

Chapter # 9:**Binomial & Hypergeometric Distribution****1. Define Bernoulli trials.**

Ans: a Bernoulli trial two possible outcomes i.e., only success and failure is called Bernoulli trials. For each Bernoulli trial, the probability of success remains the same and the successive trials are independent.

2. Define random experiment.

Ans: An experiment in which outcomes vary from trial to trial is called random an experiment.

3. Define Binomial Probability Distribution.

Ans: if "P" is the probability of success in a single trial and "q" is the probability of failure, then the probability of exactly x successes in "n" trials of a binomial experiment is given by:

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

$$x = 0, 1, 2, 3, 4, \dots, n$$

4. Define binomial experiment.

Ans: An experiment in which the outcomes can be classified as success or failure and in which the probability of success remains constant from trial to trial is called Binomial experiment.

5. What are the properties of binomial distribution?

Ans: A binomial experiment possesses the following properties:

- Each trial of experiment results in an outcome which can be classified into two categories i.e., success and failure.
- The probability of success remains constant from one trial of the experiment to the next.
- The repeated trials are independent.
- The experiment is repeated a fixed number of times.

6. A random variable X has a binomial distribution with $n = 5$ and $p = 0.2$, find $P(X = 2)$.

Ans: By pdf of binomial distribution:

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

$$P(X = 2) = 0.0391 \quad \text{Ans.}$$

7. Write down the formulas of mean, variance and standard deviation of binomial distribution.

Ans:

$$\text{Mean} = np, \quad \text{Variance} = npq, \quad \text{Standard Deviation} = \sqrt{npq}$$

8. A random variable 'X' is binomially distributed when $n = 15$ and $p = 0.4$. Find mean and variance of 'X'.

Ans: By using formulas of mean and variance of binomial distribution:

$$\text{Mean} = np$$

$$\text{Mean} = 6$$

$$\text{Variance} = npq$$

$$q = 1 - p = 0.6$$

$$\text{Variance} = 3.6$$

9. In binomial distribution, mean = 6 and Variance = 2.4, find parameters of binomial distribution.

Ans: As we know that;

$$\text{Mean} = np \quad \text{and} \quad \text{Variance} = npq$$

$$6 = np \text{ ----- } 1$$

$$2.4 = 6q$$

$$q = 0.4 \quad \text{Then}$$

$$p = 0.6 \text{ (Put into 1)}$$

$$n = 10$$

Hence, 'n' and 'p' are the parameters of the binomial distribution.

10. Find the number of trails of a binomial distribution which has mean = 12 and S.D = 2

Ans: As we know that;

$$\text{Mean} = np \quad \text{and} \quad \text{Variance} = npq$$

$$12 = np \text{ (By using mean)----- } 1$$

$$4 = 12q \text{ (By using variance)}$$

$$q = 0.333 \quad \text{Then}$$

$$p = 0.666 \text{ (Put into 1)}$$

$$n = 18$$

11. A coin is tossed 5 times. What is the probability of getting exactly 3 heads?

Ans: By using binomial probability distribution function;

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

Probability of head is our success; then,

$$p = 0.5, \quad q = 0.5 \quad \text{and} \quad n = 5$$

$$P(X = 3) = 0.3125 \quad \text{Ans.}$$

12. What is Binomial frequency distribution.

Ans: If the binomial probability distribution is multiplied by the number of experiments 'N' then the distribution is called binomial frequency distribution.

$$P(X = x) = N \cdot \binom{n}{x} p^x q^{n-x}$$

13. Define hyper geometric experiment.

Ans: An experiment in which a random sample is selected without replacement from a finite population is called hyper geometric experiment.

14. What are the properties of hyper geometric distribution?

Ans: A hyper geometric experiment has the following properties:

- Each trial of an experiment results in an outcome that can be classified into one of the two categories: success or failure.
- The probability of success changes from one trial of the experiment to the next.
- The repeated trials are dependent.
- The experiment is repeated a fixed number of times.



15. What is the Hypergeometric probability function?

Ans: A population of 'N' items consists 'K' items of one kind (success) and N-K items are another kind (failure) and we are interested in the probability of getting 'x' success among 'n' items selected at random from the population.

$$P(X = x) = \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}}, \quad x = 0, 1, 2, 3, 4, \dots, n.$$

16. What are the parameters of the hyper geometric distribution?

Ans: n, k, and N.

17. What is mean and variance of hyper geometric distribution with parameters N, n, K?

Ans:

$$\text{Mean} = \frac{nk}{N}, \quad \text{Variance} = \frac{nk}{N} \left(1 - \frac{k}{N}\right) \left(\frac{N-n}{N-1}\right), \quad \text{Standard Deviation} = \sqrt{\frac{nk}{N} \left(1 - \frac{k}{N}\right) \left(\frac{N-n}{N-1}\right)}$$

18. Given N = 10, n = 4 and K = 5, find E(X).

Ans: As we know that expected value of x is known as the mean;

Then,

$$E(X) = \text{Mean} = \frac{nk}{N}$$

$$E(X) = 2 \quad \text{Ans.}$$

19. In a hyper geometric distribution N = 10, n = 2 and k = 3, find P(X=0)

Ans: By using Probability density function of Hypergeometric distribution:

$$P(X = x) = \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}}$$

$$P(X = 0) = 0.467 \quad \text{Ans.}$$

20. If N = 11, n = 5, k = 7, find variance of the hyper geometric distribution?

Ans: The variance of hypergeometric distribution is :

$$\text{Variance} = \frac{nk}{N} \left(1 - \frac{k}{N}\right) \left(\frac{N-n}{N-1}\right)$$

$$\text{Variance} = 0.6942 \quad \text{Ans.}$$

21. If 'x' is a binomial random variable with n = 9 and p = 1/3, then find S.D.(3+2x)?

Ans: As we know that:

$$\text{Standard Deviation} = \sqrt{npq}$$

$$\text{Standard Deviation} = \text{S. D. (x)} = 1.4142$$

$$\text{S. D. (3 + 2x)} = 2\text{S. D. (x)}$$

$$\text{S. D. (3 + 2x)} = 2.8284$$

22. In a binomial distribution mean = 36 and q = 0.83, find 'n' and 'p'?

Ans: As we know that;

$$\text{Mean} = np$$

and

$$\text{Variance} = npq$$

$$p = 1 - q$$

$$p = 0.17$$

$$36 = n(0.17) \quad (\text{By using mean}) \text{-----} 1$$

$$n = 212$$

