left(\frac{20+100}{2}\right)\cos\left(\frac{20-100}{2}\right) + \cos 140$

$= 2\cos\left(\frac{\cancel{120}^{60}}{\cancel{2}}\right)\cos\left(\frac{-\cancel{80}^{40}}{\cancel{2}}\right) + \cos 140$

$= 2\cos 60 \cos 40 + \cos 140$

$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)\cos 40 + \cos 140$

$= \cos 40 + \cos 140$

$= 2\cos\left(\frac{40+140}{2}\right)\cos\left(\frac{40-140}{2}\right)$

$= 2\cos\left(\frac{\cancel{180}^{90}}{\cancel{2}}\right)\cos\left(\frac{-\cancel{100}^{50}}{\cancel{2}}\right)$

$= 2\cos 90 \cos 50$

$= 2(0)\cos 50$

$= 0 = R.H.S$

(ii) $\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{2}\cos 2\theta$

L.H.S:-

$$=\frac{-1}{2}\left[-2\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)\right]$$

$\because 2\sin\alpha\sin\beta=\cos(\alpha+\beta)-\cos(\alpha-\beta)$

$$=\frac{-1}{2}\left[\cos\left(\frac{\pi}{4}-\cancel{\theta}+\frac{\pi}{4}+\cancel{\theta}\right)-\cos\left(\frac{\cancel{\pi}}{\cancel{4}}-\theta-\frac{\cancel{\pi}}{\cancel{4}}+\theta\right)\right]$$

$$=\frac{-1}{2}\left[\cos\left(\frac{\pi+\pi}{4}\right)-\cos(-2\theta)\right]$$

$$=\frac{-1}{2}\left[\cos\frac{\cancel{2}\pi}{\cancel{4}2}-\cos 2\theta\right]$$

$$=+\frac{1}{2}\left[0+\cos 2\theta\right]$$

$$=\frac{1}{2}\cos 2\theta$$

$=$ R.H.S

(iii) $\frac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}=\tan 4\theta$

L.H.S:-

$$=\frac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}$$

$$=\frac{(\sin 7\theta+\theta)+(\sin 5\theta+\sin 3\theta)}{(\cos 7\theta+\theta)+(\cos 5\theta+\cos 3\theta)}$$

$$= \frac{\left[2\sin\left(\frac{\overset{4}{\cancel{8\theta}}}{\cancel{2}}\right)\cos\left(\frac{\overset{3}{\cancel{6\theta}}}{\cancel{2}}\right)\right]+\left[2\sin\left(\frac{\overset{4}{\cancel{8\theta}}}{\cancel{2}}\right)\cos\left(\frac{\cancel{2\theta}}{\cancel{2}}\right)\right]}{\left[2\cos\left(\frac{\overset{4}{\cancel{8\theta}}}{\cancel{2}}\right)\cos\left(\frac{\overset{3}{\cancel{6\theta}}}{\cancel{2}}\right)\right]+\left[2\cos\left(\frac{\overset{4}{\cancel{8\theta}}}{\cancel{2}}\right)\cos\left(\frac{\cancel{2\theta}}{\cancel{2}}\right)\right]}$$

$$= \frac{\cancel{2}\sin 4\theta\,\cancel{(\cos 3\theta + \cos\theta)}}{\cancel{2}\cos 4\theta\,\cancel{(\cos 3\theta + \cos\theta)}}$$

$$= \tan 4\theta$$

$$= R.H.S$$

Q.NO.26-

Express the following sums or differences as products

i) $\sin 5\theta + \sin 3\theta$

$$\therefore 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$= 2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)$$

$$= 2\sin\left(\frac{\overset{4}{\cancel{8\theta}}}{\cancel{2}}\right)\cos\left(\frac{\cancel{2\theta}}{\cancel{2}}\right)$$

$$= 2\sin 4\theta\cos\theta$$

$\sin 8\theta - \sin 4\theta$

$$\therefore 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$= 2\cos\left(\frac{8\theta+4\theta}{2}\right)\sin\left(\frac{8\theta-4\theta}{2}\right)$$

$$= 2\cos\left(\frac{\overset{6}{\cancel{12}}\theta}{\cancel{2}}\right)\sin\left(\frac{\overset{2}{\cancel{4}}\theta}{\cancel{2}}\right)$$

$$= 2\cos 6\theta \sin 2\theta$$

(iii) $\cos 6\theta + \cos 3\theta$

$$\therefore 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$= 2\cos\left(\frac{6\theta+3\theta}{2}\right)\cos\left(\frac{6\theta-3\theta}{2}\right)$$

$$= 2\cos\left(\frac{9\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

$$= 2\cos\frac{9\theta}{2}\cos\frac{3\theta}{2}$$

(iv) $\cos 7\theta - \cos\theta$

$$= -2\sin\left(\frac{\overset{4}{\cancel{8}}\theta}{\cancel{2}}\right)\sin\left(\frac{\overset{3}{\cancel{6}}\theta}{\cancel{2}}\right)$$

$$= -2\sin 4\theta \sin 3\theta$$

v) $\cos 12^\circ + \cos 48^\circ$

$$= 2\cos\left(\frac{12+48}{2}\right)\cos\left(\frac{12-48}{2}\right)$$

$$= 2\cos\left(\frac{\overset{30}{\cancel{60}}}{\cancel{2}}\right)\cdot\cos\left(\frac{-\overset{18}{\cancel{36}}}{\cancel{2}}\right)$$

$$= 2\cos 30 \cos 18$$

(vi) $\sin(x+30^\circ)+\sin(x-30^\circ)$

$= 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$

$= 2\sin\left(\frac{x+\cancel{30}+x-\cancel{30}}{2}\right)\cos\left(\frac{\cancel{x}+30-\cancel{x}+30}{2}\right)$

$= 2\sin\left(\frac{\cancel{2}x}{\cancel{2}}\right)\cos\left(\frac{\overset{30}{\cancel{60}}}{\cancel{2}}\right)$

$= 2\sin x\cos 30$

Q.NO. 3:-

Prove fallowing identities.

(i) $\frac{\sin 3x-\sin x}{\cos x-\cos 3x} = \cot 2x$

L.H.S:

$= \dfrac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}$

$= \dfrac{\cancel{2}\cos\left(\frac{\overset{2}{\cancel{4}}x}{\cancel{2}}\right)\sin\left(\frac{\cancel{2}x}{\cancel{2}}\right)}{+\cancel{2}\sin\left(\frac{\overset{2}{\cancel{4}}x}{\cancel{2}}\right)\sin\left(\frac{+\cancel{2}x}{\cancel{2}}\right)}$

$= \dfrac{\cos 2x\ \cancel{\sin x}}{\sin 2x\ \cancel{\sin x}}$

$$= \frac{\cos 2x}{\sin 2x}$$

$$= \cancel{\tan}\cot 2x = R.H.S$$

(ii) $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

$$= \frac{2\sin\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}{2\cos\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}$$

$$= \frac{\cancel{2}\sin\left(\frac{\cancel{10x}^{5}}{\cancel{2}}\right)\cos\left(\frac{\cancel{6x}^{3}}{\cancel{2}}\right)}{\cancel{2}\cos\left(\frac{\cancel{10x}^{5}}{\cancel{2}}\right)\cos\left(\frac{\cancel{6x}^{3}}{\cancel{2}}\right)}$$

$$= \frac{\sin 5x \,\cancel{\cos 3x}}{\cos 5x \,\cancel{\cos 3x}}$$

$$= \tan 5x$$

$$= R.H.S$$

(iii) $\frac{\sin\alpha - \sin\beta}{\sin\alpha + \sin\beta} = \tan\frac{\alpha-\beta}{2}\cot\frac{\alpha+\beta}{2}$

L.H.S:-

$$= \frac{\cancel{2}\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{\cancel{2}\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$= \cot\left(\frac{\alpha+\beta}{2}\right) \tan\left(\frac{\alpha-\beta}{2}\right)$

$= \tan\left(\frac{\alpha-\beta}{2}\right) \cot\left(\frac{\alpha-\beta}{2}\right)$

$= R.H.S$

Exercise 10.1

Q:NO.1

Without using tables, find the values of:

(i) $\sin(-780^\circ)$

$= -\cancel{780}\ -\sin 780^\circ$

$$90\,\overline{)\,780}\ = 8,\quad -720,\quad 60 \qquad\qquad 2\,\overline{)\,8}\ = 4,\quad 8,\quad 0+1=1$$

$= -\sin[90 \times 8 + 60]$

$= -\sin 60$

(ii) $\cot(-855)$

$= -\cot 855$

$$90\,\overline{)\,855}\ = 9,\quad -810,\quad 45 \qquad\qquad 2\,\overline{)\,9}\ = 4,\quad 8,\quad 1+1=2$$

$= -\cot[90 \times 9 + 45]$

$= -(-\tan 45)$

$= \tan 45^\circ$

(iii) cosec 2040

$$\begin{array}{r|l} & 22 \\ \hline 90 & 2040 \\ & -1980 \\ \hline & 60 \end{array} \qquad \begin{array}{r|l} & 5 \\ \hline 4 & 22 \\ & 20 \\ \hline & 2+1=3 \end{array}$$

$= \operatorname{cosec}[22 \times 90 + 60]$

$= -\operatorname{cosec} 60$

(iv) $\sec(-960^\circ)$

$= -\sec 960^\circ$

$$\begin{array}{r|l} & 10 \\ \hline 90 & 960 \\ & 900 \\ \hline & 60 \end{array} \qquad \begin{array}{r|l} & 2 \\ \hline 4 & 10 \\ & -8 \\ \hline & 2+1=3 \end{array}$$

$= -\sec[90 \times 10 + 60]$

$= -\sec 60$

(v) $\tan(1110^\circ)$

$= \tan 1110^\circ$

$$\begin{array}{r|l} & 12 \\ \hline 90 & 1110 \\ & -1080 \\ \hline & 30 \end{array} \qquad \begin{array}{r|l} & 3 \\ \hline 4 & 12 \\ & 12 \\ \hline & 0+1=1 \end{array}$$

$= \tan[90 \times 12 + 30]$

$= \tan 30$

(vi) $\sin(-300°)$

$= -\sin 300°$

$$\begin{array}{r|l} & 3 \\ \hline 90 & 300 \\ & -270 \\ \hline & 30 \end{array}$$

$= -\sin(90 \times 3 + 30)$

$= \cos 30$

Q.NO. 2:-

Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.

(i) $\sin 196°$

$$\begin{array}{r|l} & 2 \\ \hline 90 & 196 \\ & -180 \\ \hline & 16 \end{array}$$

$= \sin[90 \times 2 + 16]$

$= -\sin 16$

(viii) $\cos(-435)$

$= \cos 435$

$$\begin{array}{r|l} & 5 \\ 90 & 435 \\ & -450 \\ & -15 \end{array} \qquad \begin{array}{r|l} & 1 \\ 4 & 5 \\ & 4 \\ & 1+1 = 2 \end{array}$$

$= \cos[90 \times 5 - 15]$

$= \sin 15$

(ii) $\cos 147^\circ$

$$\begin{array}{r|l} & 2 \\ 90 & 147 \\ & -180 \\ & -33 \end{array}$$

$= \cos[90 \times 2 - 33]$

$= -\cos 33^\circ$

(iii) ~~319~~ $\sin 319$

$= \sin 319^\circ$

$$\begin{array}{r|l} & 4 \\ 90 & 319 \\ & -360 \\ & -41 \end{array} \qquad \begin{array}{r|l} & 1 \\ 4 & 4 \\ & 4 \\ & 0+1=1 \end{array}$$

$= \sin[90 \times 4 - 41]$

$= -\sin 41^\circ$

(iv) $\cos 254^\circ$

$= \cos 254$

$$\begin{array}{r|l} & 3 \\ 90 & 254 \\ & -270 \\ \hline & -16 \end{array}$$

$= \cos[90 \times 3 + (-16)]$

$= -\sin 60^\circ$

(v) $\tan 294^\circ$

$= \tan 294$

$$\begin{array}{r|l} & 3 \\ 90 & 294 \\ & -270 \\ \hline & 24 \end{array}$$

$= \tan[90 \times 3 + 24]$

$= -\cot 24$

ii) $\cos 728^\circ$

$= \cos 728^\circ$

$$\begin{array}{r|l} & 8 \\ 90 & 728 \\ & 720 \\ \hline & 8 \end{array} \qquad \begin{array}{r|l} & 2 \\ 4 & 8 \\ & 8 \\ \hline & 0+1=1 \end{array}$$

$= \cos[90 \times 8 + 8]$

$= \cos 8^\circ$

(viii) $\sin(-625^\circ)$

$= -\sin 625$

$$\begin{array}{r} 7 \\ 90\overline{)625} \\ -630 \\ \hline -5 \end{array} \qquad \begin{array}{r} 1 \\ 4\overline{)7} \\ 4 \\ \hline 3 \end{array}$$

$= -\sin[90 \times 7 - 5]$

$= \cos 5^\circ$

(ix) $\sin 150$

$= \sin 150$

$$\begin{array}{r} 2 \\ 90\overline{)150} \\ -180 \\ \hline -30 \end{array}$$

$= \sin[90 \times 2 - 30]$

$= \sin 30^\circ$

Q.NO.3:-

Prove the following:

(i) $\sin(180 + \alpha)\ \sin(90 - \alpha) = -\sin\alpha \cos\alpha$

L.H.S:-

$= \sin[90 \times 2 + \alpha]\ \sin[90 \times 1 - \alpha]$

$= (-\sin\alpha)(\cos\alpha)$

$= -\sin\alpha \cos\alpha$

$= R.H.S$

(iii) $\sin 780 \sin 480 + \cos 120 + \cos 30 = \frac{1}{2}$

L.H.S:-

$$90\overline{)780} = 8, \quad -720, \quad 60$$
$$90\overline{)480} = 5, \quad -450, \quad 30$$
$$90\overline{)120} = 1, \quad -90, \quad 30$$

$$4\overline{)8} = 2, \quad 8, \quad 0+1=1$$
$$4\overline{)5} = 1, \quad -4, \quad 1+1=2$$

$= \sin[90\times8+60]\sin[90\times5+30]+\cos[90\times1+30]\sin(30)$

$=(\sin 60)(\cos 30)+(-\sin 30)(\sin 30)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{(\sqrt{3})^2}{4} - \frac{1}{4}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{\cancel{2}^{1}}{\cancel{4}_{2}}$

$= \frac{1}{2} = R.H.S$

(iii) $\cos 306 + \cos 234 + \cos 162 + \cos 18 = 0$

L.H.S:-

$$90\overline{)306}^{\,3} \qquad 90\overline{)234}^{\,3} \qquad 90\overline{)162}^{\,2}$$

270	-270	-180
36	-36	-18

$= \cos[90\times 3 + 36] + \cos[90\times 3 - 36] + \cos[90\times 2 - 18]$
$\quad + \cos 18$

$= \sin \cancel{36} - \sin 36 - \cos \cancel{18} + \cos \cancel{18}$

$= 0$

$=$ R.H.S

(iv) $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$

L.H.S:-

$$90\overline{)330}^{\,4} \qquad 90\overline{)600}^{\,7} \qquad 90\overline{)120}^{\,1} \qquad 90\overline{)150}^{\,2}$$

-270	-540	-90	-180
-30	-30	130	-30

$$4\overline{)6}^{\,1}$$
4

$2+1=3$

$= \cos[90\times 4 - 36]\sin[90\times 7 - 30] + \cos[90\times 1 + 30]$
$\sin[90\times 2 - 30]$

$= (\cos 30)(-\cos 30) + (-\sin 30)(\sin 30)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{(-\sqrt{3})^2}{4} - \frac{1}{4}$

$= \frac{-3-1}{4}$

$= \frac{-\cancel{4}}{\cancel{4}}$

$= -1$

$= R.H.S$

Q.NO. 4:-

Prove that.

(i) $\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\operatorname{cosec}(2\pi-\theta)}$

L.H.S:-

$= \frac{[\sin(180+\theta)]^2 \tan(270+\theta)}{[\cot(270+\theta)^2][\cos(180+\theta)]^2 \operatorname{cosec}(360-\theta)}$

$= \frac{(-\sin\theta)^2(\cot\theta)}{(\tan\theta)^2(-\cos\theta)^2(\operatorname{cosec}\theta)}$

$$= \frac{\sin^2\theta\left(\frac{\cos\theta}{\sin\theta}\right)}{\frac{\sin^2\theta}{\cos^2\theta}(\cos^2\theta)\left(\frac{1}{\sin\theta}\right)}$$

$$= \cos\theta$$

$$= R.H.S$$

(ii) $$\frac{\cos(90+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90-\theta)} = -1$$

L.H.S:-

$$= \frac{\cos[90\times1+\theta]\sec(-\theta)\tan[90\times2-\theta]}{\sec[90\times4+\theta]\sin[90\times2+\theta]\cot[90\times1-\theta]}$$

$$= \frac{\sin\theta\sec(-\theta)\tan\theta}{-\sec\theta\sin(-\theta)\tan\theta}$$

$$= \frac{\cancel{-\sin\theta\sec\theta\tan\theta}}{\cancel{\sin\theta\sec\theta\tan\theta}}$$

$$= -1$$

Q.NO.5:-

If α, β, γ are the angles of a triangle ABC, then prove that:

i) $\sin(\alpha+\beta) = \sin\gamma$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\alpha+\beta = 180-\gamma$$

Taking sin on both sides

$$\sin(\alpha+\beta) = \sin(180^\circ-\gamma)$$

$= \sin 180 + \sin \gamma$

$\therefore \sin(\alpha+\beta) = \sin \gamma$

(ii) $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin \frac{\gamma}{2}$

$\frac{\alpha}{2} + \frac{\beta}{2} = 90^\circ - \frac{\gamma}{2}$

Taking cos on both sides

$\cos\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \cos\left(90^\circ - \frac{\gamma}{2}\right)$

$\cos\left(\frac{\alpha+\beta}{2}\right) = \sin \frac{\gamma}{2}$

(iii) $\cos(\alpha+\beta) = -\cos \gamma$

$\alpha+\beta = 180 - \gamma$

Taking cos on both sides

$\cos(\alpha+\beta) = \cos(180-\gamma)$

$= 0 - \gamma\cos\gamma$

$\cos(\alpha+\beta) = -\cos\gamma$

v) $\tan(\alpha+\beta) + \tan\gamma = 0$

$\alpha+\beta = 180-\gamma$

$\tan(\alpha+\beta) = \tan(180-\gamma)$

$\tan(\alpha+\beta) = 0 - \tan\gamma$

$\tan(\alpha+\beta) + \tan\gamma = 0$