1st Year Physics

Chapter # 08

Waves

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Ch # 08
Waves

Waves:
A wave is a disturbance in the medium which causes the particles of the medium to undergo vibratory motion about their mean position in equal intervals of time.

Types:
There are two types of waves.

i- Mechanical waves: The waves which require medium for their propagation are called mechanical waves.

For example, sound waves, water waves, string waves etc.

ii- Electromagnetic waves:

The waves which do not require any medium for their propagation are called electromagnetic waves. These waves are produced due to electric and magnetic fields which are mutually perpendicular.

For example, X-rays, light waves, radio waves etc.

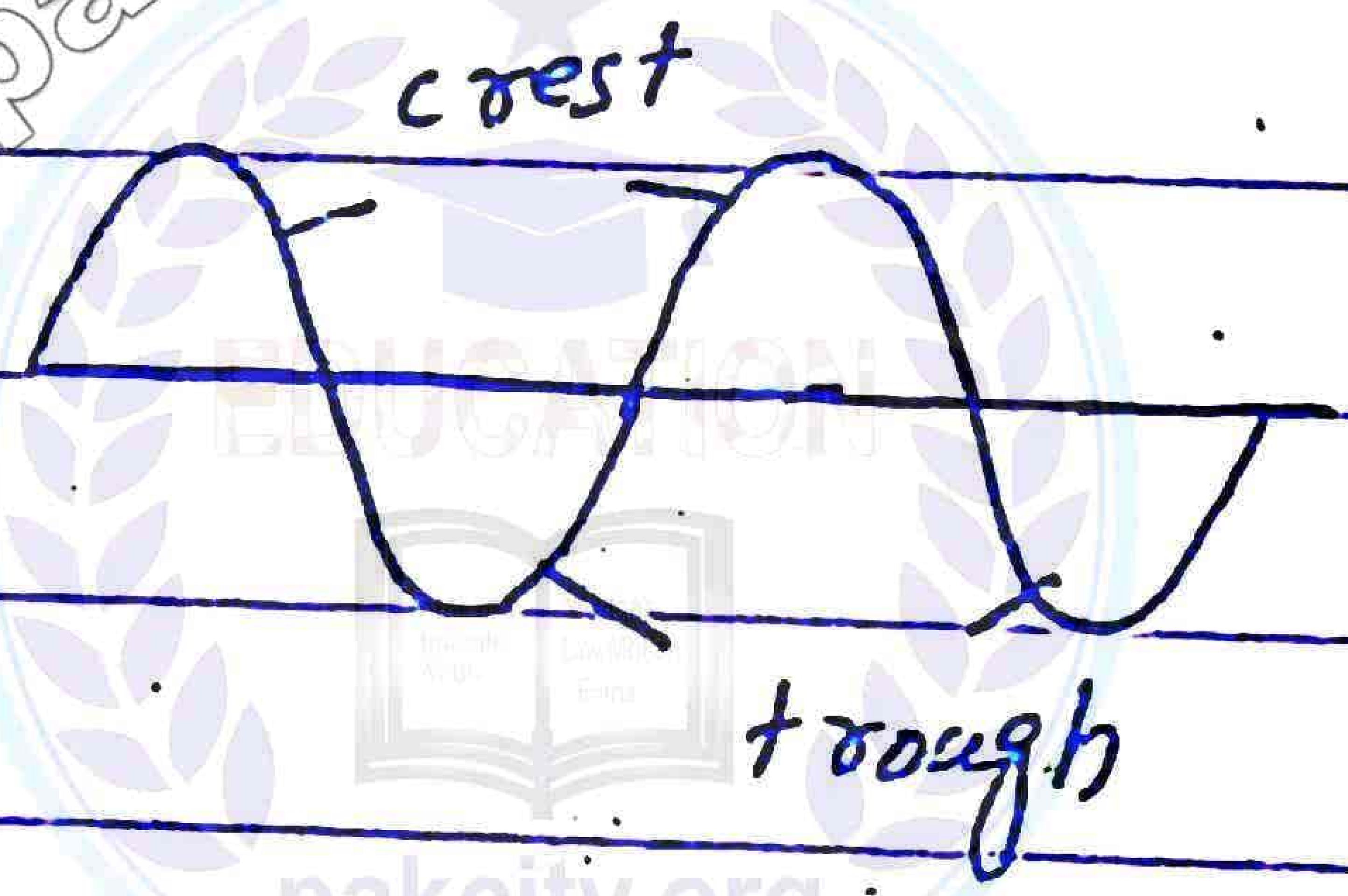
Progressive wave:

A wave, which transfers energy by moving away from the source of disturbance, is called progressive or travelling wave. There are two kinds of progressive waves transverse waves and longitudinal wave. **For example:** The ripples are the example of progressive waves because they carry energy across the water surface.

Transverse waves:

The waves in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves are called transverse waves.

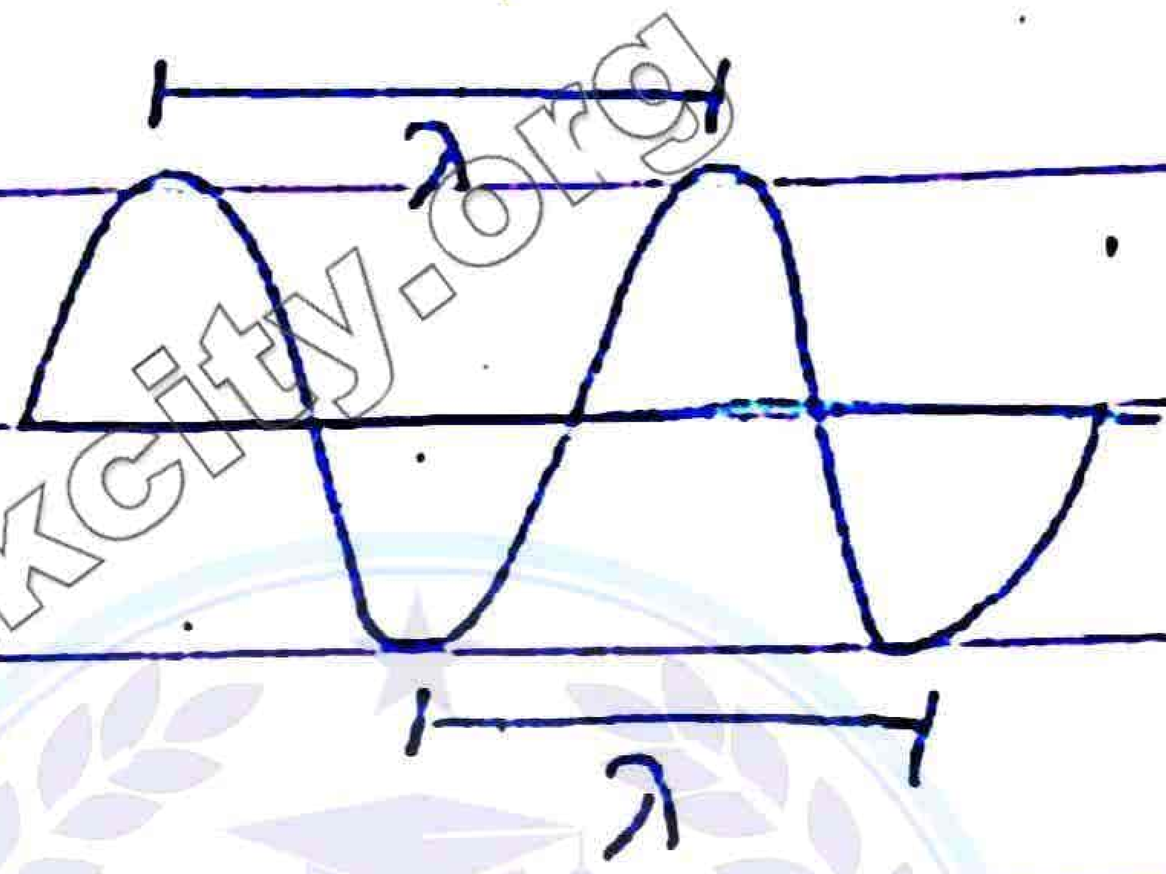
For example, water waves, light waves etc.



Crest: In the transverse wave, the portion above the mean position is called crest.

Trough: In the transverse wave, the portion below the mean position is called trough.

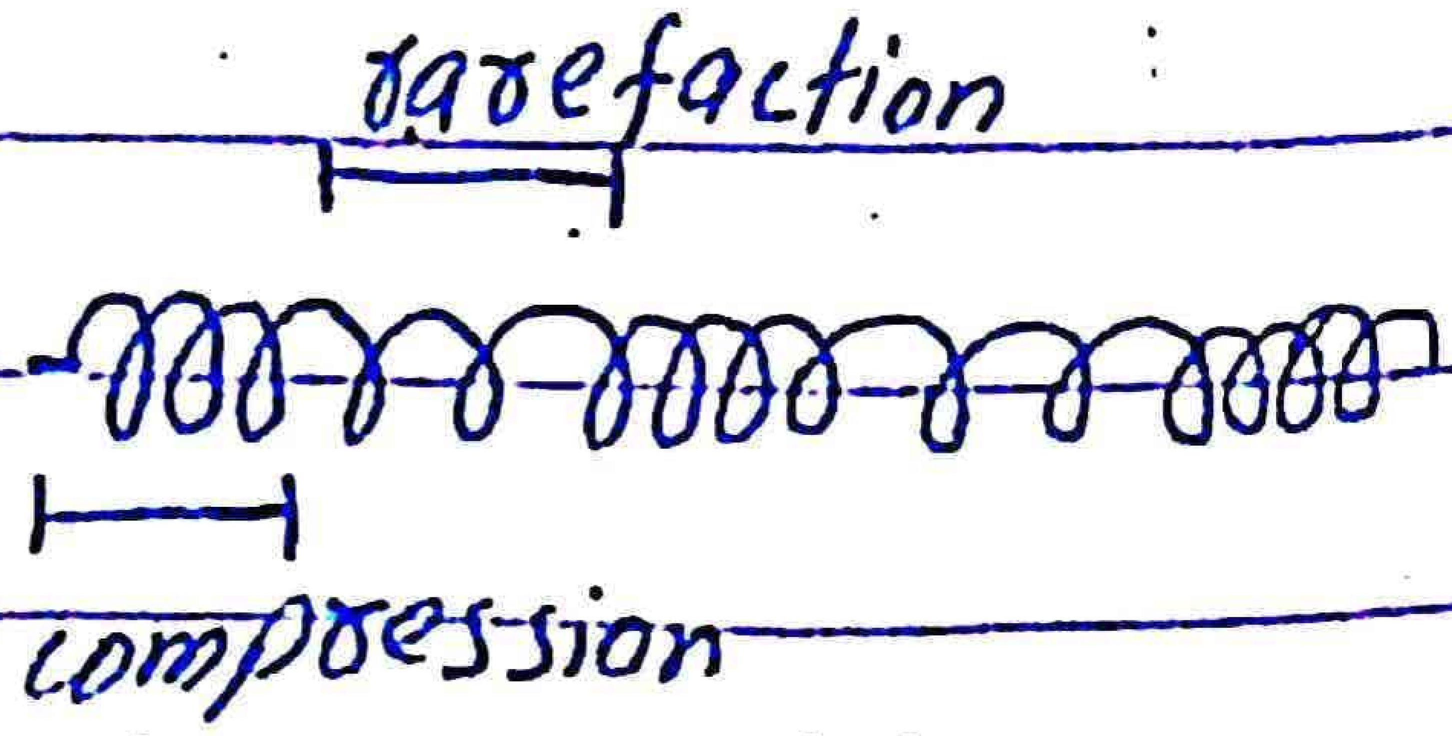
Wavelength: The distance between two consecutive crests or trough in transverse wave is called wavelength. It is represented by lemda (λ). Its SI unit is meter (m).



Longitudinal waves:

The waves in which particles of the medium have displacement along the direction of propagation of waves are called longitudinal waves.

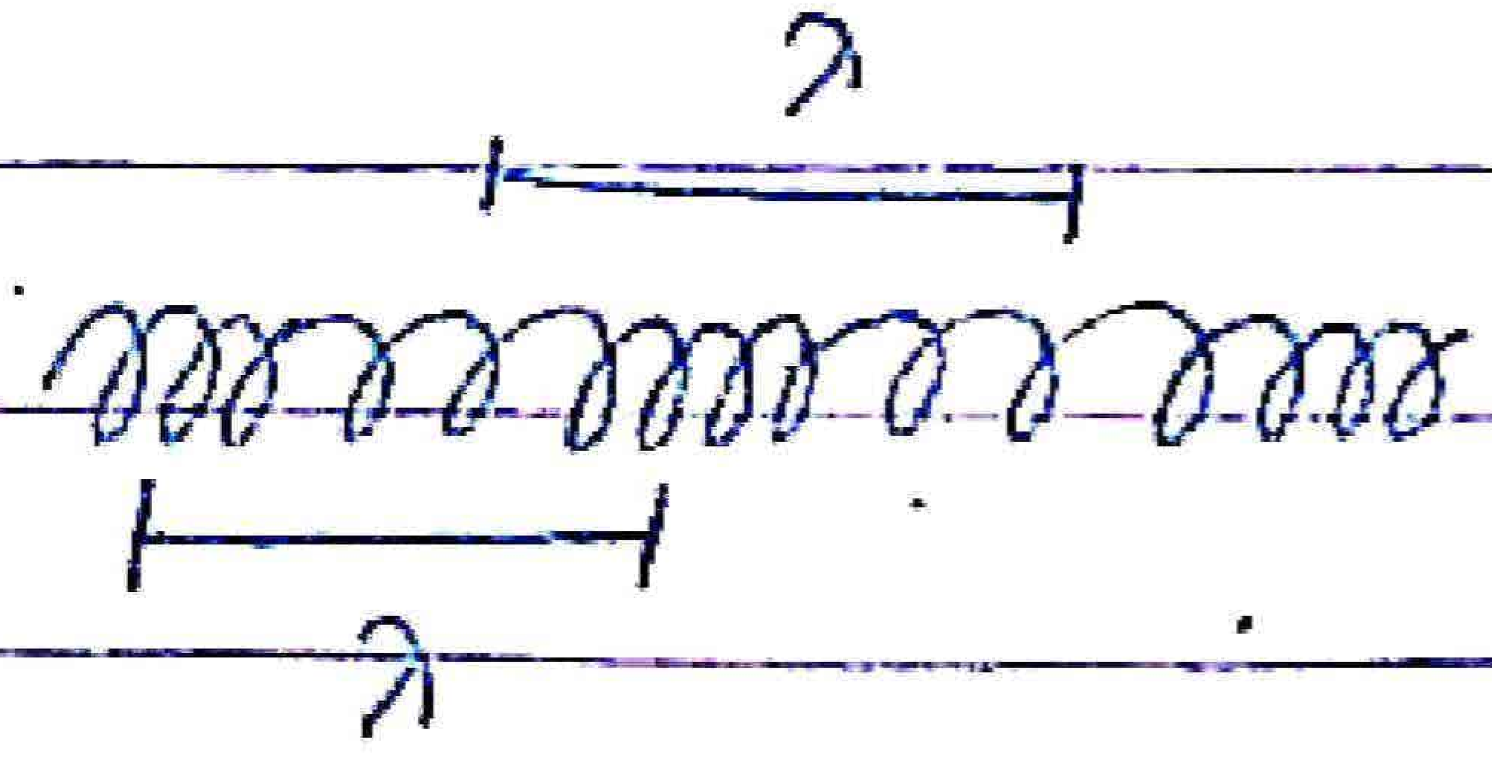
For example, sound waves, waves produced in spring etc



Compression: In the longitudinal wave, the portion where particles of the medium are close together is called compression.

Rarefaction: In the longitudinal wave, the portion where particles of the medium are far apart is called rarefaction.

wavelength: For longitudinal wave, the distance between two consecutive compression or rarefaction is called wavelength. It is represented by lemda (λ), its S.I unit is meter (m)



Periodic waves: Continuous regular and rhythmic disturbances in a medium result from periodic vibrations of a source cause periodic waves in that medium. A good example of a periodic vibrator is an oscillating mass-spring system.

Prove that $v = f\lambda$

we know that rate of change of distance is called speed.

$$speed = \frac{distance}{time}$$

For one wave.

$$speed = \frac{wavelength}{time\ period}$$

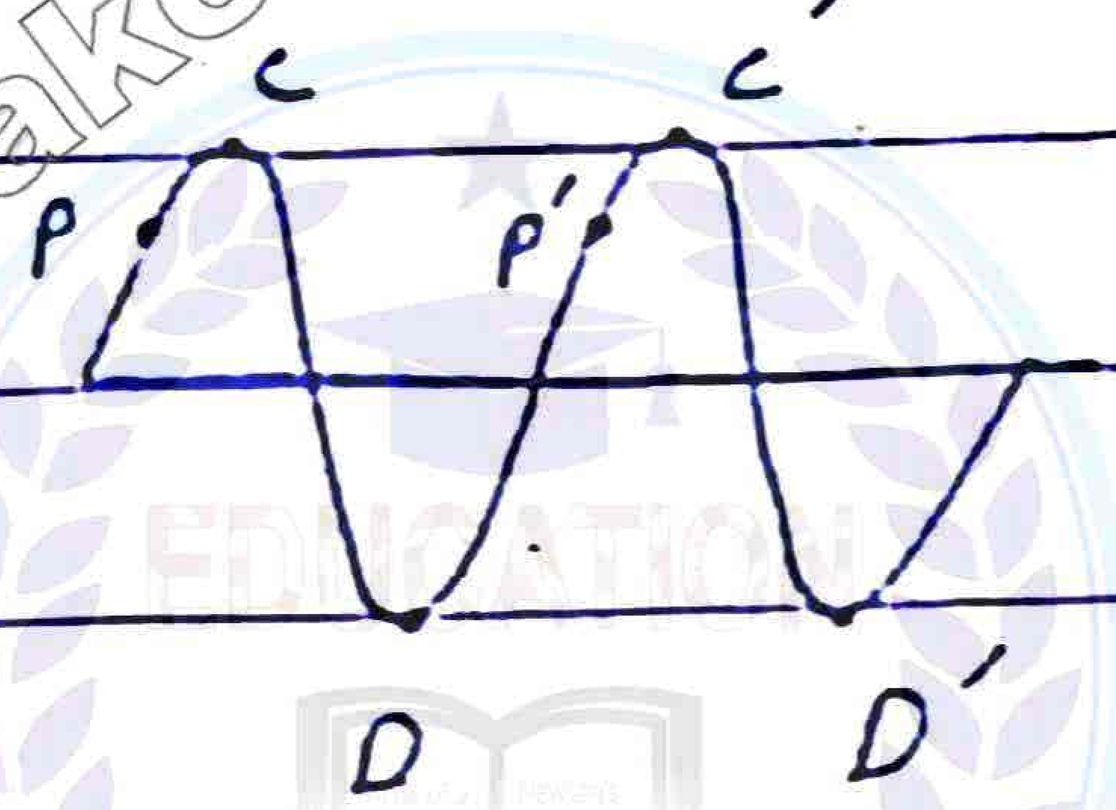
$$v = \frac{\lambda}{T}$$

$$v = \frac{1}{T}(\lambda)$$

$$v = f\lambda$$

In phase:

In a wave, the points seperated from one another through distance of λ, 2λ, 3λ, ---- are called in phase. These points can be anywhere along the wave and need not to be highest or lowest points.



For the shown wave: points P, P' and C, C' and D, D' are in phase

Out of phase:

In a wave, the points seperated from one another through distance of $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$, ---- are called out of phase. For the shown wave C, D, D' and C', D' are out of phase.

Speed Of Sound In Air:

The speed of sound through any medium is:

$$v = \sqrt{\frac{E}{\rho}}$$

Here E is the modulus of elasticity and ρ is the density of medium.

The speed of sound through solids is greater than through gases. Because in solids, the molecules are closely packed and transfer the sound energy faster.

As the density of solids is greater than gases but the modulus of elasticity for solids is also greater.

so, the ratio $\frac{E}{\rho}$ for the solids is greater than gases. Therefore, speed of sound through solids is greater.

$$V_{solid} > V_{gases}$$

Newton's calculation:

According to Newton sound travels through the air adiabatically. When sound travels, the atmospheric pressure increases from P to $P+\Delta P$ and volume decreases from V to $V-\Delta V$. But the temperature remains constant. According to Boyle's law:

$$PV = (P+\Delta P)(V-\Delta V)$$

$$\cancel{PV} = \cancel{PV} - P\Delta V + \Delta P V - \Delta P \Delta V$$

$$0 = -P\Delta V + \Delta P V - \Delta P \Delta V$$

Here $\Delta P \Delta V$ is a very small quantity and we neglect it.

$$0 = -P\Delta V + \Delta P V$$

$$P\Delta V = \Delta P V$$

$$P = \frac{\Delta P V}{\Delta V}$$

$$P = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$= \frac{\text{stress}}{\text{strain}}$$

P = modulus of elasticity

$$P = E$$

So, the speed of sound through air can be written as:

$$V = \sqrt{\frac{P}{\rho}}$$

By putting the values of atmospheric pressure P and density of air ρ at S.T.P. we get:

$$V = 280\ ms^{-1}$$

This value is different

from the experimental value of speed of sound through air which is $332\ ms^{-1}$.

Laplace Correction:

According to Laplace the temperature of air does not remain same when sound passes through, it increases slightly. when compression forms temperature increases and during rarefaction it does not decrease to same level. so, pressure of air increases from P to $P+\Delta P$ and volume decreases from V to $V-\Delta V$. Now according to Boyle's law.

$$PV^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

Here γ is the ratio of molar specific heat of gas at constant pressure to molar specific heat at constant volume.

$$\gamma = \frac{\text{molar specific heat at constant pressure}}{\text{molar specific heat at constant volume}}$$

$$\gamma = \frac{C_p}{C_v}$$

Take

$$PV^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

$$P\cancel{V^{\gamma}} = (P+\Delta P)\cancel{V^{\gamma}}\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

According to Binomial theorem:

$$(1+x)^n = 1+nx+\cdots\cdots$$

$$P = (P+\Delta P)\left(1-\frac{\gamma\Delta V}{V} + \text{neglecting heigher terms}\right)$$

$$P = (P+\Delta P)\left(1-\frac{\gamma\Delta V}{V}\right)$$

$$\cancel{P} = \cancel{P} - \frac{\gamma P \Delta V}{V} + \Delta P - \frac{\Delta P \gamma \Delta V}{V}$$

$$0 = -\frac{\gamma P \Delta V}{V} + \Delta P - \frac{\gamma \Delta P \Delta V}{V}$$

Here $\frac{\gamma \Delta P \Delta V}{V}$ is a very small quantity and we neglect it.

$$0 = -\frac{\gamma P \Delta V}{V} + \Delta P$$

$$\frac{\gamma P \Delta V}{V} = \Delta P$$

$$\gamma P = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$= \frac{\text{stress}}{\text{strain}}$$

= modulus of elasticity

$$\gamma P = E$$

So, the speed of sound through air is:

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

$$V = \sqrt{\gamma}\sqrt{\frac{P}{\rho}}$$

for air $\gamma = 1.4$ so,

at S.T.P

$$V = \sqrt{1.4}\,(280)$$

$$V = 333\ ms^{-1}$$

This value is very close to the experimental value of $332\ ms^{-1}$.

Effect of pressure on speed of sound in gas:

There is no effect of pressure on speed of sound in a gas.

Mathematically,

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

When atmospheric pressure increases, density of gas also increases with the same ratio. So,

$\frac{P}{\rho}$ remains constant. Therefore, speed of sound also remains same.

Effect of density on speed of sound in a gas:

The speed of sound is inversely proportional to the square root of density at constant pressure.

$$V \propto \frac{1}{\sqrt{\rho}}$$

For any gas having greater density, speed of sound will be smaller.

Effect of temperature on speed of sound in a gas:

The speed of sound varies directly with the temperature of gas.

When the temperature of gas increases, the energy of gas molecules also increases and they vibrate with greater amplitude and hence transfer sound faster.

Moreover, speed of sound is given by:

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

When temperature of gas increases, it expands and density decreases. Due to decrease in density speed of sound increases.

Prove that

$$v_t = v_o + 0.61t$$

If v_o is the speed of sound through air at $0^\circ c$ and v_t is the speed at temperature $t^\circ c$. Then at constant pressure:

$$v_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad \text{and} \quad v_o = \sqrt{\frac{\gamma P}{\rho_o}}$$

By dividing the above equations:

$$\frac{v_t}{v_o} = \frac{\sqrt{\frac{\gamma P}{\rho_t}}}{\sqrt{\frac{\gamma P}{\rho_o}}}$$

$$= \sqrt{\frac{\cancel{\gamma P}}{\rho_t} \times \frac{\rho_o}{\cancel{\gamma P}}}$$

$$\frac{v_t}{v_o} = \sqrt{\frac{\rho_o}{\rho_t}} \rightarrow (1$$

According to expression for volume thermal expansion:

$$V_t = V_o (1 + \beta t)$$

Here V_t is volume at $t^\circ C$, V_o is volume at $0^\circ C$ and β is the coefficient of volume thermal expansion.

For all gases, $\beta = \frac{1}{273}$

So,

$$V_t = V_o\left(1+\frac{t}{273}\right) \rightarrow (2$$

We know that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

now

$$V_t = \frac{m}{\rho_t} \quad \text{and} \quad V_o = \frac{m}{\rho_o}$$

Now eq. (2 will become.

$$\frac{\cancel{m}}{\rho_t} = \frac{\cancel{m}}{\rho_o}\left(1+\frac{t}{273}\right)$$

$$\rho_o = \rho_t\left(1+\frac{t}{273}\right)$$

Put in eq. (1

$$\frac{v_t}{v_o} = \sqrt{\frac{\cancel{\rho_t}\left(1+\frac{t}{273}\right)}{\cancel{\rho_t}}}$$

$$\frac{v_t}{v_o} = \sqrt{1+\frac{t}{273}}$$

$$\frac{v_t}{v_0} = \sqrt{\frac{273+t}{273}}$$

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}}$$

Here T is the absolute temperature at $t°c$ and T_0 is at $0°c$.
Above equation shows that speed is directly proportional to the temperature.
Take

$$\frac{v_t}{v_0} = \left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

According to Binomial theorem

$$(1+x)^n = 1+nx+----$$

$$\frac{v_t}{v_0} = 1+\frac{1}{2}\left(\frac{t}{273}\right) + \text{neglecting higher terms}$$

$$\frac{v_t}{v_0} = 1+\frac{t}{546}$$

$$v_t = v_0\left(1 + \frac{t}{546}\right)$$

$$v_t = v_0 + \frac{v_0 t}{546}$$

For air at STP.

$$v_0 = 332\ ms^{-1}$$

$$v_t = v_0 + \frac{332}{546} t$$

$$v_t = v_0 + 0.61\ t$$

This is the required mathematical expression.

Principle of Superposition:

If a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves by $y_1, y_2 ----, y_n$ then the resultant displacement of the particle, under the simultaneous action of these n waves is the algebric sum of all the displacements i-e;

$$Y = Y_1 + Y_2 + --- + Y_n.$$

This is called principle of superposition. Again if two waves which cross each other have opposite phases, their resultant displacement will be

$$Y = Y_1 - Y_2$$

Particularly if $Y_1 = Y_2$ then resultant displacement $Y = 0$.

Interference:

Superposition of two waves having the same frequency and travelling in the same direction results in the phenomenon called interference.

CRO:

A sensitive cathode ray oscilloscope (CRO) acts as a detector of sound waves. The CRO is a device to display the input signal into waveform on its screen.

Constructive interference:

When a path difference is an integral multiple of wavelength displacements, the two waves are added up. This effect is called constructive interference. Therefore, the condition for constructive interference can be written as:

Path difference $= \Delta S = n\lambda$

$n = 0, \pm 1, \pm 2, \pm 3, \ldots$

Destructive interference:

At points where the displacements of two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength. This effect is called destructive interference. Therefore, the condition for destructive interference can be written as:

$$\Delta S = (2n+1)\frac{\lambda}{2}$$

$n = 0, \pm 1 \pm 2 \pm 3, \ldots$

Beats:

Two waves of slightly different frequencies and travelling in the same direction produce beats. Number of beats per second is equal to the difference between the frequencies of the tuning forks.

Number of beats $= f_1 - f_2$.

When the difference between the frequencies of the two sounds is more than about 10 Hz, then it becomes difficult to recognize the beats.

How are beats useful in tuning musical instruements:

We can use beats to tune a string instruement, such as piano or violin, by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening until no beats are heard.

Slinky spring:

A slinky spring is a loose spring which has small intial length but a relatively large extended length.

Reflection of wave:

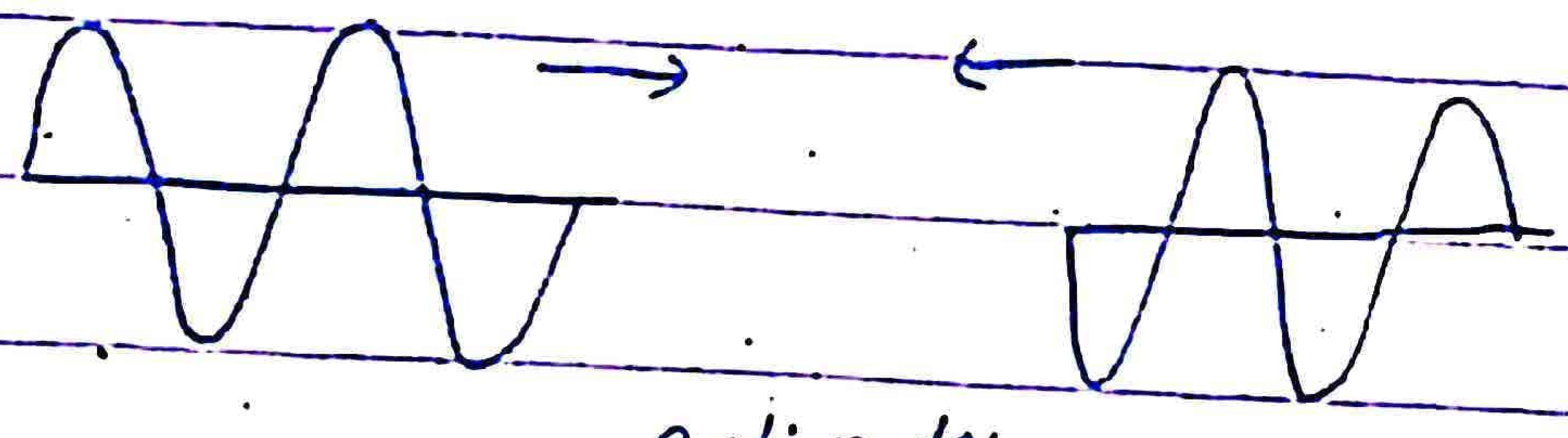
1, Whenever, a transverse wave, travelling in a rarer medium, encounters a denser medium, it bounces back such that the direction of its displacement is reversed. An incident crest on reflection becomes a trough. It is reflected such that it undergoes a phase change of

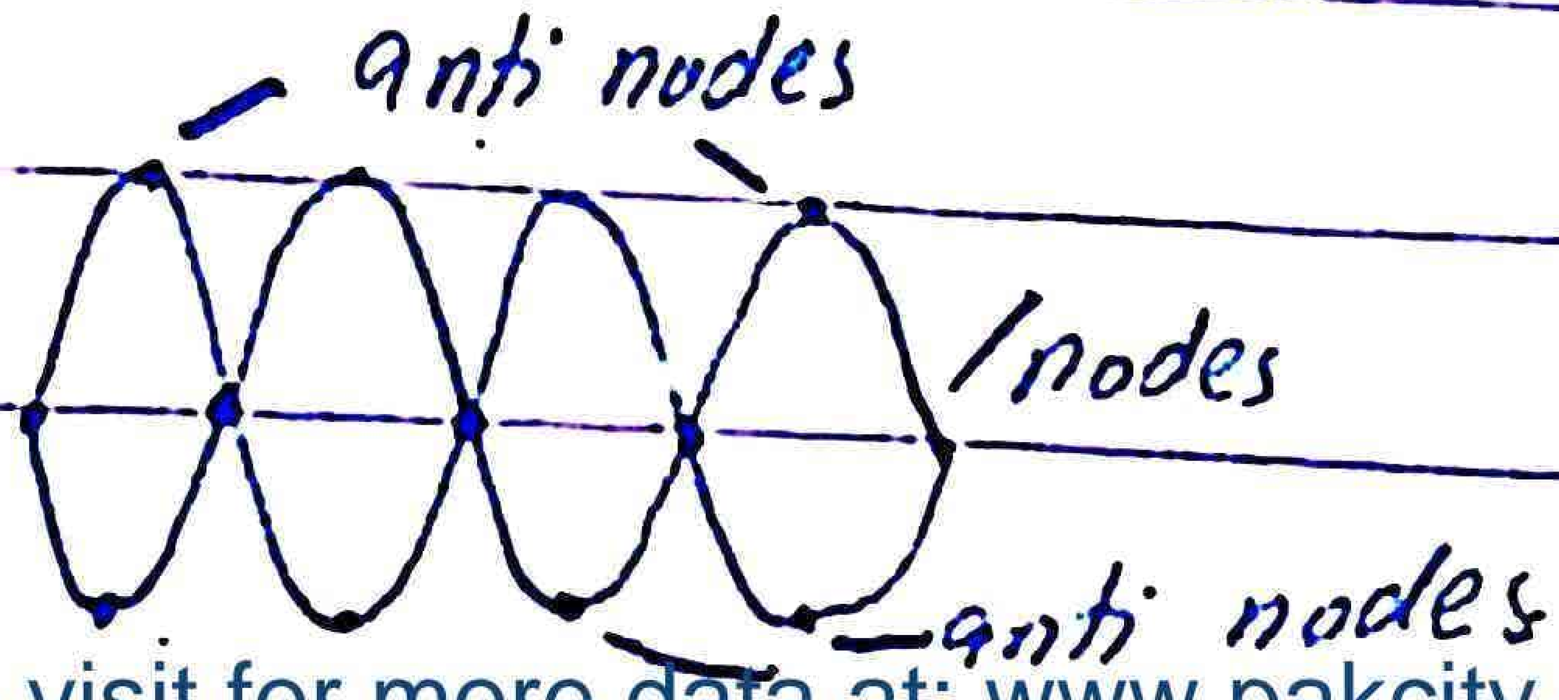
180°.

2) When a transverse wave travelling in a denser medium, is reflected from the boundary of a rare medium, the direction of its displacement remains the same. An incident crest is reflected as a crest. It is reflected without any change in phase.

Stationary waves:

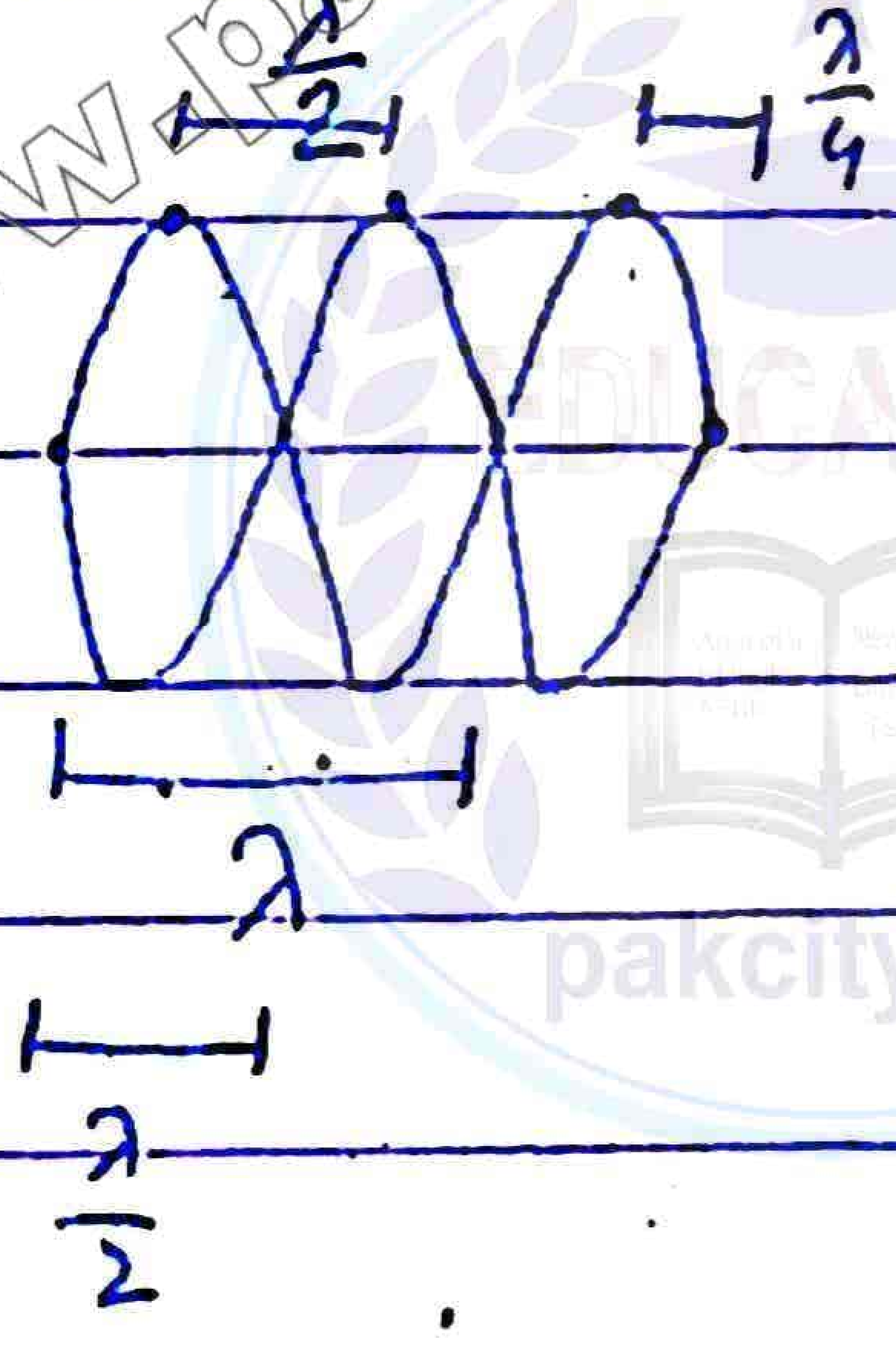
When two waves of equal frequency travelling in opposite direction superpose they produce stationary waves.





Nodes: In the stationary wave, the points that show zero displacement are called nodes. These points lie on the mean position.

Anti-nodes: In the stationary wave, the points that show maximum displacement are called anti-nodes.



MCQ's:

i- The distance between two consecutive nodes or anti-nodes is $\frac{\lambda}{2}$

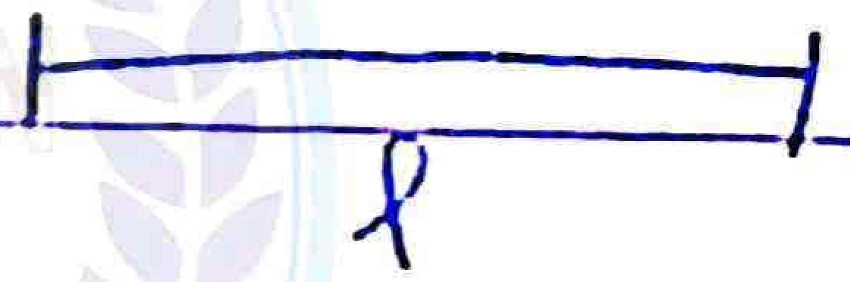
ii- The distance between a node and anti-node is $\frac{\lambda}{4}$

Stationary waves In A Stretched String:

Defination: when two waves of equal frequency travelling in opposite direction superpose they produce stationary wave.

Explanation:

Consider a string of length "l" connected from its two ends having tension F in it as shown in figure.



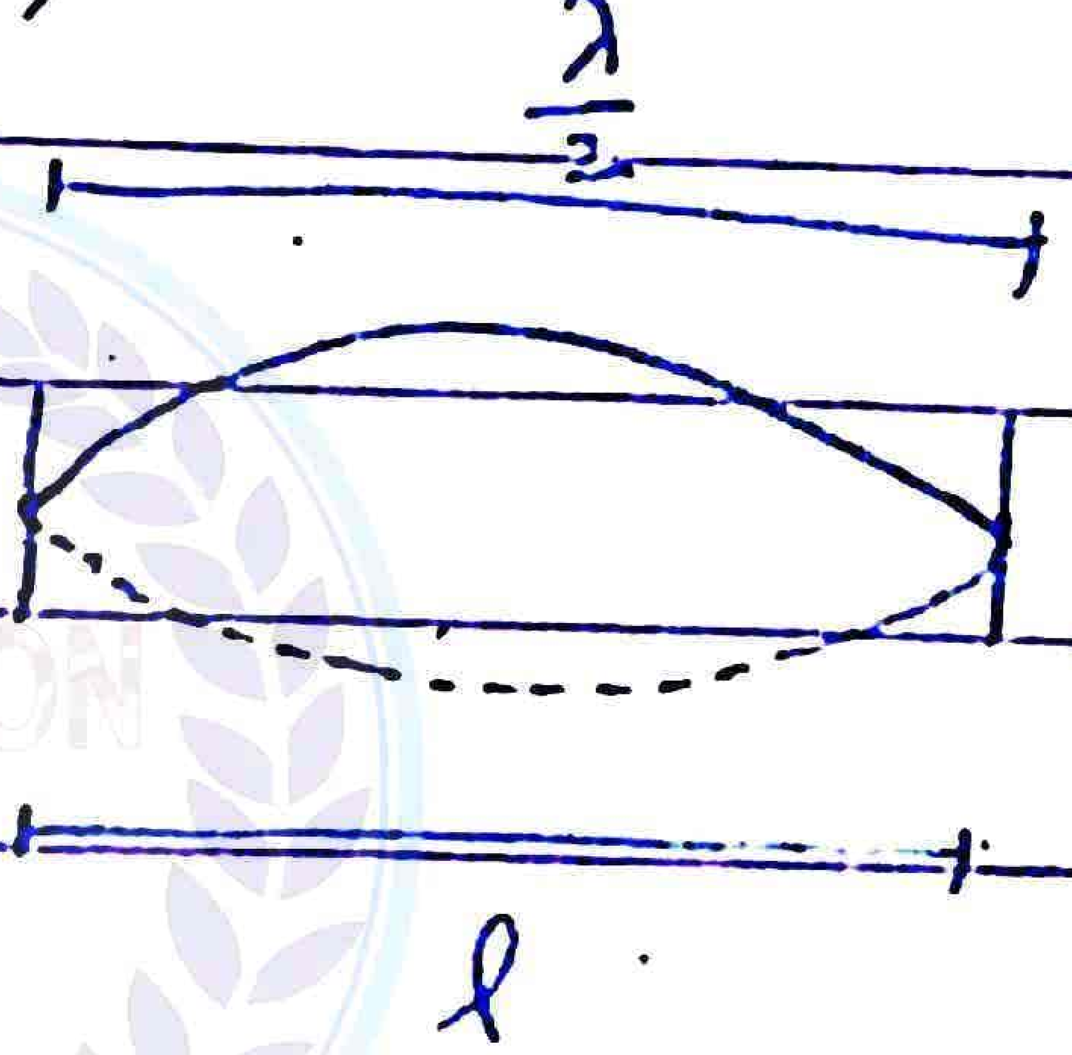
When the string is plucked from different points, different number of loops for stationary waves will be produced. The speed of wave is given by

$$v = \sqrt{\frac{F}{m}}$$

Here: F is tension in the string and m is mass per unit length.

Now we will discuss different modes of vibrations.

First mode of vibration:



when the string is plucked from its middle, a single loop of stationary wave is produced. The distance between two consecutive nodes is $\frac{\lambda}{2}$.

So,

$$l = \frac{\lambda_1}{2}$$

$$2l = \lambda_1$$

$$\lambda_1 = 2l$$

According to wave equation

$$v = f_1 \lambda_1$$

$$\frac{v}{\lambda_1} = f_1$$

$$f_1 = \frac{v}{\lambda_1}$$

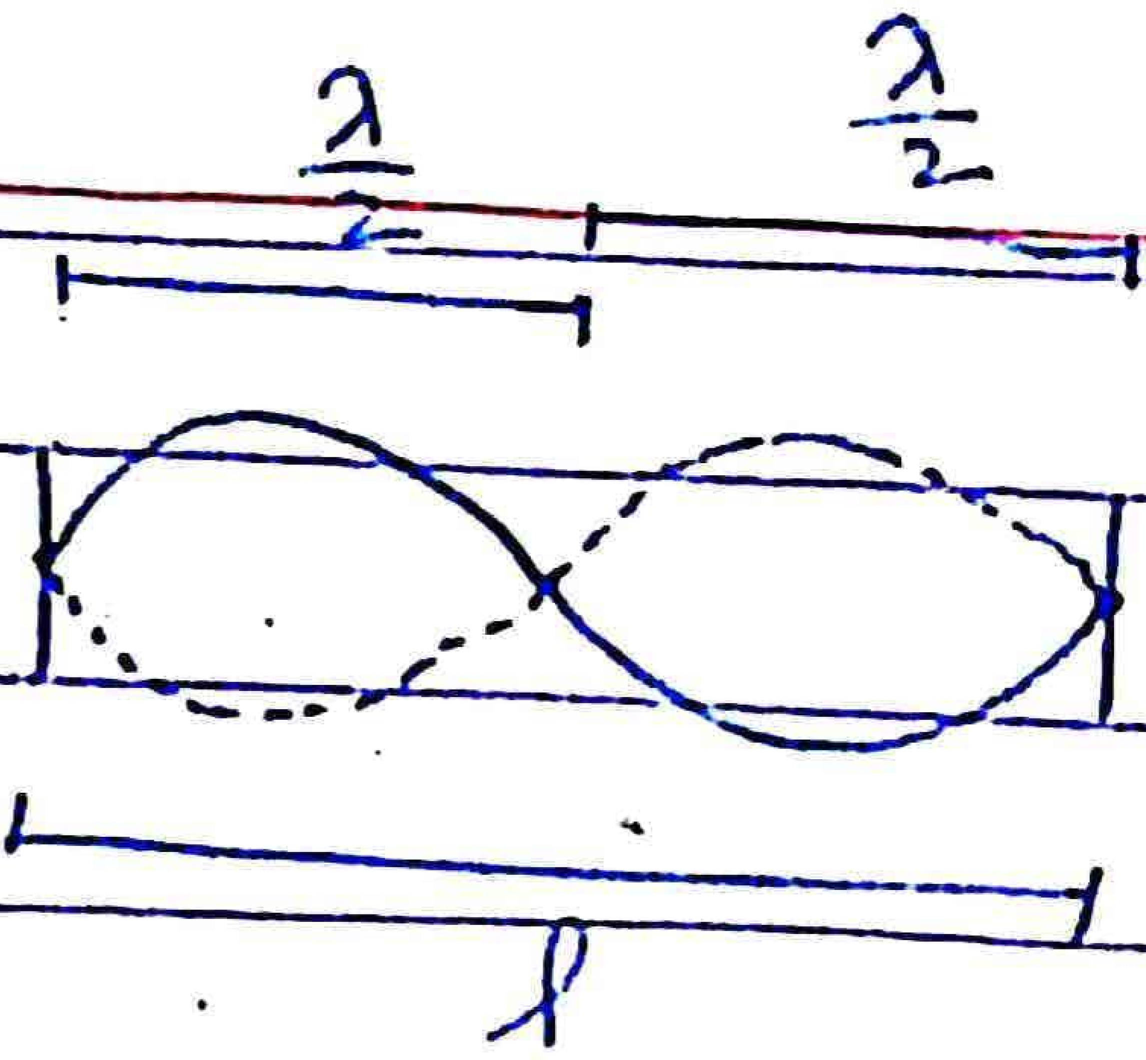
$$= \frac{1}{\lambda_1}(v) = \frac{1}{2l}(v)$$

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

This is the expression for first mode of vibration. This frequency is also called fundamental frequency.

Second mode of vibration:

For the second mode of vibration, the string is plucked such that two loops of stationary waves are produced.



The distance between two consecutive nodes is $\frac{\lambda}{2}$. So

$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

$$= \frac{\lambda_2 + \lambda_2}{2}$$

$$l = \frac{2\lambda_2}{2}$$

$$\frac{2l}{2} = \lambda_2$$

$$\lambda_2 = \frac{\lambda_1}{2}$$

According to wave equation

$$v = f_2 \lambda_2$$

$$\frac{v}{\lambda_2} = f_2$$

$$f_2 = \frac{v}{\frac{2l}{2}}$$

$$f_2 = \frac{2v}{2l}$$

$$f_2 = 2\left(\frac{v}{2l}\right)$$

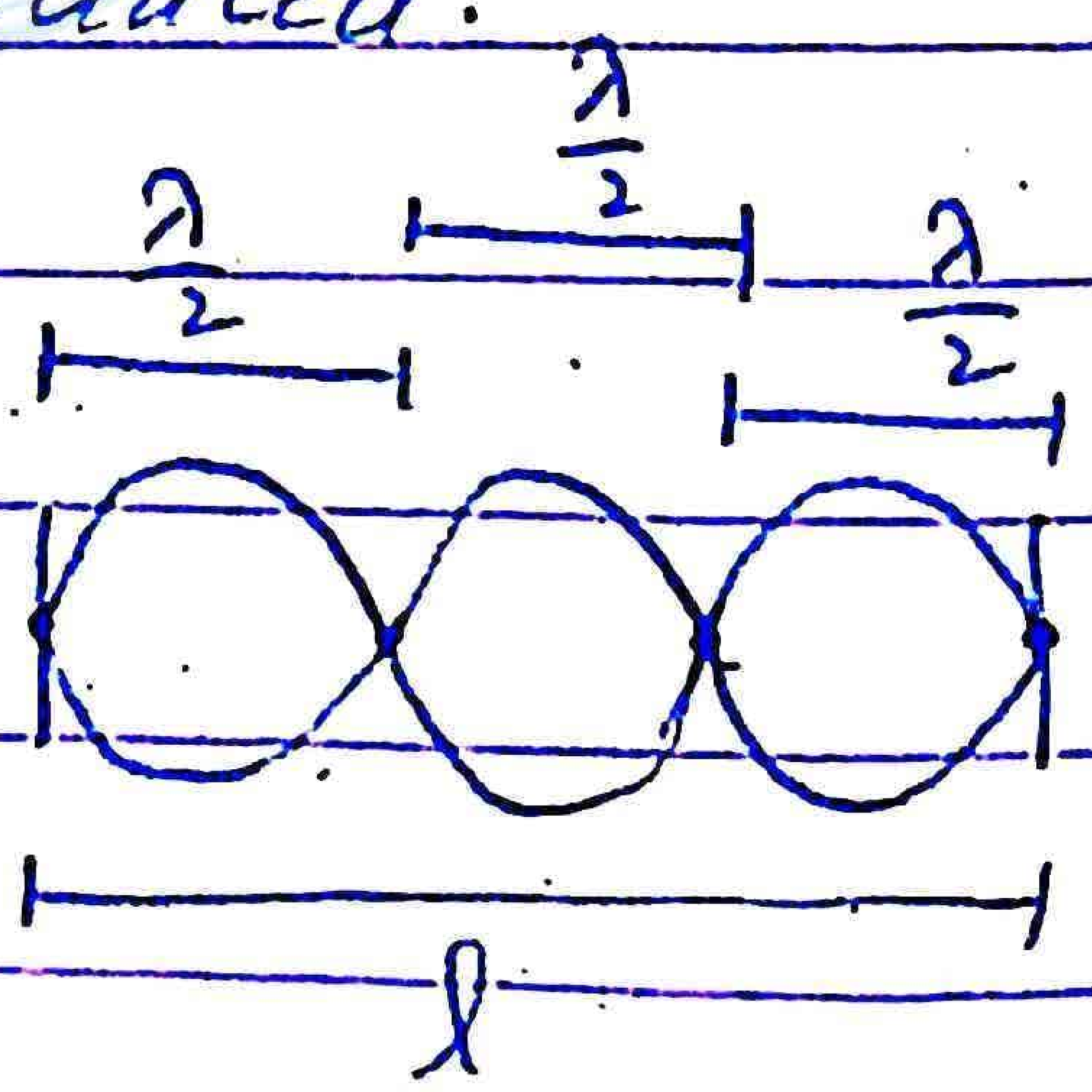
$$f_2 = 2f_1$$

This is frequency for second harmonic.

Third mode of vibration:

For the third mode of vibration, the string is plucked such that three loops of stationary waves are produced.

The distance between two consecutive nodes is $\frac{\lambda}{2}$. So,



$$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$l = \frac{\lambda_3 + \lambda_3 + \lambda_3}{2}$$

$$l = \frac{3\lambda_3}{2}$$

$$2l = 3\lambda_3$$

$$\frac{2l}{3} = \lambda_3$$

$$\lambda_3 = \frac{2l}{3}$$

$$\lambda_3 = \frac{\lambda_1}{3}$$

According to wave equation:

$$v = f_3 \lambda_3$$

$$\frac{v}{\lambda_3} = f_3$$

$$f_3 = \frac{v}{\frac{2l}{3}}$$

$$f_3 = \frac{3v}{2l}$$

$$f_3 = 3\left(\frac{v}{2l}\right)$$

$$f_3 = 3f_1$$

This is the frequency for third mode of vibration.

nth mode of vibration:

We know that

$$f_1 = \frac{v}{2l} \quad \text{and} \quad \lambda_1 = 2l$$

$$f_2 = 2f_1 \qquad \lambda_2 = \frac{\lambda_1}{2}$$

$$f_3 = 3f_1 \qquad \lambda_3 = \frac{\lambda_1}{3}$$

$$\vdots \qquad \vdots$$

$$f_n = nf_1 \qquad \lambda_n = \frac{\lambda_1}{n}$$

$$f_n = \frac{nv}{2l} \qquad \lambda_n = \frac{2l}{n}$$

Here $n = 1, 2, 3, ----$

The frequency for first harmonic f_1 is called fundamental frequency while frequencies $f_1, 2f_1, 3f_1 -- nf_1$

are called harmonic series. From the above relations we can conclude that the frequency increases as the number of loops increases but wavelength decreases. However, the product of frequency and wavelength remains same that is equal to speed.

$$v = f_n \lambda_n$$

Stationary Waves In Air Columns

Organ Pipe:

The instrument in which stationary waves are formed due to the motion of air molecules is called organ pipe.

Types: There are two types of organ pipe.

i- Open organ pipe:

The organ pipe for which both the ends are open is called open organ pipe.

ii- Closed organ pipe:

The organ pipe for which one end is closed is called closed organ pipe.

Formation of nodes and anti nodes:

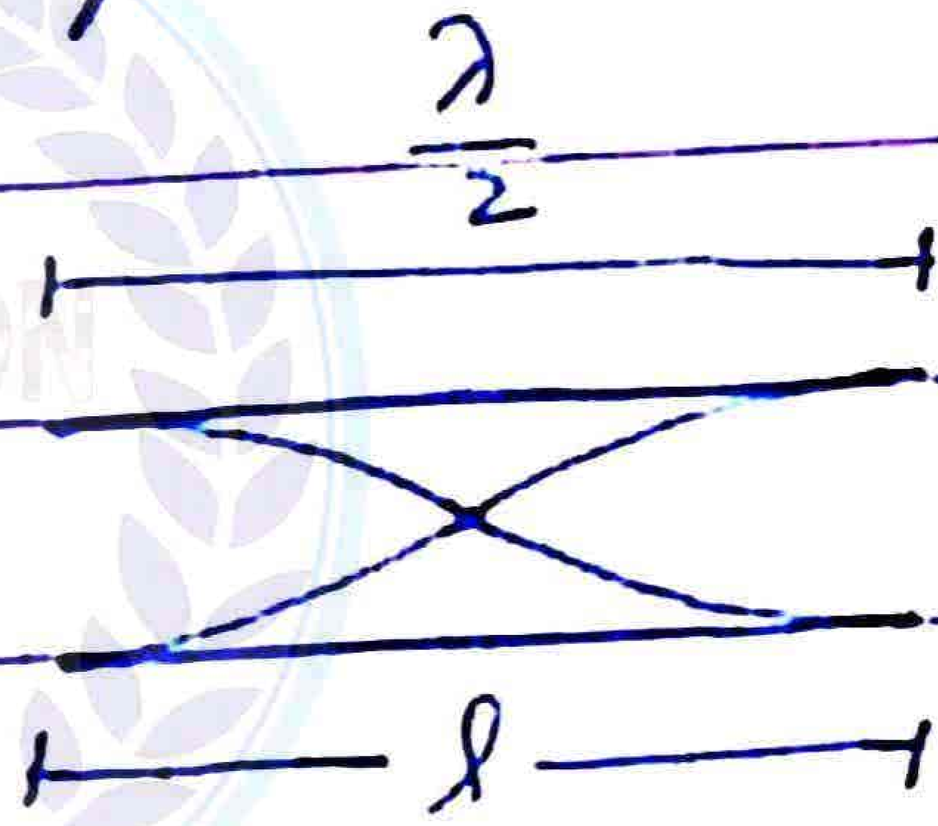
For the organ pipe nodes are formed at the closed end because air molecules are bounded to move there. while anti-nodes are formed at the open end because air molecules are free to move there.

Open Organ Pipe:

Consider an open organ pipe of length "l" in which stationary waves are formed due to the motion of air molecules. Now we will discuss different modes of vibration.

First mode of vibration:

For the first mode of vibration, anti-nodes are formed at the open ends while a node is present at the middle.



The distance between two consecutive anti-nodes is $\frac{\lambda}{2}$. So,

$$l = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2l$$

According to wave equation.

$$v = f_1 \lambda_1$$

$$\frac{v}{\lambda_1} = f_1$$

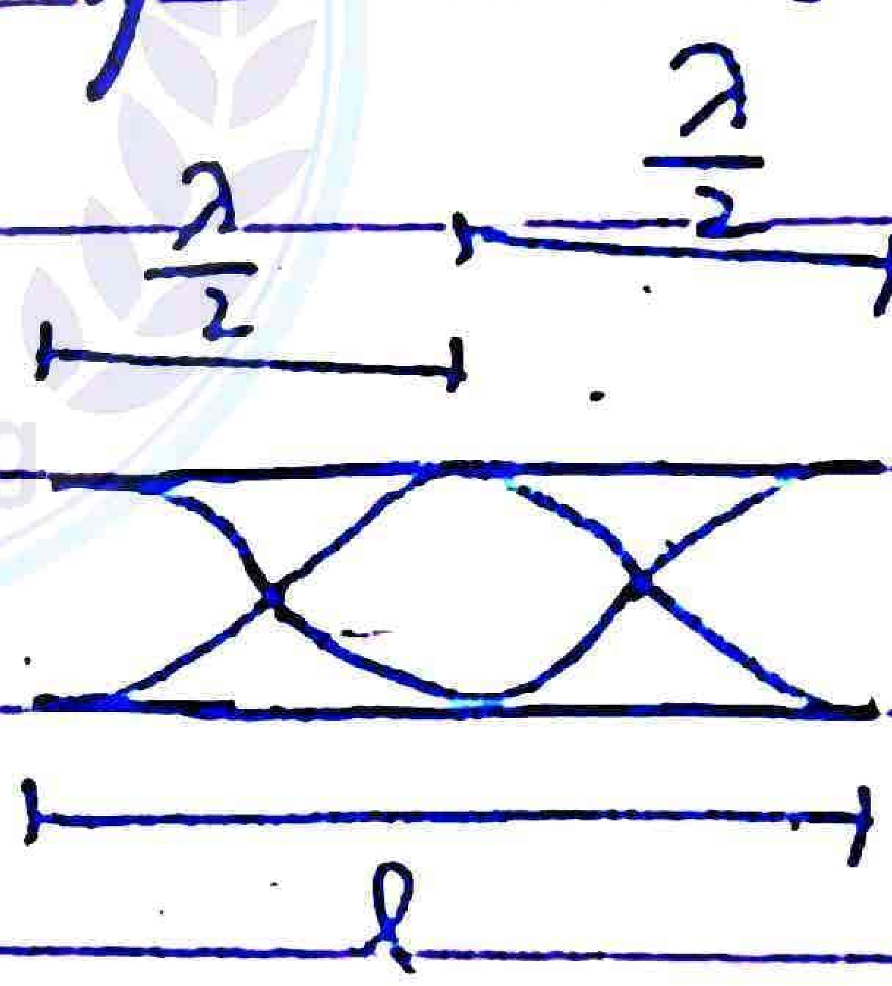
So,

$$f_1 = \frac{v}{2l}$$

This is the frequency for first mode of vibration called fundamental frequency.

Second mode of vibration:

For the second mode of vibration. anti-nodes



are formed at the open ends while a loop is present at the middle. The distance between two consecutive anti-nodes is $\frac{\lambda}{2}$.

$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

$$= \frac{\lambda_2 + \lambda_2}{2}$$

$$l = \frac{2\lambda_2}{2}$$

$$\frac{2l}{2} = \lambda_2$$

$$\lambda_2 = \frac{\lambda_1}{2}$$

According to wave equation

$$v = f_2 \lambda_2$$

$$\frac{v}{\lambda_2} = f_2$$

$$f_2 = \frac{v}{\frac{2l}{2}}$$

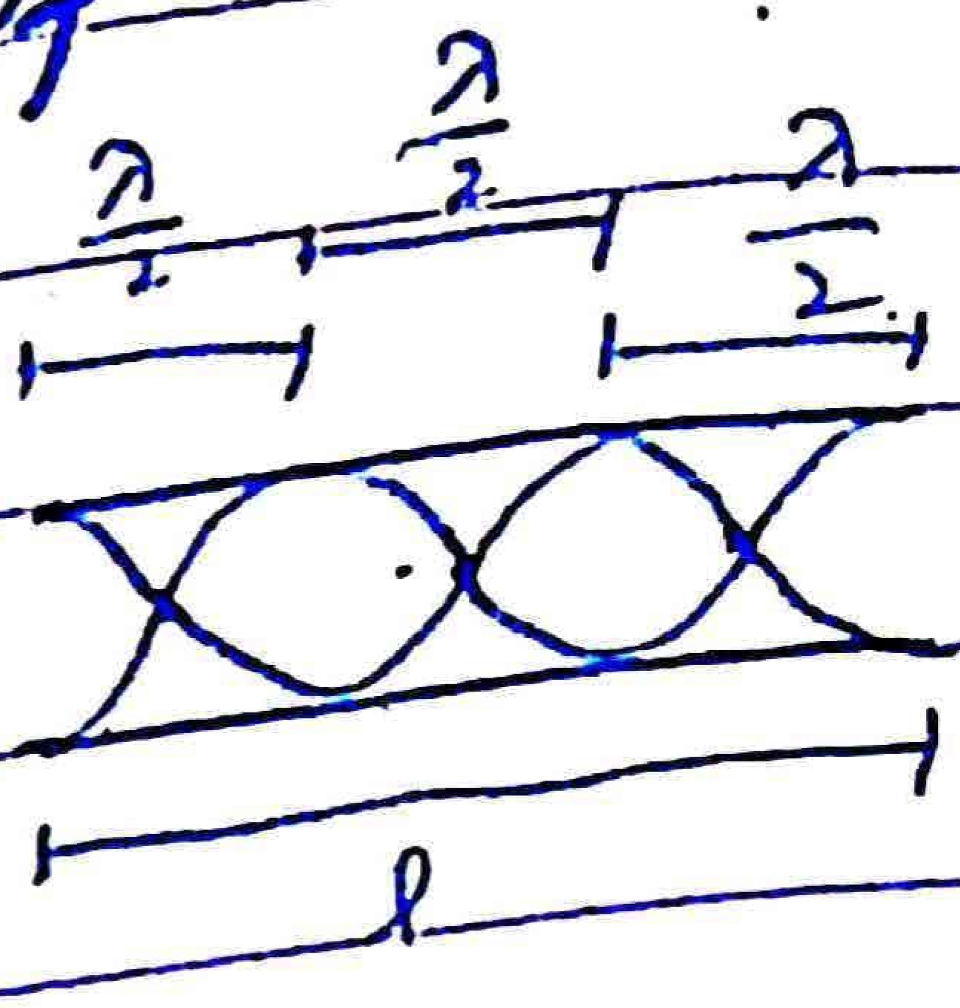
$$f_2 = \frac{2v}{2l}$$

$$f_2 = 2f_1$$

This is the frequency for second mode of vibration.

Third mode of vibration.

For the third mode of vibration, anti nodes are present at the open ends while two loops are present in between. The distance between two consecutive anti-nodes is $\frac{\lambda}{2}$. So,



$$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$l = \frac{\lambda_2 + \lambda_3 + \lambda_3}{2}$$

$$l = \frac{3\lambda_3}{2}$$

$$2l = 3\lambda_3$$

$$\frac{2l}{3} = \lambda_3$$

$$\lambda_3 = \frac{2l}{3}$$

$$\lambda_3 = \frac{\lambda_1}{3}$$

According to wave equation.

$$V = f_3 \lambda_3$$

$$\frac{V}{\lambda_3} = f_3$$

$$f_3 = \frac{V}{\frac{2l}{3}}$$

$$f_3 = \frac{3V}{2l}$$

$$f_3 = 3\left(\frac{V}{2l}\right)$$

$$f_3 = 3f_1$$

This is the frequency for third mode of vibration.

nth mode of vibration:

We know that

$f_1 = \frac{v}{2l}$ and $\lambda = 2l$

$f_2 = 2f_1$ $\lambda_2 = \frac{\lambda_1}{2}$

$f_3 = 3f_1$ $\lambda_3 = \frac{\lambda_1}{3}$

$\vdots$

$f_n = nf_1$ $\lambda_n = \frac{\lambda_1}{n}$

$f_n = \frac{nv}{2l}$ $\lambda_n = \frac{2l}{n}$

Here $n = 1, 2, 3, \ldots$

The frequency for first harmonic f_1 is called fudamental frequency while frequencies $f_1, 2f_1, 3f_1 \ldots nf_1$ are called harmonic series.

From the above relations We can conclude that the frequency increases as the number of loops increases but wavelength decreases. However, the product of frequency and wavelength remains same that is equal to speed.

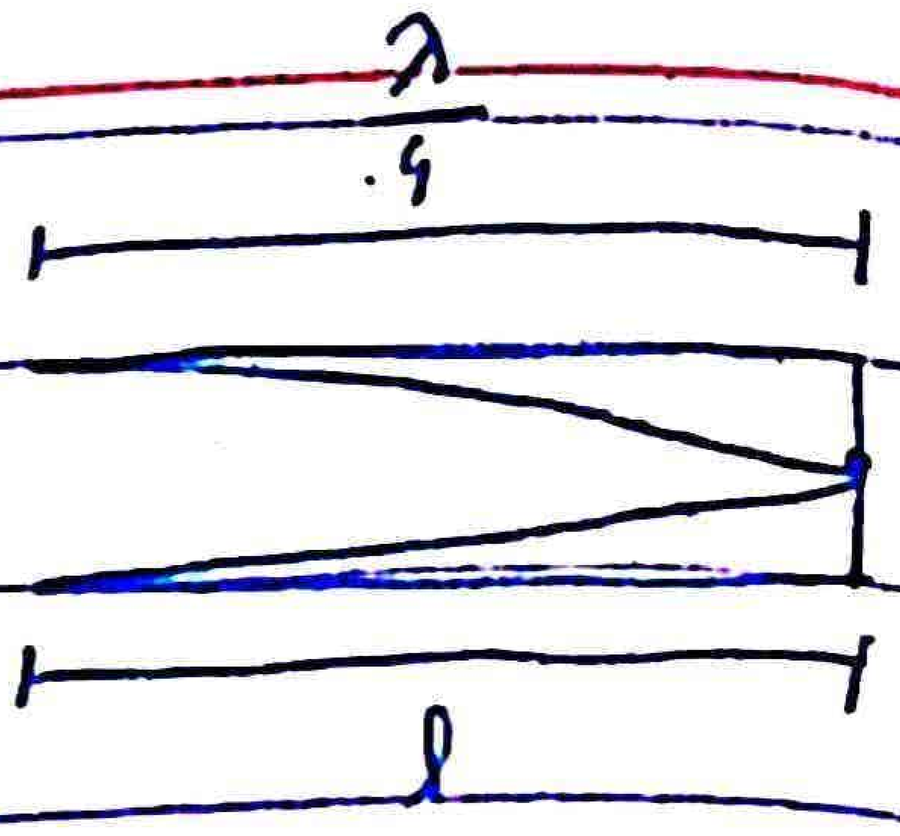
$$V = f_n \lambda_n$$

Closed Organ Pipe:

Consider a closed organ pipe having length "l" in which stationary waves are formed due to the motion of air molecules. Now we will discuss different modes of vibration.

First mode of vibration:

For the first mode of vibration, a node is



formed at the closed end while anti-node is formed at open end. The distance between node and anti-node is $\frac{\lambda}{4}$. so,

$$l = \frac{\lambda_1}{4}$$

$$4l = \lambda_1$$

$$\lambda_1 = 4l$$

According to wave equation

$$v = f_1 \lambda_1$$

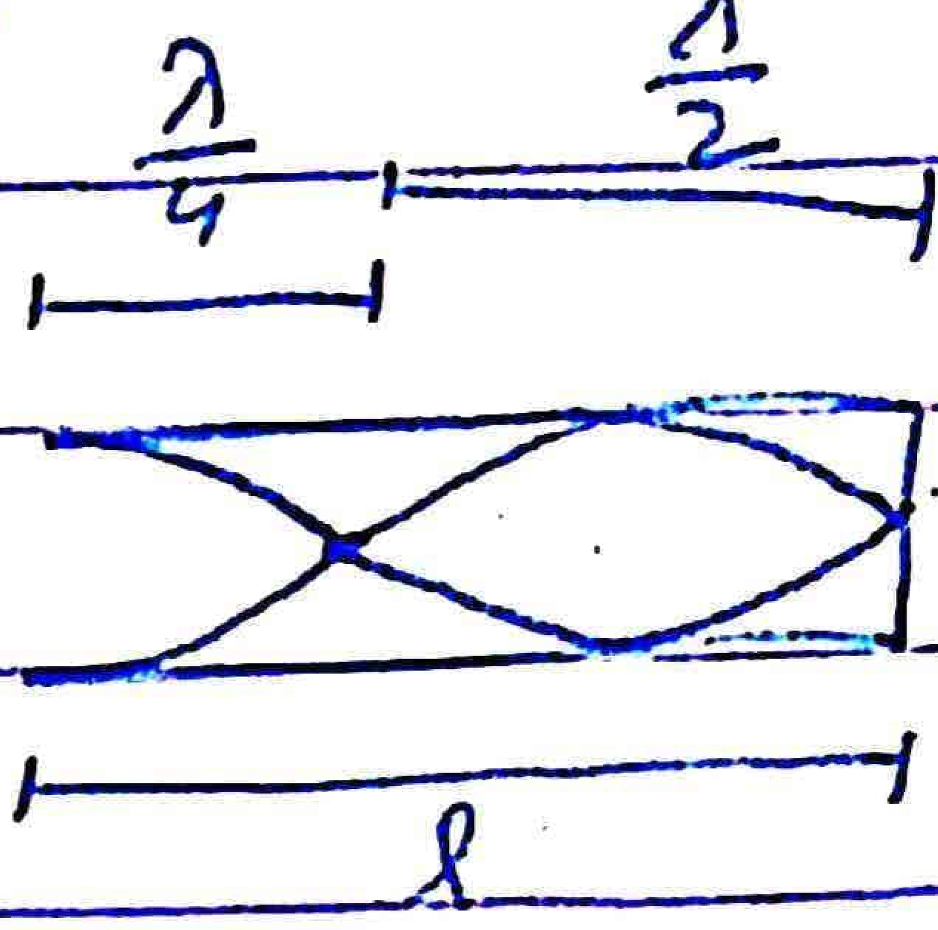
$$\frac{v}{\lambda_1} = f_1$$

$$f_1 = \frac{v}{4l}$$

This is the frequency for first mode of vibration also called fundamental frequency.

Second mode of vibration:

For the second mode of vibration, anti node is formed at open end, node is formed at closed end while a loop is also present. The distance between two consecutive nodes is $\frac{\lambda}{2}$ while distance between node and anti-node is $\frac{\lambda}{4}$.



$$l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2}$$

$$= \frac{\lambda_3 + 2\lambda_3}{4}$$

$$l = \frac{3\lambda_3}{4}$$

$$4l = 3\lambda_3$$

$$\frac{4l}{3} = \lambda_3$$

$$\lambda_3 = \frac{4l}{3}$$

$$\lambda_3 = \frac{\lambda_1}{3}$$

According to wave equation

$$v = f_3 \lambda_3$$

$$\frac{v}{\lambda_3} = f_3$$

$$f_3 = \frac{v}{\frac{4l}{3}}$$

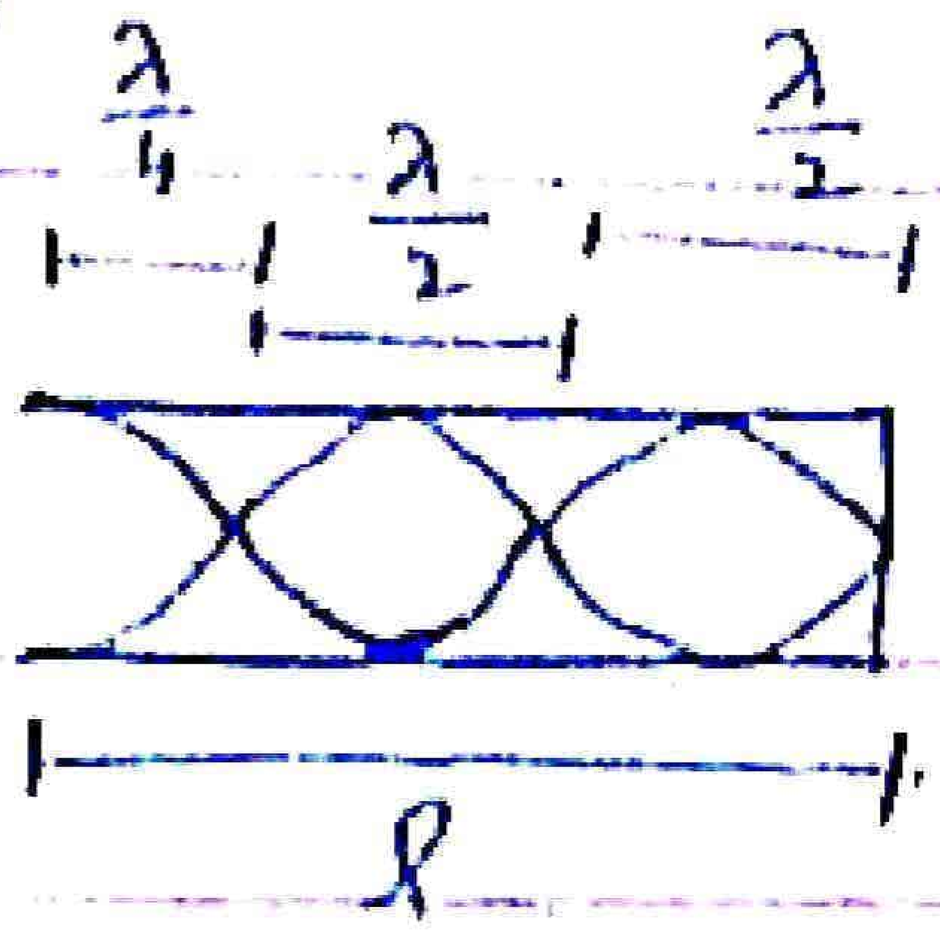
$$= \frac{3v}{4l}$$

$$f_3 = 3f_1$$

This is the frequency for second mode of vibration.

Third mode of vibration:

For the third mode of vibration, anti-node is formed at open end. node is formed at closed end while two loops are also present there.



The distance between two consecutive nodes is $\frac{\lambda}{2}$ and distance between node and anti-node is $\frac{\lambda}{4}$. So,

$$l = \frac{\lambda_5}{4} + \frac{\lambda_5}{2} + \frac{\lambda_5}{2}$$

$$= \frac{\lambda_5 + 2\lambda_5 + 2\lambda_5}{4}$$

$$l = \frac{5\lambda_5}{4}$$

$$4l = 5\lambda_5$$

$$\frac{4l}{5} = \lambda_5$$

$$\lambda_5 = \frac{\lambda_1}{5}$$

According to wave equation

$$v = f_5 \lambda_5$$

$$\frac{v}{\lambda_5} = f_5$$

$$f_5 = \frac{\frac{v}{4l}}{5}$$

$$f_5 = \frac{5v}{4l}$$

$$f_5 = 5f_1$$

This is the frequency for third mode of vibration.

nth mode of vibration:-

We know that

$f_1 = \frac{v}{4l}$, $\lambda_1 = 4l$

$f_3 = 3f_1$, $\lambda_3 = \frac{\lambda_1}{3}$

$f_5 = 5 f_1$, $\lambda_5 = \frac{\lambda_1}{5}$

$\vdots$ $\vdots$

$f_n = n f_1$, $\lambda_n = \frac{\lambda_1}{n}$

$f_n = \frac{nv}{4l}$, $\lambda_n = \frac{4l}{n}$

Here $n = 1, 3, 5, \ldots$

MCQ:

The open organ pipe is richer in harmonics.

Doppler Effect

Introduction: Doppler effect shows that if there is some relative motion between the source of waves and the observer, an apparent change in frequency of the waves is observed. This effect was observed by Johann Doppler while he was observing the frequency of light emitted from distant stars.

Explanation:

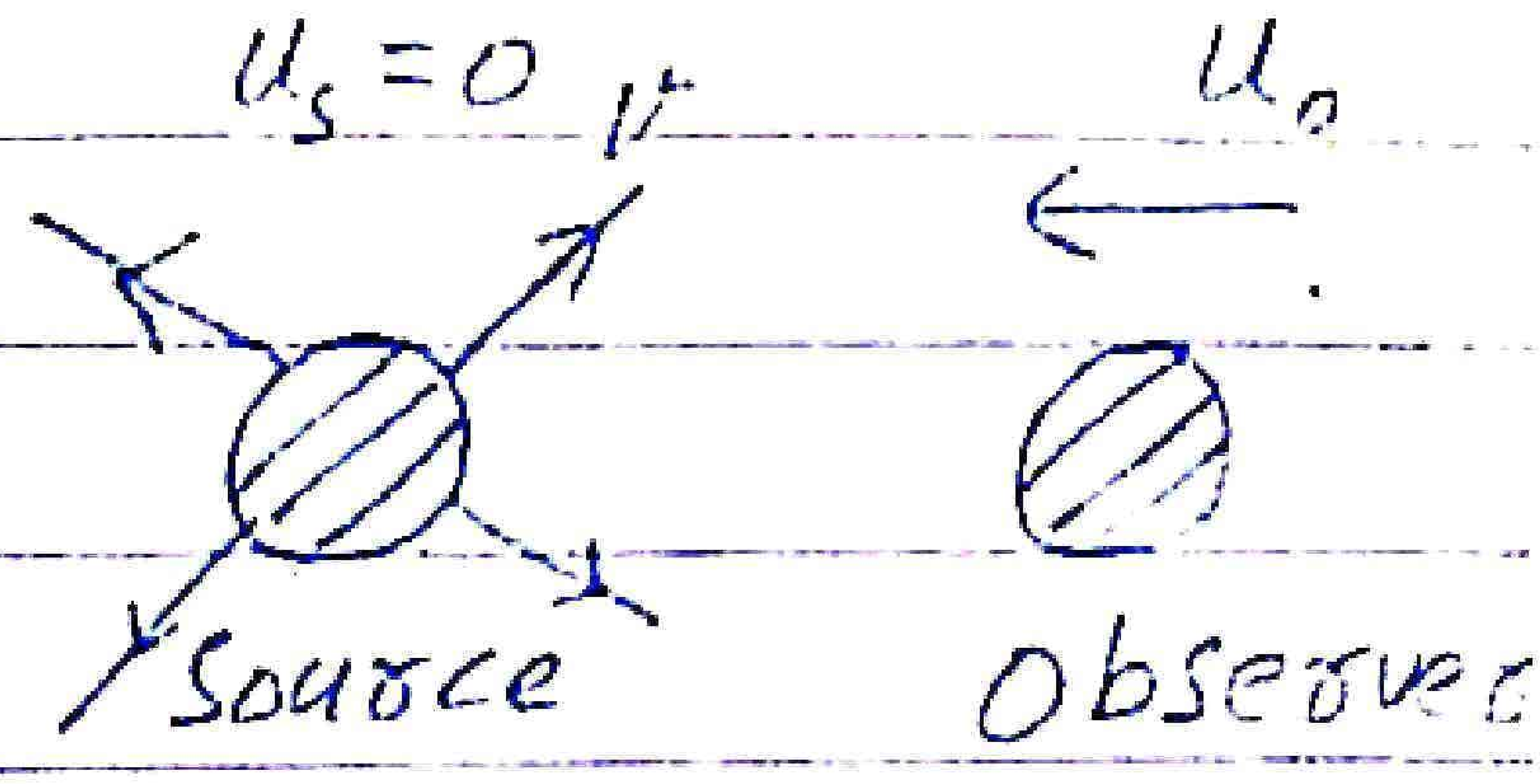
Consider a source of sound through which sound waves are produced having velocity v, the frequency is f, wavelength is λ. An observer is present at some distance. When both the source and observer are at rest: then frequency of sound waves can be determined by wave equation.

$$f = \frac{v}{\lambda}$$

Now we will discuss four different cases due to relative motion of source and observer to calulate the apparent frequency.

Case 1: When observer moves towards the stationary source.

When the observer is moving towards the stationary source with speed u_o. Then the relative velocity will be: $v + u_o$. Now the apparent frequency will be:



$$f_n = \frac{v + u_o}{\lambda}$$

According to wave equation the wavelength for stationary source and observer will be:

$$\lambda = \frac{v}{f}$$

So,

$$f_n = \frac{v + u_o}{\frac{v}{f}}$$

$$f_n = \left(\frac{v+u_o}{v}\right) f$$

As $\frac{v+u_o}{v} > 1$

so,

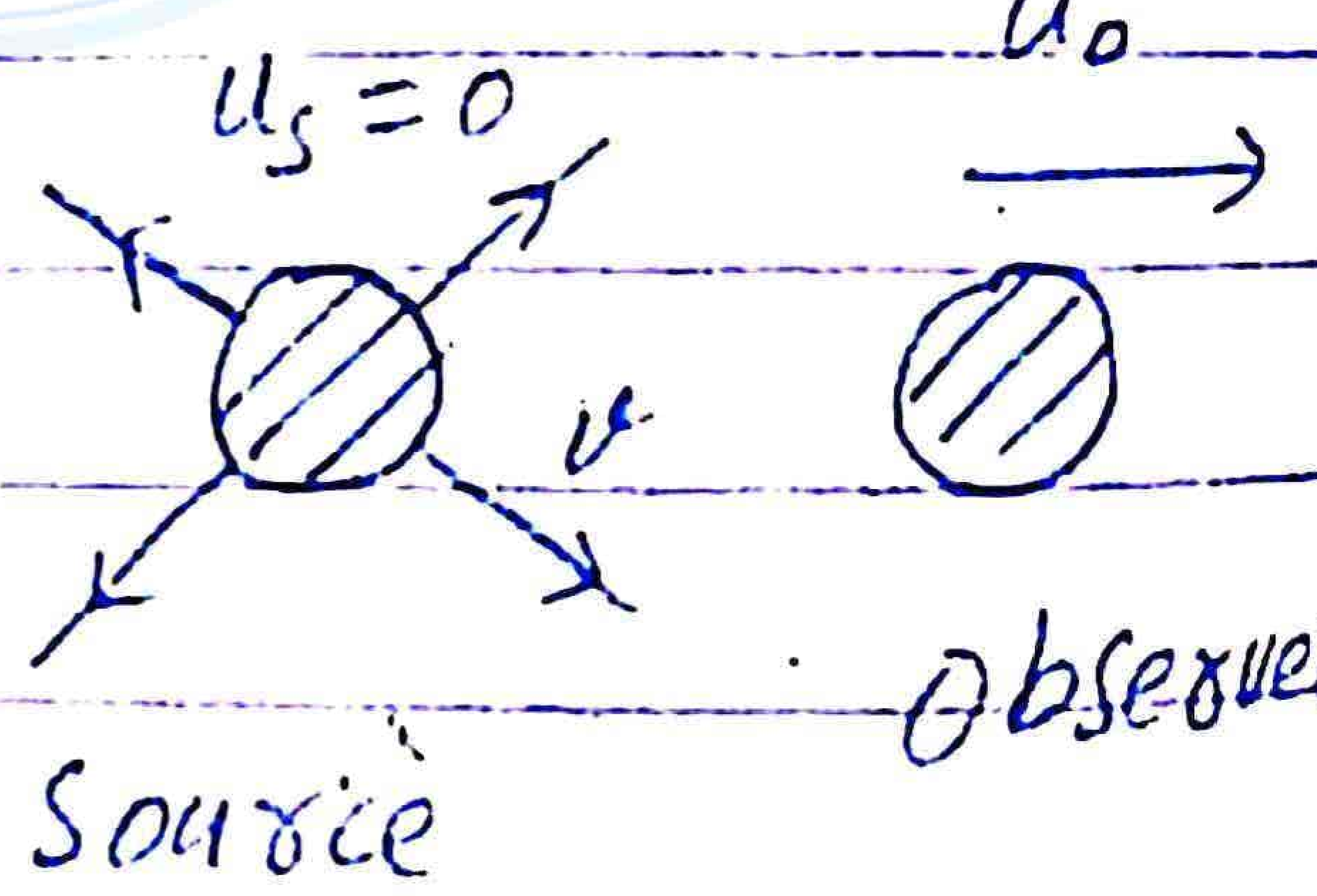
$$f_n > f$$

Result:

when observer moves towards the stationary source, the apparent frequency will increase.

Case.2: When observer moves away from the stationary source.

when the observer is moving away from the stationary source with speed u_o. Then the relative velocity will be $v - u_o$. Then the apparent



frequency will be:

$$f_B = \frac{v - u_o}{\lambda}$$

According to wave equation, the wavelength for stationary source and observer will be:

$$\lambda = \frac{v}{f}$$

So,

$$f_B = \frac{v - u_o}{\frac{v}{f}}$$

$$\therefore f_B = \left(\frac{v - u_o}{v}\right) f,$$

As $\frac{v - u_o}{v} < 1$

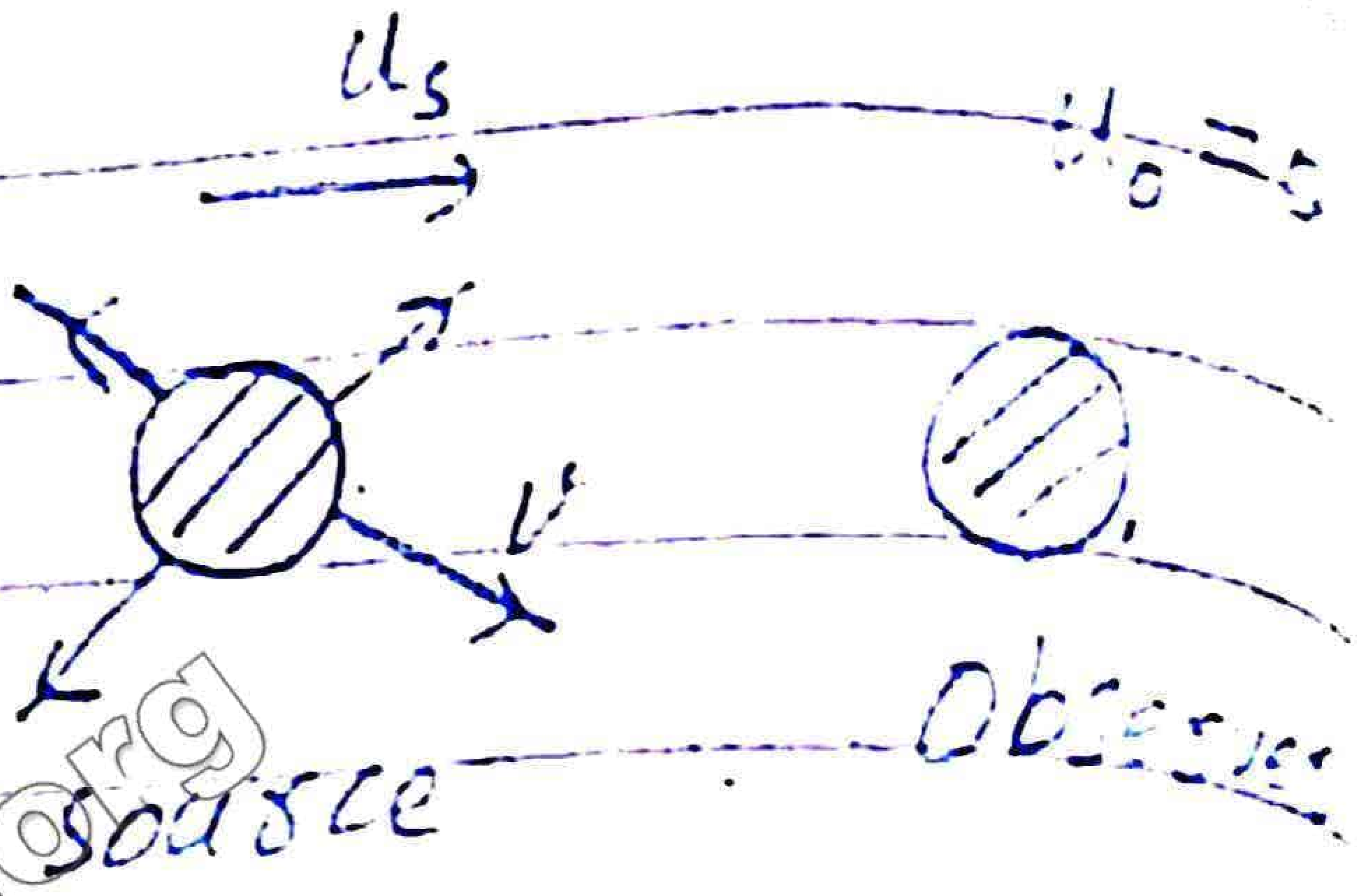
So,

$$f_B < f$$

Result: when observer moves away from the stationary source, the apparent frequency will decrease.

Case. 3: when source of sound moves towards the stationary observer.

when source of sound moves towards the stationary observer with speed u_s. Then the waves between the source and observer will be compressed by an amount called Doppler shift ($\Delta\lambda$) given by:



$$\Delta\lambda = \frac{u_s}{f}$$

The wavelength for the observer after compression is:

$$\lambda_c = \lambda - \Delta\lambda$$

$$\lambda_c = \frac{v}{f} - \frac{u_s}{f}$$

$$\lambda_c = \frac{v - u_s}{f}$$

according to wave equation:

$$f_c = \frac{v}{\lambda_c}$$

$$f_c = \frac{v}{\frac{v - u_s}{f}}$$

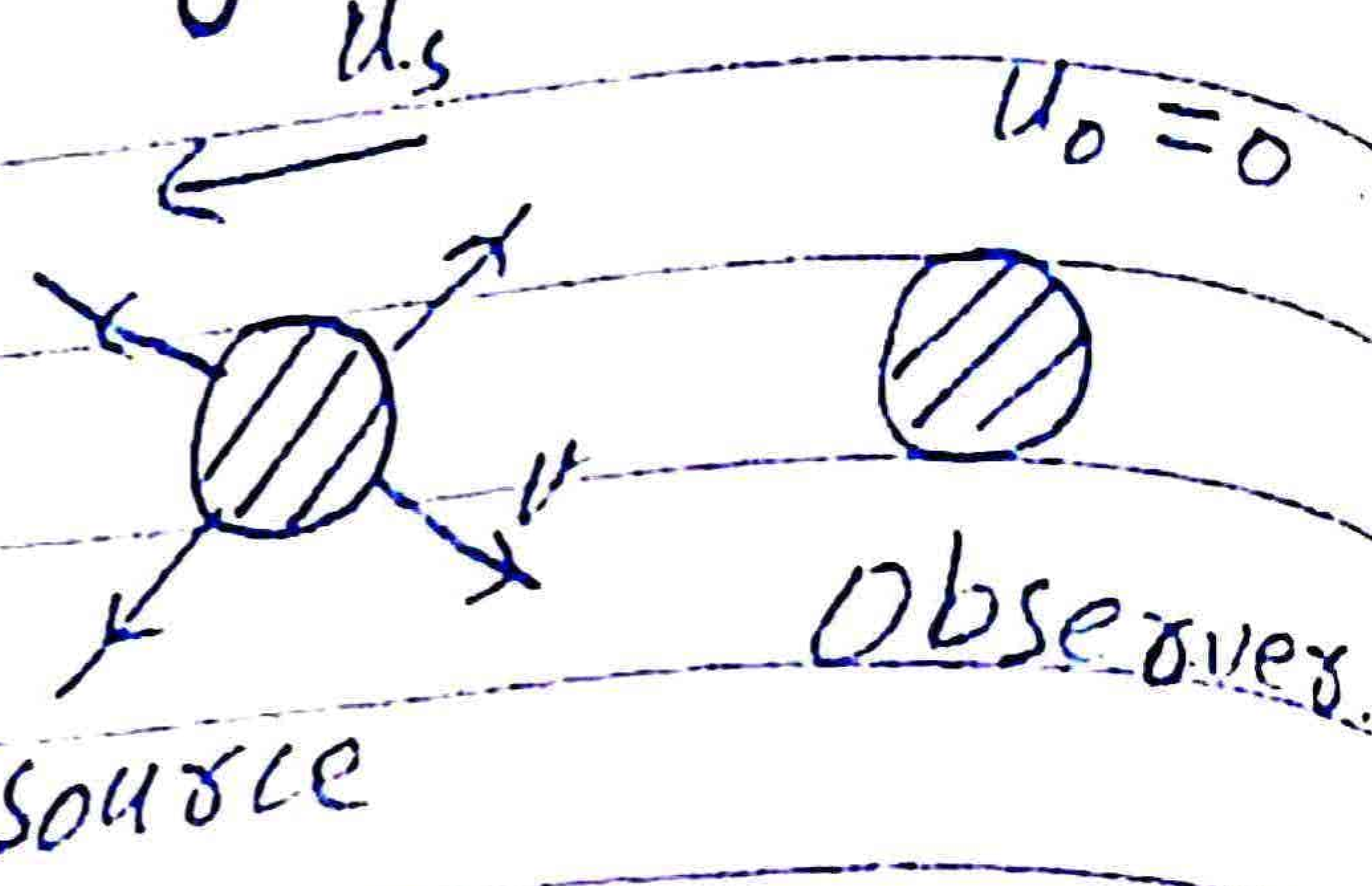
$$f_c = \left(\frac{v}{v - u_s}\right) f$$

As $\frac{v}{v - u_s} > 1$

so, $f_c > f$

Result: when source moves towards the stationary observer, the apparent frequency will increase.

Case. 4: When source of sound moves away from the stationary observer:



when source of sound moves away from the stationary observer with speed u_s. Then the waves between source and observer will be stretched by an amount called Doppler shift ($\Delta\lambda$) given by:

$$\Delta\lambda = \frac{u_s}{f}$$

The wavelength for the observer after stretch will be:

$$\lambda_D = \lambda + \Delta\lambda$$

$$\lambda_D = \frac{v}{f} + \frac{u_s}{f}$$

$$\lambda_D = \frac{v + u_s}{f}$$

According to wave equation

$$f_D = \frac{v}{\lambda_D}$$

$$= \frac{v}{\frac{v+u_s}{f}}$$

$$f_D = \left(\frac{v}{v+u_s}\right) f$$

As $\frac{v}{v+u_s} < 1$

So, $f_D < f$

Result: When source of sound moves away from the stationary observer, the apparent frequency will decrease.

Q: What is radar? How is it useful to determine the motion of aeroplane?

Radar is a device, which transmits and recieves radio waves. If an aeroplane approaches towards the radar, then the wavelength of the wave reflected from the aeroplane would be shorter and if it moves away, then the wavelength would be larger.

Q: What is Sonar? Write its uses.

Sonar is an acronym derived from "Sound navigation and ranging". The general name for sonic or ultrasonic underwater echo-ranging and echo sounding system. Sonar is the name of technique for detecting the presence of objects underwater by acoustical echo.

Its known military applications include the detection and location of submarines, control of antisubmarine weapons, mine hunting and depth measurement of sea.

Q: What is blue shift?

Stars moving towards the Earth show a blue-shift. This is because the wavelength of light emitted by the star are shorter than if the star had been at rest. So, the spectrum is shifted towards shorter wavelength, i-e, to the blue end of the spectrum.

Red shift:

Stars moving away from the earth show a red shift. The emitted waves have a longer wavelength than if the star had been at rest. So the spectrum is shifted towards longer wavelength, i-e towards the red end of the spectrum.

M.C.Q:

Astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speeds.

Q: What is Radar speed trap?

Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off by any car in the path of microwaves in between sending out bursts. The transmitter is open to detect reflected microwaves. If the reflection is caused by a moving obstacle, the reflected microwaves are Doppler shifted. By measuring the Doppler shift, the speed at which the car moves is calculated by computer programme.

Short Questions

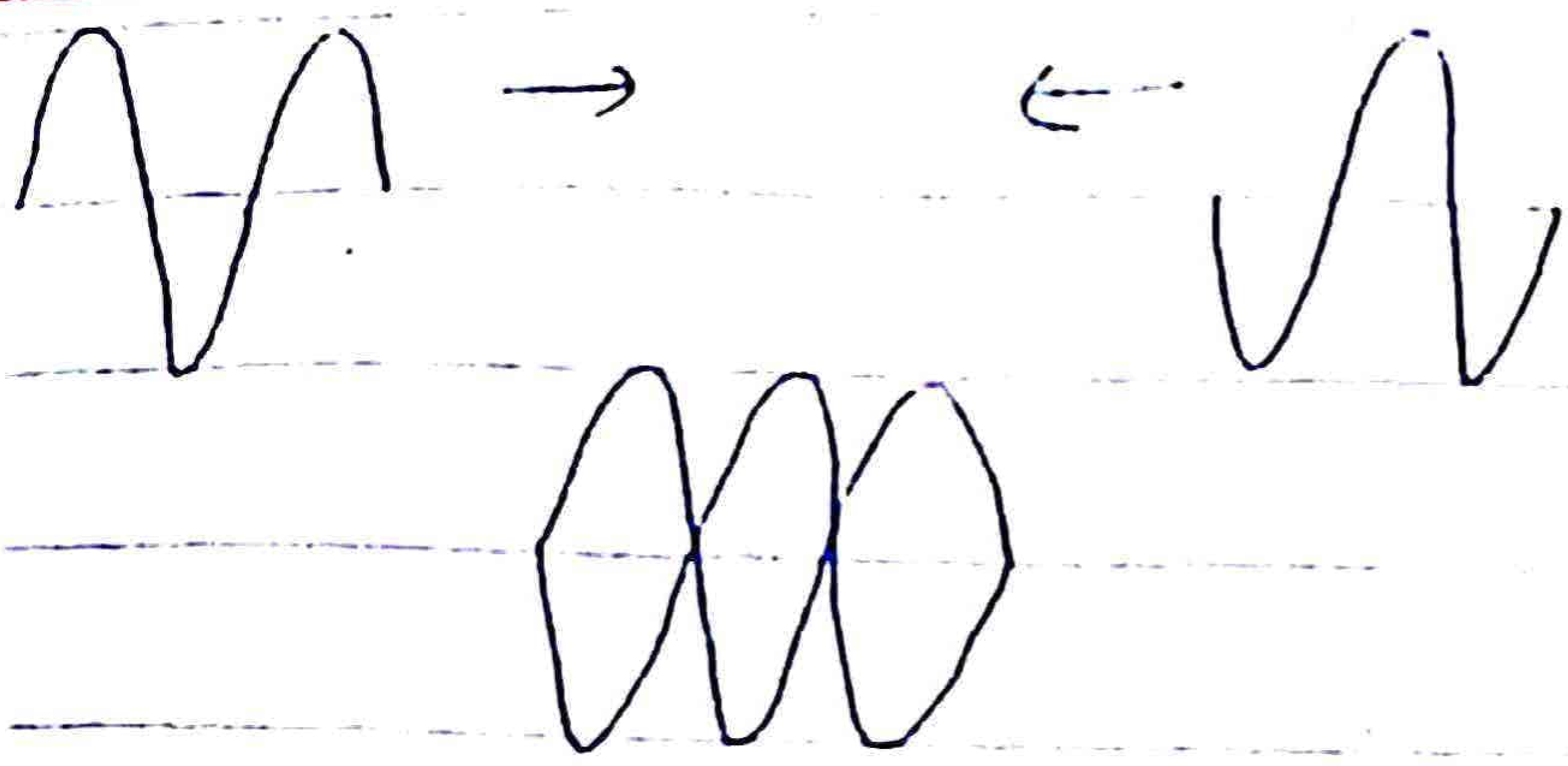
8.1: Longitudinal waves have some common features with transverse wave such as:

i- Both the waves transfer energy.

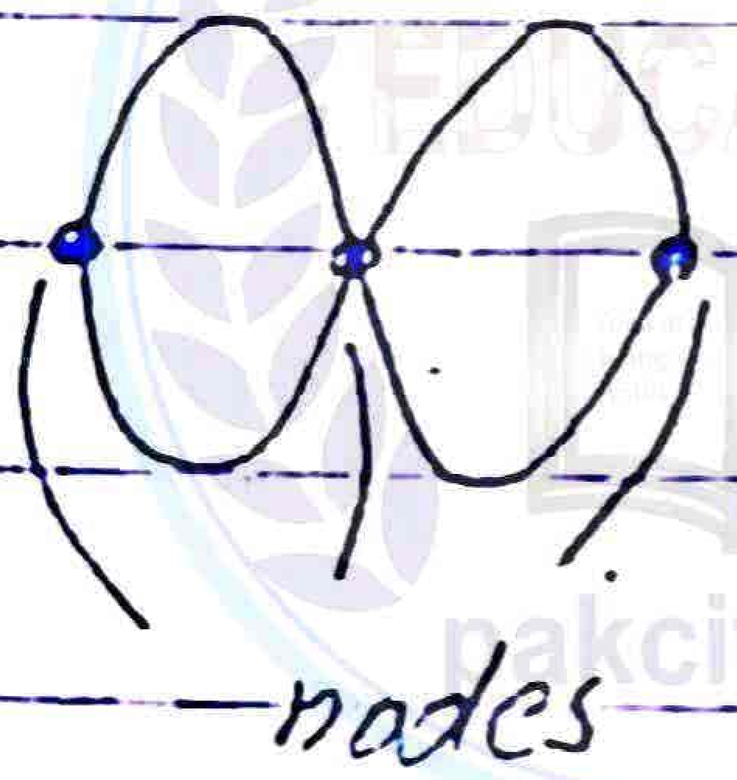
ii- Both the waves are progressive.

iii- Both the waves follow the wave equation $v = f\lambda$.

8.3: No, it is not possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave. Stationary waves are formed when two waves of equal frequency travelling in opposite direction superpose.

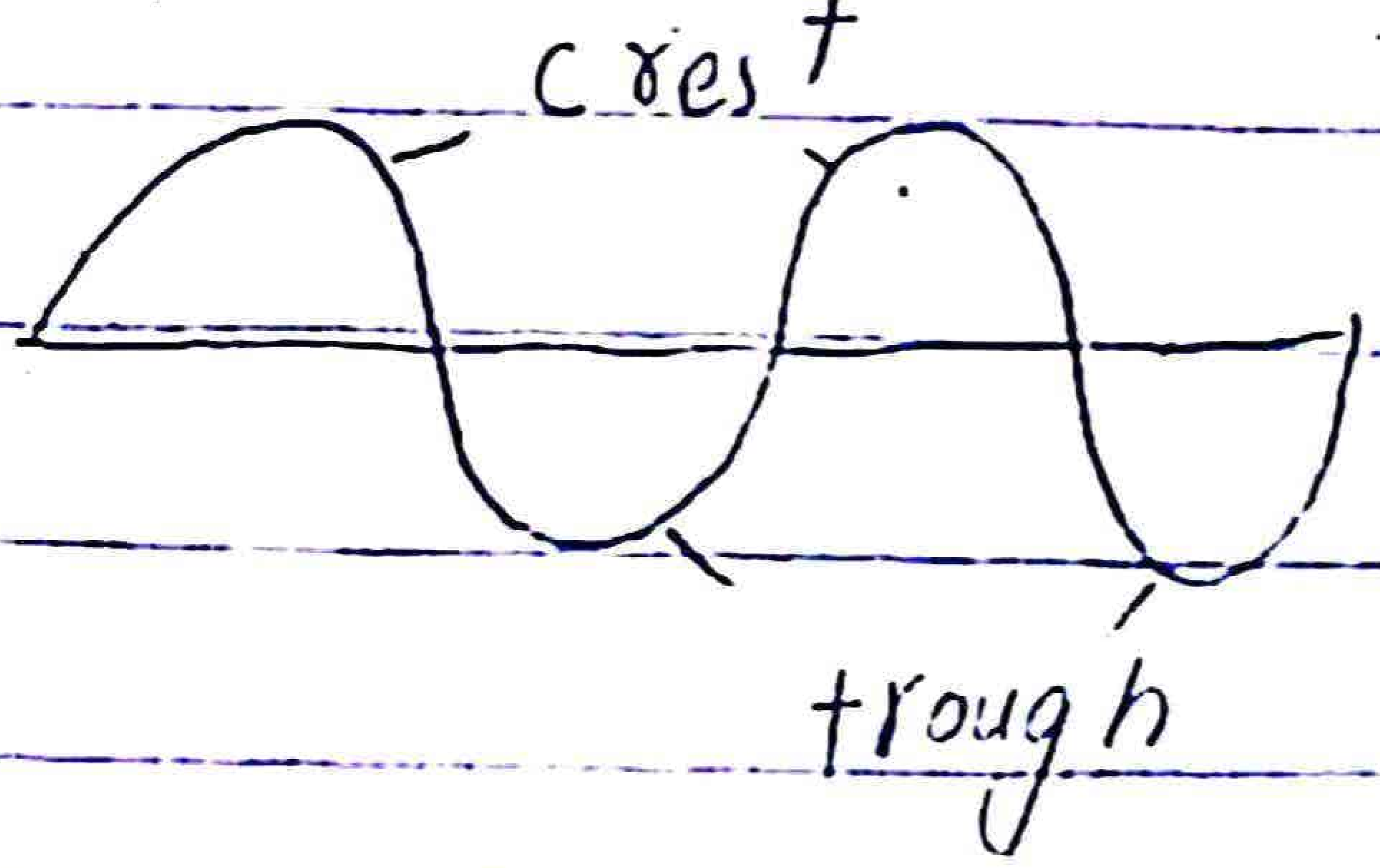


8.4: It is stationary wave. Because in stationary wave nodes are the points that always show zero displacement of particles.



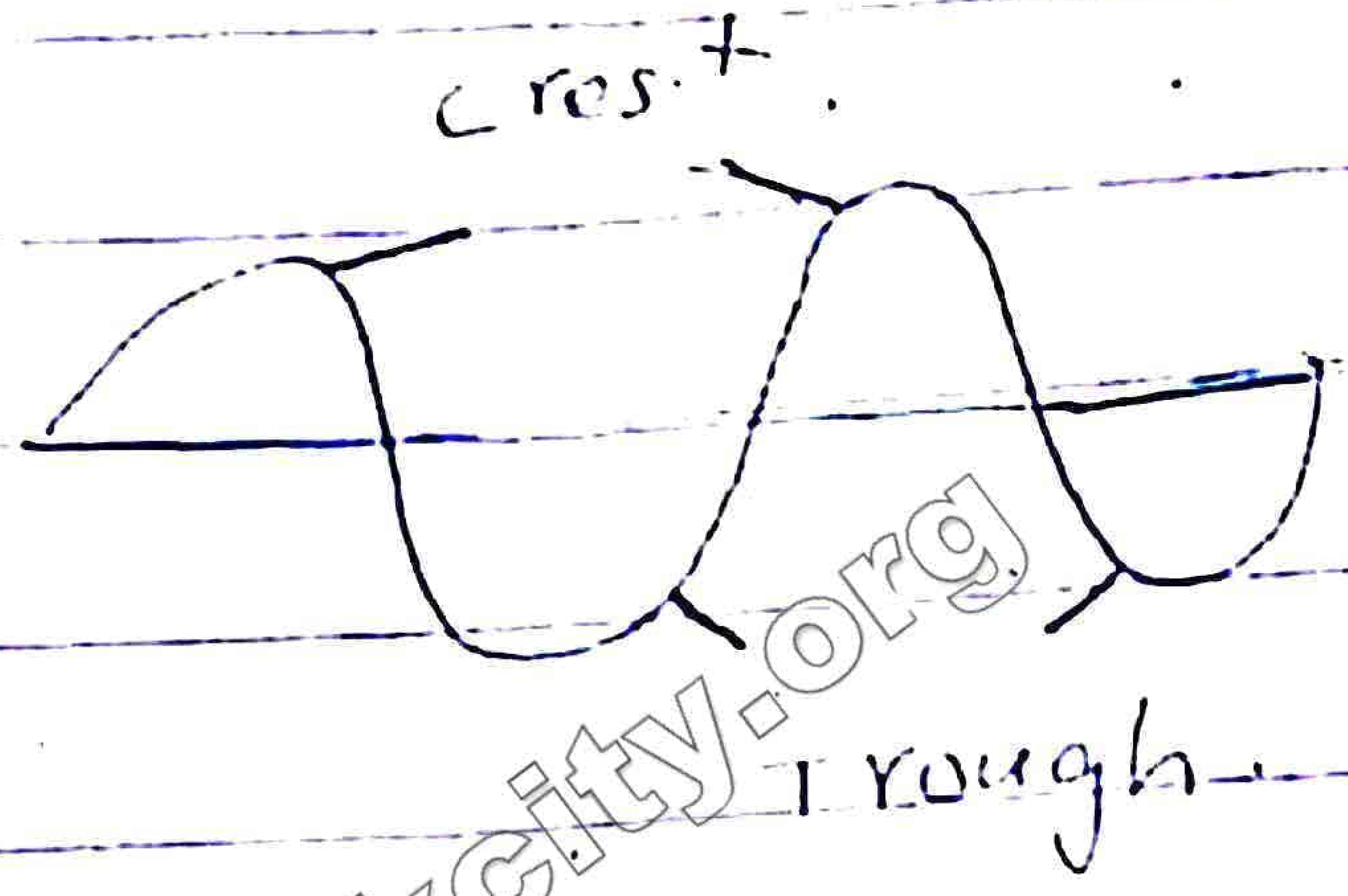
8.5: Crest:

In the transverse wave, the portion above the mean position is called crest.



Trough:

In the transverse wave, the portion below the mean position is called trough.

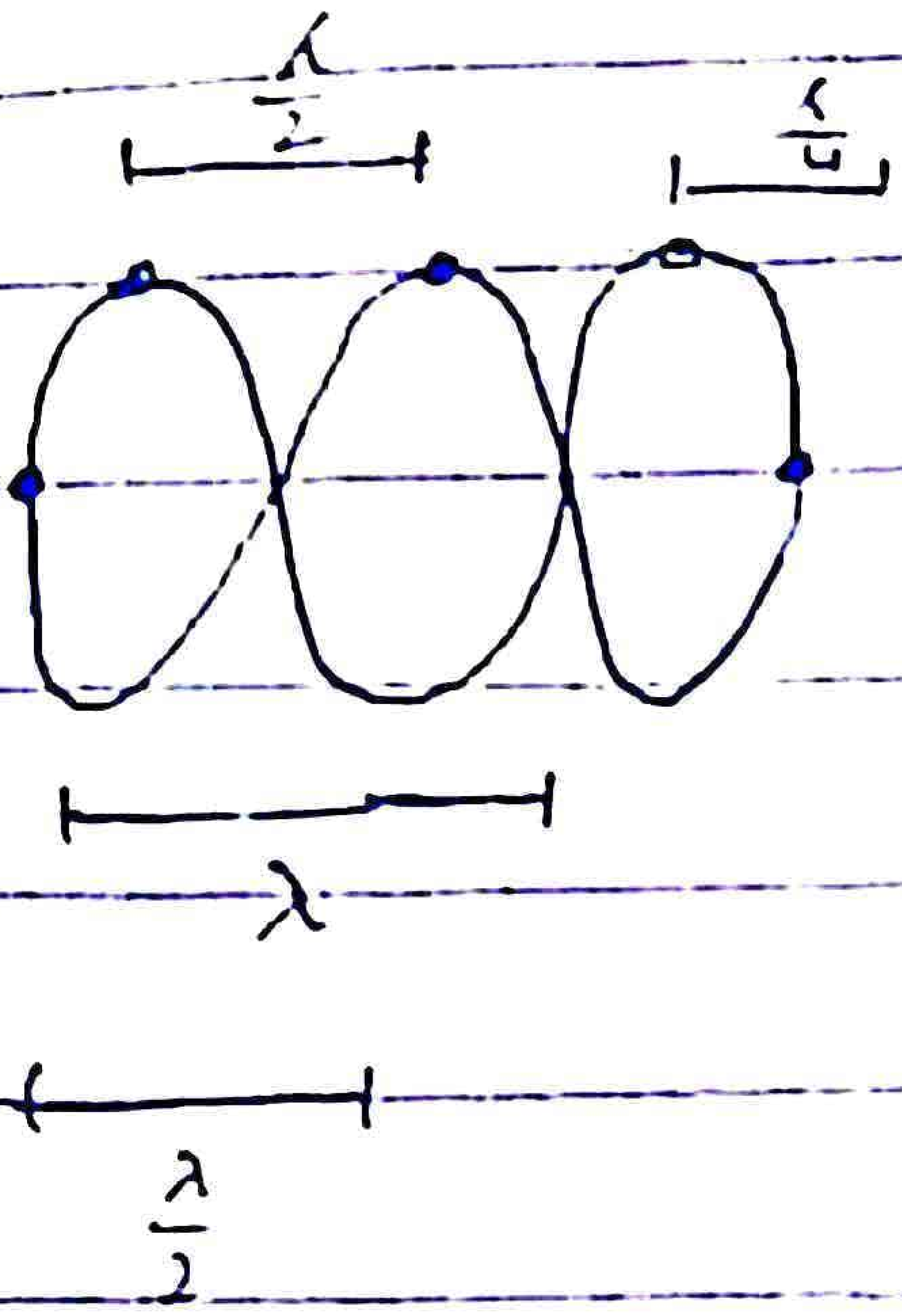


Nodes:

In the stationery wave, the points that show zero displacement are called nodes. These points lie on the mean position.

Anti-nodes:

In the stationery wave, the points that show maximum displacement are called anti-nodes.



86: Speed of Sound in Air:

The speed of sound through any medium is

$$V = \sqrt{\frac{E}{\rho}}$$

Here E is the modulus of elasticity and ρ is the density of medium.

The speed of sound through solids is greater than through gases. Because in solids, the molecules are closely packed, and transfer sound energy faster. As the density of solid is greater than gases but the modulus of elasticity for solids is also greater. So the ratio of $\frac{E}{\rho}$ for the solid is greater than gases. therefore, speed of sound through solids is greater.

$$V_{solid} > V_{gases}.$$

8.7. How are beats useful in tuning musical instruements?

We can use beats to tune a String instruement, such as piaone or violin, by beating a note against a note of known frequency. The string can then be adjusted to the derived frequency by tightening or loosing until no beats are heard.

8.9: As a result of a distant explosion, an observer senses a ground tremor and then hears the explosion, it is due to the difference of sound waves moving through ground and air. The speed of sound through solid is greater than speed through air.

$$V_{solid} > V_{air}$$

8.10: Sound travels faster in warm air than in cold air because in warm air molecules are vibrating with greater amplitude moreover, the speed of sound is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

For warm air, the air density will be smaller. So speed of sound will be greater.

8.11: Following two conditions can be made such that source of sound moves with respect to an observer so that the frequency of its sound does not change

i- when both source and observer move in same direction with the same speed.

ii- When observer is at centre and source moves around it in a circle.

Numerical Problems

8.1 Data

wavelength $= \lambda_1 = 1500\ m$

frequency $= f_1 = 200\ KHZ$

$f_1 = 200000\ HZ$

second wavelength $= \lambda_2 = ?$

second frequency $= f_2 = 1000\ KHZ$

$f_2 = 1000000\ HZ$

speed $= v = ?$

Solution According to wave equation

$$v = f\lambda$$

so, $$f_1 \lambda_1 = f_2 \lambda_2$$

$$\frac{f_1 \lambda_1}{f_2} = \lambda_2$$

$$\lambda_2 = \frac{(200000)(1500)}{1000000}$$

$$\boxed{\lambda_2 = 300\ m}$$

Now

$$v = f_1 \lambda_1$$
$$= (200000)(1500)$$

$$\boxed{v = 3 \times 10^{8}\ ms^{-1}}$$

8.3 Data

length of string $= l = 120cm$

$$l = \frac{120}{100}\ m = 1.2\ m$$

number of segments $= n = 4$

frequency $= f_4 = 120\ HZ$

wave length $= \lambda = ?$

fundamental frequency $= f_1 = ?$

Solution $\quad f_n = n f_1$

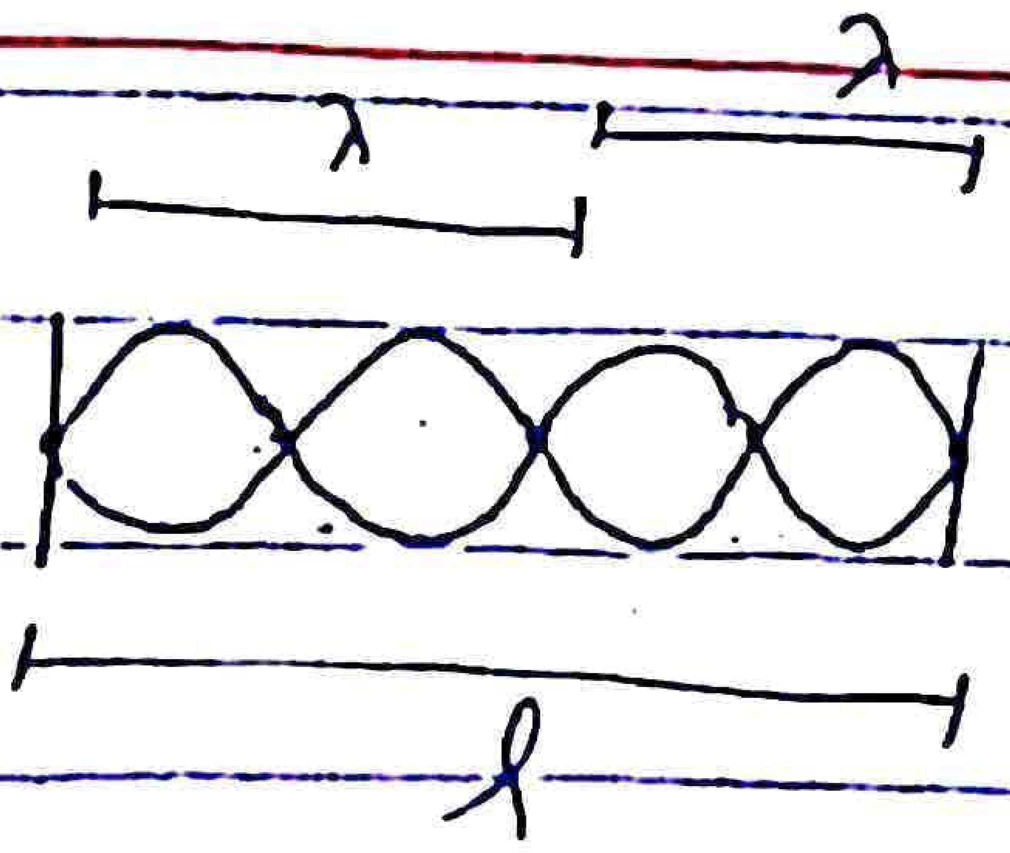
$$f_4 = 4 f_1$$

$$\frac{f_4}{4} = f_1$$

$$f_1 = \frac{120}{4}$$

$$\boxed{f_1 = 30\ HZ}$$

For four segments



$$l = 2\lambda$$

$$\frac{l}{2} = \lambda \Rightarrow \lambda = \frac{1.2}{2}$$

$$\boxed{\lambda = 0.6\ m}$$

8.4 Data

frequency $= f = 300\ Hz$

a) changed frequency $= f' = ?$

$$\lambda' = \lambda - \frac{\lambda}{3}$$

$$\lambda' = \frac{3\lambda - \lambda}{3} = \frac{2\lambda}{3}$$

b) changed frequency $= f' = ?$

$$F' = F + \frac{F}{3} = \frac{3F + F}{3} = \frac{4F}{3}$$

Solution According to wave equation

$$v = f\lambda$$

when the wavelength changes frequency will also change but speed will remain same so,

$$f\lambda = f'\lambda'$$

$$f\cancel{\lambda} = f'\left(\frac{2\cancel{\lambda}}{3}\right)$$

$$f = \frac{2f'}{3}$$

$$\frac{3f}{2} = f'$$

$$f' = \frac{3(300)}{2}$$

$$\boxed{f' = 450\ Hz}$$

b) As

$$f = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

Now

$$f' = \frac{1}{2l}\sqrt{\frac{F'}{m}}$$

$$f' = \frac{1}{2l}\sqrt{\frac{\frac{4F}{3}}{m}}$$

$$f' = \frac{1}{2l}\sqrt{\frac{4F}{3m}}$$

$$f' = \sqrt{\frac{4}{3}}\left(\frac{1}{2l}\sqrt{\frac{F}{m}}\right)$$

$$= \sqrt{\frac{4}{3}}\,(f)$$

$$f' = \sqrt{\frac{4}{3}}\,(300)$$

$$\boxed{f' = 346\ HZ}$$

8.5 Data

length of organ pipe $= l = 50cm$

$l = \frac{50}{100} m = 0.5m$

fundamental frequency $= f_1 = ?$

next harmonic $= ?$

a) when pipe is open at both ends

b) when pipe is closed at one end

speed of sound $= v = 350 ms^{-1}$

Solution

a) when pipe is open at both ends

$$f_n = \frac{nv}{2l}$$

$$f_1 = \frac{(1)v}{2l}$$

$$= \frac{350}{2(0.5)}$$

$$\boxed{f_1 = 350\ Hz}$$

next harmonic $= f_2 = 2f_1$

$$f_2 = 2(350)$$

$$\boxed{f_2 = 700\ Hz}$$

b) when pipe is closed at one end

$$f_n = \frac{nv}{4l}$$

$$f_1 = \frac{(1)(350)}{4(0.5)}$$

$$\boxed{f_1 = 175\ Hz}$$

next harmonic $= f_3 = 3f_1$

$$f_3 = 3(175)$$

$$\boxed{f_3 = 525\ Hz}$$

8.6 Data

minimum length $= l_{min} = 30\ mm$

$l_{min} = 30 \times 10^{-3}\ m$

maximum length $= l_{max.} = 4m$

frequency range $= ?$

speed of sound $= v = 340\ ms^{-1}$

Solution

For organ pipe whose one end is open and other is closed.

$$f = \frac{v}{4l}$$

$$f_{max.} = \frac{v}{4l_{min.}}$$

$$= \frac{340}{4(30 \times 10^{-3})}$$

$$f_{min} = \frac{v}{4l_{max.}}$$

$$= \frac{340}{4(4)}$$

$f_{max} = 2833\ Hz$, $f_{min} = 21\ Hz$

So, the required frequency range is 21 Hz to 2833 Hz.

8.7 Data

beat frequency = 3 Hz

frequency of one fork = $f_1 = 256\ Hz$

beat frequency = 1 Hz
after wax

frequency of other tuning fork = f_2 = ?

Solution

beat frequency = difference of frequencies

$$\pm 3 = f_1 - f_2$$

$$f_2 = f_1 \pm 3$$

$$f_2 = 256 \pm 3$$

$f_2 = 256 + 3$, $f_2 = 256 - 3$

$f_2 = 259 \text{ HZ}$, $f_2 = 253 \text{ HZ}$

As the frequency is lowered.

so,

$$\boxed{f_2 = 253 \text{ HZ}}$$

8.10 Data

wavelength $= \lambda' = 478 \text{ nm}$

$\lambda' = 478 \times 10^{-9} \text{ m}$

wavelength measured in laboratory $= \lambda = 397 \text{ nm}$

$\lambda = 397 \times 10^{-9} \text{ m}$

a) Is the galaxy moving towards or away from the earth?

b) speed of galaxy $= u_s = ?$

speed of light $= c = 3 \times 10^8 \text{ ms}^{-1}$

Solution a) As the wavelength from galaxy is greater than wavelength measured in laboratory.

$$\lambda' > \lambda$$

So, galaxy is moving

away from the earth.

b) As the source is moving away from observer.

$$f' = \left(\frac{v}{v+u_s}\right) f$$

$$\frac{c}{\lambda'} = \left(\frac{c}{c+u_s}\right)\frac{c}{\lambda}$$

$$\lambda(c+u_s) = c\lambda'$$

$$c\lambda + \lambda u_s = c\lambda'$$

$$\lambda u_s = c\lambda' - c\lambda$$

$$= c(\lambda' - \lambda)$$

$$\lambda u_s = (3\times10^8)(478\times10^{-9} - 392\times10^{-9})$$

$$(397\times10^{-9})\, u_s = 24.3$$

$$u_s = \frac{24.3}{397\times10^{-9}} \Rightarrow \boxed{u_s = 6.1\times10^7\, ms^{-1}}$$