Ch # 07

Oscillations

Oscillatory / vibratory motion:

The to and fro motion of a body about a fixed position is called oscillatory or vibratory motion.

Periodic motion:

The vibratory motion of a body that repeats itself after a specific interval of time is called periodic motion.

Examples:

a: a mass, suspended from a spring, when pulled down and then released, starts oscillating.

b: The bob of a simple pendulum when displaced from its rest position and released, vibrates.

c: a streel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.

d: a steel ball rolling in a curved dish, oscillates about its rest position.

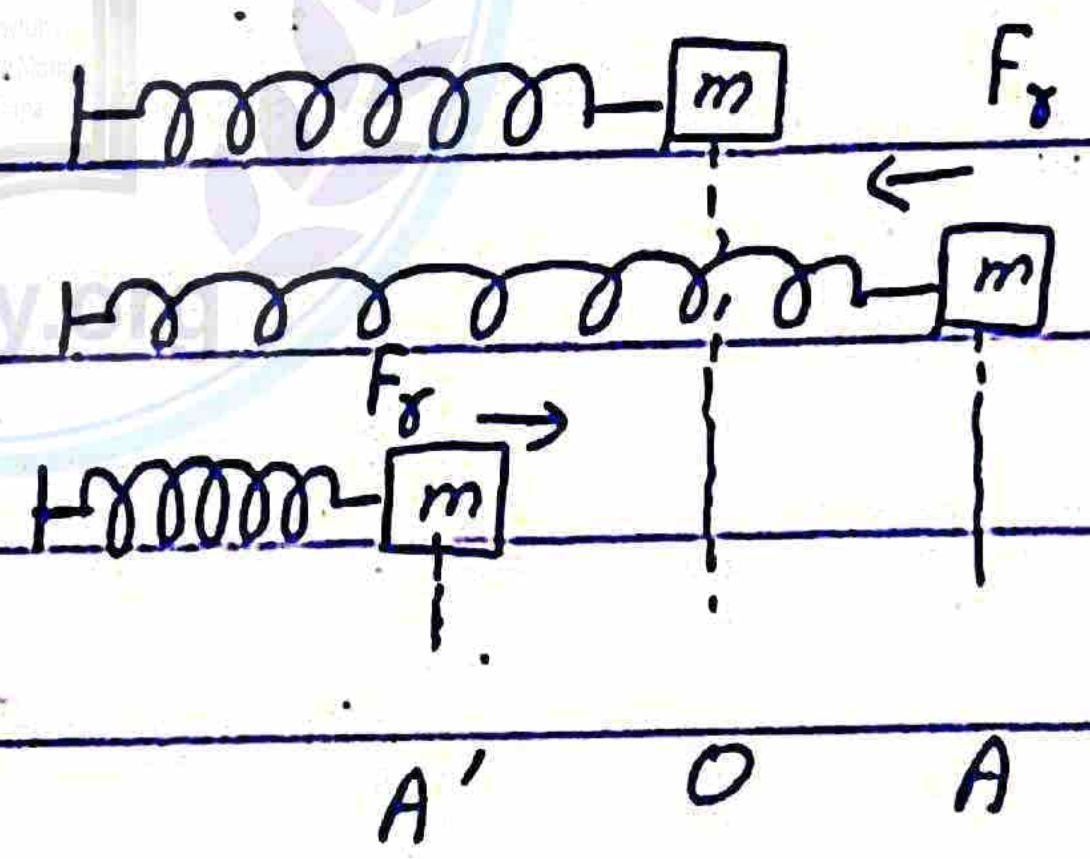
Simple harmonic motion:

Defination:

The oscillatory or vibratory motion of a body in which acceleration is directly proportional to displacement and always directed towards the mean position is called simple harmonic motion.

Explanation:

Consider a body of mass "m" attached to a spring of spring constant "k" placed on a frictionless surface



as shown in figure. We take the mass to extreme position A and release, the mass will start oscillating about the mean position O. According to Hook's law, restoring force is given by:

$$F_r = -Kx \rightarrow (1$$

Here K is spring constant and its value depends upon the stifness of the spring. For a soft spring K will be small and for a hard spring K will be greater. Its SI unit is Nm^{-1}.

According to Newton's second law:

$$F = ma \rightarrow (2$$

Comparing eq.(1 and eq.(2

$$ma = -Kx$$

$$a = -\frac{K}{m}x$$

$$a = -(\text{constant})\,x$$

$$a \propto -x$$

This is the mathematical relation for simple harmonic motion.

Variation in restoring force:

For the SHM, the force that push or pull the object towards mean position is called restoring force.

i- Restoring force will be zero at mean position.

ii- Restoring force will be maximum at extreme positions.

iii- Restoring force increases when object moves from mean position to extreme positions.

iv- Restoring force decreases when object moves from extreme to mean position.

Variation in speed:

The speed of object is maximum at mean position and zero at extreme positions.

1- Instantaneous displacement:-

The value of the distance from the mean position at any time is known as instantaneous displacement. It is zero at the instant when the body is at the mean position and is maximum at the extreme position.

Amplitude:

The maximum value of displacement is known as amplitude.

2 - Vibration: A vibration means one complete round trip of the body in motion. It is the motion of mass from its mean position to the upper extreme position from upper extreme position to lower extreme position and back to its mean position.

3- Time period: It is the time T required to complete one vibration.
Its SI unit is second (s).

4. Frequency:

Frequency f is the number of vibrations executed by a body in one second and is expressed as vibrations per second or cycles per second or hertz (Hz).
The definitions of T and f shows that the two quantities are related by the equation.

$$f = \frac{1}{T}$$

v) Angular Frequency:

If T is the time period of a body executing SHM, its angular frequency

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

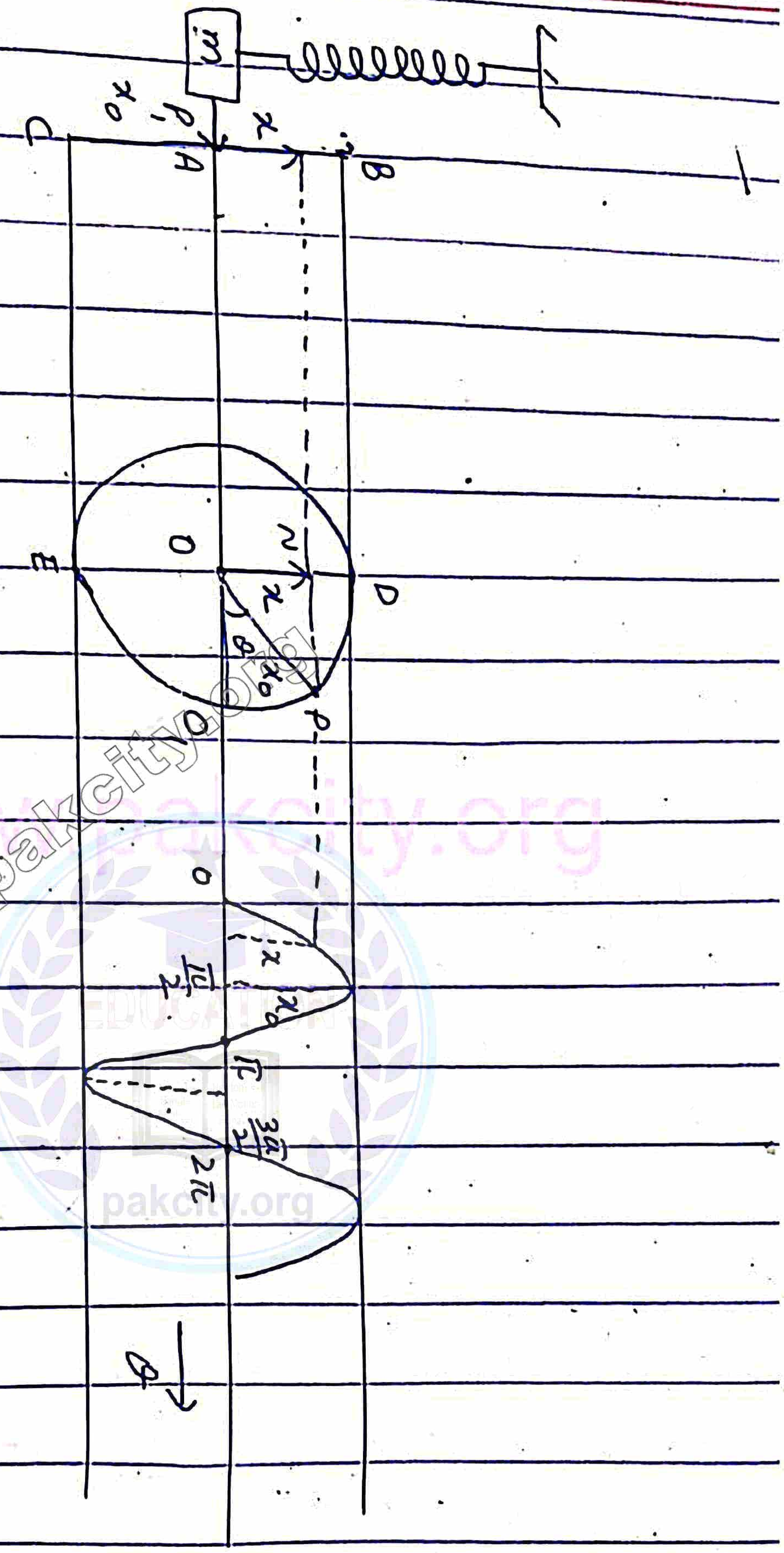
Angular frequency "ω" is basically a

characteristic of circular motion.

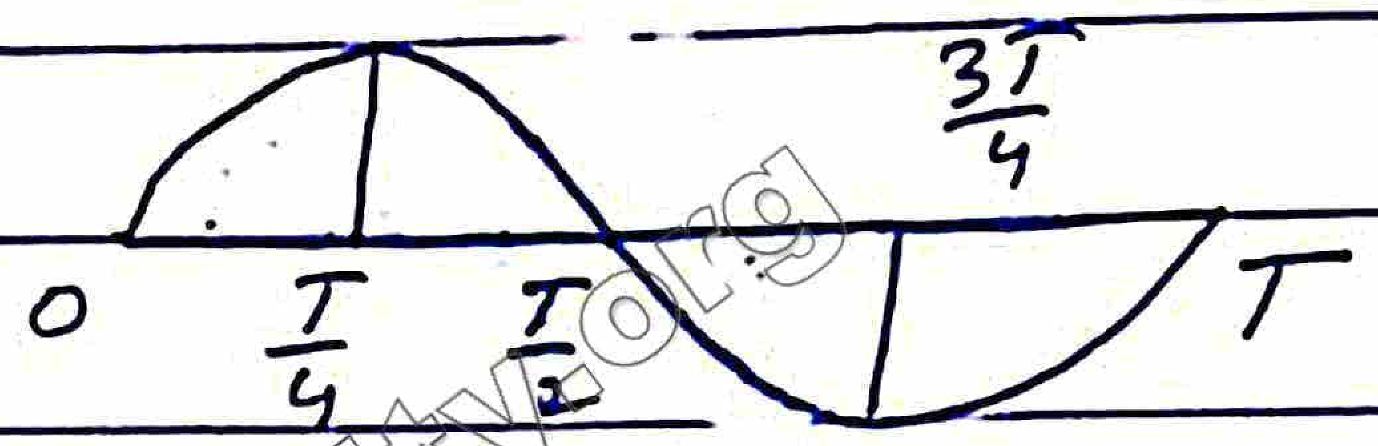
SHM And Uniform Circular Motion:

The vibratory or oscillatory motion of a body in which acceleration is directly proportional to the displacement and directed towards the mean position is called simple harmonic motion.

We will take the SHM for circular motion and wave form as well. Consider a body of mass "m" attached to a spring which is suspended vertically as shown in figure. We take the mass at extreme position and release. In this way, the body starts vibrating about its mean position. The instantaneous displacement is "x" while maximum displacement, amplitude

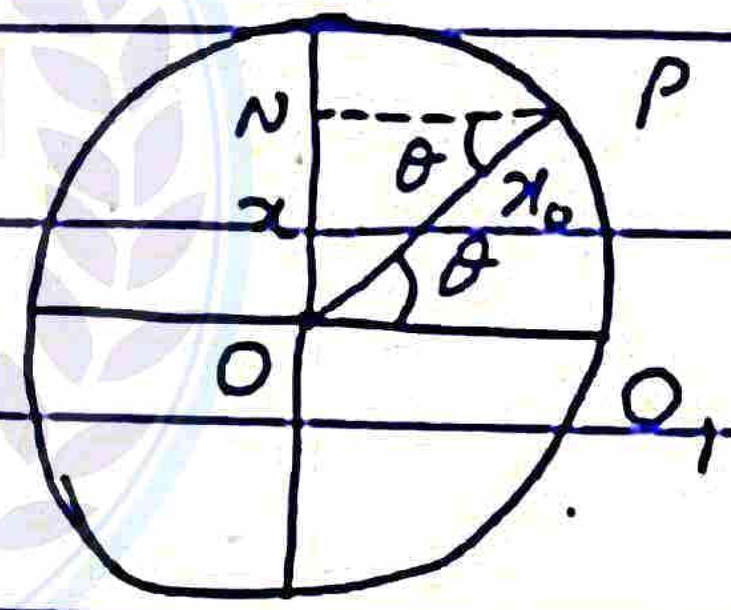


is x_0. We take the SHM in the form of circular motion for which point P is rotating in circle. Then we take the wave form for the SHM. The phase changes as $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ according to the time period which changes as $0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$



Displacement:

Consider the simple harmonic motion in circular form for which instantaneous displacement is "x" while maximum displacement amplitude is x_0. For the triangle ONP.



$$\sin\theta = \frac{\text{Perp.}}{\text{hyp.}}$$

$$\sin\theta = \frac{|ON|}{|OP|}$$

$$\sin\theta = \frac{x}{x_0}$$

$$x_0 \sin\theta = x$$

$$x = x_0 \sin\theta = x_0 \sin \omega t$$

This is the mathematical expression for instantaneous displacement. It depends upon the phase angle θ.

i- For $\theta = 0^\circ$

$$x = x_0 \sin 0^\circ$$
$$= x_0 (0)$$
$$x = 0$$

The body is at mean position.

ii- For $\theta = 90^\circ, \frac{\pi}{2}$

$$x = x_0 \sin 90^\circ$$
$$= x_0 (1)$$
$$x = x_0$$

The body is at extreme position.

iii- For $\theta = 180^\circ, \pi$

$$x = x_0 \sin 180^\circ$$
$$= x_0 (0)$$
$$x = 0$$

The body is again at the mean position.

iv- For $\theta = 270°, \frac{3\pi}{2}$

$$x = x_0 \sin 270°$$
$$= x_0 (-1)$$
$$x = -x_0$$

The body is at other extreme position.

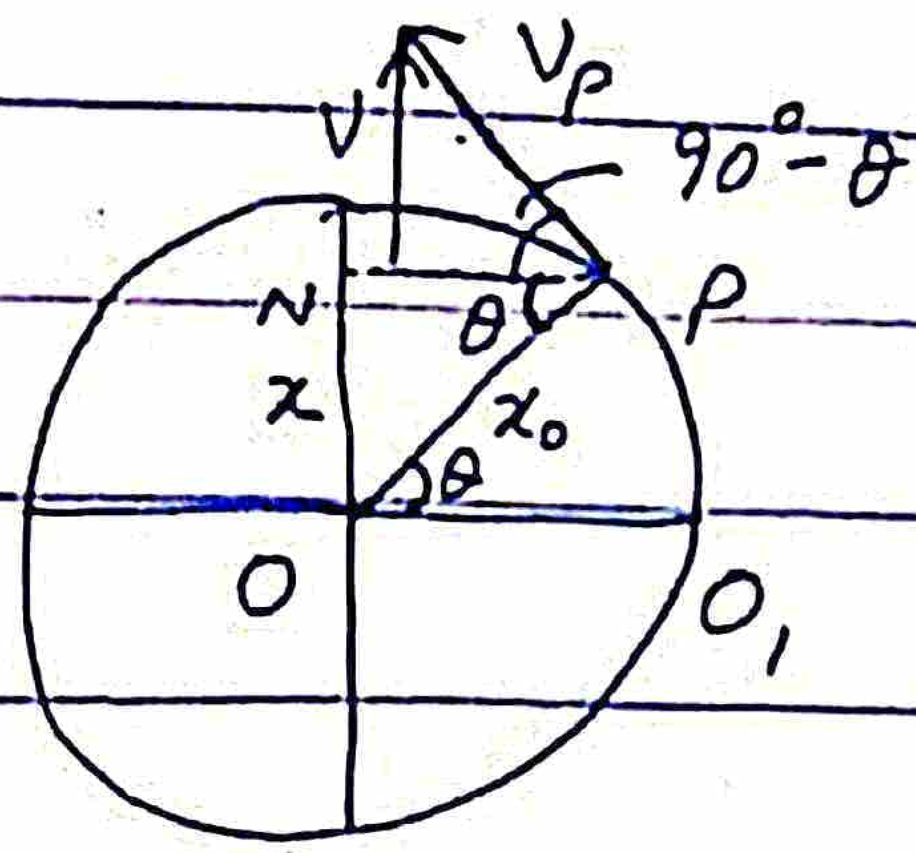
v- For $\theta = 360°, 2\pi$

$$x = x_0 \sin 360°$$
$$= x_0 (0)$$
$$x = 0$$

The body is at mean position after completing one vibration.

Instantaneous velocity:

Consider the SHM in circular form for which instantaneous displacement is x while maximum displacement amplitude is x_0. The tangential velocity is v_p at point P and instantaneous velocity is v as shown in the figure. The tangential velocity and angular velocity are related as:



$$v = r\omega$$

So,

$$V_p = x_0 \omega$$

For the triangle formed above the circle.

$$\sin(90^\circ - \theta) = \frac{\text{perp}}{\text{hyp.}}$$

$$\cos\theta = \frac{v}{V_p}$$

$$V_p \cos\theta = v$$

$$v = V_p \cos\theta$$

As $V_p = x_0 \omega$

So,

$$v = x_0 \omega \cdot \cos\theta \longrightarrow (1$$

$$v = x_0 \omega \cdot \cos \omega t$$

For ΔONP

$$\sin\theta = \frac{Perp.}{Hyp.} = \frac{|ON|}{|OP|}$$

$$\sin\theta = \frac{x}{x_0}$$

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$= 1 - \left(\frac{x}{x_0}\right)^2$$

$$= 1 - \frac{x^2}{x_0^2}$$

$$\cos^2\theta = \frac{x_0^2 - x^2}{x_0^2}$$

$$\sqrt{\cos^2\theta} = \sqrt{\frac{x_0^2 - x^2}{x_0^2}}$$

$$\cos\theta = \frac{\sqrt{x_0^2 - x^2}}{x_0}$$

Put this value in eq. (1

$$v = \cancel{x_0}\,\omega \frac{\sqrt{x_0^2 - x^2}}{\cancel{x_0}}$$

$$v = \omega\sqrt{x_0^2 - x^2}$$

This is the expression for

instantaneous velocity of a body performing SHM.

At mean position:

For the mean position.

$x = 0$

so, $v = \omega\sqrt{x_0^2 - (0)^2}$

$= \omega\sqrt{x_0^2}$

$v = \omega x_0$

so, the velocity is maximum at mean position.

At extreme positions:

For the extreme positions.

$x = x_0$

so, $v = \omega\sqrt{x_0^2 - x_0^2}$

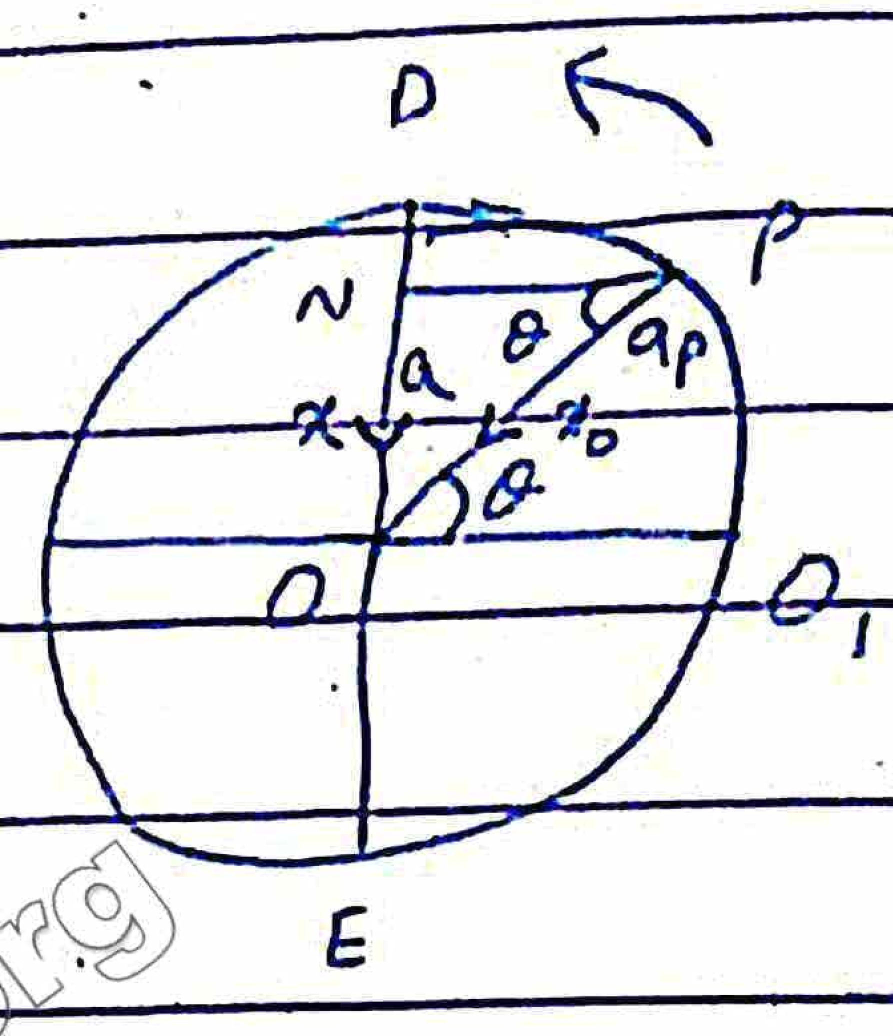
$= \omega(0)$

$v = 0$

so, the velocity is zero at extreme positions.

Acceleration In Terms Of ω:

consider the simple harmonic motion of a body in the form of circular motion



for which instantaneous displacement is x, instantaneous acceleration is a. While maximum displacement amplitude is x_0, acceleration is a_p.

For the body moving in circle, the centripetal acceleration is:

$$a_c = \frac{v^2}{r}$$

$$= \frac{(r\omega)^2}{r}$$

$$= \frac{r^{\cancel{2}}\omega^2}{\cancel{r}}$$

$$a_c = r\omega^2$$

so, $a_p = x_0 \omega^2$

Now for $\triangle ONP$

$$\sin\theta = \frac{\text{Perp.}}{\text{hyp.}} = \frac{|ON|}{|OP|}$$

$$= \frac{a}{a_p}$$

$$a_p \sin\theta = a$$

$$a = a_p \sin\theta$$

Put the value of $a_p = x_0\omega^2$.

$$a = x_0 \omega^2 \sin\theta \rightarrow (1)$$

For $\triangle ONP$

$$\sin\theta = \frac{\text{Perp.}}{\text{hyp.}} = \frac{|ON|}{|OP|}$$

$$\sin\theta = \frac{x}{x_0}$$

so, eq. (1) becomes

$$a = \cancel{x_0}\omega^2 \left(\frac{x}{\cancel{x_0}}\right)$$

$$a = \omega^2 x$$

This is the expression

for instantaneous acceleration in SHM which is always directed towards the mean position.

$$a = -\omega^2 x$$

At mean position:

At the mean position

$x = 0$

so, $a = -\omega^2 (0)$

$a = 0$

so, at the mean position the acceleration is zero.

At extreme positions:

At the extreme positions.

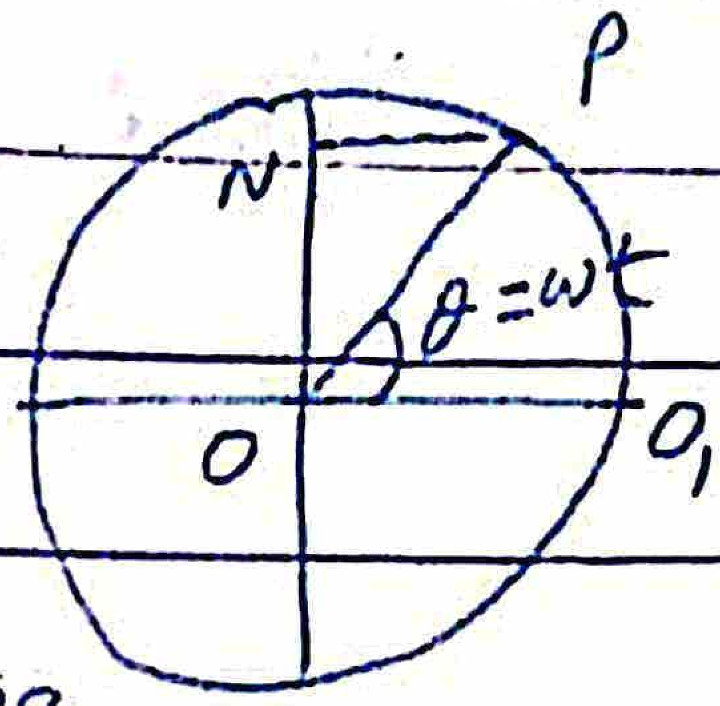
$x = x_0$

so, $a = -\omega^2 x_0$

so, at the extreme positions the acceleration is maximum.

Phase:

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM is known as phase.



The phase determines the state of motion of the vibrating point. If a body starts its motion from mean position, its phase at this point would be 0 or π. Similarly at the extreme position its phase would be $\pi/2$ and $3\pi/2$.

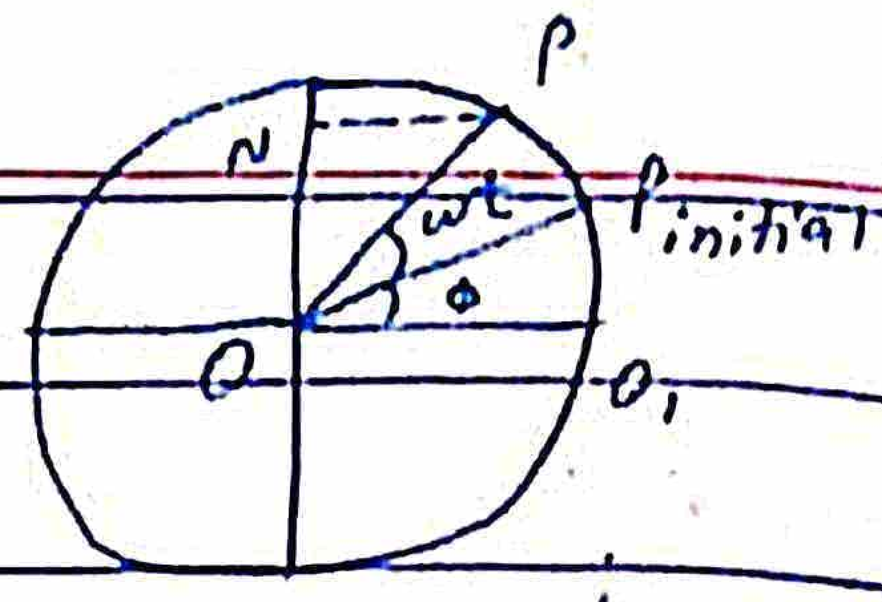
What is initial phase? What will be displacement when initial phase is 90°?

$$x = x_o \sin(\omega t + \phi)$$

Now phase angle is $\omega t + \phi$ i.e.,

$$\theta = \omega t + \phi$$

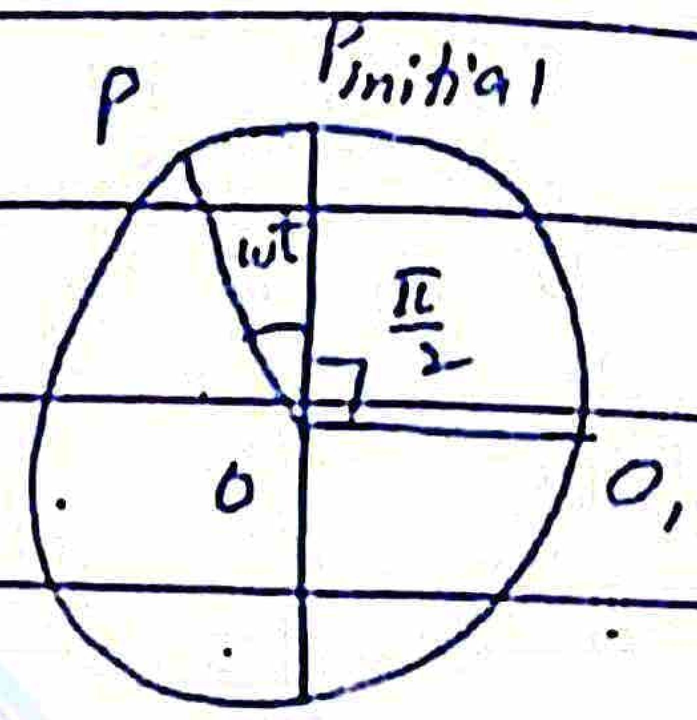
when $t = 0$, $\theta = \phi$, So ϕ is the initial phase.



Condition: If we take initial phase as $\pi/2$ or $90°$, the displacement.

$$x = x_0 \sin(\omega t + 90°)$$

$$x = x_0 \cos \omega t$$



A Horizontal Mass Spring System:

For a body of mass 'm' attached to a spring placed on a horizontal surface. producing simple harmonic motion, acceleration is directly proportional to the displacement and always directed towards the mean position. Mathematically,

$$a = -\frac{K}{m} x \rightarrow (1)$$

Here a is instantaneous acceleration, K is spring constant, m is mass of the body and x is the displacement.

Moreover, the acceleration is given by:

$$a = -\omega^2 x \rightarrow (2$$

comparing eq. (1 and eq. (2

$$-\omega^2 x = -\frac{K}{m} x$$

$$\omega^2 = \frac{K}{m}$$

$$\boxed{\omega = \sqrt{\frac{K}{m}}}$$

And

$$2\pi f = \sqrt{\frac{K}{m}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

And

$$\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

The displacement for the body executing SHM is

$$x = x_0 \sin \omega t$$

so,

$$\boxed{x = x_0 \sin \sqrt{\frac{K}{m}}\, t}$$

The instantaneous velocity is:

$$v = \omega \sqrt{x_0^2 - x^2}$$

$$= \sqrt{\frac{K}{m}} \sqrt{x_0^2 - x^2}$$

$$v = \sqrt{\frac{K}{m}(x_0^2 - x^2)}$$

$$= \sqrt{\frac{K}{m}(x_0^2)\left(1 - \frac{x^2}{x_0^2}\right)}$$

$$\boxed{v = x_0 \sqrt{\frac{K}{m}\left(1 - \frac{x^2}{x_0^2}\right)}}$$

The velocity is maximum at mean position when $x = 0$

$$v_0 = x_0 \sqrt{\frac{K}{m}\left(1 - \frac{0}{x_0}\right)}$$

$$\boxed{v_0 = x_0 \sqrt{\frac{K}{m}}}$$

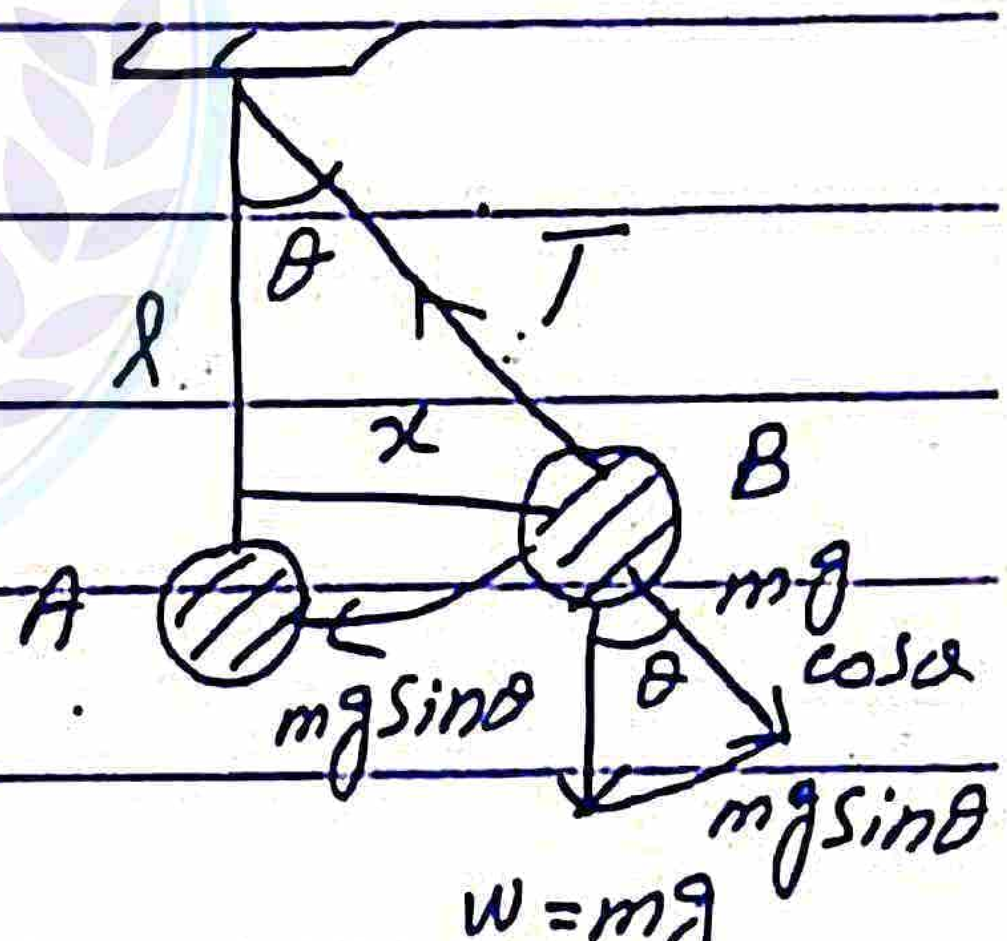
so, $\boxed{V = V_0\sqrt{1-\frac{x^2}{x_0^2}}}$

Simple Pendulum

Defination:

Simple pendulum consists of a small but heavy mass attached to an inextensible light string that is attached to a frictionless support.

Explanation:



Consider the mass "m" attached to a light and inextensible string of length "l" through a frictionless support. Now the mass is moved to extreme position. B making angle θ and release. Then the mass starts performing simple harmonic motion.

we resolve the weight into its rectangular components $mg \sin\theta$ and $mg \cos\theta$. Here the component $mg\cos\theta$ is acting opposite to the tension T and cancel each other while component $mg \sin\theta$ is acting towards the mean position and it acts as the restoring force. so,

$$F = -mg \sin\theta$$

For small angle $\sin\theta \approx \theta$

so,

$$F = -mg\theta$$

According to Newton's second law:

$$F = ma$$

comparing the above equations:

$$\cancel{m}a = -\cancel{m}g\theta$$

$$a = -g\theta \rightarrow (1$$

From the shown diagram

$$\theta = \frac{\text{arc } AB}{l}$$

$$\theta = \frac{x}{l}$$

Put in eq. (1)

$$a = -g\frac{x}{l}$$

$$a = -\frac{g}{l}x$$

Comparing with expression for instantaneous acceleration

$$a = -\omega^2 x$$

$$\Rightarrow \quad \omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

Now

$$2\pi f = \sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$T = \frac{1}{f}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Result:

The time period for simple pendulum depends upon the length of pendulum(l) and acceleration due to gravity (g). It is independent of the mass attached.

What is second pendulum?

The simple pendulum whose time period is 2 seconds is called second pendulum.

$$T = 2\,s \quad , \quad f = \frac{1}{T} = \frac{1}{2}$$

$$f = 0.5\ Hz$$

And

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\cancel{2} = \cancel{2}\pi\sqrt{\frac{l}{9.8}}$$

$$\frac{1}{\pi} = \sqrt{\frac{l}{9.8}}$$

$$\left(\frac{1}{\pi}\right)^2 = \left(\sqrt{\frac{l}{9.8}}\right)^2$$

$$\frac{1}{\pi^2} = \frac{l}{9.8}$$

$$\frac{9.8}{\pi^2} = l$$

$$l = 0.99\,m$$

Energy conservation In SHM:

Statement:

Energy can neither be created nor destroyed, it can only be changed from one form to the other. However, total amount of energy at any point remains constant.

Explanation:

Consider a body of mass "m" attached to a spring performing simple harmonic motion with instantaneous displacement "x" and maximum displacement amplitude x_0. During the SHM, elastic P.E and K.E change into each other. but total amount of energy at any point will be same.

According to Hook's law the magnitude of restoring force is:

$$F = Kx$$

At mean position when $x=0$

$$F = K(0)$$

$$F = 0$$

At extreme positions when $x = x_0$

$$F = Kx_0$$

Average force $= F = \frac{0 + Kx_0}{2}$

$$F = \frac{Kx_0}{2}$$

The amount of work done in displacing the object will be

$$W = Fd$$

$$= \left(\frac{Kx_0}{2}\right)(x_0)$$

$$W = \frac{1}{2}Kx_0^2$$

This amount of work done will be stored as elastic P.E in the object.

P.E at extreme positions:

The elastic P.E will be maximum at extreme positions, given by:

$$P.E = \frac{1}{2} K x_0^2$$

P.E at mean position:

The elastic P.E will be zero at mean position.

$$P.E = \frac{1}{2} K (0)$$

$$P.E = 0$$

P.E at any instant:

The elastic P.E at any instant of time t is:

$$P.E = \frac{1}{2} K x^2$$

For K.E.

The K.E of the body is given by:

$$K.E = \frac{1}{2} m v^2$$

And

$$v = x_0 \sqrt{\frac{K}{m}\left(1 - \frac{x^2}{x_0^2}\right)}$$

So,

$$K.E = \frac{1}{2} m \left(x_0 \sqrt{\frac{K}{m}\left(1 - \frac{x^2}{x_0^2}\right)}\right)^2$$

$$= \frac{1}{2} \cancel{m} x_0^2 \left(\frac{K}{\cancel{m}}\right)\left(1 - \frac{x^2}{x_0^2}\right)$$

$$K.E = \frac{1}{2} K x_0^2 \left(1 - \frac{x^2}{x_0^2}\right)$$

K.E at any instant:

The K.E at any instant of time is

$$K.E = \frac{1}{2} K x_0^2 \left(1 - \frac{x^2}{x_0^2}\right)$$

K.E at mean position:

At mean position, $x = 0$

So,

$$K.E = \frac{1}{2} K x_0^2 \left(1 - \frac{0}{x_0^2}\right)$$

$$K.E = \frac{1}{2} K x_0^2$$

It is maximum.

K.E at extreme positions:

At extreme positions

$x = x_0$

so,
$$K.E = \frac{1}{2} K x_0^2 \left(1 - \frac{x_0^2}{x_0^2}\right)$$

$$K.E = \frac{1}{2} K x_0^2 (1 - 1)$$

$$= \frac{1}{2} K x_0^2 (0)$$

$$K.E = 0$$

Total energy at mean position:

$$\text{Total energy} = P.E + K.E$$

$$= 0 + \frac{1}{2} K x_0^2$$

$$T.E = \frac{1}{2} K x_0^2$$

Total energy at extreme positions:

$$\text{Total energy} = P.E + K.E$$
$$= \frac{1}{2} K x_0^2 + 0$$

$$T.E = \frac{1}{2} K x_0^2$$

Total energy at any instant:

Total energy = P.E + K.E

$$T.E = \frac{1}{2} K x^2 + \frac{1}{2} K x_0^2 \left(1 - \frac{x^2}{x_0^2}\right)$$

$$= \frac{1}{2} K x^2 + \frac{1}{2} K x_0^2 - \frac{1}{2} K \cancel{x_0^2} \left(\frac{x^2}{\cancel{x_0^2}}\right)$$

$$= \cancel{\frac{1}{2} K x^2} + \frac{1}{2} K x_0^2 - \cancel{\frac{1}{2} K x^2}$$

$$T.E = \frac{1}{2} K x_0^2$$

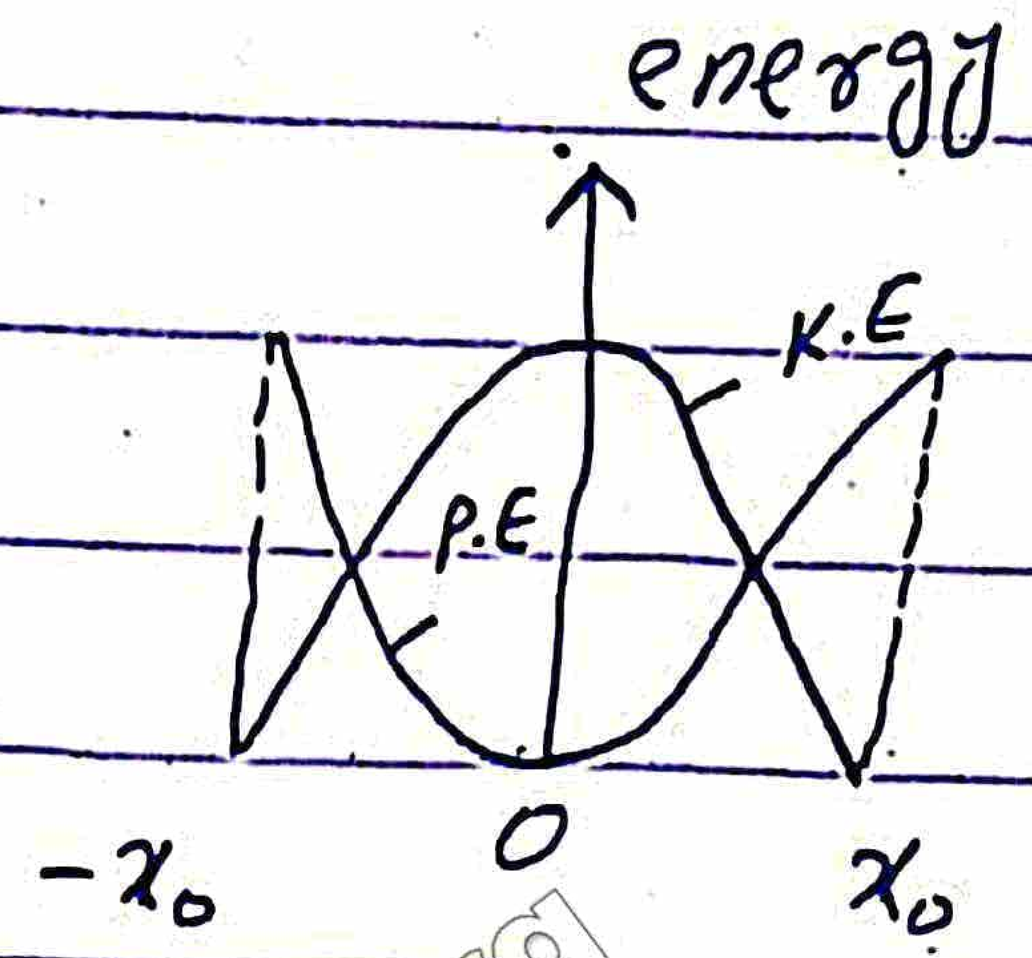
Result:

From the above discussion, we can conclude that

i- P.E is maximum at extreme positions and zero at the mean position.

ii- K.E is maximum at mean position and zero at extreme positions.

iii- Total energy at any point remains same.



Free oscillation:

A body is said to be executing free vibrations when it oscillates without the interference of an external force. The frequency of these vibrations is known as its natural frequency.

For example:

When a simple pendulum is slightly displaced from its mean position freely with its natural frequency that depends only upon the length of the pendulum.

Forced oscillations:

If a freely oscillating system is subjected to an external force, then force vibrations will take place.
Such as when the mass of a vibrating pendulum is struck repeatedly then forced vibrations are produced.

For example:

The vibrations of a factory floor caused by the running of heavy machinery is an example of forced vibration.

Driven harmonic oscillator:

A physical system under going forced vibrations is known as driven harmonic oscillator.

Resonance

Defination:

Resonance occurs when the frequency of the applied force is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator.

Mechanical resonance

Resonance in swing:

If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

→ Resonance for column of Soldiers:

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might setup oscillations of dangerously large amplitude in the bridge structure.

Electrical resonance

→ Tuning the radio:

When we turn the knob of a radio to tune a station, we are changing the natural frequency of the electrical circuit of the reciever, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

→ Microwave Oven:

The wave produced in oven have a wavelength of 12cm at a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

Damped Oscillation:

Such oscillations in which the amplitude decreases steadily with time, are called damped oscillation.

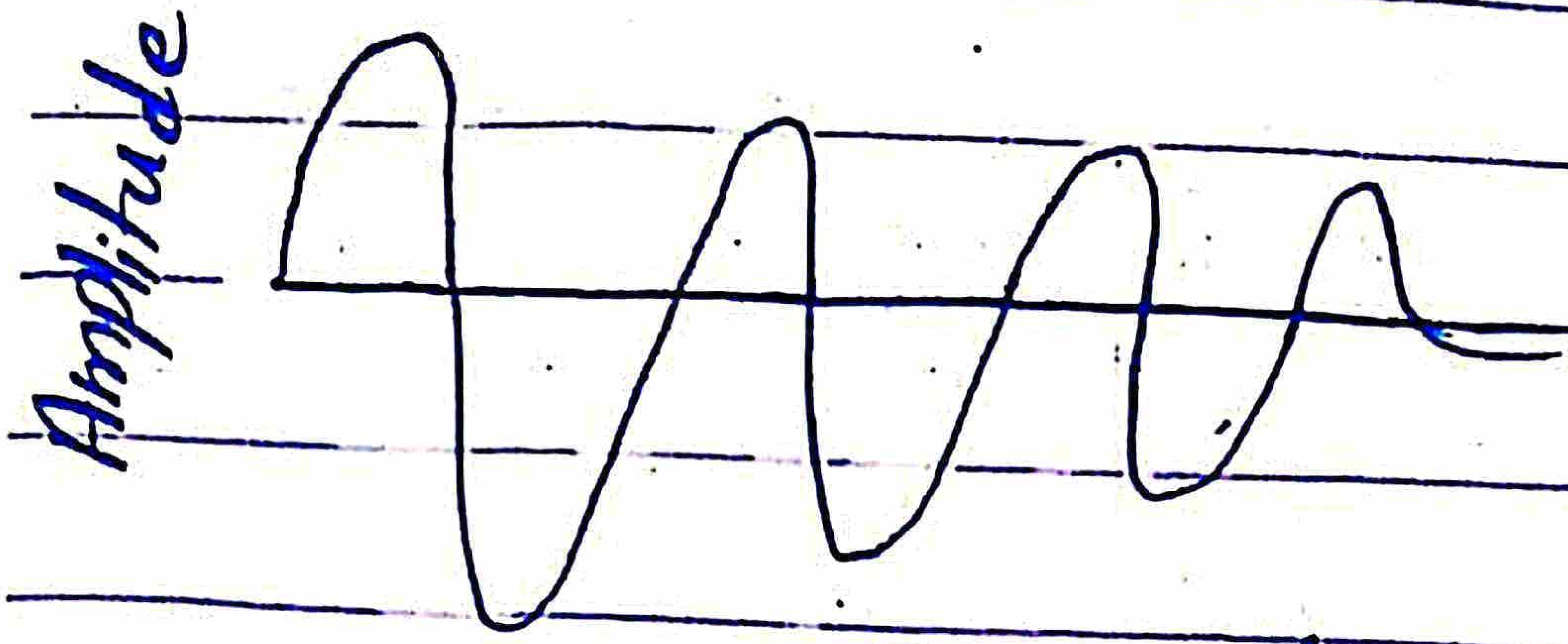
e.g The amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero.

Q: What is the reason of Damping?

In practice, the amplitude of this motion gradually becomes smaller and smaller because of friction and air resistance because the energy of the Oscillator is used up in doing work against the resistive forces.

Damping:

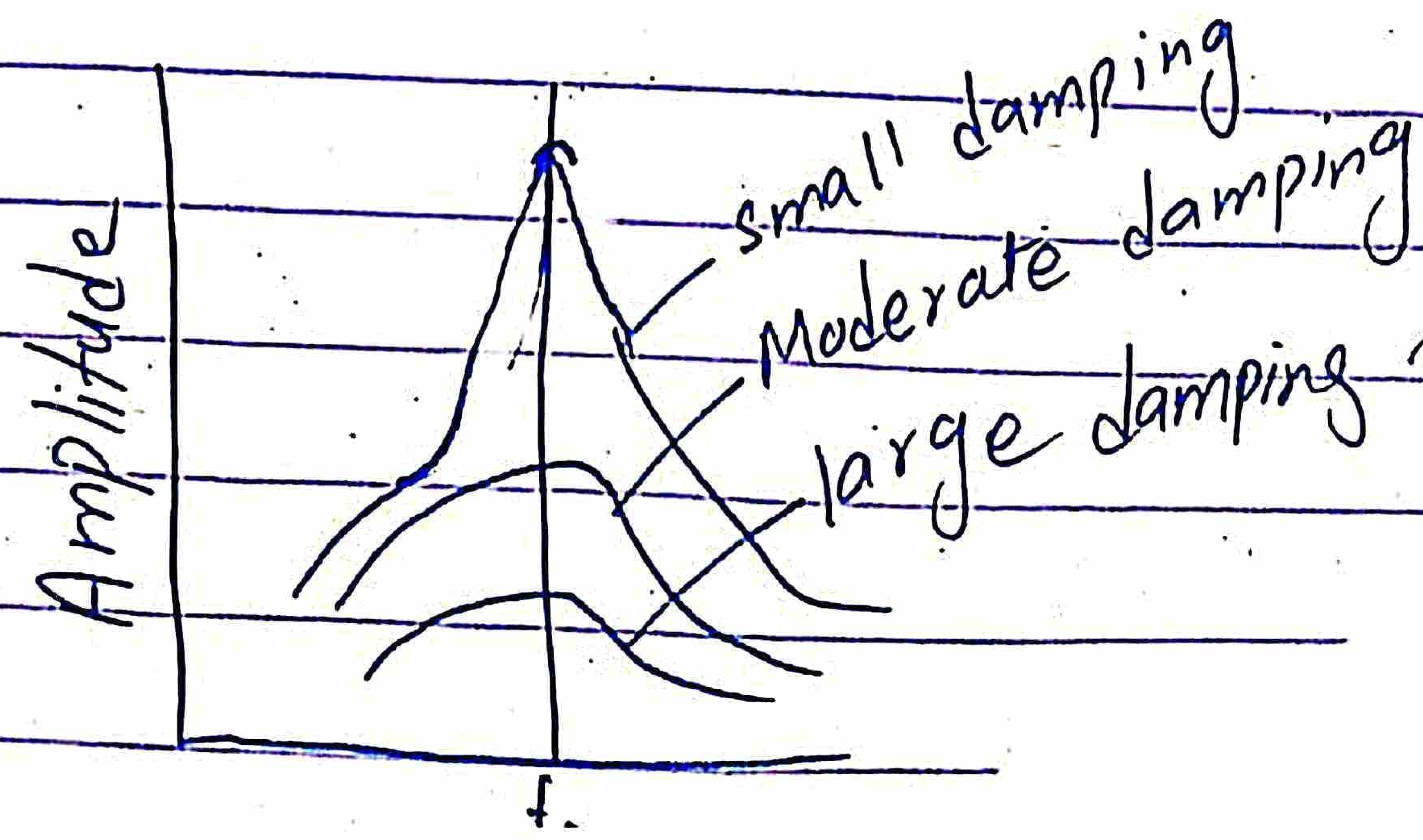
Damping is the process whereby energy is dissipated from the oscillating system.



Damped

→ Amplitude, Sharpness and damping:

The amplitude as well as its sharpness, both depend upon the damping. Smaller the damping, greater will be the amplitude and more sharp will be the resonance.



Short Questions

7.1: Two characteristics of simple harmonic motion are

i- The acceleration of the vibrating body is always directly proportional to the displacement and directed towards the mean position.

$$a \propto -x$$

ii- The velocity is maximum at mean position and zero at extreme positions.

7.2: No, the frequency for harmonic oscillators does not depend upon the amplitude. mathematically, for mass spring system:

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

for simple pendulum

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

7.3: No, we can not realize an ideal simple pendulum. Because for ideal simple pendulum following conditions should be done.

i- The bob should be small. but heavy.

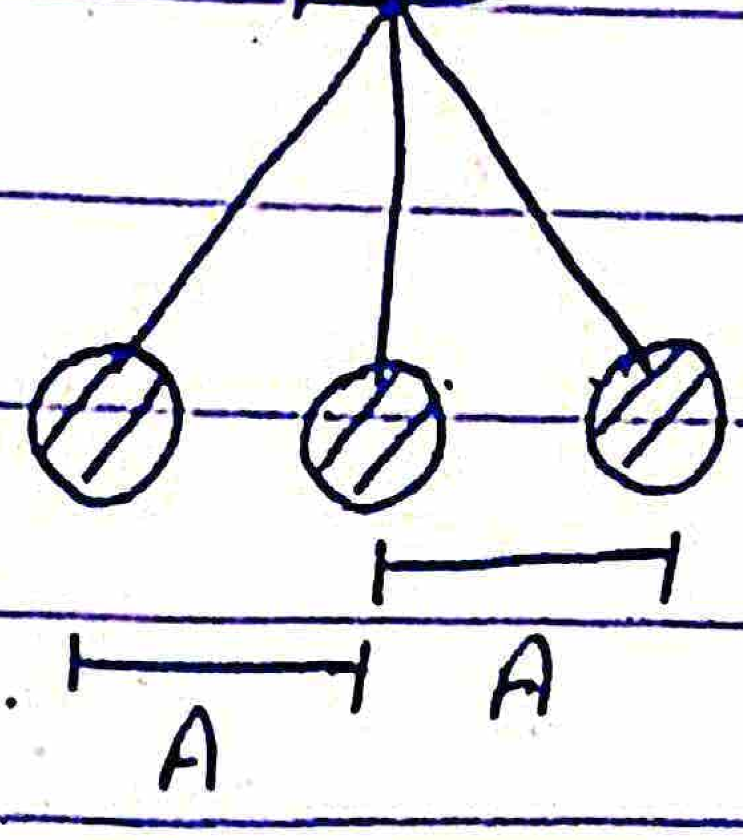
ii- The support should be frictionless.

iii- The string should be light and inextensible.

iv- There should not be air friction or gravitational acceleration their.

The above conditions can not be acheived 100% to realize an ideal simple pendulum.

7.4: The time required to complete one vibration for SHM



is called time period. So, for one vibration, total distance will be:

$$A + A + A + A = 4A$$

7.5: The time period of simple is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If the length is doubled

$$T' = 2\pi\sqrt{\frac{2l}{g}}$$

$$= \sqrt{2}\ 2\pi\sqrt{\frac{l}{g}}$$

$$T' = \sqrt{2}\,(T)$$

So, time period will be increased $\sqrt{2}$ times when length is doubled.

The time period does not depend upon the suspended mass. So, there will be no effect on time period by doubling the mass.

7.6: No, the acceleration of a simple harmonic oscillator does not remain constant during its motion. It is directly proportional to the displacement and directed towards the mean position.

$$a \propto -x$$

Yes, the acceleration will be zero at mean position.

when $x = 0$

Then $a = 0$

7.7: The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM is known as phase. The phase determines the state of motion of the vibrating point. If a body starts its motion from mean position, its phase at this point would be 0 or π. Similarly at the extreme positions its phase would be $\pi/2$ and $3\pi/2$.

7.8: The addition of two simple harmonic motions will produce a resultant which is also simple harmonic, if

i- Both the systems are parallel.

ii- Both have same frequency.

iii- Both have constant phase difference.

7.9: The acceleration and velocity for the simple harmonic motion are:

$$a = -\omega^2 x \quad \text{and} \quad v = \omega\sqrt{x_0^2 - x^2}$$

For mean position when $x = 0$

$$a = -\omega^2(0) \quad ; \quad v = \omega\sqrt{x_0^2 - 0}$$

$a = 0$, $v = \omega\sqrt{x_0^2}$

$v = \omega x_0$

So, acceleration is zero and velocity is maximum at mean position.

For extreme position when $x = x_0$

$a = -\omega^2 x_0$, $v = \omega\sqrt{x_0^2 - x_0^2}$

$= \omega(0)$

$v = 0$

So, acceleration is maximum and velocity is zero at extreme position.

7.10: (i) $y = A \sin(\omega t + \phi)$

We know that

$x = x_0 \sin(\omega t + \phi)$

So, for the given equation y is the instantaneous displacement, A is the maximum displacement amplitude, ω is the angular frequency, t is the time and φ is initial phase.

ii) $a = -\omega^2 x$

Above relation shows that acceleration for SHM is directly proportional to the displacement and always directed towards the mean position.

7.12: Resonance in swing:

If a series of regular pushes are given to the swing, its motion can be built up enormously, If pushes are given irregularly the swing will hardly vibrate.

Microwave oven:

The wave produced in oven have wavelength of 12cm at a frequency of 2450Hz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

7.11: 1- P.E is maximum at extreme position and zero at mean position.

2- K.E is maximum at mean position and zero at extreme positions.

3- Total energy at any point remains same.

7.13: If a mass spring system is hung vertically and set into oscillation, the motion eventually stops due to

i- The friction of the support.

ii- The air resistance.

iii- The restoring force of spring.

Numerical Problems

7.1 Data

mass $= m = 100\ g = \frac{100}{1000}\ kg$

$m = 0.1\ kg$, $g = 9.8\ ms^{-2}$

displacement $= x = 4\ cm = \frac{4}{100}\ m$

$x = 0.04\ m$

time period $= T = 0.568\ s$

other mass $= m' = ?$

Solution The weight of body will be balanced by restoring force.

$$mg = kx$$

$$\frac{mg}{x} = k$$

$$K = \frac{(0.1)(9.8)}{0.04}$$

$$K = 24.5 \ Nm^{-1}$$

Now

$$T = 2\pi \sqrt{\frac{m'}{K}}$$

$$0.568 = 2\pi \sqrt{\frac{m'}{24.5}}$$

$$\frac{0.568}{2\pi} = \sqrt{\frac{m'}{24.5}}$$

$$\left(\sqrt{\frac{m'}{24.5}}\right)^2 = (0.09)^2$$

$$\frac{m'}{24.5} = 8.17 \times 10^{-3}$$

$$m' = (8.17 \times 10^{-3})(24.5)$$

$$\boxed{m' = 0.2 \ Kg}$$

7.2 Data

load $= m = 15 \ g = \frac{15}{1000} \ Kg$

$m = 0.015 \ Kg$

displacement $= x = 2 \ cm = \frac{2}{100} m$

$x = 0.02\ m$, $g = 9.8\ ms^{-2}$

other mass $= m' = 294\ g$

$m' = \frac{294}{1000}\ kg = 0.294\ kg$

amplitude $= x_0 = 10\ cm$

$x_0 = \frac{10}{100}\ m = 0.1\ m$

time period $= T = ?$

spring constant $= K = ?$

maximum speed $= v_0 = ?$

Solution The weight of body will be balanced by restoring force.

$$mg = Kx$$

$$\frac{mg}{x} = K$$

$$K = \frac{(0.015)(9.8)}{0.02}$$

$$\boxed{K = 7.35\ Nm^{-1}}$$

Now

$$T = 2\pi\sqrt{\frac{m'}{K}}$$

$$= 2\pi \sqrt{\frac{0.294}{7.35}}$$

$$\boxed{T = 1.26\ s}$$

And

$$V_0 = x_0 \sqrt{\frac{K}{m}}$$

$$= (0.1) \sqrt{\frac{7.35}{0.294}}$$

$$V_0 = 0.5\ ms^{-1}$$

$$= (0.5)(100)\ cms^{-1}$$

$$\boxed{V_0 = 50\ cms^{-1}}$$

7.3 Data

mass $= m = 8\ kg$

amplitude $= x_0 = 30\ cm$

$$x_0 = \frac{30}{100}\ m = 0.3\ m$$

restoring force $= F_r = 60\ N$

time period $= T = ?$

acceleration $= a = ?$, speed $= v = ?$

$K.E = ?$, $P.E = ?$

displacement $= x = 12\ cm = \frac{12}{100}\ m$

$x = 0.12\ m$

Solution The restoring force is.

$$F_r = Kx_o$$

$$60 = K(0.3)$$

$$\frac{60}{0.3} = K$$

$$K = 200\ Nm^{-1}$$

Now

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{8}{200}}$$

$$\boxed{T = 1.3\ s}$$

Now

$$a = \omega^2 x$$

$$= \left(\frac{2\pi}{T}\right)^2 x$$

$$= \frac{4\pi^2 x}{T^2}$$

$$a = \frac{4\pi^2(0.12)}{(1.3)^2}$$

$$\boxed{a = 3\ ms^{-2}}$$

Now

$$v = \omega \sqrt{x_0^2 - x^2}$$

$$v = \sqrt{\frac{K}{m}} \sqrt{(0.3)^2 - (0.12)^2}$$

$$= \sqrt{\frac{200}{8}} (0.27)$$

$$\boxed{v = 1.4\ ms^{-1}}$$

Now

$$K.E = \frac{1}{2} K x_0^2 \left(1 - \frac{x^2}{x_0^2}\right)$$

$$K.E = \frac{1}{2}(200)(0.3)^2 \left(1 - \frac{(0.12)^2}{(0.3)^2}\right)$$

$$= \frac{1}{2}(200)(0.3)^2 (0.84)$$

$$\boxed{K.E = 7.6\ J}$$

Now

$$P.E = \frac{1}{2} K x^2$$

$$= \frac{1}{2}(200)(0.12)^2$$

$$\boxed{P.E = 1.44\ J}$$

7.4 **Data**

mass of block $= m = 4\ kg$

height $= h = 0.8\ m$, $g = 9.8\ ms^{-2}$

spring constant $= K = 1960\ Nm^{-1}$

maximum distance $=$ amplitude $= x_o = ?$

Solution The gravitational P.E of the block will be converted into elastic P.E of spring.

$$\text{elastic P.E} = \text{gravitational P.E}$$

$$\frac{1}{2} K x_o^2 = mgh$$

$$K x_o^2 = 2mgh$$

$$x_o^2 = \frac{2mgh}{K}$$

$$= \frac{2(4)(9.8)(0.8)}{1960}$$

$$x_o^2 = 0.032$$

$$\boxed{x_o = 0.18\ m}$$

7.5 Data

length of pendulum $= l = 50\,cm$

$l = \frac{50}{100}\,m = 0.5\,m$

frequency $= f = ?$, $g = 9.8\,ms^{-2}$

Solution

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{9.8}{0.5}}$$

$$\boxed{f = 0.7\ Hz}$$

7.6 Data

mass of block $= m = 1.6\,kg$

spring constant $= K = 1000\,Nm^{-1}$

amplitude $= x_0 = 2\,cm = \frac{2}{100}\,m$

$x_0 = 0.02\,m$

velocity $= v = ?$, $x = 0$

Solution

$$v = \omega\sqrt{x_0^2 - x^2}$$

$$v = \sqrt{\frac{K}{m}}\,\sqrt{x_0^2 - x^2}$$

$$= \sqrt{\frac{1000}{1.6}} \sqrt{(0.02)^2 - (0)^2}$$

$$= (25)(0.02)$$

$$\boxed{v = 0.5 \ ms^{-1}}$$

7.7 Data

mass of car $= m_1 = 1300$ kg

spring constant of one spring $= K' = 20000 \ Nm^{-1}$

total spring constant $= K = 4K'$

$$K = 4(20000)$$

$$K = 80000 \ Nm^{-1}$$

mass of people $= m_2 = 160$ kg

total mass $= m = m_1 + m_2$

$$m = 1300 + 160$$

$$m = 1460 \text{ kg}$$

frequency $= f = ?$

Solution

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{80000}{1460}}$$

$$\boxed{f = 1.18\ Hz}$$

7.8 Data

amplitude = x_o = ?

frequency = f = ?

time period = T = ?

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t$$

displacement = x = ?

when $t = 2\ s$

Solution Given that

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t$$

comparing with

$$x = x_o \cos \omega t$$

$$\boxed{x_o = 0.25\,m}$$

$$\omega = \frac{\pi}{8}$$

$$2\pi f = \frac{\pi}{8}$$

$$f = \frac{\cancel{\pi}}{8(2\cancel{\pi})} \Rightarrow \boxed{f = \frac{1}{16}\ Hz}$$

$$T = \frac{1}{f}$$

$$= \frac{1}{\frac{1}{16}}$$

$$T = \frac{16}{1} \Rightarrow \boxed{T = 16\ s}$$

NOW

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)(2)$$

$$= 0.25 \cos\left(\frac{\pi}{4}\right)$$

$$= 0.25 \cos 45^\circ$$

$$\boxed{x = 0.18\ m}$$