07 Random Variables
08 Probability Distributions

Chapter – 7, 8

1. What is a random variable?

Ans: A variable whose values are determined by the outcomes of a random experiment is called random variable.

2. Define continuous and discrete random variable.

Ans:

Continuous Random Variable:

A random variable which takes on only a infinite number of values on a continuous scale in a given interval is called continuous variable. For example, the distance covered by a car between two points.

Discrete Random Variable: A random variable which takes on only a infinite number of values or a sequence of whole numbers is called a discrete random variable. For example, Number of people in a room, number of schools in a city Lahore.

3. What is distribution function of a discrete random variable X?

Ans: The distribution function of a X or the cumulative distribution function of X, denoted by F(x), is the probability that X will assume a value less than or equal to x, i.e.

$$F(x) = P(X \leq x)$$

4. Define probability density function.

Ans: The probability density function of a continuous random variable X is specified by a smooth curve such that the total area under the curve is one.

5. What are the properties of discrete probability distribution?

Ans: A discrete probability distribution has following two properties:

a) $0 \leq P(x_i) \leq 1$
b) $\sum P(x_i) = 1$

6. What is meant by mathematical expectation of a random variable?

Ans: If a discrete random variable *X* assumes the values 1, 2… *n* with respective probabilities *P*(1), *P*(2), … *P*(n) such that the sum of the probabilities is equal to 1, then the mathematical expectation or expected value of *X* denoted by *E(X)* is defined as, *E(X)* = $\Sigma x P(x)$. *E(X)* is also called mean of *X*, denoted by μ. Enlist two properties of expectation.

7. Enlist properties of expectations.

Ans: Properties of Expectation (Laws of Expectation)

1) $E(a) = a$
2) $E(aX + b) = aE(X) + b$
3) $E(XY) = E(X)E(Y)$
4) $E(X + Y) = E(X) + E(Y) \quad OR \quad E(X - Y) = E(X) - E(Y)$
5) $E[X - E(X)] = 0 \quad OR \quad E(X - \mu) = 0$

8. Given *X* = 0, 1, 2 and *P(X)* = 9/16, 6/16, 1/16, find variance of *X*.

Ans:

x	P(x)	x.P(x)	x^2.P(x)
0	0.56	0	0
1	0.38	0.375	0.375
2	0.06	0.125	0.25
Total	**1.00**	**0.50**	**0.63**

$$Mean = \mu = E(X) = \sum xP(x)$$

$$\boldsymbol{\mu = E(X) = 0.50}$$

$$Variance = \sigma_x^2 = Var(X) = \sum x^2\, P(x) - [E(x)]^2$$

$\boldsymbol{\sigma_x^2 = Var(X) = 0.38}$ **Ans.**

9. **Given the probability distribution. Find K.**
   X = 0, 1, 2, 3, 4 and *P(X)* = 1/210, 20/210, *K*, 70/210, 10/210.

Ans:

X	P(X)
0	1/210
1	20/210
2	K
3	70/210
4	10/210

By using the following property:

$$\sum P(x_i) = 1$$

1/210 + 20/210 + K +70/210 + 10/210 = 1

K = 109/210 **Ans.**

10. **Given that *f(x)* = x/10, x = 1, 2, 3, 4. Show that *f(x)* is a probability function.**

Ans: By using the Property:

$$\sum P(x_i) = 1$$

Putting the values of x into f(x);

1/10 + 2/10 + 3/10 + 4/10 = 1

10/10 = 1

1 = 1 Hence, proved that f(x) is a probability function.

11. Given *f(x) = k/x²*, *x* = 1, 2, find *k*.

Ans: putting the values of x into f(x) and then put equals to one;

$k/(1)^2 + k/(2)^2 = 1$

k/1 + k/4 = 1

5k/4 = 1

k = 4/5 **Ans.**

12. Given *x* = 0, 2, 3 and *f(x)* = |1 – *X*|/4, find *E(X)*.

Ans: Putting the values of x into f(x), we get…

x	f(x)	x.f(x)
0	1/4	0
2	1/4	1/2
3	2/4	3/2
Total		**2**

$$E(X) = \sum x.f(x)$$

$E(X) = 2$ **Ans.**

13. Given *E(X)* = 0.63 and *Var(X)* = 0.2331 then find *E(X²)*.

Ans:

$$Var(X) = E(X^2) - (E(X))^2$$

$E(X^2) = 0.63$ **Ans.**

14. Given *X* = 1, 2, 3, 4, 5 and *P(X)* = 1/10, 3/10, *P*, 2/10, 1/10. Find the value of *P*.

Ans:

X	P(X)
1	1/10
2	3/10
3	P
4	2/10
5	1/10

By using the following property:

$$\sum P(x_i) = 1$$

1/10 + 3/10 + P +2/10 + 1/10 = 1

P = 3/10 **Ans.**

15. Find the probability distribution of the number of heads when two coins are tossed.

Ans: Probability distribution of number of heads:

Sample space = {HH, HT, TH, TT}

X	f	P(x)
0	1	1/4
1	2	2/4
2	1	¼

16. Define random experiment.

Ans: An experiment in which outcomes vary from trial to trial is called random experiment.

17. Enlist properties of probability mass function.

Ans: Properties of Probability mass function (PMF):

a) $0 \leq P(x_i) \leq 1$
b) $\sum P(x_i) = 1$

18. If E(X) = 1.4, then find E(5x – 4).

Ans: By using the law of expectation:

$$E(aX + b) = aE(X) + b$$

$$E(5x - 4) = 5E(x) - 4$$

$$E(5x - 4) = 3$$ **Ans.**

19. Given: E(x) = 0 and E(x^2) = 8/9. Find E($3x^2$ – 2x + 5).

Ans: By using the laws of expectation:

$$E(aX + b) = aE(X) + b$$

$$E(3x^2 - 2x + 5) = E(3x^2) - E(2x) + 5$$

$$E(3x^2 - 2x + 5) = 3E(x^2) - 2E(x) + 5$$

$$E(3x^2 - 2x + 5) = 7.667$$ **Ans.**

20. Given: E(x) = 0.56, var(x) = 1.36 and if y =2x + 1, then find E(y) and var(y).

Ans: By using of Laws of expectation:

$$Mean = E(y) = E(2x + 1)$$

$$E(y) = 2E(x) + 1$$

$$\boldsymbol{E(y) = 2.12}$$

$$Var(y) = Var(2x + 1)$$

$$Var(y) = Var(2x)$$

$$\boldsymbol{Var(y) = 5.44}$$