Chapter # 3: **Measures of central location**

1. What is meant by measures of central tendency?

Ans: The averages tend to lie in the center of a distribution they are called measures of central tendency. They are also called measures of location because they locate the center of a distribution.

2. Write the types of averages.

Ans: The most commonly used averages are:

i. Arithmetic mean
ii. Geometric mean
iii. Harmonic mean
iv. Median
v. Mode

3. What are two qualities of a good average?

Ans: Properties of a good average are given below:
i. It is well defined
ii. It is easy to calculate
iii. It is easy to understand
iv. It is based on all the values
v. It is capable of mathematical treatment

4. Mean of 5 values is 70. Find the sum of values.

Ans: $\bar{X} = \frac{\sum x}{n}$

$\sum x = 350$ **Ans.**

5. In a moderately skewed distribution, the values of mean and median are 120 and 110 respectively, find the value of mode.

Ans: $Mode = 3Median - 2Mean$

$Mode = 3(110) - 2(120)$

$Mode = 90$ **Ans.**

6. Given *u = (x – 170)/5, ∑fu = 100, ∑f = 200*, find arithmetic mean.

Ans: By coding method:

$\bar{X} = A + \frac{\sum fU}{\sum f} \times h$ Where; $\left(U = \frac{x-A}{h}\right)$

$\bar{X} = 172.5$ **Ans.**

7. Write down any two mathematical properties of arithmetic mean.

Ans: There are following mathematical properties of Arithmetic Mean:

i. The sum of deviations of all observations from their mean is zero. i.e. $\sum(X - \bar{X}) = 0$
ii. The sum of squares of deviations of all observations from their mean is minimum. i.e. $\sum(X - \bar{X})^2$ is minimum

iii. The mean of a constant is constant itself. i.e. If $X = a$ then $\bar{X} = a$

iv. The mean is affected by change of origion and scale. If we add or subtract a constant from all the values or multiply or divide all the values by a constant, the mean is affected by the respective change.

i.e. If $Y = X \pm a$ then $\bar{Y} = \bar{X} \pm a$, If $Y = a \pm bX$ then $\bar{Y} = a \pm b\bar{X}$, If $Y = \frac{X}{a}$ then $\bar{Y} = \frac{\bar{X}}{a}$

8. Define mode and give its formula in case of grouped data.

Ans: The most repeated value in a data is called mode. It is denoted by $\hat{X}$

Formulas;

For ungrouped data: $\hat{X} = \text{The most frequent value in a data}$

For grouped data: $\hat{X} = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$

9. Find the mode of 3, 3, 7, 8, 10, 11, 10, 12, and 10.

Ans: 10 is most frequent value in the given data and then called the mode.

10. Define the median with formula.

Ans: The value which divides the ordered data into two equal parts is called as median. It is denoted by $\tilde{X}$.

11. Write down the properties of median.

Ans:

1. a constant “a” is added to each of the n observations $y_1, y_2, \ldots\ldots Y_n$ having median M, then the median of y_1+a, y_2+a, ……. y_n+a would be “a+M”.
2. The sum of the absolute deviations of the observations from their median is minimum i.e.,

$\sum|y - \text{median}|$ is minimum

3. For a symmetrical distribution median is equidistant from the first and third quartiles i.e.,

Q_3 – Median = Median = Median – Q_1

12. What are the advantages and disadvantages of median?

Ans:

Merits of Median:

1. It is quick to find.
2. It is not much affected by exceptionally large or small values in a data.
3. It is suitable for skewed distribution.

Demerits of Median:

1. It is not rigidly defined.
2. It is not readily suitable for algebraic development.
3. It is less stable in repeated sampling experiments than the mean.
4. It is not based on all the observations.

13. Define quartiles; also write down its formulas.

Ans: The values which divide the ordered data into four equal parts are called quartiles. There are 3 quartiles;
Q_1 is called first quartile or lower quartile.

For ungrouped data: $Q_1 = \left(\frac{n+1}{4}\right)^{th}$ $value$

For grouped data: $Q_1 = l + \frac{h}{f}\left[\frac{n}{4} - c\right]$

Q_2 is called second quartile or median.

For ungrouped data: $Q_2 = 2\left(\frac{n+1}{4}\right)^{th}$ $value$

For grouped data: $Q_2 = l + \frac{h}{f}\left[\frac{2n}{4} - c\right]$

Q_3 is called third quartile or upper quartile.

For ungrouped data: $Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ $value$

For grouped data: $Q_3 = l + \frac{h}{f}\left[\frac{3n}{4} - c\right]$

14. If the value of Q_2, D_5 and P_{50} are equal to 72.32 then find the median of the distribution.

Ans: Median = 72.32

15. Define geometric mean with formula.

Ans: The geometric mean of 'n' positive values is defined as the n^{th} root of the product.
If $X_1, X_2, X_3, \ldots\ldots, X_n$ are '**n**' values of a variable **"x"** and none of them being Zero then the geometric defined as:

$$G = \sqrt[n]{x_1.x_2.x_3.\ldots\ldots x_n}$$

16. If geometric mean of 3 items is 7, find the product of all items?

Ans:

$$G.M. = \sqrt[n]{x_1.x_2.x_3.\ldots.x_n}$$

$$G.M. = \sqrt[3]{a.b.c}$$

$$7 = (a.b.c)^{1/3}$$

$$\boldsymbol{a.b.c = 343}$$

17. Find harmonic mean of 5, 10, and 20.

Ans: By definition;

$$H.M = \frac{n}{\Sigma\frac{1}{x}}$$

X	**1/x**
5	0.2
10	0.1
20	0.05

Total	0.35

H.M. = 8.571 Ans.

18. Define weighted mean.

Ans: When all the values in the data are not of equal importance then we assign them certain numerical values to express their relative importance. These assigned values are called weights. The average of these weights with values is called weighted mean. The weights may be the quantities consumed or the numerical coefficient and are generally denoted by ω. The weighted mean denoted by "$\bar{y}_w$" of a set of 'n' values say

$y_1, y_2, \ldots . y_n$ With weights $\omega_1, \omega_2, \ldots . \omega_n$ is then given by:

$$\bar{y}_w = \frac{w_1 y_1 + w_1 y_1 + \ldots . + w_n y_n}{w_1 + w_2 + \ldots \ldots + w_n}$$

$$\bar{y}_w = \frac{\Sigma w_i y_i}{\Sigma w_i}$$

Where; i = 1, 2, 3, 4 …n

19. Define harmonic mean.

Ans: Harmonic mean is defined as the reciprocal of the mean of the reciprocals of the items in a series. It is the ratio of the number of items and the sum of reciprocal of items.

$$H = \frac{\Sigma f}{\Sigma f \frac{1}{x}}$$

20. Calculate geometric mean of X = 1, 1, 27.

Ans: by definition;

$$G.M. = \sqrt[n]{x_1 . x_2 . x_3 . \ldots . x_n}$$

$$G.M. = \sqrt[3]{1.1.27}$$

$G.M. = 3$ **Ans.**

21. Write down properties of geometric mean.

Ans:

1. G.M is always less then A.M. i.e GM < A.M.
2. Geometric mean of constant variable is always constant.

22. Write down merits of geometric mean.

Ans:

1. It is rigidly defined by a mathematical formula.
2. It is based on all values.
3. It is less affected by extremely large values.

23. Write down demerits of geometric mean.

Ans:

1. It is not calculated if any of the observations is zero or negative.
2. In case of negative values, it cannot be computed at all.
3. It is not easy to understand.

24. Write down the merits of A.M.

Ans:

1. It is rigidly defined by mathematical formula.
2. It is easy to calculate.
3. It is easy to understand.
4. It is based upon all the values.
5. It is stable statistics in repeated sampling experiments.

25. Write demerits of A.M.

Ans:

1. It is greatly affected by extreme value.
2. It cannot be calculated for open-end classes without assuming open ends.
3. It gives fallacious and misleading conclusions when there is too much variation in data.

26. Find A.M. if $\sum fx = 500$ and $\sum f = 50$.

Ans: By definition;

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$\bar{X} = 10$ **Ans.**

27. If G.M. of two values is 3. Find the product of two values.

Ans: by definition;

$$G.M. = \sqrt[n]{x_1.x_2.x_3. \dots . x_n}$$

$$G.M. = \sqrt[2]{a.b}$$

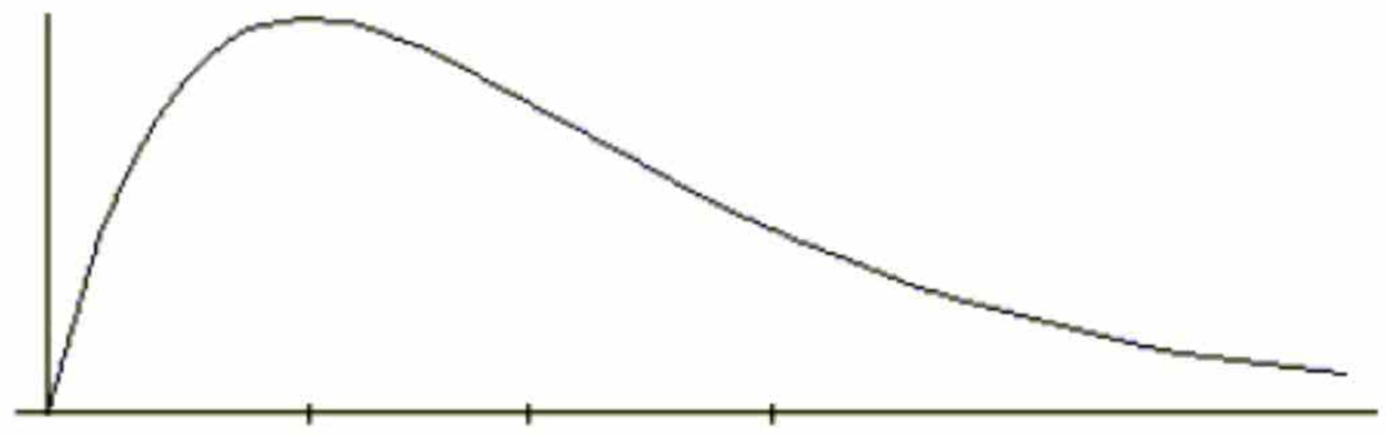
$$G.M. = (a.b)^{1/2}$$

$$\boldsymbol{a.b = 9}$$

28. Illustrate the graphically positions of mean, median and mode for frequency curve which are skewed to the right and left.

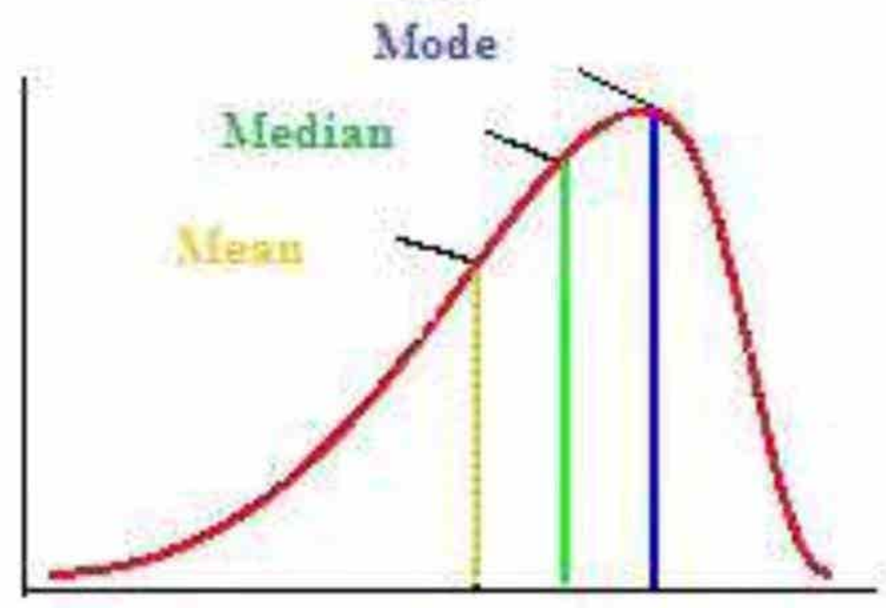
Ans.

a) For moderately positively skewed distributions, the following empirical relation holds.
Mean > Median > Mode



b) For moderately negatively skewed distributions, the following empirical relation holds.

Mean < Median < Mode



Left-Skewed (Negative Skewness)

29. What are the properties of mode?

Ans:

1. If a constant a is added to each of the n observations $y_1 . y_2 \ldots y_n$ having mode m, then the mode of $a + y_1, a + y_2 \ldots .. a + y_n$ would be $a + m$.
2. If a is multiplied with each of the n observations $y_1 . y_2 \ldots y_n$ having mode m then the mode of $ay_1 . ay_2 \ldots ay_n$ would be am.

30. Write down the merits of mode.

Ans:

1. It is easily located.
2. It is not affected by extreme values.
3. It can be located in case of open end classes.
4. It is simple to understand.

31. What are the demerits of mode?

Ans:

1. It is ill defined.
2. It is not based on all the values.
3. It is not capable of further algebraic treatment.
4. When the distribution has more than one mode then mode should not be calculated.

32. If $\overline{y}_1 = 3$ with $n_{1=}$ 3and $\overline{y}_2 = 4$ with $n_{2=}2$, then find $\overline{y}_c$.

Ans: By definition;

$$\bar{y}_c = \frac{n_1\overline{y_1} + n_2\overline{y_2}}{n_1 + n_2}$$

$\bar{y}_c = 17/5$ **Ans.**

33. Define average.

Ans: An Average is a single value which represents all values of data in some definite way.

For Example: The average income of middle class families is Rs.17000/per month.

34. For a certain frequency distribution, the mean was 40.5 and median was 36. Find mode by using of empirical relation.

Ans: Mode = 3Median – 2 mean

Mode = 3(36) – 2(40.5)

Mode = 27 **Ans.**