Unit 13 Practical Geometry - Circles

Introduction:

The word geometry is derived from two Greek words namely Geo (earth) and Metron (measurement). In fact, geometry means measurement of earth.

Definition of Geometry

Geometry is the branch of mathematics, which deals with the shape, size and position of geometric figures.

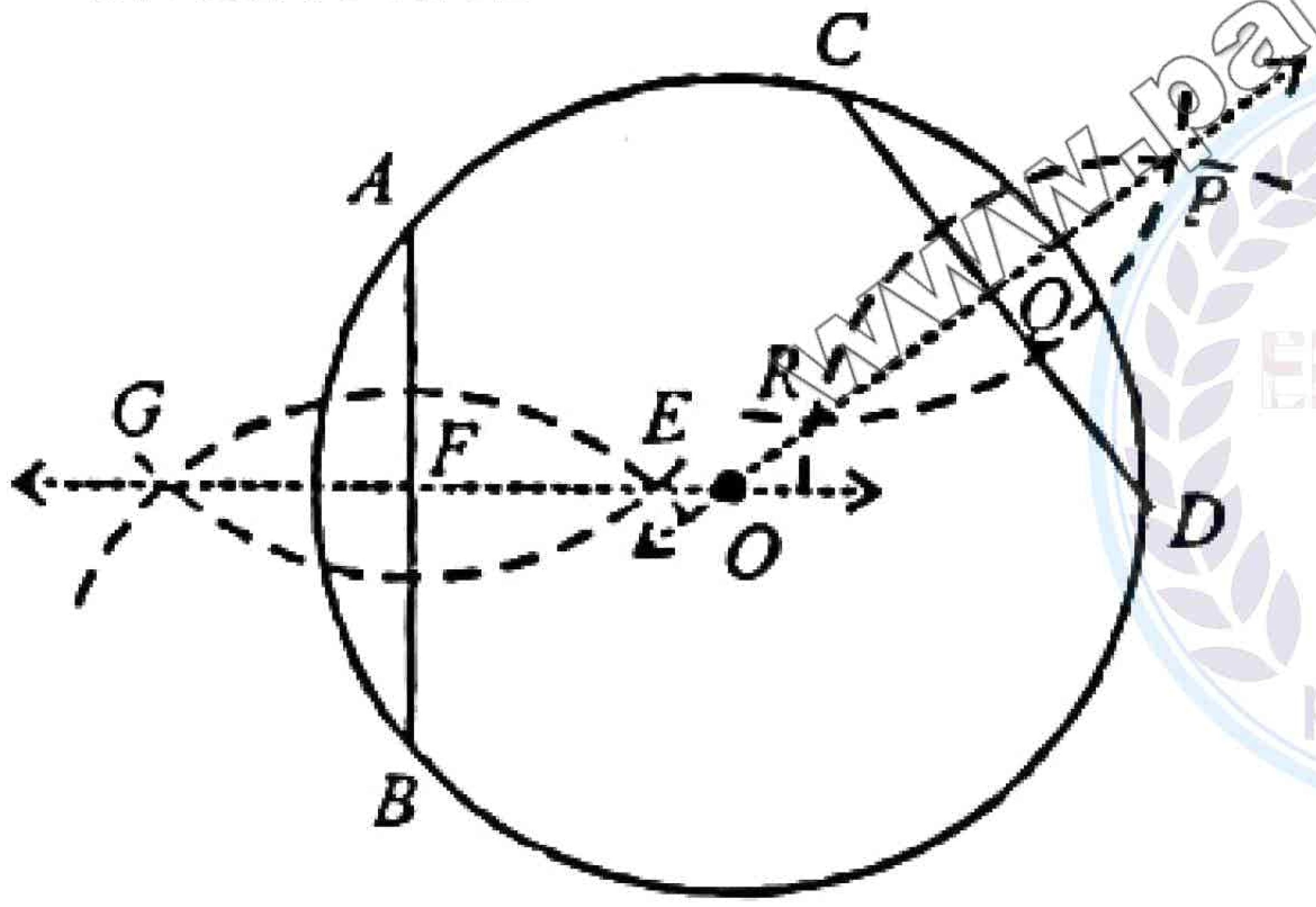
The Greek mathematicians (600 -300 BC) contributed a lot. In particular "Euclid's Elements" have been taught all over the world.

CONSTRUCTION OF A CIRCLE

A circle of any radius can be constructed by rotating a compass about a fixed point.

1. To locate the centre of a given circle

Given: A circle



Steps of Construction:

i. Draw two chords $\overline{AB}$ and $\overline{CD}$

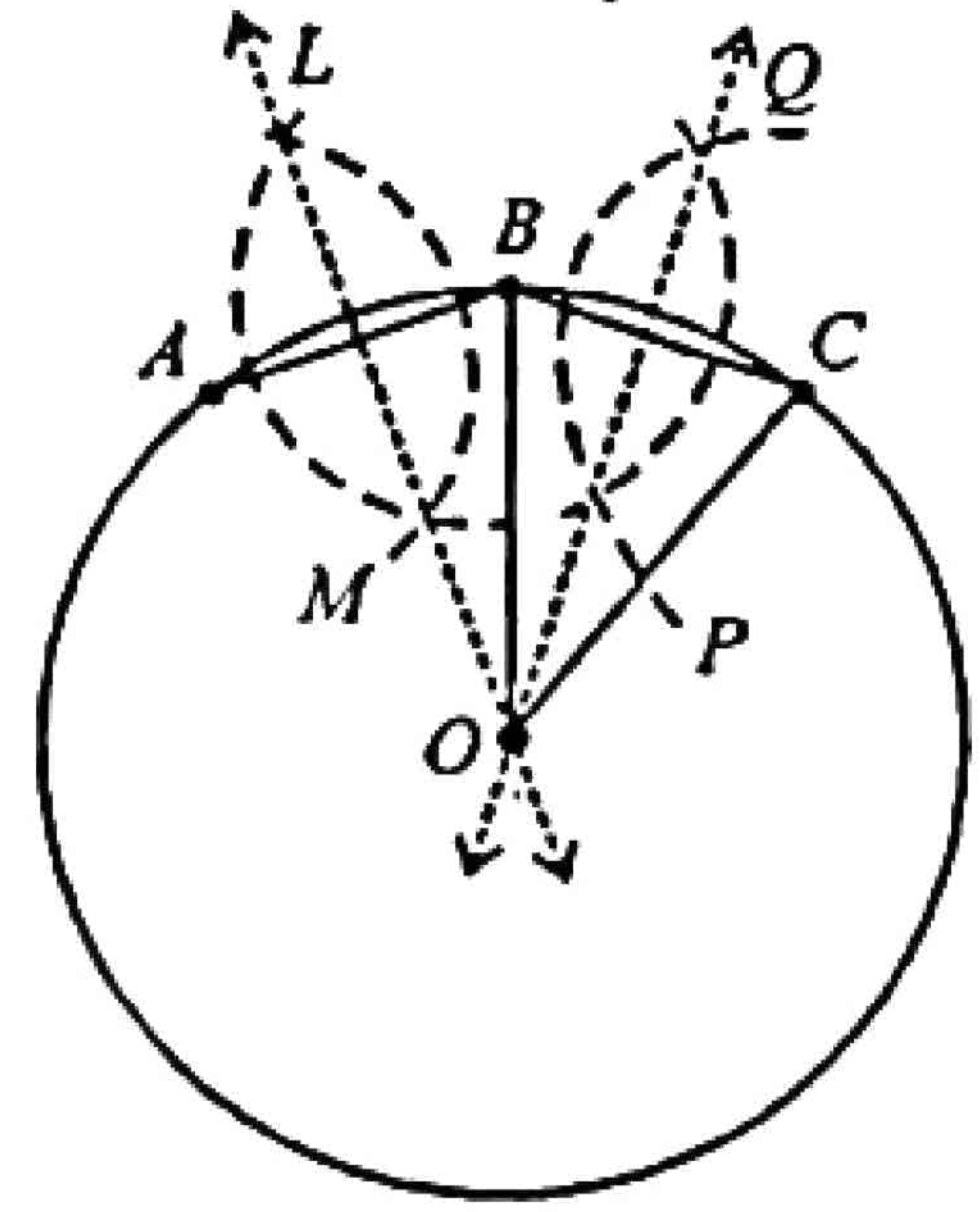
ii. Draw $\overleftrightarrow{EFG}$ as perpendicular bisector of chord $\overline{AB}$.

iii. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of chord $\overline{CD}$.

iv. Perpendicular bisectors $\overleftrightarrow{EFG}$ and $\overleftrightarrow{PQR}$ intersect each other at O. Here O is the centre of circle.

2. To draw a circle passing through three given non-collinear points:

Given: Three non-collinear points A, B and C.



Steps of Construction:

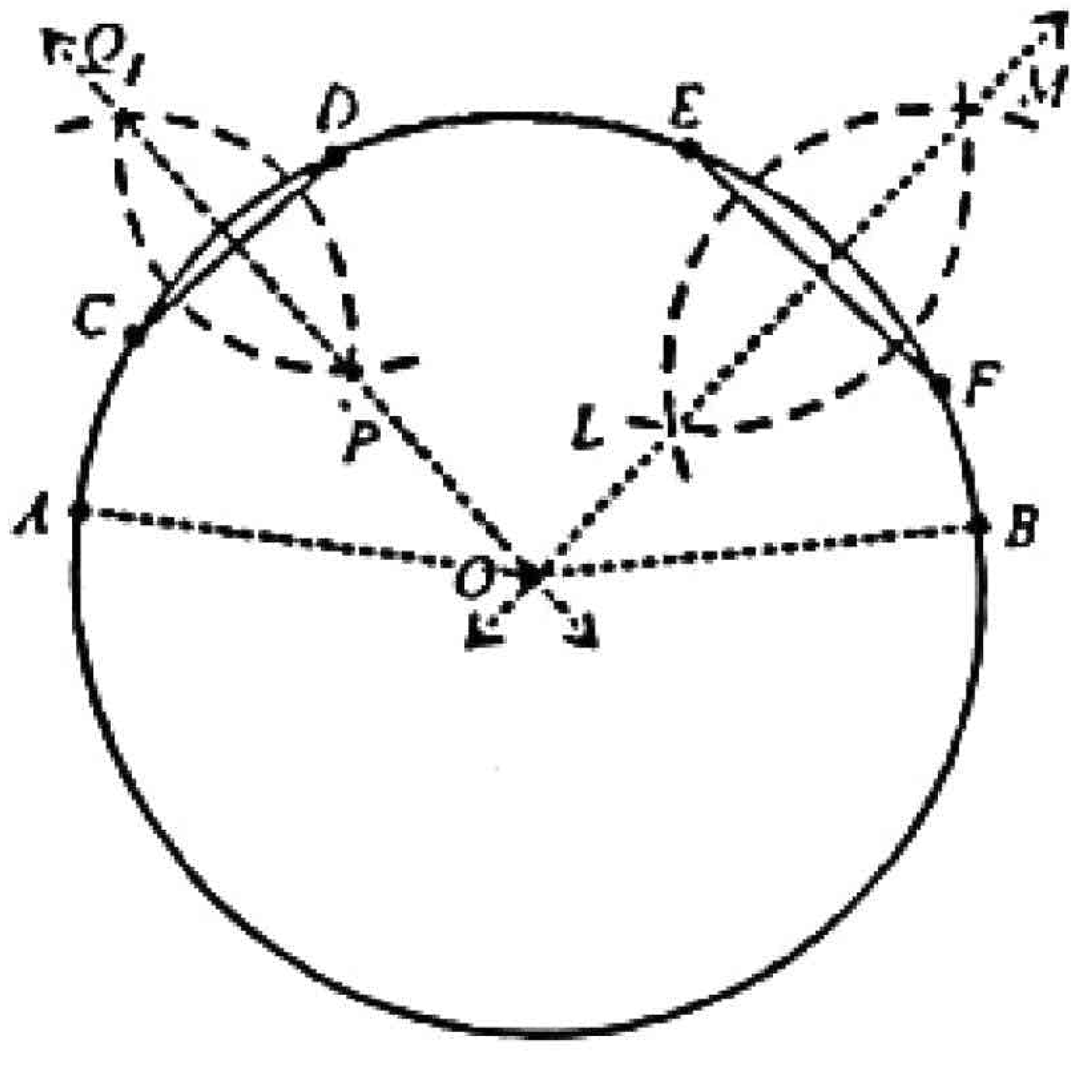
i. Join A with B and B with C.

ii. Draw $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at point O.

iii. Draw a circle with radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$ having centre at O, which is the required circle.

3. To complete the circle by finding the centre when a part of a circumference is given.

Given: $\overparen{AB}$ is Part of circumference of a circle.



Steps of Construction:

i. Let C, D, E and F be the four points on the given arc AB.

ii. Draw chord $\overline{CD}$ and $\overline{EF}$.

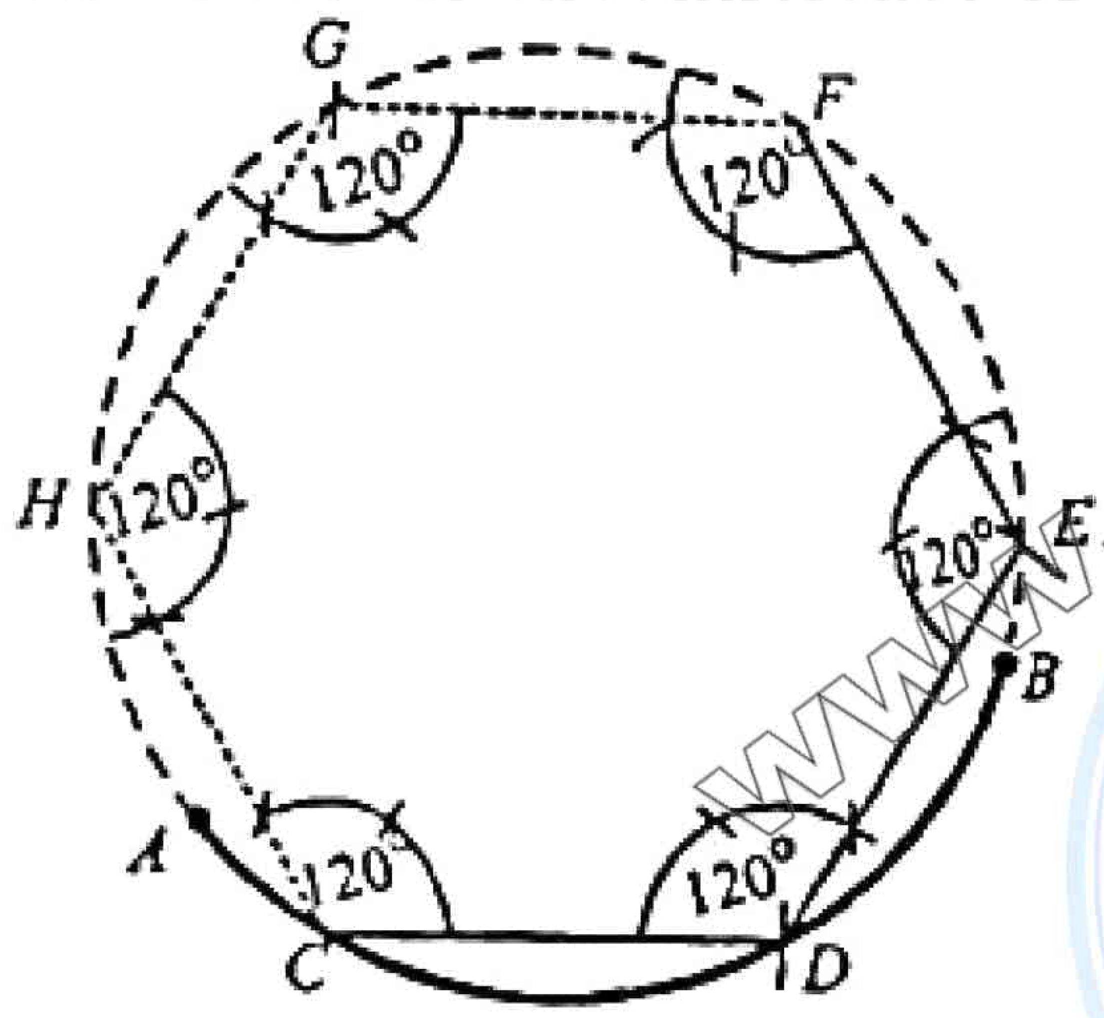
iii. Draw $\overleftrightarrow{PQ}$ as perpendicular bisector of $\overline{CD}$ and $\overleftrightarrow{LM}$ as perpendicular bisector of $\overline{EF}$

iv. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at O. Therefore, O is equidistant from points A, B, C, D, E and F.

v. Complete the circle with centre O and radius ($m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$). This will pass through all the points A, B, C, D, E and F on the given part of the circumference.

4. To complete the circle without finding the center when a part of its circumference is given.

Given: $\widehat{AB}$ is Part of circumference of a circle.



Steps of Construction:

i. Take a chord $\overline{CD}$ of reasonable length on the arc AB.

ii. Construct an internal angle of 120° at point D and draw a line segment $\overline{DE}$ equal to the length of $\overline{CD}$.

iii. At point E again construct an internal angle of 120° and from point E draw line segment $\overline{EF}$ of length equal to $\overline{CD}$ etc.

iv. Continue this practice until we reach at the starting point.

v. Now join the points D, E, F, G, H and C by arcs DE, EF and FG, GH and HC all having length equal to the length of arc CD.

As a result we get a circle including the given part of circumference.

EXERCISE 13.1

Q.1 Divide an arc of any length

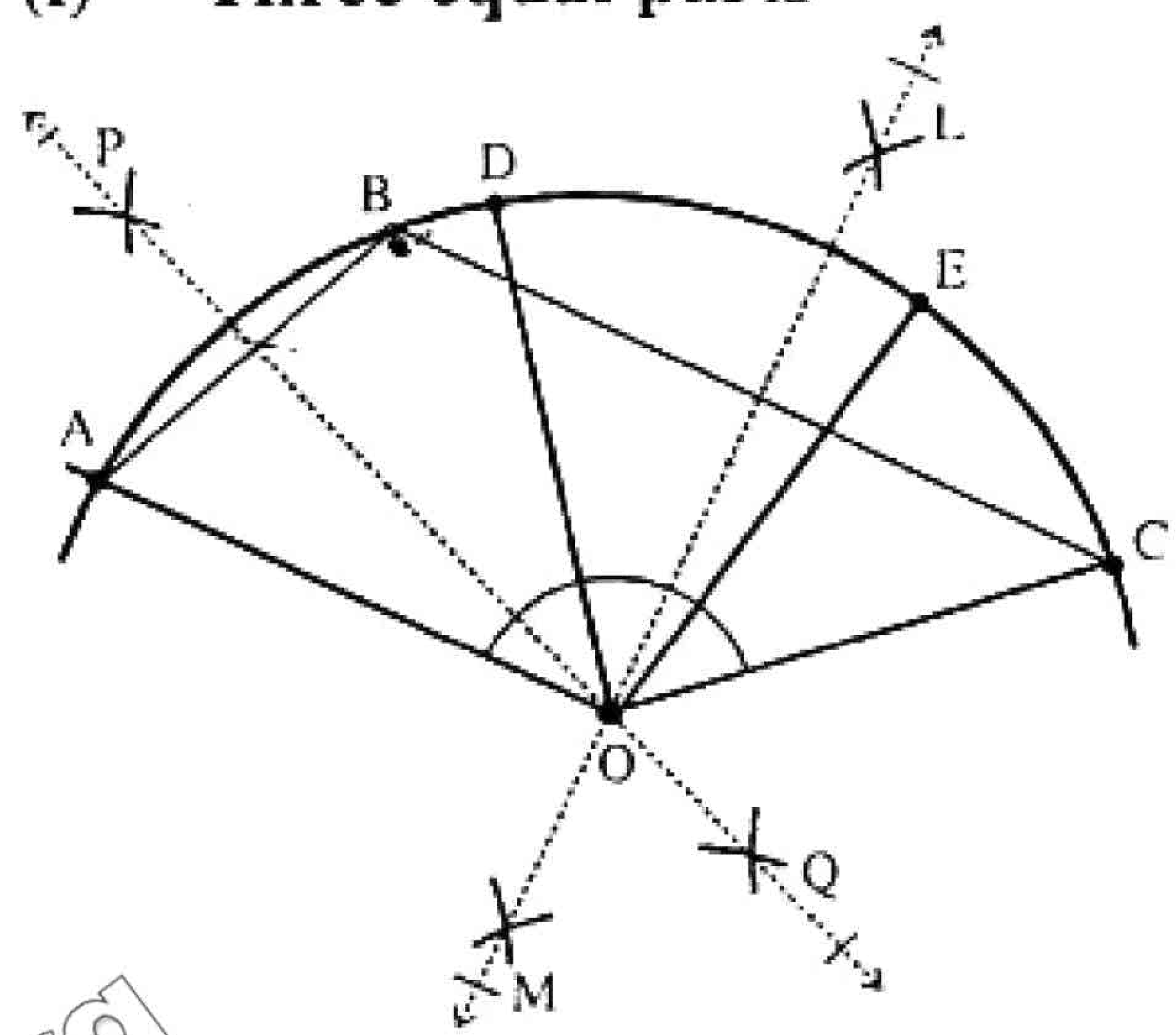
(i) Into three equal parts

(ii) Into four equal parts

(iii) Into six equal parts

Solution:

(i) Three equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

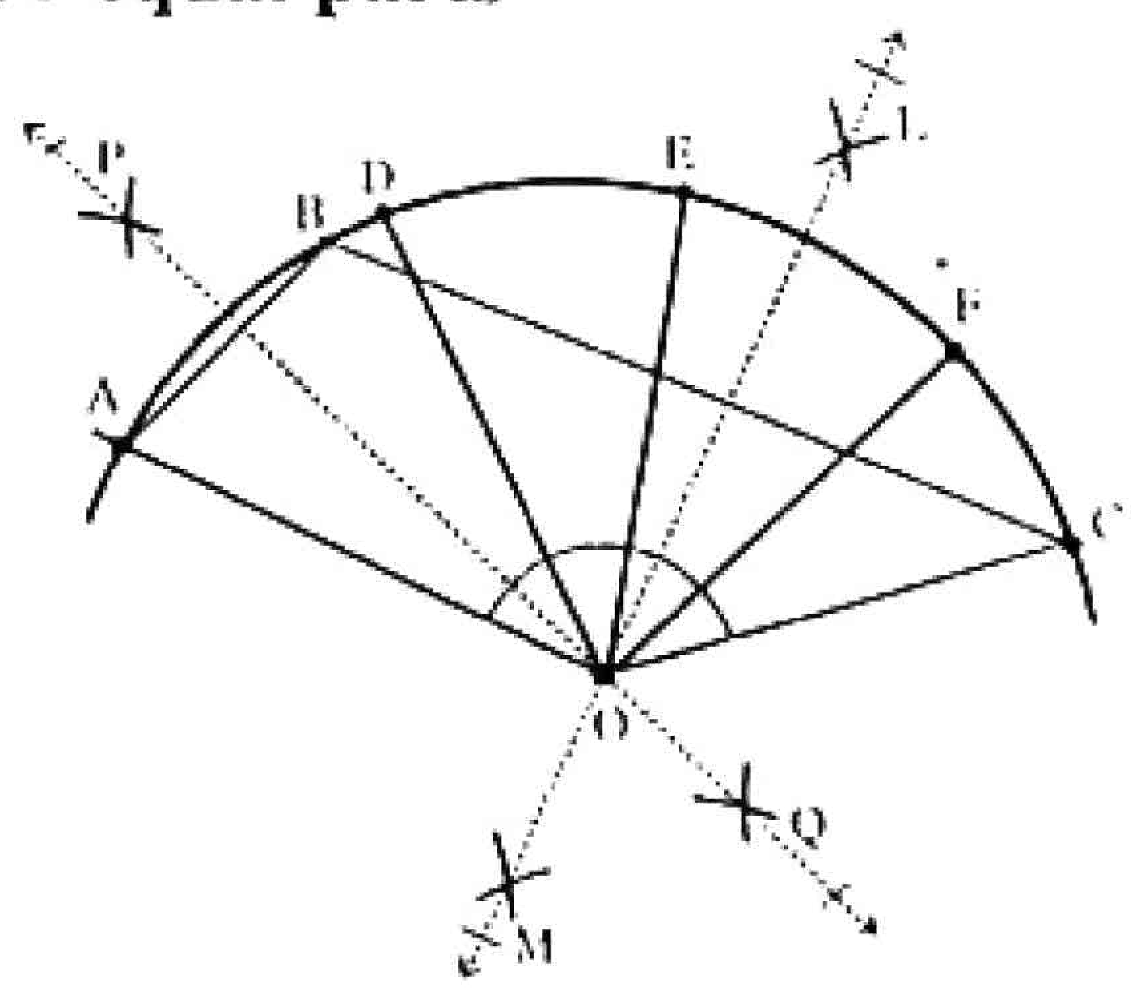
iii. Draw right bisectors $\overline{PQ}$ and $\overline{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into three equal central angles cutting the arc AC at points D and E.

vi. Arcs of same radii corresponding to equal central angles are equal. Thus three equal parts of the arc ABC are $m\widehat{AD} = m\widehat{DE} = m\widehat{EC}$.

(ii) Four equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

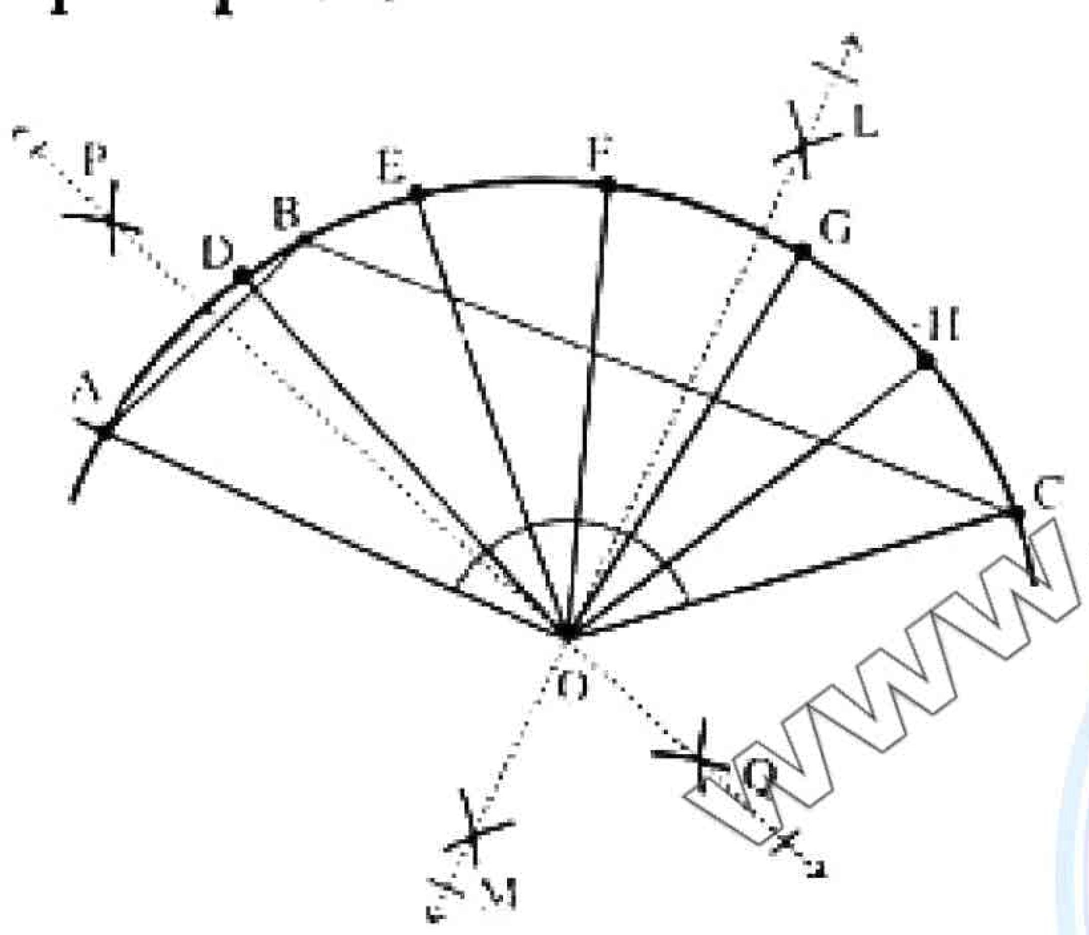
iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into four equal central angles cutting the arc AC at points D, E and F.

vi. Arcs of same radii corresponding to equal central angles are equal. Thus four equal parts of the arc ABC are $m\widehat{AD} = m\widehat{DE} = m\widehat{EF} = m\widehat{FC}$.

(iii)Six equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

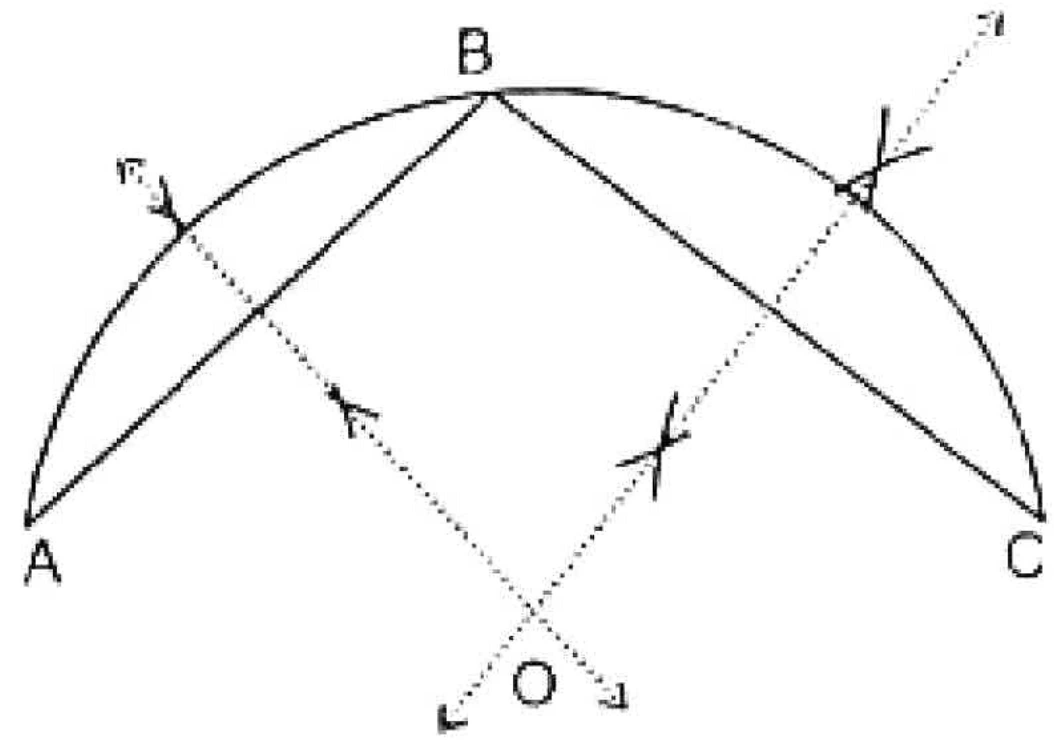
iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into six equal central angles cutting the arc AC at points D, E, F, G and H.

Arcs of same radii corresponding to equal central angles are equal. Thus six equal parts of the arc ABC are $m\widehat{AD} = m\widehat{DE} = m\widehat{EF} = m\widehat{FG} = m\widehat{GH} = m\widehat{HC}$

Q.2 Practically find the centre of an arc ABC



Steps of Construction:

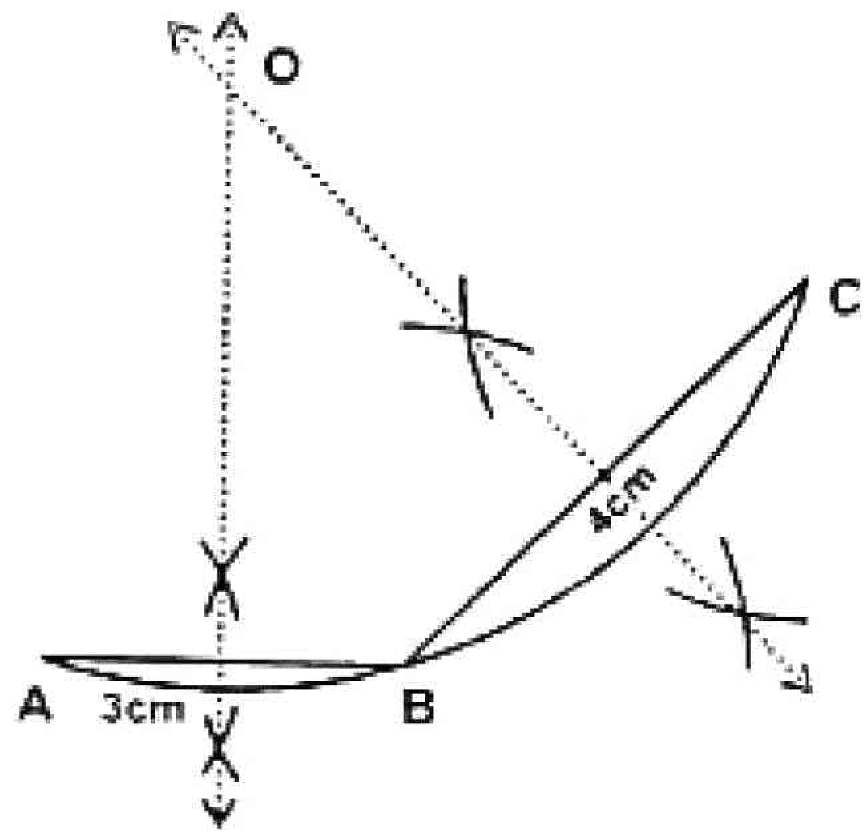
i. We draw an arc ABC of any length.

ii. We draw line segments $\overline{AB}$ and $\overline{BC}$.

iii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$, intersecting each other at point O.

iv. Point 'O' is the required centre of arc ABC.

Q. 3 (i) If $|\overline{AB}| = 3cm$ and $|\overline{BC}| = 4cm$ are the lengths of two chords of an arc, then locate the centre of the arc.



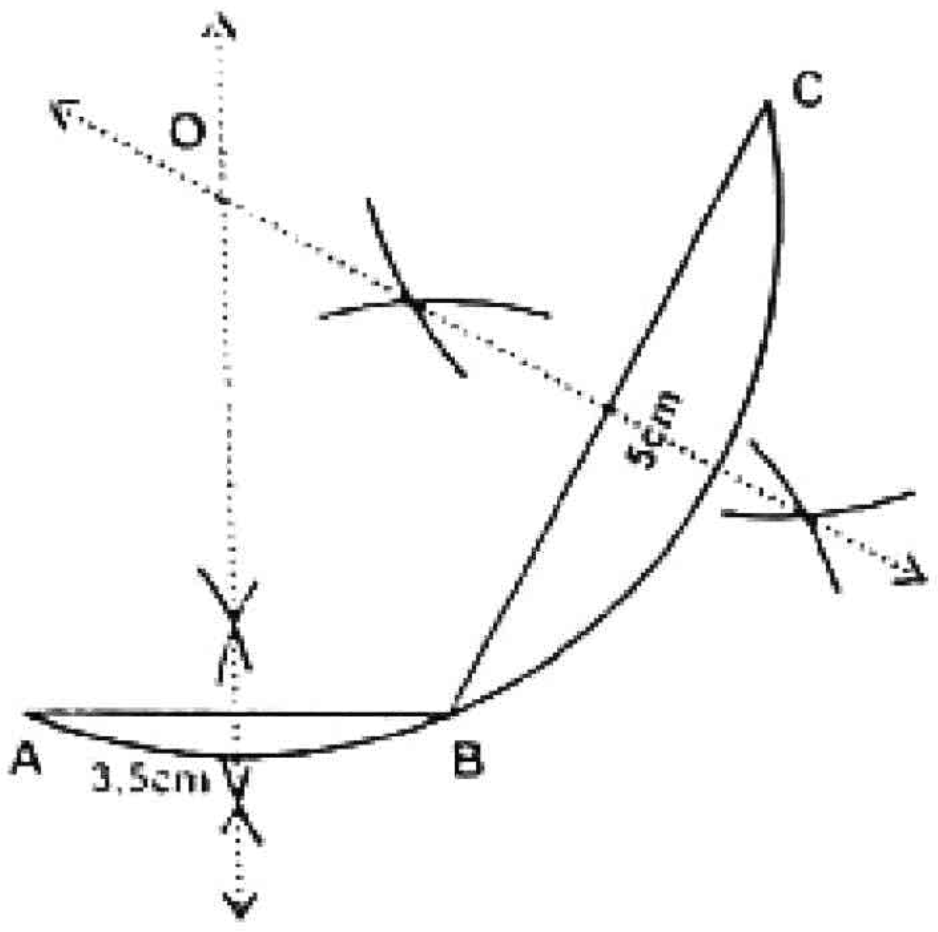
Steps of Construction:

i. We draw $|\overline{AB}| = 3cm$ and $|\overline{BC}| = 4cm$, inclined at any angle.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$ intersecting each other at point O, which is the required centre of arc ABC.

iii. Taking centre 'O', we draw an arc ABC of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$.

(ii) If $|\overline{AB}| = 3.5cm$ and $|\overline{BC}| = 5cm$ are the lengths of two chords of an arc, then locate the centre of the arc.



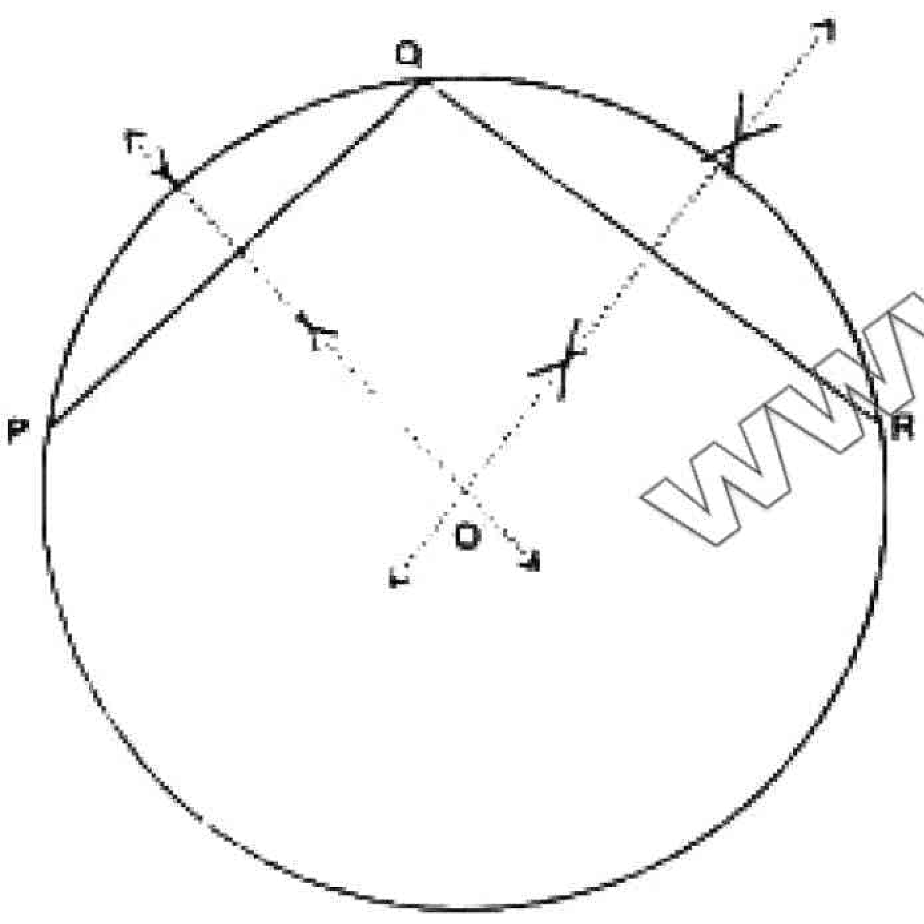
Steps of Construction:

i. We draw $|\overline{AB}| = 3.5cm$ and $|\overline{BC}| = 5cm$, inclined at any angle.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$ intersecting each other at point O, which is the required centre of arc ABC.

iii. Taking centre 'O', we draw an arc ABC of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$.

4. For an arc draw two perpendicular bisectors of the chords $\overline{PQ}$ and $\overline{QR}$ of this arc, construct a circle through P, Q and R.



Steps of construction:

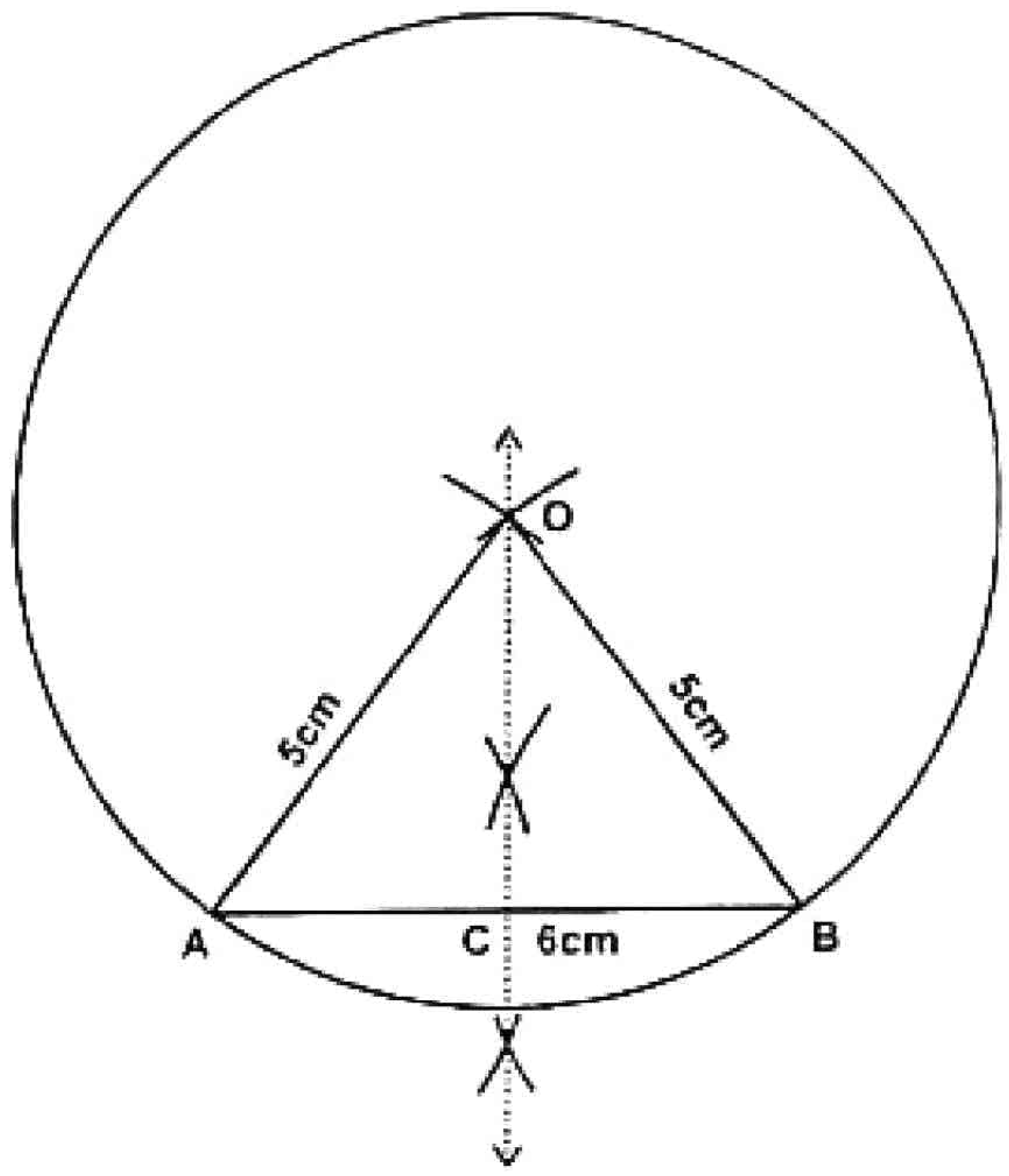
i. We take an arc PQR of any length.

ii. We take two chords $\overline{PQ}$ and $\overline{QR}$ of any lengths of arc PQR.

iii. We draw right bisectors of $\overline{PQ}$ and $\overline{QR}$, intersecting each other at point 'O', which is the centre of arc PQR.

iv. Taking 'O' as centre, we complete the required circle passing through P, Q and R.

5. Describe a circle of radius 5 cm passing through points A and B, 6 cm apart. Also find distance from the centre to line AB.



Steps of Construction:

i. We draw a line segment $\overline{AB}$ of length 6cm.

ii. We draw right bisector of $\overline{AB}$ intersecting it at point 'C'.

iii. From points A and B we draw arcs of radius 5cm each, intersecting the bisector at point O.

iv. Taking 'O' as centre we draw a circle of radius 5 cm passing through the points A and B.

v. To find the distance of centre O from $\overline{AB}$, we consider right angle ΔOAC.

By Pythagorean Theorem

$$(m\overline{OC})^2 + (m\overline{AC})^2 = (m\overline{OA})^2$$

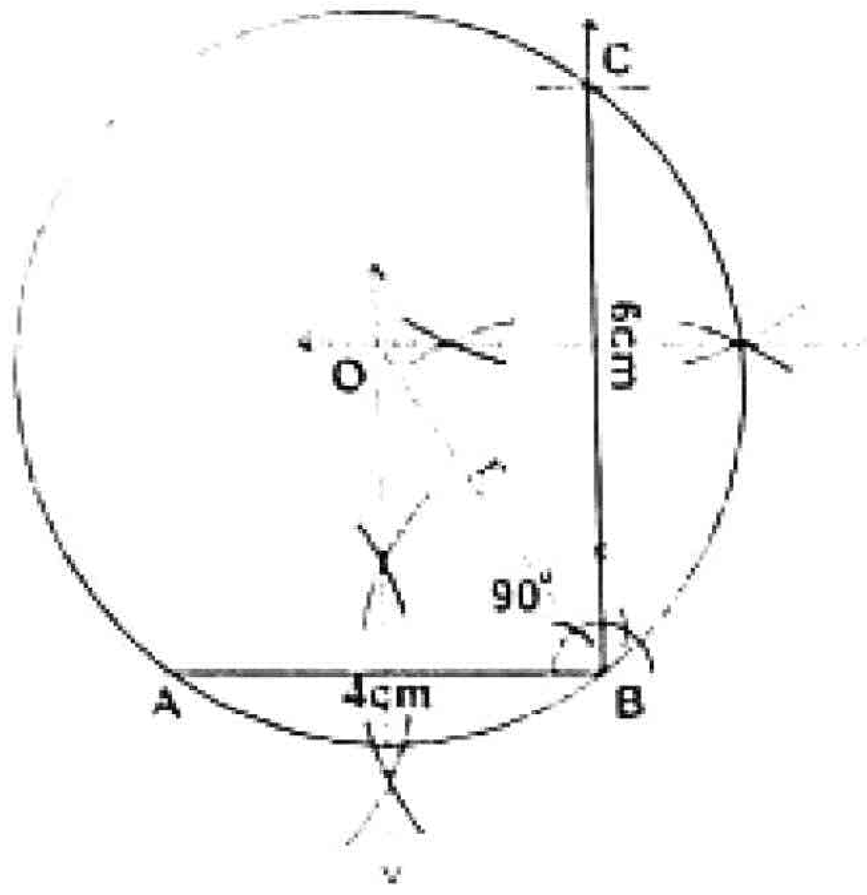
$$(m\overline{OC})^2 + (3)^2 = (5)^2$$

$$(m\overline{OC})^2 = 25 - 9$$

$$(m\overline{OC})^2 = 16$$

$$= 4 \text{ cm } m\overline{OC}$$

6. If $|\overline{AB}| = 4cm$ and $|\overline{BC}| = 6cm$, such that $\overline{AB}$ is perpendicular to $\overline{BC}$, construct a circle through points A, B and C. Also measure its radiu



Steps of construction:

i. We draw $\overline{AB}$ and $\overline{BC}$, 4 cm and 6 cm long respectively, perpendicular to each other.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$, intersecting each other at point 'O'.

iii. Taking 'O' as centre we draw a circle of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$ passing through the points A, B and C.

iv. The radius of this circle is measured to be 3.6 cm.

v. By Pythagoras theorem

$r^2 = 2^2 + 3^2$

$r^2 = 4 + 9$

$\sqrt{r^2} = \sqrt{13}$

$r = 3.6cm$

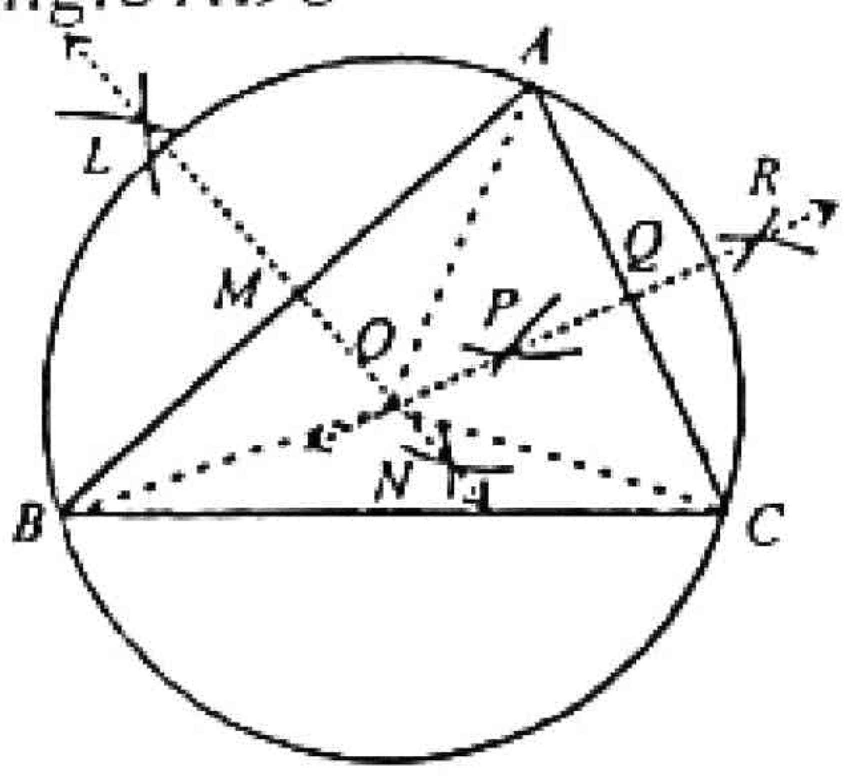
CIRCLES ATTACHED TO POLYGONS

1. Circum circle:

The circle passing through the vertices of triangle ABC is known as circum circle, its radius as circum radius and centre as circum centre.

Circumscribe a circle about a given triangle.

Given: Triangle ABC



Steps of Construction:

i. Draw $\overleftrightarrow{LMN}$ as perpendicular bisector of side $\overline{AB}$.

ii. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{AC}$.

iii. $\overleftrightarrow{LN}$ and $\overleftrightarrow{PR}$ intersect at point O.

iv. With centre O and radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

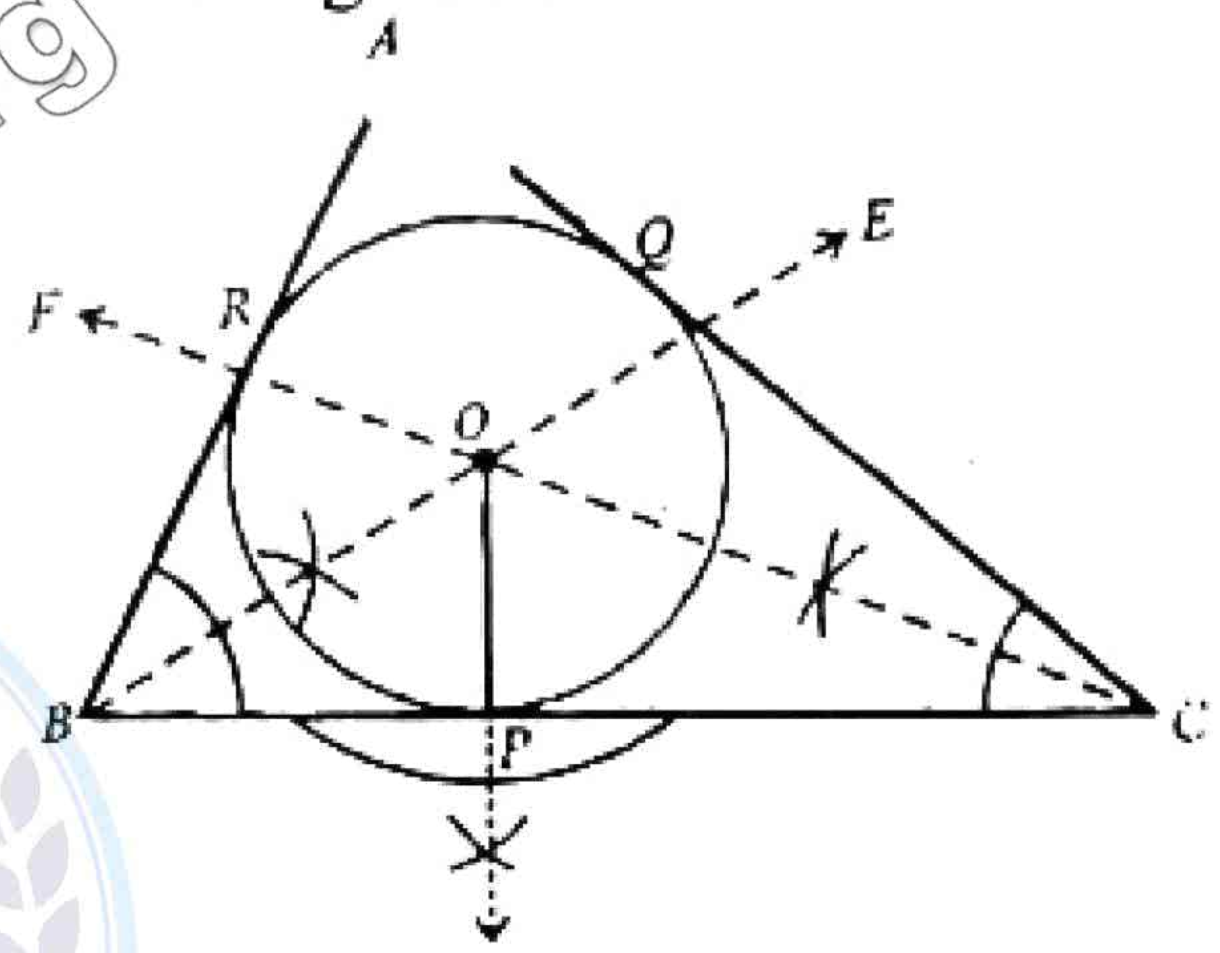
This circle will pass through A, B and C whereas O is the circum centre of the circumscribed circle.

2. Inscribed circle or In-circle:

A circle which touches the three sides of a triangle internally is known as in-circle, its radius as in-radius and centre as in-centre.

Inscribe a circle in a given triangle.

Given: A Triangle ABC



Steps of Construction:

i. Draw $\overrightarrow{BE}$ and $\overrightarrow{CF}$ to bisect the angles ABC and ACB respectively. Rays $\overrightarrow{BE}$ and $\overrightarrow{CF}$ intersect each other at point O.

ii. O is the centre of the inscribed circle.

iii. From O draw $\overrightarrow{OP}$ perpendicular to $\overline{BC}$.

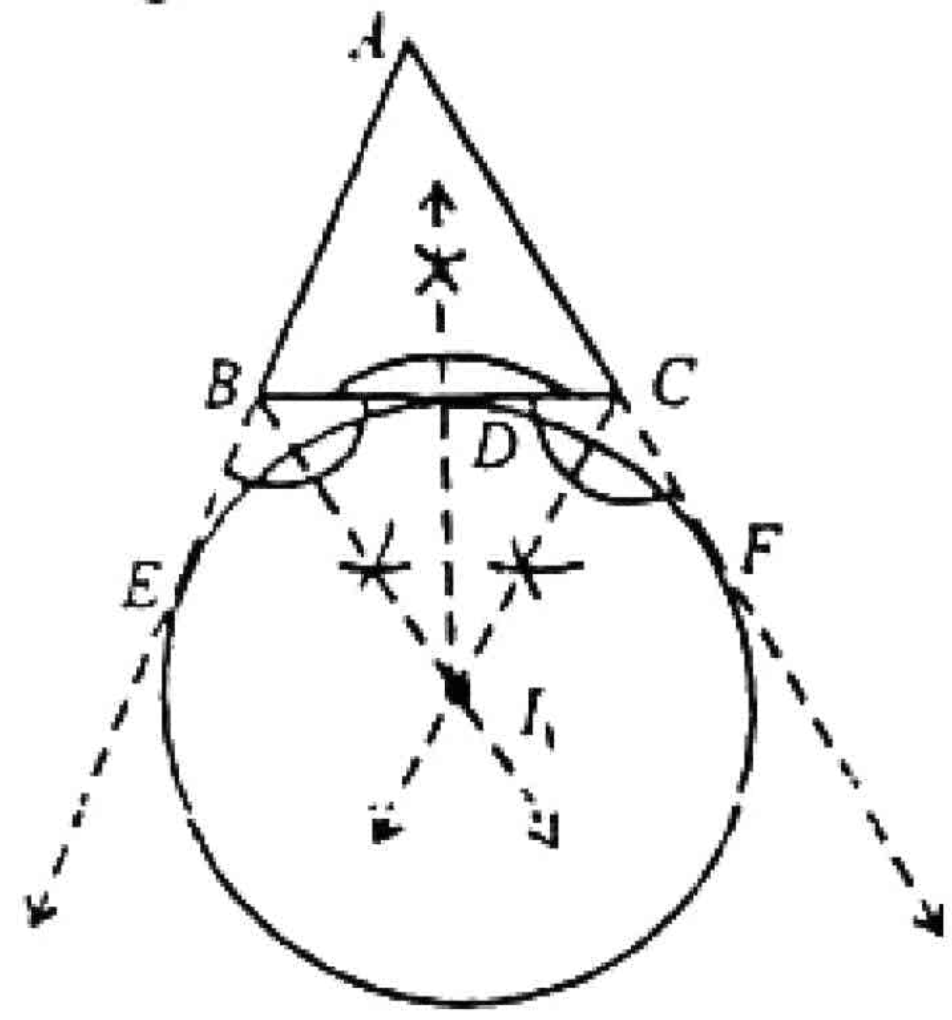
iv. With centre O and radius $\overline{OP}$ draw a circle. This circle is the inscribed circle of triangle ABC.

3. Escribed Circle:

The circle touching one side of the triangle externally and other two produced sides internally is called escribed circle (e-circle). The centre of e-circle is called e-centre and radius is called e-radius.

Escribe a circle to a given triangle.

Given: A Triangle ABC



Steps of Construction:

i. Produce the sides $\overline{AB}$ and $\overline{AC}$ of ΔABC.

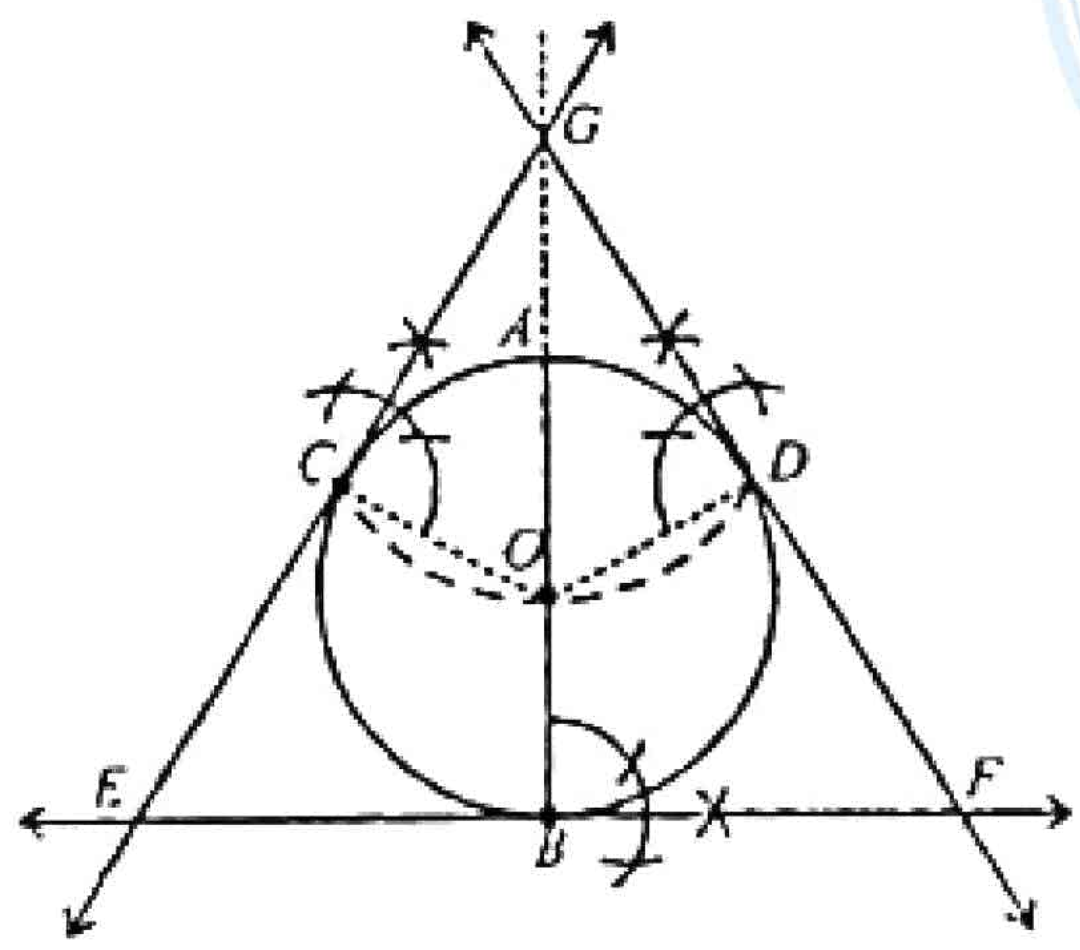
ii. Draw bisectors of exterior angles EBC and FCB. These bisectors of exterior angles meet at I_1

iii. From I_1 draw perpendicular on side $\overline{BC}$ of ΔABC intersecting $\overline{BC}$ at D. $\overline{I_1D}$ is the radius of the escribed circle with centre at I_1.

iv. Draw the circle with radius $\overline{I_1D}$ and centre at I_1 that will touch the side BC of the ΔABC externally and the produced sides AB and AC internally.

4. Circumscribe an equilateral triangle about a given circle.

Given: A circle with centre O of reasonable radius.



Steps of Construction:

i. Draw $\overline{AB}$ the diameter of the circle for locating.

ii. Draw an arc of radius m$\overline{OA}$ with centre at A, to locate points C and D on the circle.

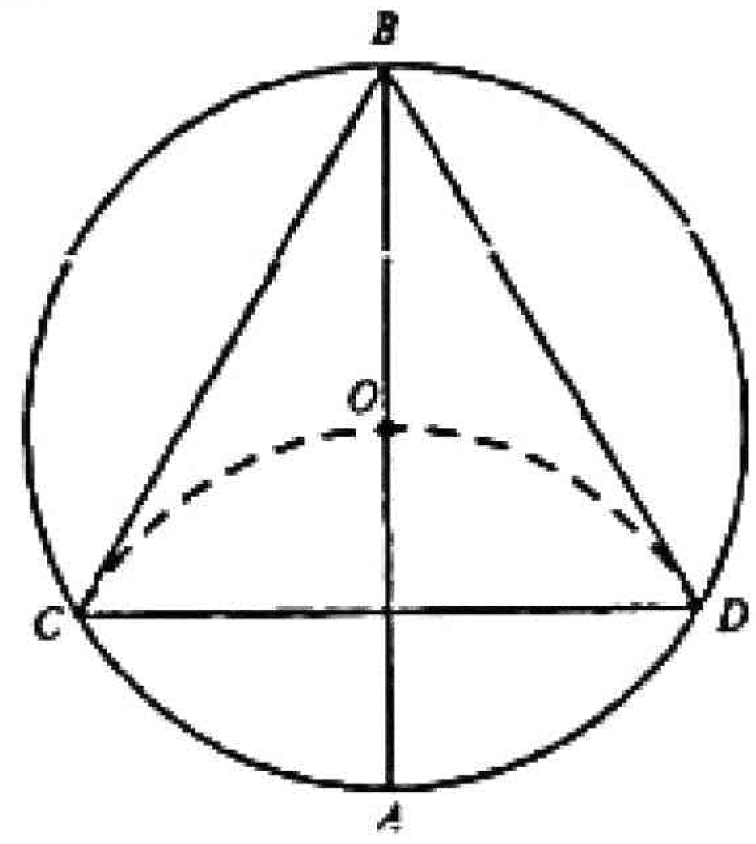
iii. Join O to the points C and D.

iv. Draw tangents to the circle at points B, C and D.

v. These tangents intersect at point E, F and G. Thus ΔEFG is required equilateral triangle.

5. Inscribe an equilateral triangle in a given circle.

Given: A circle with centre at O.



Steps of Construction:

i. Draw any diameter $\overline{AB}$ of circle.

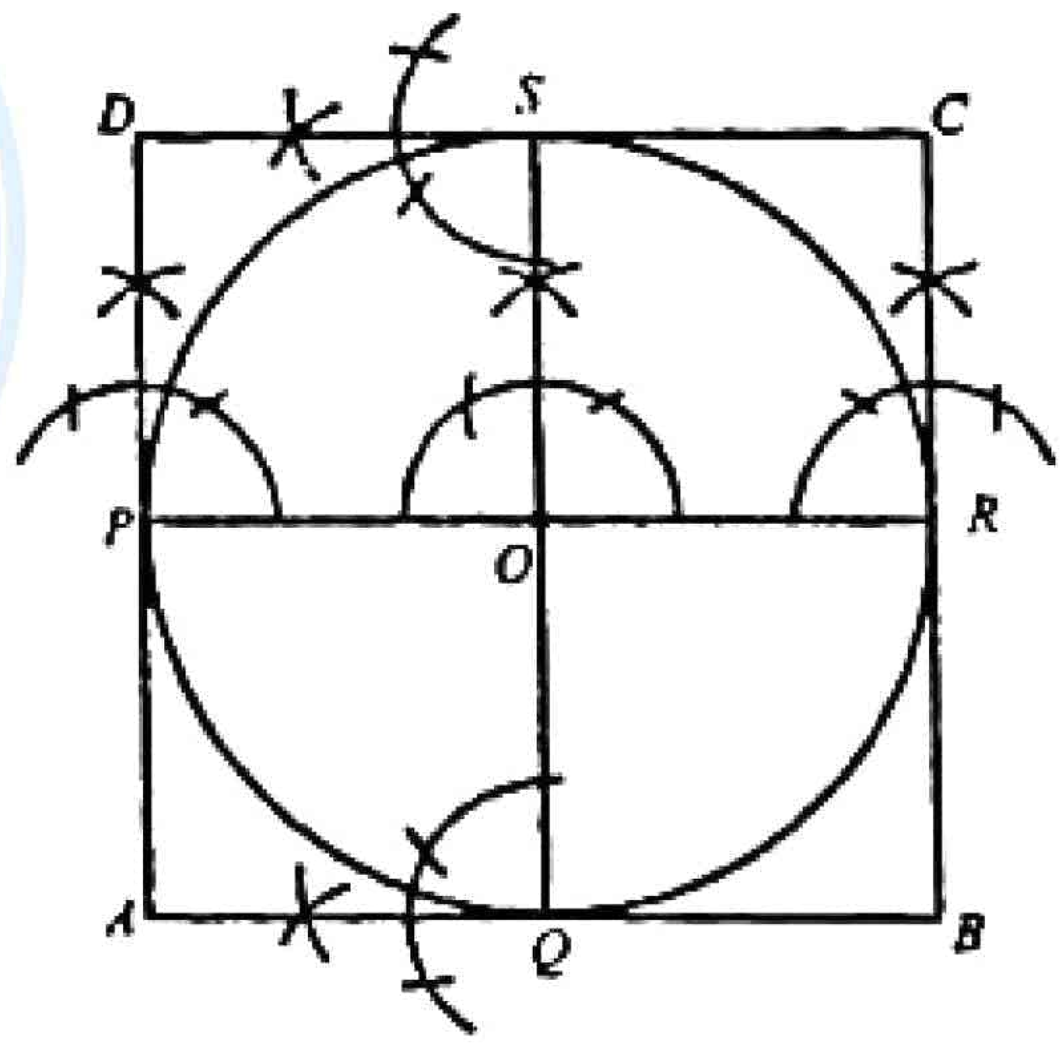
ii. Draw an arc of radius m$\overline{OA}$ from point A. The arc cuts the circle at points C and D.

iii. Join the points B, C and D to form straight line segments $\overline{BC}$, $\overline{CD}$ and $\overline{BD}$.

iv. Triangle BCD is the required inscribed equilateral triangle.

6. Circumscribe a square about a given circle.

Given: A circle with centre at O.



Steps of Construction:

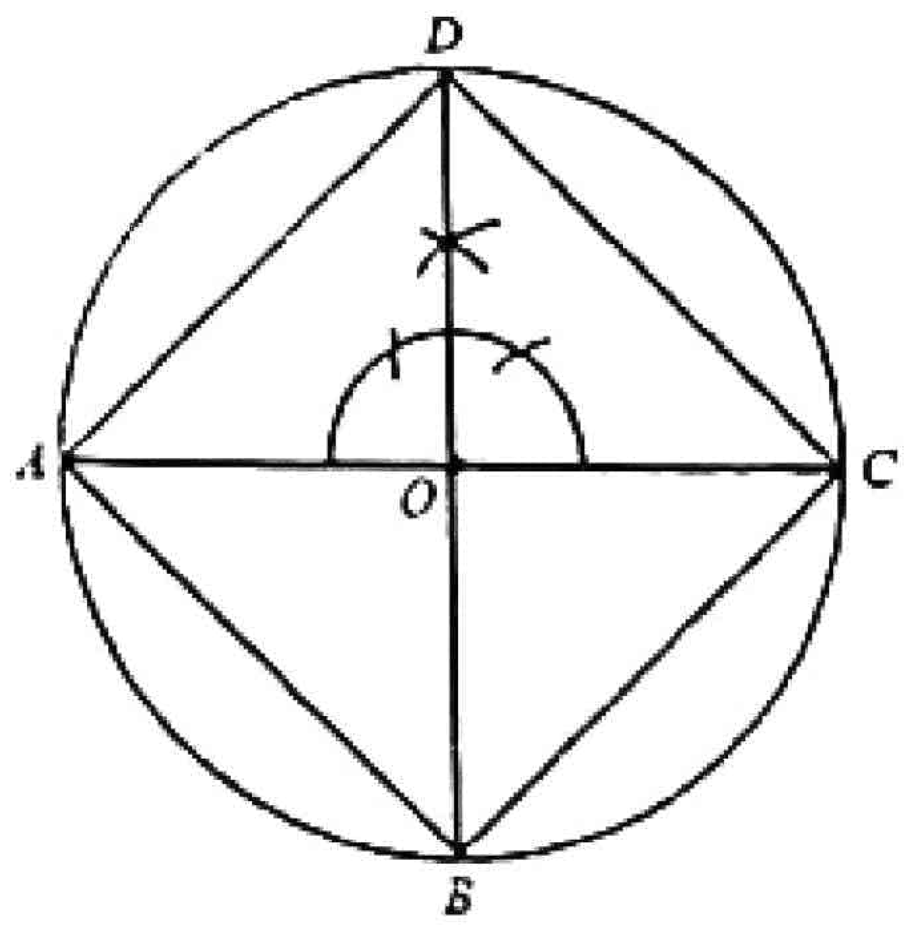
i. Draw two diameters $\overline{PR}$ and $\overline{QS}$ which bisect each other at right tangle.

ii. At points P, Q, R and S draw tangents to the circle.

iii. Produce the tangents to meet each other at A, B, C and D. ABCD is the required circumscribed square.

7. Inscribe a square in a given circle.

Given: A circle with centre at O.



Steps of Construction:

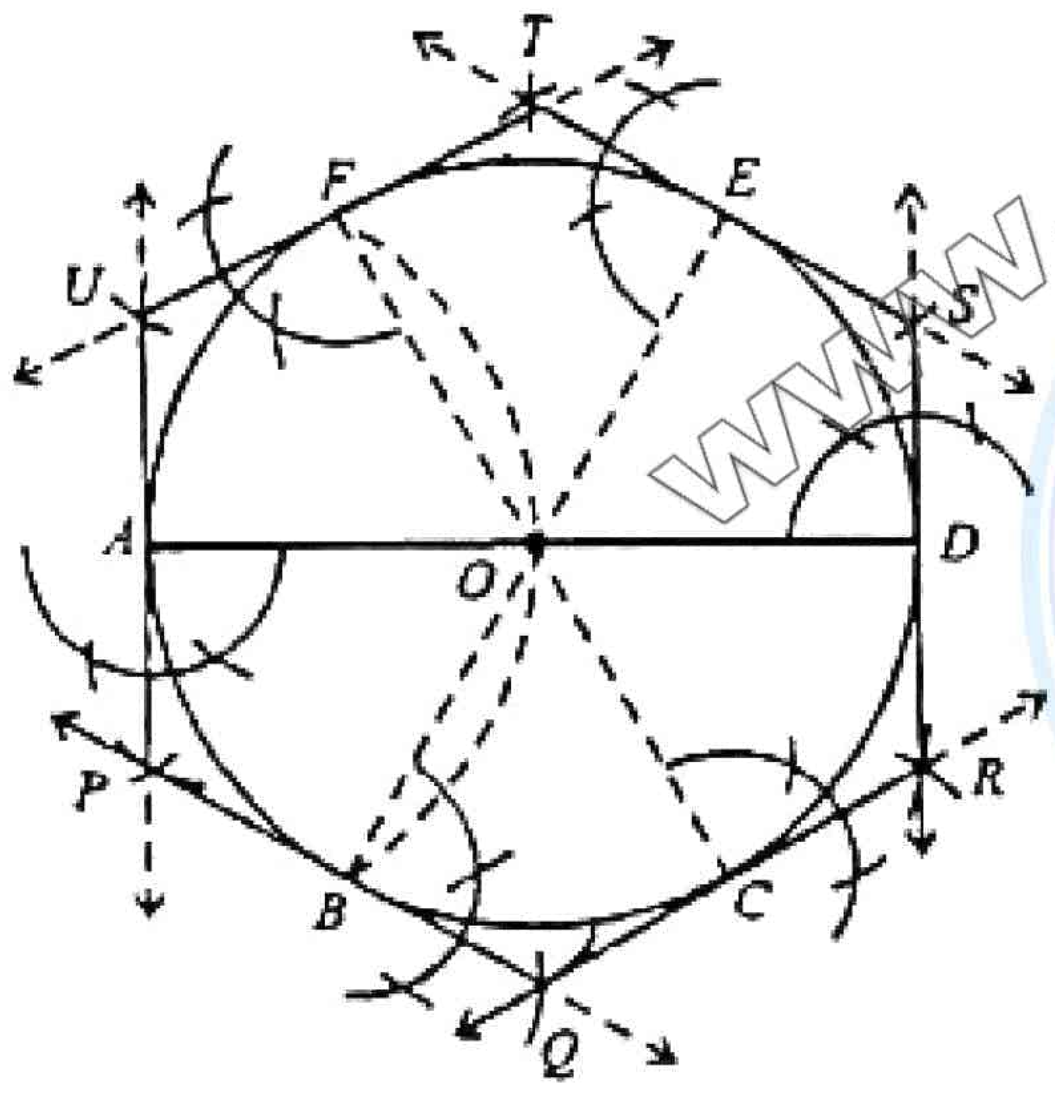
i. Through O draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.

ii. Join *A* with *B*, *B* with *C*, *C* with *D*, and *D* with *A*.

iii. *ABCD* is the required square inscribed in the circle.

8. Circumscribe a regular hexagon about a given circle.

Given: A circle, with centre at *O*.



Steps of Construction:

i. Draw any diameter $\overline{AD}$.

ii. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at points B and F.

iii. Join B with O and extend it to meet the circle at E.

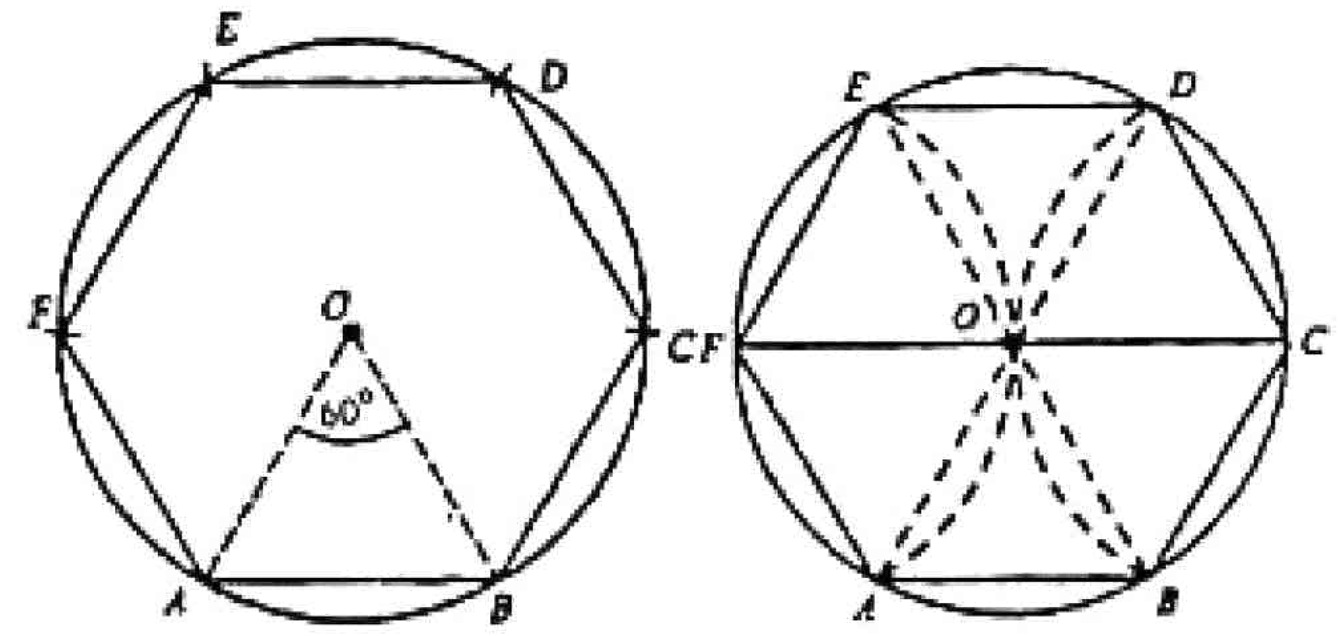
iv. Join F with O and extend it to meet the circle at C.

v. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U respectively.

vi. Thus PQRSTU is the circumscribed regular hexagon.

9. Inscribe a regular hexagon in a given circle:

Given: A circle, with centre at *O*.



Steps of Construction:

i. Take any point A on the circle with centre O.

ii. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at point B and F.

iii. Join O and A with points B and F.

iv. ΔOAB and ΔOAF are equilateral triangles therefore $\angle AOB$ and $\angle AOF$ are of measure 60° i.e., $m\overline{OA} = m\overline{AB} = m\overline{AF}$.

v. Produce $\overline{FO}$ to meet the circle at C. Join B to C. Since $m\angle BOC = 60$ therefore $m\overline{BC} = m\overline{OA}$.

vi. From C and F, draw arcs of radius $\overline{OA}$, which intersect the circle at points D and E.

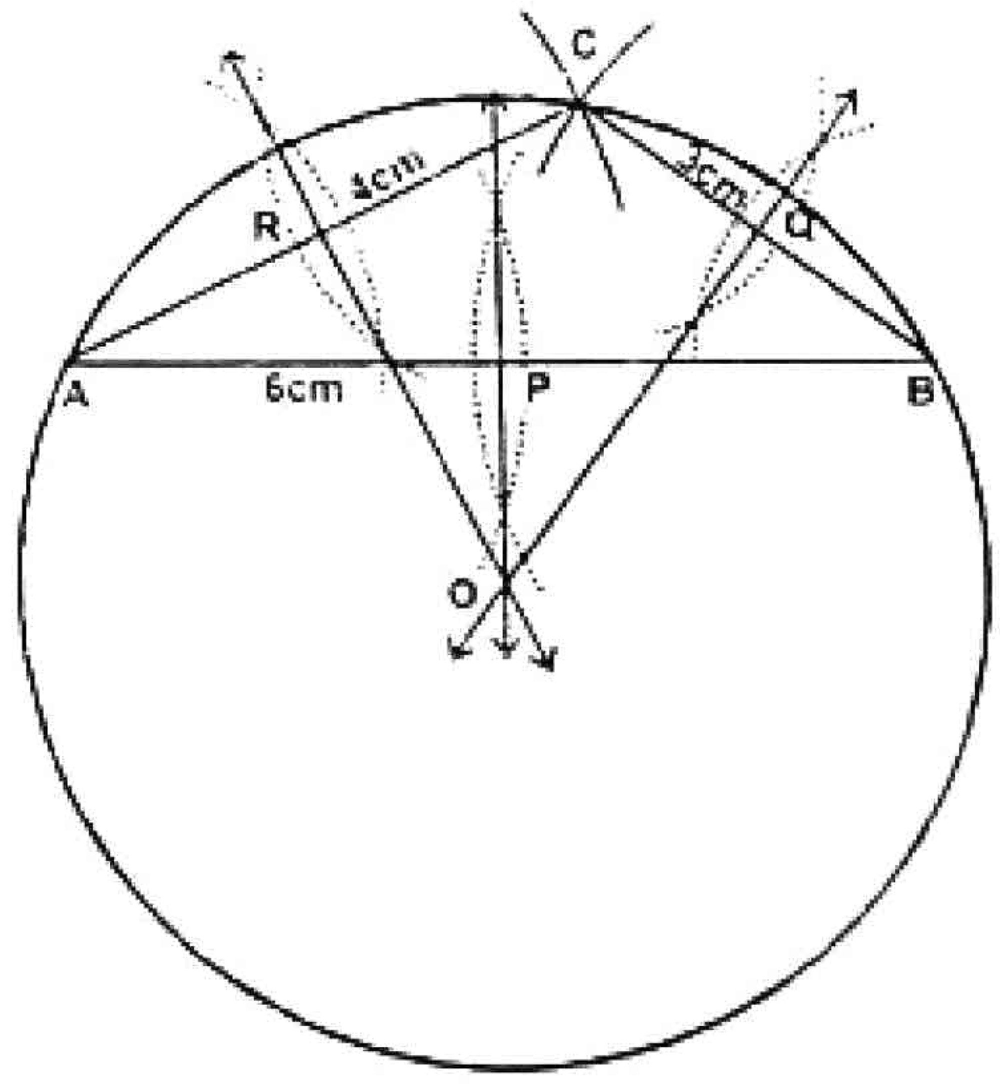
vii. Join C to D, D to E to F. So, we have $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$
Thus the figure ABCDEF is a regular hexagon inscribed in the circle.

EXERCISE 13.2

Q. 1 Circumscribe a circle about a triangle ABC with sides $|\overline{AB}| = 6cm$, $|\overline{BC}| = 3cm$ and $|\overline{CA}| = 4cm$. Also measure its circum radius.

Solution:

Data: $|\overline{AB}| = 6cm, |\overline{BC}| = 3cm, |\overline{CA}| = 4cm$



Steps of construction:

i. We construct triangle ABC according to given condition.

ii. We draw right bisectors $\overleftrightarrow{OP}, \overleftrightarrow{OQ}$ and $\overleftrightarrow{OR}$ of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively, concurrent at point 'O'.

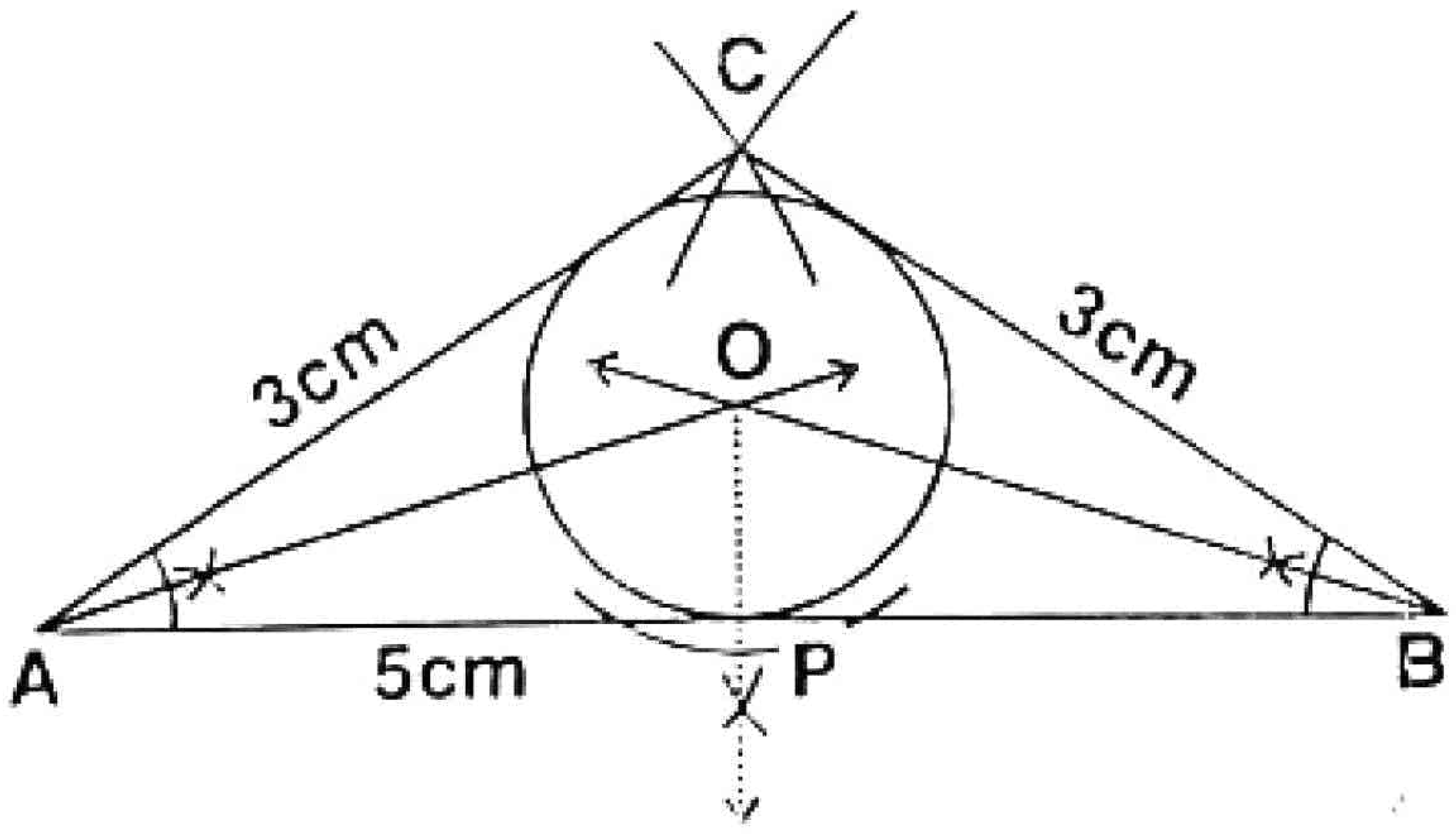
iii. Taking 'O' as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the vertices A, B and C.

iv. This is the required circum circle, whose radius is measured to be 3.3 cm.

Q. 2 Inscribe a circle in a triangle ABC with side $|\overline{AB}| = 5cm$, $|\overline{BC}| = 3cm$ and $|\overline{CA}| = 3cm$. Also measure its in-radius.

Solution:

Data: $|\overline{AB}| = 5cm$, $|\overline{AB}| = 5cm$, $|\overline{CA}| = 3cm$



Steps of construction:

i. We construct triangle ABC according to given condition.

ii. We draw bisectors of $\angle A$ and $\angle B$ intersecting each other at point 'O'.

iii. From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.

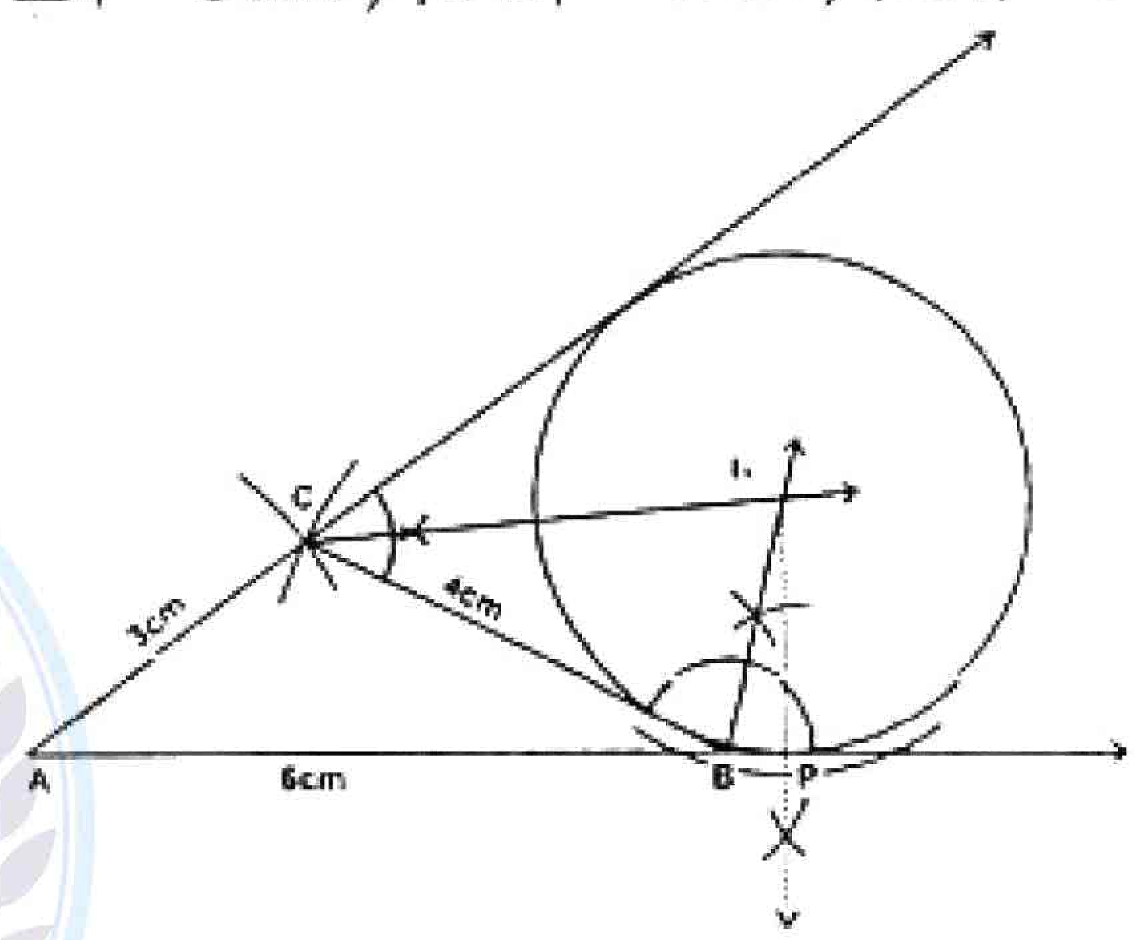
iv. Taking 'O' as centre and radius equal to $m\overline{OP}$, we draw a circle, touching three sides of triangle internally.

v. This is the required in-circle whose radius is measured to be 0.8 cm.

Q. 3 Escribe a circle opposite to vertex A to a triangle ABC with sides $|AB| = 6cm$, $|BC| = 4cm$ and $|CA| = 3cm$. Find its radius also.

Solution:

Data: $|AB| = 6cm$, $|BC| = 4cm$, $|CA| = 3cm$



Steps of construction:

i. We construct a triangle ABC according to given condition.

ii. We produce the sides $\overline{AB}$ and $\overline{AC}$ beyond B and C respectively.

iii. We draw, bisectors of exterior angles at points B and C, intersecting each other at point I_1.

iv. From point I_1, we draw $\overrightarrow{I_1P}$ perpendicular to $\overline{AB}$ produced.

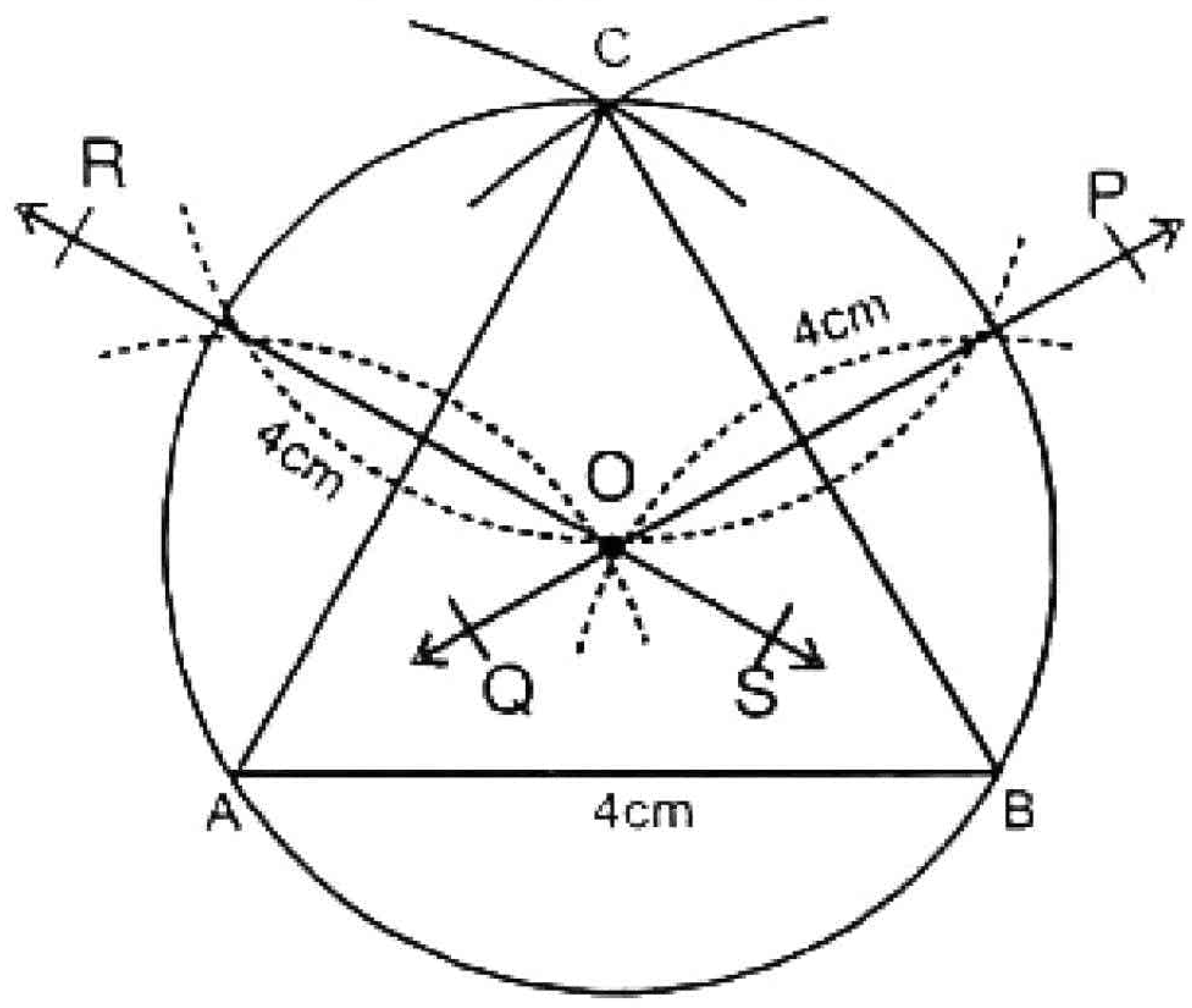
v. Taking I_1, as centre and radius equal to I_1P, we draw a circle, touching one side of ΔABC externally and other two produced sides internally.

vi. This is the required escribed circle, whose radius is measured to be 2.2 cm.

Q. 4 Circumscribe a circle about an equilateral triangle ABC with each side of length 4cm.

Solution:

Data: $m\overline{AB} = m\overline{BC} = m\overline{CA} = 4cm$



Steps of construction:

i. We construct equilateral triangle ABC with each side 4cm long.

ii. We draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of side $\overline{BC}$ and $\overline{AC}$ respectively intersecting each other at point O.

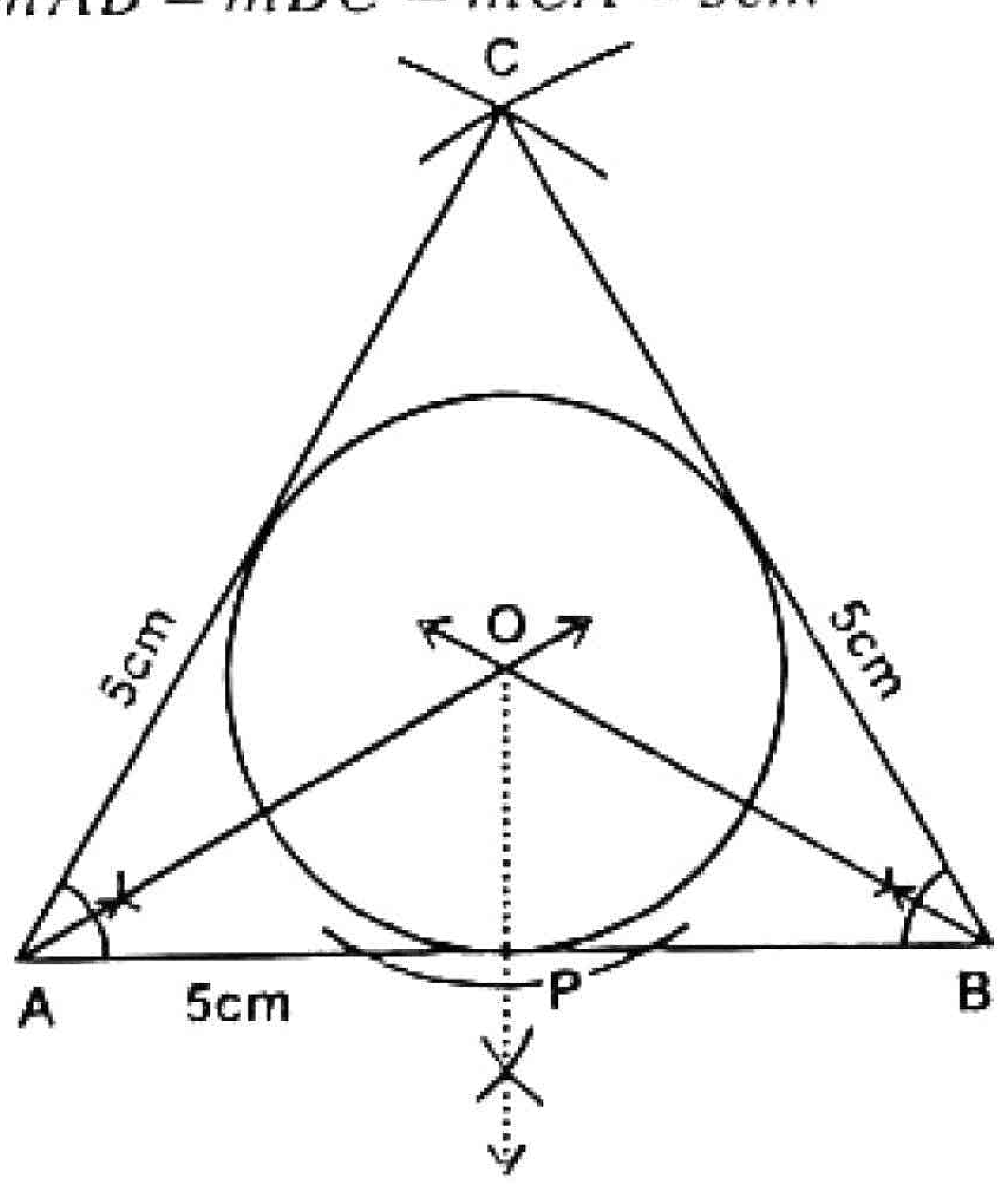
iii. Taking O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the points A, B and C.

iv. This is our required circum circle whose radius is measured to be 2.3 cm.

Q. 5 Inscribe a circle in an equilateral triangle ABC with each side of length 5cm.

Solution:

Data: $m\overline{AB} = m\overline{BC} = m\overline{CA} = 5cm$



Steps of construction:

i. We construct equilateral triangle ABC with each side 5cm long.

ii. We draw bisectors of ∠A and ∠B intersecting each other at point 'O'.

iii. From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.

iv. Taking 'O' as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.

v. This is the required in-circle whose radius is measured to be 1.4 cm.

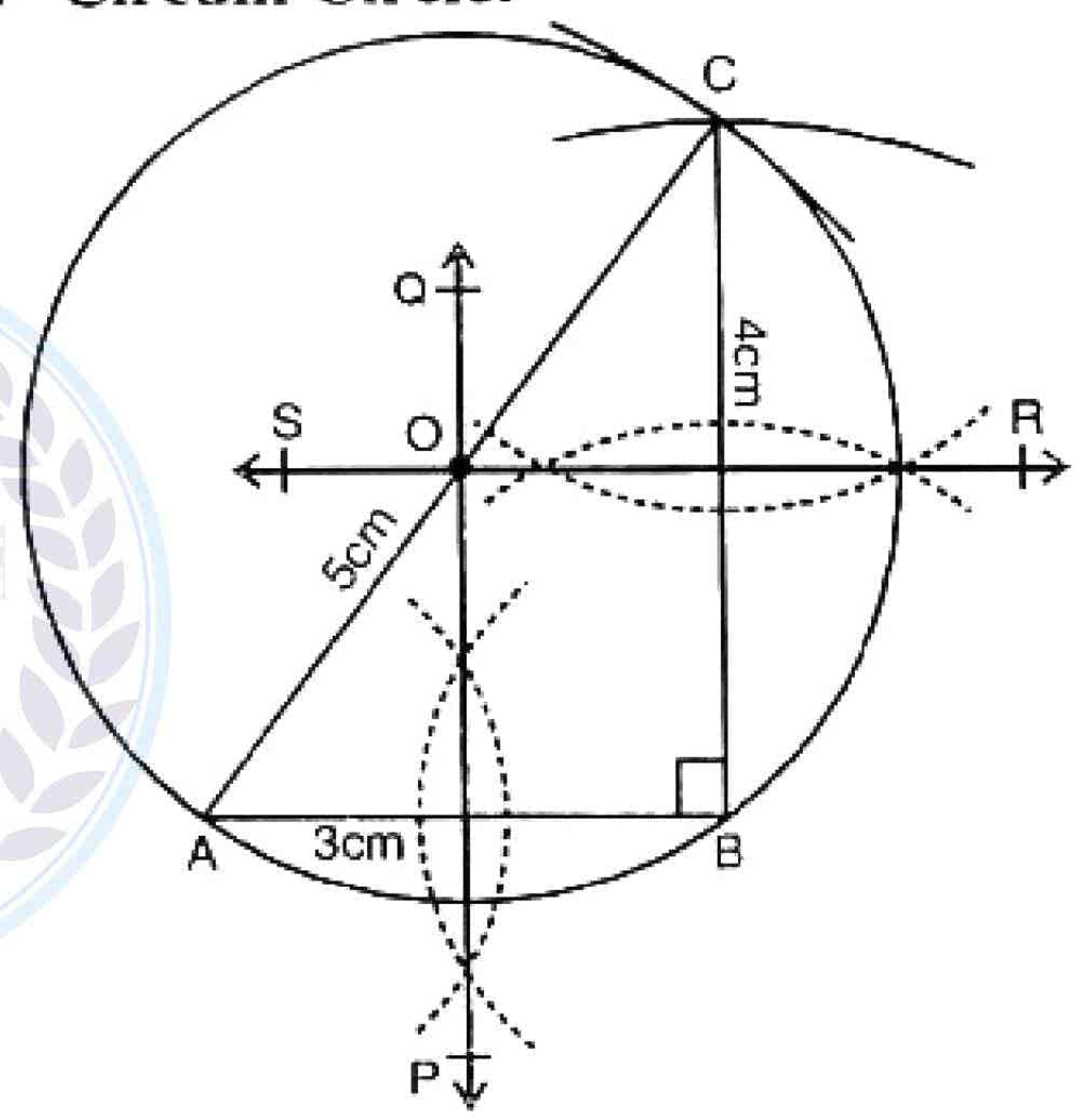
Q. 6 Circumscribe and inscribe circles with regard to a right angle triangle with sides 3cm, 4cm and 5cm.

Solution:

Let

$m\overline{AB} = 3cm, m\overline{BC} = 4cm$ and $m\overline{CA} = 5cm$

a. Circum Circle:



Steps of construction:

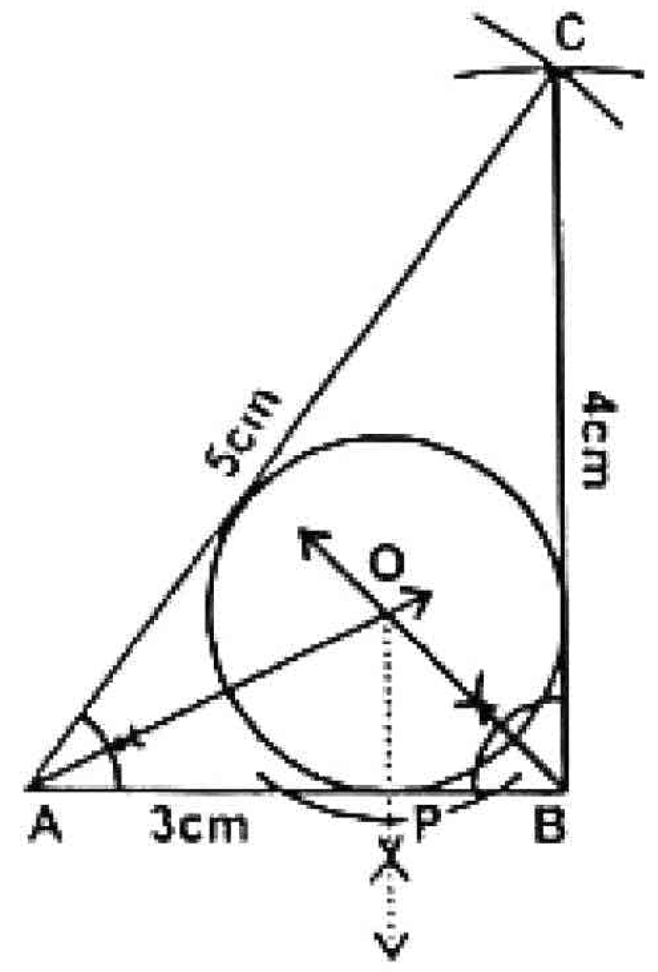
i. We construct right angle triangle ABC with sides 3cm, 4cm and 5cm.

ii. We draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of side $\overline{AB}$ and $\overline{BC}$ respectively intersecting each other at point O.

iii. Taking O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the points A, B and C.

iv. This is our required circum circle whose radius is measured to be 2.5 cm.

b. Inscribed Circle



Steps of construction:

i. We construct right angle triangle ABC according to given condition.

ii. We draw bisectors of ∠A and ∠B intersecting each other at point 'O'.

iii. From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$

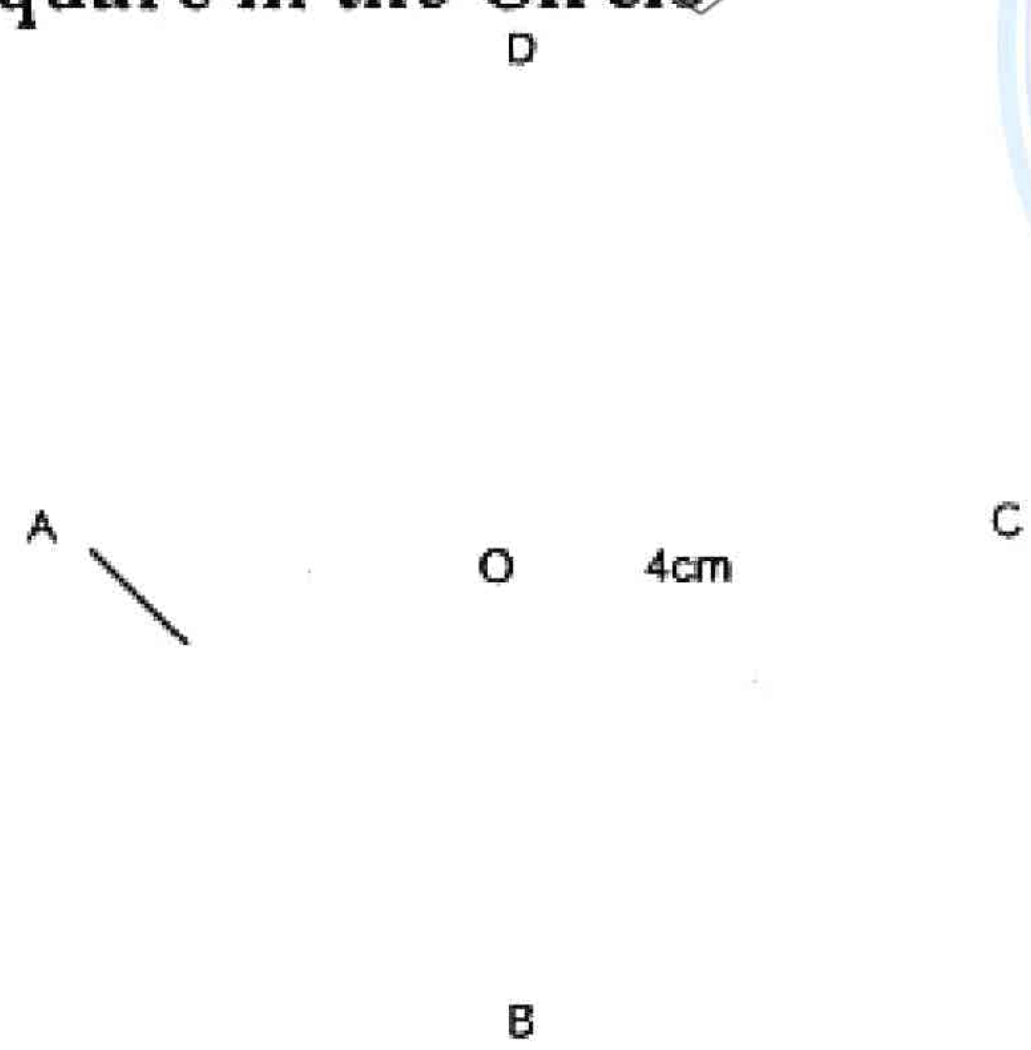
iv. Taking 'O' as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.

v. This is the required in-circle whose radius is measured to be 1 cm.

Q. 7 In and about a circle of radius 4 cm describe a square.

Solution:

a. Square in the Circle



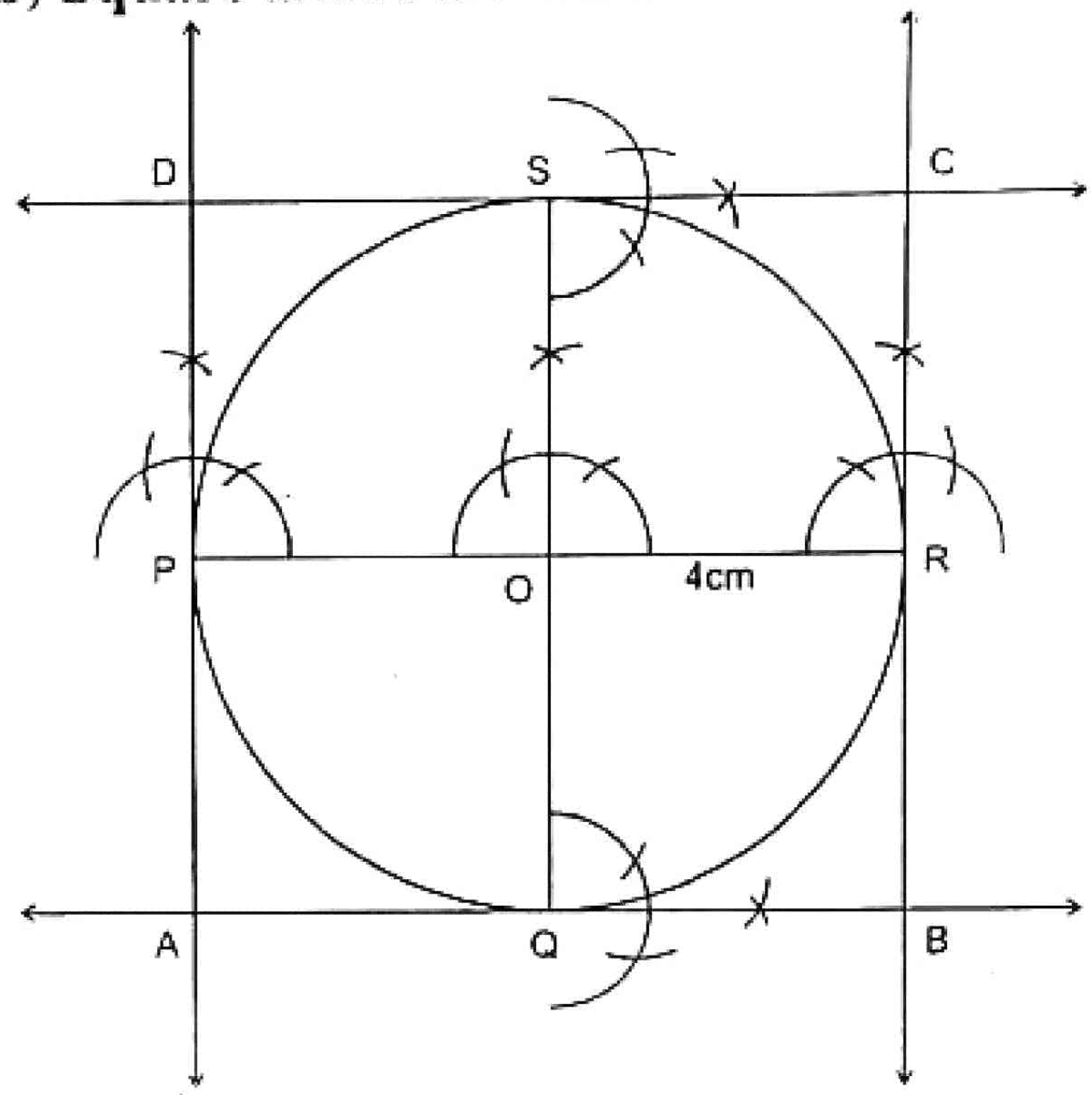
Steps of construction:

i. We draw a circle with centre 'O' of radius 4cm.

ii. We draw two diameters $\overline{AC}$ and $\overline{BD}$ of circle perpendicular to each other.

iii. By joining points A with B, B with C, C with D and D with A, we get the required square inscribed in the given circle.

(b) Square about the Circle



Steps of Construction:

i. We draw a circle with centre "O' and a radius 4cm.

ii. We draw two diameters $\overline{PR}$ and $\overline{QS}$ of circle perpendicular to each other.

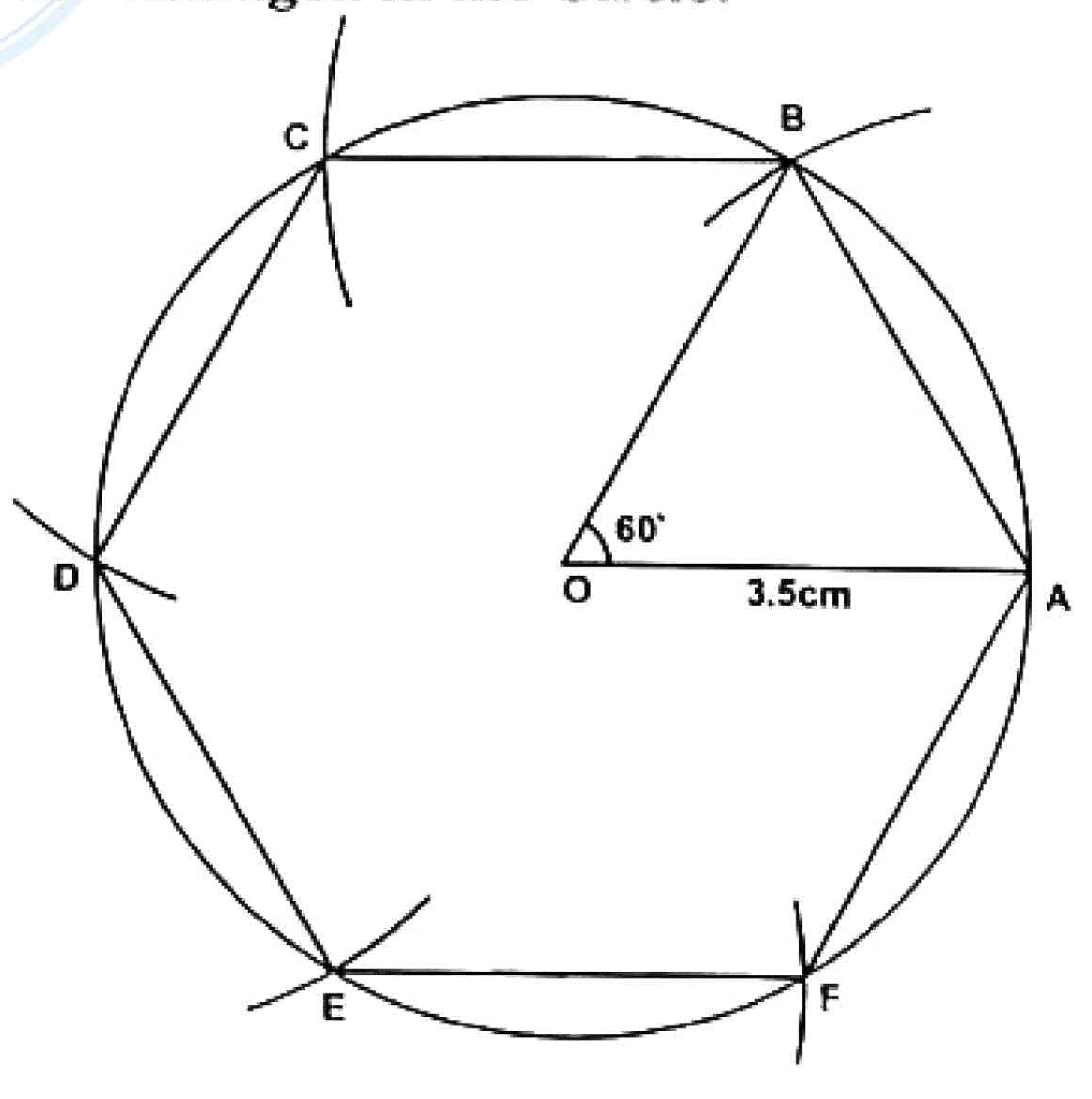
iii. We draw tangents to the circle at points P, Q, R and S.

iv. We produce the tangents to meet each other at point A, B, C and D.

v. ABCD is the required circumscribed square.

Q. 8 In and about a circle of radius 3.5 cm describe a hexagon.

a. Hexagon in the Circle:



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Steps of Construction:

i. We draw a circle with centre 'O' of radius 3.5 cm.

ii. We take a point A anywhere on the circle and draw the radial segment $\overline{OA}$.

iii. From point A, we draw an arc of radius $\overline{OA}$ which intersects the circle at point B.

iv. By joining 'O' with A and B we get an equilateral triangle OAB, so that the angle subtended by the chord at the centre is 60°.

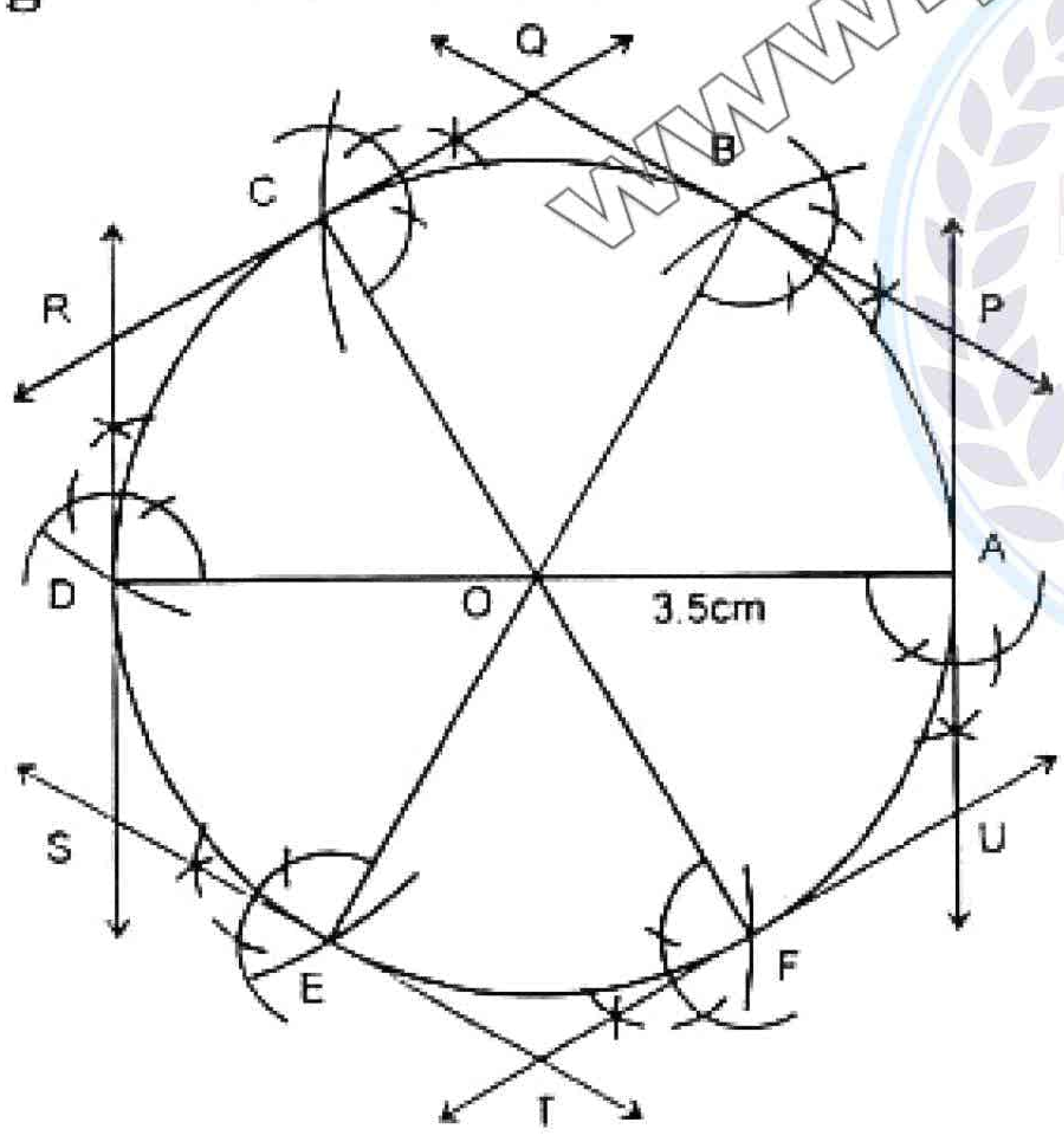
v. From point B, we draw an arc of same radius intersecting the circle at point C, then joining B to C we get another chord $\overline{BC}$.

vi. We continue to draw the arcs, which cut the circle at points D, E and F, such that

$$m\overline{OA} = m\overline{AB} = m\overline{BC} = m\overline{CD}$$
$$= m\overline{DE} = m\overline{EF} = m\overline{FA}$$

vii. We draw end to end on the circle the six chords $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$ and $\overline{FA}$, which completes the required hexagon.

b. Hexagon about the Circle:



Steps Construction:

i. We draw a circle with centre 'O' of radius 3.5 cm.

ii. We take a point A anywhere on the circle and draw the radial segment $\overline{OA}$.

iii. From point A, we draw an arc of radius $\overline{OA}$, which intersects the circle at point B.

iv. From point B, we draw an arc of same radius intersecting the circle at point C.

v. We continue to draw the arcs, which cut the circle at points D, E and F.

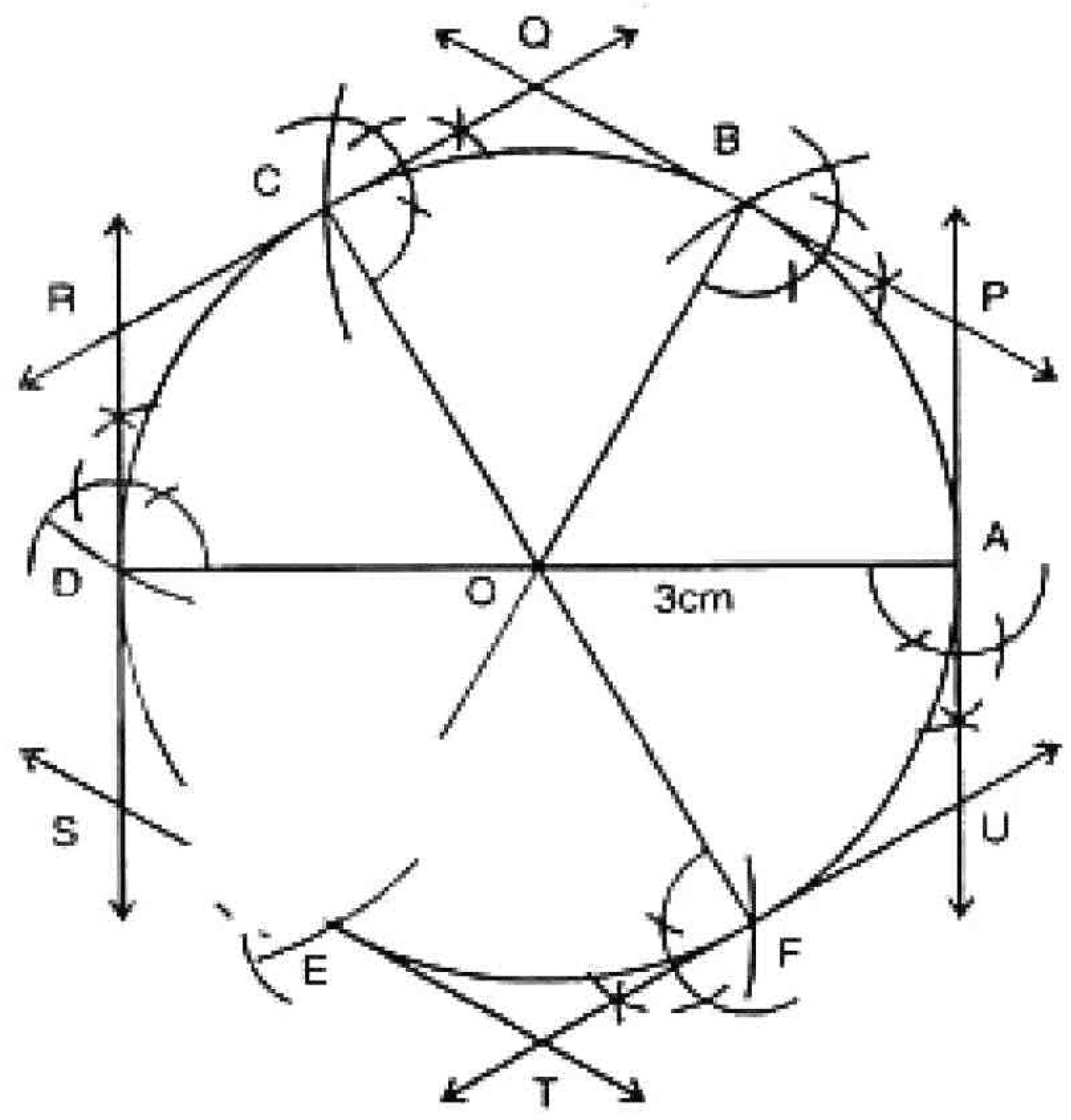
vi. We draw the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$.

vii. We draw tangents at points A, B, C, D, E and F to the circle.

viii. We produce the tangents to meet each other at points P, Q, R, S, T and U.

ix. PQRSTU is the required circumscribed hexagon.

Q. 9 Circumscribe a regular hexagon about a circle of radius 3 cm.



Steps Construction:

i. We draw a circle with centre 'O' of radius 3 cm.

ii. We take a point A anywhere on the circle and draw radial segment $\overline{OA}$.

iii. From point A, we draw an arc of radius $\overline{OA}$, which intersects the circle at point B.

iv. From point B, we draw an arc of same radius intersecting the circle at point C.

v. We continue to draw the arcs, which cut the circle at points D, E and F.

vi. We draw diameter $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$.

vii. We draw tangents at points A, B, C, D, E and F to the circle.

viii. We produce the tangents to meet each other at points P, Q, R, S, T and U.

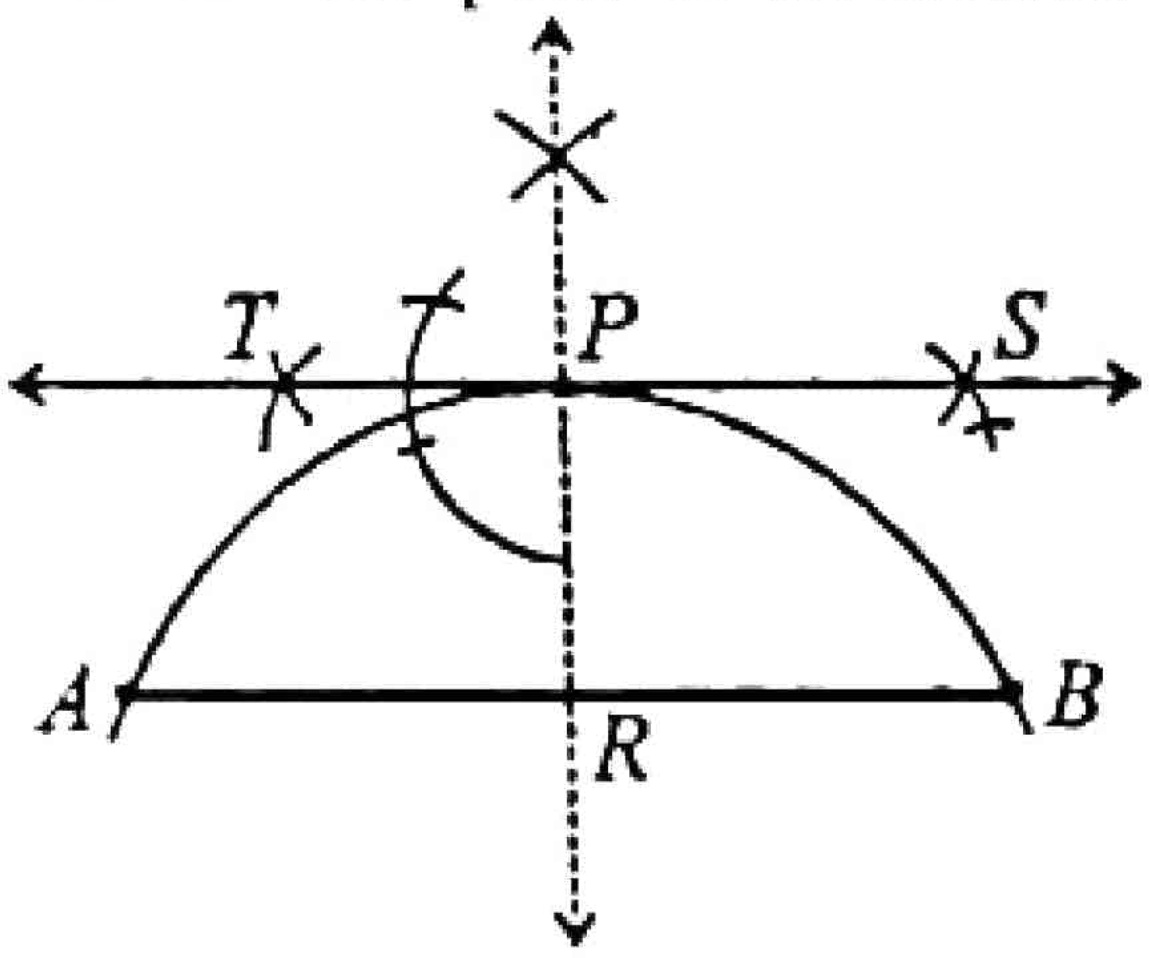
ix. PQRSTU is the required circumscribed hexagon.

TANGENT TO THE CIRCLE

(1) Draw a tangent to a given arc without using the centre through a given point *P*:

Case (i) When *P* is the middle point of the arc.

Given: *P* is the mid-point of an arc *AB*.



Steps of Construction:

i. Join A and B, to form chord $\overline{AB}$.

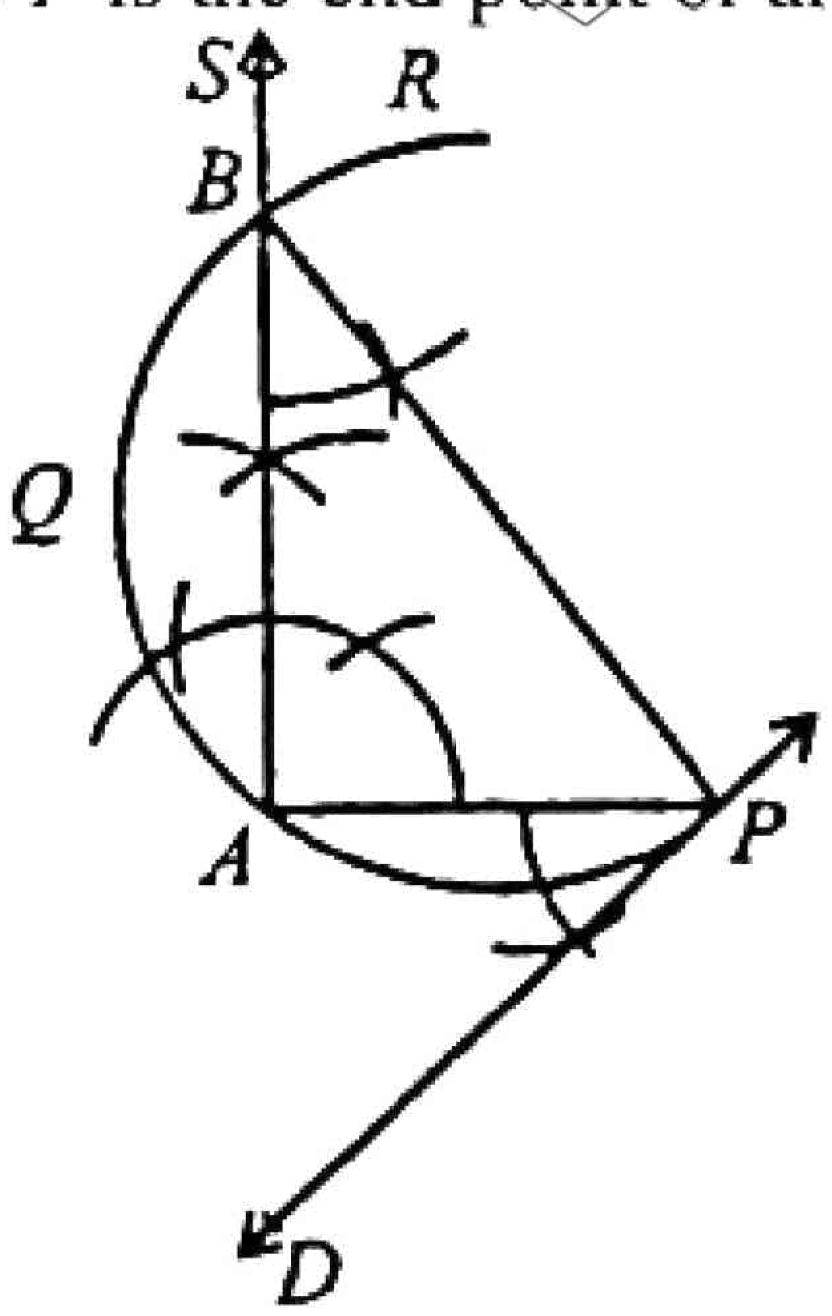
ii. Draw the perpendicular bisector of chord $\overline{AB}$ which passes through mid point *P* of arc AB and midpoint *R* of $\overline{AB}$.

iii. At point *P* construct a right angle TPR.

iv. Produce $\overrightarrow{PT}$ in the direction of *P* beyond point *S*. Thus $\overleftrightarrow{TP}$ is the required tangent to the arc AB at point *P*.

Case (ii) When *P* is at end point of the arc

Given: *P* is the end point of arc PQR.



Steps of Construction:

i. Take a point *A* on the arc PQR.

ii. Join the point *A* and *P*.

iii. Draw perpendicular $\overrightarrow{AS}$ at *A* which intersect the arc PQR at *B*.

iv. Join the points *B* and *P*.

v. Draw ∠APD of measure equal to that of ∠ABP.

vi. Now

$m\angle BPD = m\angle BPA + m\angle APD$

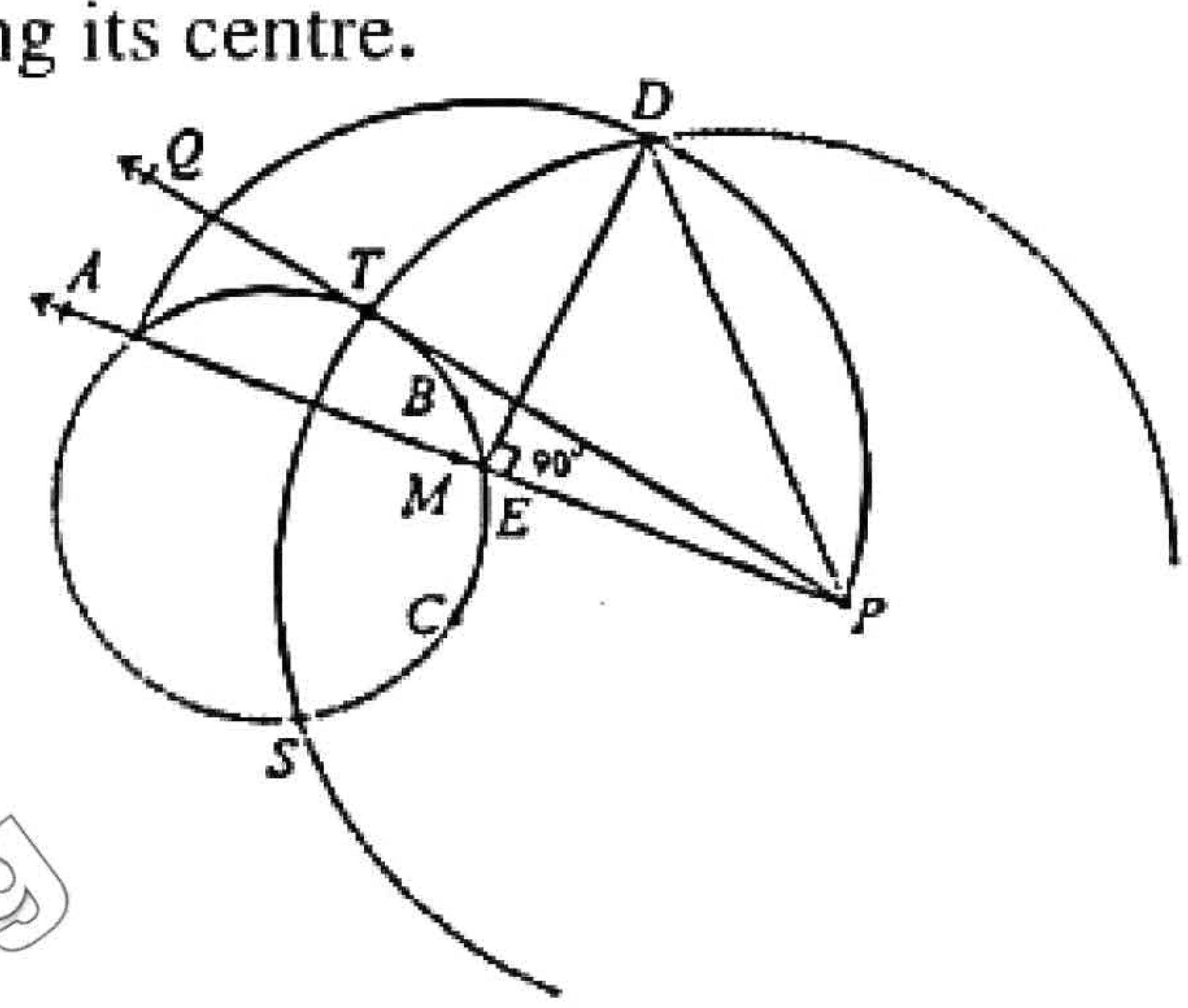
$= m\angle BPA + m\angle ABP$ [∵ $m\angle APD = m\angle ABP$]

$= 90^\circ$

vii. $\overleftrightarrow{PD}$ is the required tangent.

Case (iii) When point P is outside the arc.

Given: Point P is outside the arc ABC without knowing its centre.



Steps of Construction:

i. Join A to P. $\overline{AP}$ cuts the arc at E.

ii. Find mid-point M of $\overline{AP}$.

iii. Draw a semi circle of radius $|AM| = |MP|$ with center at M.

iv. Draw perpendicular at point *E* which meets the semi circle at *D*.

v. Draw an arc of radius $\overline{PD}$ with P as its center.

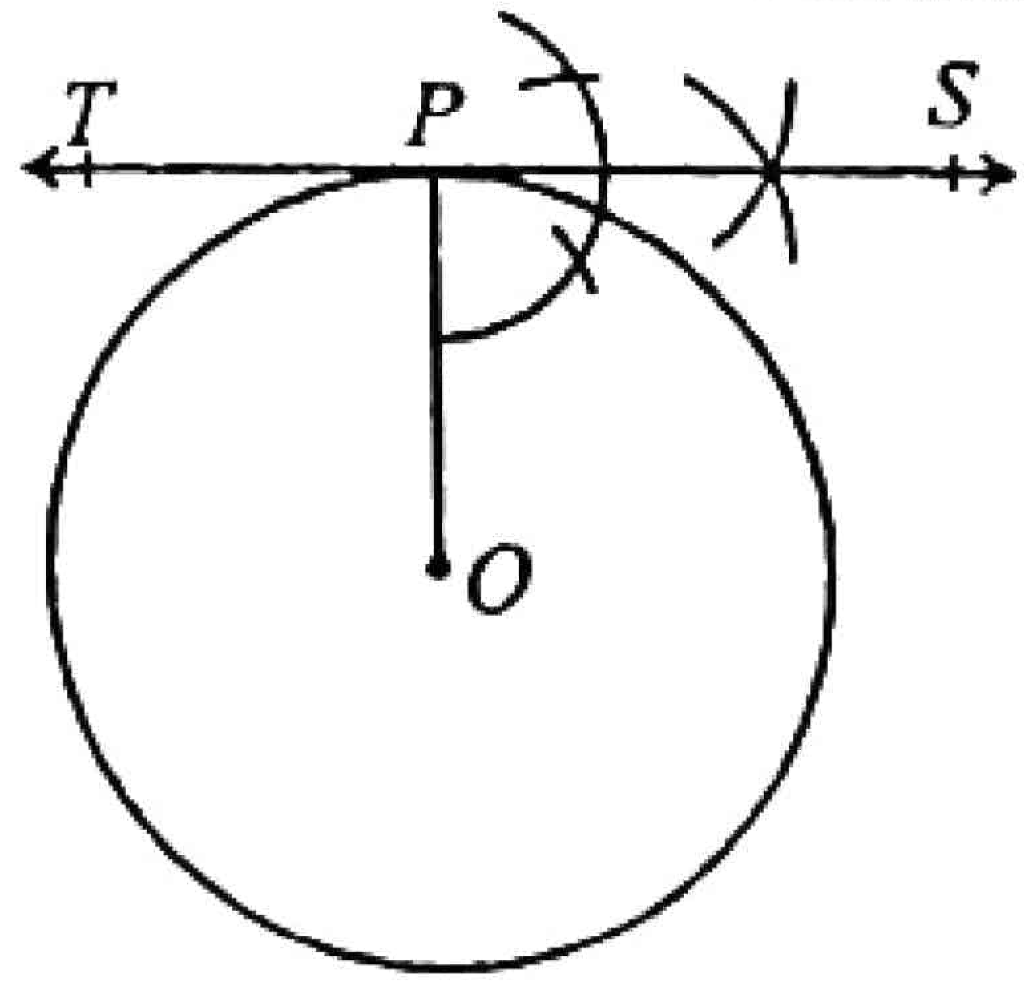
vi. This arc cuts the given arc ABC at point *T*.

vii. Join *P* with *T*.

viii. $\overrightarrow{PTQ}$ is the required tangent.

(2) To draw a tangent to a circle at a given point P on the circumference:

Given: A circle with the centre *O* and some point P lies on the circumference.



Steps of Construction:

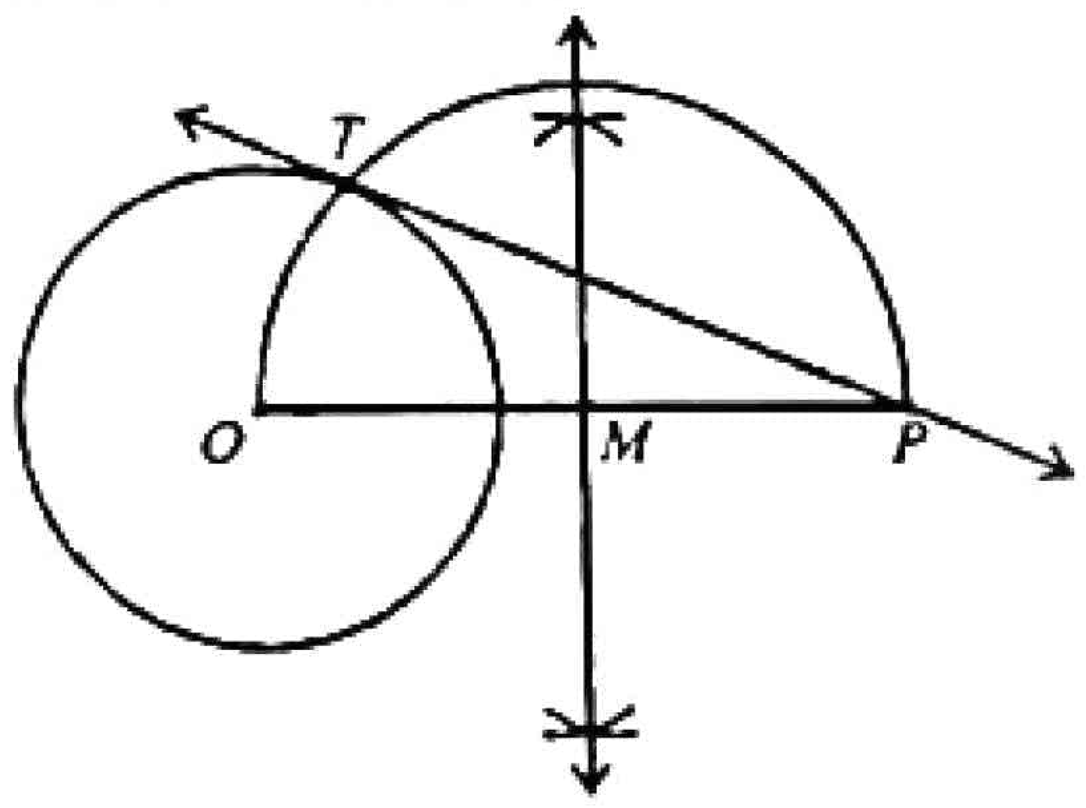
i. Join point P to the centre O, so that $\overline{OP}$ is the radius of the circle.

ii. Draw a line TPS which is perpendicular to the radius $\overline{OP}$.

$\therefore$ $\overleftrightarrow{TPS}$ is the required tangent to the circle at given point P.

(3) To draw a tangent to a circle from a given point P which lies outside the circle.

Given: A circle with centre O and some point P outside the circle.



Steps of Construction:

i. Join point P to the centre O.

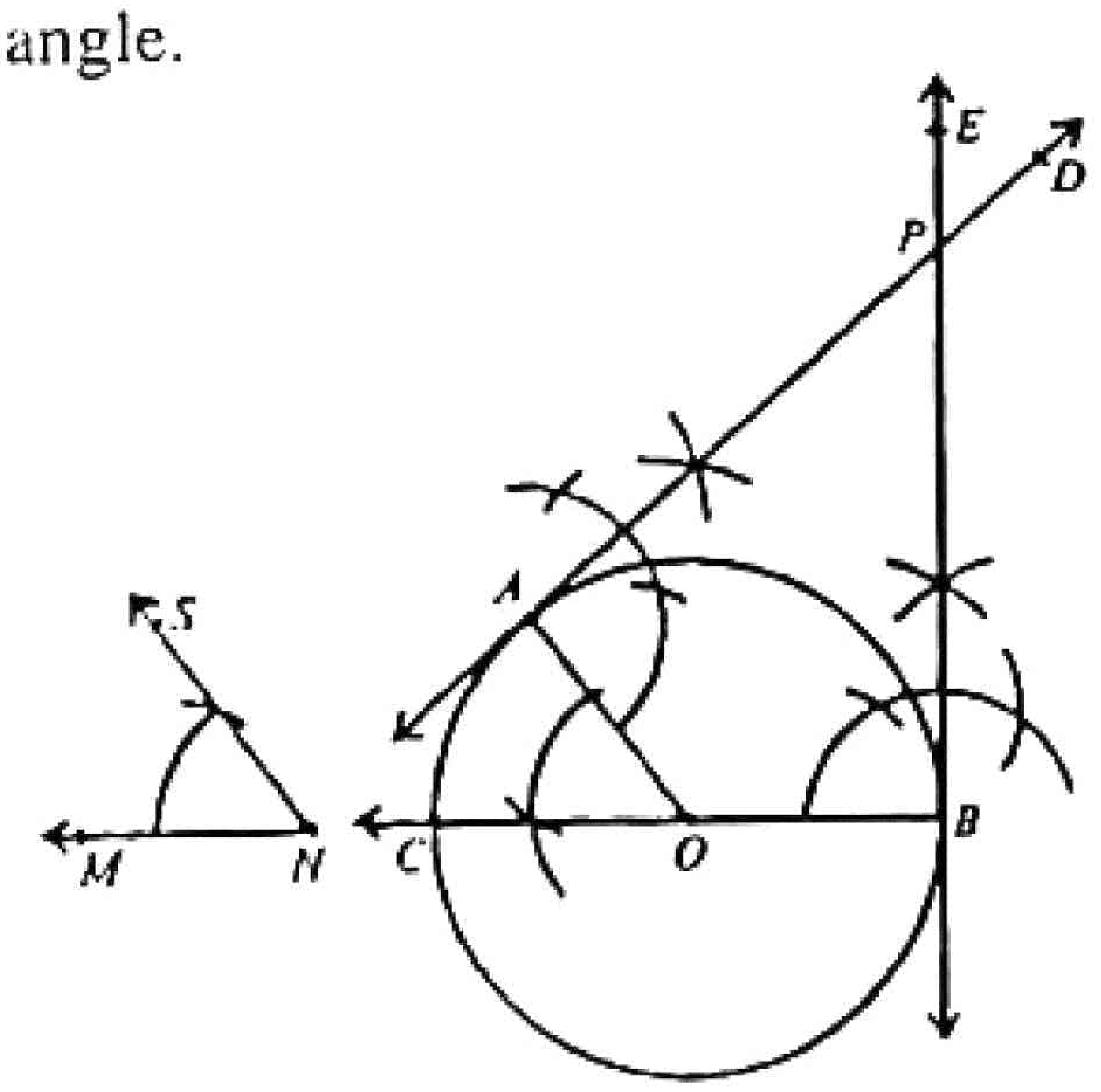
ii. Find M, the midpoint of a $\overline{OP}$ by drawings its right bisector.

iii. Construct a semi circle of radius $\overline{OM}$, with M as its centre. This semi circle cuts the given circle at T.

iv. Join P with T and produce $\overline{PT}$ on both sides, then $\overleftrightarrow{PT}$ is the required tangent.

(4) To draw two tangents to a circle meeting each other at a given angle:

Given: A circle with centre O, $\angle MNS$ is a given angle.



Steps of Construction:

i. Take a point A on the circumference of circle having centre O.

ii. Join the points O and A.

iii. Draw $\angle COA$ of measure equal to that of $\angle MNS$.

iv. Produce $\overline{CO}$ to meet the circle at B.

v. $m\angle AOB = 180^0 - m\angle COA$.

vi. Draw $\overleftrightarrow{AD}$ perpendicular to $\overline{OA}$.

vii. Draw $\overleftrightarrow{BE}$ perpendicular to $\overline{OB}$.

viii. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BE}$ intersect at P.

ix. $m\angle AOB + m\angle APB = 180^0$, that is, $m\angle AOB = 180^0 - m\angle APB$.

x. From step (v) and step (ix), we have.

$$180^0 - m\angle COA = 180^0 - m\angle APB$$

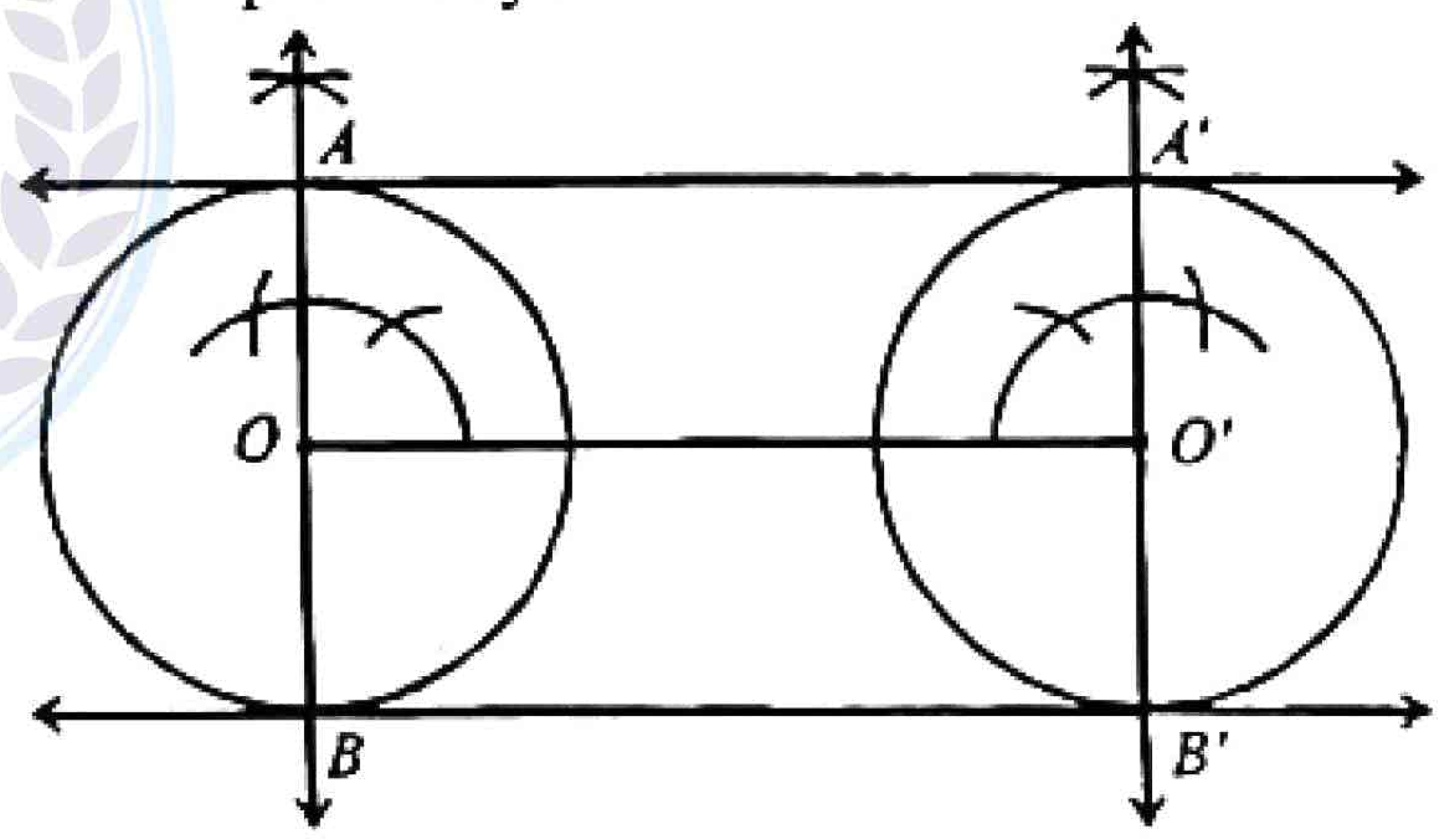
$$\Rightarrow m\angle COA = m\angle APB$$

$$\Rightarrow m\angle APB = m\angle MNS \quad (\because m\angle COA = m\angle MNS)$$

xi. $\overleftrightarrow{AP}$ and $\overleftrightarrow{BP}$ are the required tangents meeting at the given $\angle MNS$.

(5) To draw direct (or external) common tangents to equal circlers.

Given: Two circles of equal radii with centres O and O′ respectively.



Steps of Construction:

i. Join the centre O and O′.

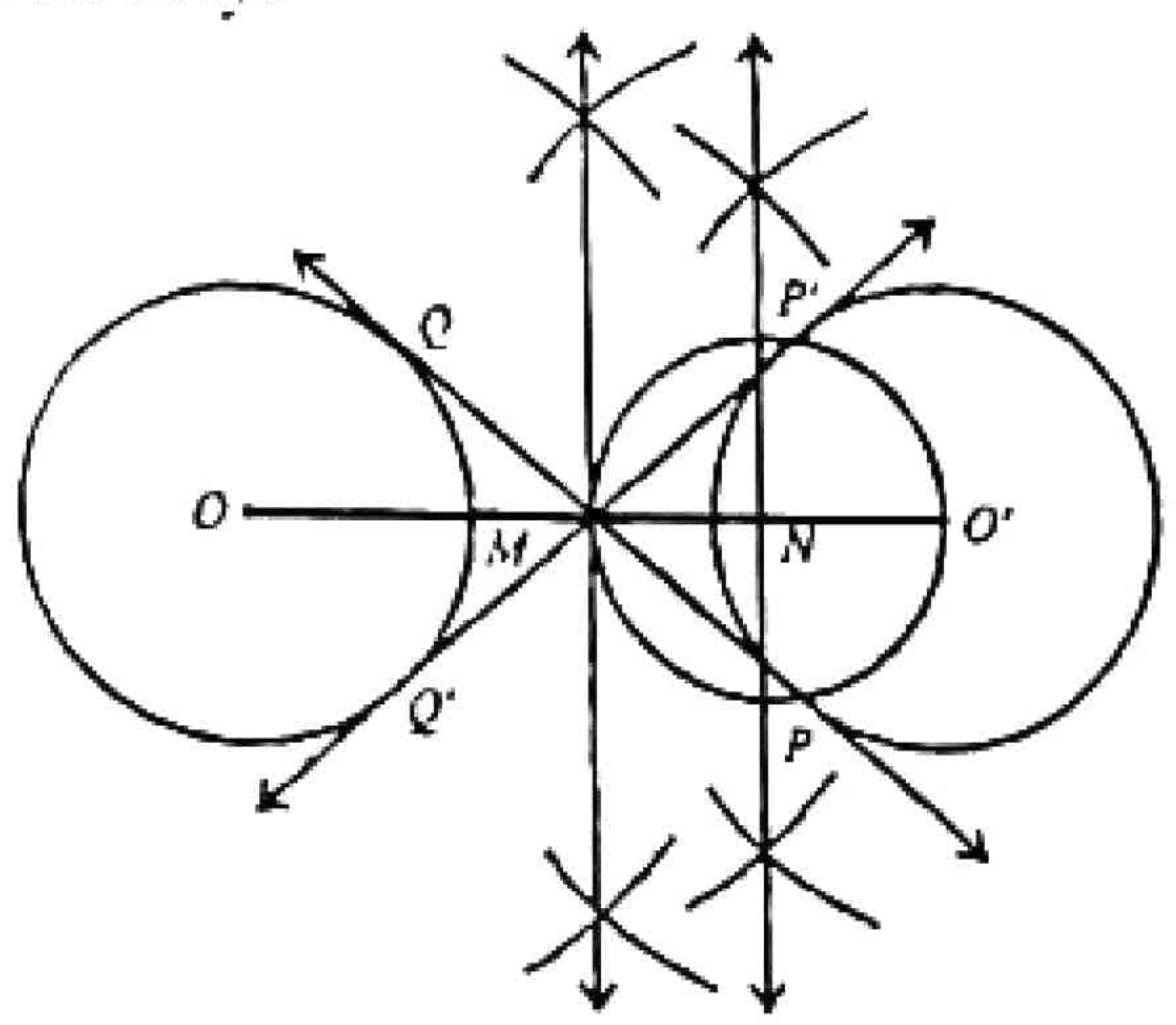
ii. Draw diameter AOB of the first circle so that $\overline{AOB} \perp \overline{OO'}$.

iii. Draw diameter A′O′B′ of the second circle so that $\overline{A'O'B'} \perp \overline{OO'}$.

iv. Draw $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$ which are the required common tangents.

(6) To draw transverse or (internal) common tangents to two equal circles:

Given: Two equal circles with centres O and O′ respectively.



Steps of Construction:

i. Join the centre O and O′.

ii. Find mid-point M of $\overline{OO'}$.

iii. Find mid-point N of $\overline{MO'}$.

iv. Taking point N as centre and radius equal to m $\overline{MN}$, draw a circle intersecting the circle with centre O′ at point P and P′.

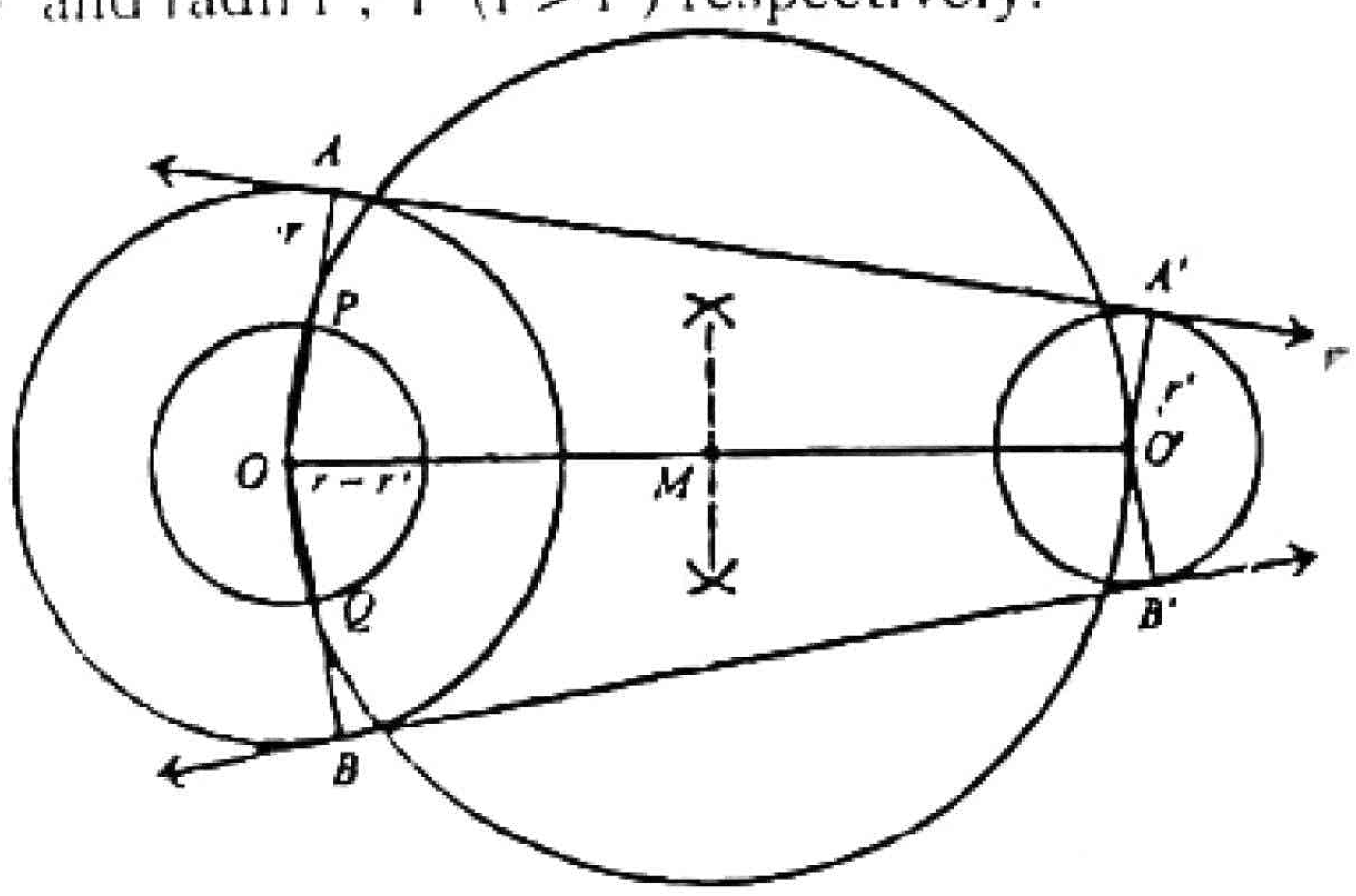
v. Draw a line through the points M and P touching the second circle at the point Q.

vi. Draw a line through the points M and P′ touching the second circle at the point Q′.

Thus $\overleftrightarrow{PQ}$ and $\overleftrightarrow{P'Q'}$ are the required transverse common tangents to the given circles.

(7) To draw direct (or external) common tangents to (two) unequal circles:

Given: Two unequal circles with centres O, O′ and radii r, r′ (r > r′) respectively.



Steps of Construction:

i. Join the points O and O′.

ii. On diameter $\overline{OO'}$, construct a new circle with centre M, the mid-point of $\overline{OO'}$.

iii. Draw another circle with centre at O and radius = r − r′, cutting the circle with diameter $\overline{OO'}$ at P and Q.

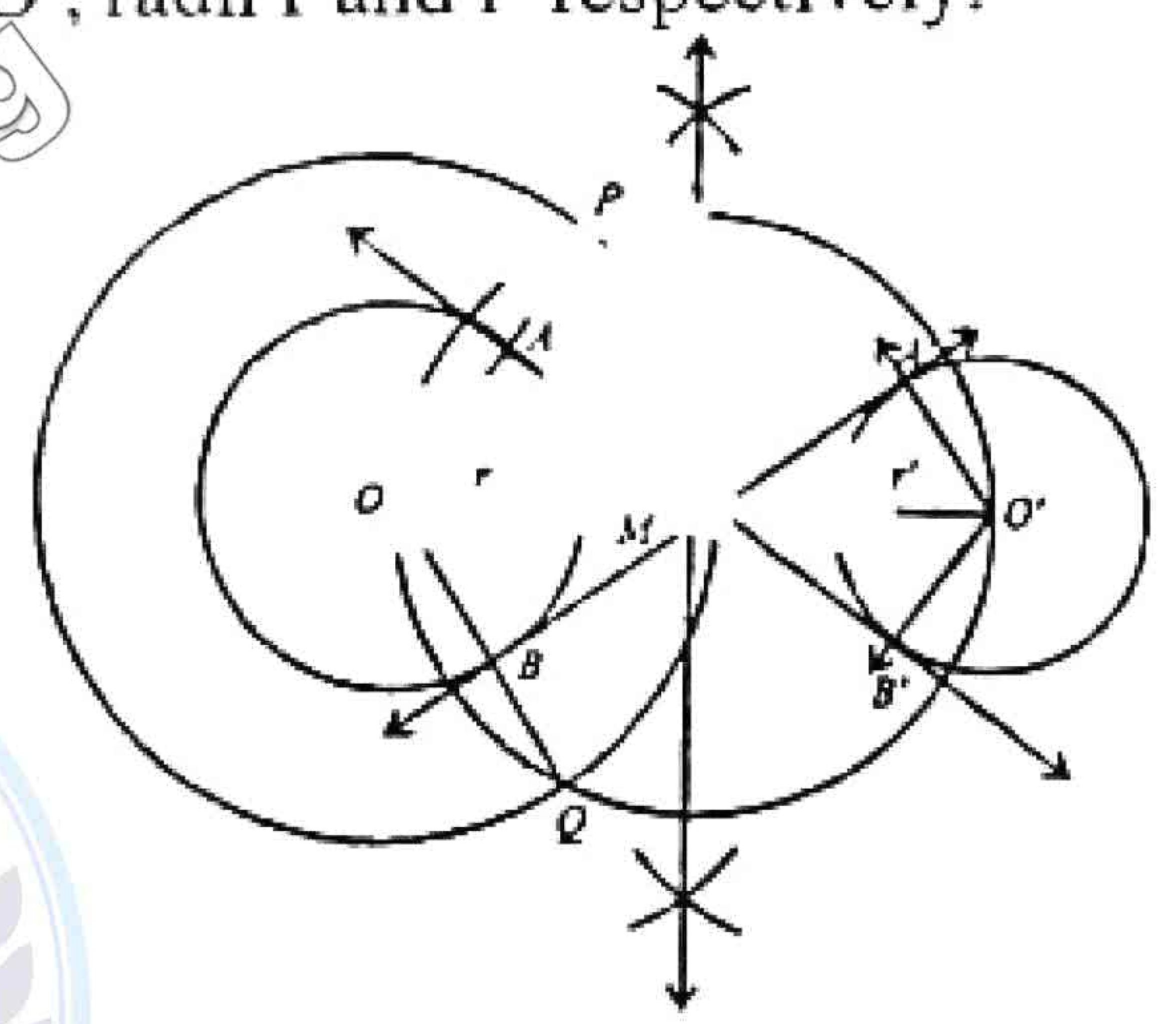
iv. Produce $\overline{OP}$ and $\overline{OQ}$ to meet the first circle at A and B respectively.

v. Draw $\overrightarrow{O'A'} \parallel \overline{OA}$ and $\overrightarrow{O'B'} \parallel \overline{OB}$.

vi. Join AA′ and BB′ which are the required direct common tangents.

(8) To draw transverse or internal common tangents to two unequal circles:

Given: Two unequal circle with centres O and O′, radii r and r′ respectively.



Steps of Construction:

i. Join the centres O and O′ of the given circles.

ii. Find the midpoint M of $\overline{OO'}$.

iii. On diameter $\overline{OO'}$, construct a new circle of radius $\overline{OM}$ with centre M.

iv. Draw another circle with centre at O and radius = r + r′ intersecting the circle of diameter $\overline{OO'}$ at P and Q.

v. Join O with P and Q. $\overline{OP}$ and $\overline{OQ}$ meet the circle with radius r at A and at B respectively.

vi. Draw $\overrightarrow{O'B'} \parallel \overline{OA}$ and $\overrightarrow{O'A'} \parallel \overline{OB}$.

vii. Join A with B′ and A′ with B. Thus $\overleftrightarrow{AB'}$ and $\overleftrightarrow{A'B}$ are the required transverse common tangents.

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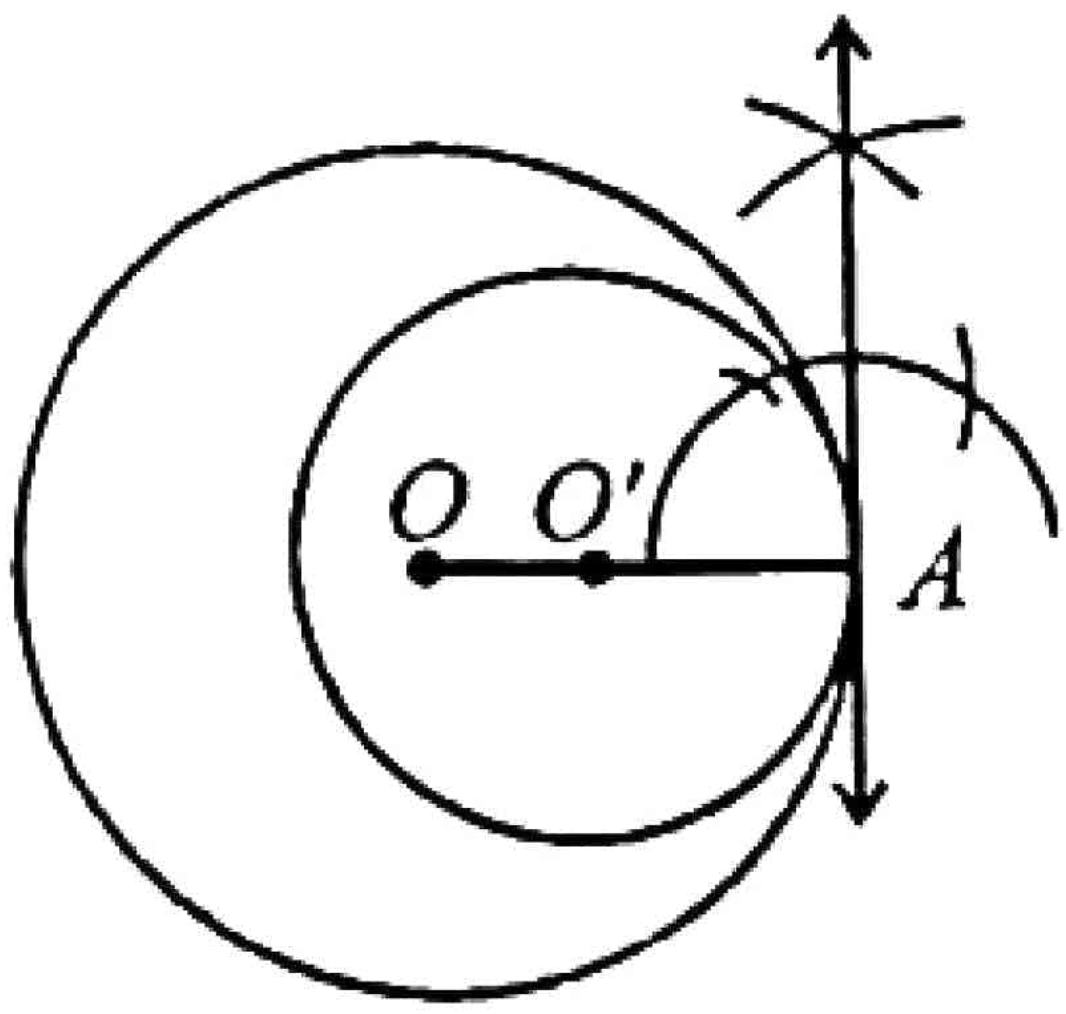
(9) To draw a tangent to two unequal touching circles:

Case I:

Given: Two unequal touching circles with centres O and O′.



Steps of Construction:

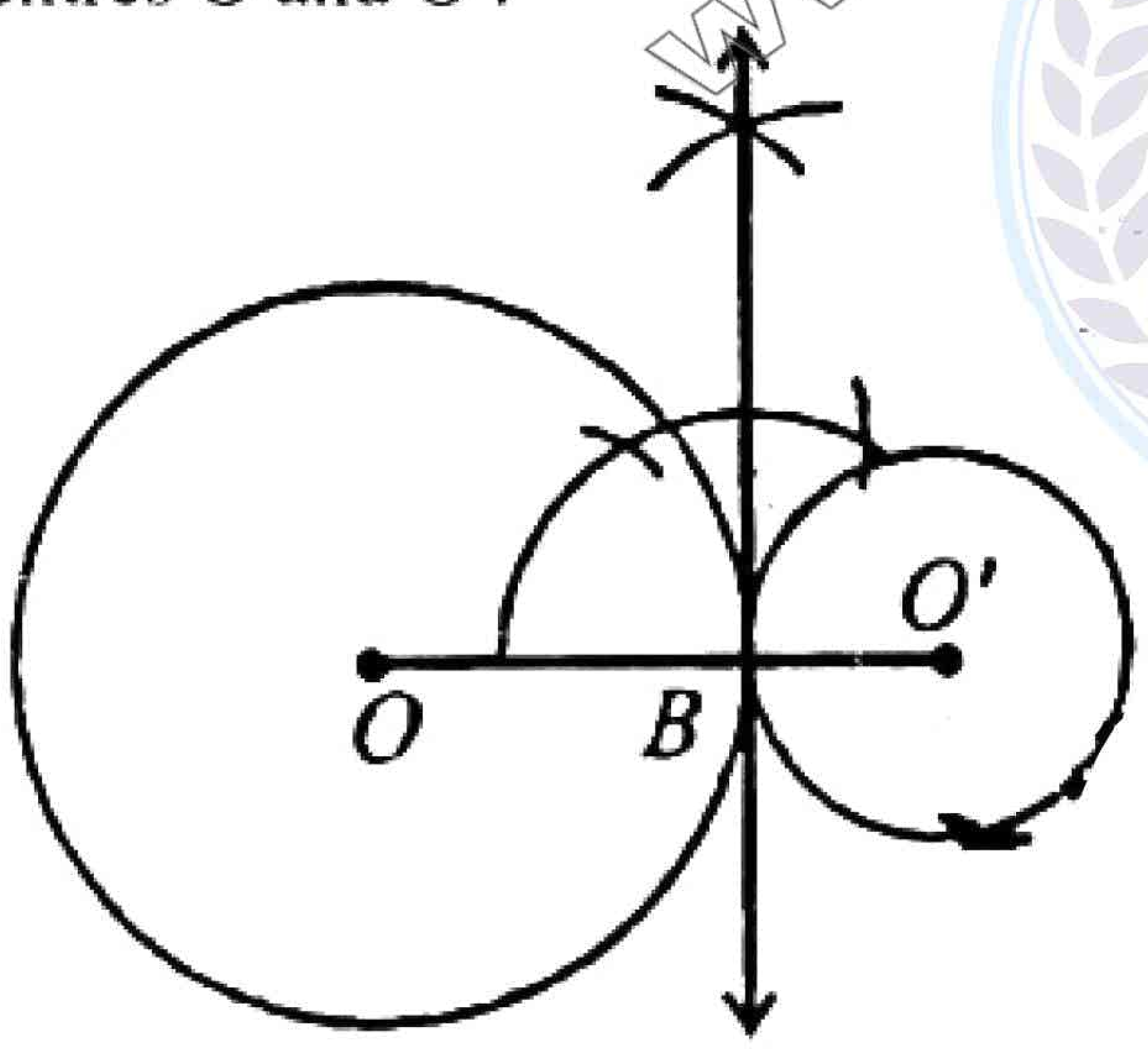
i. Join O with O′ and produce $\overline{OO'}$ to meet the circles at the point A where these circles touch each other.

ii. Tangent is perpendicular to the line segment $\overline{OA}$.

iii. Draw perpendicular to $\overline{OA}$ at the point A which is the required tangent.

Case II :

Given: Two unequal touching circles with centres O and O′.



Steps of Construction:

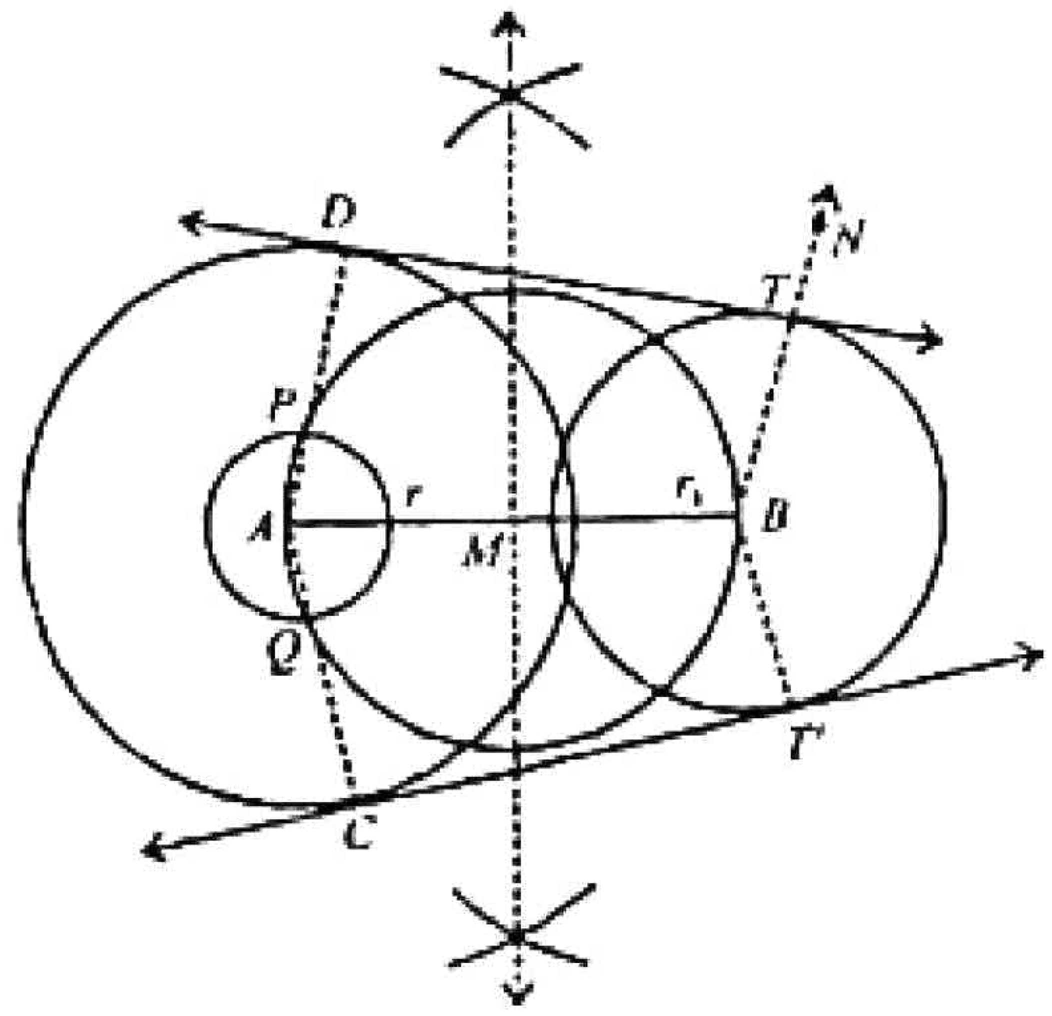
i. Join O with O′. $\overline{OO'}$ intersects the circles at the point B where these circles touch each other.

ii. Tangent is perpendicular to line segment containing the centre of the circles.

iii. Draw perpendicular to $\overline{OO'}$ at the point B which is the required tangent.

(10) To draw a tangent to two unequal intersecting circles:

Given: Two intersecting circles with centres A and B.



Steps of Construction:

i. Take a line segment $\overline{AB}$.

ii. Draw two circles of radii r and r_1 (where $r > r_1$) with centre at A and B respectively.

iii. Taking centre at A, draw a circle of radius $r - r_1$.

iv. Bisect the line segment AB at point M.

v. Taking centre at M and radius = m$\overline{AM}$ = m$\overline{BM}$, draw a circle intersecting the circle of radius $r - r_1$ at P and Q.

vi. Join the point A with P and produce it to meet the circle with centre A at D. Also join A with Q and produce it to meet the circle with centre A at C.

vii. Draw $\overrightarrow{BN}$ parallel to $\overline{AD}$, which intersects the circle with centre B at T.

viii. Draw a line joining the points D and T. $\overleftrightarrow{DT}$ is a common tangent to the given two circles.

ix. Repeat the same process on the other side of $\overline{AB}$. $\overleftrightarrow{CT'}$ is also a common tangent to the given two circles.

(11) To Draw a circle which touches both the arms of a given angle:

Given: An angle ∠BAC.