Chapter = 06

Gravitation

THEORY NOTES

NEWTON'S LAW OF GRAVITATION:

STATEMENTS:

This is law that everybody in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers."

EXPLANATION:

Consider tow bodies of masses 'm_1' and 'm_2' with their centers distance 'r' apart.
The magnitude of force of attraction between them is,

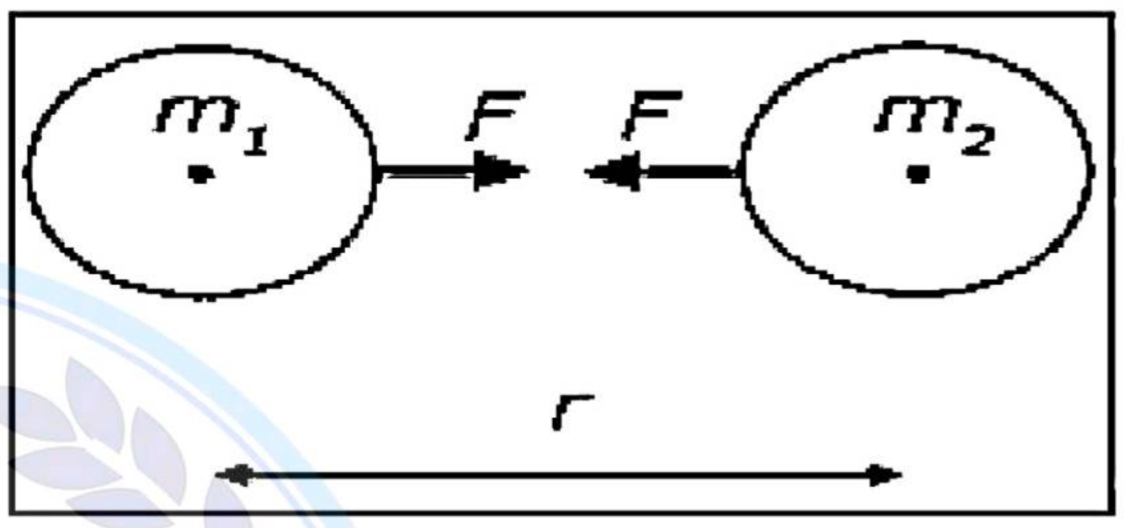
$F \propto m_1 . m_2 \longrightarrow (1)$

Also $F \propto \frac{1}{r^2} \longrightarrow (2)$

Combining these factors,

$F \propto m_1 . m_2 / r^2$

Or $F = G m_1 . m_2 / r^2 \longrightarrow (3)$



Where 'G' is a constant of proportionality and is called the Universal Gravitational Constant. In S. I system, its value is 6.673 10^{-11} $N.m^2 Kg^{-2}$

The gravitational force between two bodies form a pair of action and reaction forces. The body of mass 'm_1' attracts the body of mass 'm_2' by a force $\vec{F}_{12}$, while the body of mass 'm_2' is attracted by the body of mass 'm_1' with a force $\vec{F}_{21}$. These forces are equal and opposite thus,

$$\vec{F}_{12} = \vec{F}_{21}$$

MASS OF EARTH:

Consider a body of mass 'm' placed on the surface of the earth. Let the mass of the earth is 'M_e' and radius of the earth is 'R_e' .Neglecting the radius of body if compared with that of the earth.

Gravitational force of attraction between earth and body is

$$F = G\,m\,M_e / R_e^2 \text{.......}(1)$$

We know that the force of attraction of the earth on a body is equal to weight the weight of body. i.e

$$F = W \text{ ....}(2)$$

Therefore Comparing (1) and (2), we get,

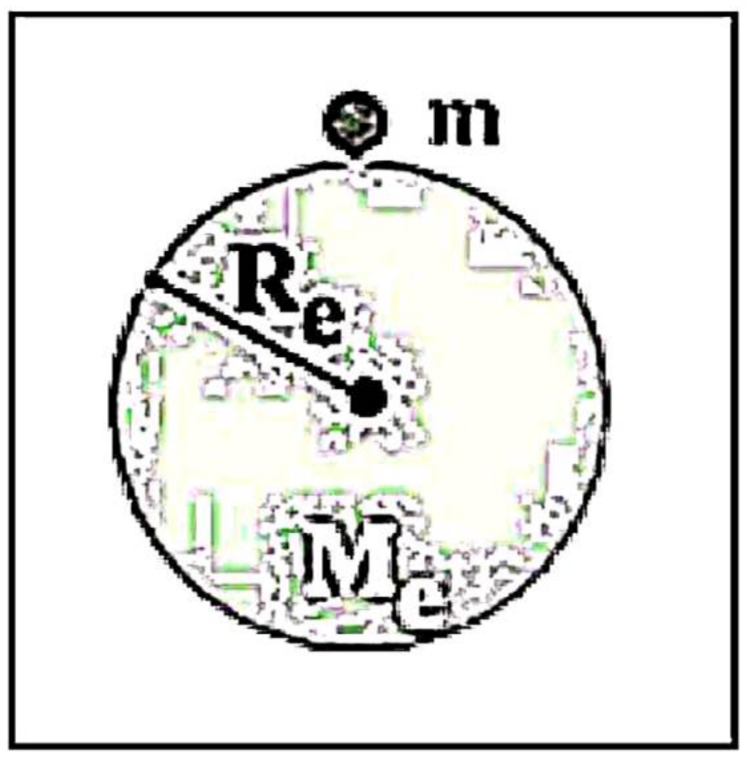
$$W = G\,m\,M_e / R_e^2$$

But $W = mg$

$$mg = G\,m\,M_e / R_e^2$$

or $g = G\,M_e/R_e^2$

or $M_e = g \times R_e^2/G$



From astronomical data:

$g = 9.8\ m/s^2$

$R_e = 6.38 \times 10^6$ m

$G = 6.67 \times 10^{-11}$ N-m^2/kg^2

Putting these values in the above equation.

$$M_e = 9.8\,(6.38 \times 10^6)^2/6.67 \times 10^{-11}$$

or

$$\boxed{M_e = 5.98 \times 10^{24}\ Kg}$$

DENSITY OF EARTH:

Let the average density be 'ρ' of the earth, which is given by,

$$\rho = \frac{M_e}{v_e} \qquad \left(\rho = \frac{Mass}{Volume}\right)$$

Since,

$$V_e = \frac{4}{3}\pi R_e^{\,3}$$

Thus,

$$\rho = \frac{M_e}{\frac{4}{3}\pi R_e^{\,3}}$$

$$\rho = \frac{5.98 \times 10^{24}}{\frac{4}{3} \times 3.14 \times (6.38 \times 10^6)^3}$$

$$\rho = 5.5 \times 10^3 \text{ Kg / m}^3$$

VARIATION OF 'g' WITH ALTITUDE:

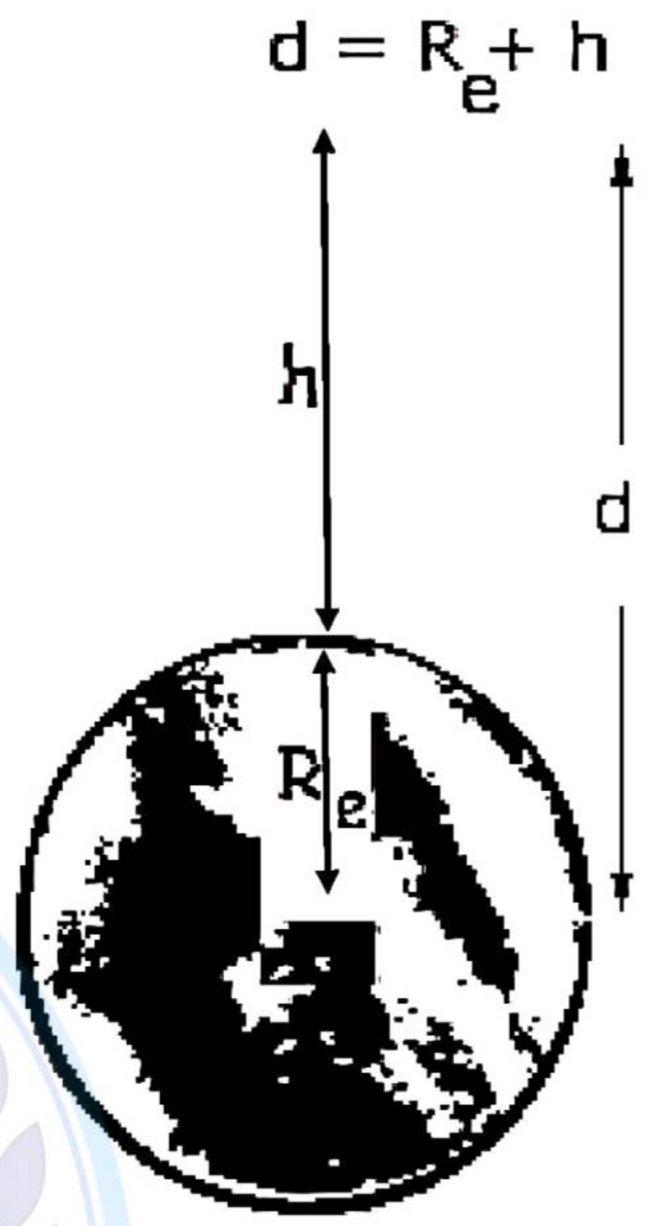
We know that value of "g" at the surface of earth is given by

$$g = \frac{GM_e}{{R_e}^2} \text{-------(i)}$$

If the earth be considered as a sphere of homogenous composition then 'g' any point above its surface will vary inversely as the square of the distance from that point to its centre, which is as below. At a distance (R_e+h),

$$g' = \frac{GM_e}{(R_e+h)^2} \text{---------(ii)}$$

Dividing eq (i) by (ii)



$$\frac{g}{g'} = \frac{\frac{GM_e}{{R_e}^2}}{\frac{GM_e}{(R_e+h)^2}}$$

$$\frac{g}{g'} = \frac{(R_e+h)^2}{{R_e}^2}$$

$$\frac{g}{g'} = \left(\frac{R_e+h}{R_e}\right)^2$$

$$\frac{g}{g'} = \left(\frac{R_e}{R_e} + \frac{h}{R_e}\right)^2$$

$$\frac{g}{g'} = \left(1 + \frac{h}{R_e}\right)^2$$

or $$\frac{g'}{g} = \left(1 + \frac{h}{R_e}\right)^{-2}$$

Expanding by binomial expansion, $(1 + x)^{-n} = 1 - \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} - \cdots$

$$\left(1 + \frac{h}{R_e}\right)^{-2} = 1 - \frac{2\frac{h}{R_e}}{1} + \frac{2(2-1)\frac{h}{R_e}^2}{2\times1} - \cdots$$

Neglecting higher powers of $\frac{h}{R_e}$

$$\frac{g'}{g} = 1 - \frac{2h}{R_e}$$

$$g' = g\left(1 - \frac{2h}{R_e}\right)$$

From this equation, we conclude that the greater the value of 'h' the smaller is the value of 'g' or ' value of 'g' decreases with altitude".

For example at h 16000m, g = 9.757 m/sec^2

VARIATION OF 'g' WITH DEPTH:

Let g' be the acceleration due to gravity at a depth 'd' below the surface of earth, that is at a distance (R_e-d) from the center or earth.

$$M_e = V \times \rho$$

$$M_e = \frac{4}{3}\rho\pi {R_e}^3$$

Where 'ρ' is the density of the earth which is supposed to be uniform everywhere,

The value of "g" at the surface of earth is given by ,

$$g = \frac{GM_e}{{R_e}^2}$$

Putting the value of M_e

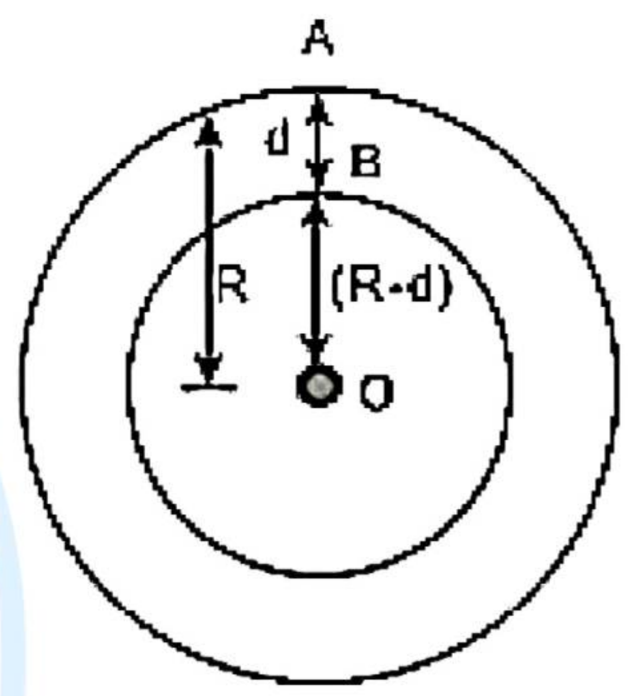
$$g = \frac{G\frac{4}{3}\rho\pi {R_e}^3}{{R_e}^2}$$

$$g = G\frac{4}{3}\rho\pi R_e \text{ ----------(i)}$$

And at depth 'd' the value of "g" is given by

$$g' = G\frac{4}{3}\rho\pi(R_e - d) \text{ ----------(ii)}$$

Dividing eq(ii) by eq(i)



$$\frac{g'}{g} = \frac{G\frac{4}{3}\rho\pi(R_e - d)}{G\frac{4}{3}\rho\pi R_e}$$

$$\frac{g'}{g} = \frac{R_e - d}{R_e}$$

$$\frac{g'}{g} = \frac{R_e}{R_e} - \frac{d}{R_e}$$

$$\frac{g'}{g} = 1 - \frac{d}{R_e}$$

or

$$g' = g\left(1 - \frac{d}{R_e}\right)$$

That is value of 'g' decreases with depth from surface of the earth.

For example, At d = 4000 Km, g = 8 m/sec^2

WEIGHTLESSNESS

DEFINITION:

"The condition in which the apparent weight of the body becomes zero then it is said to be in state of weightlessness".

EXPLANATION:

To discuss weightlessness in artificial satellites, let us take the example of an elevator having a block of mass (m) suspended by a spring balance attached to the coiling of the elevator. The tension in the thread indicates the weight of the block. Consider following cases.

1. When Elevator is at Rest

$$\boxed{T = m g}$$

2. When Elevator is Ascending with an Acceleration 'a'

In this case

$$T > m g$$

Therefore, Net force = T - mg

$$m a = T - m g$$

$$\boxed{T = m g + m a}$$

In this case of the block appears "heavier".

3. When Elevator is Descending with Acceleration 'a'

In this case

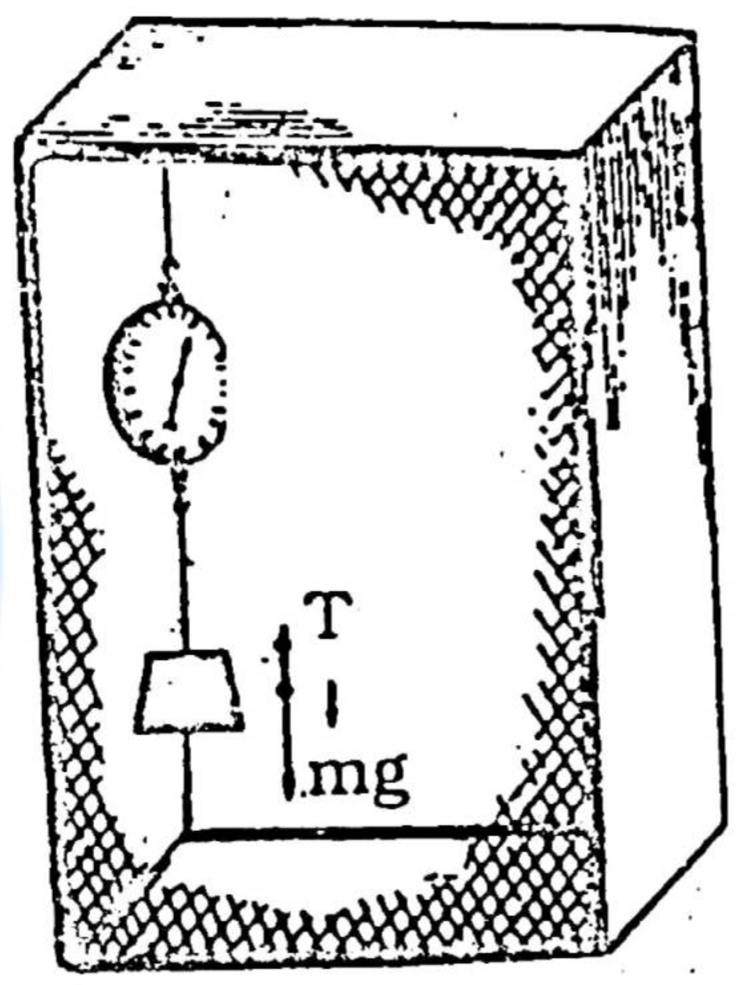
$$m g > T$$

Therefore

$$\text{Net force} = m g - T$$

$$m a = m g - T$$

$$\boxed{T = m g - m a} \text{ --------(i)}$$



In this case, the body appears lighter

4. When the Elevator is falling freely Under the Action of Gravity

If the cable supporting the elevator breaks, the elevator will fall down with acceleration equal to 'g'

From (i) => $T = mg - ma$

But $a = g$

Therefore

$$T = mg - mg$$

$$\boxed{T = 0}$$

In this case, spring balance will read zero. This is the state of "weightlessness".

In this case gravitation force still acts on the block due to the reason that elevator block, spring balance and string all have same acceleration when they fall freely, the weight of the block appears zero.

WEIGHTLESS NESS IN SATELLITES:

Similarly in case of satellite orbiting around earth, a body experiences gravitational acceleration "g" but also feels an acceleration a_c, that is centripetal acceleration towards the center of earth which equal to the value of "g". So the apparent weight is given by

Eq(i) => $T = mg - ma$

or $T = mg - ma_c$

$T = mg - mg$ $(a_c = g)$

$$\boxed{T = 0}$$

Therefore the astronaut or body inside satellite experiences weightlessness.

ARTIFICIAL GRAVITY

INTRODUCTION:

All orbiting satellites along with their astronauts and other objects are in a state of free fall and consequently will be in a state of weightlessness. Weightlessness in space craft is highly inconvenient to an astronaut in a number of ways.

For example he cannot pour liquid into a glass, neither he can drink properly. In order to overcome this problem, artificial gravity is produced in the space crafts.

EXPLANATION:

In order to produce an artificial gravity in the space craft, the laboratory of space craft is rotated with suitable frequency about its own axis. The rotation is so maintained that the astronaut do not feel weightlessness. The frequency of rotation depends on the length of laboratory of space craft.

Consider a space craft whose laboratory is 'L' meter long consisting of two chambers connected by a tunnel. Let us see how many revolutions per second must the space craft make in order to supply artificial gravity for the astronauts.

Let 'T' be the time for one revolution and 'f' be the frequency of rotation.

When the laboratory revolves, a centripetal accelaration is experienced by the astronauts.

$a_c = \frac{v^2}{r}$

Where a_c is the centripetal acceleration

Since radius of laboratory is R , therefore,

$a_c = \frac{v^2}{R}$ --------(iv)

Now we will determine the linear speed of the laboratory.

As we know that $V = R\omega$

$a_c = \frac{(R\omega)^2}{R}$

Now, $\omega = 2\pi f$

so,

$a_c = \frac{R^2(2\pi f)^2}{R}$

$a_c = 4\pi^2 f^2 R$

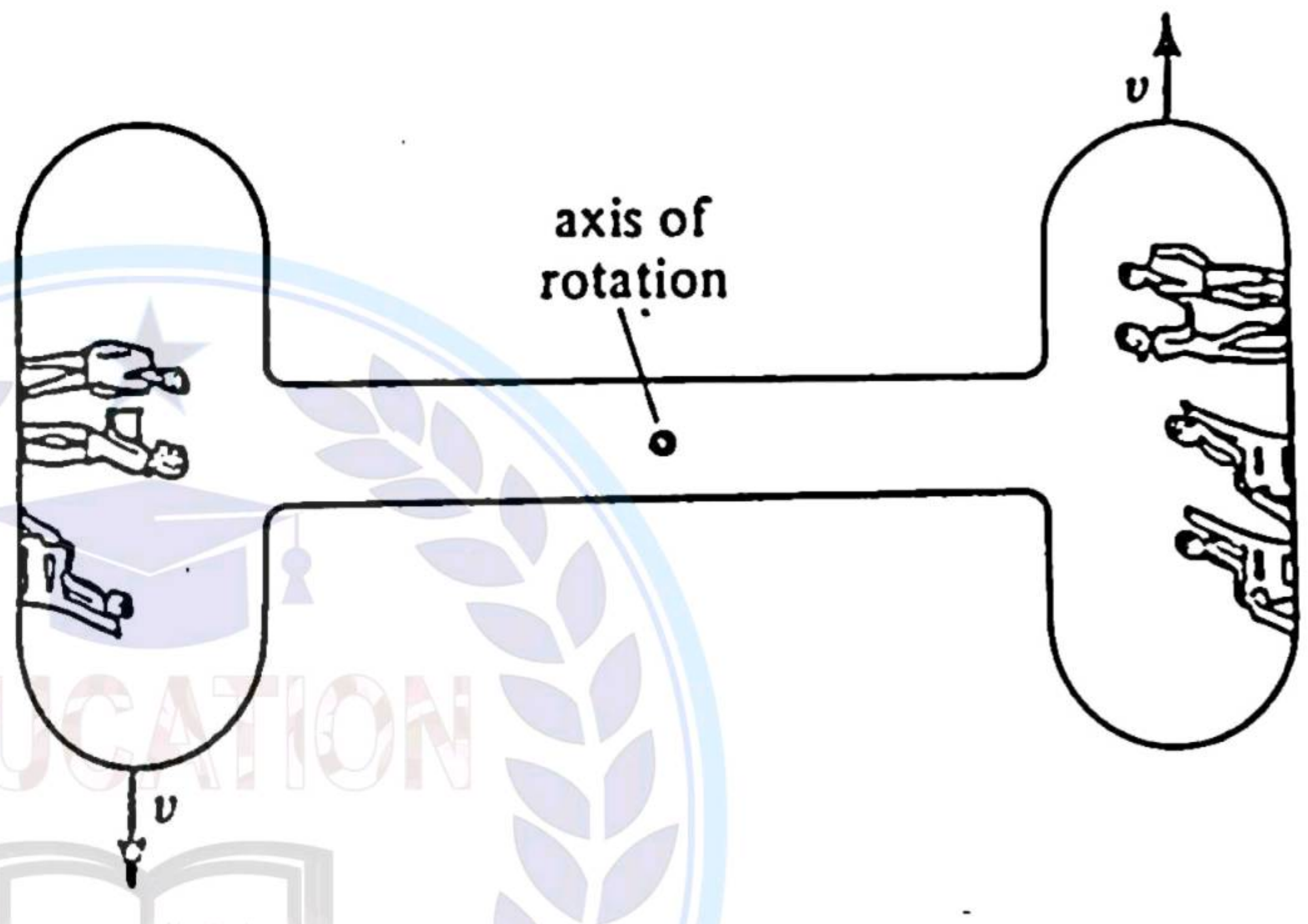
or $f^2 = \frac{a_c}{4\pi^2 R}$

Taking square roots on both sides

$f = \sqrt{\frac{a_c}{4\pi^2 R}}$

or

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$



Let, R=10 m and a_c=g=9.8 m/s²

now, f becomes => $f = \frac{1}{2(3.14)}\sqrt{\frac{9.8}{10}}$

$f = 0.158\ rev/sec$

or $f = 9.5\ rev/min$

So, in order to produce artificial gravity on a satellite of length 20m it should be rotated at a rate of 9.5 rev/min.

M.C.Qs.

1. The force of gravitation acts along the.
(a) axis of rotation.
(b) Line joining the interacting bodies.
(c) Line perpendicular to the interacting
(d) None of these

2. According to the law of universal Gravitation:
(a) Everybody in the universes attracts everybody.
(b) The force of attraction is directly proportional to the product of their masses
(c) The force of attraction is inversely proportional to the squire of their distance.
(d) All of the above

3. Force of gravitational attraction of earth on other bodies is given by:
(a) $F = \frac{GM_e m}{r^2}$ (b) $F = \frac{GM_e}{r^2}$
(c) $F = \frac{Gm}{r^2}$ (d) $F = \frac{M_e m}{r^2}$

4. The force of attraction or repulsion between two bodies is:
(a) Inversely proportional to the distance
(b) Directly proportional to the distance
(c) Inversely proportional to the square of the distance (d) None of the above

5. A hole is drilled through the earth along the diameter and a stone is dropped into it. When the stone is at the centre of the earth it has:
(a) Mass (b) Weight
(c) Acceleration (d) Both a and b

6. The acceleration due to gravity on moon is 1/6th of that on earth, what will be the mass of the body on moon, if its mass on earth is m:
(a) m/6 (b) 6 m
(c) m (d) m/3

7. The value of 'g' at the centre of earth is:
(a) Maximum (b) Minimum
(c) Zero (d) None of them

8. The value of g at a certain height above the earth is:
(a) Nearly the same as an the surface of earth
(b) Nearly the same as an the center of earth
(c) Estimated to decreases with altitude
(d) Estimated to depend on the variation of the earth radius

9. If the mass of the earth becomes four times large, the value of g will:
(a) remain unchanged
(b) Becomes four times larger
(c) Be double
(d) sixteen times larger

10. When a lift is moving upward with a uniform velocity, the apparent weight of a body inside the lift will be:
(a) Equal to its actual weight
(b) Less than the actual weight
(c) More than the actual weight
(d) Zero

11. Artificial gravity can be created in the space craft by:
(a) Revolving it around the earth
(b) Spinning it around its own axis
(c) Increasing its velocity
(d) decreasing its velocity

12. The gravitational constant was determined

experimentally by:
(a) Newton (b)Einstein
(c) Cavendish (d) Maxwell

13. If man a goes above the earth's surface to a distance equal to thrice the earth's radius, the value of acceleration due to gravity at that point becomes:

(a) 1/3 g (b) 1/4 g (c)1/9 g (d)1/16 g

14. The approximate value of the average density of the earth is:
(a)5.5 x 10^3 kg /m^3 (b) 6.5 x 10^3 kg /m^3
(c) 7.5 x 10^3 kg /m^3 (d)8.673 x 10^3 kg /m^3

15. If a planet existed whose mass and radius were both twice that of the earth, then acceleration due to gravity at its surface would be:
(a)4.9 m/s^2 (b) 19.6 m/s^2
(c)2.45 m/s^2 (d) same as that of earth

16. When the space ship is at a distance equal to twice of the earth's radius from its centre then the gravitational acceleration is:
(a)4.9 m/s^2 (b) 19.6 m/s^2
(c)2.45 m/s^2 (d) 9.8 m/s^2

17. If both masses of two bodies and distance between them are doubled then the gravitational force between them will be:
(a) doubled (b) halved
(c) four times (d) remain same

18. If radius of earth is doubled then the value of gravitational acceleration will be:
(a) doubled (b) halved
(c)four times (d) one fourth

19. Unit of gravitational constant is:
(a) m /s^2 (b) N m^2 / kg^2
(c) N (d) N kg^2 / m^2

20. The value of 'g' for an object falling on any planet is independent of which of the following quantities:
(a) Mass of planet (b) Radius of planet
(c) Mass of falling object (d) all of these

21. If the radius of the Earth was to shrink and their masses were to remains the same, the acceleration due to gravity on the surface of Earth will
(a) Decreases (b) Remains same
(c) Increases (d) None

22. Newton's law does not hold good for particles
(a) moving slowly (b) at rest
(c) moving with high velocity (d) moving with velocity comparable to velocity of light

23. At what depth, the weight of the body becomes three fourth, as compared to the body weight on the surface of earth.
(a) d=R/2 (b) d=R/4
(c) d=2R/3 (d) d=4R/5

24. The distance from the centre of the earth at which the gravitational acceleration becomes one half the value that it has on the earth's surface is calculated as:
(a) 14.1 earth's radius (b) 141.1 earth's radius
(c) 1.41 earth's radius (d) 1.5 earth's radius

25. If distance between two masses is halved, then the force of attraction between them will be
(a) four times (b) doubled
(c) reduced to 1 /2 (d) reduced to 1 /4

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2022

9. The weight of a man is 600 N at the earth, his weight on the moon, where $g_m = g/6$ will be:
*3600N *600N *300N ***100N**

28.If elevator moves upward with the acceleration a , the apparent weight of body of mass m is:
* mg-ma * mg *** mg+ma** * zero

2021

(ix) If the radius of the earth were to shrink by 1% while its mass remains same, the acceleration due to gravity on the earth surface would:
* Decrease *** Increase** *Remain the same *become half

(xxxvi) The value of the gravitational constant (G) was first determined experimentally by:
*Newton ***Cavendish** * Einstein *Maxwell

2019

7. The gravitational constant was determined experimentally by:
*Newton *Einstein ***Cavendish** *Maxwell

2018

1.Artificial gravity in spacecraft can be created by:
*translatory Motion *vibratory Motion ***spin motion** *orbital motion

2017

7. If man a goes above the earth's surface to a distance equal to thrice the earth's radius, the value of acceleration due to gravity at that point becomes:
*1/3 g *1/4 g *1/9 g ***1/16 g**

2016

4. At a distance, equal to twice of the radius of the earth above the surface of the earth, the value of gravitational acceleration will be:
*one half *one fourth *Four times ***one ninth**

2015

8. The value of gravitational constant 'G' was determined experimentally by:
***Cavendish** *Newton *Joules *Huygen

2014

1. If a person ascends form the surface of the earth to a distance equal to the radius of the earth, the value of g will be:
* 1/2 g *** 1/4g** *2g *4g

11. Artificial gravity can be created in the spaceship by producing:
*translatory motion *vibratory motion ***spin motion** *orbital motion

2013

8. The weight of a man is 600 N at the earth, his weight on the moon, where $g_m = g/6$ will be:

*3600N *600N *300N ***100N**

2012

8. At a distance equal to the radius of earth above the earth's surface, the value of gravitational acceleration becomes:

*half *** one forth** * double * four times

2011

17. If one moves up from the surface of earth to a distance equal to the radius of the earth the value of acceleration due to gravity will be:

*1/2 g ***1/4 g** *2 g *4 g

2010

1. If we go up from the surface of earth to a distance equal to the radius of the earth the value of acceleration due to gravity will be:

***one fourth** * one eighth * one ninth * double

TEXTBOOK NUMERICALS

Q.1: A 10 kg mass is at a distance of 1 m from a 100 kg mass. Find the gravitational force of attraction when (i) 10 kg mass exerts force on the 100 kg mass (ii) 100 kg mass exerts force on the 10 kg mass

Data:

First Mass $= m_1 = 10\ kg$

Second Mass $= m_2 = 100\ kg$

Distance between masses $= r = 1\ m$

Force of m_1 onto $m_2 = F_{12} = ?$

Force of m_2 onto $m_1 = F_{21} = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Then,

Force of m_1 onto m_2:

$$F_{12} = G\frac{m_1 m_2}{r^2}$$

$$F_{12} = 6.67 \times 10^{-11}\frac{10 \times 100}{(1)^2}$$

$$\boxed{F_{12} = 6.67 \times 10^{-8} N}$$

Force of m_2 onto m_1:

$$F_{21} = G\frac{m_2 m_1}{r^2}$$

$$F_{21} = 6.67 \times 10^{-11}\frac{100 \times 10}{(1)^2}$$

$$\boxed{F_{21} = 6.67 \times 10^{-8} N}$$

Result: The force of m_1 onto m_2 and m_2 onto m_1 is equal to $6.67 \times 10^{-8} N$.

Q.2: Compute gravitational acceleration at the surface of the planet Jupiter which has a diameter as 11 times as compared with that of the earth and a mass equal to 318 times that of earth.

Data:

$Diameter\ of\ Jupiter = D_j = 11 \times\ D_E$

$Mass\ of\ Jupiter = M_j = 318 \times\ M_E$

$Value\ of\ g\ on\ Jupiter = g_j = ?$

Solution:

Since

$$Diameter\ of\ Jupiter = D_j = 11 \times D_E$$

Dividing by "2" O.B.S.

$$\frac{D_j}{2} = 11 \times \frac{D_E}{2}$$

$$Radius\ of\ Jupiter = R_j = 11 \times R_E$$

The acceleration due to gravity is given by

$$g = \frac{GM_E}{{R_E}^2}$$

On the surface of moon

$$g_j = \frac{GM_j}{{R_j}^2}$$

$$g_j = \frac{G(318 \times M_E)}{(11 \times R_E)^2}$$

$$g_j = \frac{(318)}{(11)^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_j = \frac{(318)}{121}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_j = 2.62\ (g)$$

$$g_j = 2.62 \times 9.8$$

$$\boxed{g_j = 25.6\ m/s^2}$$

Result: The value of g on the surface of Jupiter is $25.6 \frac{m}{s^2}$.

Q.3: The mass of the planet Jupiter 1.9 x 10^27 kg and that of the sun is 2.0 x 10^30 kg. If the average distance between them is 7.8 x 10^11 m, find the gravitational force of the sun on Jupiter.

Data:

Mass of Jupiter $= m_1 = 1.9 \times 10^{27}\ kg$

Mass of Sun $= m_2 = 2 \times 10^{30}\ kg$

Average Distance $= r = 7.8 \times 10^{11} m$

Force of Sun on jupiter $= F = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Then,

$$F_{12} = 6.67 \times 10^{-11}\frac{1.9 \times 10^{27} \times 2 \times 10^{30}}{(7.8 \times 10^{11})^2}$$

$$\boxed{F_{12} = 6.24 \times 10^{33} N}$$

Result: The force of Sun on Jupiter is equal to $6.24 \times 10^{33} N$.

Q.4: The radius of the moon is 27% of the earth's radius and its mass is 1.2% of the earth's mass. Find the acceleration due to gravity on the surface of the moon. How much will a 424N body weight there?

Data:

$$Radius\ of\ moon = R_m = 27\%\ of\ R_E$$

$$= \frac{27}{100} R_E = 0.27\ R_E$$

$$Mass\ of\ moon = M_m = 1.2\%\ of\ M_E$$

$$= \frac{1.2}{100} M_E = 0.012\ M_E$$

$Value\ of\ g\ on\ moon = g_m = ?$

$Weight\ of\ body = W = 424\ N$

$Weight\ of\ body\ on\ moon = W_m = ?$

Solution:

The acceleration due to gravity is given by

$$g = \frac{GM_E}{{R_E}^2}$$

On the surface of moon

$$g_m = \frac{GM_m}{{R_m}^2}$$

$$g_m = \frac{G(0.012\ M_E)}{(0.27\ R_E)^2}$$

$$g_m = \frac{(0.012)}{(0.27)^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_m = \frac{(0.012)}{(0.27)^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_m = 0.165\ (g)$$

$$g_m = 0.165 \times 9.8$$

$$\boxed{g_m = 1.62\ m/s^2}$$

The weight of body is given by

$W = mg$----(i)

The weight of body on moon given by

$W_m = mg_m$----(ii)

Dividing eq (ii) by eq (i)

$$\frac{W_m}{W} = \frac{mg_m}{mg}$$

$$W_m = \frac{W \times g_m}{g} = \frac{424 \times 1.62}{9.8}$$

$$\boxed{W_m = 70\ N}$$

Result: The value of g on the surface of moon is

$1.62\ \frac{m}{s^2}$ and weight of the body will be 70 N.

Q.5: What is the value of the gravitational acceleration at a distance of (i) earth's radius above the earth's surface? (ii) Twice earth's radius above the earth's surface?

Data:

Value of "g" at height $= g' = ?$

(a) Distance from centre of earth $= R_e' = 2R_e$

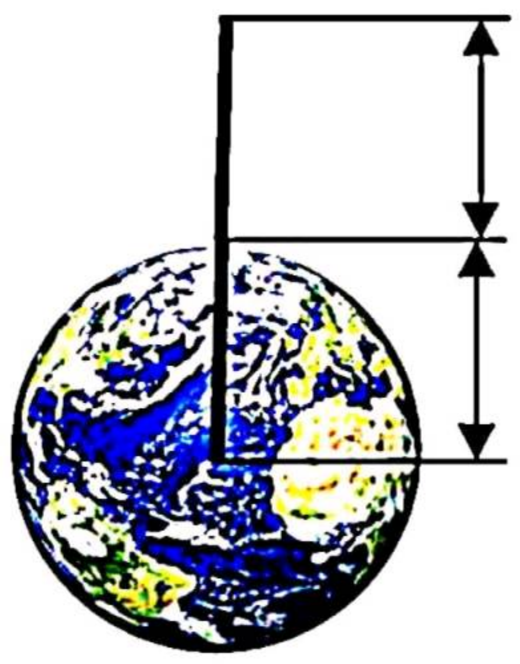
(b) Distance from centre of earth $= R_e' = 3R_e$

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

(a) When distance from centre of earth $= R_e' = 2R_e$



$$g' = \frac{GM_e}{R_e'^2}$$

$$g' = \frac{GM_e}{(2R_e)^2}$$

or $$g' = \frac{1}{4}\frac{GM_e}{R_e^2}$$

Since

$$g = \frac{GM_e}{R_e^2}$$

Therefore

$$g' = \frac{1}{4} \times g$$

$$g' = \frac{1}{4} \times 9.8$$

$$\boxed{g' = 2.45\ m/s^2}$$

(b) When distance from centre of earth $= R_e' = 3R_e$

$$g' = \frac{GM_e}{R_e'^2}$$

$$g' = \frac{GM_e}{(3R_e)^2}$$

or $$g' = \frac{1}{9}\frac{GM_e}{R_e^2}$$

Since

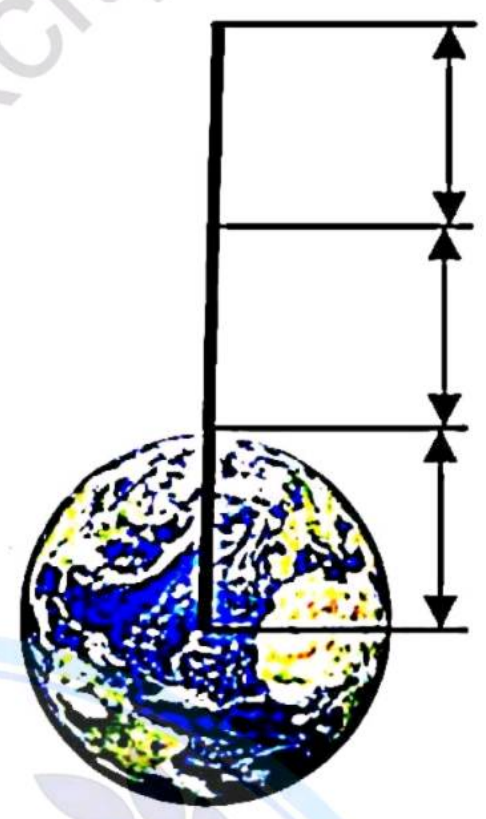
$$g = \frac{GM_e}{R_e^2}$$

Therefore

$$g' = \frac{1}{9} \times g$$

$$g' = \frac{1}{9} \times 9.8$$

$$\boxed{g' = 1.08\ m/s^2}$$



<u>Result:</u> (a) When distance from centre of earth $= R_e' = 2R_e$ value of "g" is $2.45\ m/s^2$ (a) When distance from centre of earth $= R_e' = 3R_e$ value of "g" is $1.08\ m/s^2$

Q.6: At what distance from the center of the earth does the gravitational acceleration have one half the value that it has on the earth's surface?

Data:

Distance from centre of earth = Re' = ?

Value of 'g' at Re' = g' = (1/2) g

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

$$R_e^2 = \frac{GM_e}{g} \quad \text{---- (i)}$$

At Distance Re' from centre of earth

$$g' = \frac{GM_e}{R_e'^2}$$

$$\frac{1}{2}g = \frac{GM_e}{R_e'^2}$$

$$R_e'^2 = \frac{2GM_e}{g}$$

Or $$R_e'^2 = 2R_e^2$$

Taking Sq.root O.B.S

$$R_e'^2 = \sqrt{2}\,R_e = 1.41 \times 6.38 \times 10^6 = 9.022 \times 10^6 m$$

Result:

At $9.022 \times 10^6\ m$ the value of g becomes one hlaf of the value of g on the surface.

Q.7: Compute the gravitational attraction between two college students of mass 80 & 50 kg respectively, 2m apart from each other. Is this force worth worrying about?

Data:

Mass of 1st student $= m_1 = 80\ kg$

Mass of Second Student $= m_2 = 50\ kg$

Distance b/w students $= r = 2\ m$

Force b/w Students $= F = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Then,

$$F_{12} = 6.67 \times 10^{-11}\frac{80 \times 50}{(2)^2}$$

$$\boxed{F_{12} = 6.67 \times 10^{-8} N}$$

Result: The force between students is equal to $6.67 \times 10^{-8} N$ and since this force is negligible therefore this force is not at all worth worying about.

Q.8: Determine the gravitation between the proton and the electron in a hydrogen atom, assuming that the electron describes a circular orbit with a radius of 0.53x10^{-10} m (mass of proton = 1.67x10^{-27} kg, mass of electron = 9.1x10^{-31} kg).

Data:

Mass of Proton $= m_1 = 1.67 \times 10^{-27}\ kg$

Mass of Electron $= m_2 = 9.1 \times 10^{-31}\ kg$

Average Distance $= r = 0.53 \times 10^{-10} m$

Force Between Proton and Electron $= F = ?$

Solution:

According to Newton's law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

then,

$$F_{12} = 6.67 \times 10^{-11}\frac{1.67\times10^{-27}\times9.1\times10^{-31}}{(0.53\times10^{-10})^2}$$

$$\boxed{F_{12} = 3.6 \times 10^{-47} N}$$

Result: The force between Proton and Electron is equal to $3.6 \times 10^{-47} N$.

Q.9: A woman with a mass of 45 kg is standing on a scale in an elevator. The elevator accelerates with a constant acceleration of 1.2 m/s^2. What is the women's weight as measured by her in the elevator.

Data:

Mass of Woman = m = 45 kg

Apparent Weight of Women = W_{app} = ?

Acceleration of elevator = a = 1.2 m/s^2

Solution:

When elevator is moving upward then its acceleration is given by

$$W_{app} = m(a + g)$$

$$W_{app} = 45(1.2 + 9.8)$$

$$\boxed{W_{app} = 495\ N}$$

Result: The women's weight as measured by her in the elevator is 495 N.

PAST PAPER NUMERICALS

2022

vii) At what height from center of the earth, the value of g becomes 36% of its value on the surface of earth?

Data:

Distance from centre of earth = Re′=?

Value of 'g' at Re′ = g′ = $\frac{36}{100}$ g

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

$$R_e^2 = \frac{GM_e}{g} \quad \text{---- (i)}$$

At Distance Re′ from centre of earth

$$g' = \frac{GM_e}{R_e'^2}$$

$$\frac{36}{100}\ g = \frac{GM_e}{R_e'^2}$$

$$R_e'^2 = \frac{100 \times GM_e}{36 \times g}$$

Or

$$R_e'^2 = \frac{25}{9}\ R_e^2$$

Taking Sq.root O.B.S

$$\boxed{R_e'^2 = \frac{5}{3}\ R_e = 1.66 \times 6.38 \times 10^6 = 10.5 \times 10^6 m}$$

Result: At $10.5 \times 10^6\ m$ the value of g becomes 36%of the value of g on the surface

2021

Q.2 (vii) Calculate the centripetal force acting on a man whose mass is 64kg when resting on the ground at the equator. (Radius of the earth = 6.4 x10^6 m.)

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Data:

Mass of man= m =64 kg

Radius of Earth = Re = 6.4 x 10^6 m

Centripetal Force = F_c =?

Solution:

The centripetal Acceleration is given by

$$a_c = R\omega^2$$

And

$$\omega = \frac{2\pi}{T}$$

So,

$$a_c = R\left(\frac{2\pi}{T}\right)^2 = \frac{6.4 \times 10^6 \times 4 \times 3.14^2}{(86400)^2}$$

$$\boxed{a_c = 0.034\ m/s^2}$$

Now, Centripetal force is given by

$$\boxed{F_c = ma_c = 64 \times 0.034 = 2.17\ N}$$

Result: the centripetal acceleration is $0.034\ m/s^2$ and centripetal force is $2.17\ N$.

2019

Q.2(vi) How many times in a second does a spaceship of diameter 30 m, needed to be rotated in order to create artificial gravity?

Data:

Frequency= f = ?

Diameter = d = 30 m

Radius = d/2 = 30/2 =15 m

Solution:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

$$f = \frac{1}{(2 \times 3.14)}\sqrt{\frac{9.8}{15}}$$

$$\boxed{f = 0.128\ rev/sec}$$

Result: The spaceship must be rotated 0.128 rev per second.

2018

Q.2(xi) A man weighing 60 kg is standing on the floor of an elevator. Calculate the force exerted by the man when the elevator is ascending at a rate of 2 m/s^2.

Data:

Mass of person = m = 60 kg

Force exerted by the person = F = ?

Acceleration of elevator = a = 2m/s^2

Solution:

When elevator is moving upward then its acceleration is given by

$$F = m(a + g)$$

$$F = 60(2 + 9.8)$$

$$\boxed{F = 708\ N}$$

Result: The force exerted by the man when the elevator is ascending is 708 N.

2017

Q.2 (iv) The ratio of acceleration due to gravity inside a deep mine to that on the surface of earth is 0.99.Find the depth of the mine, assuming that the density of earth is uniform. Take the radius of earth as $6.38 \times 10^6 m$.

Data:

Acceleration due to gravity inside a deep mine = g'

Acceleration due to gravity on the surface of earth = g

$$\frac{g'}{g} = 0.99$$

Radius of earth $= R_E = 6.38 \times 10^6 m$

Solution:

The acceleration due to gravity at a depth is given by

$$g' = g\left(1 - \frac{d}{R_E}\right)$$

Or we can write as

$$\frac{g'}{g} = \left(1 - \frac{d}{R_E}\right)$$

$$0.99 = \left(1 - \frac{d}{6.38 \times 10^6}\right)$$

$$1 - 0.99 = \frac{d}{6.38 \times 10^6}$$

$$0.01 = \frac{d}{6.38 \times 10^6}$$

$$d = 0.01 \times 6.38 \times 10^6$$

$$\boxed{d = 6.38 \times 10^4 m}$$

Result: The depth of the mine is $6.38 \times 10^4 m$.

2016

Q.2 (v) Textbook Numerical 4

2015

Q.2 ii) At what distance from centre of earth does the gravitational acceleration have one third of the value that it has on the earth's surface?

Data:

Distance from centre of earth = Re'=?

Value of 'g' at Re' = g' = (1/3) g

Solution:

As we know that

$$g = \frac{GM_e}{R_e^2}$$

$$R_e^2 = \frac{GM_e}{g} \quad \text{---- (i)}$$

At Distance Re' from centre of earth

$$g' = \frac{GM_e}{R_e'^2}$$

Or

$$\frac{1}{3}g = \frac{GM_e}{R_e'^2}$$

$$R_e'^2 = \frac{3GM_e}{g}$$

$$R_e'^2 = 3R_e^2$$

Taking Sq.root O.B.S

$$\boxed{R_e'^2 = \sqrt{3}\,R_e = 1.73 \times 6.38 \times 10^6 = 11.03 \times 10^6 m}$$

Result:

At $11.03 \times 10^6 m$ the value of g becomes one third of the value of g on the surface.

2014

Q.2 (xiii) Textbook Numerical 4

2013

Q.2 (ii) .The mass of the moon is approximately one eightieth (1/80) of the earth's mass and its radius is one fourth (1/4) that of earth. Find the acceleration due to gravity on the surface of the moon.

Data:

$Radius\ of\ moon = R_m = (\frac{1}{4})\ R_E$

$= 0.25\ R_E$

$Mass\ of\ moon = M_m = (\frac{1}{80})\ M_E$

$= 0.0125\ M_E$

$Value\ of\ g\ on\ moon = g_m = ?$

Solution:

The acceleration due to gravity is given by

$$g = \frac{GM_E}{{R_E}^2}$$

On the surface of moon

$$g_m = \frac{GM_m}{{R_m}^2}$$

$$g_m = \frac{G(0.0125\ M_E)}{(0.25\ R_E)^2}$$

$$g_m = \frac{(0.0125\ )}{(0.25\ )^2}\left(\frac{GM_E}{{R_E}^2}\right)$$

$$g_m = 0.2\ (g)$$

$$g_m = 0.2 \times 9.8$$

$$\boxed{g_m = 1.96\ m/s^2}$$

Result: The value of g on the surface of moon is $1.96\ \frac{m}{s^2}$

2012

Q.2 (x) Same as 2019 Q.2(vi)

2011, 2010 No Numerical