MATHEMATICS 2nd YEAR

UNIT # 05

LINEAR INEQUALITIES & LINEAR PROGRAMMING

Muhammad Salman Sherazi
M.Phil (Math)

Contents

Sherazi Mathematics

اچھی باتیں

1۔جو کسی کا برا نہیں چاہتے ان کے ساتھ کوئی برا نہیں کر سکتا یہ میرے رب کا وعدہ ہے۔

2۔برے سلوک کا بہترین جواب اچھا سلوک اور جہالت کا جواب "خاموشی" ہے۔

3۔کوئی مانے یا نہ مانے لیکن زندگی میں دو ہی اپنے ہوتے ہیں ایک خود اور ایک خدا۔

4۔جو دوگے وہی لوٹ کے آئے گا عزت ہو یا دھوکہ۔

5۔جس سے اس کے والدین خوشی سے راضی نہیں اس سے اللہ بھی راضی نہیں۔

Linear Inequalities in one variable:-

The inequalities of the form $ax+b<c$, $ax+b>c$, $ax+b\leq c$, $ax+b\geq c$ are called linear inequalities in one variable.

Linear Inequalities in two variables:-

The inequalities of the form $ax+by<c$, $ax+by>c$, $ax+by\leq c$, $ax+by\geq c$ are called linear inequalities in two variables x and y. where a, b and c are constants.

Corresponding Equation/Associated Equation:-

The corresponding equation to any inequality is an equation formed by replacing the inequality symbol with an equal sign. For example
(i) Corresponding equation of $x+2y<6$ is $x+2y=6$. (ii) Corresponding equations of $x\geq 0$ and $2x+y\geq 2$ are $x=0$ and $2x+y=2$ respectively.

Graphing of a Linear Inequality in two variables:-

(i) The corresponding equation is useful for graphing inequalities, because this equation forms the boundary line to the graph of given inequality.

(ii) A vertical line (line ∥ to y-axis) divides the xy-plane into two regions called half planes. (Left half plane and right half plane) (iii) A non-vertical line (line ∥ to x-axis) divides xy-plane into two regions called "half planes". (Upper Half plane and Lower half plane) (iv) If the inequality is strict ($<$ or $>$) then we draw dashed or broken boundary line.

(v) If the inequality is non-strict ($\leq$ or $\geq$) then we draw a solid boundary line.

Note:- $x\geq 0$ means right half plane, $x\leq 0$ means left half plane, $y\geq 0$ means upper half plane, $y\leq 0$ means lower half plane.

Procedure for Graphing a linear Inequality in two variables:-

(i) Graph the corresponding equation of given inequality. (ii) Select any test point (not on the graph of corresponding equation of inequality). The origin $(0,0)$ is most convenient point to choose as a test pt.

(iii) Put the coordinates of the test pt. in the inequality.

(iv) If the test pt. satisfies the given inequality, then shade the half plane containing the test pt.

(v) If the test pt. does not satisfy the given inequality then the shade the half plane that does not contain the test pt.

Example 1:- Graph the inequality $x+2y<6$.

Solution:- $x+2y<6$ —— (i)

The corresponding eq. of (i) is

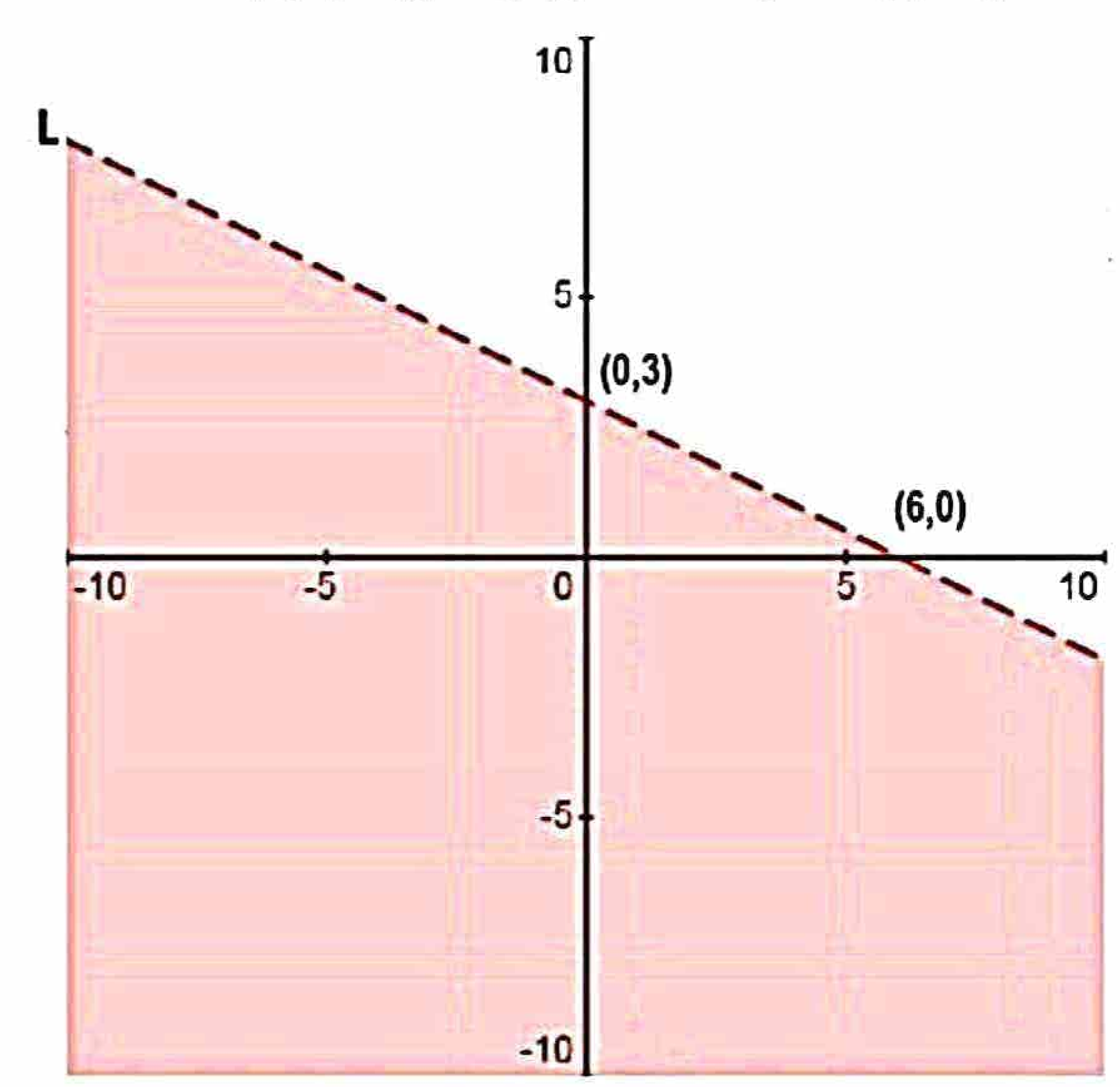
$L: x+2y=6$ —— (ii)

Put $x=0$, $y=3$ so the pt. $(0,3)$

Put $y=0$, $x=6$ so the pt $(6,0)$

Test pt $(0,0)$:- we test (i) at $(0,0)$

(i) $\to 0<6 \longrightarrow$ True



Example 2. Graph the following inequalities in xy-plane; (i) $2x \geq -3$ (ii) $y \leq 2$

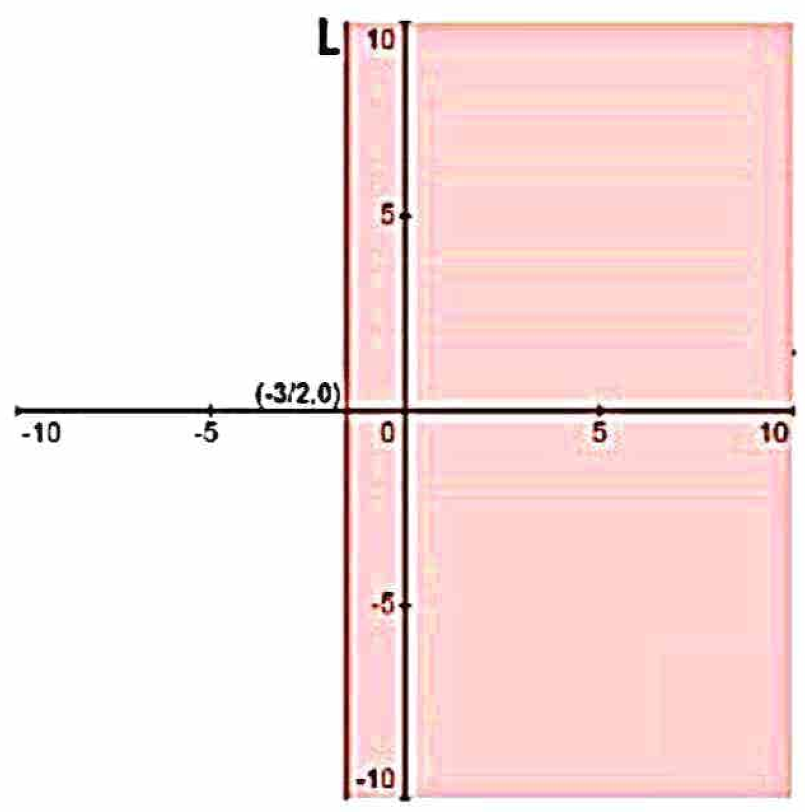
Solution:- (i) $2x \geq -3$ —— (i)

The corresponding eq. of (i) is

L; $2x = -3 \rightarrow x = -\frac{3}{2}$ (line ∥ to y-axis passing through $(-\frac{3}{2}, 0)$)

Test pt (0,0):- We test (i) at (0,0)

(i) $\rightarrow 0 \geq -3 \longrightarrow$ True

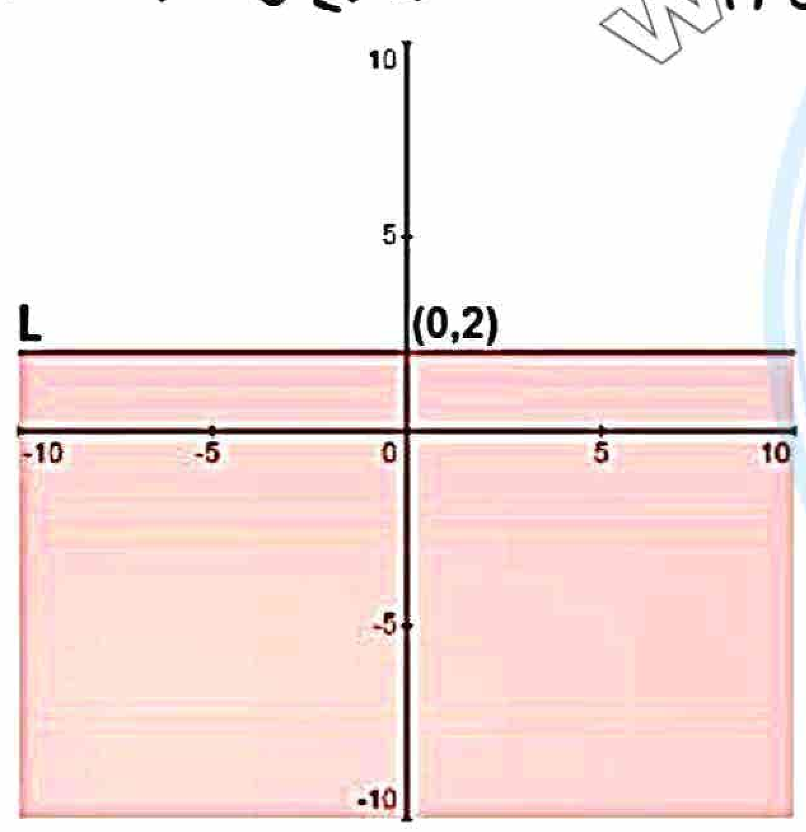


(ii) $y \leq 2$ —— (i)

The corresponding eq./associated eq. of (i) is L; $y = 2$ (line ∥ to x-axis passing throug (0,2))

Test pt (0,0):- We test (i) at (0,0)

(i) $\rightarrow 0 \leq 2 \longrightarrow$ True



Solution Set of linear Inequalities:- The ordered pair (a, b) which satisfy the linear inequalit in two variables x and y form the solution.

Solution Region:- Solution region of system of inequalities is the common region that satisfies all given inequalities in the system.

Corner point/Vertex:- A point of the solution region where two of its boundary lines intersect is called the corner point or vertex of the solution region.

Example 1:- Graph the system of inequalities $x - 2y \leq 6$, $2x + y \geq 2$

Solution:- $x - 2y \leq 6$ —— (i)

$2x + y \geq 2$ —— (ii)

The associated eqs. of (i) and (ii) are

L1; $x - 2y = 6$ —— (iii) L2; $2x + y = 2$ —— (iv)

(iii) $\rightarrow$ Put $x = 0$, $y = -3$ so the pt $(0, -3)$

Put $y = 0$, $x = 6$ so the pt $(6, 0)$

(iv) $\rightarrow$ Put $x = 0$, $y = 2$ so the pt $(0, 2)$

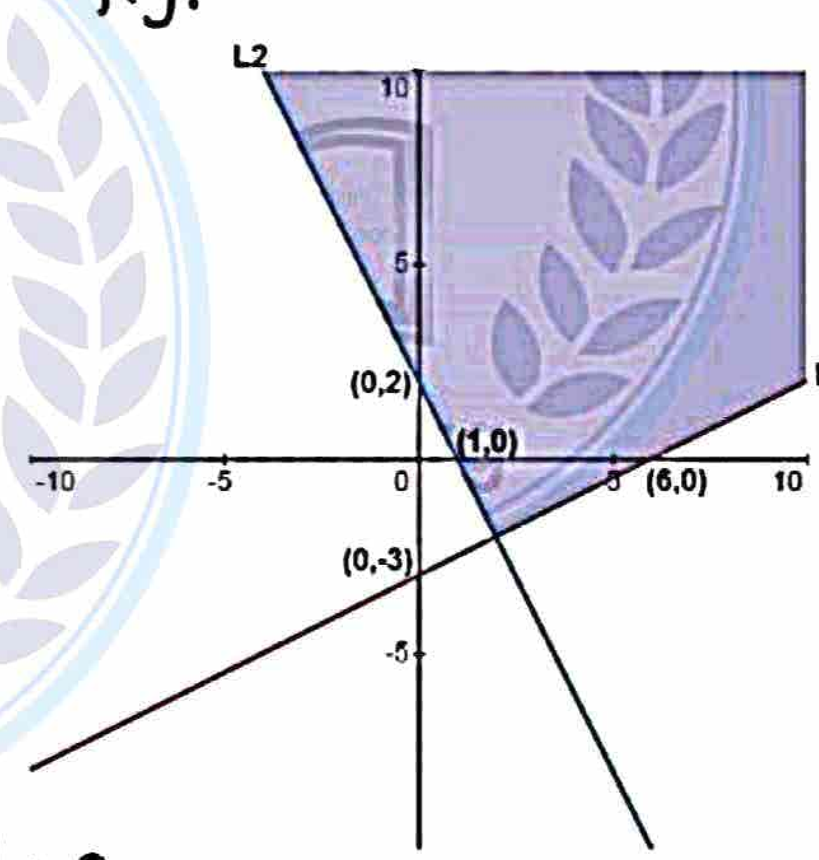
Put $y = 0$, $x = 1$ so the pt $(1, 0)$

Test pt (0,0):- We test (i) and (ii) at (0,0) so

(i) $\rightarrow 0 \leq 6$ —— True

(ii) $\rightarrow 0 \geq 2$ —— False

Solution region:- The solution of the given system of graph of (i) and (ii). So solution region is shaded area as shown in fig.



Example 2. Graph the solution region for the following system of inequalities: $x - 2y \leq 6$, $2x + y \geq 2$, $x + 2y \leq 10$

Solution:- $x - 2y \leq 6$ —— (i)

$2x + y \geq 2$ —— (ii)

$x + 2y \leq 10$ —— (iii)

The associated eqs. of (i), (ii) and (iii) are

L1; $x - 2y = 6$ —— (iv) L2; $2x + y = 2$ —— (v)

L3; $x + 2y = 10$ —— (vi)

(iv) → Put $x=0$, $y=-3$ so the pt. $(0,-3)$
Put $y=0$, $x=6$ so the pt. $(6,0)$

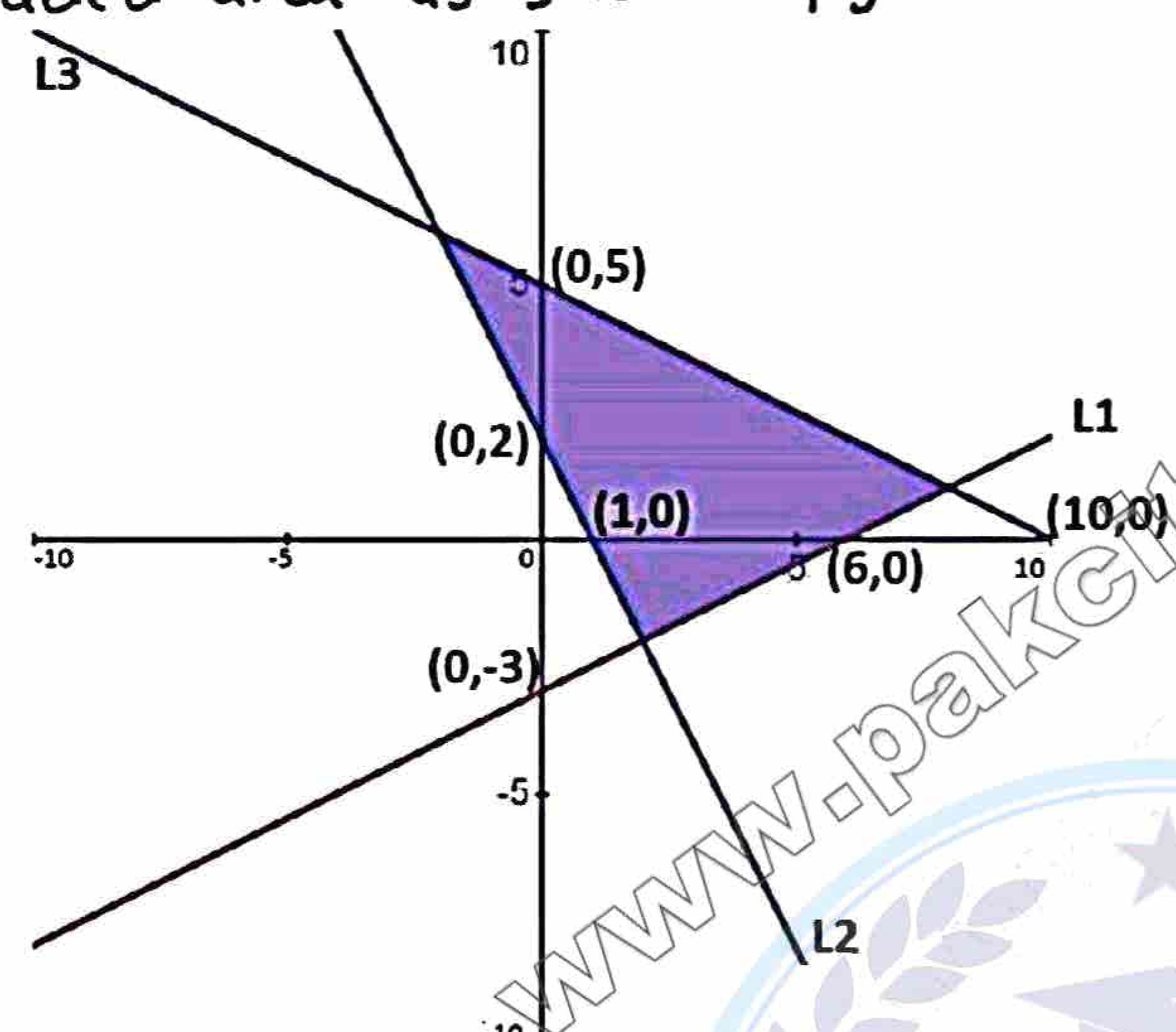
(v) → Put $x=0$, $y=2$ so the pt $(0,2)$
Put $y=0$, $x=1$ so the pt $(1,0)$

(vi) → Put $x=0$, $y=5$ so the pt $(0,5)$
Put $y=0$, $x=10$ so the pt $(10,0)$

Test pt $(0,0)$:- We test (i), (ii) and (iii) at $(0,0)$. so

(i) → $0 \le 6$ —— True (ii) → $0 \ge 2$ —— False
(iii) → $0 \le 10$ —— True

Solution region:- The solution of the given system is intersection of the graphs of (i), (ii) and (iii). So solution region is shaded area as show in fig.



Example 3. Graph the following systems of inequalities. (i) $2x+y \ge 2$, $x+2y \le 10$, $y \ge 0$
(ii) $2x+y \ge 2$, $x+2y \le 10$, $x \ge 0$ (iii) $2x+y \ge 2$, $x+2y \le 10$
$x \ge 0$, $y \ge 0$

Solution:- (i) $2x+y \ge 2$ —— (i)
$x+2y \le 10$ —— (ii)
$y \ge 0$

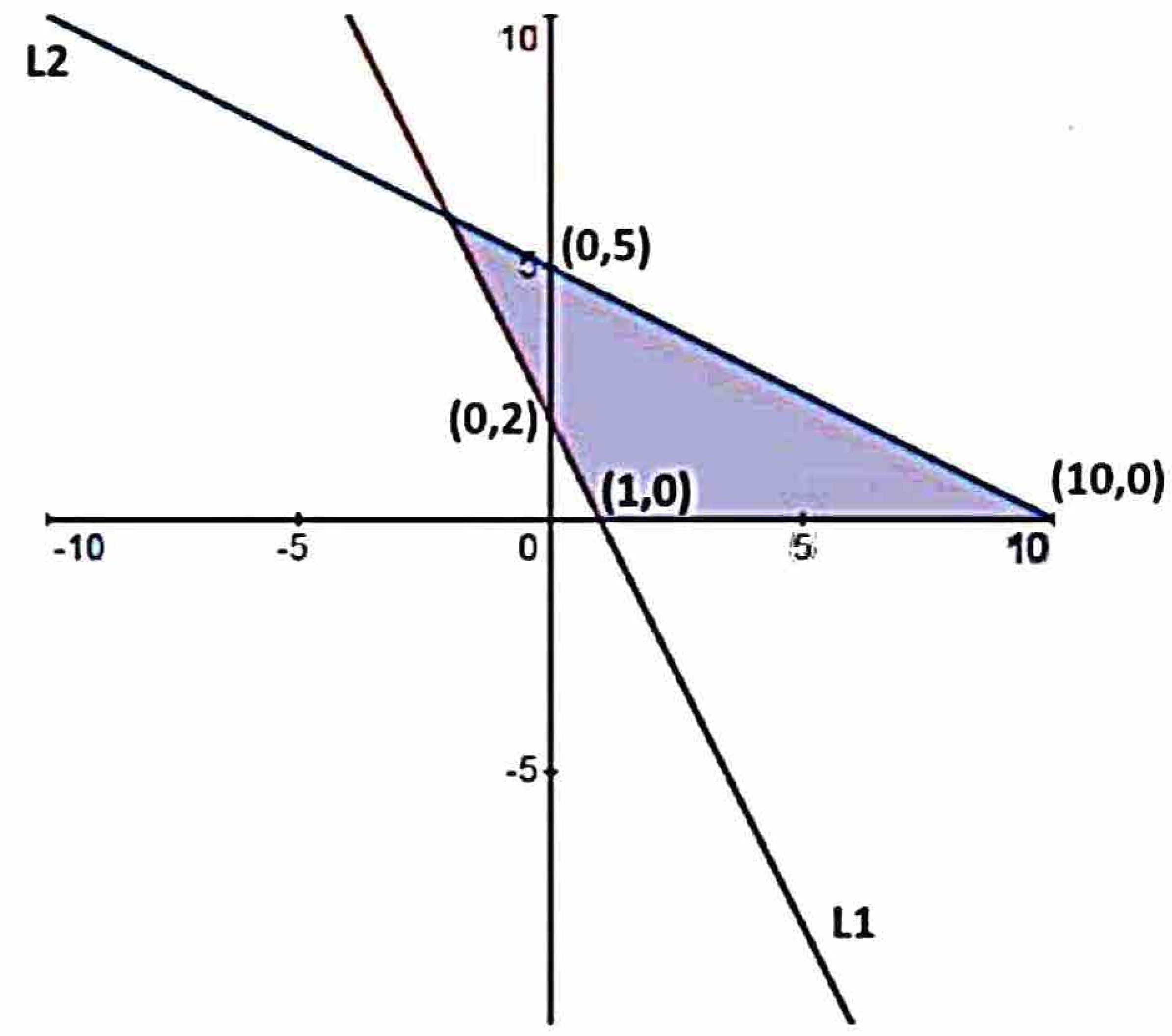
The associated eqs. of (i) and (ii) are
L1; $2x+y = 2$ —— (iii) L2; $x+2y = 10$ —— (iv)

(iii) → Put $x=0$, $y=2$ so the pt $(0,2)$
Put $y=0$, $x=1$ so the pt $(1,0)$

(iv) → Put $x=0$, $y=5$ so the pt $(0,5)$
→ Put $y=0$, $x=10$ so the pt $(10,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$. so
(i) → $0 \ge 2$ —— False (ii) → $0 \le 10$ —— True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). So solution region is shaded area as shown in fig.



(ii) $2x+y \ge 2$ —— (i), $x+2y \le 10$ —— (ii)
$x \ge 0$

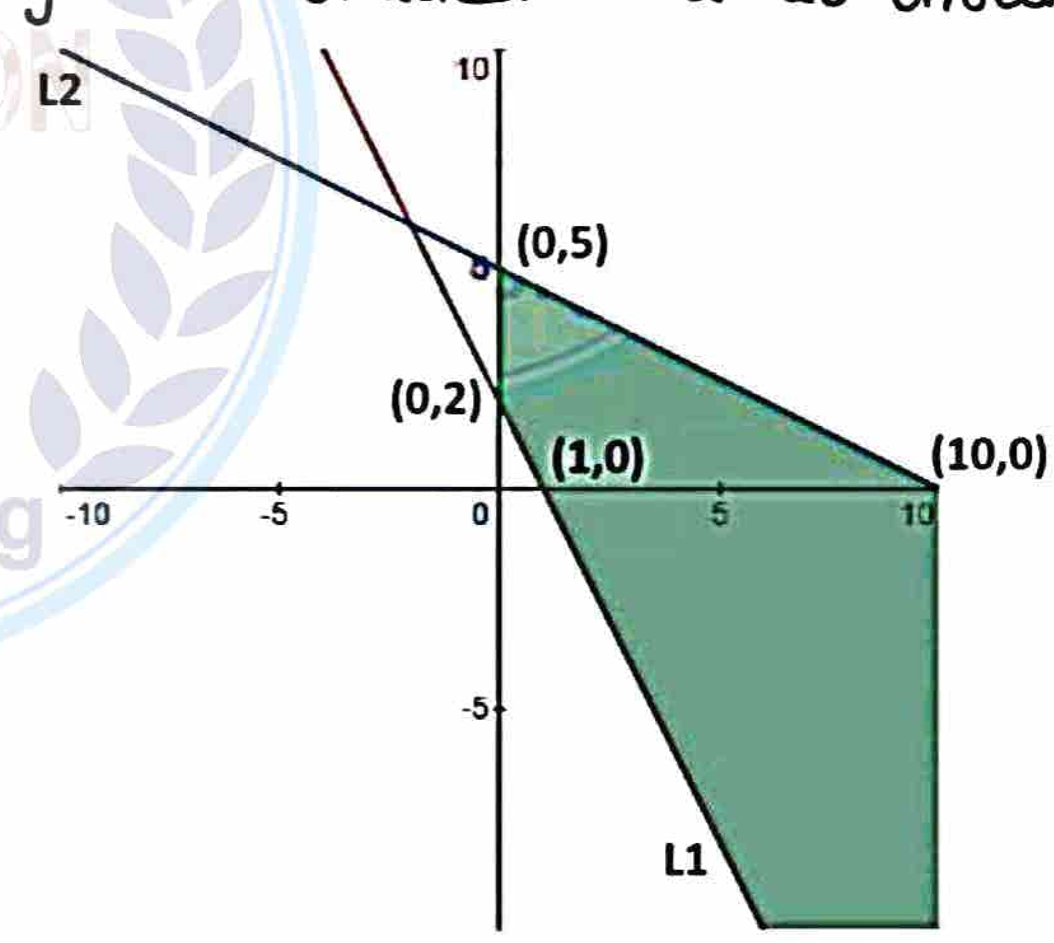
The associated eqs of (i) and (ii) are
L1; $2x+y = 2$ —— (iii) L2; $x+2y = 10$ —— (iv)

(iii) → Put $x=0$, $y=2$ so the pt $(0,2)$
Put $y=0$, $x=1$ so the pt $(1,0)$

(iv) → Put $x=0$, $y=5$ so the pt $(0,5)$
Put $y=0$, $x=10$ so the pt $(10,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$
so (i) → $0 \ge 2$ —— False (ii) → $0 \le 10$ —— True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). so solution region is shaded area as shown in fig.



(iii) $2x+y \ge 2$ —— (i) $x+2y \le 10$ —— (ii)
$x \ge 0$, $y \ge 0$

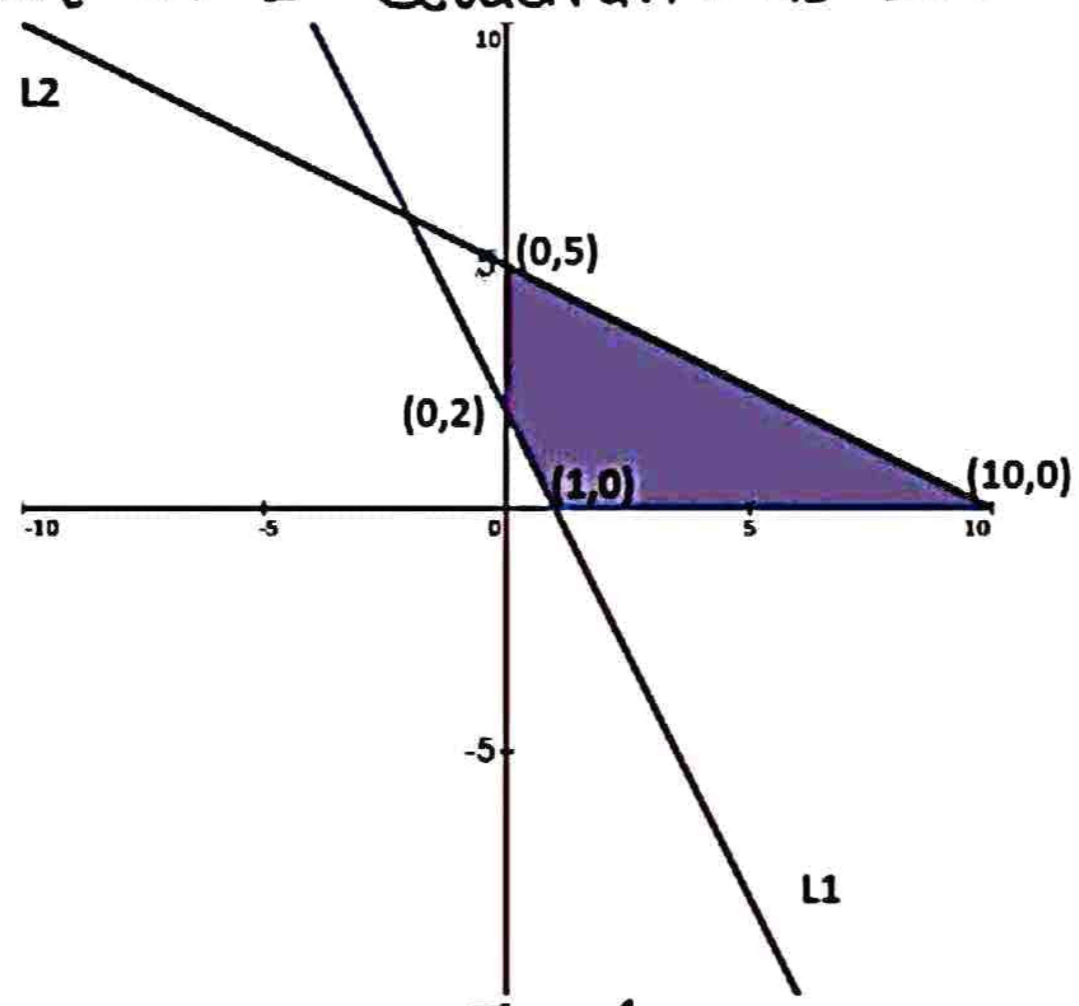
The associated eqs of (i) and (ii) are
L1; $2x+y = 2$ —— (iii) L2; $x+2y = 10$ —— (iv)

(iii) → Put $x=0$, $y=2$, so the pt $(0,2)$
Put $y=0$, $x=1$ so the pt $(1,0)$

(iv) → Put $x=0$, $y=5$ so the pt $(0,5)$
Put $y=0$, $x=10$ so the pt $(10,0)$

Test pt(0,0):- We test (i) and (ii) at (0,0)
so (i) → $0 \geq 2$ —— False (ii) → $0 \leq 10$ —— True

Solution region:- The solution of the given system of graph of (i) and (ii). Also $x \geq 0$, $y \geq 0$ indicates that graph of solution set in I[st] Quadrant as shown



Exercise 5.1

Q1. Graph the solution set of each of the following linear inequality in xy-plane.

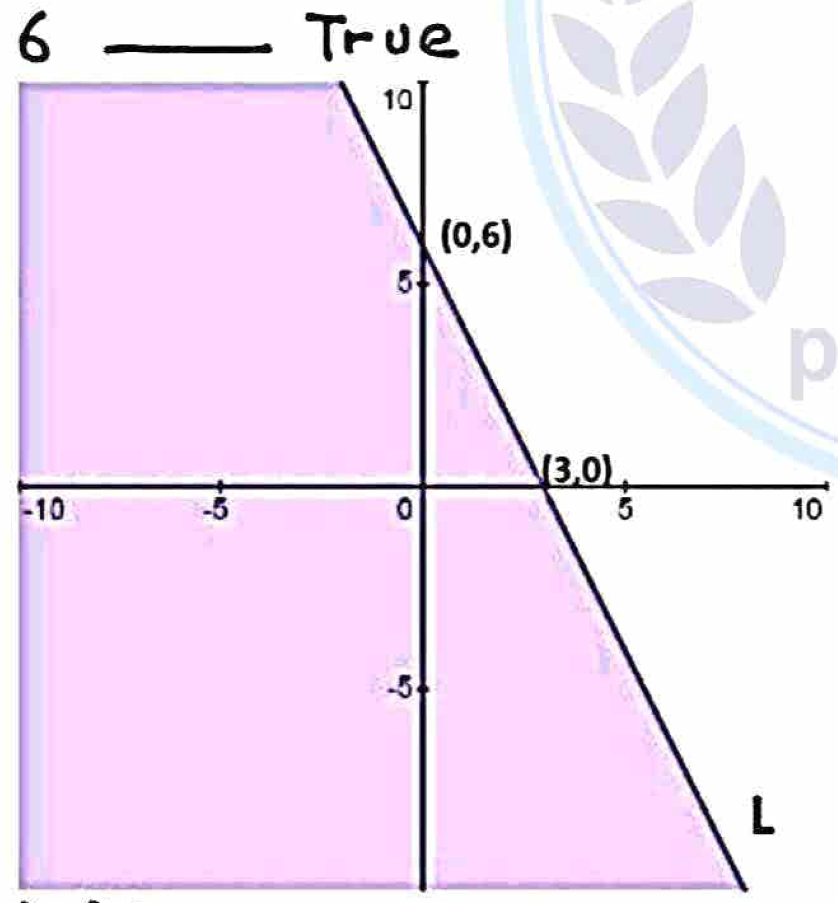
(i) $2x + y \leq 6$

Solution:- $2x + y \leq 6$ —— (i)

The associated eq. of (i) is L; $\underline{2x + y = 6}$ —— (ii)

(ii) → Put $x = 0$, $y = 6$ so the pt $(0, 6)$
Put $y = 0$, $x = 3$ so the pt $(3, 0)$

Test pt(0,0):- We test (i) at (0,0)
(i) → $0 \leq 6$ —— True



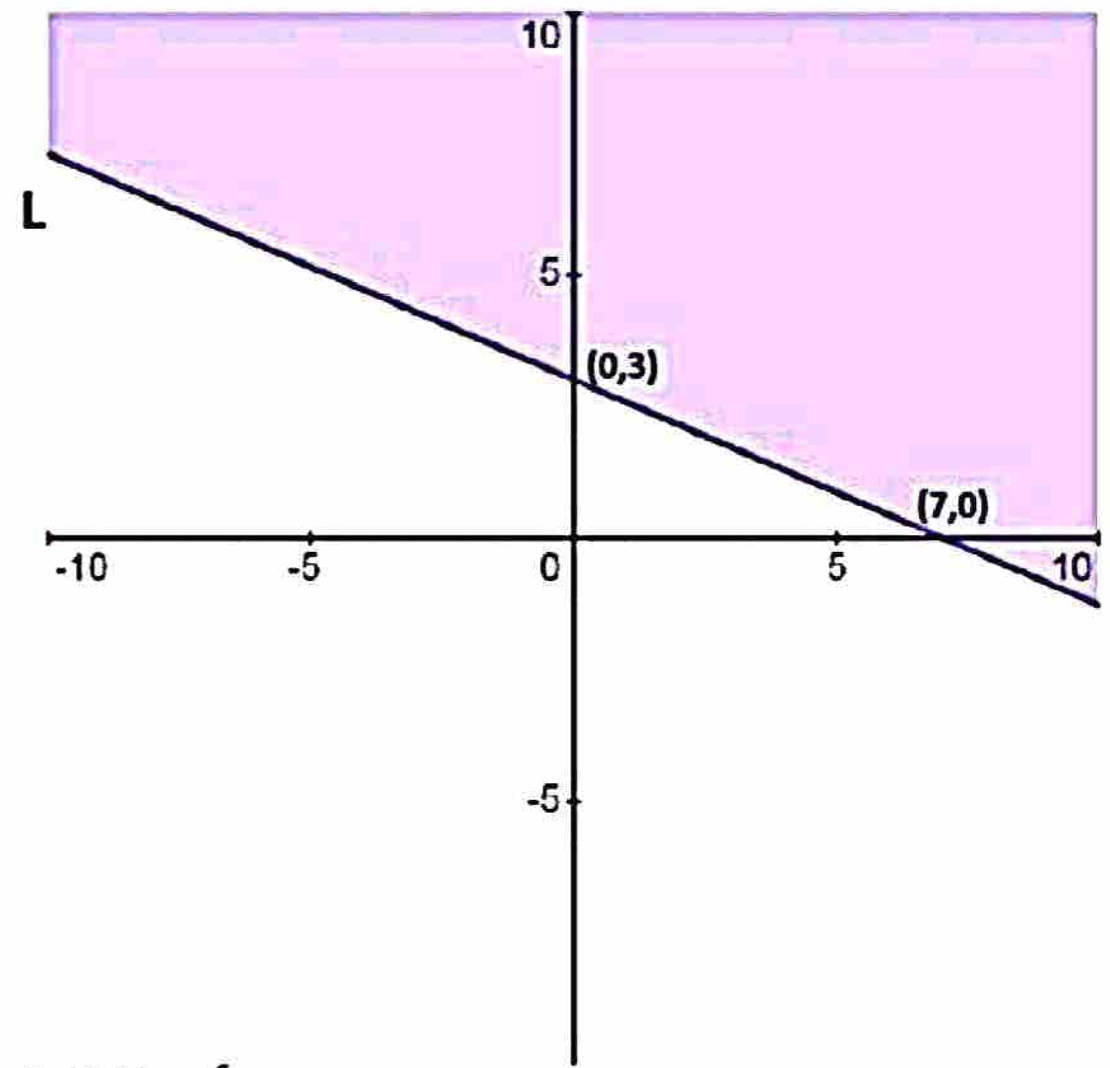
(ii) $3x + 7y \geq 21$

Solution:- $3x + 7y \geq 21$ —— (i)

The associated eq. of (i) is L; $\underline{3x + 7y = 21}$ —— (ii)

(ii) → Put $x = 0$, $y = 3$ so the pt $(0, 3)$
→ Put $y = 0$, $x = 7$ so the pt $(7, 0)$

Test pt(0,0):- We test (i) at (0,0)
(i) → $0 \geq 21$ —— False



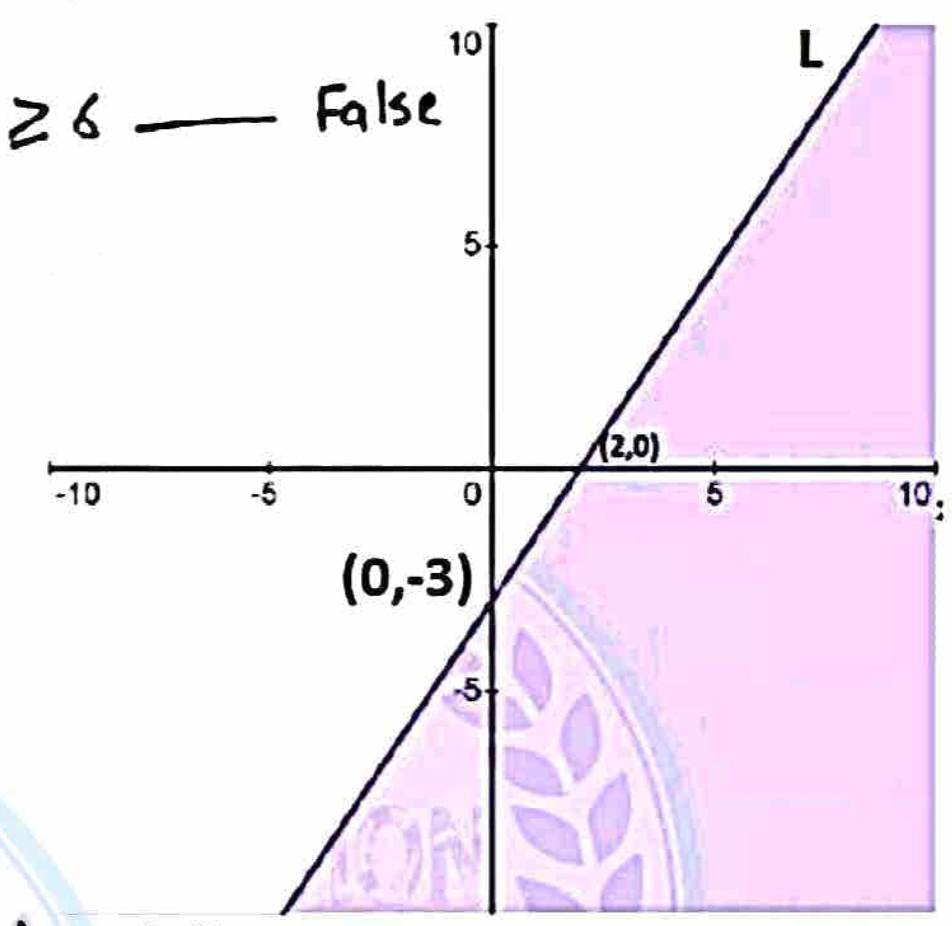
(iii) $3x - 2y \geq 6$

Solution:- $3x - 2y \geq 6$ —— (i)

The associated eq. of (i) is L; $\underline{3x - 2y = 6}$ —— (ii)

(ii) → Put $x = 0$, $y = -3$ so the pt. $(0, -3)$
Put $y = 0$, $x = 2$ so the pt. $(2, 0)$

Test pt(0,0):- We test (i) at (0,0). so
(i) → $0 \geq 6$ —— False



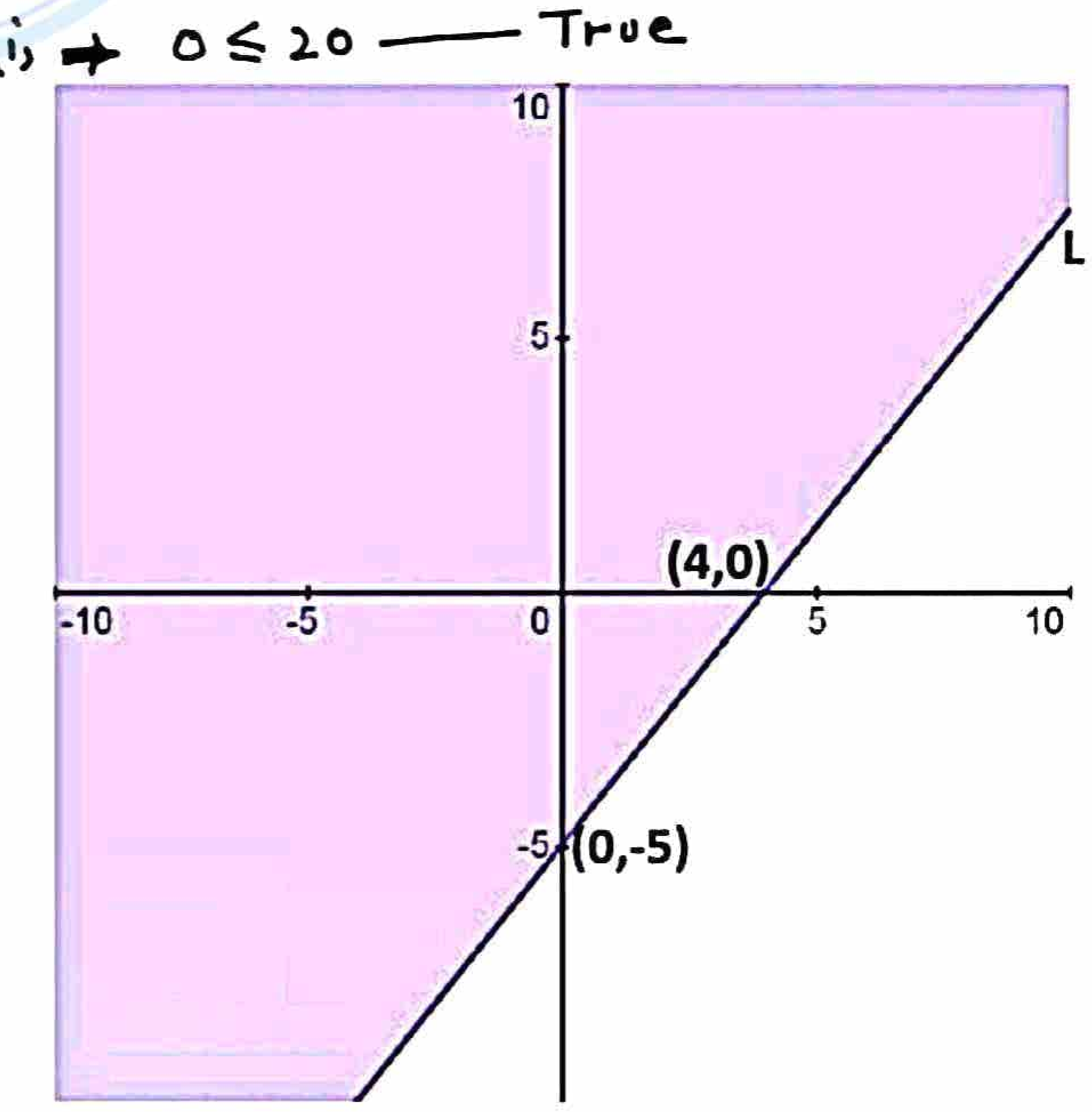
(iv) $5x - 4y \leq 20$

Solution:- $5x - 4y \leq 20$ —— (i)

The associated eq. of (i) is L; $\underline{5x - 4y = 20}$ —— (ii)

(ii) → put $x = 0$, $y = -5$ so the pt $(0, -5)$
Put $y = 0$, $x = 4$ so the pt $(4, 0)$

Test pt(0,0):- We test (i) at (0,0). so
(i) → $0 \leq 20$ —— True



(v) $2x+1 \geq 0$

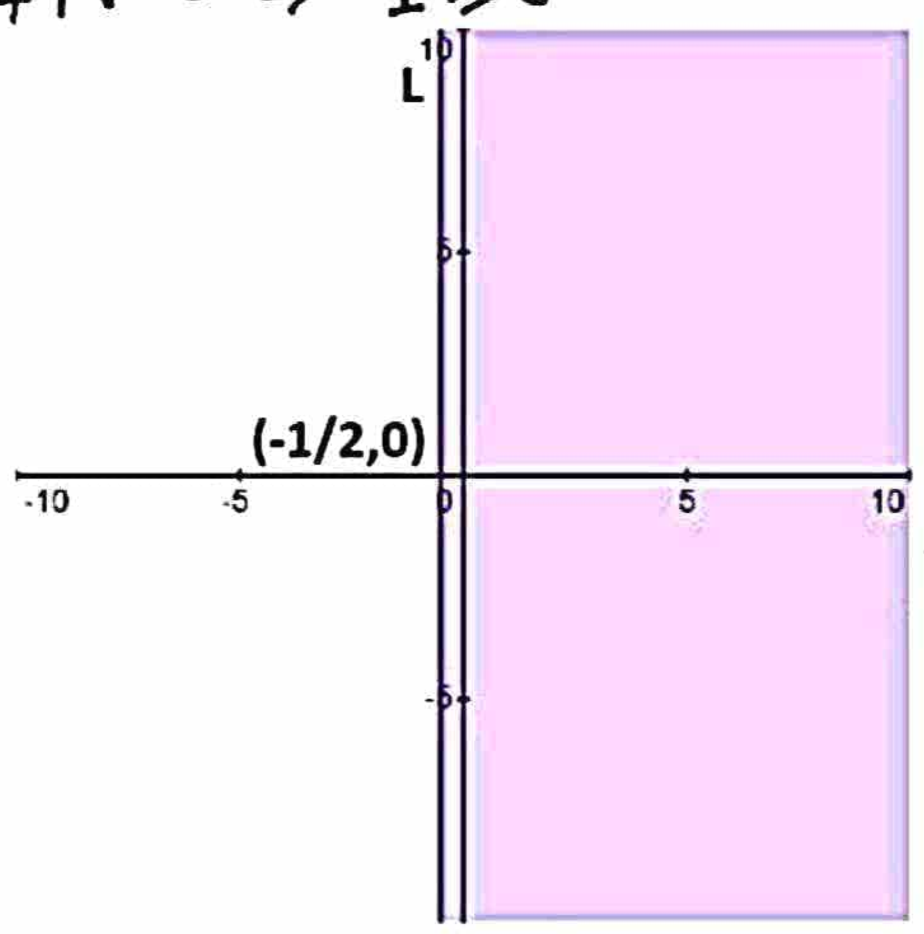
Solution:- $2x+1 \geq 0$ ——— (i)

The associated eq. of (i) is $L; 2x+1=0$

$\Rightarrow 2x=-1 \Rightarrow x=-\frac{1}{2}$ (line ∥ to y-axis passing through $(-\frac{1}{2},0)$)

Test pt (0,0):- We test (i) at (0,0). so

(i) $\Rightarrow 2(0)+1 \geq 0 \Rightarrow 1 \geq 0$ —— True



(vi) $3y-4 \leq 0$

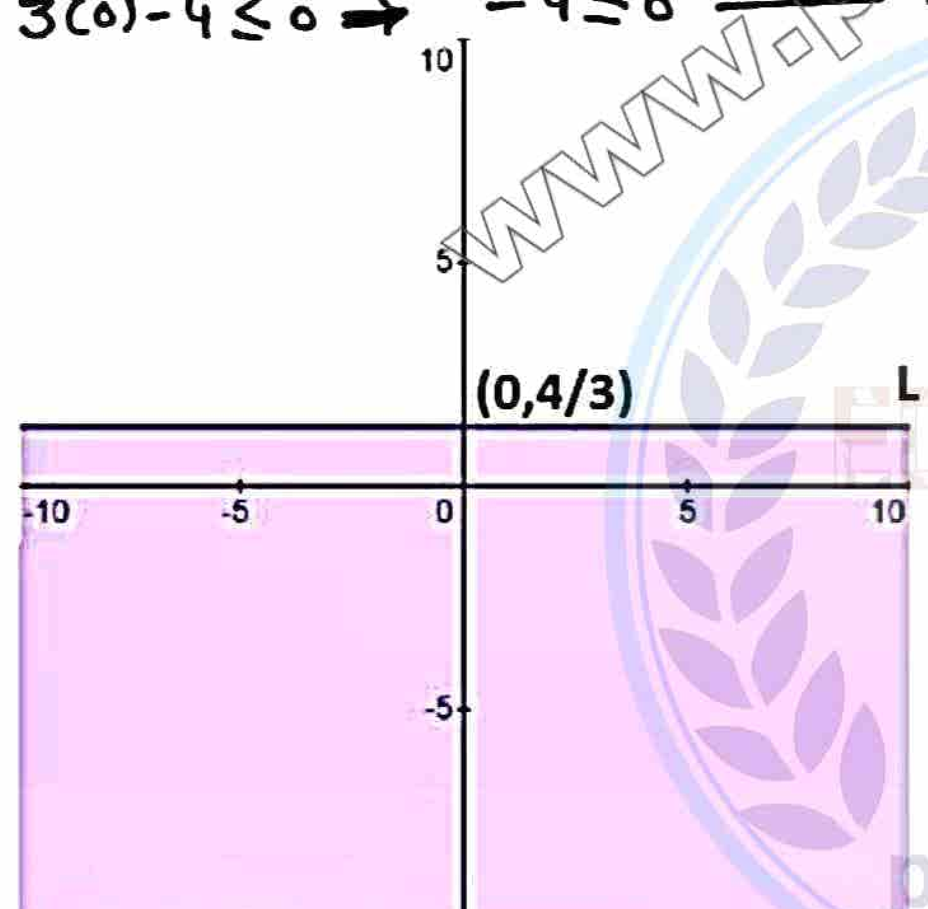
Solution:- $3y-4 \leq 0$ ——— (i)

The associated eq. of (i) is $L; 3y-4=0$

$\Rightarrow 3y=4 \Rightarrow y=\frac{4}{3}$ (line ∥ to x-axis passing through $(0,\frac{4}{3})$)

Test pt (0,0):- We test (i) at (0,0). so

(i) $\Rightarrow 3(0)-4 \leq 0 \Rightarrow -4 \leq 0$ —— True



Q2. Indicate the solution set of the following systems of linear inequalities by shading:

(i) $2x-3y \leq 6$
$2x+3y \leq 12$

Solution: $2x-3y \leq 6$ —— (i)
$2x+3y \leq 12$ —— (ii)

The associated eqs. of (i) and (ii) are

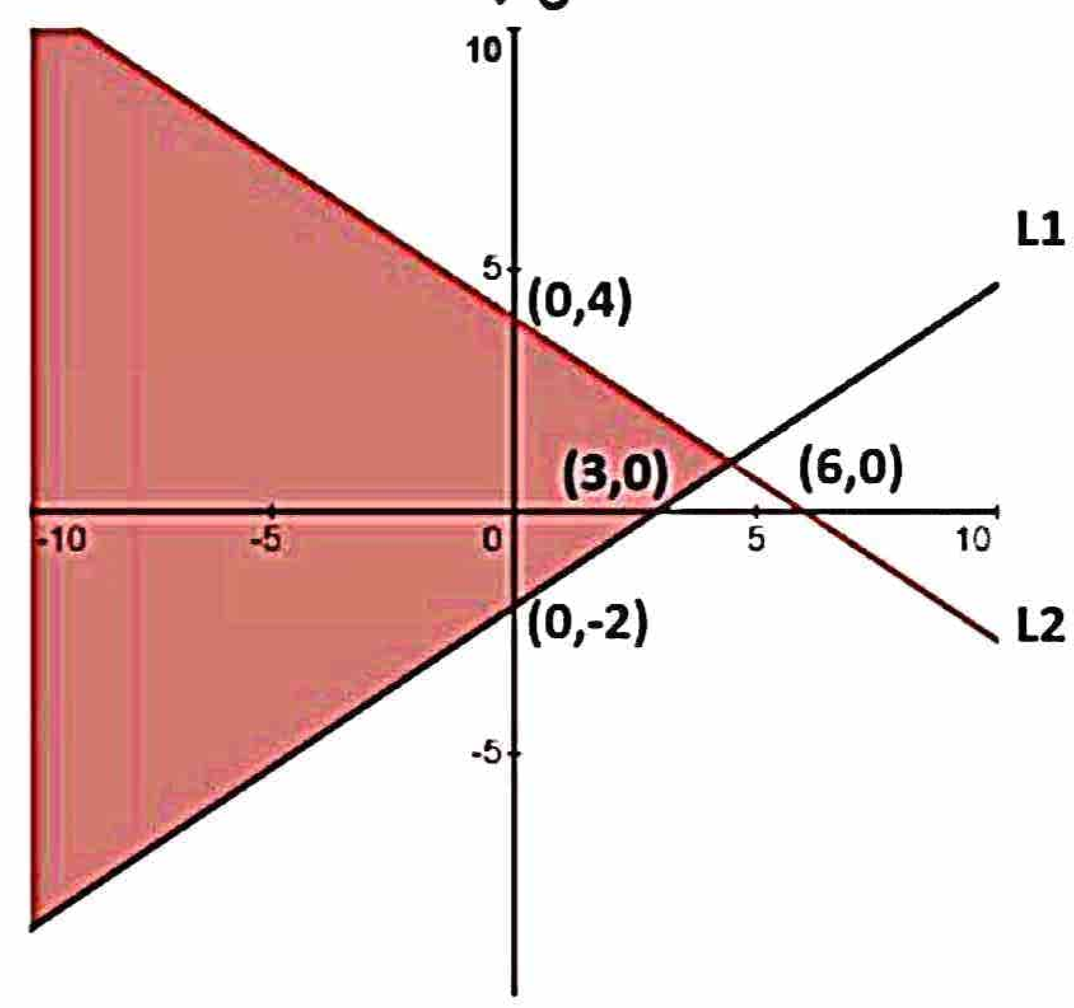
$L1; 2x-3y=6$ —— (iii) $L2; 2x+3y=12$ —— (iv)

(iii) $\Rightarrow$ Put $x=0$, $y=-2$ so the pt $(0,-2)$
Put $y=0$, $x=3$ so the pt $(3,0)$

(iv) $\Rightarrow$ Put $x=0$, $y=4$ so the pt $(0,4)$
Put $y=0$, $x=6$ so the pt $(6,0)$

Test pt (0,0):- We test (i) and (ii) at (0,0). so (i) $\Rightarrow 0 \leq 6$ — True
(ii) $\Rightarrow 0 \leq 12$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). So solution region is shaded area as shown in fig.



(ii) $x+y \geq 5$
$-y+x \leq 1$

Solution:- $x+y \geq 5$ —— (i) $-y+x \leq 1$ —(ii)

The associated eqs. of (i) and (ii) are

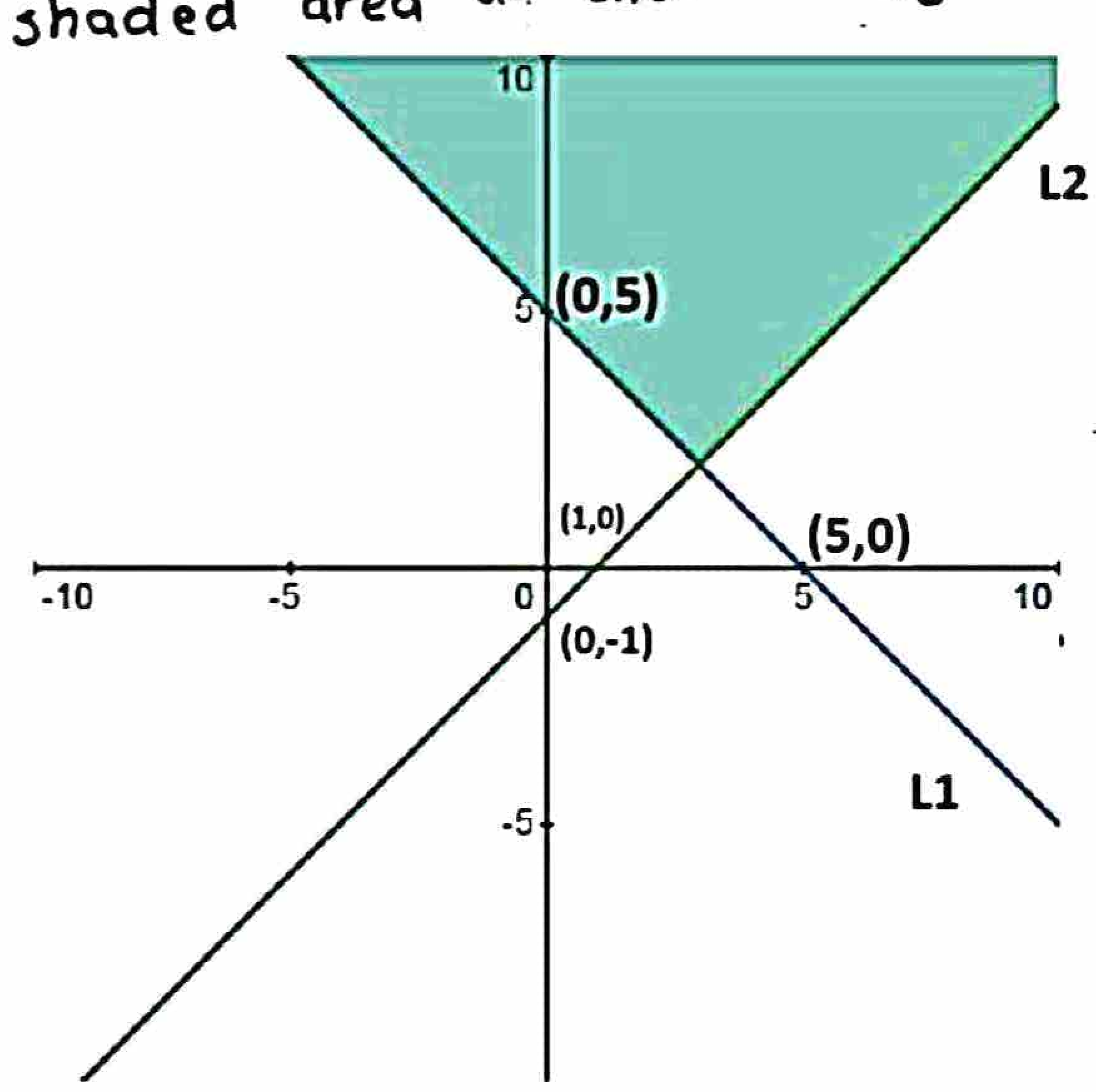
$L1; x+y=5$ —— (iii) $L2; -y+x=1$ —— (iv)

(iii) $\Rightarrow$ Put $x=0$, $y=5$ so the pt $(0,5)$
Put $y=0$, $x=5$ so the pt $(5,0)$

(iv) $\Rightarrow$ Put $x=0$, $y=-1$ so the pt $(0,-1)$
Put $y=0$, $x=1$ so the pt $(1,0)$

Test pt (0,0):- We test (i) and (ii) at (0,0). so (i) $\Rightarrow 0 \geq 5$ —— False
(ii) $\Rightarrow 0 \leq 1$ —— True

Solution region:- The solution region of given system is intersection of the graphs of (i) and (ii). So solution region is shaded area as shown in fig.



(iii) $3x+7y \geq 21$, $x-y \leq 2$

Solution:- $3x+7y \geq 21$ — (i) $x-y \leq 2$ — (ii)

The associated eqs. of (i) and (ii) are

l_1; $3x+7y=21$ — (iii) l_2; $x-y=2$ — (iv)

(iii) → Put $x=0$, $y=3$ so the pt $(0,3)$

Put $y=0$, $x=7$ so the pt $(7,0)$

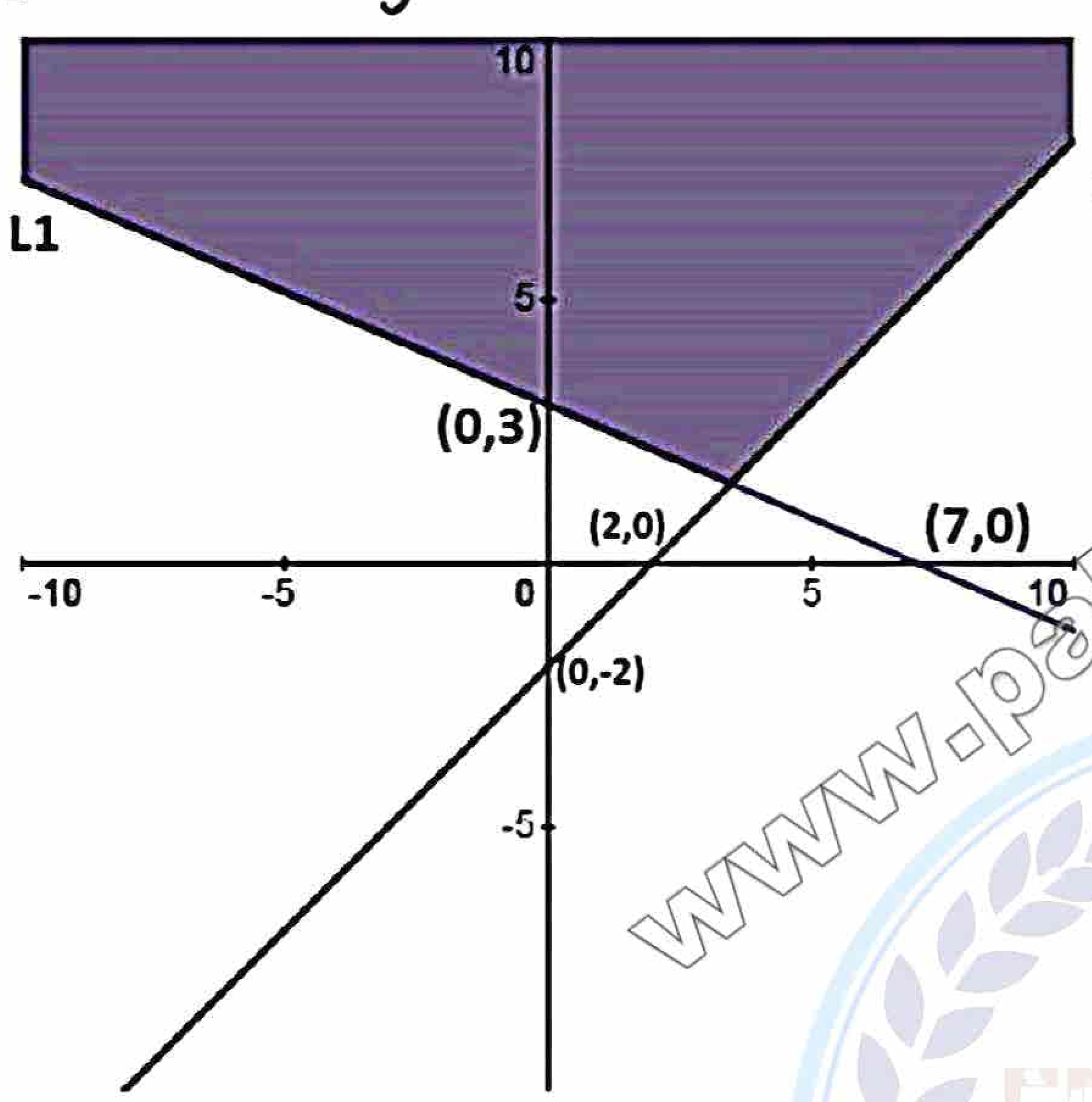
(iv) → $x=0$, $y=-2$ so the pt $(0,-2)$

Put $y=0$, $x=2$ so the pt $(2,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$. So (i) → $0 \geq 21$ — False

(ii) → $0 \leq 2$ — True

Solution region:- The solution of the given system of graph of (i) and (ii). So solution region is shaded area as shown in fig.



(iv) $4x-3y \leq 12$, $x \geq -\frac{3}{2}$

Solution:- $4x-3y \leq 12$ — (i) , $x \geq -\frac{3}{2}$ — (ii)

The associated eqs. of (i) and (ii) are

L1; $4x-3y=12$ — (iii) , L2; $x=-\frac{3}{2}$ — (iv)

(iii) → Put $x=0$, $y=-4$ so the pt $(0,-4)$

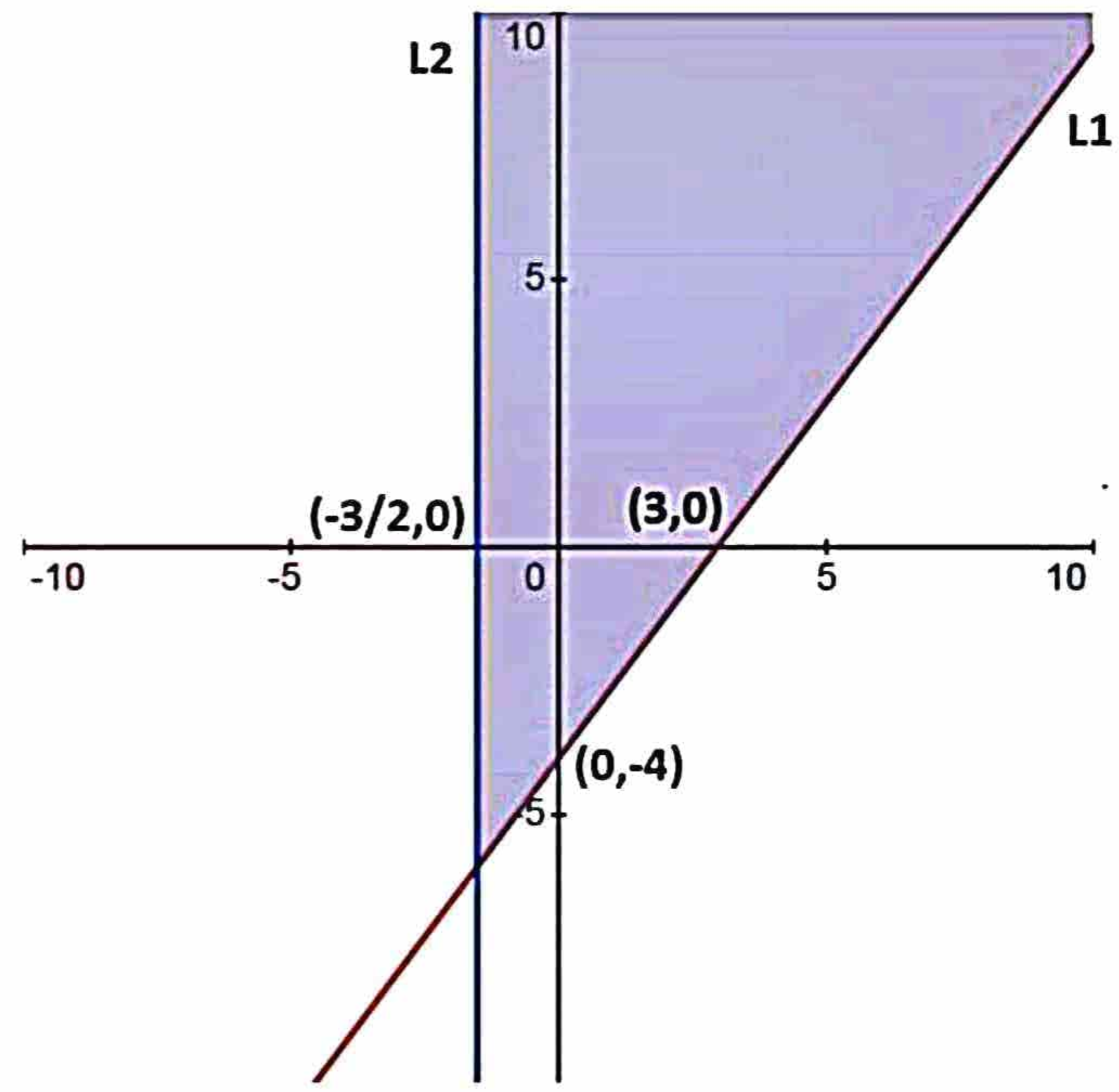
Put $y=0$, $x=3$ so the pt $(3,0)$.

(iv) → $x=-\frac{3}{2}$ (line ∥ to y-axis through pt. $(-\frac{3}{2},0)$)

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$

so (i) → $0 \leq 12$ — True (ii) → $0 \geq -\frac{3}{2}$ — True

Solution region:- The solution of the given system is intersection of graph of (i) and (ii). So solution region is shaded area as shown in fig.



(v) $3x+7y \geq 21$, $y \leq 4$

Solution:- $3x+7y \geq 21$ — (i) , $y \leq 4$ — (ii)

The associated eqs. of (i) and (ii) are

L1; $3x+7y=21$ — (iii) L2; $y=4$ — (iv)

(iii) → Put $x=0$, $y=3$ so the pt $(0,3)$

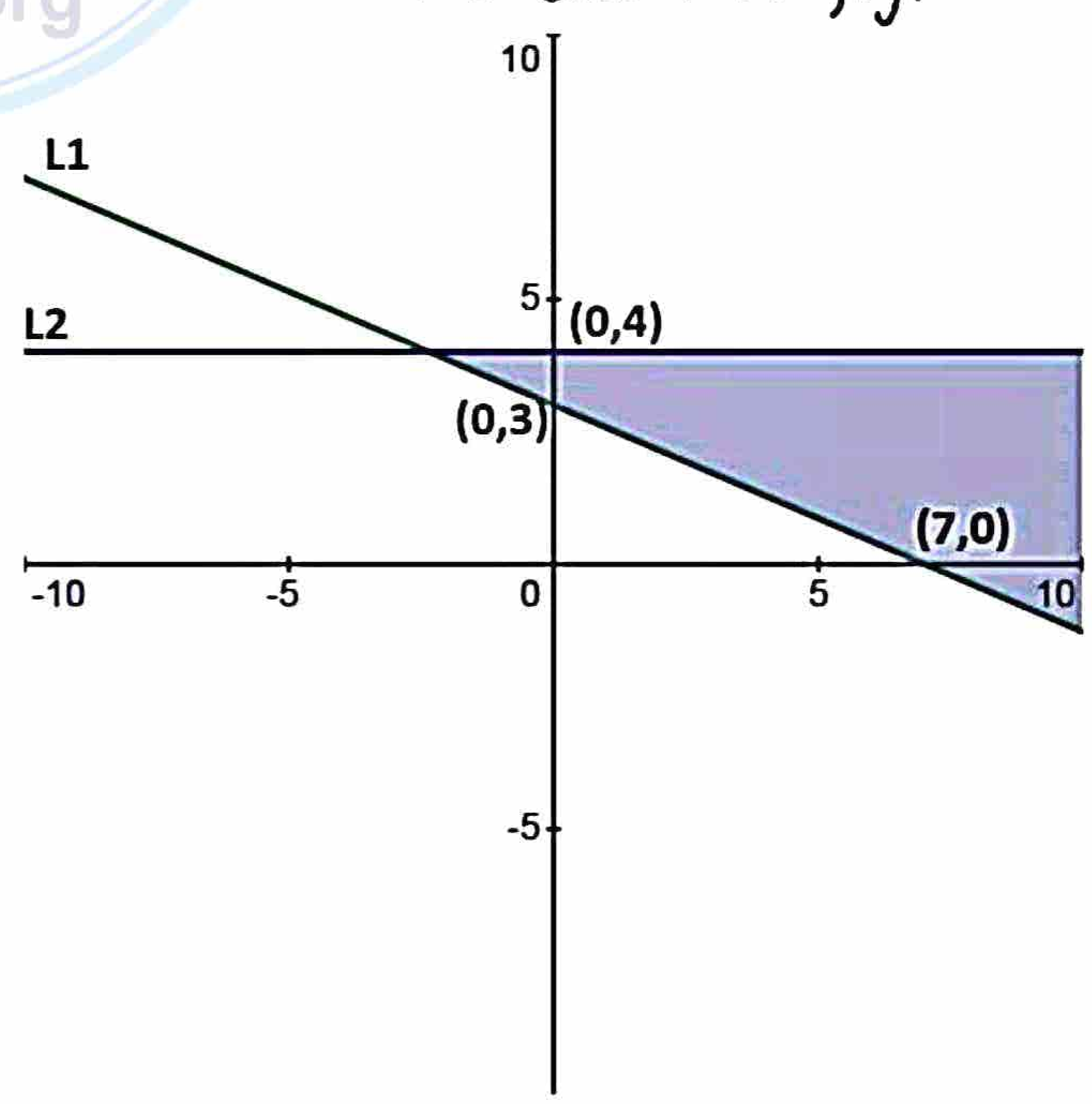
Put $y=0$, $x=7$ so the pt $(7,0)$

(iv) → $y=4$ (line ∥ to the x-axis, through pt $(0,4)$)

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$

so (i) → $0 \geq 21$ — False, (ii) → $0 \leq 4$ — True

Solution region:- The solution of the given system is intersection of the graph of (i) and (ii). So Solution region is shaded area as shown in fig.



Q3. Indicate the solution region of the following systems of linear inequalities by shading:

(i) $2x-3y \le 6$; $2x+3y \le 12$; $y \ge 0$

Solution:- $2x-3y \le 6$ —(i) $2x+3y \le 12$ —(ii)

The associated eqs. of (i) and (ii) are

L1; $2x-3y=6$ —(iii) L2; $2x+3y=12$ —(iv)

(iii) → $x=0$, $y=-2$ so the pt $(0,-2)$

Put $y=0$, $x=3$ so the pt $(3,0)$

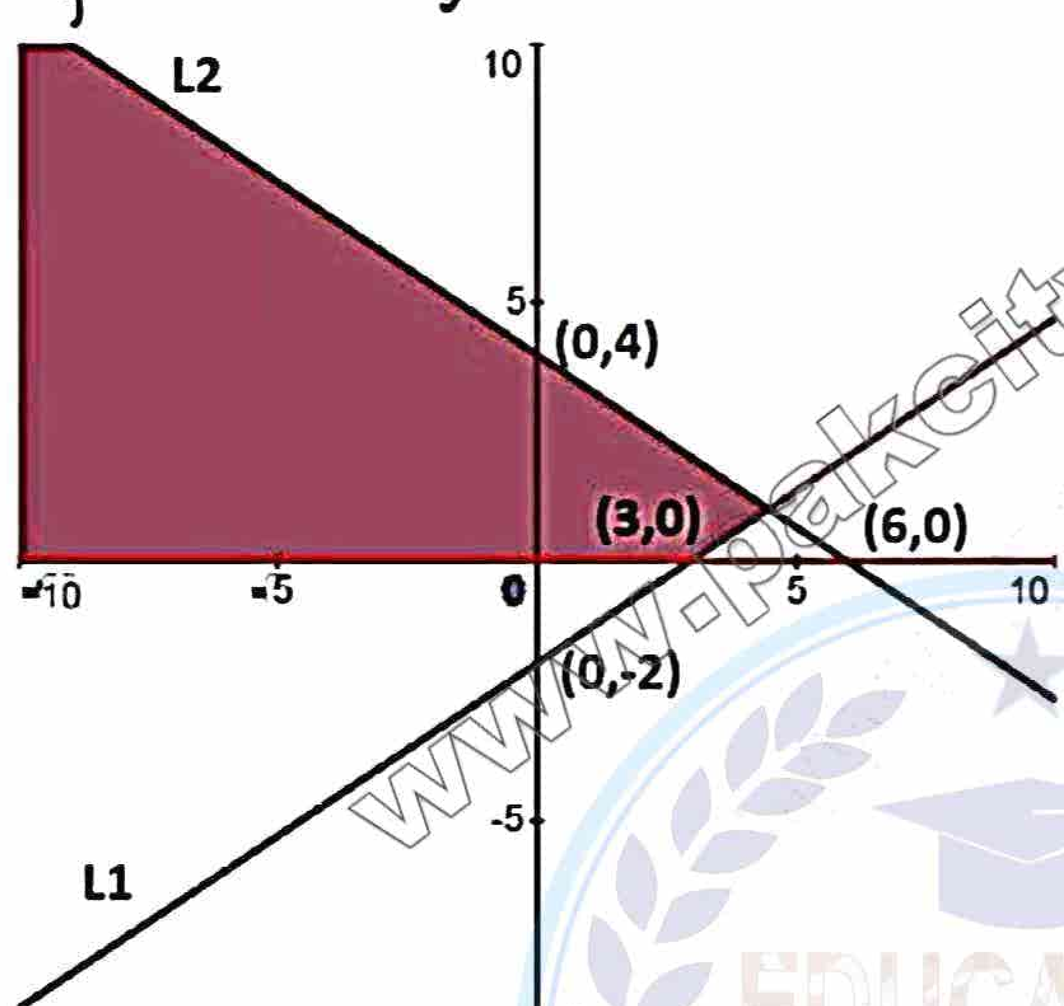
(iv) → Put $x=0$, $y=4$ so the pt $(0,4)$

Put $y=0$, $x=6$ so the pt $(6,0)$

Test pt (0,0):- we test (i) and (ii) at (0,0)

so (i) → $0 \le 6$ — True (ii) → $0 \le 12$ — True

Solution region:- The solution of the given system is intersection of graphs of (i) and (ii). Also $y \ge 0$ shows that the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



(ii) $x+y \le 5$; $y-2x \le 2$; $x \ge 0$

Solution:- $x+y \le 5$ —(i) $y-2x \le 2$ —(ii)

The associated eqs. of (i) and (ii) are

L1; $x+y=5$ —(iii) L2; $y-2x=2$ —(iv)

(iii) → Put $x=0$, $y=5$ so the pt $(0,5)$
$y=0$, $x=5$ so the pt $(5,0)$

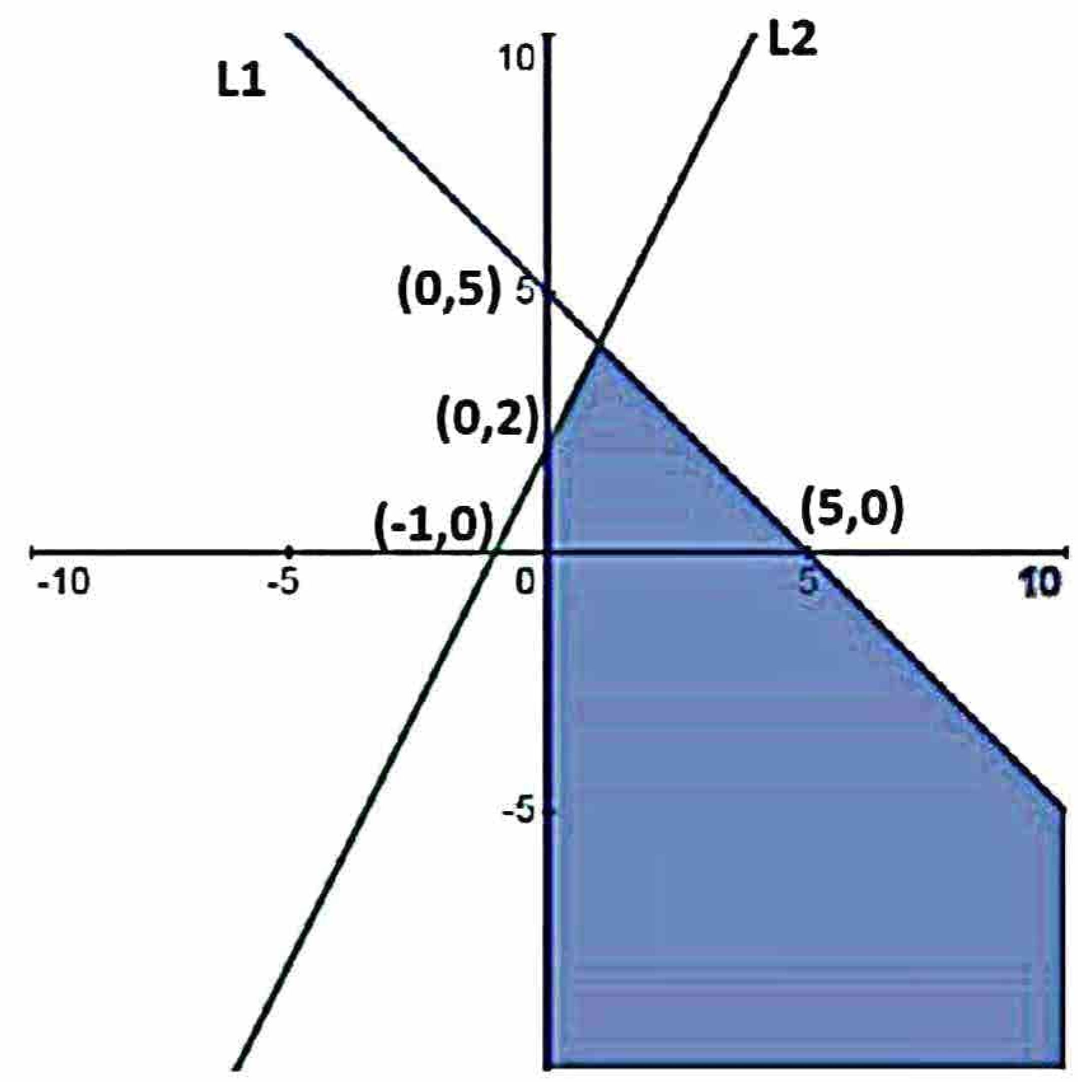
(iv) → Put $x=0$, $y=2$ so the pt $(0,2)$

Put $y=0$, $x=-1$ so the pt $(-1,0)$

Test pt (0,0):- we test (i) and (ii) at (0,0)

(i) → $0 \le 5$ — True (ii) → $0 \le 2$ — True

Solution region:- The solution of the given system is the intersection of the graphs of (i) and (ii). Also $x \ge 0$ shows that the solution set is right half plane including the graph of boundary line $x=0$ as shown in fig.



(iii) $x+y \ge 5$; $x-y \ge 1$; $y \ge 0$

Solution:- $x+y \ge 5$ —(i) $x-y \ge 1$ —(ii)

The associated eqs. of (i) and (ii) are

L1; $x+y=5$ —(iii) L2; $x-y=1$ —(iv)

(iii) → put $x=0$, $y=5$ so the pt $(0,5)$

put $y=0$, $x=5$ so the pt $(5,0)$

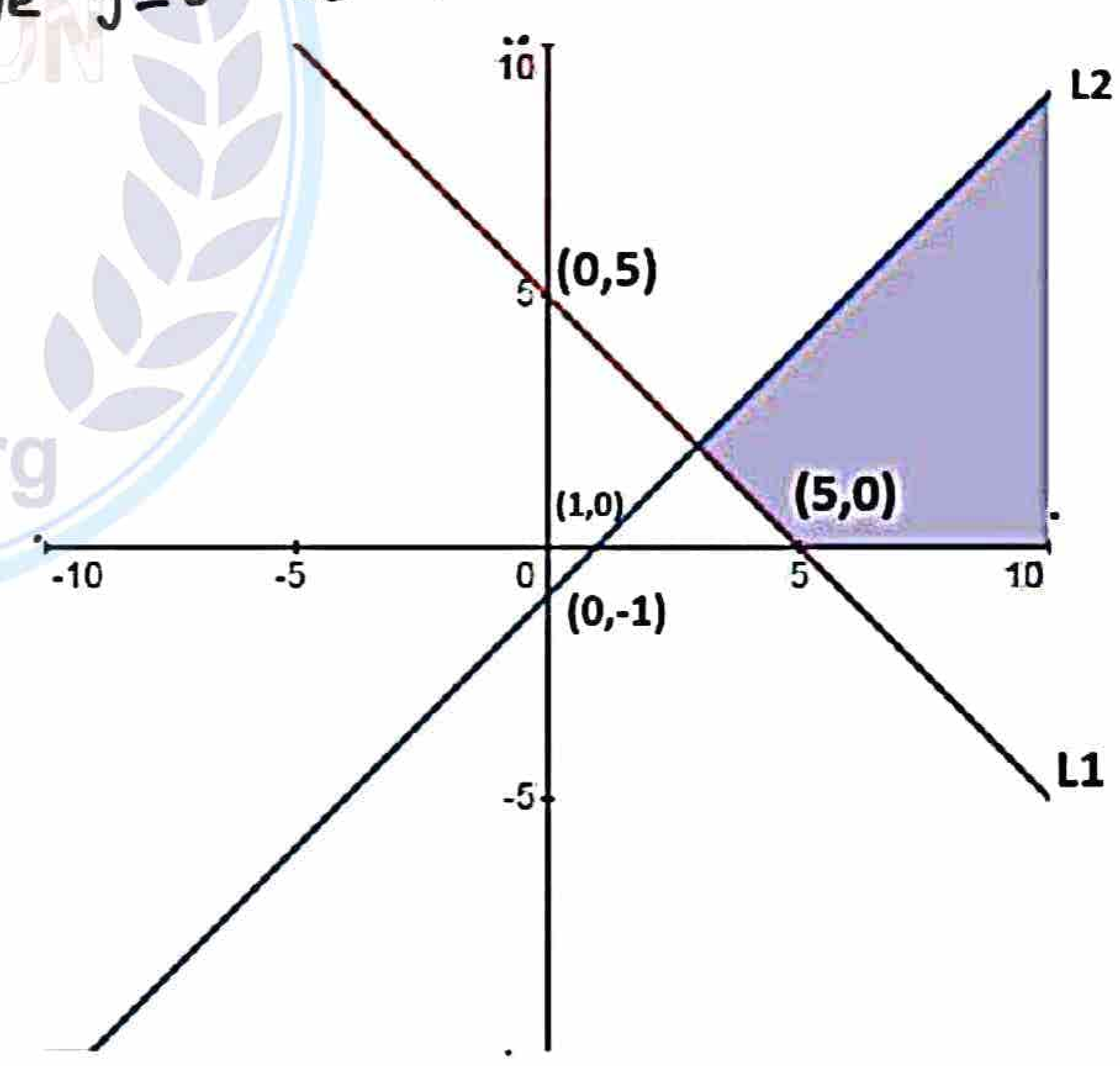
(iv) → put $x=0$, $y=-1$ so the pt $(0,-1)$

Put $y=0$, $x=1$ so the pt $(1,0)$

Test pt (0,0):- we test (i) and (ii) at (0,0)

(i) → $0 \ge 5$ — False (ii) → $0 \ge 1$ — False

Solution region:- The solution of the given system is the intersection of the graphs of (i) and (ii). Also $y \ge 0$ shows that the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



(iv) $3x+7y \le 21$; $x-y \le 2$; $x \ge 0$

Solution:- $3x+7y \le 21$ —(i) , $x-y \le 2$ —(ii)

The associated eqs. of (i) and (ii) are

L1; $3x+7y=21$ —(iii) L2; $x-y=2$ —(iv)

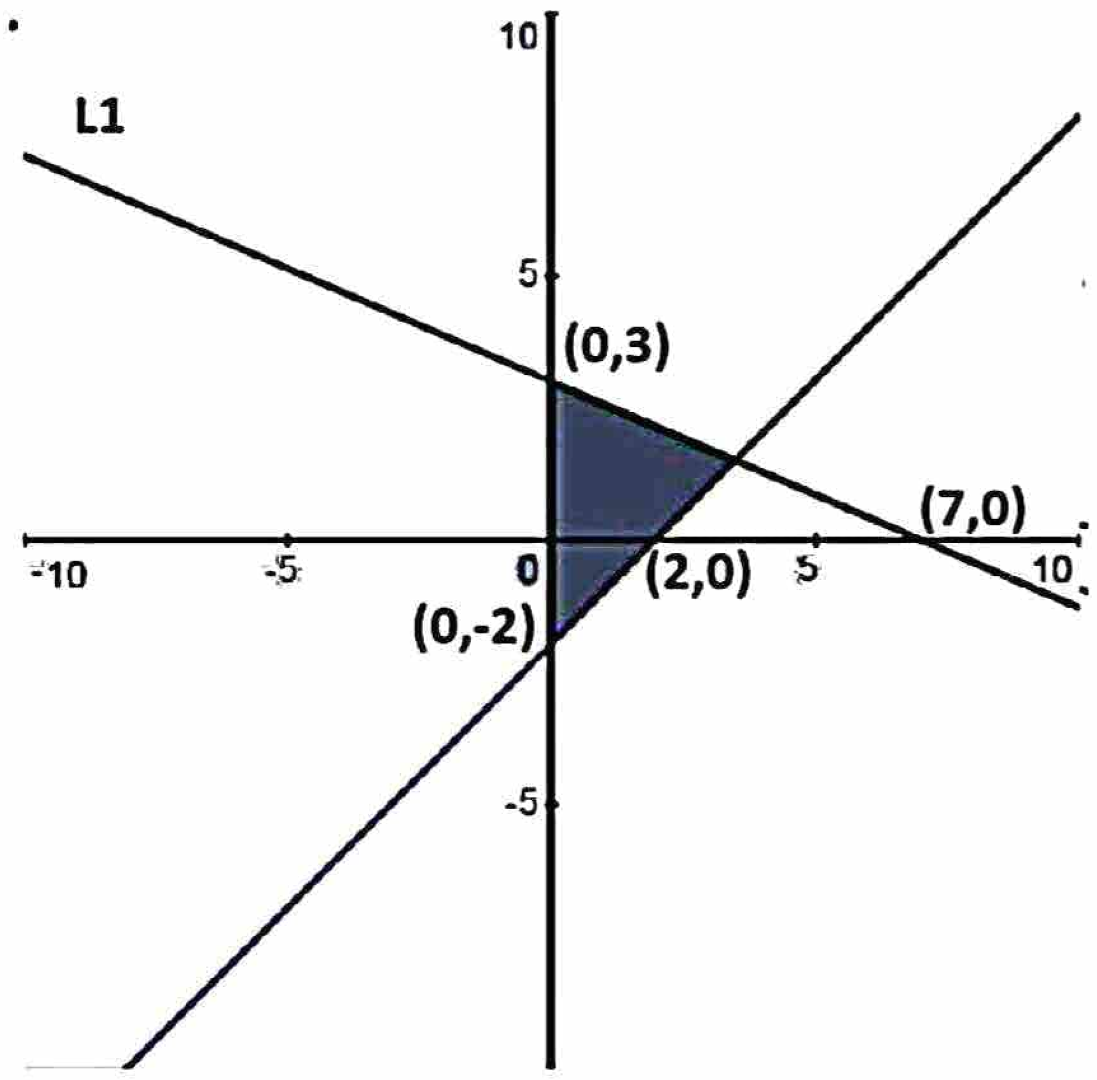
(iii) → put $x=0$, $y=3$ so the pt $(0,3)$

put $y=0$, $x=7$ so the pt $(7,0)$

(iv) → put $x=0,\ y=-2$ so the pt $(0,-2)$
Put $y=0,\ x=2$ so the pt $(2,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$
so (i) → $0 \le 21$ — True (ii) $0 \le 2$ — True

Solution region:- The solution of the given system is intersection of graphs of (i) and (ii). Also $x \geq 0$ shows that the solution set is right half plane including the graph of boundary line $x=0$ as shown in fig.



(v) $3x+7y \le 21$; $x-y \le 2$; $y \geq 0$

Solution:- $3x+7y \le 21$ — (i) , $x-y \le 2$ — (ii)

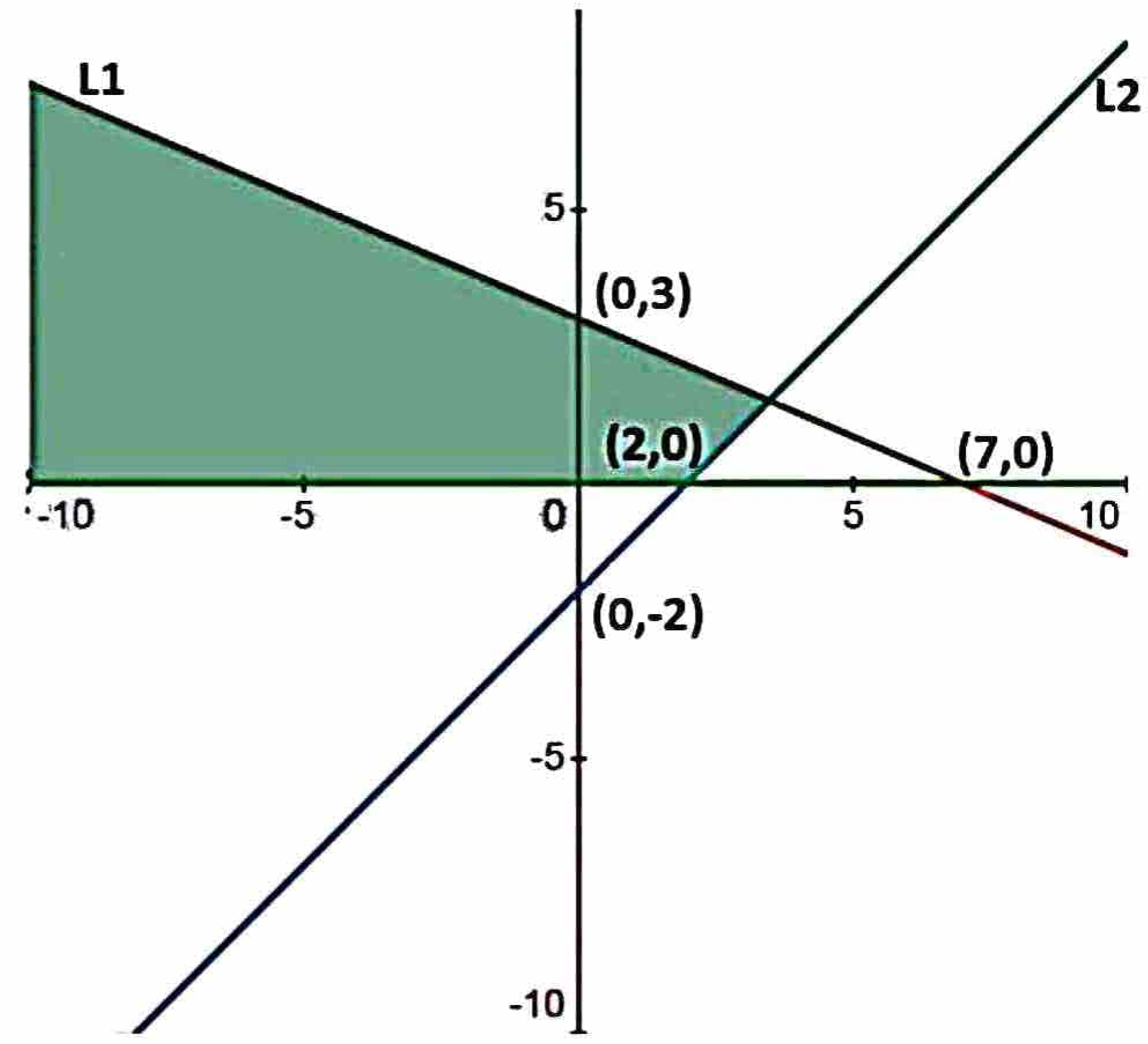
The associated eqs. of (i) and (ii) are
L1; $3x+7y = 21$ ——— (iii) L2; $x-y = 2$ —— (iv)

(iii) → Put $x=0,\ y=3$ so the pt $(0,3)$
Put $y=0,\ x=7$ so the pt $(7,0)$

(iv) → Put $x=0,\ y=-2$ so the pt $(0,-2)$
Put $y=0,\ x=2$ so the pt $(2,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$
So (i) → $0 \le 21$ — True (ii) → $0 \le 2$ — True

Solution region:- The solution of the given system is intersection of graphs of (i) and (ii). Also $y \geq 0$ shows that the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



(vi) $3x+7y \le 21$, $2x-y \geq -3$, $x \geq 0$

Solution:- $3x+7y \le 21$ — (i) $2x-y \geq -3$ (ii)

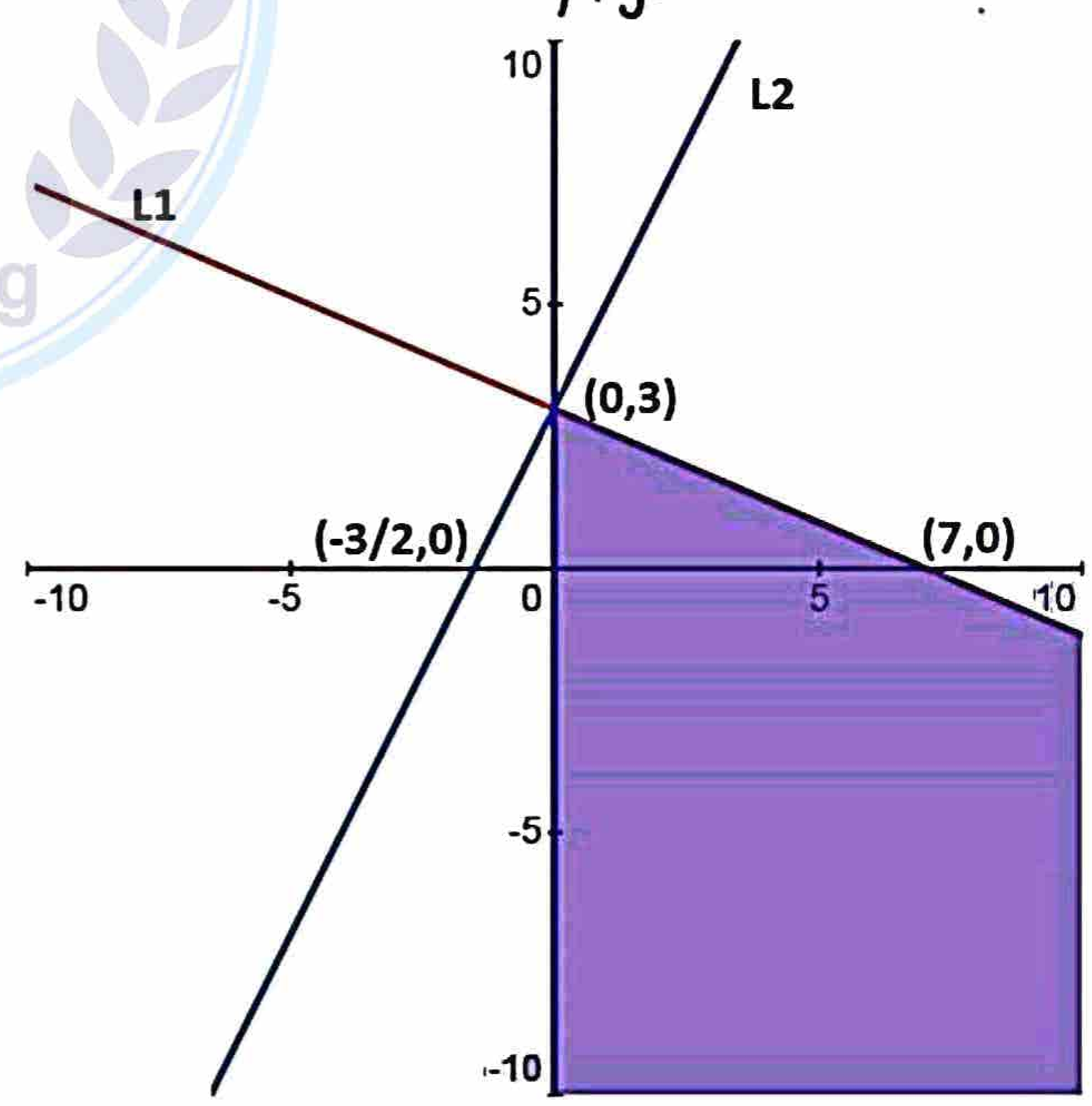
The associated eqs. of (i) and (ii) are
L1; $3x+7y = 21$ —— (iii) , L2; $2x-y = -3$ (iv)

(iii) → put $x=0,\ y=3$ so the pt $(0,3)$
Put $y=0,\ x=7$ so the pt $(7,0)$

(iv) → Put $x=0,\ y=3$ so the pt $(0,3)$
Put $y=0$ $x=-\frac{3}{2}$ so the pt $(-\frac{3}{2},0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$
So (i) → $0 \le 21$ —— True (ii) → $0 \geq -3$ — True

Solution region:- The solution of the given system is intersection of graphs of (i) and (ii). Also $x \geq 0$ shows that the solution set is right half plane including the graph of boundary line $x=0$ as shown in fig.



Q4. Graph the solution region of the following system of linear inequalities and find the corner points in each case.

(i) $2x-3y\le 6$; $2x+3y\le 12$; $x\ge 0$

Solution:- $2x-3y\le 6$ — (i) , $2x+3y\le 12$ — (ii)

The associated eqs. of (i) and (ii) are

$L_1; 2x-3y=6$ — (iii) , $L_2; 2x+3y=12$ — (iv)

(iii) → put $x=0$, $y=-2$ so the pt $(0,-2)$

put $y=0$, $x=3$ so the pt $(3,0)$

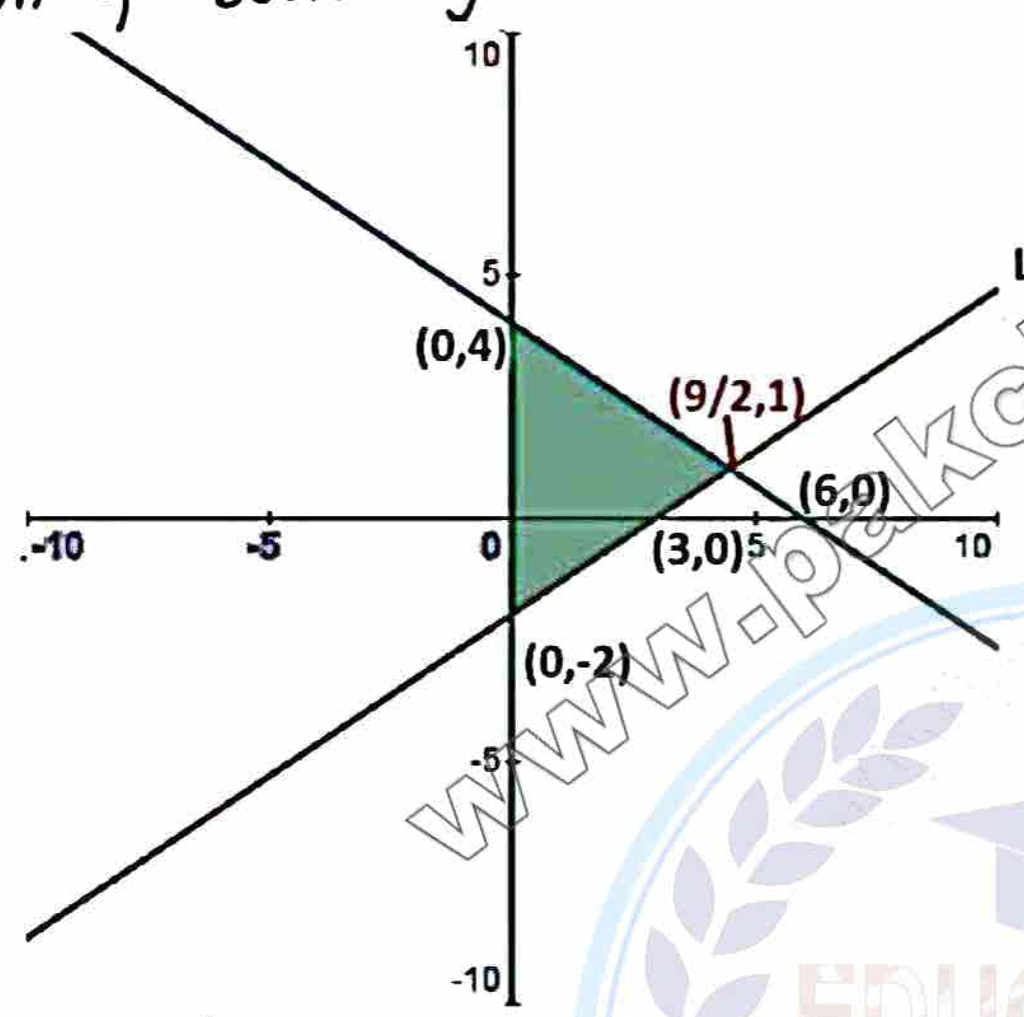
(iv) → put $x=0$, $y=4$ so the pt $(0,4)$

put $y=0$, $x=6$ so the pt $(6,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$

so (i) → $0\le 6$ — True (ii) → $0\le 12$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). Also $x\ge 0$ shows that the solution set is right half plane including the graph of boundary line $x=0$ as shown in fig.



Corner point:- As $2x-3y=6$ → (i)

$2x+3y=12$ → (ii)

By (i) + (ii) → $4x=18$ → $x=9/2$ Put in (ii)

$2(9/2)+3y=12$ → $3y=12-9$ → $y=1$

so $(\frac{9}{2},1)$ is pt. of intersection of lines L1 and L2. Hence corner pts. are $(0,-2),(0,4),(\frac{9}{2},1)$

(ii) $x+y\le 5$; $-2x+y\le 2$, $y\ge 0$

Solution:- $x+y\le 5$ — (i) , $-2x+y\le 2$ — (ii)

The associated eqs. of (i) and (ii) are

$L_1; x+y=5$ — (iii) , $L_2; -2x+y=2$ — (iv)

(iii) → put $x=0$, $y=5$ so the pt $(0,5)$

Put $y=0$, $x=5$ so the pt $(5,0)$

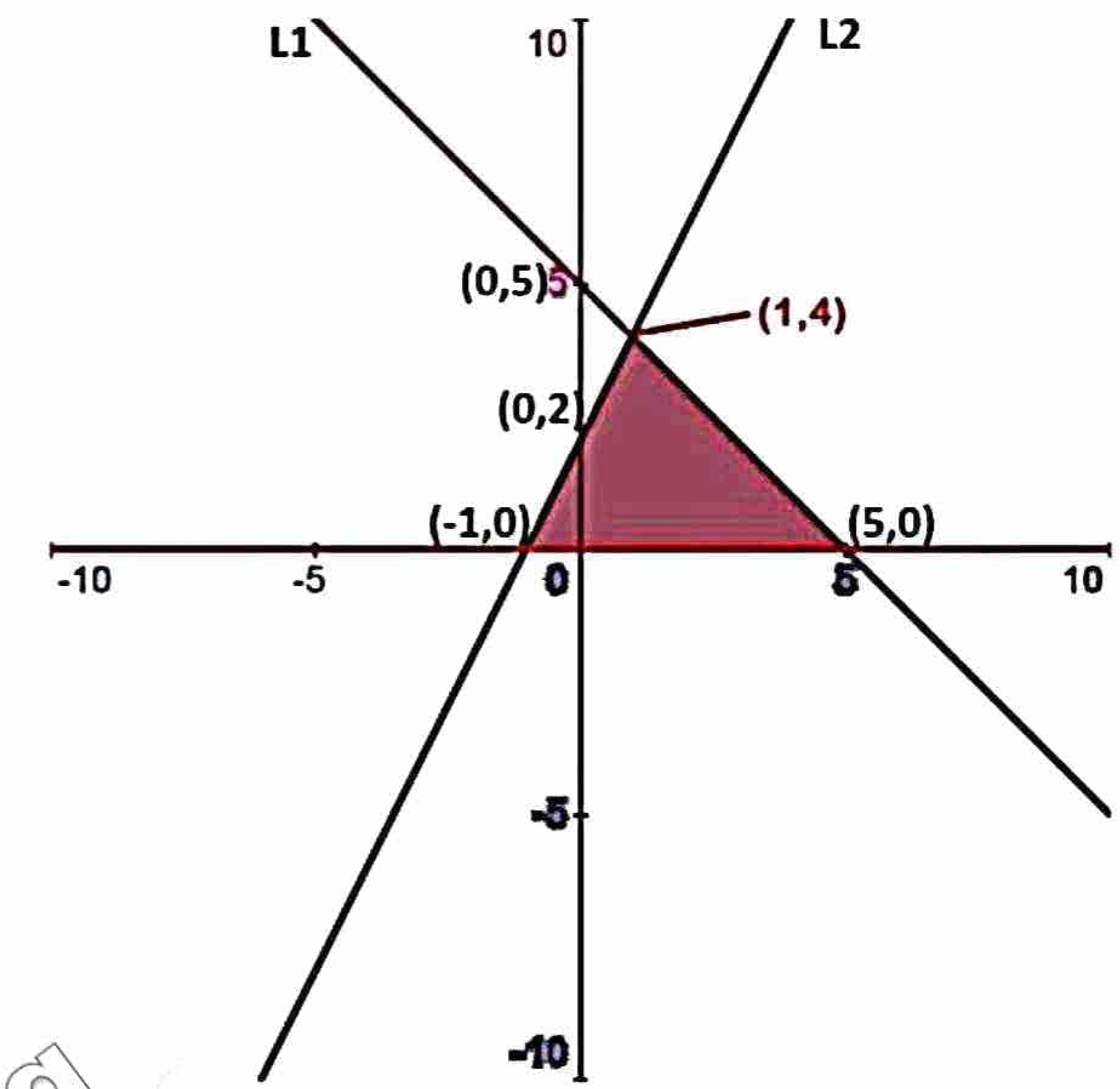
(iv) → Put $x=0$, $y=2$ so the pt $(0,2)$

Put $y=0$, $x=-1$ so the pt $(-1,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$

so (i) → $0\le 5$ — True , $0\le 2$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). Also $y\ge 0$ shows that the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



Corner point:- As $x+y=5$ — (i)

$-2x+y=2$ — (ii)

By (ii) − (i) → $3x=3$ → $x=1$ put in (i)

$1+y=5$ → $y=4$. So $(1,4)$ is the pt. of intersection of lines L1 and L2.

Hence corner pts. are $(-1,0)$, $(5,0)$, $(1,4)$

(iii) $3x+7y\le 21$; $2x-y\le -3$, $y\ge 0$

Solution:- $3x+7y\le 21$ — (i) , $2x-y\le -3$ (ii)

The associated eqs. of (i) and (ii) are

$L_1; 3x+7y=21$ — (iii), $L_2; 2x-y=-3$ (iv)

(iii) → Put $x=0$, $y=3$ so the pt $(0,3)$

Put $y=0$, $x=7$ so the pt $(7,0)$

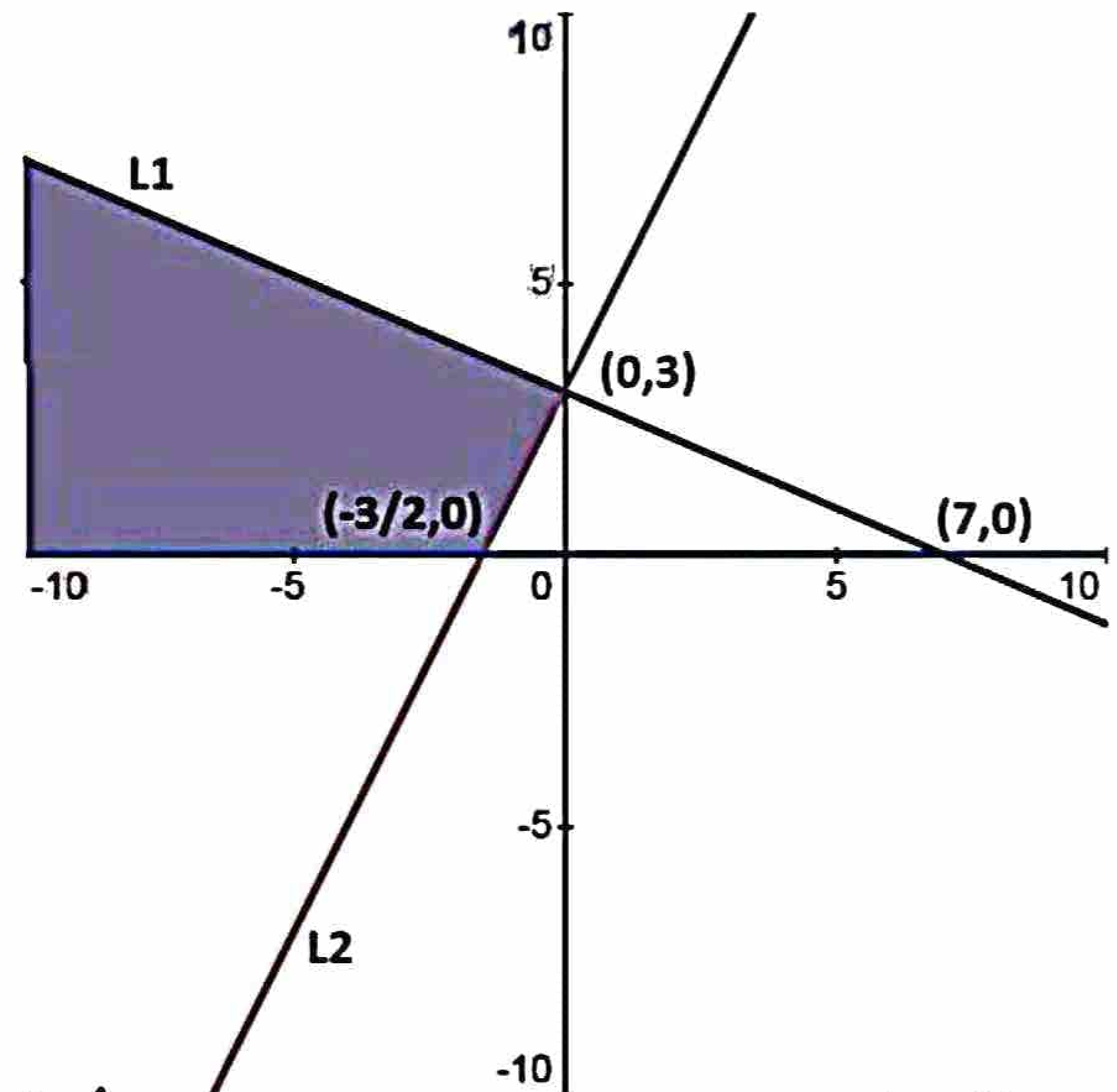
(iv) → Put $x=0$, $y=3$ so the pt $(0,3)$

Put $y=0$, $x=-\frac{3}{2}$ so the pt $(-\frac{3}{2},0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$

so (i) → $0\le 21$ — True , (ii) → $0\le -3$ — False

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). Also $y\ge 0$ Shows that the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



Corner point :- corner pts are $(0,3)$ and $(-\frac{3}{2},0)$.

(iv) $3x+2y \geq 6$; $x+3y \leq 6$; $y \geq 0$

Solution:- $3x+2y \geq 6$ — (i) , $x+3y \leq 6$ — (ii)

The associated eqs. of (i) and (ii) are

L1; $3x+2y=6$ — (iii) L2; $x+3y=6$ — (iv)

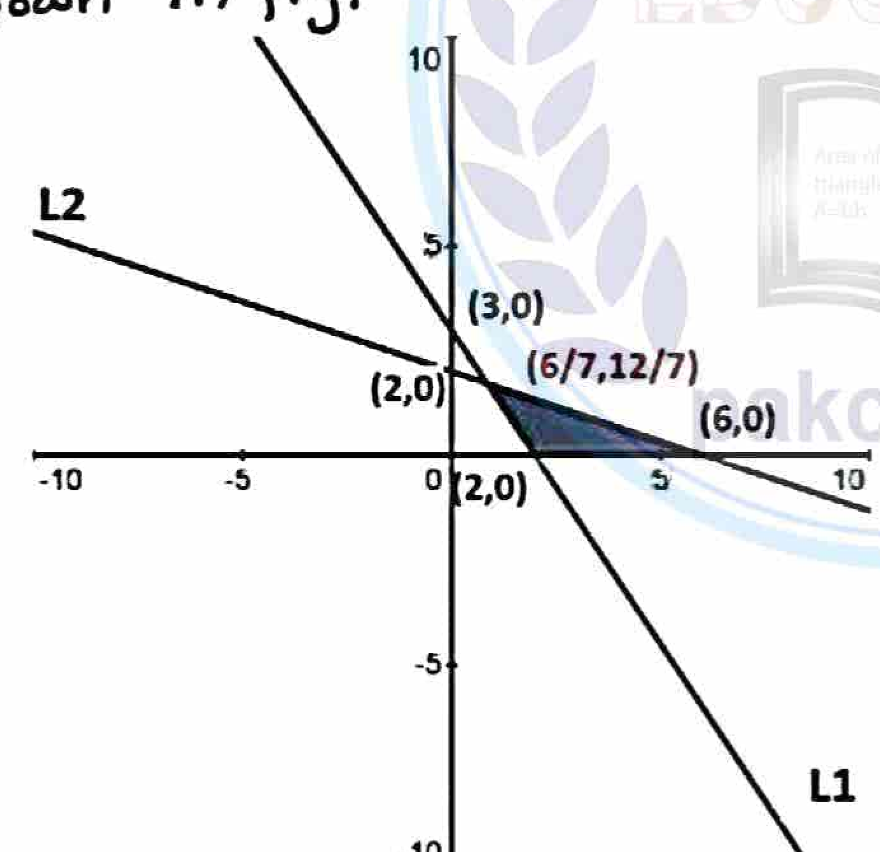
(iii) → put $x=0$, $y=3$ so the pt $(0,3)$
put $y=0$, $x=2$ so the pt $(2,0)$

(iv) → put $x=0$, $y=2$ so the pt $(0,2)$
put $y=0$, $x=6$ so the pt $(6,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$. So

(i) → $0 \geq 6$ — False , (ii) → $0 \leq 6$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). Also $y \geq 0$ shows that the solution set is upper half plane including the graph of boundary line $y=0$ as shown in fig.



Corner point:- As $3x+2y=6$ — (i)
$x+3y=6$ — (ii)

By 3(ii) − (i) →
$$\begin{aligned}3x+9y&=18\\ 3x+2y&=6\\ \hline 7y&=12\end{aligned}$$
→ $y=\frac{12}{7}$ Put in (ii)

→ $x = 6-\frac{36}{7} = \frac{42-36}{7} = \frac{6}{7}$

So pt. of intersection of lines (i) and (ii) is $(\frac{6}{7},\frac{12}{7})$

Hence corner pts are $(2,0)$, $(6,0)$, $(\frac{6}{7},\frac{12}{7})$.

(v) $5x+7y \leq 35$; $-x+3y \leq 3$; $x \geq 0$

Solution:- $5x+7y \leq 35$ — (i) , $-x+3y \leq 3$ — (ii)

The associated eqs. of (i) and (ii) are

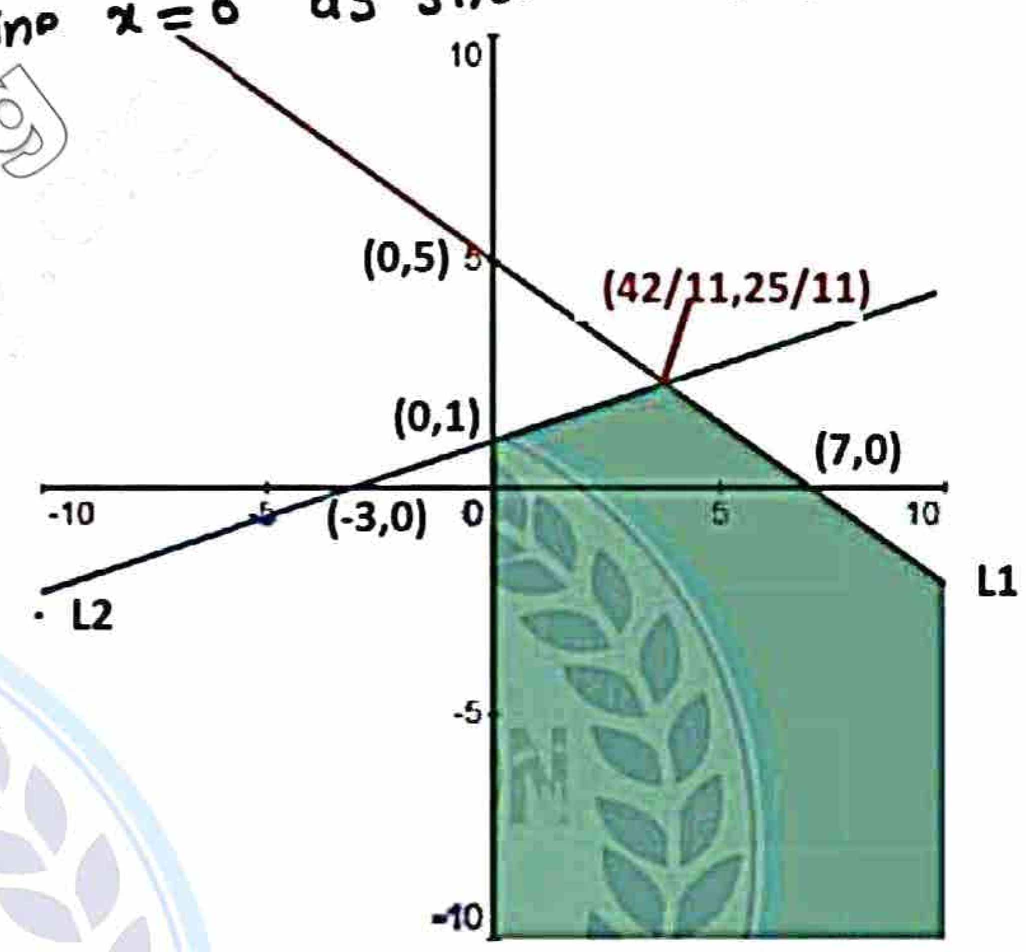
L1; $5x+7y=35$ — (iii) L2; $-x+3y=3$ — (iv)

(iii) → put $x=0$, $y=5$ so the pt $(0,5)$
put $y=0$, $x=7$ so the pt $(7,0)$

(iv) → put $x=0$, $y=1$ so the pt $(0,1)$
put $y=0$, $x=-3$ so the pt $(-3,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$. So (i) → $0 \leq 35$ True , (ii) → $0 \leq 3$ True

Solution region:- The solution of given system is intersection of the graphs of (i) and (ii). Also $x \geq 0$ shows that the solution set is right half plane including the graph of boundary line $x=0$ as shown in fig.



Corner point :- As $5x+7y=35$ — (i)
$-x+3y=3$ — (ii)

By 5(ii) + (i) → $22y=50$ → $y=\frac{50}{22}$ Put in (ii)

$-x+3(\frac{50}{22})=3$

→ $-x = 3-\frac{150}{22} = 3-\frac{75}{11} = \frac{33-75}{11}$

→ $-x=-\frac{42}{11}$ → $x=\frac{42}{11}$.

So pt. of intersection of lines (i) and (ii) is $(\frac{42}{11},\frac{25}{11})$. So cornerpoints are $(0,1)$ and $(\frac{42}{11},\frac{25}{11})$

(vi) $5x+7y \leq 35$; $x-2y \leq 2$; $x \geq 0$

Solution:- $5x+7y \leq 35$ — (i), $x-2y \leq 2$ — (ii)

The associated eqs. of (i) and (ii) are

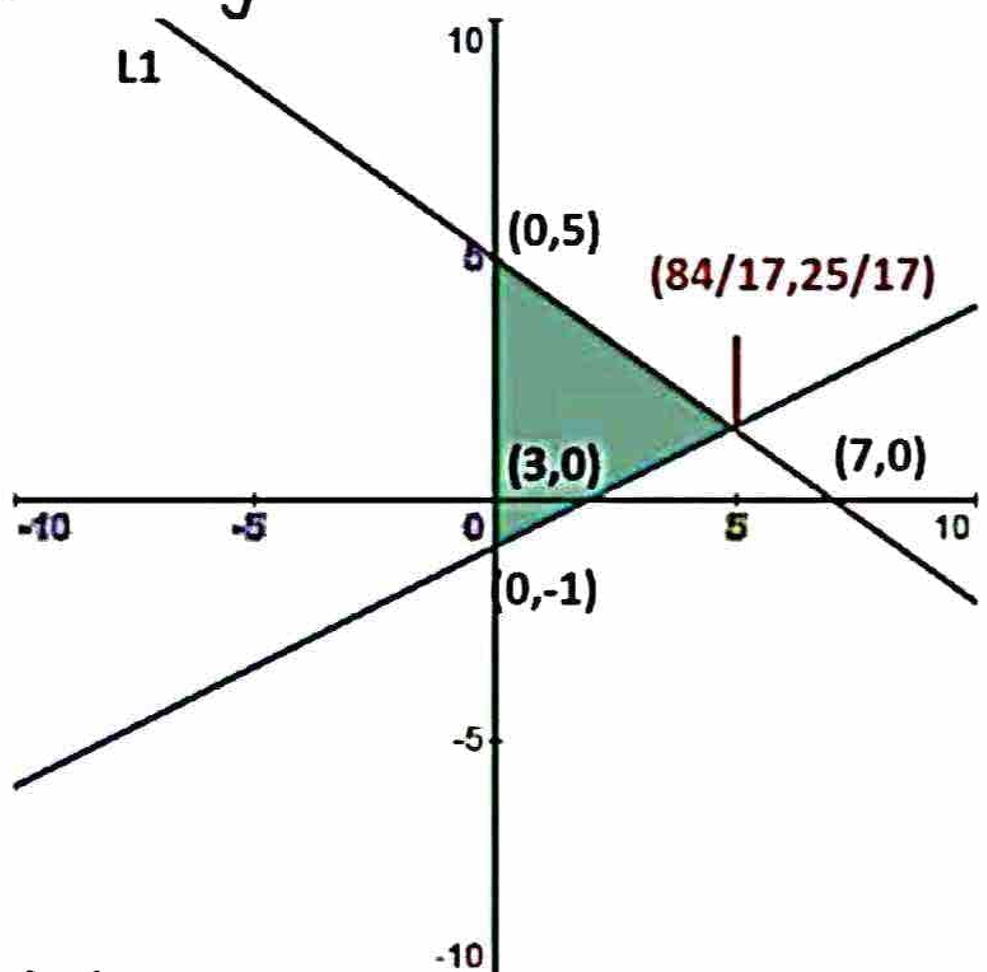
L1; $5x+7y=35$ — (iii) , L2; $x-2y=2$ — (iv)

(iii) Put $x=0$, $y=5$ so the pt $(0,5)$
put $y=0$, $x=7$ so the pt $(7,0)$

(iv) → Put $x=0$, $y=-1$ so the pt $(0,-1)$
Put $y=0$, $x=2$ so the pt $(2,0)$

Test pt $(0,0)$:– we test (i) and (ii) at $(0,0)$. So
(i) $\rightarrow 0 \le 35$ —— True (ii) $\rightarrow 0 \le 2$ —— True

Solution region:- The solution of the given system is intersection of the graphs of (i) and (ii). Also $x \geq 0$ shows that the solution set is right half plane including the graph of boundary line $x = 0$ as shown in fig.



Corner Point:- As $5x + 7y = 35$ —— (i)
$x - 2y = 2$ —— (ii)

By $5(ii) - (i) \rightarrow$
$$5x - 10y = 10$$
$$5x + 7y = 35$$
$$-17y = -25 \Rightarrow y = \frac{25}{17} \text{ put in (ii)}$$
$$\Rightarrow x - 2\left(\frac{25}{17}\right) = 2 \Rightarrow x = 2 + \frac{50}{17}$$
$$\Rightarrow x = \frac{84}{17} \qquad x = \frac{34+50}{17}$$

So pt. of intersection of lines L1 and L2 are $\left(\frac{84}{17}, \frac{25}{17}\right)$. So corner pts are $(0,-2)$, $(0,5)$ and $\left(\frac{84}{17}, \frac{25}{17}\right)$

Q5. Graph the solution region of the following system of linear inequalities by shading. (i) $3x - 4y \le 12$; $3x + 2y \geq 3$; $x + 2y \le 9$

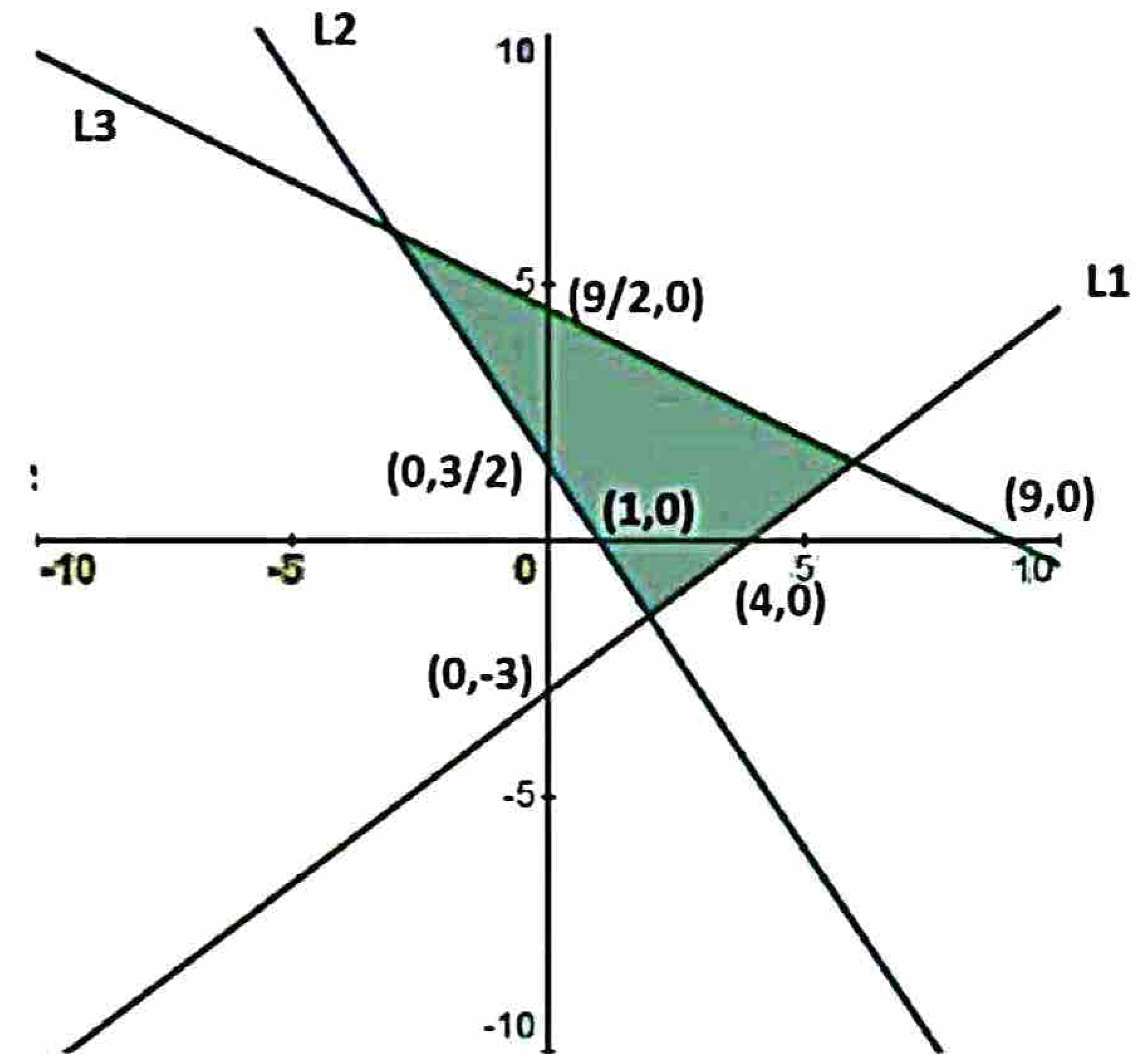
Solution:- $3x - 4y \le 12$ —— (i), $3x + 2y \geq 3$ —— (ii)
$x + 2y \le 9$ —— (iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $3x - 4y = 12$ —— (iv), L2; $3x + 2y = 3$ —— (v)
L3; $x + 2y = 9$ —— (vi)

(iv) $\rightarrow$ put $x = 0$, $y = -3$ so the pt $(0, -3)$
Put $y = 0$, $x = 4$ so the pt $(4, 0)$
(v) $\rightarrow$ put $x = 0$, $y = \frac{3}{2}$ so the pt $(0, 3/2)$
put $y = 0$, $x = 1$ so the pt $(1, 0)$
(vi) $\rightarrow$ put $x = 0$, $y = 9/2$ so the pt $(0, 9/2)$
Put $y = 0$, $x = 9$ so the pt $(9, 0)$

Test pt $(0,0)$:- we test (i), (ii) and (iii) at $(0,0)$
so (i) $\rightarrow 0 \le 12$ —— True, (ii) $\rightarrow 0 \geq 3$ —— False
(iii) $\rightarrow 0 \le 9$ —— True

Solution region:- The solution of the given system is intersection of (i), (ii) and (iii). So solution region is shaded area as shown in fig.



(ii) $3x - 4y \le 12$; $x + 2y \le 6$; $x + y \geq 1$

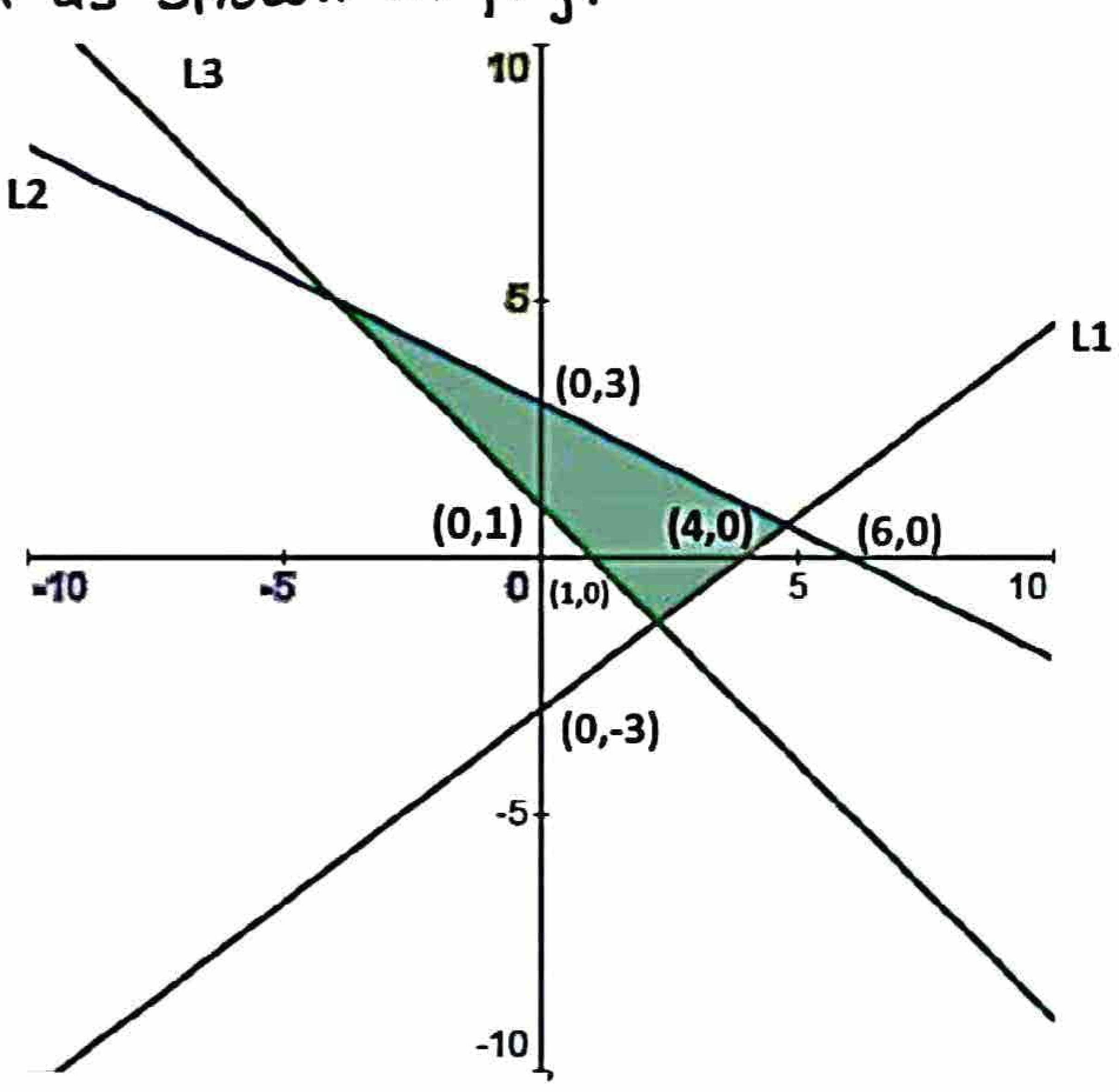
Solution:- $3x - 4y \le 12$ —— (i), $x + 2y \le 6$ —— (ii)
$x + y \geq 1$ —— (iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $3x - 4y = 12$ —— (iv), L2; $x + 2y = 6$ —— (v)
L3; $x + y = 1$ —— (vi)

(iv) $\rightarrow$ Put $x = 0$, $y = -3$ so the pt $(0, -3)$
Put $y = 0$, $x = 4$ so the pt $(4, 0)$
(v) $\rightarrow$ Put $x = 0$, $y = 3$ so the pt $(0, 3)$
Put $y = 0$, $x = 6$ so the pt $(6, 0)$
(vi) $\rightarrow$ Put $x = 0$, $y = 1$, so the pt $(0, 1)$
Put $y = 0$, $x = 1$ so the pt $(1, 0)$

Test pt $(0,0)$:- we test (i), (ii) and (iii) at $(0,0)$. So (i) $\rightarrow 0 \le 12$ —— True, (ii) $\rightarrow 0 \le 6$ —— True
(iii) $\rightarrow 0 \geq 1$ —— False

Solution region:- The solution of the given system is intersection of (i), (ii) and (iii). So solution region is shaded area as shown in fig.



(iii) $2x+y \le 4$; $2x-3y \ge 12$; $x+2y \le 6$

Solution:- $2x+y \le 4$ —(i) ; $2x-3y \ge 12$ —(ii)

$x+2y \le 6$ —(iii)

The associated eqs. of (i), (ii) and (iii) are

L1; $2x+y=4$ —(iv) , L2; $2x-3y=12$ —(v)

L3; $x+2y=6$ —(vi)

(iv) → Put $x=0, y=4$ so the pt $(0,4)$
Put $y=0, x=2$ so the pt $(2,0)$

(v) → Put $x=0, y=-4$ so the pt $(0,-4)$
Put $y=0, x=6$ so the pt $(6,0)$

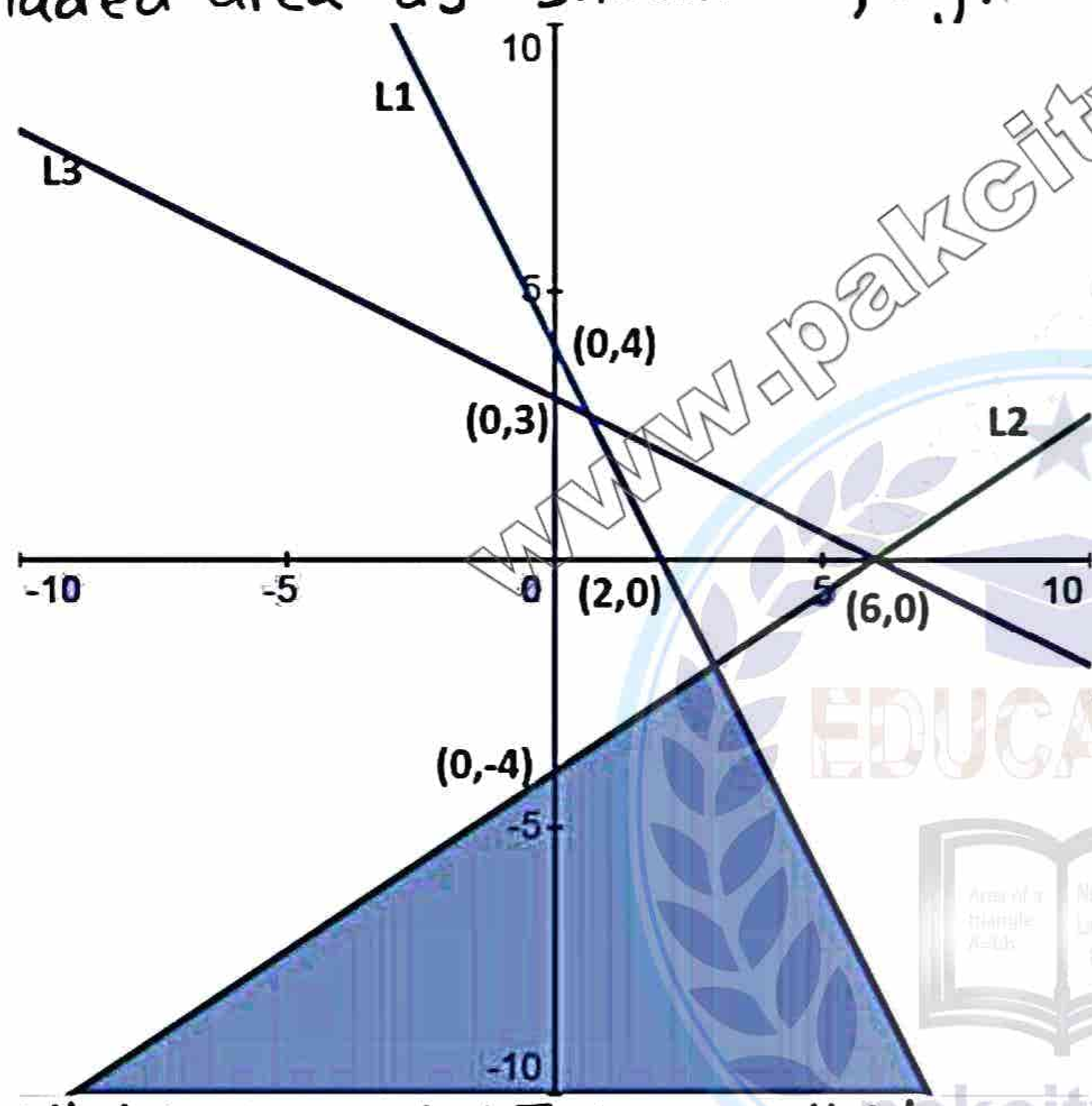
(vi) → Put $x=0, y=3$ so the pt $(0,3)$
Put $y=0, x=6$ so the pt $(6,0)$

Test pt $(0,0)$:- we test (i), (ii), (iii) at $(0,0)$

so (i) → $0 \le 4$ — True, (ii) → $0 \ge 12$ — False

(iii) → $0 \le 6$ — True

Solution region:- The solution of the given system is intersection of graphs of (i), (ii) and (iii). So solution region is shaded area as shown in fig.



(iv) $2x+y \le 10$; $x+y \le 7$; $-2x+y \le 4$

Solution:- $2x+y \le 10$ —(i) , $x+y \le 7$ —(ii)

$-2x+y \le 4$ —(iii)

The associated eqs. of (i), (ii) and (iii) are

L1; $2x+y=10$ —(iv) ; L2; $x+y=7$ —(v)

L3; $-2x+y=4$ —(vi)

(iv) → Put $x=0, y=10$ so the pt $(0,10)$
Put $y=0, x=5$ so the pt $(5,0)$

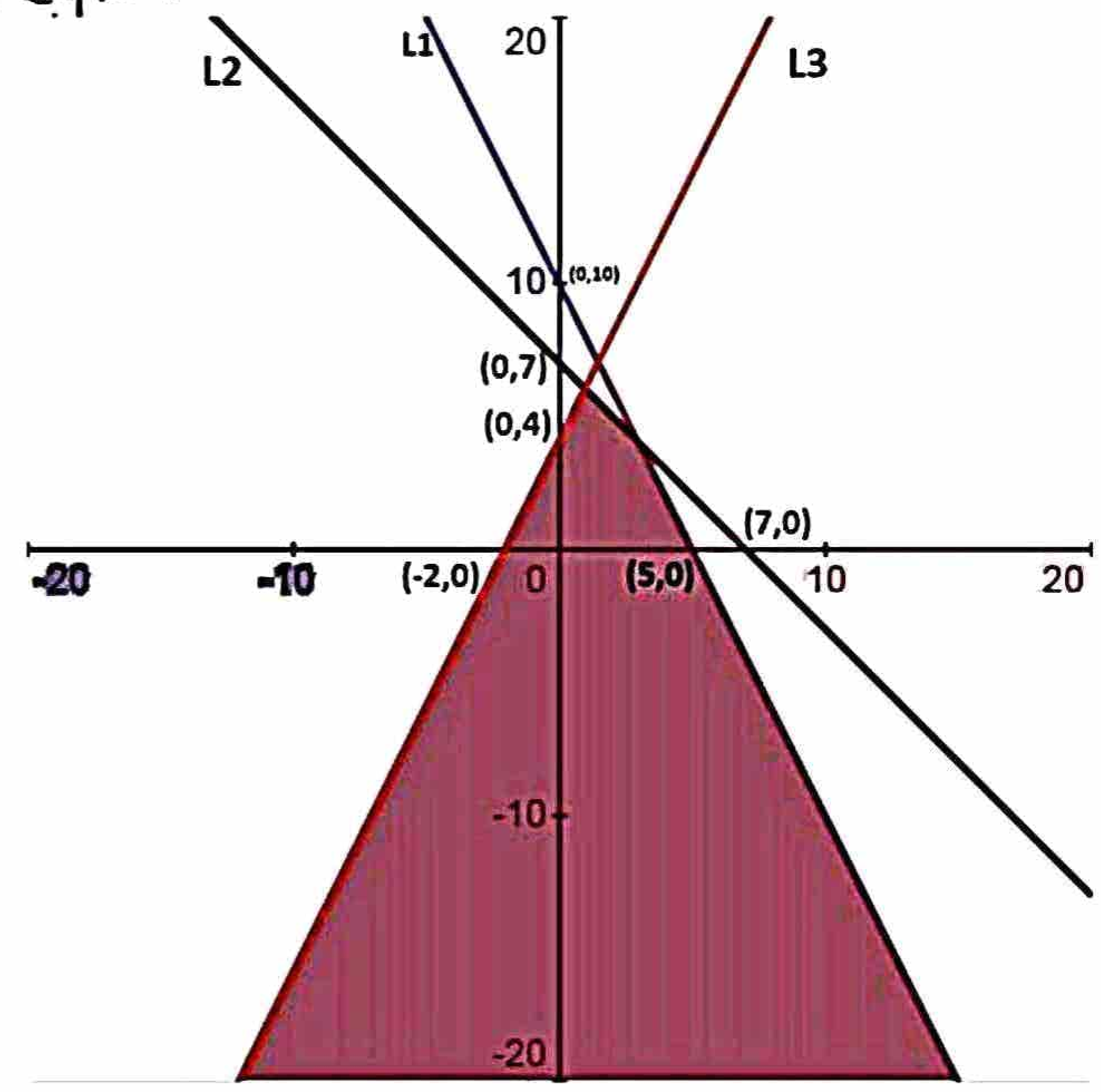
(v) → Put $x=0, y=7$ so the pt $(0,7)$
Put $y=0, x=7$ so the pt $(7,0)$

(vi) → Put $x=0, y=4$ so the pt $(0,4)$
Put $y=0, x=-2$ so the pt $(-2,0)$

Test pt $(0,0)$:- we test (i), (ii) and (iii) at $(0,0)$. so (i) → $0 \le 10$ — True, (ii) → $0 \le 7$ True

(iii) → $0 \le 4$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i), (ii) and (iii). So solution region is shaded area as shown in fig.



(v) $2x+3y \le 18$; $2x+y \le 10$; $-2x+y \le 2$

Solution:- $2x+3y \le 18$ —(i), $2x+y \le 10$ (ii)

$-2x+y \le 2$ —(iii)

The associated eqs. of (i), (ii) and (iii) are

L1; $2x+3y=18$ —(iv), L2; $2x+y=10$ (v)

L3; $-2x+y=2$ —(vi)

(iv) → Put $x=0, y=6$ so the pt $(0,6)$
Put $y=0, x=9$ so the pt $(9,0)$

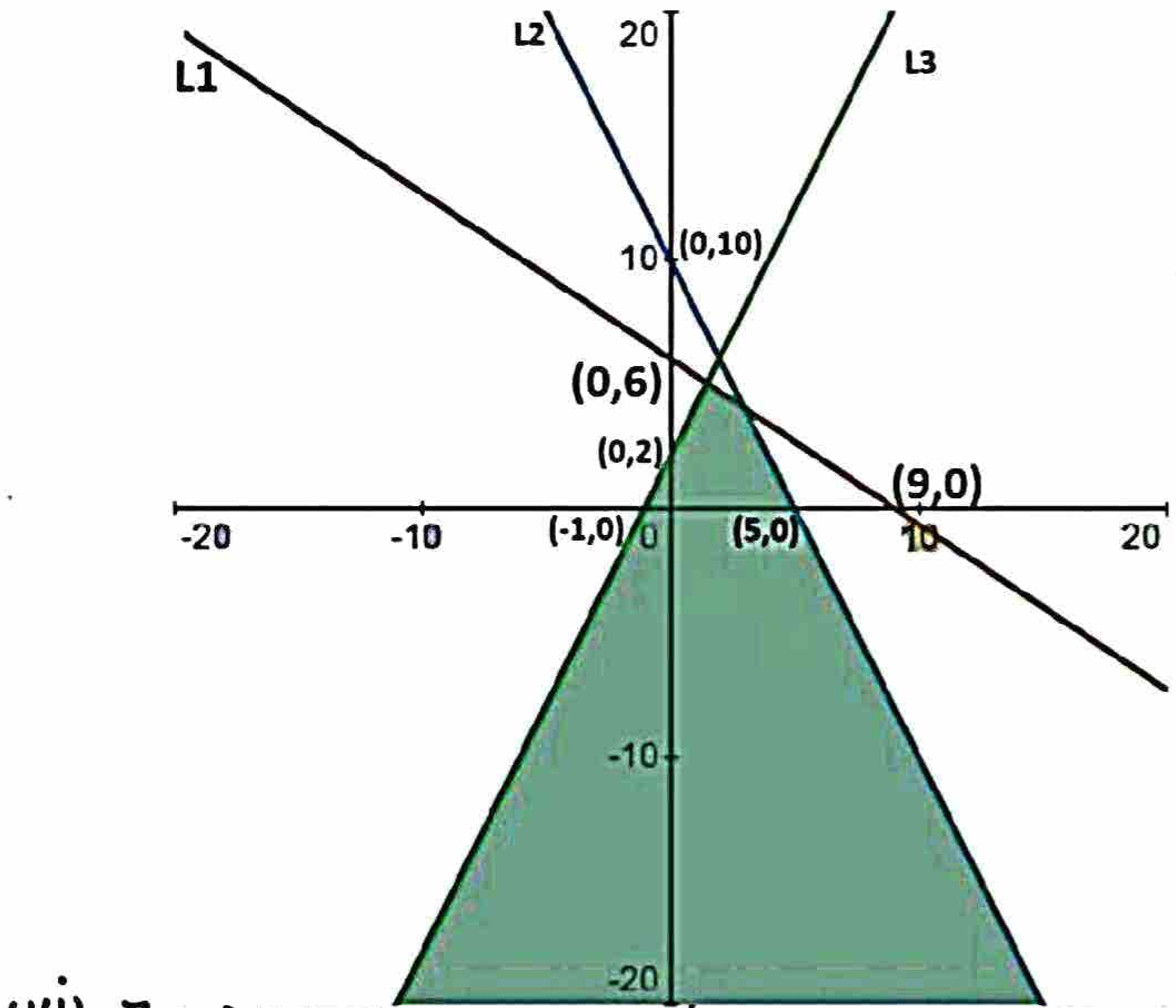
(v) → Put $x=0, y=10$ so the pt $(0,10)$
Put $y=0, x=5$ so the pt $(5,0)$

(vi) → Put $x=0, y=2$ so the pt $(0,2)$
Put $y=0, x=-1$ so the pt $(-1,0)$

Test pt $(0,0)$:- We test (i), (ii) and (iii) at $(0,0)$. so (i) → $0 \le 18$ — True, (ii) → $0 \le 10$ True

(iii) → $0 \le 2$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i), (ii) and (iii). So solution region is shaded area as shown in fig.



(vi) $3x-2y \geq 3$; $x+4y \leq 12$; $3x+y \leq 12$

Solution:- $3x-2y \geq 3$ —(i) , $x+4y \leq 12$ —(ii)
$3x+y \leq 12$ —(iii)

The associated eqs. of (i), (ii) and (iii)

L1; $3x-2y=3$ —(iv) , L2; $x+4y=12$ —(v)

L3; $3x+y=12$ —(vi)

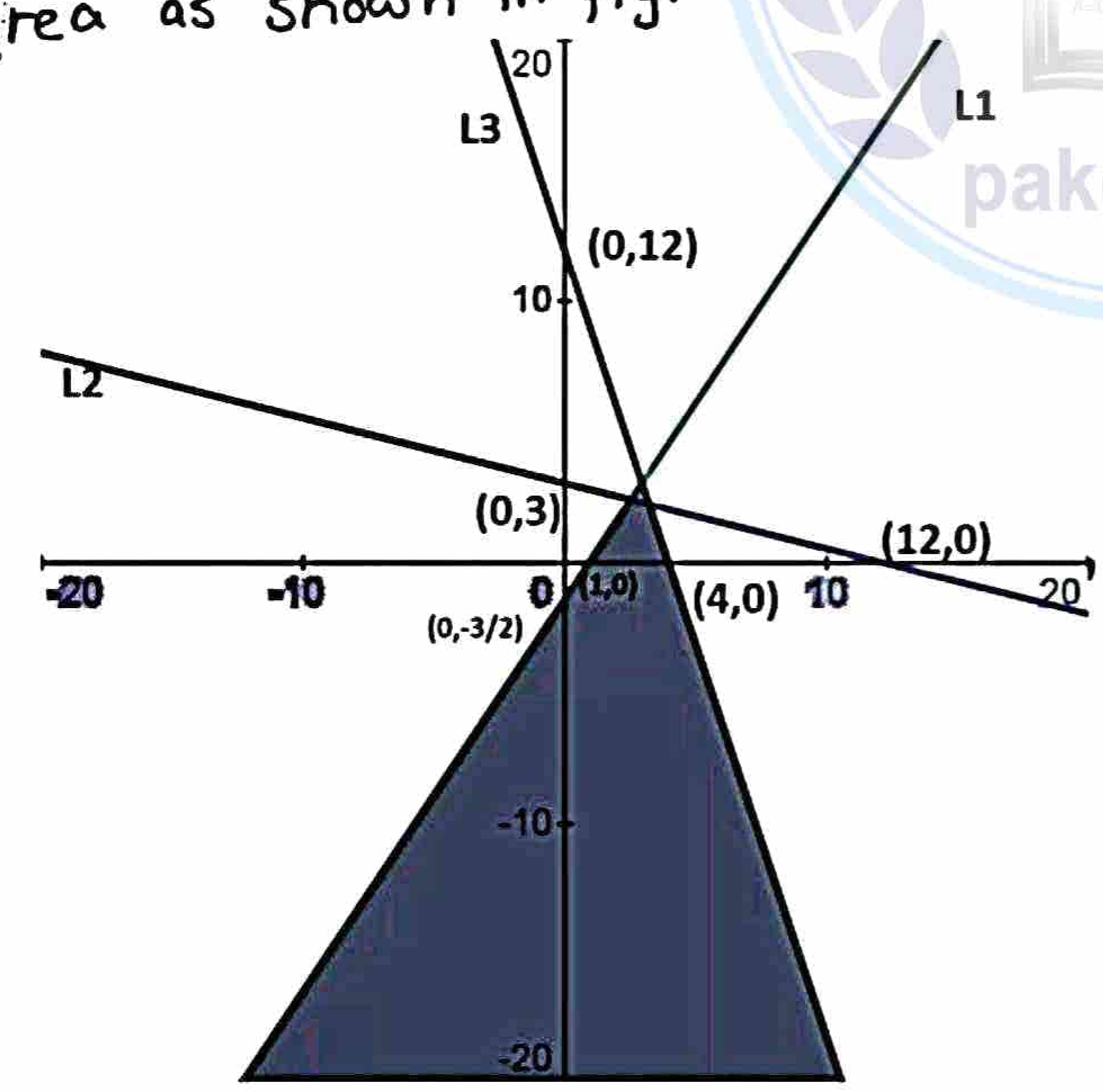
(iv) → put $x=0$, $y=-\frac{3}{2}$ so the pt $(0, -\frac{3}{2})$
put $y=0$, $x=1$ so the pt $(1, 0)$

(v) → put $x=0$, $y=3$ so the pt $(0, 3)$
put $y=0$, $x=12$ so the pt $(12, 0)$

(vi) → put $x=0$, $y=12$ so the pt $(0, 12)$
put $y=0$, $x=4$ so the pt $(4, 0)$

Test pt (0,0):- we test (i), (ii) and (iii) at (0,0)
so (i) → $0 \geq 3$ — False (ii) → $0 \leq 12$ — True
(iii) → $0 \leq 12$ — True

Solution region:- The solution of the given system is intersection of the graphs of (i), (ii) and (iii). so solution region is shaded area as shown in fig.



Problem constraints:-

The restrictions applied on the every day life problems are called problem constraints.

Non-negative constraints:-

The constraints that are always satisfied are called natural constraints or non-negative constraints.

Decision variables:- The variable used in non-negative constraints are called decision variables.

Feasible region:- The solution region which is restricted to the first quadrant is called feasible region. we restrict the solution region by using non-negative constraints $x \geq 0$ and $y \geq 0$.

Feasible solution:- Each point of feasible region is called feasible solution of the system.

Feasible solution set:- A set consisting of all the feasible solutions of the system is called feasible solution set.

Example 1:- Graph the feasible region and find the corner points for the system. $x-y \leq 3$, $x+2y \leq 6$, $x \geq 0$, $y \geq 0$

Solution:- $x-y \leq 3$ — (i) , $x+2y \leq 6$ —(ii)

The associated eqs. of (i) and (ii) are

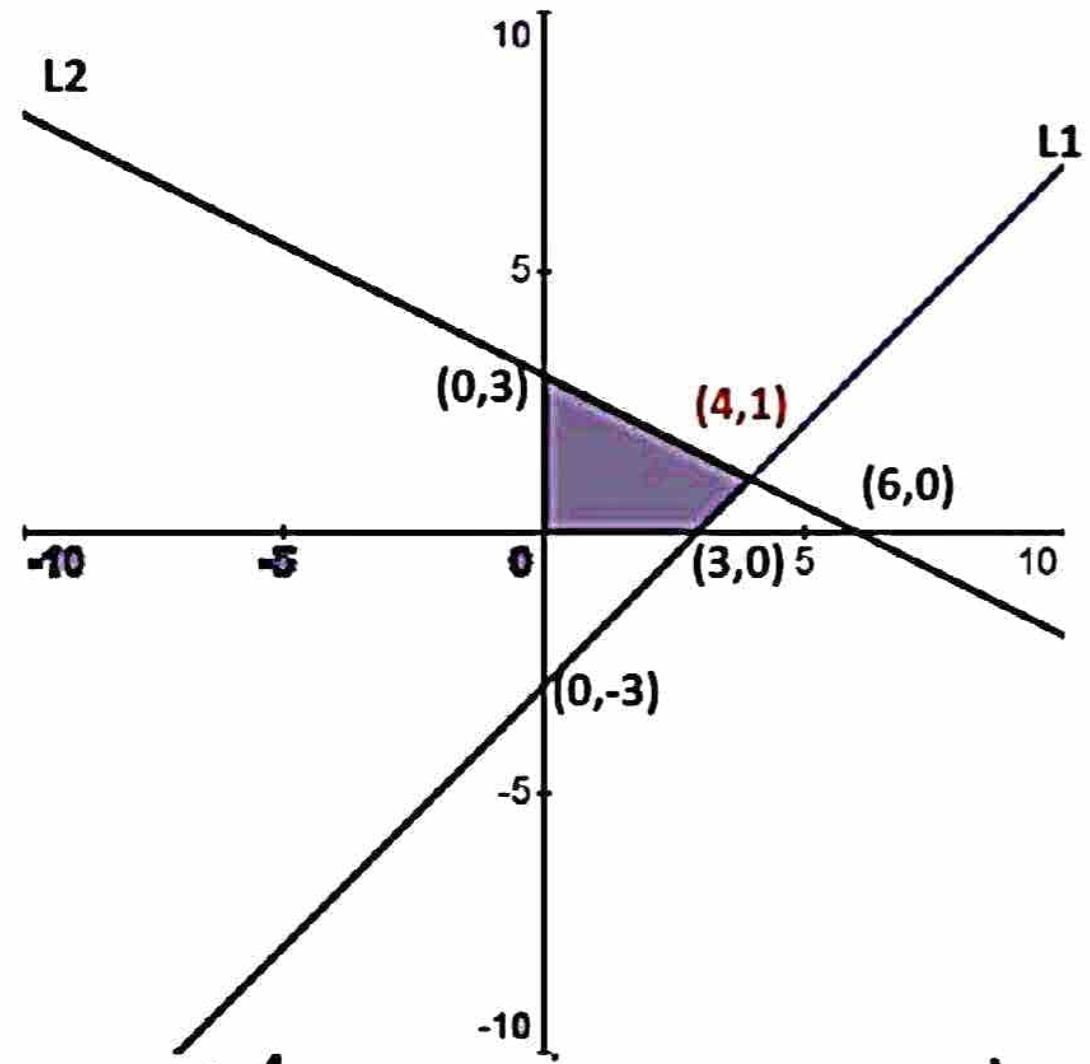
L1; $x-y=3$ —(iii) , L2; $x+2y=6$ —(iv)

(iii) → put $x=0$, $y=-3$ so the pt $(0, -3)$
put $y=0$, $x=3$ so the pt $(3, 0)$

(iv) → put $x=0$, $y=3$ so the pt $(0, 3)$
put $y=0$, $x=6$ so the pt $(6, 0)$

Test pt (0,0):- we test (i) and (ii) at (0,0)
so (i) → $0 \leq 3$ — True , (ii) → $0 \leq 6$ — True

Feasible region:- The feasible region of the given system is the intersection of the graphs of (i) and (ii). Also $x \geq 0$, $y \geq 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



Corner points:- As $x-y=3$ —(i)
$x+2y=6$ —(ii)

By (ii) − (i) → $x+2y=6$
$\underline{\begin{array}{r} x-y=3 \\ -\ +\ - \end{array}}$
$3y=3$ → $y=1$ put in (i) $x=4$

so $(4,1)$ is pt. of intersection of lines (i) and (ii). Thus corner pts. of feasible region are $(0,0)$, $(3,0)$, $(4,1)$, $(0,3)$.

Example 3. Graph the feasible regions feasible subject to the following constraints

(a) $2x-3y \le 6$
$2x+y \ge 2$
$x \ge 0,\ y \ge 0$

(b) $2x-3y \le 6$
$2x+y \ge 2$
$x+2y \le 8,\ x \ge 0,\ y \ge 0$

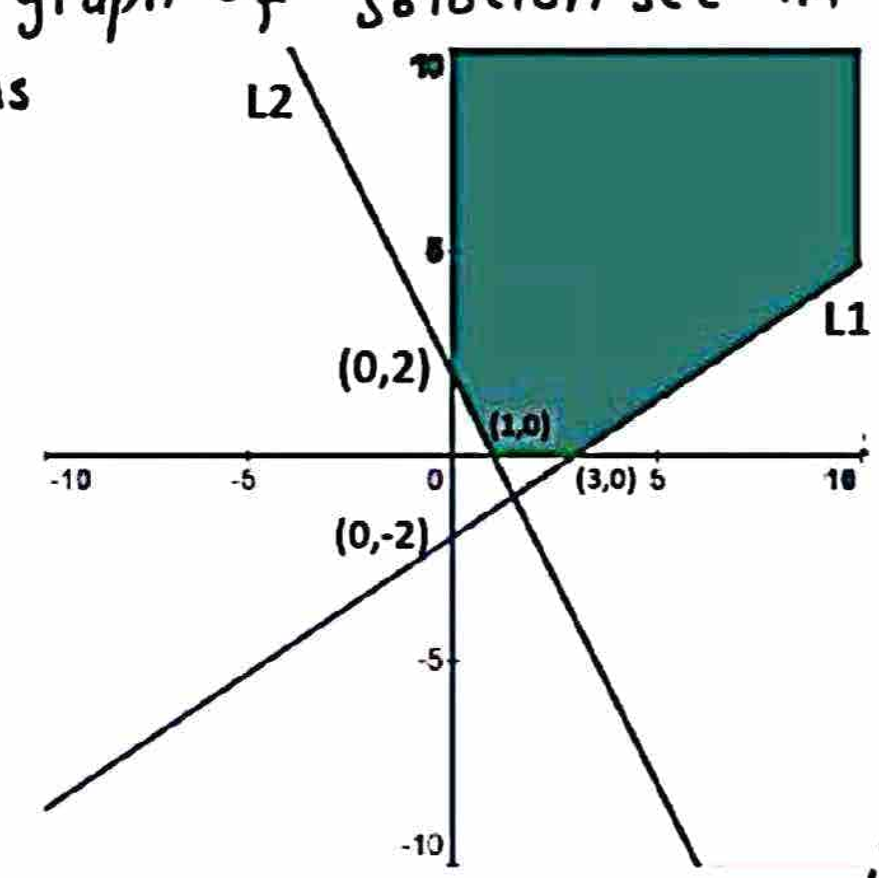
Solution:- (a) $2x-3y \le 6$ —(i); $2x+y \ge 2$ —(ii); $x \ge 0$, $y \ge 0$

The associated eqs. of (i) and (ii) are
L1; $2x-3y=6$ —(iii), L2; $2x+y=2$ —(iv)

(iii) → put $x=0$, $y=-2$ so the pt $(0,-2)$
put $y=0$, $x=3$ so the pt $(3,0)$
(iv) → put $x=0$, $y=2$ so the pt $(0,2)$
put $y=0$, $x=1$ so the pt $(1,0)$

Test pt (0,0):- we test (i) and (ii) at $(0,0)$
so (i) → $0 \le 6$ — True (ii) → $0 \ge 2$ — False

Feasible region:- The feasible region of the given system is the intersection of the graphs of (i) and (ii). Also $x \ge 0, y \ge 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



(b) $2x-3y \le 6$; $2x+y \ge 2$; $x+2y \le 8$
$x \ge 0,\ y \ge 0$

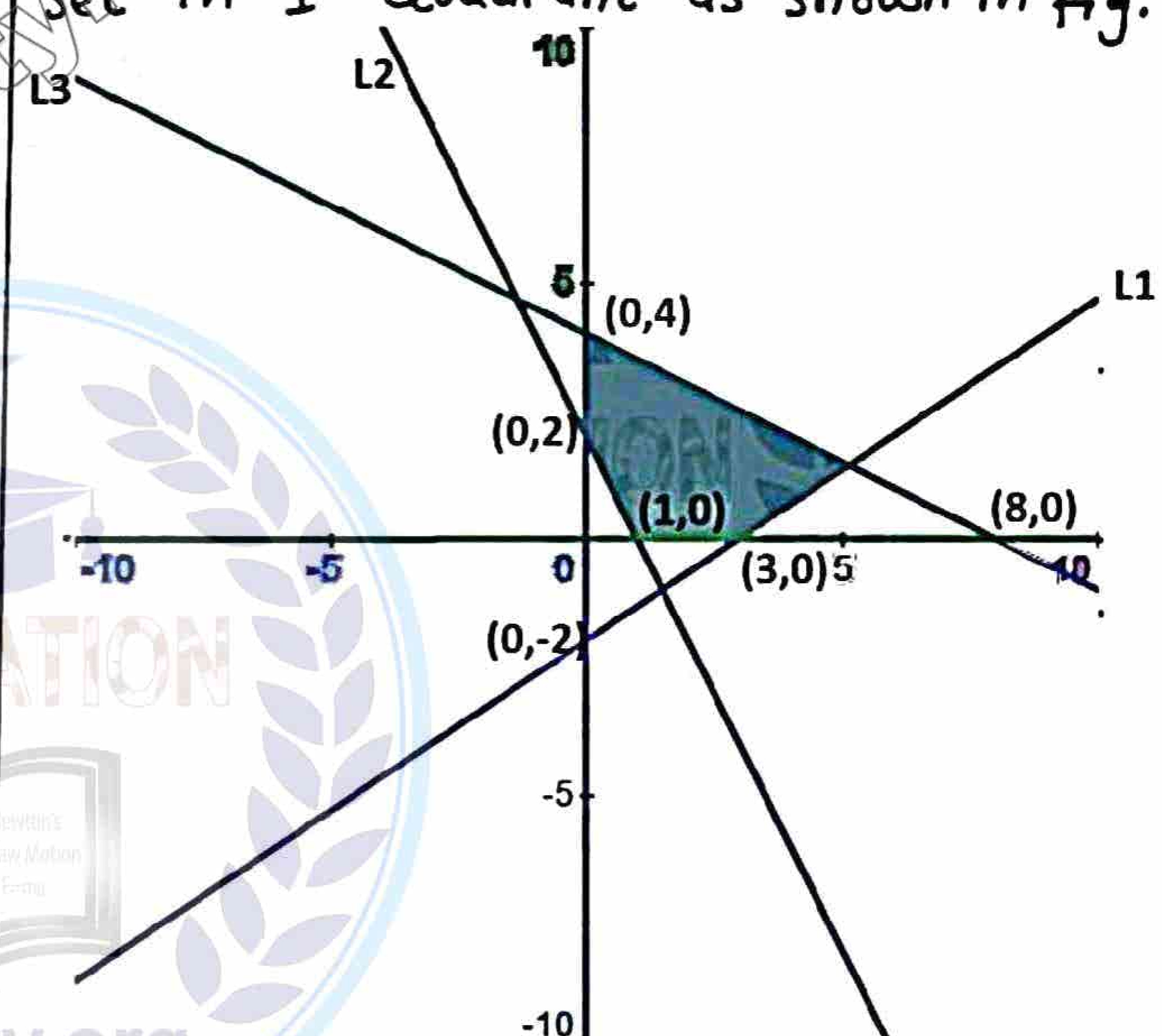
Solution:- $2x-3y \le 6$ —(i), $2x+y \ge 2$ —(ii)
$x+2y \le 8$ —(iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $2x-3y=6$ —(iv), L2; $2x+y=2$ —(v)
L3; $x+2y=8$ —(vi)

(iv) → put $x=0$, $y=-2$ so the pt $(0,-2)$
put $y=0$, $x=3$ so the pt $(3,0)$
(v) → put $x=0$, $y=2$ so the pt $(0,2)$
put $y=0$, $x=1$ so the pt $(1,0)$
(vi) → put $x=0$, $y=4$ so the pt $(0,4)$
put $y=0$, $x=8$ so the pt $(8,0)$

Test pt (0,0):- we test (i) and (ii), (iii) at $(0,0)$
(i) → $0 \le 6$ — True (ii) → $0 \ge 2$ — False.
(iii) → $0 \le 8$ — True

Feasible region:- The feasible region of the given system is the intersection of the graphs of (i), (ii) and (iii). Also $x \ge 0$, $y \ge 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



$\boxed{\textbf{Convex:-}}$ If the line segment obtained by joining any two points of a region lies entirely within the region, then the region is called convex.

Exercise 5.2

Q1. Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x-3y\le 6$; $2x+3y\le 12$; $x\ge 0, y\ge 0$

Solution:- $2x-3y\le 6$ — (i) , $2x+3y\le 12$ — (ii)

The associated eqs. of (i) and (ii) are

L1; $2x-3y=6$ — (iii) , L2; $2x+5y=12$ — (iv)

(iii) → Put $x=0$, $y=-2$ so the pt $(0,-2)$

Put $y=0$, $x=3$ so the pt $(3,0)$

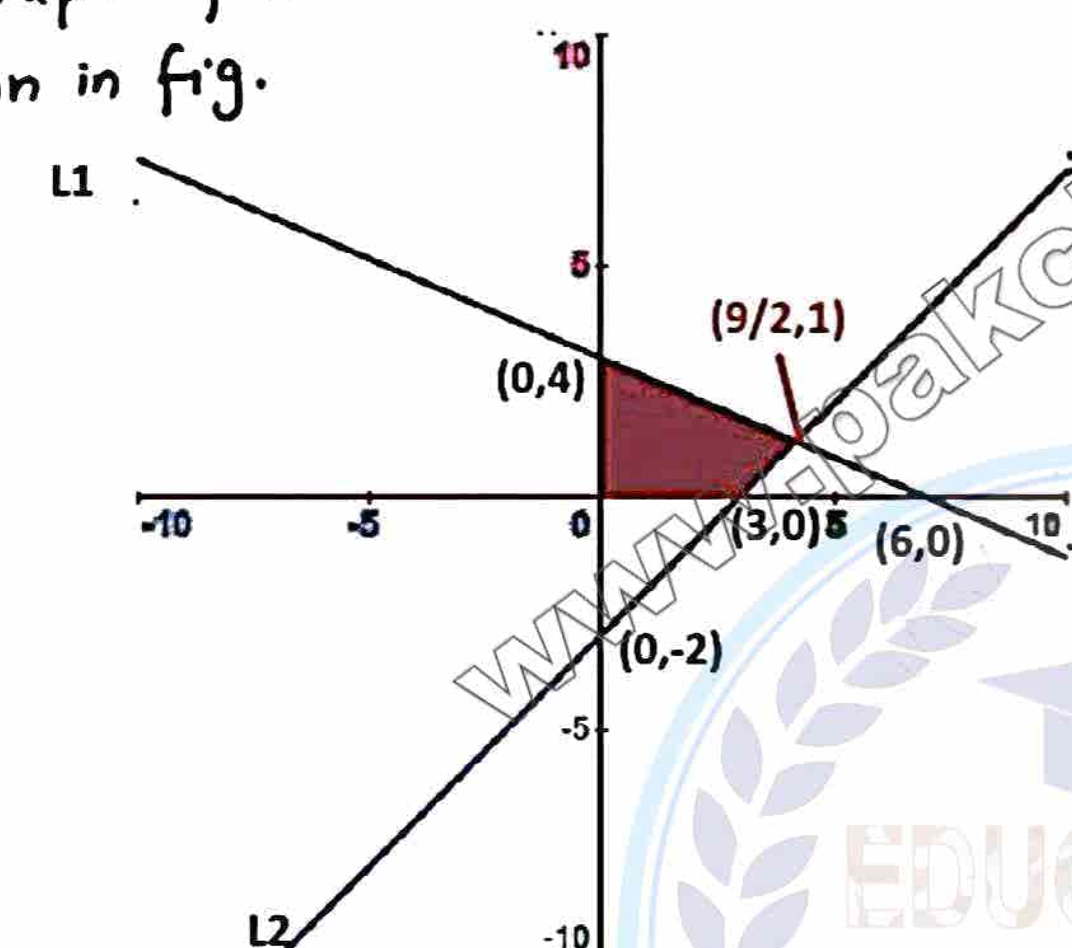
(iv) → Put $x=0$, $y=4$ so the pt $(0,4)$

Put $y=0$, $x=6$ so the pt $(6,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$

so (i) → $0\le 6$ — True , (ii) → $0\le 12$ — True

Feasible region:- The feasible region of the given system is the intersection of the graphs of (i) and (ii). Also $x\ge 0, y\ge 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- As $2x-3y=6$ — (i)

$2x+3y=12$ — (ii)

By (i) + (ii) ⇒ $4x=18$ ⇒ $x=\frac{9}{2}$ Put in (i)

⇒ $2(\frac{9}{2})-3y=6$ ⇒ $-3y=6-9$

⇒ $y=1$. so $(\frac{9}{2},1)$ is the pt. of intersection of lines L1 and L2. Thus corner points of feasible region are $(0,0)$, $(3,0)$, $(\frac{9}{2},1)$ and $(0,4)$.

(ii) $x+y\le 5$; $-2x+y\le 2$; $x\ge 0, y\ge 0$

Solution:- $x+y\le 5$ — (i) , $-2x+y\le 2$ — (ii)

The associated eqs. of (i) and (ii) are

L1; $x+y=5$ — (iii) , L2; $-2x+y=2$ — (iv)

(iii) → Put $x=0$, $y=5$ so the pt $(0,5)$

Put $y=0$, $x=5$ so the pt $(5,0)$

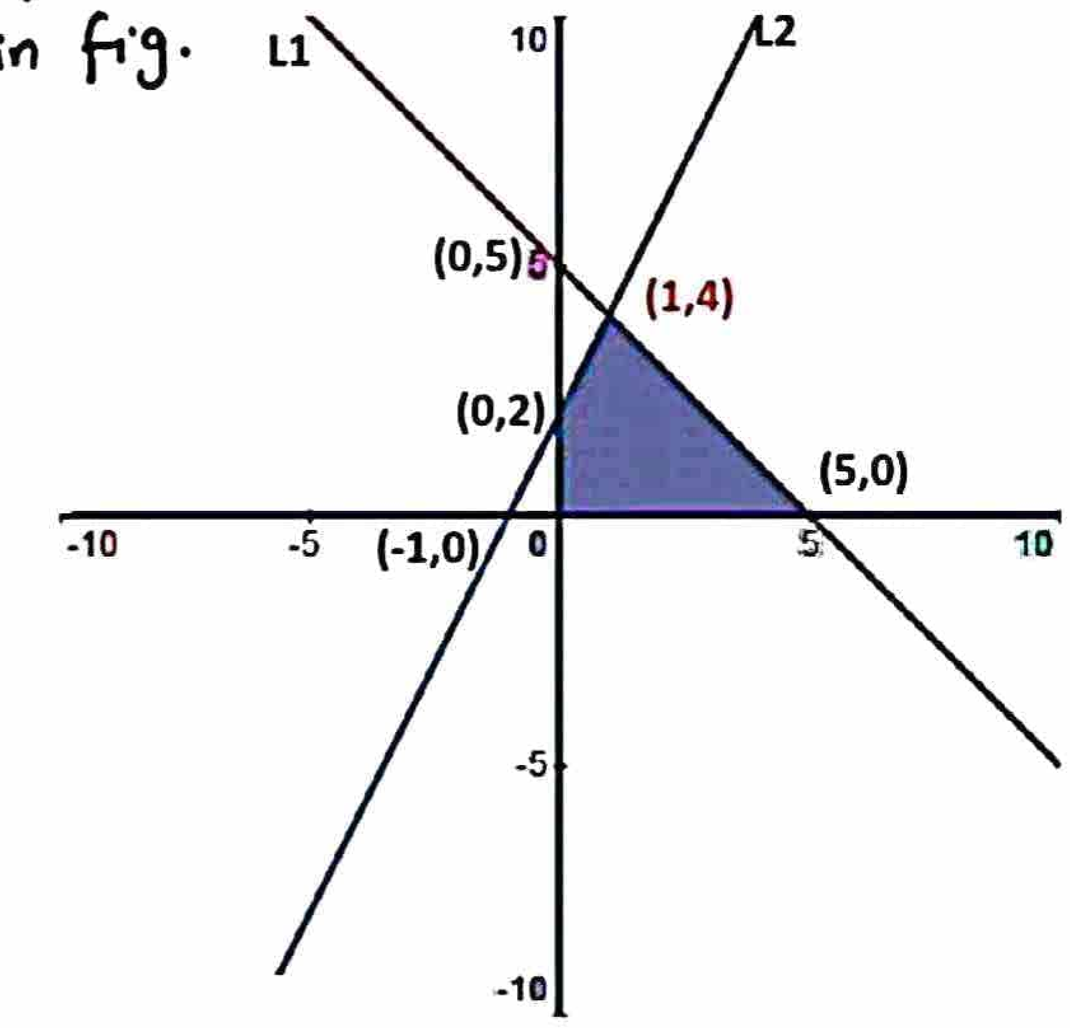
(iv) → Put $x=0$, $y=2$ so the pt $(0,2)$

Put $y=0$, $x=-1$ so the pt $(-1,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$. so

(i) → $0\le 5$ — True (ii) → $0\le 2$ — True

Feasible region:- The feasible region of the given system is the intersection of the graphs of (i) and (ii). Also $x\ge 0, y\ge 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- As $x+y=5$ — (i)

$-2x+y=2$ — (ii)

By (i) − (ii) → $3x=3$ → $x=1$ put in (i)

$1+y=5$ → $y=4$

so $(1,4)$ is pt. of intersection of lines L1 and L2. Thus corner pts. of feasible region are $(0,0)$, $(5,0)$, $(1,4)$ and $(0,2)$.

(iii) $x+y\le 5$; $-2x+y\ge 2$; $x\ge 0, y\ge 0$

Solution:- $x+y\le 5$ — (i) , $-2x+y\ge 2$ — (ii)

The associated eqs. of (i) and (ii) are

L1; $x+y=5$ — (iii) , L2; $-2x+y=2$ — (iv)

(iii) → Put $x=0$, $y=5$ so the pt $(0,5)$

Put $y=0$, $x=5$ so the pt $(5,0)$

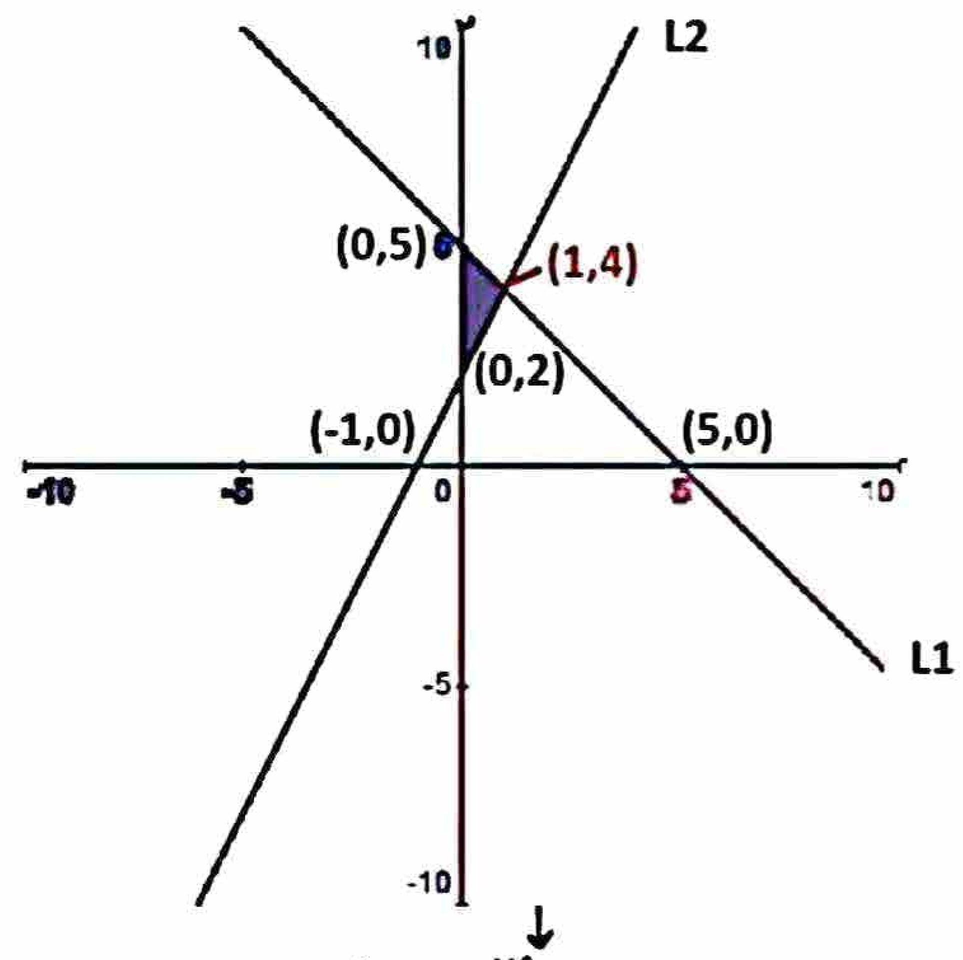
(iv) → Put $x=0$, $y=2$ so the pt $(0,2)$

Put $y=0$, $x=-1$ so the pt $(-1,0)$

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$. so

(i) → $0\le 5$ — True , (ii) → $0\ge 2$ — False

Feasible region:- The feasible region of the given system is the intersection of the graphs of (i) and (ii). Also $x\ge 0, y\ge 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- As $x+y=5$ —(i)

$-2x+y=2$ —(ii)

By (i) − (ii) → $3x=3$ → $x=1$ put in (i)

$y=4$

so $(1,4)$ is the pt. of intersection of lines L1 and L2 Hence corner pts. of feasible region are $(0,2)$, $(1,4)$ and $(0,5)$.

(iv) $3x+7y \le 21$; $x-y \le 3$; $x \ge 0$, $y \ge 0$

Solution:- $3x+7y \le 21$ —(i) , $x-y \le 3$ —(ii)

The associated eqs. of (i) and (ii) are

L1; $3x+7y=21$ —(iii) , L2; $x-y=3$ —(iv)

(iii) → put $x=0, y=3$ so the pt $(0,3)$

Put $y=0$, $x=7$ so the pt $(7,0)$

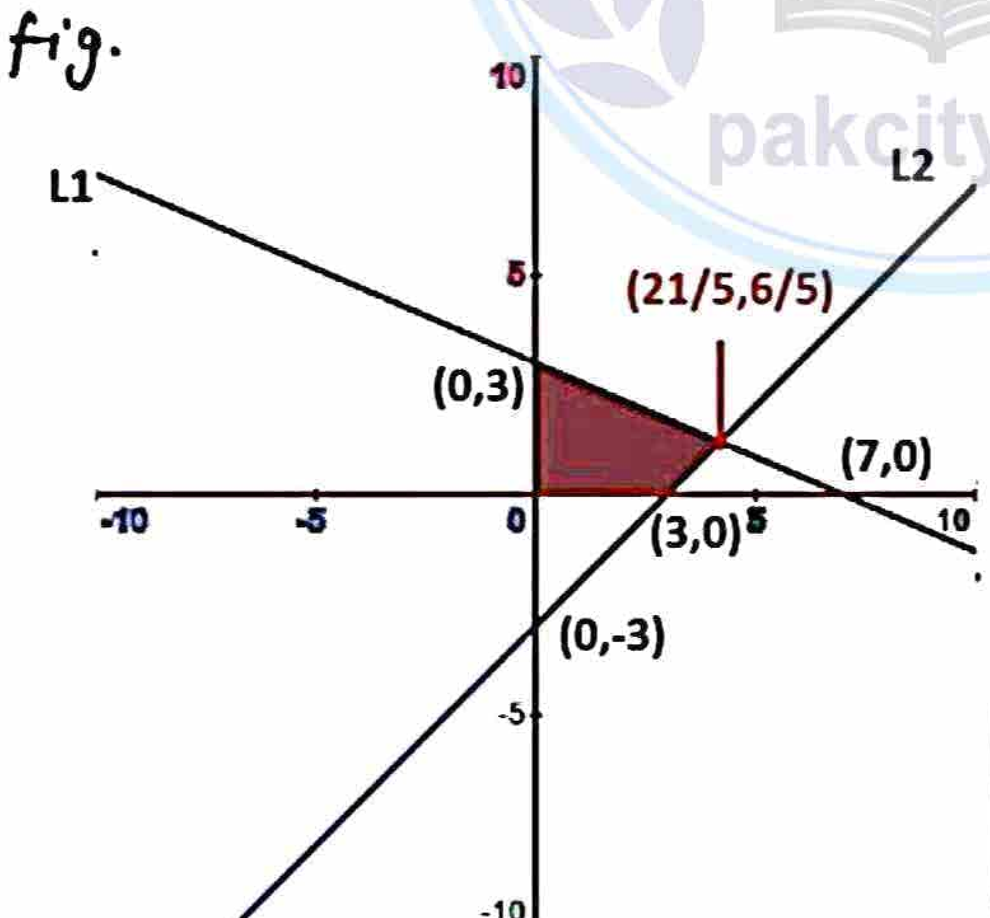
(iv) → put $x=0$, $y=-3$ so the pt $(0,-3)$

Put $y=0$, $x=3$ so the pt $(3,0)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$ so

(i) → $0 \le 21$ — True , (ii) → $0 \le 3$ — True

Feasible region:- The feasible region of given system is intersection of graphs of (i) and (ii). Also $x \ge 0$, $y \ge 0$ indicates that graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- As $3x+7y=21$ —(i)

$x-y=3$ —(ii)

By 7(ii) + (i) → $10x=42$ → $x=\frac{21}{5}$ put in (ii)

$\frac{21}{5}-y=3$ → $y=\frac{21}{5}-3=\frac{6}{5}$

so $(\frac{21}{5},\frac{6}{5})$ is the pt. of intersection of lines L1 and L2. Hence corner pts. are $(0,0)$, $(3,0)$, $(\frac{21}{5},\frac{6}{5})$ and $(0,3)$.

(v) $3x+2y \ge 6$; $x+y \le 4$; $x \ge 0$, $y \ge 0$

Solution:- $3x+2y \ge 6$ —(i) , $x+y \le 4$ —(ii)

The associated eqs. of (i) and (ii) are

L1; $3x+2y=6$ —(iii) , L2; $x+y=4$ —(iv)

(iii) → put $x=0$, $y=3$ so the pt $(0,3)$

Put $y=0$, $x=2$ so the pt $(2,0)$

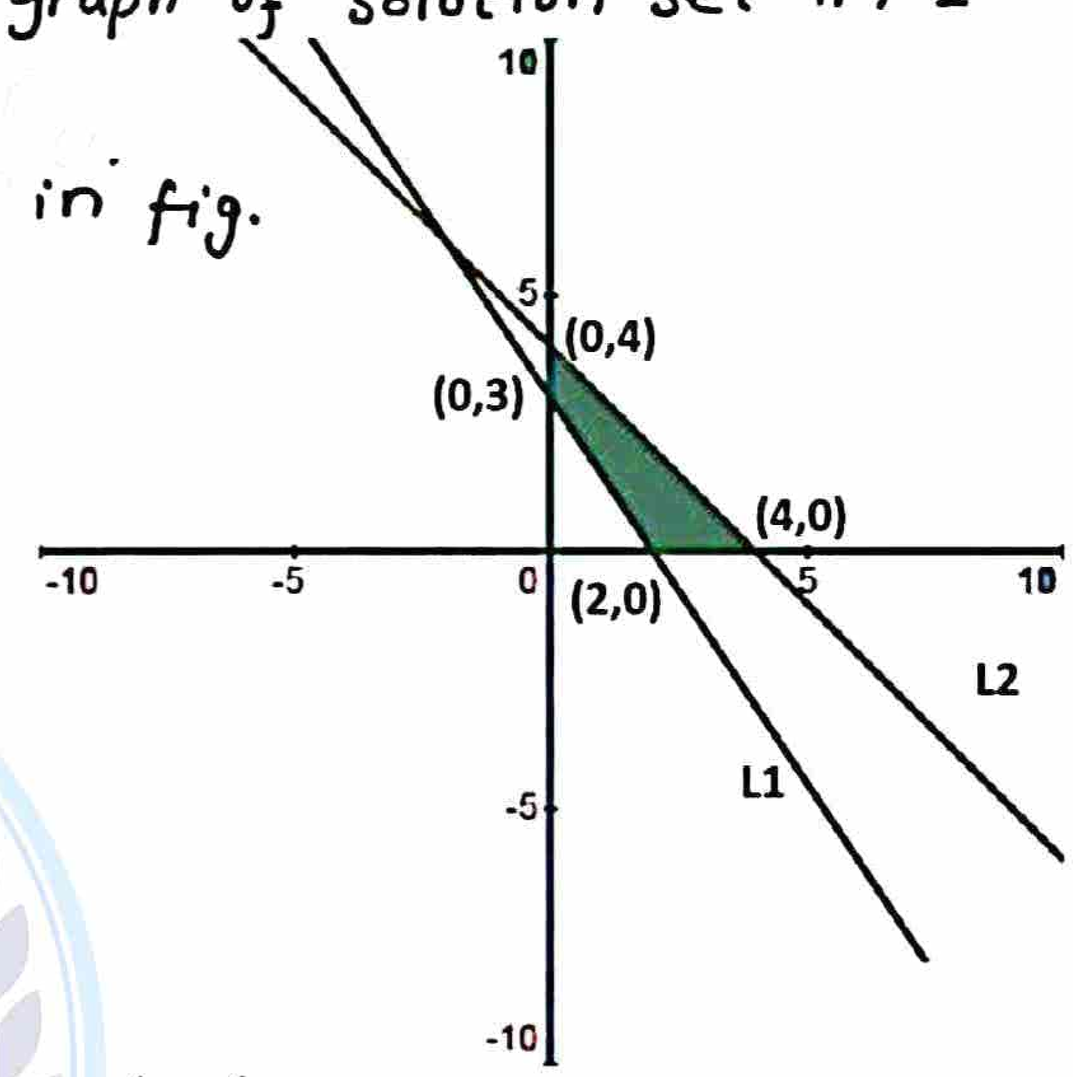
(iv) → put $x=0$, $y=4$ so the pt $(0,4)$

Put $y=0$, $x=4$ so the pt $(4,0)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$. so

(i) → $0 \ge 6$ — False , (ii) → $0 \le 4$ — True

Feasible region:- The feasible region of given system is intersection of graphs of (i) and (ii). Also $x \ge 0$, $y \ge 0$ indicates that the graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- Corner pts. of feasible region are $(2,0)$, $(4,0)$, $(0,4)$ and $(0,3)$

(vi) $5x+7y \le 35$; $x-2y \le 4$; $x \ge 0$, $y \ge 0$

Solution:- $5x+7y \le 35$ —(i), $x-2y \le 4$ —(ii)

The associated eqs. of (i) and (ii) are

l_1; $5x+7y=35$ —(iii) , l_2; $x-2y=4$ —(iv)

(iii) → put $x=0$, $y=5$ so the pt $(0,5)$

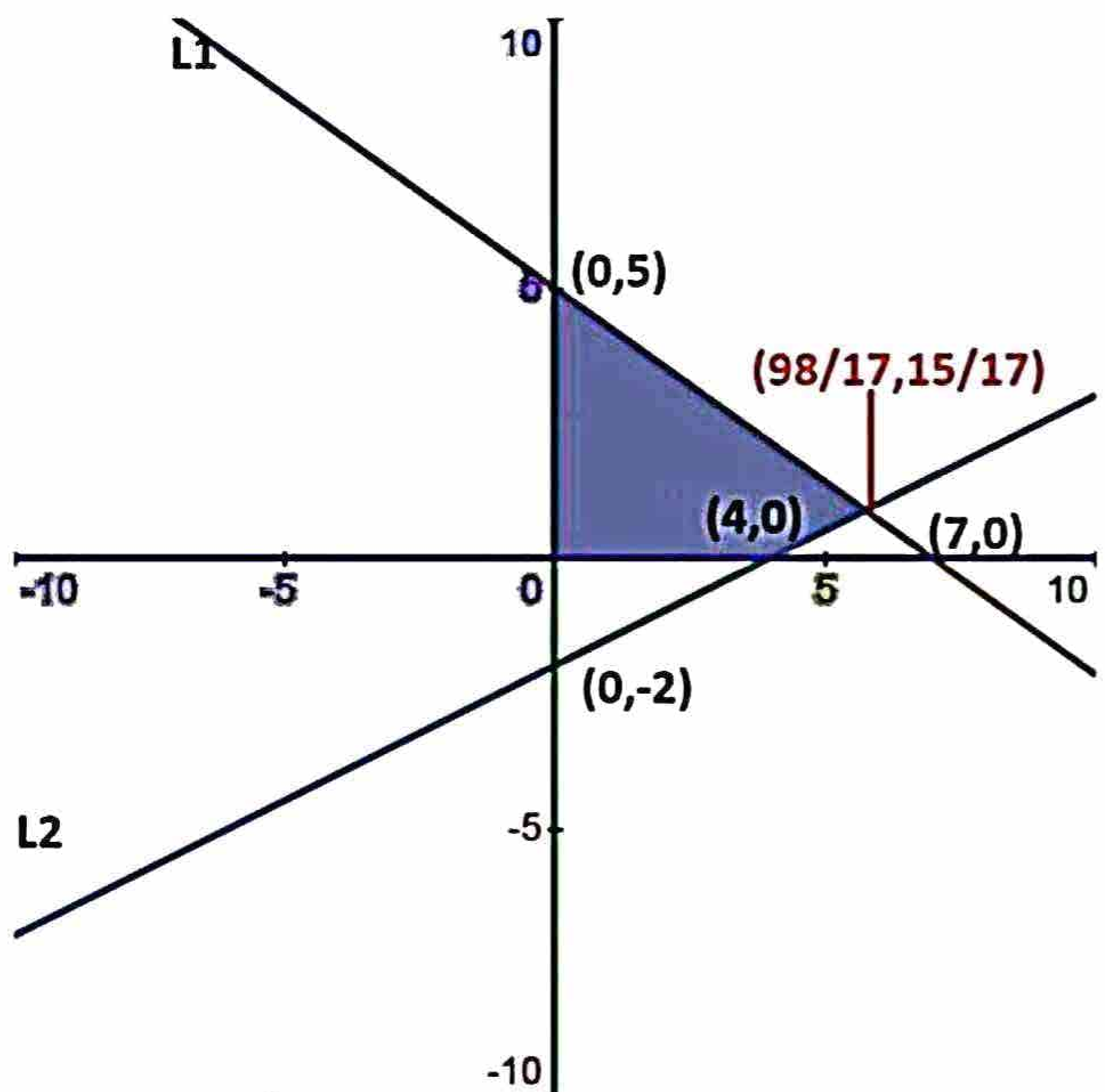
put $y=0$, $x=7$ so the pt $(7,0)$

(iv) → put $x=0$, $y=-2$ so the pt $(0,-2)$

put $y=0$, $x=4$ so the pt $(4,0)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$

Feasible region:- The feasible region of given system is intersection of graphs of (i) and (ii). Also $x \ge 0$, $y \ge 0$ indicates that the graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- As $5x+7y=35$ —(i)
$x-2y=4$ —(ii)

By 5(ii) $-$ (i) $\rightarrow$
$$5x-10y=20$$
$$\underline{\underset{-}{5x}\ \underset{-}{+}\ 7y\ \underset{-}{=}\ 35}$$
$$-17y=-15 \rightarrow y=\frac{15}{17} \text{ put in (ii)}$$
$$\rightarrow x-2\left(\frac{15}{17}\right)=4 \rightarrow x=4+\frac{30}{17}$$
$$x=\frac{98}{17}$$

So $\left(\frac{98}{17},\frac{15}{17}\right)$ is the pt. of intersection of lines (i) and (ii). Hence corner pts are $(0,0)$, $(4,0)$, $\left(\frac{98}{17},\frac{15}{17}\right)$ and $(0,5)$.

Q2. Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x+y\le 10$; $x+4y\le 12$; $x+2y\le 10$; $x\geq 0, y\geq 0$

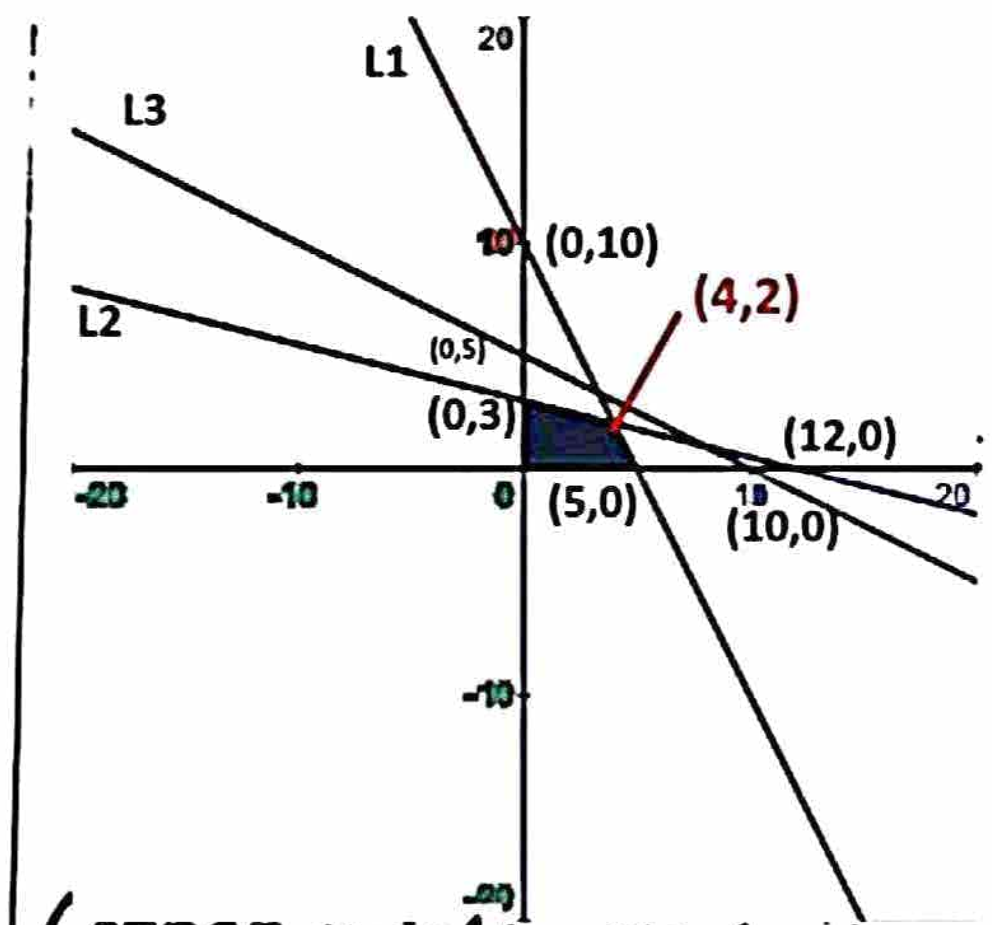
Solution:- $2x+y\le 10$ —(i), $x+4y\le 12$ —(ii)
$x+2y\le 10$ —(iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $2x+y=10$ —(iv), L2; $x+4y=12$ —(v)
L3; $x+2y=10$ —(vi)

(iv) $\rightarrow$ put $x=0$, $y=10$ so the pt $(0,10)$
put $y=0$, $x=5$ so the pt $(5,0)$
(v) $\rightarrow$ put $x=0$, $y=3$ so the pt $(0,3)$
put $y=0$, $x=12$ so the pt $(12,0)$
(vi) $\rightarrow$ put $x=0$, $y=5$ so the pt $(0,5)$
put $y=0$, $x=10$ so the pt $(10,0)$

Test pt (0,0):- We test (i), (ii) and (iii) at $(0,0)$. So (i) $\rightarrow$ $0\le 10$ — True, (ii) $\rightarrow$ $0\le 12$ — True
(iii) $\rightarrow$ $0\le 10$ — True

Feasible region:- The feasible region of given system is the intersection of graphs of (i), (ii) and (iii). Also $x\geq 0, y\geq 0$ indicates that the graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L1 and L2. so
$2x+y=10$ —(i)
$x+4y=12$ —(ii)

By 2(ii) $-$ (i) $\rightarrow$
$$2x+8y=24$$
$$\underline{\underset{-}{2x}\ \underset{-}{+}\ y\ \underset{-}{=}\ 10}$$
$$7y=14 \rightarrow y=2 \text{ put in (i)}$$
$$\rightarrow x=4$$

so $(4,2)$ is pt. of intersection of lines L1 and L2. Thus corner pts of feasible region are $(0,0)$, $(5,0)$, $(4,2)$ and $(0,3)$.

(ii) $2x+3y\le 18$; $2x+y\le 10$; $x+4y\le 12$; $x\geq 0, y\geq 0$

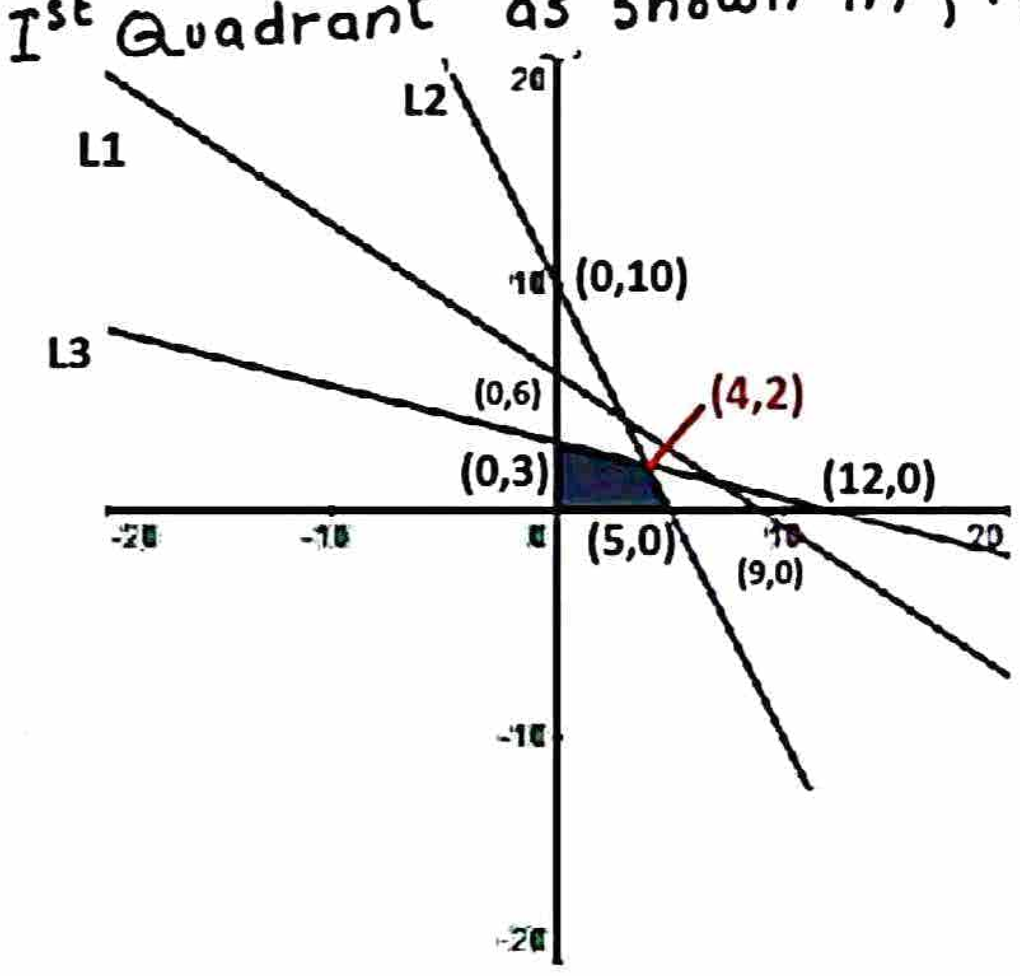
Solution:- $2x+3y\le 18$ —(i), $2x+y\le 10$ —(ii)
$x+4y\le 12$ —(iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $2x+3y=18$ —(iv), L2; $2x+y=10$ —(v)
L3; $x+4y=12$ —(vi)

(iv) $\rightarrow$ Put $x=0$, $y=6$ so the pt $(0,6)$
Put $y=0$, $x=9$ so the pt $(9,0)$
(v) $\rightarrow$ Put $x=0$, $y=10$ so the pt $(0,10)$
Put $y=0$, $x=5$ so the pt $(5,0)$
(vi) $\rightarrow$ Put $x=0$, $y=3$ so the pt $(0,3)$
Put $y=0$, $x=12$ so the pt $(12,0)$

Test pt (0,0):- We test (i), (ii) and (iii) at $(0,0)$. So (i) $\rightarrow$ $0\le 18$ — True, (ii) $\rightarrow$ $0\le 10$ — True
(iii) $\rightarrow$ $0\le 12$ — True

Feasible region:- The feasible region of given system is the intersection of graphs of (i), (ii) and (iii). Also $x\geq 0, y\geq 0$ indicates that the graph of solution set in I^{st} Quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L2 and L3. so

$$2x + y = 10 \quad \text{—— (i)}$$
$$x + 4y = 12 \quad \text{—— (ii)}$$

By $2(ii) - (i) \Rightarrow$
$$\begin{array}{r} 2x + 8y = 24 \\ \underline{\pm 2x \pm y = \pm 10} \\ 7y = 14 \end{array} \Rightarrow y = 2 \text{ put in (i)}$$
$$\Rightarrow 2x = 10 - 2 \Rightarrow x = 4$$

so $(4, 2)$ is the pt. of intersection of lines L2 and L3. Hence corner pts are $(0,0), (5,0), (4,2), (0,3)$.

(iii) $2x + 3y \le 18$; $x + 4y \le 12$; $3x + y \le 12$; $x \geq 0, y \geq 0$

Solution:- $2x + 3y \le 18$ —— (i), $x + 4y \le 12$ —— (ii)
$3x + y \le 12$ —— (iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $2x + 3y = 18$ —— (iv), L2; $x + 4y = 12$ —— (v)
L3; $3x + y = 12$ —— (vi)

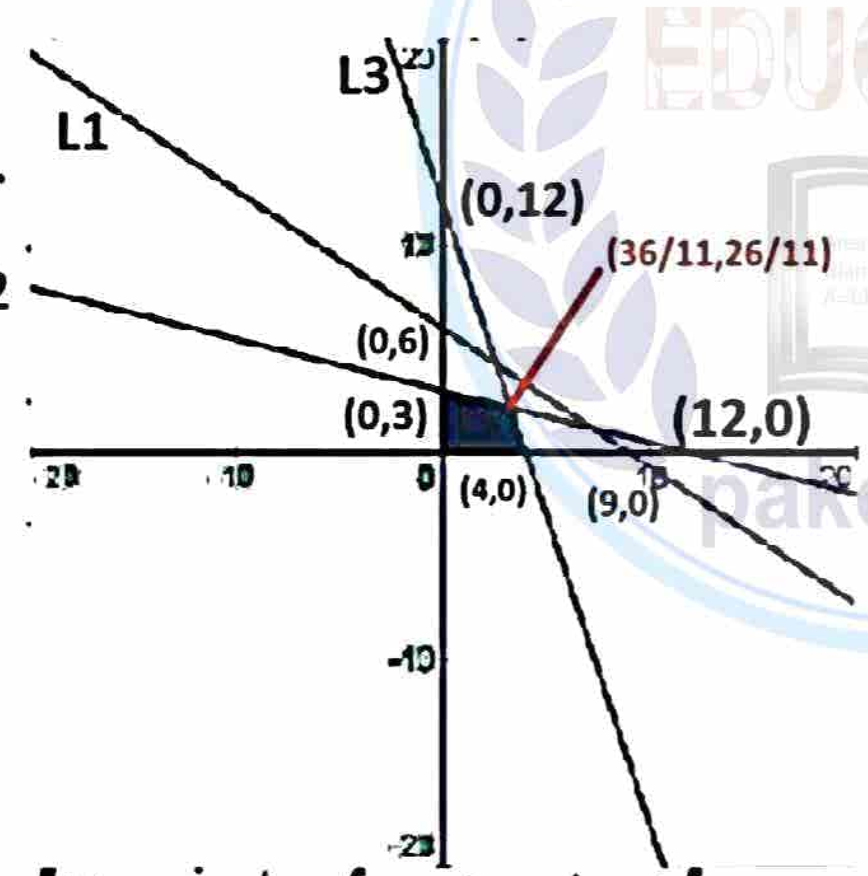
(iv) $\Rightarrow$ put $x = 0$, $y = 6$ so the pt $(0, 6)$
put $y = 0$, $x = 9$ so the pt $(9, 0)$

(v) $\Rightarrow$ put $x = 0$, $y = 3$ so the pt $(0,3)$
put $y = 0$, $x = 12$ so the pt $(12,0)$

(vi) $\Rightarrow$ put $x = 0$, $y = 12$ so the pt $(0,12)$
put $y = 0$, $x = 4$ so the pt $(4,0)$

Test pt (0,0):- We test (i), (ii) and (iii) at (0,0)
so (i) $\Rightarrow 0 \le 18$ —— True, (ii) $\Rightarrow 0 \le 12$ —— True
(iii) $\Rightarrow 0 \le 12$ —— True

Feasible region:- The feasible region of given system is intersection of graphs of (i), (ii) and (iii). Also $x \geq 0, y \geq 0$ indicates that the graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L2 and L3 so

$$x + 4y = 12 \quad \text{—— (i)}$$
$$3x + y = 12 \quad \text{—— (ii)}$$

By $3(i) - (ii) \Rightarrow$
$$\begin{array}{r} 3x + 12y = 36 \\ \underline{\pm 3x \pm y = \pm 12} \\ 11y = 24 \end{array} \Rightarrow y = \frac{24}{11} \text{ Put in (i)}$$

$$\Rightarrow x + 4\left(\frac{24}{11}\right) = 12 \Rightarrow x = 12 - \frac{96}{11}$$
$$\Rightarrow x = \frac{132 - 96}{11} = \frac{36}{11}$$

so $\left(\frac{36}{11}, \frac{26}{11}\right)$ is the pt. of intersection of lines L2 and L3. Hence corner pts. of feasible region are $(0,0), (4,0), \left(\frac{36}{11}, \frac{26}{11}\right)$ and $(0,3)$.

(iv) $x + 2y \le 14$; $3x + 4y \le 36$; $2x + y \le 10$; $x \geq 0, y \geq 0$

Solution:- $x + 2y \le 14$ —— (i), $3x + 4y \le 36$ —— (ii)
$2x + y \le 10$ —— (iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $x + 2y = 14$ —— (iv), L2; $3x + 4y = 36$ —— (v)
L3; $2x + y = 10$ —— (vi)

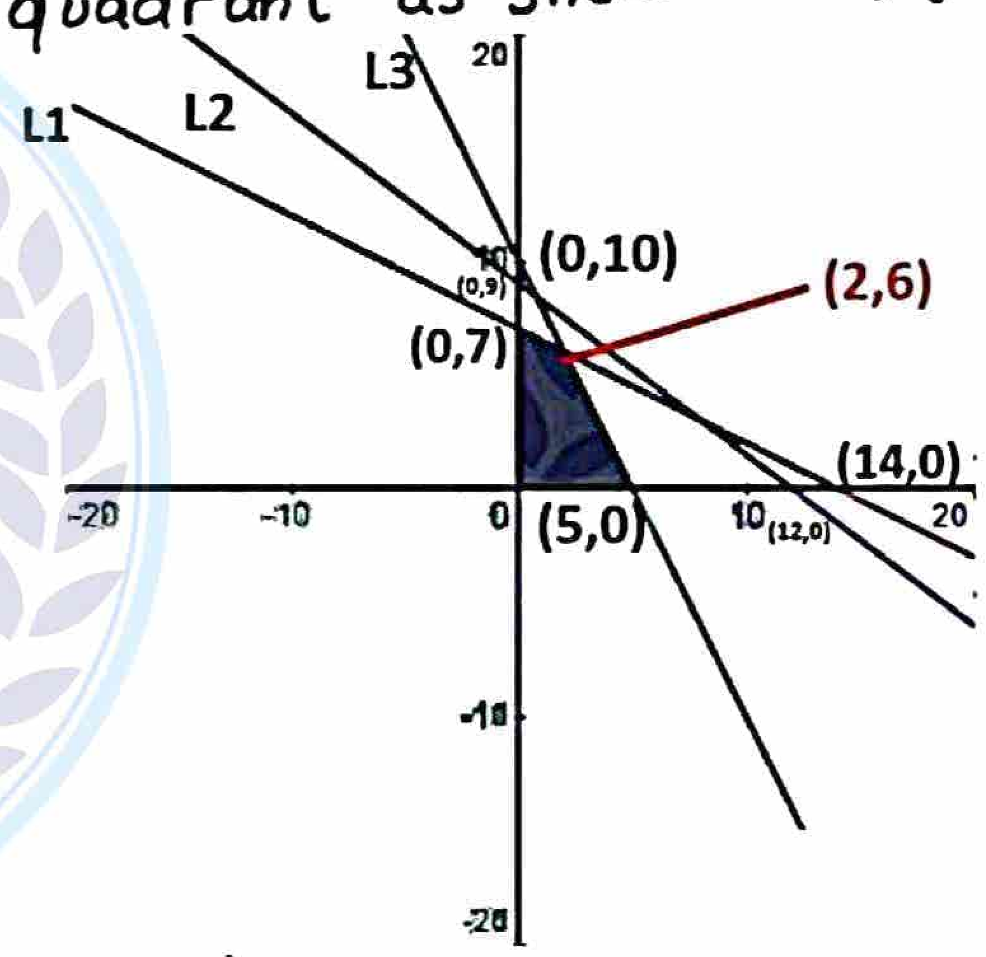
(iv) $\Rightarrow$ put $x = 0$, $y = 7$ so the pt $(0, 7)$
Put $y = 0$, $x = 14$ so the pt $(14, 0)$

(v) $\Rightarrow$ Put $x = 0$, $y = 9$ so the pt $(0, 9)$
Put $y = 0$, $x = 12$ so the pt $(12, 0)$

(vi) $\Rightarrow$ put $x = 0$, $y = 10$ so the pt $(0,10)$
Put $y = 0$, $x = 5$ so the pt $(5,0)$

Test pt (0,0):- We test (i), (ii) and (iii) at (0,0). so (i) $\Rightarrow 0 \le 14$ —— True, (ii) $\Rightarrow 0 \le 36$ —— True
(iii) $\Rightarrow 0 \le 10$ —— True

Feasible region:- The feasible region of given system is intersection of graphs of (i), (ii) and (iii). Also $x \geq 0, y \geq 0$ of solution set indicates that graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L1 and L3. so

$$x + 2y = 14 \quad \text{—— (i)}$$
$$2x + y = 10 \quad \text{—— (ii)}$$

By $2(i) - (ii) \Rightarrow$
$$\begin{array}{r} 2x + 4y = 28 \\ \underline{\pm 2x \pm y = \pm 10} \\ 3y = 18 \end{array} \Rightarrow y = 6 \text{ put in (i)}$$
$$x + 12 = 14 \Rightarrow x = 2$$

so $(2, 6)$ is pt. of intersection of lines L1 and L3. Hence corner pts. of feasible region are $(0,0), (5,0), (2,6)$ and $(0,7)$

(v) $x+3y \le 15$; $2x+y \le 12$; $4x+3y \le 24$; $x \ge 0$, $y \ge 0$

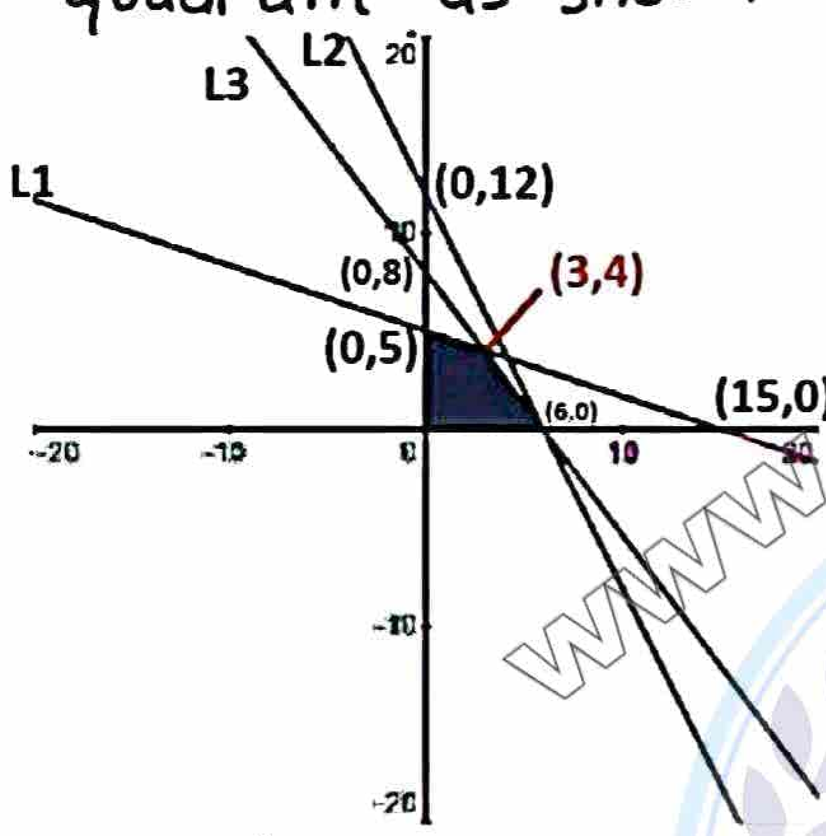
Solution:- $x+3y \le 15$ —(i), $2x+y \le 12$ —(ii)
$4x+3y \le 24$ —(iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $x+3y=15$ —(iv), L2; $2x+y=12$ —(v)
L3; $4x+3y=24$ —(vi)

(iv) → put $x=0$, $y=5$ so the pt $(0,5)$
put $y=0$, $x=15$ so the pt $(15,0)$
(v) → put $x=0$, $y=12$ so the pt $(0,12)$
put $y=0$, $x=6$ so the pt $(6,0)$
(vi) → put $x=0$, $y=8$ so the pt $(0,8)$
put $y=0$, $x=6$ so the pt $(6,0)$

Test pt (0,0):- We test (i), (ii) and (iii) at (0,0). so (i) → $0 \le 15$ — True, (ii) → $0 \le 12$ — True
(iii) → $0 \le 24$ — True

Feasible region:- The feasible region of given system is the intersection of graphs of (i), (ii) and (iii). Also $x \ge 0$, $y \ge 0$ indicates that the graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L1 and L3. so $x+3y=15$ —(i)
$4x+3y=24$ —(ii)

By (i) − (ii) → $-3x=-9$ → $x=3$ put in (i)
→ $3y=15-3=12$
→ $y=4$

so (3,4) is the pt. of intersection of lines (i) and (ii). Hence corner pts. of feasible region are $(0,0)$, $(6,0)$, $(3,4)$ and $(0,5)$.

(vi) $2x+y \le 20$; $8x+15y \le 120$; $x+y \le 11$; $x \ge 0$, $y \ge 0$

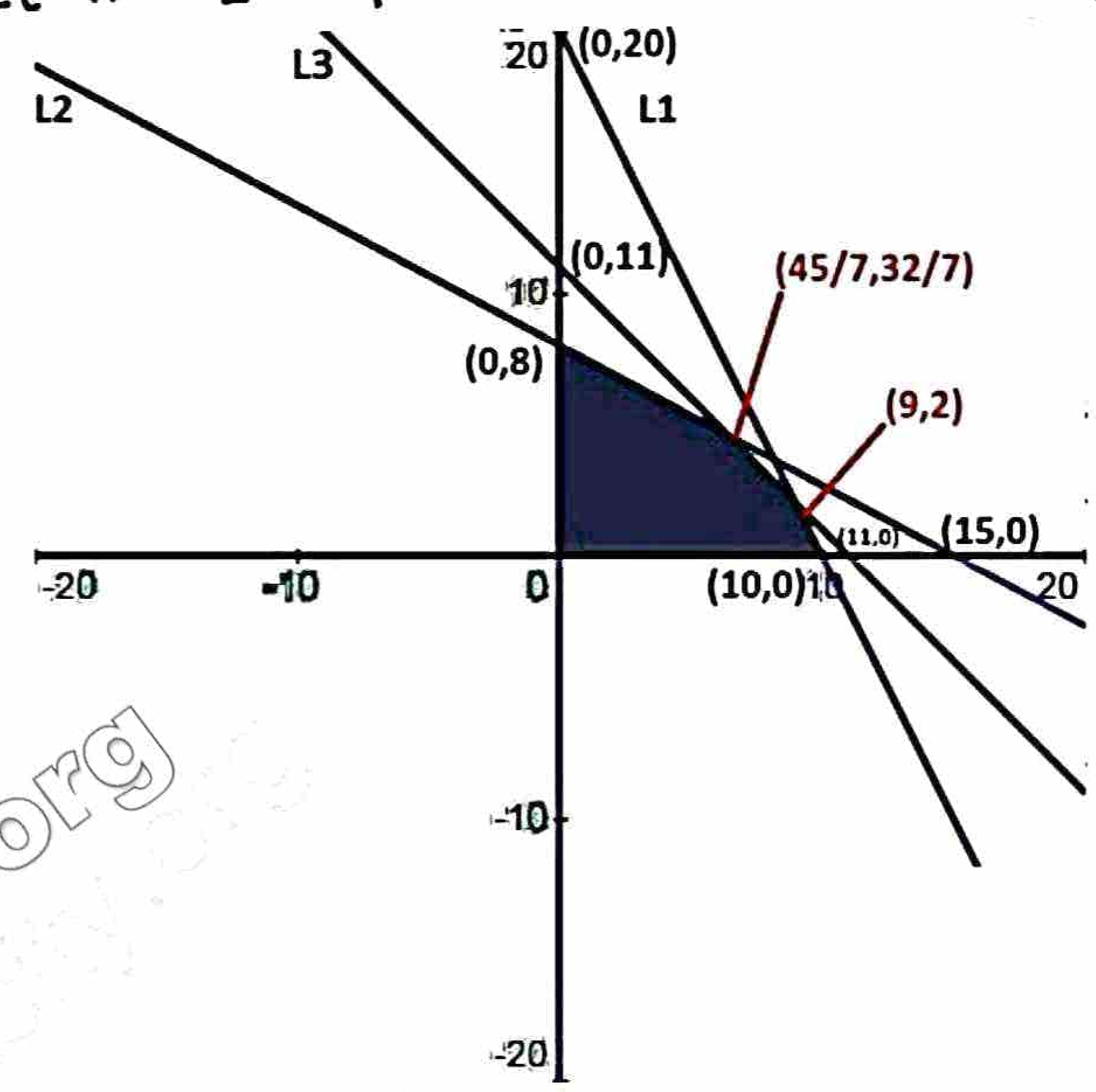
Solution:- $2x+y \le 20$ —(i); $8x+15y \le 120$ —(ii)
$x+y \le 11$ —(iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $2x+y=20$ —(iv); L2; $8x+15y=120$ —(v)
L3; $x+y=11$ —(vi)

(iv) → put $x=0$, $y=20$ so the pt $(0,20)$
put $y=0$, $x=10$ so the pt $(10,0)$
(v) → put $x=0$, $y=8$ so the pt $(0,8)$
put $y=0$, $x=15$ so the pt $(15,0)$
(vi) → put $x=0$, $y=11$ so the pt $(0,11)$
put $y=0$, $x=11$ so the pt $(11,0)$

Test pt (0,0):- We test (i), (ii) and (iii) at (0,0). so (i) → $0 \le 20$ — True (ii) → $0 \le 120$ — True
(iii) → $0 \le 11$ — True

Feasible region:- The feasible region of given system is the intersection of graphs of (i), (ii) and (iii). Also $x \ge 0$, $y \ge 0$ indicates the graph of Solution set in I^{st} quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L1 and L3. , also L2 and L3. so
$2x+y=20$ —(i)
$x+y=11$ —(ii)

By 2(ii) − (i) → $2x+2y=22$
$2x+y=20$
$y=2$ put in (ii) $x=9$

so (9,2) is the pt. of intersection of lines L1 and L3. Also
$8x+15y=120$ —(iii)
$x+y=11$ —(iv)

By 8(iv) − (iii) → $8x+8y=88$
$8x+15y=120$
$-7y=-32$ → $y=\frac{32}{7}$ put in (iv)

→ $x+\frac{32}{7}=11$ = $11-\frac{32}{7}=\frac{77-32}{7}=\frac{45}{7}$

so $(\frac{45}{7}, \frac{32}{7})$ is pt. of intersection of lines L2 and L3. Hence corner points of feasible region are $(0,0)$, $(9,2)$, $(\frac{45}{7}, \frac{32}{7})$ and $(0,8)$.

§ Linear programming

Objective function:- A function which is to be maximized or minimized is called an objective function.

Optimal solution:- The feasible solution which maximizes or minimizes the objective function is called optimal solution.

Procedure for finding optimal solution:- (i) Graph the solution set of linear inequality constraints to determine feasible region. (ii) Find the corner points of the feasible region. (iii) Evaluate the objective function at each corner point to find the optimal solution.

Example 1. Find the maximum and minimum values of the function defined as: $f(x,y) = 2x + 3y$ subject to constraints;

$x - y \le 2$; $x + y \le 4$; $2x - y \le 6$, $x \geq 0$

Solution:- $x - y \le 2$ — (i) , $x + y \le 4$ — (ii)
$2x - y \le 6$ — (iii)

The associated eqs. of (i), (ii) and (iii) are

L1; $x - y = 2$ — (iv) , L2; $x + y = 4$ — (v)
L3; $2x - y = 6$ — (vi)

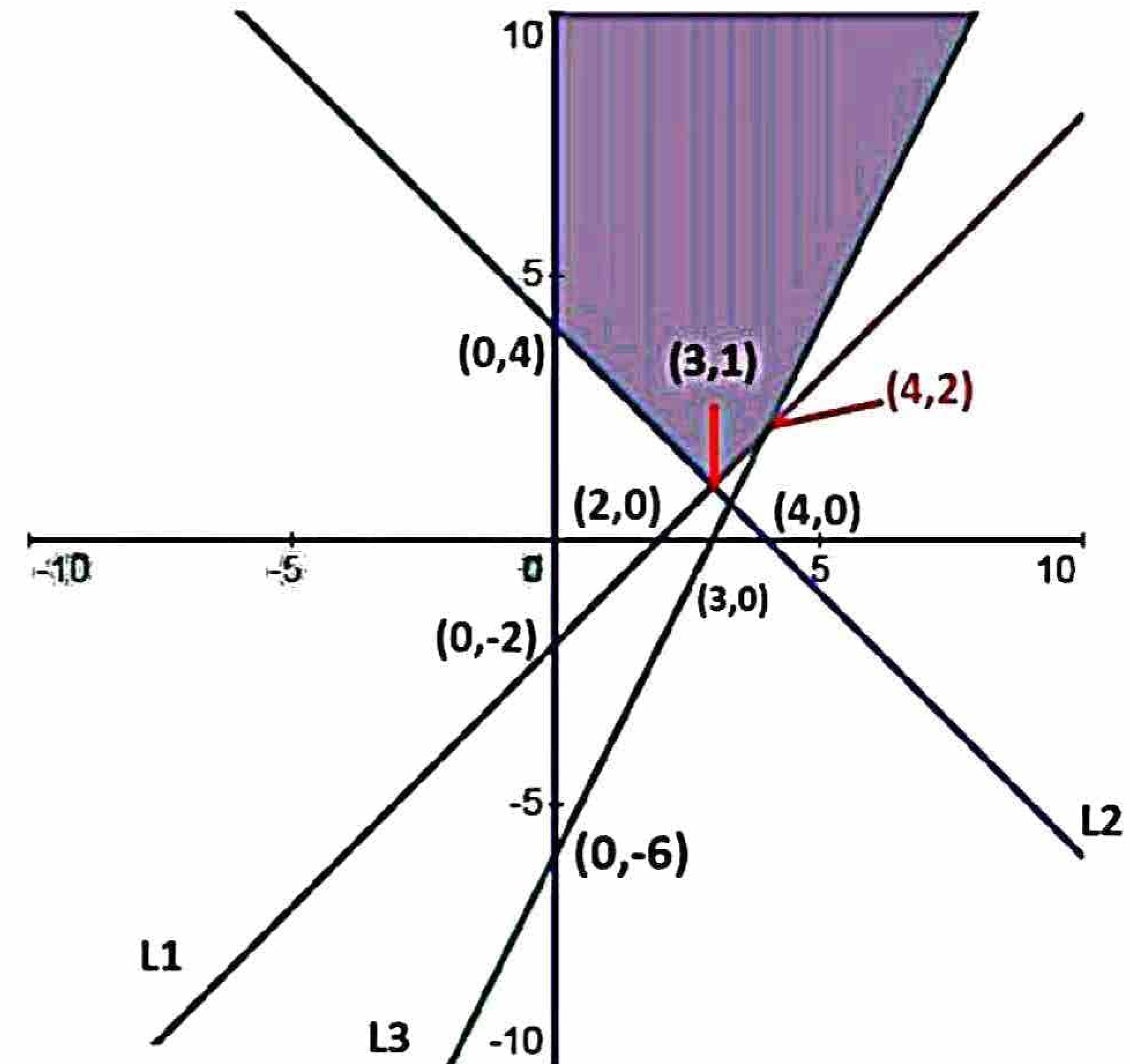
(iv) → Put $x = 0$, $y = -2$ so the pt $(0,-2)$
Put $y = 0$, $x = 2$ so the pt $(2,0)$

(v) → Put $x = 0$, $y = 4$ so the pt $(0,4)$
Put $y = 0$, $x = 4$ so the pt $(4,0)$

(vi) → Put $x = 0$, $y = -6$ so the pt $(0,-6)$
Put $y = 0$, $x = 3$ so the pt $(3,0)$

Test pt $(0,0)$:- We test (i), (ii) and (iii) at $(0,0)$. so (i) → $0 \le 2$ — True, (ii) → $0 \le 4$ — True
(iii) → $0 \le 6$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i), (ii) and (iii). Also $x \geq 0$ shows that the solution set is right half plane including the boundary line $x = 0$ as shown in fig.



Corner point:- We find pt. of intersection of lines L1 and L2, also L1 and L2. so

$x - y = 2$ — (i)
$x + y = 4$ — (ii)

By (i) + (ii) → $2x = 6$ → $x = 3$ put in (i) $y = 1$

so $(3,1)$ is pt. of intersection of lines L1 and L3. Also

$x - y = 2$ — (iii)
$2x - y = 6$ — (iv)

By (iv) − (iii) →

$2x - y = 6$
$\underline{\mp x \pm y = \mp 2}$
$x = 4$ put in (iii) → $y = 2$

so $(4,2)$ is the pt. of intersection of lines L1 and L3. Hence corner pts. of feasible region are $(0,4)$, $(3,1)$, $(4,2)$.

Optimal Solution:- We find values of $f(x,y) = 2x + 3y$ at corner pts.

$f(0,4) = 2(0) + 3(4) = 12$
$f(3,1) = 2(3) + 3(1) = 9$
$f(4,2) = 2(4) + 3(2) = 14$. So minimum value of $f(x,y)$ is 9 at $(3,1)$.

The expression $2x + 3y$ does not possess the maximum value in the feasible region because by increasing x and y, value of $2x + 3y$ can be made larger.

Example 2. Find the minimum and maximum values of f and ϕ defined as $f(x,y) = 4x + 5y$, $\phi(x) = 4x + 6y$ under the constraints

$2x - 3y \le 6$; $2x + y \geq 2$; $2x + 3y \le 12$
$x \geq 0$, $y \geq 0$.

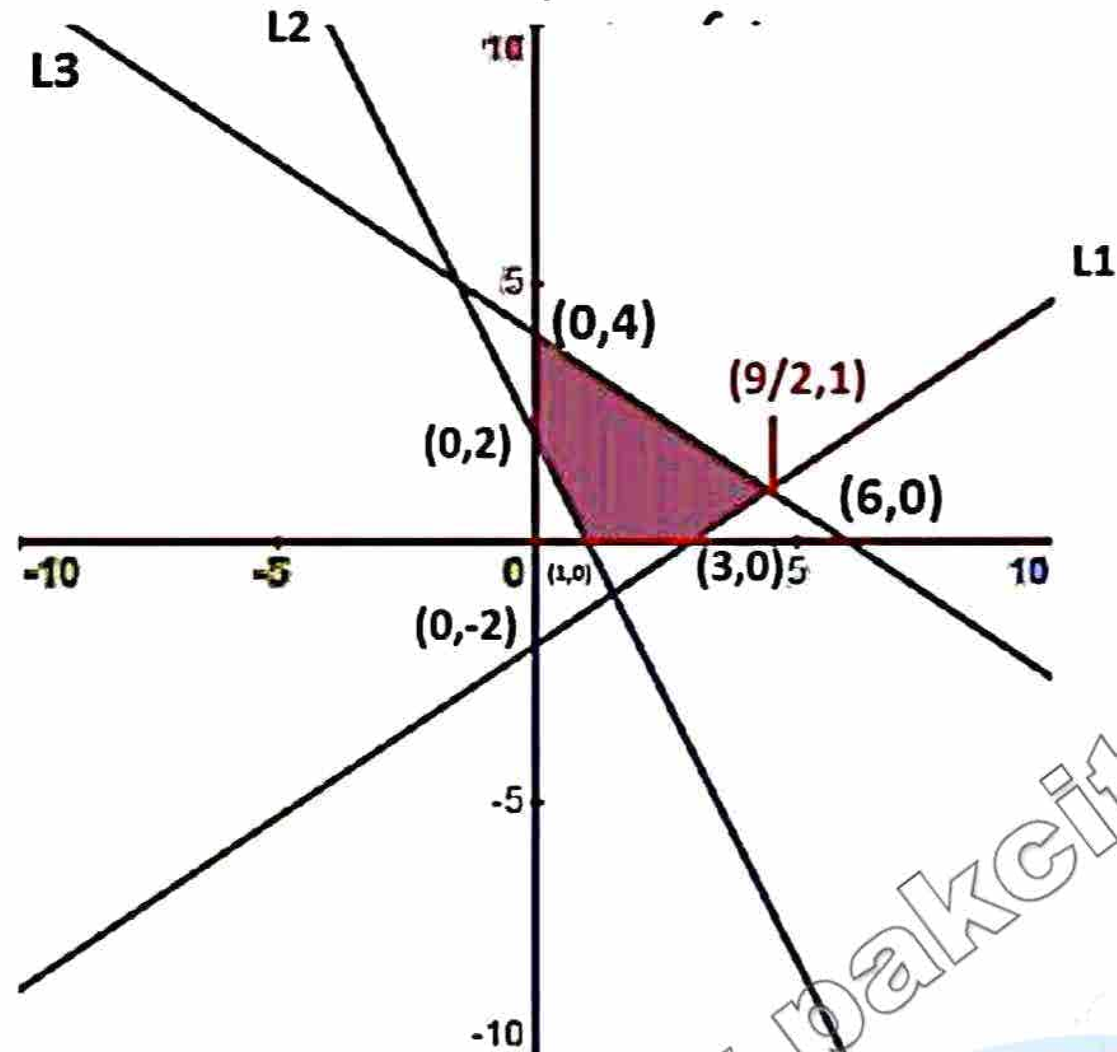
Solution:- $2x - 3y \le 6$ — (i) , $2x + y \geq 2$ — (ii)
$2x + 3y \le 12$ — (iii)

The associated eqs. of (i), (ii) and (iii) are
L1; $2x - 3y = 6$ — (iv) , L2; $2x + y = 2$ — (v) , L3; $2x + 3y = 12$ — (vi)

(iv) → put $x=0, y=-2$ so the pt $(0,-2)$
Put $y=0, x=3$ so the pt $(3,0)$
(v) → Put $x=0, y=2$ so the pt $(0,2)$
Put $y=0, x=1$ so the pt $(1,0)$
(vi) → Put $x=0, y=4$ so the pt $(0,4)$
Put $y=0, x=6$ so the pt $(6,0)$

Test pt $(0,0)$:- we test (i), (ii) and (iii) at $(0,0)$
so (i) → $0 \le 6$ — True, (ii) → $0 \geq 2$ — False
(iii) → $0 \le 12$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i), (ii) and (iii). Also $x \geq 0, y \geq 0$ indicates that the graph of solution set in



Corner point:- we find pt. of intersection of lines L1 and L3. so
$2x-3y=6$ — (i)
$2x+3y=12$ — (ii)

By (i) + (ii) → $4x = 18$ → $x = 9/2$ Put in (i)
$2(9/2) - 3y = 6$ → $-3y = 6-9 = -3$ → $y=1$
so $(9/2, 1)$ is pt. of intersection of lines L1 and L3. Hence corner pts. are $(1,0), (3,0), (9/2, 1)$, $(0,4)$ and $(0,2)$.

Optimal solution:- we find values of $f(x,y) = 4x+5y$ and $\phi(x,y) = 4x+6y$ at corner pts.

Corner pts.	$f(x,y) = 4x+5y$	$\phi(x,y) = 4x+6y$
$(1,0)$	$f(1,0) = 4(1)+5(0) = 4$	$\phi(1,0) = 4(1)+6(0) = 4$
$(3,0)$	$f(3,0) = 4(3)+5(0) = 12$	$\phi(3,0) = 4(3)+6(0) = 12$
$(9/2,1)$	$f(\frac{9}{2},1) = 4(\frac{9}{2})+5(1) = 23$	$\phi(\frac{9}{2},1) = 4(\frac{9}{2})+6(1) = 24$
$(0,4)$	$f(0,4) = 4(0)+5(4) = 20$	$\phi(0,4) = 4(0)+6(4) = 24$
$(0,2)$	$f(0,2) = 4(0)+5(2) = 10$	$\phi(0,2) = 4(0)+6(2) = 12$

from the table, it is clear that $f(x,y)$ has minimum value 4 at $(1,0)$ and maximum value 23 at $(\frac{9}{2},1)$. Similarly, $\phi(x)$ has minimum value 4 at $(1,0)$ and maximum value 24 at $(\frac{9}{2},1)$ and $(0,4)$.

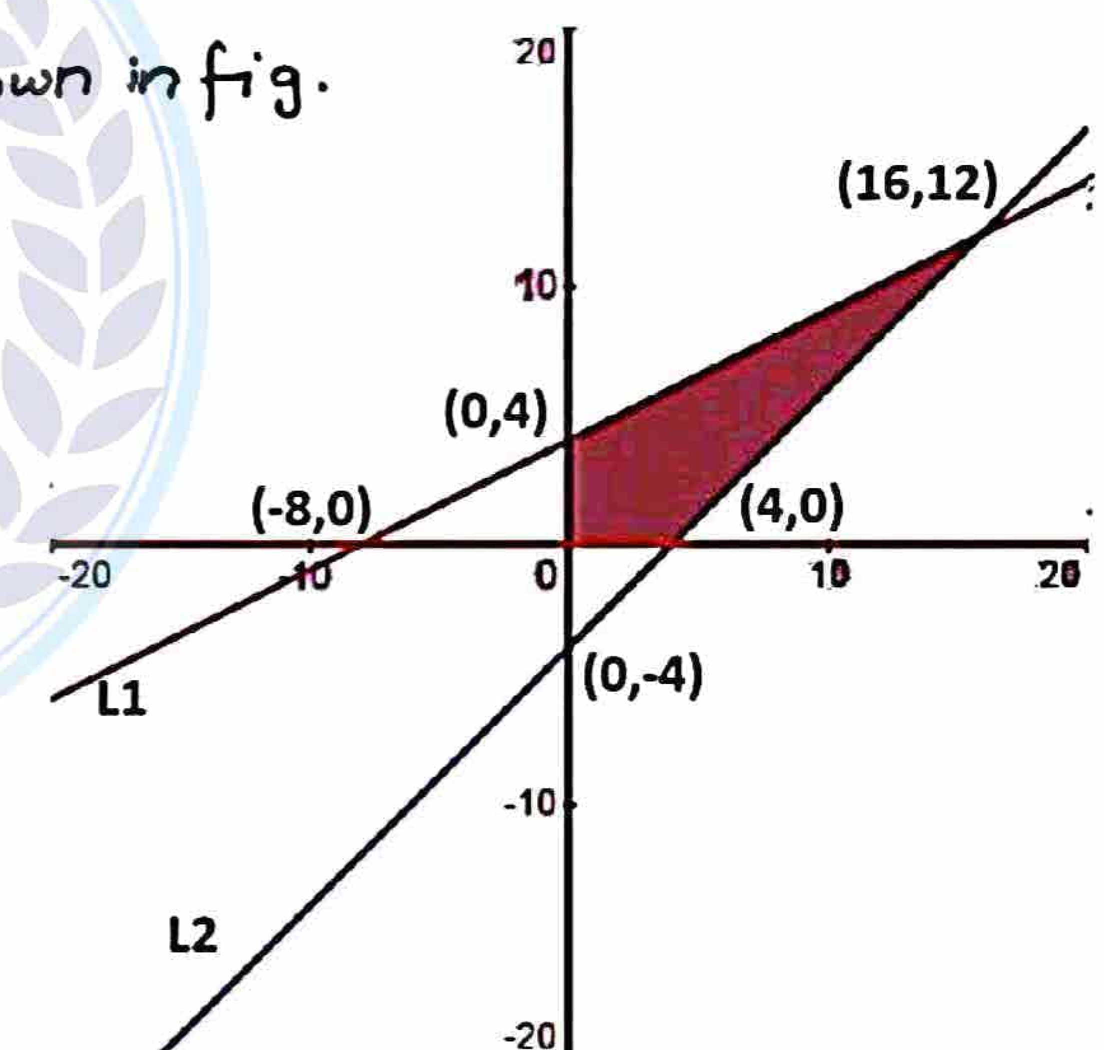
Exercise 5.3

Q1. Maximize $f(x,y) = 2x+5y$ subject to the constraints
$2y - x \le 8$; $x - y \le 4$; $x \geq 0$, $y \geq 0$

Solution:- $-x+2y \le 8$ — (i), $x-y \le 4$ — (ii)
The associated eqs. of (i) and (ii) are
L1; $-x+2y=8$ — (iii), L2; $x-y=4$ — (iv)
(iii) → put $x=0, y=4$ so the pt $(0,4)$
Put $y=0, x=-8$ so the pt $(-8,0)$
(iv) → put $x=0, y=-4$ so the pt $(0,-4)$
Put $y=0, x=4$ so the pt $(4,0)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$
So (i) → $0 \le 8$ — True (ii) → $0 \le 4$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i) and (ii). Also $x \geq 0, y \geq 0$ indicates that graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- we find pt. of intersection of lines L1 and L2 so
$-x+2y = 8$ — (i)
$x - y = 4$ — (ii)

By (i) + (ii) → $y = 12$ put in (ii) → $x = 16$
so $(16,12)$ is the pt. of intersection of lines L1 and L2. Hence corner pts. of feasible region are $(0,0), (4,0), (16,12)$ and $(0,4)$.

Optimal Solution:- We find values of $f(x,y) = 2x+5y$ at corner pts.

$f(0,0) = 2(0)+5(0) = 0$, $f(4,0) = 2(4)+5(0) = 8$

$f(16,12) = 2(16)+5(12) = 92$, $f(0,4) = 2(0)+5(4) = 20$

Thus $f(x,y)$ has maximum value 92 at $(16,12)$

Q2. Maximize $f(x,y) = x+3y$ subject to constraints $2x+5y \le 30$, $5x+4y \le 20$
$x \ge 0, y \ge 0$

Solution:- $2x+5y \le 30$ — (i) , $5x+4y \le 20$ — (ii)

The associated eqs. of (i) and (ii) are

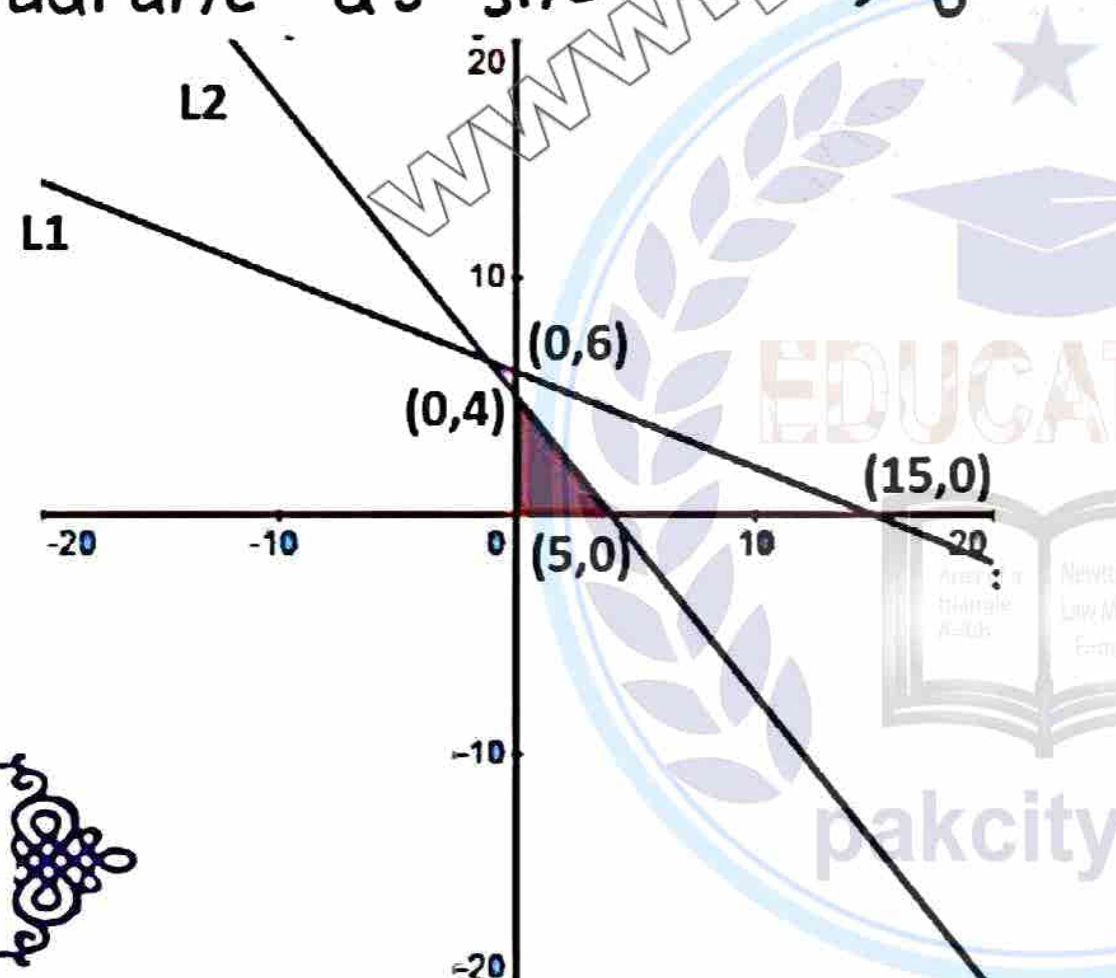
L1; $2x+5y = 30$ — (iii) , L2; $5x+4y = 20$ — (iv)

(iii) → put $x=0, y=6$ so the pt $(0,6)$
put $y=0, x=15$ so the pt $(15,0)$

(iv) → put $x=0, y=5$ so the pt $(0,5)$
put $y=0, x=4$ so the pt $(4,0)$.

Test pt $(0,0)$:- We test (i) and (ii) at $(0,0)$
so (i) → $0 \le 30$ — True , (ii) → $0 \le 20$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i) and (ii). Also $x \ge 0, y \ge 0$ indicates that graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- Corner points of feasible region are $(0,0)$, $(4,0)$ and $(0,5)$.

Optimal Solution:- We find values of $f(x,y) = x+3y$ at corner pts.

$f(0,0) = 0+3(0) = 0$, $f(4,0) = 4+3(0) = 4$

$f(0,5) = 0+3(5) = 15$. so $f(x,y)$ has maximum value 15 at $(0,5)$.

Q3. Maximize $z = 2x+3y$ subject to constraints $3x+4y \le 12$
$2x+y \le 4$; $4x-y \le 4$; $x \ge 0, y \ge 0$

Solution:- $3x+4y \le 12$ — (i) , $2x+y \le 4$ — (ii)
$2x+y \le 4$ — (iii)

The associated eqs. of (i), (ii) and (iii) are

L1; $3x+4y = 12$ — (iv) , L2; $2x+y = 4$ — (v)

L3; $4x-y = 4$ — (vi)

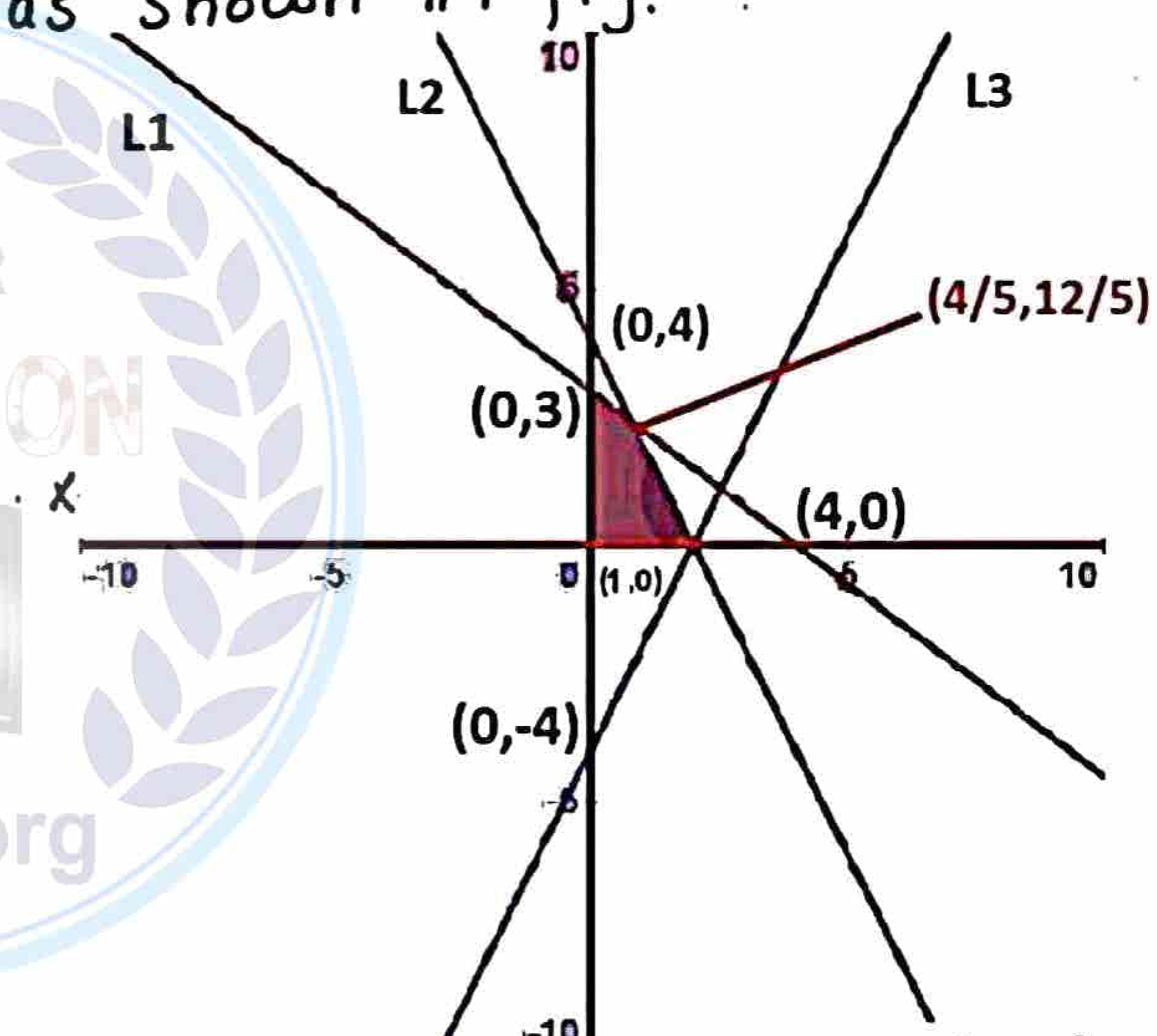
(iv) → put $x=0, y=3$ so the pt $(0,3)$
put $y=0, x=4$ so the pt $(4,0)$

(v) → put $x=0, y=4$ so the pt $(0,4)$
put $y=0, x=2$ so the pt $(2,0)$

(vi) → put $x=0, y=4$ so the pt $(0,-4)$
put $y=0, x=1$ so the pt $(1,0)$

Test pt $(0,0)$:- We test (i), (ii) and (iii) at $(0,0)$. so (i) → $0 \le 12$ — True , (ii) → $0 \le 4$ — True
(iii) → $0 \le 4$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i), (ii) and (iii). Also $x \ge 0, y \ge 0$ shows that graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L1 and L2 so

$3x+4y = 12$ — (i)
$2x+y = 4$ — (ii)

By 4(ii) − (i) → $8x+4y = 16$
$\underline{3x \pm 4y = 12}$
$5x = 4 \rightarrow x = 4/5$ put in (ii)

$2(\frac{4}{5}) + y = 4 \rightarrow y = 4 - \frac{8}{5} = \frac{12}{5}$

so $(\frac{4}{5}, \frac{12}{5})$ is the pt. of intersection of lines L1 and L2. Hence corner pts. of feasible region are $(0,0)$, $(2,0)$, $(\frac{4}{5}, \frac{12}{5})$

and $(0,3)$.

Optimal Solution:- we find values of $z = 2x+3y$ at corner pts.

$(0,0)$, $z = 2(0)+3(0) = 0$, $(2,0)$, $z = 2(2)+3(0) = 4$

$(\frac{4}{5}, \frac{12}{5})$, $z = 2(\frac{4}{5})+3(\frac{12}{5}) = \frac{8}{5}+\frac{36}{5} = \frac{44}{5} = 8.8$

$(0,3)$, $z = 2(0)+3(3) = 9$

so $z = 2x+3y$ has maximum value 9 at $(0,3)$

Q4. Minimize $z = 2x+y$ subject to the constraints: $x+y \geq 3$, $7x+5y \leq 35$, $x \geq 0$, $y \geq 0$

Solution:- $x+y \geq 3$ —(i), $7x+5y \leq 35$ —(ii)

The associated eqs. of (i) and (ii) are

$L_1;\ x+y = 3$ —(iii), $L_2;\ 7x+5y = 35$ —(iv)

(iii)→ put $x=0$, $y=3$ so the pt $(0,3)$

Put $y=0$, $x=3$ so the pt $(3,0)$

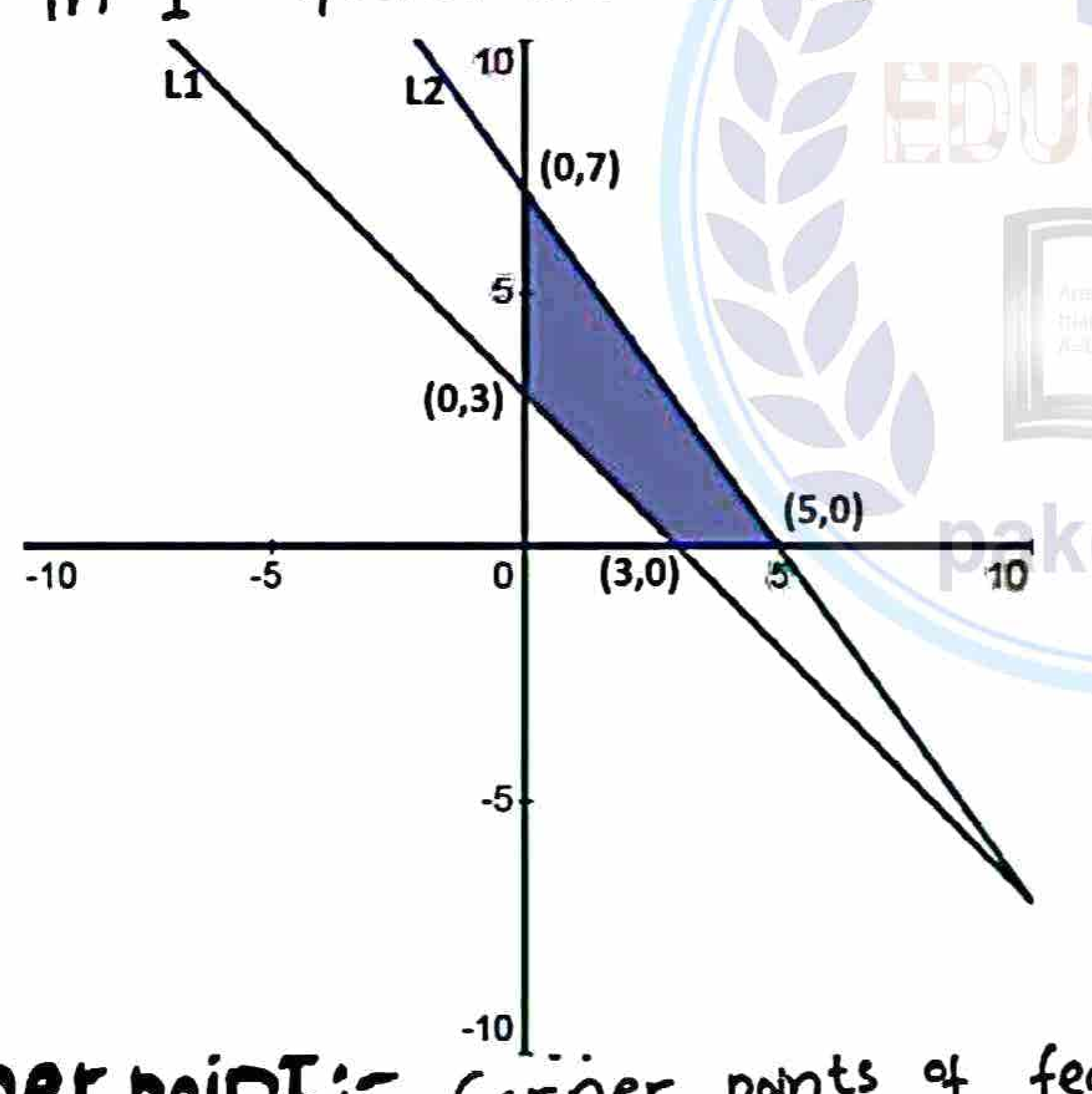
(iv)→ put $x=0$, $y=7$ so the pt $(0,7)$

put $y=0$, $x=5$ so the pt $(5,0)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$.

so (i) → $0 \geq 3$ — False (ii) → $0 \leq 35$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i) and (ii). Also $x \geq 0, y \geq 0$ indicates that the graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- Corner points of feasible region are $(3,0)$, $(5,0)$, $(0,7)$ and $(0,3)$.

Optimal Solution:- we find values of $z = 2x+y$ at corner pts.

$(3,0)$, $z = 2(3)+0 = 6$, $(5,0)$, $z = 2(5)+0 = 10$

$(0,7)$, $z = 2(0)+7 = 7$, $(0,3)$, $z = 2(0)+3 = 3$

so $z = 2x+y$ has minimum value 3 at $(0,3)$.

Q5. Maximize the function defined as; $f(x,y) = 2x+3y$ subject to constraints: $2x+y \leq 8$; $x+2y \leq 14$; $x \geq 0$, $y \geq 0$

Solution:- $2x+y \leq 8$ —(i), $x+2y \leq 14$ —(ii)

The associated eqs. of (i) and (ii) are

$L_1;\ 2x+y = 8$ —(iii), $L_2;\ x+2y = 14$ —(iv)

(iii)→ Put $x=0$, $y=8$ so the pt $(0,8)$

Put $y=0$, $x=4$ so the pt $(4,0)$

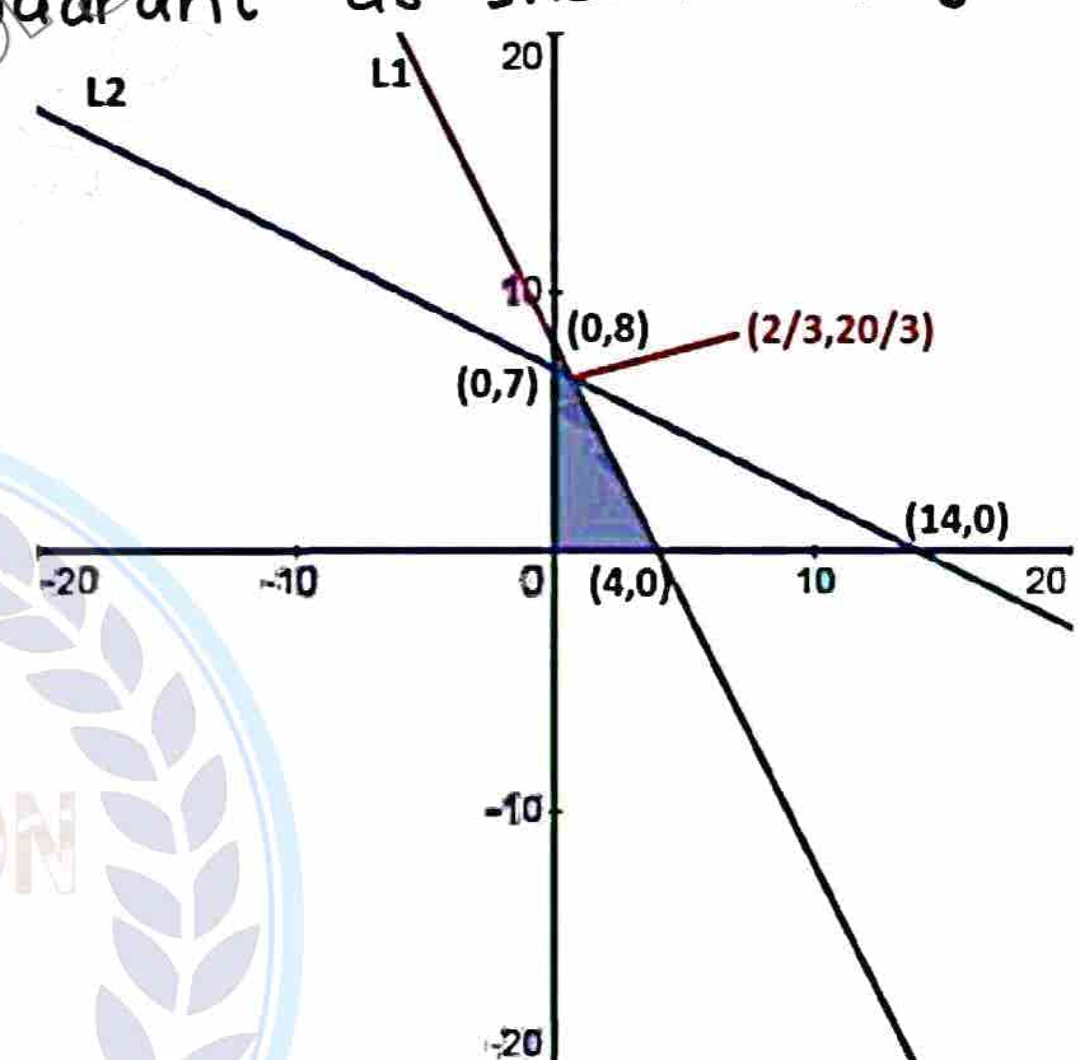
(iv)→ Put $y=0$, $x=14$ so the pt $(14,0)$

Put $x=0$, $y=7$ so the pt $(0,7)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$

(i) → $0 \leq 8$ — True (ii) → $0 \leq 14$ — True

Feasible region:- The feasible region of the given system is the intersection of graphs of (i) and (ii). Also $x \geq 0, y \geq 0$ indicates that the graph of solution set in I^{st} quadrant as shown in fig.



Corner point:- We find pt. of intersection of lines L_1 and L_2. so

$2x+y = 8$ — (i)

$x+2y = 14$ — (ii)

By $2(ii) - (i)$ → $2x+4y = 28$

$\quad\ 2x+y = 8$

$\overline{\quad 3y = 20}$ → $y = \frac{20}{3}$ put in (i)

$2x + \frac{20}{3} = 8$ → $2x = 8 - \frac{20}{3}$ → $2x = \frac{4}{3}$

→ $x = \frac{4}{6} = \frac{2}{3}$

so $(\frac{2}{3}, \frac{20}{3})$ is the pt. of intersection of lines L_1 and L_2. Hence corner pts. of feasible region are $(0,0)$, $(4,0)$, $(\frac{2}{3}, \frac{20}{3})$ and $(0,8)$.

Optimal Solution:- We find values of $f(x,y) = 2x+3y$ at corner pts.

$f(0,0) = 2(0)+3(0) = 0$, $f(4,0) = 2(4)+3(0) = 8$

$f(\frac{2}{3},\frac{20}{3}) = 2(\frac{2}{3})+3(\frac{20}{3}) = \frac{64}{3} = 21.33$

$f(0,7) = 2(0)+3(7) = 21$. So $f(x,y) = 2x+3y$ has maximum value at $(\frac{2}{3},\frac{20}{3})$.

Q6. Minimize $z = 3x+y$; subject to constraints: $3x+5y \geq 15$; $x+6y \geq 9$
$x \geq 0$, $y \geq 0$

Solution:- $3x+5y \geq 15$ — (i)
$x+6y \geq 9$ — (ii)

The associated eqs. of (i) and (ii) are

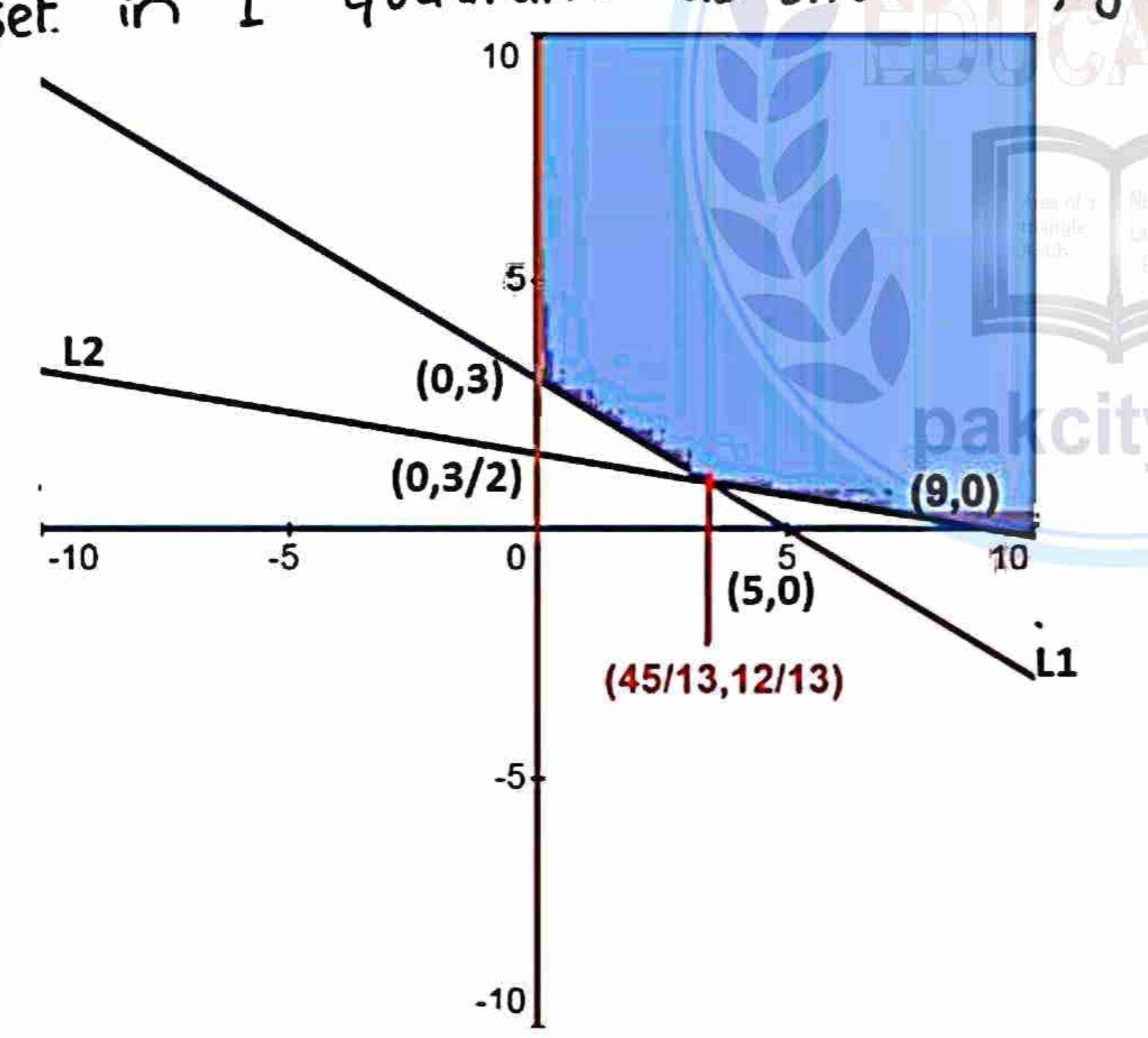
$L1;\ 3x+5y = 15$ — (iii) , $L2;\ x+6y = 9$ — (iv)

(iii) → put $x = 0$, $y = 3$ so the pt $(0,3)$
put $y = 0$, $x = 5$ so the pt $(5,0)$

(iv) → put $x = 0$, $y = \frac{3}{2}$ so the pt $(0,\frac{3}{2})$
put $y = 0$, $x = 9$ so the pt $(9,0)$

Test pt $(0,0)$:- we test (i) and (ii) at $(0,0)$
so (i) → $0 \geq 15$ — False (ii) → $0 \geq 9$ — False

Feasible region:- The feasible region of the given system is intersection of graphs of (i) and (ii). Also $x \geq 0, y \geq 0$ indicates that the graph of solution set in Ist quadrant as shown in fig.



Corner point:-

we find point of intersection of L1 and L2

so
$$3x+5y = 15 \text{ — (i)}$$
$$x+6y = 9 \text{ — (ii)}$$

By (i) − 3(ii) →
$$3x+5y = 15$$
$$\underline{-3x \; \underset{-}{+} \; 18y = \underset{-}{}27}$$
$$-13y = -12$$

→ $y = \frac{12}{13}$ put in (ii)

$$x + 6(\frac{12}{13}) = 9$$

→ $x + \frac{72}{13} = 9$

$x = 9 - \frac{72}{13}$ → $x = \frac{117-72}{13}$

→ $x = \frac{45}{13}$

so $(\frac{45}{13},\frac{12}{13})$ is the pt. of intersection of lines L1 and L2. Hence corner points are $(9,0)$, $(0,3)$ & $(\frac{45}{13},\frac{12}{13})$.

Optimal Solution:- we find values of $z = 3x+y$ at corner pts.

At	$z = 3x+y$
$(0,3)$;	$z = 3(0)+3 = 3$
$(9,0)$;	$z = 3(9)+0 = 27$
$(\frac{45}{13},\frac{12}{13})$;	$z = 3(\frac{45}{13})+\frac{12}{13}$
	$z = \frac{135}{13}+\frac{12}{13} = \frac{147}{13}$

so $z = 3x+y$ has minimum value 3 at $(0,3)$.