

Mathematics (Objective)

(For All Sessions)

(GROUP-I)

Time: 30 Minutes

Marks : 20

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

1.1	Midpoint of $A(2, 0), B(0, 2)$ is:	(A)	$(0, 2)$	(B)	$(2, 0)$	(C)	$(2, 2)$	(D)	<input checked="" type="radio"/> $(1, 1)$
2.	The ____ point satisfies $x + 2y < 6$	(A)	$(4, 1)$	(B)	<input checked="" type="radio"/> $(3, 1)$	(C)	$(1, 3)$	(D)	$(1, 4)$
3.	In a conic, the ratio of the distance from a fixed point to the distance from a fixed line is:	(A)	Focus	(B)	Vertex	(C)	<input checked="" type="radio"/> Eccentricity	(D)	Centre
4.	Standard equation of Parabola is:	(A)	<input checked="" type="radio"/> $y^2 = 4ax$	(B)	$x^2 + y^2 = a^2$	(C)	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	(D)	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
5.	Equation of tangent to circle $x^2 + y^2 = a^2$ at $P(x_1, y_1)$ is:	(A)	<input checked="" type="radio"/> $xx_1 + yy_1 = a^2$	(B)	$xx_1 - yy_1 = a^2$	(C)	$xy_1 + yx_1 = a^2$	(D)	$xy_1 - yx_1 = a^2$
6.	The volume of parallelopiped = ____.	(A)	<input checked="" type="radio"/> $(\underline{u} \times \underline{v}) \cdot \underline{w}$	(B)	$(\underline{u} \times \underline{v}) \times \underline{w}$	(C)	$\underline{u} \times (\underline{v} \times \underline{w})$	(D)	$\underline{u} \times (\underline{u} \times \underline{v})$
7.	The non-zero vectors are perpendicular when:	(A)	$\underline{u} \cdot \underline{v} = 1$	(B)	$ \underline{u} \cdot \underline{v} = 1$	(C)	<input checked="" type="radio"/> $\underline{u} \cdot \underline{v} = 0$	(D)	$\underline{u} \cdot \underline{v} \neq 0$
8.	$\underline{j} \times \underline{k} =$ ____.	(A)	<input checked="" type="radio"/> \underline{i}	(B)	$-\underline{i}$	(C)	0	(D)	\underline{k}
9.	The range of $f(x) = 2 + \sqrt{x-1}$ is:	(A)	$[1, +\infty)$	(B)	<input checked="" type="radio"/> $[2, +\infty)$	(C)	$(1, +\infty)$	(D)	$(2, +\infty)$
10.	The perimeter P of square as a function of its area A:	(A)	$3\sqrt{A}$	(B)	<input checked="" type="radio"/> $4\sqrt{A}$	(C)	\sqrt{A}	(D)	$2\sqrt{A}$
11.	If $f(x) = \frac{1}{x^2}$ then $f'(3) =$ ____.	(A)	$\frac{1}{9}$	(B)	$\frac{2}{3}$	(C)	<input checked="" type="radio"/> $-\frac{2}{27}$	(D)	$\frac{1}{27}$
12.	If $f'(c) = 0$ & $f''(c) > 0$ then C is point of:	(A)	Maxima	(B)	<input checked="" type="radio"/> Minima	(C)	Inflection	(D)	Constant
13.	$\frac{d}{dx}(\log_a x) =$ ____.	(A)	<input checked="" type="radio"/> $\frac{1}{x \ln a}$	(B)	$\frac{\ln a}{x}$	(C)	$\frac{1}{x}$	(D)	$\frac{-1}{x \ln a}$
14.	$\frac{d}{dx}(\cot ax) =$ ____.	(A)	$\csc^2 ax$	(B)	$a \csc^2 ax$	(C)	<input checked="" type="radio"/> $-a \csc^2 ax$	(D)	$-a \csc ax$
15.	$\int \frac{1}{\sqrt{1-x^2}} dx =$ ____.	(A)	<input checked="" type="radio"/> $\sin^{-1} x + c$	(B)	$\cos^{-1} x + c$	(C)	$-\sin^{-1} x + c$	(D)	$-\cos^{-1} x + c$
16.	$\int \frac{1}{x} dx =$ ____.	(A)	<input checked="" type="radio"/> $\ln x + c$	(B)	$\frac{1}{x^2} + c$	(C)	$-\frac{1}{x^2} + c$	(D)	$\frac{1}{x} + c$
17.	The solution of differential equation $\frac{dy}{dx} = -y$ is:	(A)	$y = xe^{-x}$	(B)	<input checked="" type="radio"/> $y = ce^{-x}$	(C)	$y = e^x$	(D)	$y = ce^x$
18.	$\int_0^1 \frac{1}{1+x^2} dx =$ ____.	(A)	<input checked="" type="radio"/> $\frac{\pi}{4}$	(B)	$\frac{2\pi}{3}$	(C)	$\frac{3\pi}{4}$	(D)	π
19.	x - intercept of the line $2x + 5y - 1 = 0$ is:	(A)	2	(B)	3	(C)	<input checked="" type="radio"/> $\frac{1}{2}$	(D)	$\frac{1}{5}$
20.	Slope of y - axis is:	(A)	0	(B)	1	(C)	-1	(D)	<input checked="" type="radio"/> Undefined

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Roll No _____ to be filled in by the candidate

HSSC-(P-II)-A/2024

Marks : 80

(For All Sessions)

(GROUP-I)

SECTION-I

Mathematics (Subjective)

Time: 2:30 hours



(8x2=16)

2. Write short answers of any eight parts from the following:

- If $f(x) = 2x + 1$, then find $f \circ f(x)$.
- Express the area A of a circle as a function of its circumference C .
- Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
- Define continuous function.
- Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t x
- Find $\frac{dy}{dx}$ if $y^2 - xy - x^2 + 4 = 0$
- Differentiate $x^2 \sec 4x$ w.r.t x
- Differentiate $\sin^2 x$ w.r.t $\cos^4 x$
- Find $f'(x)$ if $f(x) = e^x(1 + \ln x)$
- Find y_2 if $y = \ln(x^2 - 9)$
- Prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$
- Determine the interval in which $f(x) = \cos x$ is decreasing; $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3. Write short answers of any eight parts from the following:

(8x2=16)

- Solve the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$
- Find the area between x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$
- Evaluate: $\int_1^e x \ln x \, dx$
- Evaluate the integral $\int \frac{-2x}{\sqrt{4-x^2}} \, dx$
- Evaluate: $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \, dx$
- Evaluate the integral $\int (a + 2x)^{3/2} \, dx$
- Find the approximate change in the volume of a cube if length of its each edge changes from 5 to 5.02.
- Show that the points $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.
- Convert the equation of line $4x + 7y - 2 = 0$ into normal form.
- Find the angle from the line with slope $-\frac{7}{3}$ to the line with slope $\frac{5}{2}$.
- Find the pair of lines represented by $3x^2 + 7xy + 2y^2 = 0$.
- Find the point of intersection of lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$.

4. Write short answers of any nine parts from the following:

(9x2=18)

- Define feasible region.
- Graph the solution set of in-equality $3x + 7y \geq 21$.
- Find equation of circle with ends of diameter at $(-3, 2)$ and $(5, -6)$.
- Write down equation of tangent to the circle $x^2 + y^2 = 25$ at $(5 \cos \theta, 5 \sin \theta)$
- Find focus and vertex of Parabola $x^2 = 4(y - 1)$
- Find equation of ellipse with data Foci $(\pm 3, 0)$ Minor axis of length 10.
- Find center of hyperbola $x^2 - y^2 + 8x - 2y - 10 = 0$

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- viii. Find equation of Normal to $y^2 = 4ax$ at $(at^2, 2at)$.
- ix. Find the sum of vector \overrightarrow{AB} and \overrightarrow{CD} given four points $A(1, -1)$, $B(2, 0)$, $C(-1, 3)$ and $D(-2, 2)$
- x. Find α , so that $|\alpha \underline{i} + (\alpha + 1)\underline{j} + 2\underline{k}| = 3$ xii. If \underline{v} is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$, $\underline{v} \cdot \underline{k} = 0$, find \underline{v}
- xii. Find the area of triangle determined by the points $P(0, 0, 0)$, $Q(2, 3, 2)$ and $R(-1, 1, 4)$
- xiii. Find the value of $2\hat{i} \times 2\hat{j} \cdot \hat{k}$



SECTION-II

Note Attempt any three questions. Each question carries equal marks: (10x3=30)

5. (a) Find the values of m and n , so that given function f is continuous at $x = 3$ when
- $$f(x) = \begin{cases} mx, & \text{if } x < 3 \\ n, & \text{if } x = 3 \\ -2x + 9, & \text{if } x > 3 \end{cases} \quad (05)$$
- (b) Find $\frac{dy}{dx}$, when $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ (05)
6. (a) If $y = (\cos^{-1}x)^2$, prove that $(1-x^2)y_2 - xy_1 - 2 = 0$. (05)
- (b) Evaluate the integral $\int e^x \sin x \cos x \, dx$. (05)
7. (a) Solve the differential equation $y - x \frac{dy}{dx} = 3 \left(1 + x \frac{dy}{dx} \right)$. (05)
- (b) Graph the feasible region and corner points of the inequalities (05)
- $$2x + y \leq 10; \quad x + 4y \leq 12; \quad x + 2y \leq 10;$$
8. (a) Show that the circles: $x^2 + y^2 + 2x - 8 = 0$; $x^2 + y^2 - 6x + 6y - 46 = 0$ touch internally. (05)
- (b) Using vector method, for any triangle ABC , prove that: $c^2 = a^2 + b^2 - 2ab \cos C$. (05)
9. (a) Find the focus, vertex and directrix of the Parabola; $x^2 = 4(y - 1)$ (05)
- (b) Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$ and also find measure of the angle between them. (05)

618-12-A

☆☆	Roll No
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HSSC-(P-II)- A-2024
(For All Sessions)

Paper Code	8	1	9	4
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Mathematics (Objective)

(GROUP-II)

Time: 30 Minutes

Marks : 20

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

1.1	If $r = 0$, the circle is called:	(A)	Unit circle	(B)	Circle	(C)	Ellipse	(D)	<input checked="" type="radio"/> Point circle
2.	$[i \ i \ k] =$	(A)	\underline{i}	(B)	$-\underline{i}$	(C)	1	(D)	<input checked="" type="radio"/> 0
3.	If $\underline{u} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{v} = 4\underline{i} + 2\underline{j} - \underline{k}$ then $\underline{u} \times \underline{u} =$	(A)	u^2	(B)	<input checked="" type="radio"/> 0	(C)	1	(D)	2
4.	If $\underline{u}, \underline{v}$ are two non-zero vectors, then area of parallelogram =	(A)	<input checked="" type="radio"/> $ \underline{u} \times \underline{v} $	(B)	$\frac{1}{2} \underline{u} \times \underline{v} $	(C)	$\frac{1}{6} \underline{u} \times \underline{v} $	(D)	$\frac{1}{2} (\underline{u} \times \underline{v})$
5.	If k is any real number, $\lim_{x \rightarrow a} [kf(x)] =$	(A)	$\lim_{x \rightarrow a} f(x)$	(B)	$\lim_{x \rightarrow a} k$	(C)	<input checked="" type="radio"/> $k \lim_{x \rightarrow a} f(x)$	(D)	$f(x)$
6.	If $(f(x) = x + 3)$ then: $\lim_{x \rightarrow 3} f(x) =$	(A)	<input checked="" type="radio"/> 6	(B)	0	(C)	-3	(D)	3
7.	If $y = e^{f(x)}$ then $\frac{dy}{dx} =$	(A)	$e^{f(x)}$	(B)	$f(x)e^{f(x)}$	(C)	$f(x)e^{f(x)}$	(D)	<input checked="" type="radio"/> $f'(x)e^{f(x)}$
8.	Derivative of $x\sqrt{x^2 + 3}$ w.r. t x is:	(A)	<input checked="" type="radio"/> $\frac{2x^2 + 3}{\sqrt{x^2 + 3}}$	(B)	$\frac{3x}{2\sqrt{x^2 + 3}}$	(C)	$\frac{3x^2 + 3}{x\sqrt{x^2 + 3}}$	(D)	$\frac{3x^2 + 3}{2x\sqrt{x^2 + 3}}$
9.	Derivative of $\tanh(x^2)$ is:	(A)	$2x \operatorname{sech}^2 x$	(B)	$2 \operatorname{sech}^2 x^2$	(C)	<input checked="" type="radio"/> $2x \operatorname{sech}^2 x^2$	(D)	$\operatorname{sech}^2 x^2$
10.	Derivative of " x " w.r. t " x " is:	(A)	x^2	(B)	2	(C)	0	(D)	<input checked="" type="radio"/> 1
11.	In integration, substitution of $\sqrt{4 - x^2}$ is:	(A)	$x = \sin \theta$	(B)	<input checked="" type="radio"/> $x = 2 \sin \theta$	(C)	$x = \sin 2\theta$	(D)	$x = 2 \cos \theta$
12.	$\int \tan x \, dx =$	(A)	$\ln \cos x + c$	(B)	$\frac{1}{\ln \cos x} + c$	(C)	<input checked="" type="radio"/> $-\ln \cos x + c$	(D)	$\sec^2 x + c$
13.	Solution of differential equation: $(e^x + e^{-x}) \frac{dy}{dx} = e^x - e^{-x}$ is:	(A)	$-\ln(e^x + e^{-x}) + c$	(B)	$\ln(e^x - e^{-x}) + c$	(C)	<input checked="" type="radio"/> $\ln(e^x + e^{-x}) + c$	(D)	$\frac{(e^x + e^{-x})^2}{2}$
14.	$\int \sin x \cos x \, dx =$	(A)	<input checked="" type="radio"/> $\frac{\sin^2 x}{2} + c$	(B)	$\frac{\cos^2 x}{2} + c$	(C)	$-\sin x + c$	(D)	$\cos x + c$
15.	The line: $ay + b = 0$ is	(A)	Parallel to y-axis	(B)	<input checked="" type="radio"/> Parallel to x-axis	(C)	Passing through origin	(D)	Lies in Quad. I
16.	The slope of line joining the points $(-2, 4); (5, 11)$ is:	(A)	<input checked="" type="radio"/> 1	(B)	-1	(C)	45°	(D)	-45°
17.	The location of the plane of the point $P(x, y)$ for which $y = 0$ at:	(A)	Origin	(B)	y-axis	(C)	<input checked="" type="radio"/> x-axis	(D)	Ist Quad
18.	The maximum and minimum values occur at:	(A)	Corner point	(B)	Any point	(C)	Convex region	(D)	<input checked="" type="radio"/> Corner points of feasible region
19.	The line intersect the circle at:	(A)	One point	(B)	<input checked="" type="radio"/> Two points	(C)	Infinite points	(D)	More than two points
20.	Diameter of circle: $x^2 + y^2 = 16$ is:	(A)	<input checked="" type="radio"/> 8	(B)	4	(C)	16	(D)	32

619-12-A

Mathematics (Subjective)

(GROUP-II)

SECTION-I

2. Write short answers of any eight parts from the following:

(8×2=16)

- Define even function with example.
- Find $f \circ g(x)$ if $f(x) = 2x + 1$, $g(x) = \frac{3}{x-1}$, $x \neq 1$.
- Evaluate: $\lim_{x \rightarrow 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$.
- Prove that $\sinh 2x = 2 \sinh x \cosh x$.
- Find $\frac{dy}{dx}$ from first principles if $y = \frac{1}{\sqrt{x+a}}$.
- Differentiate w.r.t x ; $\frac{(x^2+1)^2}{x^2-1}$.
- Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$.
- Differentiate w.r.t θ ; $\tan^3 \theta \sec^2 \theta$.
- Find $f'(x)$ if $f(x) = x^3 e^{1/x}$.
- Find y_2 if $y = 2x^5 - 3x^4 + 4x^3 + x - 2$.
- Apply Maclaurin Series expansion to prove that:
 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- Find extreme values for $f(x) = 3x^2$.

3. Write short answers of any eight parts from the following:

(8×2=16)

- Evaluate $\int x\sqrt{x^2-1} dx$
- Use differentials to approximate the value of $(31)^{\frac{1}{5}}$
- Evaluate: $\int \frac{x}{\sqrt{4+x^2}} dx$
- Evaluate the integral $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$
- Evaluate: $\int_1^2 \frac{x}{x^2+2} dx$
- Find the area between x -axis and the curve $y = 4x - x^2$
- Solve the differential equation $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2} (1 + y^2)$
- The points $A(-5, -2)$ and $B(5, -4)$ are ends of a diameter of a circle. Find the centre and radius of circle.
- The coordinates of a point p are $(-6, 9)$. The axes are translated through the point $O(-3, 2)$. Find the coordinates of p referred to the new axes.
- Check whether the origin and the point $p(5, -8)$ lies on the same side or on the opposite sides of the line $3x + 7y + 15 = 0$
- By means of slopes, show that the following points lie on the same line $(-4, 6)$; $(3, 8)$; $(10, 10)$.
- Determine the value of p such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.

4. Write short answers of any nine parts from the following:

(9×2=18)

- Graph the solution set of $3y - 4 \leq 0$ in xy -plane.
- Define convex region.
- Find an equation of circle of radius a and lying in the second quadrant tangent to both the axes.
- Find center and radius of circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$.
- Write down equation of normal to the circle $x^2 + y^2 = 25$ at $(4, 3)$.
- Find vertex and directrix of the parabola $y^2 = -12x$.
- Find the point of intersection of conics $x^2 + y^2 = 8$ and $x^2 - y^2 = 1$.
- Find center and foci of hyperbola $\frac{y^2}{4} - x^2 = 1$.
- Find a vector of magnitude 4 and is parallel to $2\hat{i} - 3\hat{j} + 6\hat{k}$.
- Find direction cosines of \overrightarrow{PQ} where $P = (2, 1, 5)$ and $Q = (1, 3, 1)$.
- Find volume of parallelepiped whose edges are $\underline{u} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\underline{v} = 2\hat{i} - \hat{j} - \hat{k}$ and $\underline{w} = \hat{j} + \hat{k}$
- Find the value of $\left[\begin{matrix} \hat{k} & \hat{i} & \hat{j} \end{matrix} \right]$.
- Find α so that $\underline{u} = \alpha \hat{i} + 2\alpha \hat{j} - \hat{k}$ and $\underline{v} = \hat{i} + \alpha \hat{j} + 3\hat{k}$ are perpendicular.

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SECTION-II

Note Attempt any three questions. Each question carries equal marks: (10x3=30)

5. (a) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$ (b) Differentiate $\cos \sqrt{x}$ from the first principle. (5+5)
6. (a) Show that $y = \frac{mx}{x}$ has maximum value at $x = e$ (b) Evaluate: $\int x^3 \cos x \, dx$ (5+5)
7. (a) Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x \, dx}{\sin x (2 + \sin x)}$ (b) Minimize $z = 2x + y$ subject to constraints (5+5)
 $x + y \geq 3$ $7x + 5y \leq 35$
 $x \geq 0$ $y \geq 0$
8. (a) Find the coordinates of the points of intersection of the line $x + 2y = 6$ with the circle: $x^2 + y^2 - 2x - 2y - 39 = 0$ (5)
(b) If $\underline{a} = 4\underline{i} + 3\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} - \underline{j} + 2\underline{k}$. Find a unit vector perpendicular to both \underline{a} and \underline{b} . Also find the sine of the angle between them. (5)
9. (a) Find the focus, vertex and directrix of the Parabola $x + 8 - y^2 + 2y = 0$ (5)
(b) Find coordinates of the circumcenter of the triangle whose vertices are $A(-2, 3)$, $B(-4, 1)$ and $C(3, 5)$. (5)

620-12-A

Roll No

to be filled in by the candidate

(For All Sessions)

(Group-I)

Time: 30 Minutes

Marks : 20

Mathematics (Objective)

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1 $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = ?$ (A) 1 (B) 2 (C) 0 (D) -2
2. $\cos hx + \sin hx = ?$ (A) e^x (B) e^{-x} (C) e^{2x} (D) $2e^x$
3. $\frac{d}{dx} [\ln(2^x)] = ?$ (A) $\ln 2$ (B) 2^x (C) $\frac{1}{2^x}$ (D) $\frac{\ln 2}{2^x}$
4. $\frac{d}{dx} (\cos hx) = ?$ (A) $-\sin hx$ (B) $\sin hx$ (C) $\operatorname{sech} x$ (D) $\operatorname{cosec} hx$
5. If $f(x) = \sqrt{x}$, then $f'(0) = ?$ (A) 0 (B) 1 (C) Undefined (D) $1/2$
6. $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = ?$ (A) 1 (B) 0 (C) -1 (D) 2
7. $\int x dx = ?$ (A) x (B) $1/x$ (C) x^2 (D) C
8. $\int e^x(x+1) dx = ?$ (A) $xe^x + c$ (B) $e^x + c$ (C) $x + c$ (D) $x^2 + c$
9. $\int_0^{\pi/2} \cos x dx = ?$ (A) 0 (B) -1 (C) 1 (D) 2
10. $\int \frac{\sin 2x}{\sin x} dx = ?$ (A) $2 \cos x + c$ (B) $2 \sin x + c$ (C) $\frac{1}{2} \sin x + c$ (D) $\frac{1}{2} \cos x + c$
11. The slope of a line $x = 5$ is: (A) 0 (B) 1 (C) -1 (D) Infinite
12. Midpoint of (0, -2) and (-2, 0) is: (A) (0, 0) (B) (-1, -1) (C) (-2, -2) (D) (0, -1)
13. Distance between (-1, 2) & (7, 5) is: (A) $\sqrt{73}$ (B) 7 (C) $2\sqrt{73}$ (D) 73
14. The solution of inequality $x + 2y < 6$ is: (A) (1, 4) (B) (1, 3) (C) (1, 1) (D) (1, 5)
15. Equation of Tangent to $x^2 + y^2 = 4$ at (2, 0) is: (A) $x = 1$ (B) $y = 1$ (C) $y = 2$ (D) $x = 2$
16. Slope of tangent to parabola $y^2 = 4ax$ at (a, 2a) is: (A) 2 (B) -1 (C) 1 (D) 3
17. Eccentricity e of a circle is: (A) $e = 0$ (B) $e = 1$ (C) $0 < e < 1$ (D) $e > 1$
18. Radius of a circle $x^2 + y^2 = 2$ is: (A) 2 (B) 1 (C) $1/2$ (D) $\sqrt{2}$
19. If $P = (2, 3)$, $Q = (6, -2)$, then $|PQ|$ is (A) $\sqrt{40}$ (B) $\sqrt{42}$ (C) $\sqrt{41}$ (D) $\sqrt{43}$
20. For a vector $\vec{V} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, then $\cos \beta = ?$ (A) $\frac{3}{7}$ (B) $\frac{2}{7}$ (C) $-\frac{6}{7}$ (D) $-\frac{3}{7}$

Mathematics (Subjective)

(For All Sessions)
(GROUP-I)

Time: 2:30 hours

SECTION-I Rawalpindi Board-2023

(8x2=16)



2. Write short answers of any eight parts from the following:

- Express perimeter P of a square as a function of its area A .
- Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$
- Define even function with example.
- Find derivative by definition $\frac{1}{\sqrt{x}}$
- If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
- Differentiate w.r.t x , $y = x^2 \sec 4x$
- Find $\frac{dy}{dx}$, $xy + y^2 = 2$
- Find $\frac{dy}{dx}$ if $y = x\sqrt{\ln x}$
- Differentiate w.r.t x , $y = \cot^{-1}\left(\frac{x}{a}\right)$
- Apply the Maclaurin series to prove that: $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$
- Define feasible region.
- Graph the solution set of $2x + y \leq 6$.

3. Write short answers of any eight parts from the following:

- Evaluate $\int \tan^2 x dx$.
- Evaluate $\int \frac{(a-b)x}{(x-a)(x-b)} dx$
- Evaluate $\int x \sin x dx$.
- Evaluate $\int_{\pi/6}^{\pi/3} \cos t dt$
- Solve the differential equation $y dx + x dy = 0$
- Evaluate $\int \frac{x^2}{4+x^2} dx$
- Find the areas between the x -axis and the curve $y = x^2 + 1$ from, $x = 1$ to $x = 2$
- Find a unit vector in the direction of $\underline{V} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
- Find direction cosines of $\underline{V} = 4\underline{i} - 5\underline{j}$
- Find α , so that vector $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$, $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$ are perpendicular.
- Find the area of parallelogram whose vertices are: $A(0, 0, 0)$, $B(1, 2, 3)$, $C(2, -1, 1)$, $D(3, 1, 4)$
- A force $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$ is applied at $p(1, -2, 3)$. Find its amount about the point $Q(2, 1, 1)$

4. Write short answers of any nine parts from the following:

- Is $(\sqrt{176}, 7)$ at a distance of 15 units from the origin?
- By means of slopes, show that the point $(-4, 6)$, $(3, 8)$, $(10, 10)$ lie on the same line.
- Find K so that the line joining $A(7, 3)$, $B(k, -6)$ and the line joining $C(-4, 5)$, $D(-6, 4)$ are parallel.
- Find the equation of the line having y -intercept -7 and slope -5 .
- Find the point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- Find equation of lines represented by $2x^2 + 3xy - 5y^2 = 0$
- Find the measure of the angle between the lines represented by $9x^2 + 24xy + 16y^2 = 0$
- Find an equation of the circle with ends of diameter at $(-3, 2)$ and $(5, -6)$
- Show that the line $2x + 3y - 13 = 0$ is tangent to the circle $x^2 + y^2 + 6x - 4y = 0$
- Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$.
- Find focus and directrix of the parabola $x^2 = -16y$
- Find an equation of ellipse if foci $(-3\sqrt{3}, 0)$ and vertices $(\pm 6, 0)$.
- Find equation of hyperbola with given data foci $(0, \pm 9)$, directrices $y = \pm 4$

SECTION-II

Note Attempt any three questions. Each question carries equal marks:

(10x3=30)

- Evaluate: $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$
 - If $y = \tan(2 \tan^{-1} \frac{x}{2})$, then show that $\frac{dy}{dx} = 4 \frac{1+y^2}{4+x^2}$
- Evaluate: $\int \frac{dx}{\frac{1}{2} \sin x + \frac{\sqrt{2}}{2} \cos x}$
 - Find equation of line through intersection of $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercepts on the axes.
- Find the area bounded by the curve $f(x) = x^3 - 2x^2 + 1$ and x -axis in the 1st quadrant.
 - Minimize $Z = 3x + y$ subject to the constraints $3x + 5y \geq 15$, $x + 6y \geq 9$, $x \geq 0$, $y \geq 0$
- If $y = a \cos(\ln x) + b \sin(\ln x)$ prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$
 - Find the coordinates of the points of intersection of the line $2x + y = 5$ and the circle $x^2 + y^2 + 2x - 9 = 0$, also find the length of intercepted chord.
- Find the centre foci, eccentricity and vertices of the ellipse $x^2 + 16x + 4y^2 - 16y + 76 = 0$

Rawalpindi Board-2023



HSSC-(P-II)-A/2023

Paper Code	8	1	9	6
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Roll No. _____ to be filled in by the candidate

(For All Sessions)

(Group-II)

Time: 30 Minutes

Marks : 20

Mathematics (Objective)

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1 Midpoint of A(1,2) & B(3,8) is: (A) (2, 5) (B) (4, 10) (C) (2, 6) (D) (2, 8)
2. (1, -3) is in the solution of _____ (A) $x + y \geq 1$ (B) $x + y \leq 0$ (C) $x + y = 0$ (D) $x - y = 0$
3. Centre of circle $x^2 + y^2 - 6x + 4y + 13 = 0$ (A) (3, 2) (B) (-3, 2) (C) (3, -2) (D) (-3, -2)
4. Focus of parabola $x^2 = 4ay$ is: (A) (-a, 0) (B) (0, -a) (C) (a, 0) (D) (0, a)
5. Eccentricity e for hyperbola is: (A) $e = 1$ (B) $e = 0$ (C) $e < 1$ (D) $e > 0$
6. Length of major axis of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (A) 03 (B) 06 (C) 02 (D) 04
7. Which one is not scalar quantity: (A) Work (B) Time (C) Magnetic field (D) Speed
8. $\begin{bmatrix} k & i & j \end{bmatrix}$ (A) 2 (B) 0 (C) 1 (D) -1
9. $\lim_{x \rightarrow 2} \sqrt{x^3 + 1} - \sqrt{x^2 + 5}$ (A) -1 (B) 0 (C) 2 (D) -2
10. Area of circle of unit radius is: (A) π (B) 2π (C) π^2 (D) $2\pi^2$
11. $\frac{d}{dx}(3^x) =$ _____ (A) $3^{x \ln 3}$ (B) $3^x \ln 2$ (C) $3^x \ln 3$ (D) $x 3^{x-1}$
12. Lagrange used _____ notation for derivative. (A) $D f(x)$ (B) $f'(x)$ (C) $\frac{d}{dx} f(x)$ (D) $\dot{f}(x)$
13. $\frac{d}{dx} \cos 7x =$ _____ (A) $7 \sin 7x$ (B) $-7 \sin 7x$ (C) $7 \cos 7x$ (D) $-7 \cos 7x$
14. Minimum value of function $f(x) = x^2 + 2x - 3$ is at $x =$ _____ (A) -3 (B) -2 (C) 0 (D) -1
15. $\int \frac{1}{1+x^2} dx =$ _____ (A) $\sin^{-1} x + c$ (B) $\cos^{-1} x + c$ (C) $\tan^{-1} x + c$ (D) $\cot^{-1} x + c$
16. $\int \frac{1}{x^2} dx =$ _____ (A) $-\frac{1}{x} + c$ (B) $\frac{1}{x} + c$ (C) $\frac{2}{x} + c$ (D) $-\frac{2}{x} + c$
17. Solution of $\frac{dy}{dx} = 1$ is _____ (A) $y = x^2 + c$ (B) $y = e^x + c$ (C) $y = \ln x + c$ (D) $y = x + c$
18. $\int_0^1 3x^2 dx =$ _____ (A) 3 (B) 1 (C) 2 (D) 0
19. Equation of line through origin with slope 2: (A) $2x - y = 0$ (B) $2x + y = 0$ (C) $x + 2y = 0$ (D) $x - 2y = 0$
20. Slope of line parallel to y-axis: (A) -1 (B) 0 (C) ∞ (D) 1

SECTION-I

Rawalpindi Board-2023

(8x2=16)

Write short answers of any eight parts from the following:

- Express perimeter P of a square as a function of its area A .
- If $f(x) = (-x + 9)^3$, find $f^{-1}(x)$
- Find $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- Differentiate w.r.t "x" $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$
- If $y = \sqrt{x + \sqrt{x}}$ find $\frac{dy}{dx}$
- Find $\frac{dy}{dx}$ if $x = y \sin y$
- Find $f'(x)$ if $f(x) = x^3 \cdot e^{1/x}$
- If $y = x^2 \cdot \ln\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$
- If $y = \sin h^{-1}\left(\frac{x}{2}\right)$, Find $\frac{dy}{dx}$
- Apply the Maclaurin series to prove that: $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$
- Graph the solution set of linear inequality in xy -plane, $2x + y \leq 6$
- What is a feasible solution?

3. Write short answers of any eight parts from the following:

(8x2=16)

- Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ for $x^2 + 2y^2 = 16$
- Evaluate: $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$
- Evaluate: $\int \frac{x+2}{\sqrt{x+3}} dx$
- Evaluate: $\int \tan^{-1} x dx$
- Evaluate: $\int \frac{5x+8}{(x+3)(2x-1)} dx$
- Evaluate: $\int_{-2}^0 \frac{1}{(2x-1)^2} dx$

- Solve the differential equation $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
- Find sum of \overline{AB} and \overline{CD} where $A(1, -1)$, $B(2, 0)$, $C(-1, 3)$ and $D(-2, 2)$
- Find direction Cosines of vector $\underline{V} = 3\underline{i} - \underline{j} + 2\underline{k}$
- Find α so that $\underline{U} = 2\alpha \underline{i} + \underline{j} - \underline{k}$ and $\underline{V} = \underline{i} + \alpha \underline{j} + 4\underline{k}$ are perpendicular.
- Compute $\underline{a} \times \underline{b}$ and $\underline{b} \times \underline{a}$ for $\underline{a} = \underline{i} + \underline{j}$, $\underline{b} = \underline{j} - \underline{k}$
- Find volume of parallelepiped determined by $\underline{U} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{V} = \underline{i} - 2\underline{j} + 3\underline{k}$ and $\underline{W} = \underline{i} - 7\underline{j} - 4\underline{k}$

4. Write short answers of any nine parts from the following:

(9x2=18)

- The point $C(-5, 3)$ is the center of the circle and $P(7, 2)$ lies on the circle. What is the radius of the circle.
- Show that the points $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.
- The points $P(-2, 6)$ and $Q(-3, 2)$ are given in xy -coordinate system. Find the XY -Coordinate of P referred to the translated axes OX' and OY' .
- Find an equation of the line through $(-5, -3)$ and $(9, -1)$.
- Convert $4x + 7y - 2 = 0$ in slope-intercept form.
- Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$
- Find the point of intersection of the lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$
- Find center and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- Find focus and vertex of parabola $y^2 = -12x$
- Find foci of an ellipse $9x^2 + y^2 = 18$
- Find eccentricity of hyperbola, $\frac{y^2}{4} - x^2 = 1$
- Write parametric equations of hyperbola.
- Write down equation of tangent to the circle $x^2 + y^2 = 25$ at $(4, 3)$.

SECTION-II

(10x3=30)

Note Attempt any three questions. Each question carries equal marks:

- (a) Evaluate: $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$ (b) Find $\frac{dy}{dx}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$.
- (a) Evaluate: $\int \frac{x}{x^4 + 2x^2 + 5} dx$ (b) Find equation of the line through $(5, -8)$ and perpendicular to the join of $A(-15, -8)$ and $B(10, 7)$.
- (a) Solve the differential equation $(y - x \frac{dy}{dx}) = 2(y^2 + \frac{dy}{dx})$ (b) Graph the feasible region of the following system of linear inequalities and find the corner points.
 $2x + y \leq 10$, $x + 4y \leq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$
- (a) Show that $y = \frac{\ln x}{x}$ has maximum value at $x = e$. (b) Write an equation of the circle that passes through the given points $A(4, 5)$, $B(-4, -3)$, $C(8, -3)$
- (a) Find the focus, vertex and directrix of the parabola $x^2 - 4x - 8y + 4 = 0$

Mathematics (Objective Type)

(For all Sessions)

Paper Code 8 1 9 4

Time: 30 Minutes

Marks: 20

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.

1.1. $\int \sec x \, dx =$ _____

(A) $\ln |\sec x - \tan x| + c$

(C) $\ln |\sec x + \operatorname{cosec} x| + c$

(B) $\ln |\sec x + \cot x| + c$

(D) $\ln |\sec x + \tan x| + c$

2. $\int_0^1 |x| \, dx =$ _____

(A) 1

(B) 2

(C) 0

(D) $\frac{1}{2}$

3. Solve $\frac{1}{y} dy = \frac{1}{x} dx$

(A) $y = xc$

(B) $y = -xc$

(C) $y = x^2 + c$

(D) $xy = c$

4. Distance between A(-1, 2) and C(2, -6) is _____

(A) $\sqrt{73}$

(B) $\sqrt{70}$

(C) 7

(D) 8

5. If $m_1 = m_2$ then lines are _____

(A) perpendicular

(B) not parallel

(C) parallel

(D) neither parallel nor perpendicular

6. Slope of $12x + 35y - 7 = 0$ is _____

(A) $\frac{12}{35}$

(B) $-\frac{12}{35}$

(C) $\frac{1}{35}$

(D) 12

7. Normal form of $x + y = 1$ is _____

(A) $x \cos \frac{\pi}{3} + y \sin \frac{\pi}{3} = \frac{1}{\sqrt{2}}$

(B) $x \cos \frac{\pi}{2} + y \sin \frac{\pi}{2} = 1$

(C) $x \cos \frac{\pi}{4} + y \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

(D) $x + y = 2$

8. If $P(x, y) = 40x + 50y$ then $P(1, -1) =$ _____

(A) 10

(B) 40

(C) 50

(D) -10

9. Centre of $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ is _____

(A) (-12, -18)

(B) $\left(-\frac{12}{5}, -\frac{18}{5}\right)$

(C) (12, 18)

(D) (-12, 18)

10. Axis of $y^2 = -4ax$ is _____

(A) $y = 0$

(B) $y = a$

(C) $x = 0$

(D) $x = a$

11. Vertices of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is _____

(A) $(\pm b, 0)$

(B) (a, b)

(C) $(\pm a, 0)$

(D) $(-a, -b)$

12. Scalar triple product of coplanar vectors is _____

(A) 1

(B) 0

(C) 2

(D) -1

13. $2\mathbf{i} \times 2\mathbf{j} \cdot 2\mathbf{k} =$ _____

(A) 4

(B) 2

(C) 8

(D) 16

14. Which one is even function

(A) $\sin x$

(B) $\cos x$

(C) $\tan x$

(D) x^{101}

15. If $f(x) = \sqrt{x^2 - 9}$; then range of $f(x)$ is _____

(A) $(0, -\infty)$

(B) $(-\infty, \infty)$

(C) $(-5, 5)$

(D) $(0, +\infty)$

16. $\frac{d}{dx} 2^x =$ _____

(A) $2^x \ln 2$

(B) $2^x \ln e$

(C) $2^x \ln 4$

(D) $x 2^{x-1}$

17. Leibniz used _____ notation for derivative.

(A) $f'(x)$

(B) $f'(x)$

(C) $D f(x)$

(D) $\frac{dy}{dx}$

18. $\frac{d}{dx} (\operatorname{cosec} 7x) =$ _____

(A) $\operatorname{cosec} 7x \cot 7x$

(B) $-\operatorname{cosec} x \cot x$

(C) $-7 \operatorname{cosec} 7x \cot 7x$

(D) $\operatorname{cosec} 7x \tan 7x$

19. Which one is decreasing function

(A) $2 - 4x$

(B) $4x - 2$

(C) $4x$

(D) $4x + 5$

20. $d(xy) =$ _____

(A) $x dx + y dy$

(B) $(x + y) dx$

(C) $x dy + y dx$

(D) $x dy - y dx$

MATHEMATICS (Essay Type)

(For All Sessions)

Time: 2:30 Hours

SUBJECTIVE

Marks: 80

SECTION - I

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

i- Search the domain and range from the real numbers of $g(x) = \sqrt{x^2 - 4}$ ii- The real valued functions f and g are defined below. Find (a) $f^2(x)$ (b) $g^2(x)$,

$$f(x) = \frac{1}{\sqrt{x-1}}; \quad x \neq 1, \quad g(x) = (x^2 + 1)^2$$

iii- Evaluate $\lim_{x \rightarrow \infty} \frac{5x^4 - 10x^2 + 1}{-3x^3 + 10x^2 + 50}$ iv- Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$

v- Give any example and sketch graphically discontinuous function.

vi- Differentiate w.r. to 'x'; $\frac{(1 + \sqrt{x})(x - x^{3/2})}{\sqrt{x}}$ vii- Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{a^2 + x^2}{a^2 - x^2}}$ viii- Find the derivative w.r.t. variable involved $\cos \sqrt{x} + \sqrt{\sin x}$ ix- Find $f'(x)$ if $f(x) = \ln(\sqrt{e^{2x} + e^{-2x}})$ x- Produce y_2 from $y = e^{ax} \sin bx$ xi- Determine the intervals in which f is increasing or decreasing;

$$f(x) = \cos x \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

xii- The perimeter of a triangle is 16 centimeters. If one side is of length 6 cm, what are lengths of the other sides for maximum area of the triangle?

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

i- Use differential find $\frac{dy}{dx}$; $x^4 + y^2 = xy^2$ ii- Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$ iii- Evaluate $\int \sec x dx$ iv- Evaluate $\int \sin^{-1} x dx$ v- Evaluate $\int e^x \left(\frac{1}{x} + \ln x \right) dx$ vi- Evaluate $\int \frac{5x + 8}{(x+3)(2x-1)} dx$ vii- Evaluate $\int_1^2 \frac{x}{x^2 + 2} dx$ viii- Find the area between the x-axis and the curve $y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{3}$

ix- Show that the points A (-1, 2); B(7, 5) and C(2, -6) are vertices of a right triangle.

x- Find an equation of vertical line through (-5, 3)

xi- Convert $15y - 8x + 3 = 0$ in slope-intercept form.xii- Find the lines represented by; $x^2 - 2xy \sec \alpha + y^2 = 0$

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- i- Indicate the solution set of inequality $3x - 2y \geq 6$
- ii- What is objective function?
- iii- Write an equation of circle with centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
- iv- Check the position of the point (5, 6) with respect to the circle $x^2 + y^2 = 81$
- v- Find an equation of parabola with focus (-3, 1) and directrix $x = 3$
- vi- Determine the equation of ellipse having foci $(\pm 3, 0)$ and minor axis of length 10.
- vii- Calculate the eccentricity of $\frac{y^2}{16} - \frac{x^2}{49} = 1$
- viii- Find an equation of the normal line to $y^2 = 4ax$ at $(at^2, 2at)$
- ix- If O is origin and $\vec{OP} = \vec{AB}$, find the point P when A and B are (-3, 7) and (1, 0) respectively
- x- Write the direction cosines of $\vec{u} = 2\vec{i} + 3\vec{j} + 4\vec{k}$
- xi- Prove that in any triangle ABC, $a^2 = b^2 + c^2 - 2bc \cos A$
- xii- If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
- xiii- A force $\vec{F} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ is applied at a point (1, -1, 2). Find the moment of \vec{F} about the point (2, -1, 3)

SECTION - II

Note: Attempt any three questions from the following.

10 x 3 = 30

- 5- (a) Show that $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$
- (b) Show that $y = x^x$ has maximum value at $x = \frac{1}{e}$
- 6- (a) Integrate $\int \frac{4 + 7x}{(1+x)^2(2+3x)} dx$
- (b) Find the point which is equidistant from the point A(5, 3), B(-2, 2) and C(4, 2). What is radius of circumcircle of triangle ABC.
- 7- (a) Find $\int_{\pi/6}^{\pi/4} \cos^2 \theta \cot^2 \theta d\theta$
- (b) Minimize $Z = 2x + y$ subject to constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$
- 8- (a) Find the area of the triangular region. Whose vertices are A(5, 3), B(-2, 2), C(4, 2)
- (b) Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$
- 9- (a) Find equations of the common tangents to the two conics $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and $\frac{x^2}{25} + \frac{y^2}{9} = 1$
- (b) Use vectors, prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long.

Rawalpindi Board-2021

Inter-(Part-II) -A-2021

Roll No. _____ to be filled in by the candidate.

(For all sessions)

Paper Code

8

1

9

1

Mathematics (Objective Type)

Time: 30 Minutes

Marks:20

NOTE: Write answers to the questions on objective answer sheet provided. Four possible answers A,B,C & D to each question are given. Which answer you consider correct, fill the corresponding circle A,B,C or D given in front of each question with Marker or pen ink on the answer sheet provided.

1-1. If $g(x) = \frac{1}{x^2}$, $x \neq 0$ then $g \circ g(x)$ equals.

(A) x

(B) x^2

(C) x^4

(D) x^3

2. $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$ equals.

(A) zero

(B) 1

(C) 2

(D) 3

3. The derivative of \sqrt{x} at $x = 1$ is:

(A) $\frac{1}{2}$

(B) 2

(C) 1

(D) $-\frac{1}{2}$

4. $\frac{d}{dx} \left[\frac{1}{g(x)} \right]$ equals.

(A) $\frac{1}{g^2(x)}$

(B) $\frac{-g'(x)}{(g(x))^2}$

(C) $-g(x)$

(D) $\frac{1}{g(x)}$

5. If $y = 5e^x$ then y_3 equals.

(A) $25e^x$

(B) $15e^x$

(C) $15e^x$

(D) $5e^x$

6. If $f(x+h) = \cos(x+h)$ then $f'(x)$ equals.

(A) $\cos x$

(B) $-\cos x$

(C) $-\sin x$

(D) $\sin x$

7. Inverse of $\int \dots dx$ is:

(A) $\frac{d}{dy}$

(B) $\frac{d}{dx}$

(C) $\frac{dy}{dx}$

(D) $\frac{dx}{dy}$

8. $\int_a^b f(x) dx$ equals:

(A) $-\int_b^a f(x) dx$

(B) $\int_{-b}^a f(x) dx$

(C) $\int_b^{-a} f(x) dx$

(D) $\int_a^{-b} f(x) dx$

9. The general solution of $\frac{dy}{dx} = \frac{-y}{x}$ is:

(A) $xy = c$

(B) $x^2 y^2 = c$

(C) $\frac{x}{y} = c$

(D) $\frac{y}{x} = c$

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10. $\int e^{-x} (\cos x - \sin x) dx$ equals:
 (A) $-\bar{e}^x \sin x + c$ (B) $\bar{e}^x \cos x + c$ (C) $\bar{e}^x + c$ (D) $\bar{e}^x \sin x + c$
11. The distance of point (3,7) from x-axis is:
 (A) 3 (B) 7 (C) -3 (D) -7
12. Slope of Y-axis is:
 (A) zero (B) 1 (C) 2 (D) undefined
13. Equation of horizontal line through (7,-9) is:
 (A) $y = -9$ (B) $y = 7$ (C) $x = -9$ (D) $x = 7$
14. (0,2) is solution of inequality.
 (A) $3x + 5y > 7$ (B) $3x + 5y < 7$ (C) $x < 0$ (D) $x > 0$
15. Centre of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:
 (A) (g, f) (B) $(-g, -f)$ (C) $(0, 0)$ (D) $(-g, -f)$
16. Equation of Latus rectum of parabola $x^2 = 4ay$ is:
 (A) $y = -a$ (B) $y = a$ (C) $x = -a$ (D) $x = a$
17. Vertices of $\frac{x^2}{16} - \frac{y^2}{25} = 1$ are:
 (A) $(0, \pm 4)$ (B) $(0, \pm 5)$ (C) $(\pm 4, 0)$ (D) $(\pm 5, 0)$
18. The non zero vectors \underline{a} and \underline{b} are parallel if $\underline{a} \times \underline{b}$ is:
 (A) zero (B) 1 (C) 2 (D) 3
19. $\cos \theta$ equals:
 (A) $\underline{a} \cdot \underline{b}$ (B) $\underline{a} \times \underline{b}$ (C) $|\underline{a} \times \underline{b}|$ (D) $\hat{\underline{a}} \cdot \hat{\underline{b}}$
20. If any two vectors of scalar triple product are equal then its value is:
 (A) -1 (B) zero (C) 1 (D) 2

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Inter - (Part-II) -A-2021

Roll No. _____ to be filled in by the candidate.

(For all sessions)

Mathematics (Essay Type)

Time: 2:30 Hours



Section -I

Marks: 80

2. Write short answers of any eight parts from the following.

2x8=16

- If $f(x) = x^2 - x$, find (a). $f(-2)$ (b). $f(x-1)$
- Find $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 + x - 6}$.
- Find $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$
- Differentiate w.r.t "x". $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$.
- Find $\frac{dy}{dx}$ if $3x + 4y + 7 = 0$
- Differentiate w.r.t "x" $\cot^{-1}\left(\frac{x}{a}\right)$.
- If $y = x^2 \cdot e^x$, then find $\frac{d^2y}{dx^2}$.
- Apply Maclaurin series, Prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- If $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2}$, then find (a). $(f \circ g)(x)$ (b). $(g \circ f)(x)$.
- Find the intervals in which $f(x)$ is increasing or decreasing $f(x) = \cos x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3. Write short answers of any eight parts from the following.

2x8=16

- Using differential find $\frac{dy}{dx}$, if $x^2 + 2y^2 = 16$.
- Evaluate $\int x\sqrt{x^2-1} dx$.
- Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$.
- Evaluate $\int \sin^2 x dx$.
- Evaluate $\int \frac{ax+b}{ax^2+2bx+c} dx$.
- Evaluate $\int e^{3x} \left(\frac{3 \sin x - \cos x}{\sin^2 x} \right) dx$.
- Solve $\frac{dy}{dx} = \frac{y^2+1}{e^x}$.
- Find an equation of the vertical line through (-5,3).
- Find an equation of the line through (-5,-3), (9,-1)
- Convert $4x + 7y - 2 = 0$ in normal form.
- Find the area below the curve $y = 3\sqrt{x}$ and above the x-axis between $x = 1$ and $x = 4$.
- Find the mid point of the line segment joining the points A(3,1), B(-2,-4).

4. Write short answers of any nine parts from the following.

2x9=18

- Graph the solution set by shading of inequality $5x - 4y \leq 20$.
- Find equation of circle with centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$.
- Write equation of tangent to the circle $3x^2 + 3y^2 + 5x - 13y + 2 = 0$ at $\left(1, \frac{10}{3}\right)$.

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- iv. Find vertex of $x^2 - 4x - 8y + 4 = 0$.
- v. Find point of intersection of conics $3x^2 - 4y^2 = 12$ and $3y^2 - 2x^2 = 7$
- vi. Find equation of parabola whose focus is $F(-3,4)$ and directrix is $3x - 4y + 5 = 0$
- vii. Find the unit vector in the same direction of vector $\underline{V} = [3, -4]$.
- viii. If $\overline{AB} = \overline{CD}$ find the co-ordinate of the point A when points B, C, D are (1,2), (-2,5) and (4,11) respectively
- ix. Find $|3\underline{v} + \underline{w}|$ if $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$, $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$.
- x. Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$.
- xi. Compute $\underline{b} \times \underline{a}$ if $\underline{b} = \underline{i} - \underline{j} + \underline{k}$, $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$.
- xii. Find the work done if the point at which the constant force $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object, moves from $p_1(3,1,-2)$ to $p_2(2,4,6)$.
- xiii. If $\underline{a} + \underline{b} + \underline{c} = 0$ then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$.

Section -II

Note: Attempt any three questions from the following.

10x3=30

5. (a) If $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$, then show $f(x)$ is continuous at $x = 1$.

(b) If $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$, then find $\frac{dy}{dx}$.

6. (a) Find the approximate increase in the volume of a cube of the length of its each edge changes from 5 to 5.02.

- (b) Determine the value of P such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.

7. (a) Evaluate $\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$.

- (b) Minimize $z = 2x + y$ subject to the constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$.

8. (a) Write equations of two tangents from (2,3) to the circle $x^2 + y^2 = 9$

- (b) Prove by vector method $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

9. (a) Show that $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots$

- (b) Show that an equation of the parabola with focus at $(a \cos \alpha, a \sin \alpha)$ and

directrix $x \cos \alpha + y \sin \alpha + a = 0$ is $(x \sin \alpha - y \cos \alpha)^2 = 4a(x \cos \alpha + y \sin \alpha)$



622-12-A-



Roll No. _____ to be filled in by the candidate.

(For all sessions)

Paper Code

8

1

9

1

Mathematics (Objective Type)

Time: 30 Minutes

Marks: 20

NOTE: Write answers to the questions on objective answer sheet provided. Four possible answers A,B,C & D to each question are given. Which answer you consider correct, fill the corresponding circle A,B,C or D given in front of each question with Marker or pen ink on the answer sheet provided.

 1-1. The domain of $g(x) = 2x - 5$ is:

(A) IR

(B) the set of positive No.

(C) The set of negative real No.

(D) The set of non-negative real No.

2. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\frac{n}{2}} =$

(A) e (B) e^2 (C) $e^{\frac{1}{2}}$ (D) e^3

3. $\frac{d}{dx}(x-5)(3-x) =$

(A) $2x+8$ (B) $-2x+8$ (C) $2x-8$ (D) $x+8$

4. If $3x+4y+7=0$, then $\frac{dy}{dx} =$

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $-\frac{4}{3}$ (D) $-\frac{3}{4}$

5. $\frac{d}{dx}(\sec x) =$

(A) $\sec x \tan x$ (B) $\sec x$ (C) $\csc x$ (D) $-\sec x \tan x$

6. If $f(x) = \sin x$, then $f'(0) =$

(A) 0

(B) 1

(C) -1

(D) 2

 7. Differential of y is denoted by:
(A) dy' (B) $\frac{dy}{dx}$ (C) dy (D) dx

8. $\int \frac{1}{1+x^2} e^{\tan^{-1} x} dx =$

(A) $e^{\sec^{-1} x} + c$ (B) $e^{\tan^{-1} x} + c$ (C) $e^{-\tan^{-1} x} + c$ (D) $e^{-\sec^{-1} x} + c$

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9. $\int_1^e \ln x \, dx =$
- (A) -1 (B) 0 (C) 1 (D) e
10. The order of differential equation $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 3x = 0$ is:
- (A) 2 (B) 1 (C) 0 (D) 3
11. If a line " ℓ " is parallel to x -axis, then inclination =
- (A) 90° (B) 0° (C) 30° (D) 45°
12. If a line " ℓ " intersect x -axis at $(a, 0)$, then " a " is called _____ of line " ℓ ".
- (A) y-intercept (B) x-intercept (C) slope (D) inclination
13. $y = mx + c$ is _____ form of equation of line:
- (A) point slope (B) intercept (C) normal (D) slope intercept
14. An equation of line bisecting I and III quadrant is:
- (A) $x = y$ (B) $x = -y$ (C) $x + 2y = 0$ (D) $x - 2y = 0$
15. $x = 0$ is the solution of the inequality.
- (A) $2x + 1 > 0$ (B) $2x + 1 < 0$ (C) $2x + 1 \leq 0$ (D) $2x - 1 < 0$
16. The centre of circle $(x + 1)^2 + (y - 2)^2 = 26$ is:
- (A) (1, 2) (B) (-1, 2) (C) (-1, -2) (D) (1, -2)
17. The equation of directrix of the parabola $x^2 = 4ay$ is:
- (A) $x = a$ (B) $x = -a$ (C) $y = -a$ (D) $y = a$
18. The centre of Ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 16$ is:
- (A) (-4, 1) (B) (1, 4) (C) (-1, 4) (D) (0, 0)
19. If \vec{u} is any vector then $-\vec{u} =$
- (A) $\frac{-\vec{u}}{|\vec{u}|}$ (B) $\frac{\vec{u}}{|\vec{u}|}$ (C) $\frac{-\vec{u}}{|\vec{u}|}$ (D) $\frac{\vec{u}}{|\vec{u}|}$
20. If $2\vec{i} + \alpha\vec{j} + 5\vec{k}$ and $3\vec{i} - \vec{j} - \alpha\vec{k}$ are perpendicular, then $\alpha =$
- (A) 1 (B) 1 (C) -1 (D) 2



Roll No. _____ to be filled in by the candidate.

(For all sessions)

Paper Code

8

1

9

1

**Mathematics** (Objective Type)

Time: 30 Minutes

Marks: 20

NOTE: Write answers to the questions on objective answer sheet provided. Four possible answers A,B,C & D to each question are given. Which answer you consider correct, fill the corresponding circle A,B,C or D given in front of each question with Marker or pen ink on the answer sheet provided.

1-1. If $f(x) = \sqrt{x+4}$, then $f(x^2+4)$ is equal to:

(A) $x^2 - 8$

(B) $\sqrt{x^2 - 8}$

(C) $\sqrt{x^2 + 8}$

(D) $x^2 + 8$

2. $\lim_{x \rightarrow 0} \frac{\sin 7x}{x}$ is equal to:

(A) 1

(B) 7

(C) $\frac{1}{7}$

(D) 0

3. $\frac{d}{dx} \cos^2 x$ is equal to:

(A) $-\sin^2 x$

(B) $2 \sin x$

(C) $2 \sin x \cos x$

(D) $-2 \cos x \sin x$

4. $1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$ is Maclaurin series of:

(A) e^x

(B) $\sin x$

(C) $\cos x$

(D) $\ln(1+x)$

5. If $x = at^2$, $y = 2at$, then $\frac{dy}{dx}$ is equal to:

(A) t

(B) $\frac{1}{t}$

(C) t^2

(D) $\frac{1}{t^2}$

6. $\frac{d}{dx} \left(\frac{1}{ax+b} \right)$ is equal to:

(A) $ax+b$

(B) $\frac{-1}{(ax+b)^2}$

(C) $\frac{-a}{(ax+b)^2}$

(D) $\ln(ax+b)$

7. If $y = \sin 3x$, then y_2 is equal to:

(A) $9 \sin 3x$

(B) $-9 \sin 3x$

(C) $9 \cos 3x$

(D) $-9 \cos 3x$

8. $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ is equal to:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

9. Solution of the differential equation $\frac{dy}{dx} = \cos x$, is:

- (A) $y = \sin x + c$ (B) $y = -\sin x + c$ (C) $y = \cos x + c$ (D) $y = \ln(\sin x) + c$

10. $\int e^{\tan x} (\sec^2 x) dx$ is equal to:



- (A) $e^{\tan x} + c$ (B) $e^x \cdot \tan x + c$ (C) $e^x \cdot \sec x + c$ (D) $e^{\cot x} + c$

11. $\int_0^2 (x^2 + 1) dx$ is equal to:

- (A) $\frac{3}{10}$ (B) $\frac{14}{3}$ (C) $\frac{5}{3}$ (D) $\frac{8}{3}$

12. Point of concurrency of medians of a triangle is called:

- (A) orthocentre (B) in-centre (C) ex-centre (D) centroid

13. The lines represented by $ax^2 + 2hxy + by^2 = 0$, are real and coincident if:

- (A) $h^2 > ab$ (B) $h^2 \neq ab$ (C) $h^2 < ab$ (D) $h^2 = a + b$

14. Equation of the line bisecting the first and third quadrant is:

- (A) $y = x$ (B) $y = -x$ (C) $y = x + c$ (D) $xy = c$

15. Slope of the line which is perpendicular to the line $2x - 4y + 11 = 0$ is:

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) -2

16. Point (1, 2), satisfies the inequality.

- (A) $2x + y > 5$ (B) $2x + y \geq 5$ (C) $2x + y < 3$ (D) $2x + y < 5$

17. The centre of the circle $(x+3)^2 + (y-2)^2 = 16$, equals.

- (A) (3, -2) (B) (-3, 2) (C) (3, 2) (D) (-3, -2)

18. The eccentricity of $\frac{y^2}{4} - x^2 = 1$, equals.

- (A) $\frac{2}{\sqrt{5}}$ (B) $\frac{-2}{\sqrt{5}}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{-\sqrt{5}}{2}$

19. $2i \cdot (3j \times k)$ is equal to:

- (A) 0 (B) 2 (C) 4 (D) 6

20. $\cos \theta$, equals to:

- (A) $\hat{a} \cdot \hat{b}$ (B) $|\hat{a} \times \hat{b}|$ (C) $\hat{a} \times \hat{b}$ (D) $\frac{|\hat{a} \times \hat{b}|}{|\hat{a}|}$

Roll No. _____ to be filled in by the candidate.

(For all sessions)

Mathematics (Essay Type)

Time: 2:30 Hours

Marks: 80

Section -I



2x8=16

2. Write short answers of any eight parts from the following.

- i. Prove the identity $\sec^2 x = 1 + \tan^2 x$.
- ii. Find $f^{-1}(x)$ if $f(x) = 3x^3 + 7$.
- iii. Evaluate $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$.
- iv. Differentiate w.r.t x , $y = \frac{2x-1}{\sqrt{x^2+1}}$.
- v. Find $\frac{dy}{dx}$ if $xy + y^2 = 2$.
- vi. Differentiate $\sin^2 x$ w.r.t $\cos^4 x$.
- vii. Differentiate $\cos^{-1}\left(\frac{x}{a}\right)$ w.r.t x .
- viii. Differentiate $(\ln x)^x$ w.r.t x .
- ix. Find $f'(x)$ if $f(x) = x^3 e^{\frac{1}{x}}$.
- x. Find $\frac{dy}{dx}$ if $y = x\sqrt{\ln x}$.
- xi. Find y_2 , if $y = \sqrt{x} + \frac{1}{\sqrt{x}}$.
- xii. Determine the interval in which function is increasing or decreasing

for the mentioned domain. $f(x) = \cos x : x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. Write short answers of any eight parts from the following.

2x8=16

- i. Evaluate: $\int x(\sqrt{x}+1)dx$.
- ii. Evaluate: $\int \frac{1-x^2}{1+x^2} dx$.
- iii. Evaluate: $\int \frac{-2x}{4-x^2} dx$.
- iv. Evaluate: $\int e^x \left(\frac{1}{x} + \ln x \right) dx$.
- v. Evaluate: $\int \frac{2x}{1-\sin x} dx$.
- vi. Evaluate: $\int_{-1}^1 \left(x^{\frac{1}{3}} + 1 \right) dx$.
- vii. Define the definite integral.
- viii. Solve the differential equation $ydx + xdy = 0$.
- ix. Define the corner point.
- x. Graph the solution set of linear inequality $2x + y \leq 6$.
- xi. Find δy and dy in $y = x^2 + 2x$, when x changes from 2 to 1.8.
- xii. Find the area between the x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$.

4. Write short answers of any nine parts from the following.

2x9=18

- i. Find h such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.
- ii. Find the centroid of the triangle having vertices $(-2, 3)$, $(-4, 1)$ and $(3, 5)$.
- iii. Find an equation of the line through $(-5, -3)$ and $(9, -1)$.
- iv. Find the lines represented by the homogeneous equation $3x^2 + 7xy + 2y^2 = 0$.
- v. Find measure of the angle between the lines represent by $x^2 - xy - 6y^2 = 0$.
- vi. Find the equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$.
- vii. Find the condition that the line $y = mx + c$ may touch the circle $x^2 + y^2 = a^2$.
- viii. Derive equation of ellipse in standard form.
- ix. Find centre and foci of the $x^2 - y^2 = 9$.
- x. Let $\underline{U} = \underline{i} + 2\underline{j} - \underline{k}$ and $\underline{V} = 3\underline{i} - 2\underline{j} + 2\underline{k}$ find $|\underline{U} + 2\underline{V}|$.
- xi. Find α , so that $|\alpha\underline{i} + (\alpha + 1)\underline{j} + 2\underline{k}| = 3$.
- xii. Find a vector perpendicular to each of the vectors $\underline{a} = 2\underline{i} + \underline{j} + \underline{k}$ and $\underline{b} = 4\underline{i} + 2\underline{j} - \underline{k}$.
- xiii. Find the value of $2\underline{i} \times 2\underline{j} \cdot \underline{k}$.



Section -II

Note: Attempt any three questions from the following.

10x3=30

5. (a) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$

(b) Show that $\frac{dy}{dx} = \frac{y}{x}$ if $\frac{y}{x} = \tan^{-1} \frac{x}{y}$.

6. (a) Evaluate: $\int \sqrt{x^2 + 4} dx$.

(b) Find the lines represented by equation. Also find measure of the angle between them. $2x^2 + 3xy - 5y^2 = 0$.

7. (a) Evaluate: $\int_{1/8}^1 \frac{(x^{1/3} + 2)^2}{x^{2/3}} dx$.

(b) Minimize $z = 5x + y$ subject to the constraints $3x + 5y \geq 15$, $x + 6y \geq 9$, $x \geq 0$, $y \geq 0$.

8. (a) Find an equation of parabola if focus is $(-3, 1)$, directrix $y = 1$.

(b) Use vectors to prove that the diagonals of a parallelogram bisect each other.

9. (a) Find the centre, foci, eccentricity, vertices and directrices of $9x^2 + y^2 = 18$.

(b) Prove that $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ by using vector method.