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F.Sc Math Part-II

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M.Phil Math

Unit #5

Linear Inequalities And Linear Programming

M.C.Q's

(1) A Vertical line divides a plane into — half Planes. (LHR-2011)
(a) left And Right (b) Upper And Lower (c) Upper and Right (d) None

(2) The Solution Set of the Inequality $ax+by<c$ is = (LHR-2019)
(a) Parabola (b) half Plane (c) Circle (d) hyperbola

(3) The Inequality $ax+by \leq c$ when $a=0$ Represents — half Plane.
(a) left or Right (b) Upper and Lower (c) Open (d) None (LHR-2012)

(4) A Region which is Restricted to the first Quadrant = (LHR Group-2011)
(a) Feasible Region (b) Minimum Region (c) Maximum Region (d) Objective function

(5) System of Linear Inequalities Involved in the Problem is called
(a) Coefficients (b) Solution (c) Problem Constraints (d) Boundries (LHR-2018)

(6) The Feasible Solution which Maximizes or Minimizes the Objective Function is called = (LHR-2010,2018)
(a) Final Solution (b) Optimal Solution (c) objective function (d) feasible Solution

(7) (2,1) is in the Solution of Inequality = (LHR-2011)
(a) $2x+y>5$ (b) $x+3y>5$ (c) $2x+y<3$ (d) $2x+y>6$

(8) (1,0) is the Solution of the Inequality = (LHR-2015,2017)
(a) $7x+2y<8$ (b) $x-3y<0$ (c) $10x+5y<6$ (d) $-3x+5y>2$

(9) Which one of the following Points satisfied $x+2y<6$? (LHR-2010,2012)
(a) (0,1) (b) (3,3) (c) (1,3) (d) (0,4)

(10) The Solution of the Inequality $x+2y<6$ = (LHR-2019)
(a) (1,1) b(1,3) (c) (1,4) (d) (1,5)

(11) Solution Set of Inequality $2x<3$ = (LHR-2016)

(a) $(-\infty, \frac{3}{2})$ (b) $(\frac{3}{2}, \infty)$ (c) $(-\infty, \infty)$ (d) $(-\frac{3}{2}, \frac{3}{2})$

(12) Solution Region of Inequality $2x+y>2$ = (D.G Khan 2017)

(a) In 1st Quadrant (b) In 2nd Quadrant (3) Outwards the origion (d) None

(13) $x=5$ is Solution of the Inequality = (AJK 2016, 2017)

(a) $2x-3>0$ (b) $2x+3<0$ (c) $x+4<0$ (d) $x<0$

(14) $x=-3$ is the Solution of the Inequality (Gujrawala 2017)

(a) $2x-1>0$ (b) $2x+1>0$ (c) $x+4<0$ (d) $2x-1<0$

(15) The Non-Negative Constraints are called = DG Khan (2019)

(a) Free Variables (b) Decision Variables (c) Vertex (d) Convex

Chapter: 5

linear Inequalities and linear programming

Introduction:-

Many real life problems involve linear inequlities. Those problem (relating to trade, industry and agriculture etc.) which involve system of linear inequlities in two varriables. Linear inequlities in such problems are used to prescribe limitations or restrictions on allocation of available resources (material, captial, machine capacities, labour hours, land etc.).

linear inqulities:

Inequlities are expressed by the following four symbds;

(i) $>$ (Greater then). (ii) $<$ (less then).

(iii) $\geq$ (Greater equal to) (iv) $\leq$ (less equal to).

Example:-

$ax < b$ and $ax \geq b$ are the examples of linear inequlities.

The following operations will not affect the order or (sence of inequalities while changing it to simpler equivalent form:

(i) Adding or subtracting a constant to each side of it.

(ii) Multiplying or dividing each side of it by a positive constant.

Key point:

The order or ssence of an inequality is changed by multiplying or dividing its each side by a **negative** constant.

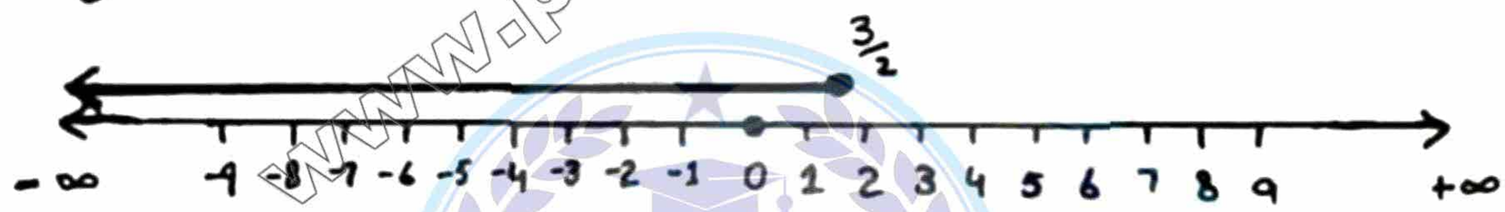
Let: $x < \frac{3}{2}$ ___(A)

All real numbers $< 3/2$ are in the solution set of **(A).**

Thus, The interval $\left[-\infty, \frac{3}{2}\right]$ or $-\infty < x < \frac{3}{2}$

is the solution set of inequality **(A).**

Figure:



So; The solution set of an inequality consists of all solutions of the inequality.

linear inequality in two variables:

" A linear inequality in two variables **x** and **y** can be one of the following forms;

i $ax+by < c$ ii $ax+by > c$

iii $ax+by \leq c$ iv $ax+by \geq c$

Where a, b, c are constants and a, b are not both zero.

Half planes:

(i) The set of ordered pairs (x,y) such that

$$ax+by < c$$

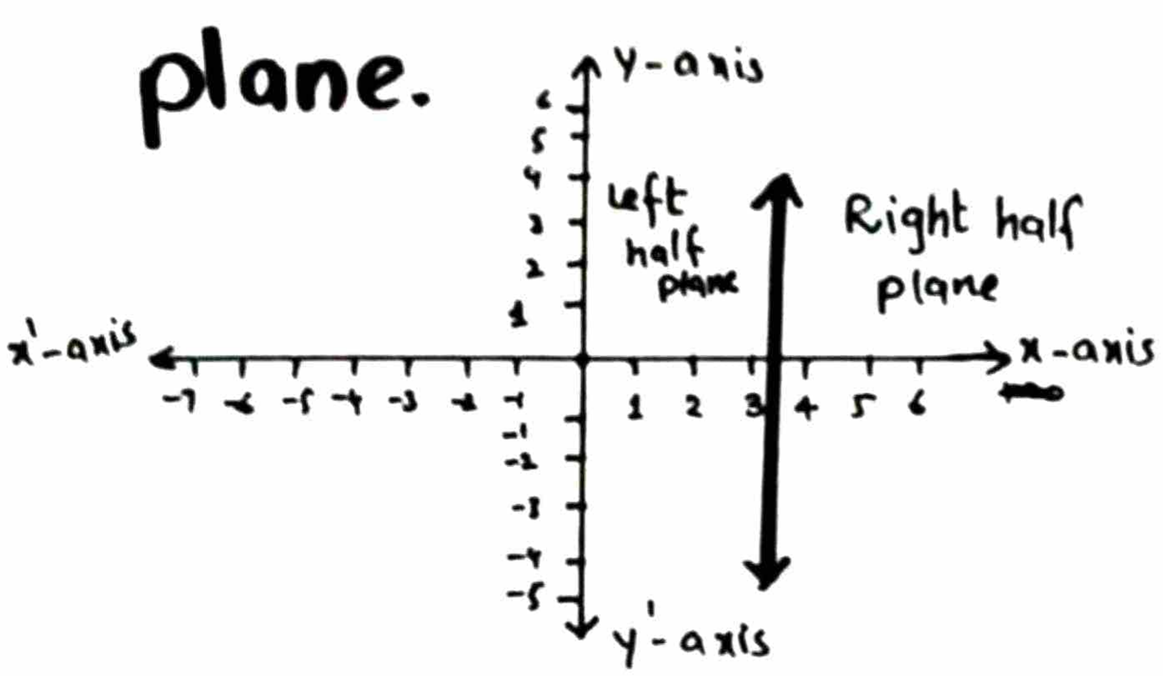
(ii) The set of ordered pairs (x,y) such that

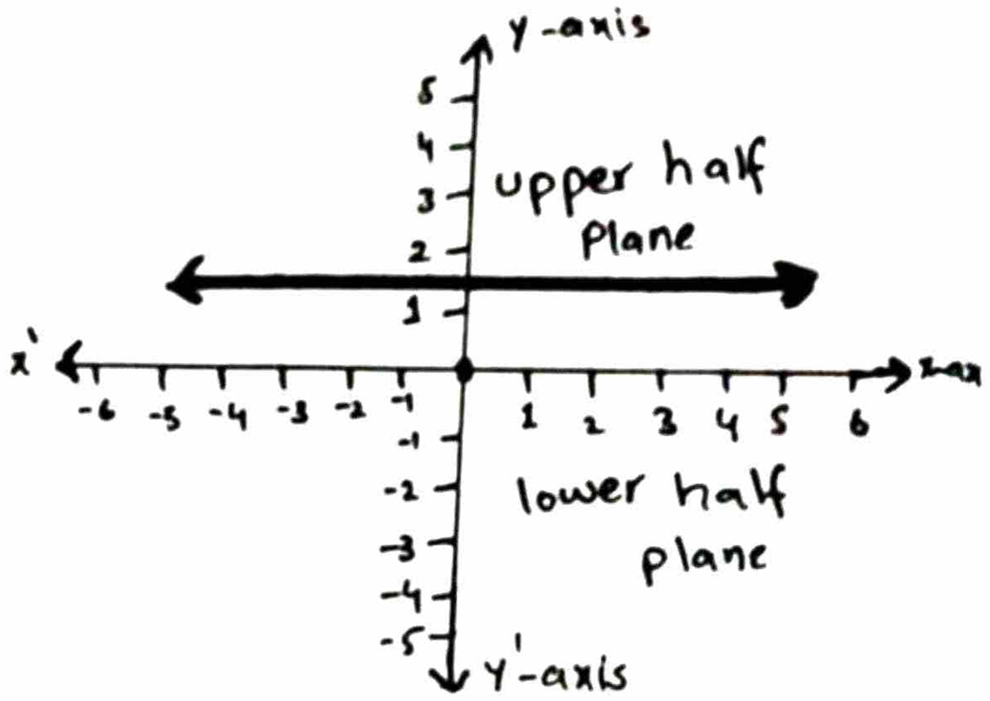
$$ax+by > c$$

In graph of linear equation these two regions (1) and (2) are called **half plane** and the line $ax+by=c$ is called the **boundary of each half plane.**

Key point: A vertical line divides into **left and right half planes** while a non-vertical (Horizontal) line divides

the planes into **upper and lower half plane.**





MCQs

A solution of a linear inequality in x and y is an ordered pair of numbers which satisfies the inequality.

Associated Equation:

"In linear inequlities, the linear equation $ax+by=c$ is called **associated or corresponding** equation."

Example:

For example inequality: $x+2y < 6$

Its associated equation: $x+2y=6$

Corner point:

"A point of a solution region where two of its boundary lines interest, is called **corner point** or **vertex** of the solution region.

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Exercise: 5.1

Question: 1

Graph the solution set of the following Linear inequality in xy-plane.

$$2x+y \leq 6$$

$$2x+y \leq 6$$

Its associated equation.

$$2x+y=6 \quad \text{---} \quad (i)$$

Put $x=0$ in eq (i)

$$2(0)+y=6$$

$$y=6$$

$$A(0,6)$$

Put $y=0$ in eq (i)

$$2x+(0)=6$$

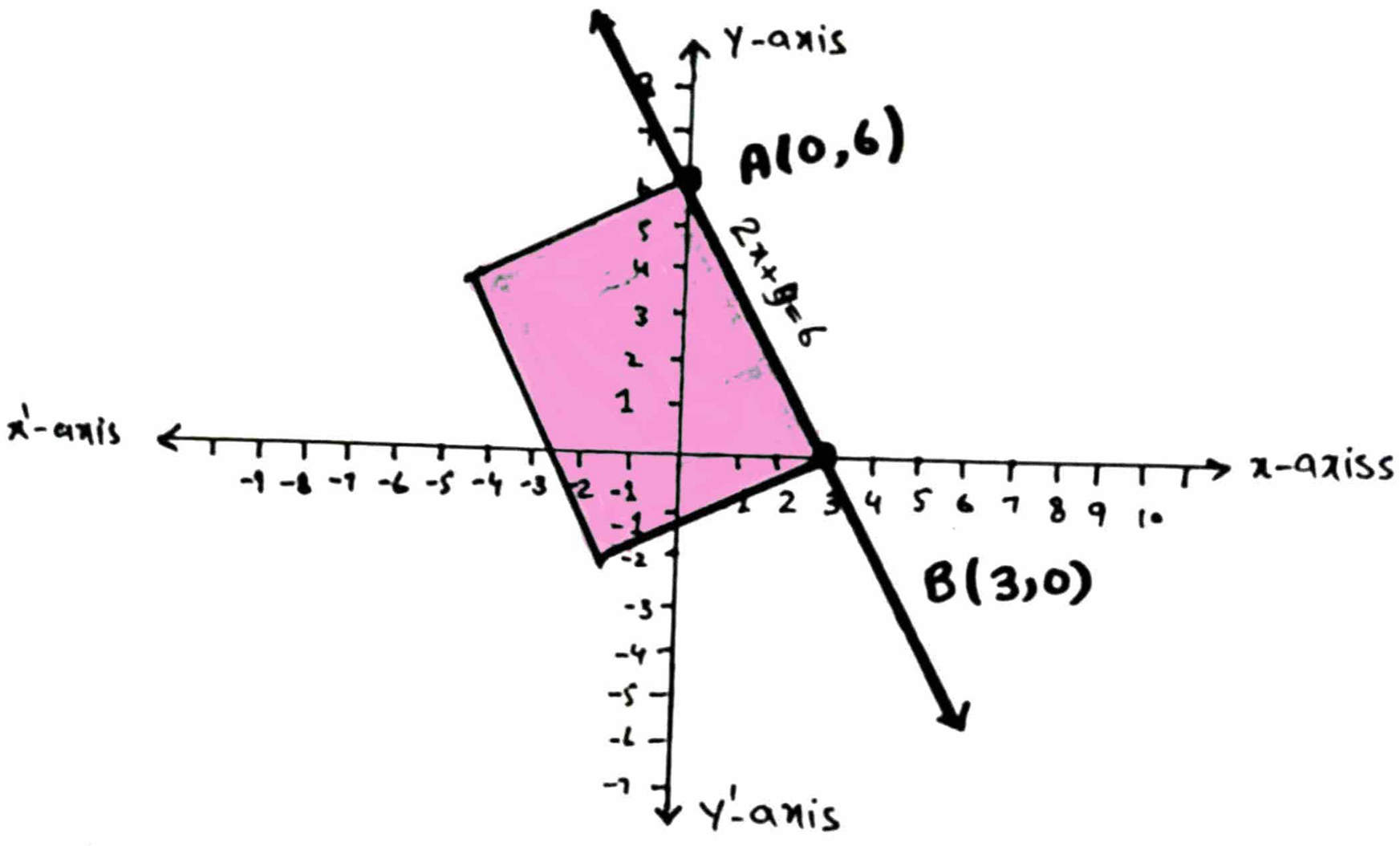
$$x=\frac{6}{2} \Rightarrow x=3$$

$$B(3,0)$$

Point test: O(0,0)

$$2x+y \leq 6 \Rightarrow 2(0)+(0) \leq 6$$

$$0 \leq 6 \quad \text{(True)}.$$



ii

$3x+7y \geq 21$

Its associated Equation.

$3x+7y = 21$ ...(i)

Put $x=0$ in eq (i)

$3(0)+7y = 21$

$y = \frac{21}{7} \Rightarrow y=3$

$A(0,3)$

Put $y=0$ in eq (i)

$3x+7(0)=21$

$x = \frac{21}{3} \Rightarrow x=7$

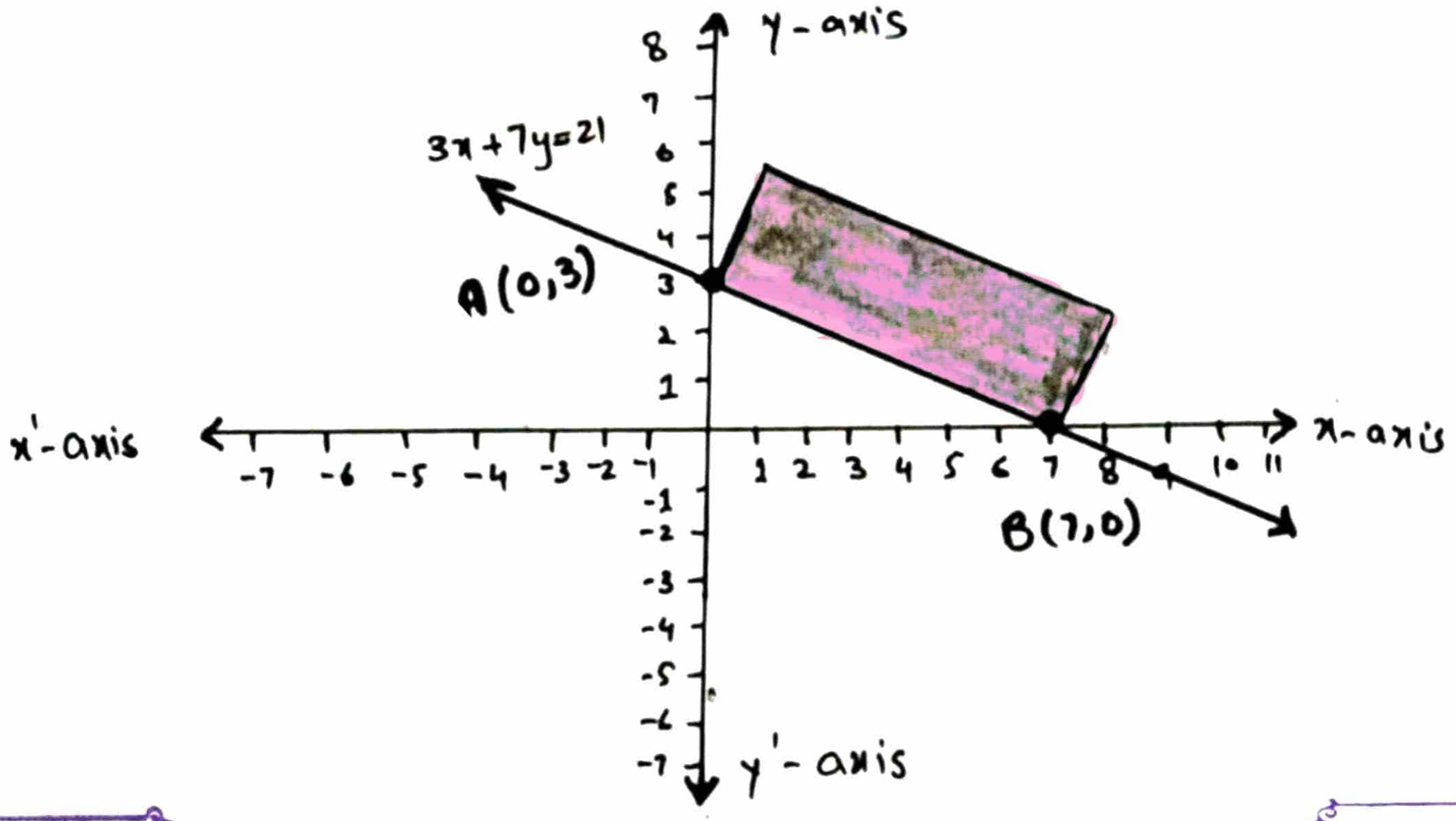
$B(7,0)$

Point test: O(0,0).

$3x+7y > 21$

$3(0)+7(0) > 21$

$0 > 21$ (False).



$$3x - 2y \geq 6$$

Its associated Equation.

$$3x - 2y = 6 \quad \text{--- i)}$$

Put $x = 0$ in eq i)

$$3(0) - 2y = 6$$

$$y = \frac{-6}{2} \Rightarrow y = -3$$

$$A(0, -3)$$

Put $y = 0$ in i)

$$3x - 2(0) = 6$$

$$x = \frac{6}{3} \Rightarrow x = 2$$

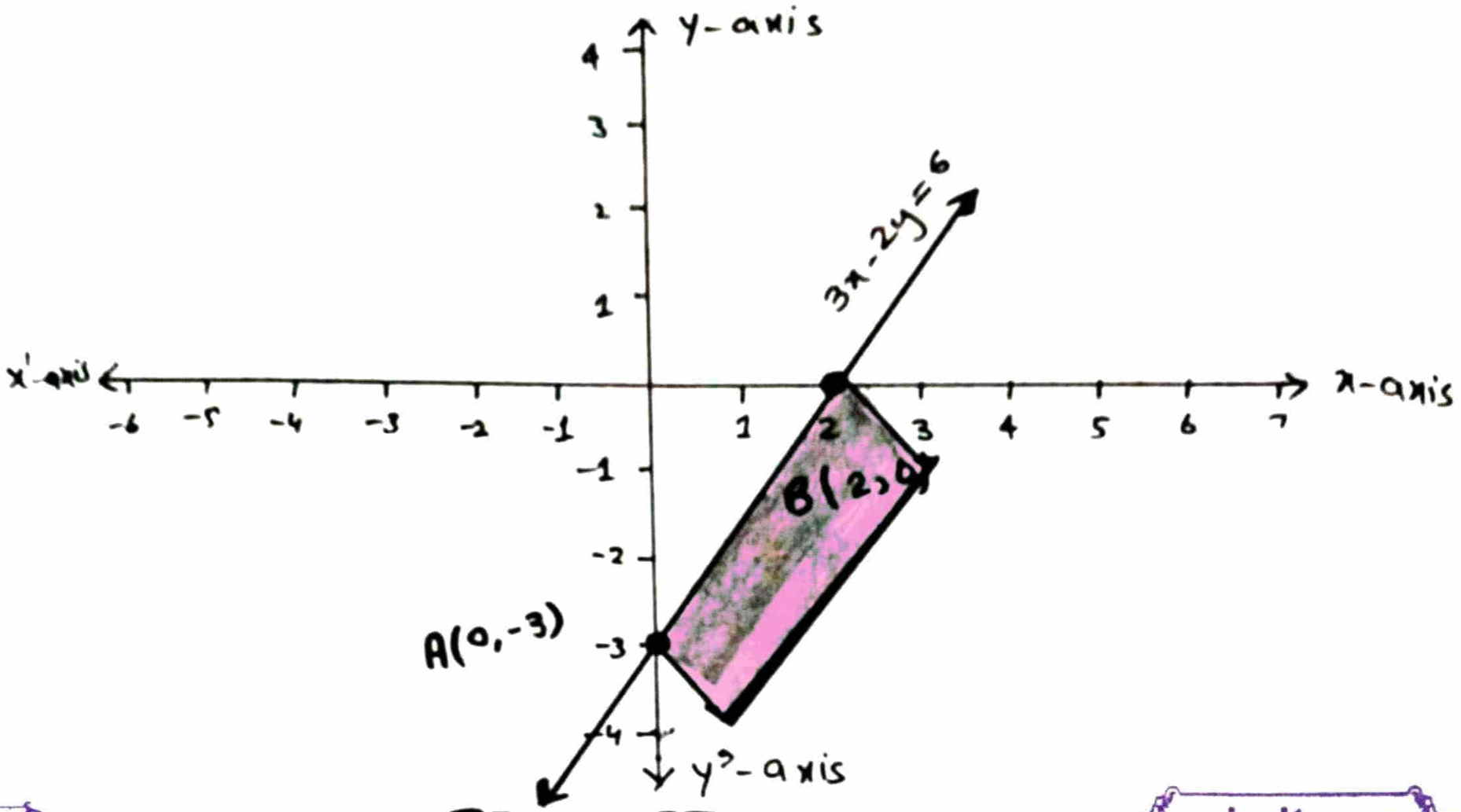
$$B(2, 0).$$

Point test: O(0,0)

$$3x - 2y \geq 6$$

$$3(0) - 2(0) \geq 6$$

$$0 \geq 6 \quad \text{(False)}.$$



$$5x - 4y \leq 20$$

$5x - 4y \leq 20$

Its associated Equation.

$$5x - 4y = 20 \quad \text{--- (i)}$$

Put $x = 0$ in eq (i)

$5(0) - 4y = 20$

$y = \frac{-20}{4} \Rightarrow y = -5$

$A(0, -5)$

Put $y = 0$ in (i)

$5x - 4(0) = 20$

$x = \frac{20}{5} \Rightarrow x = 4$

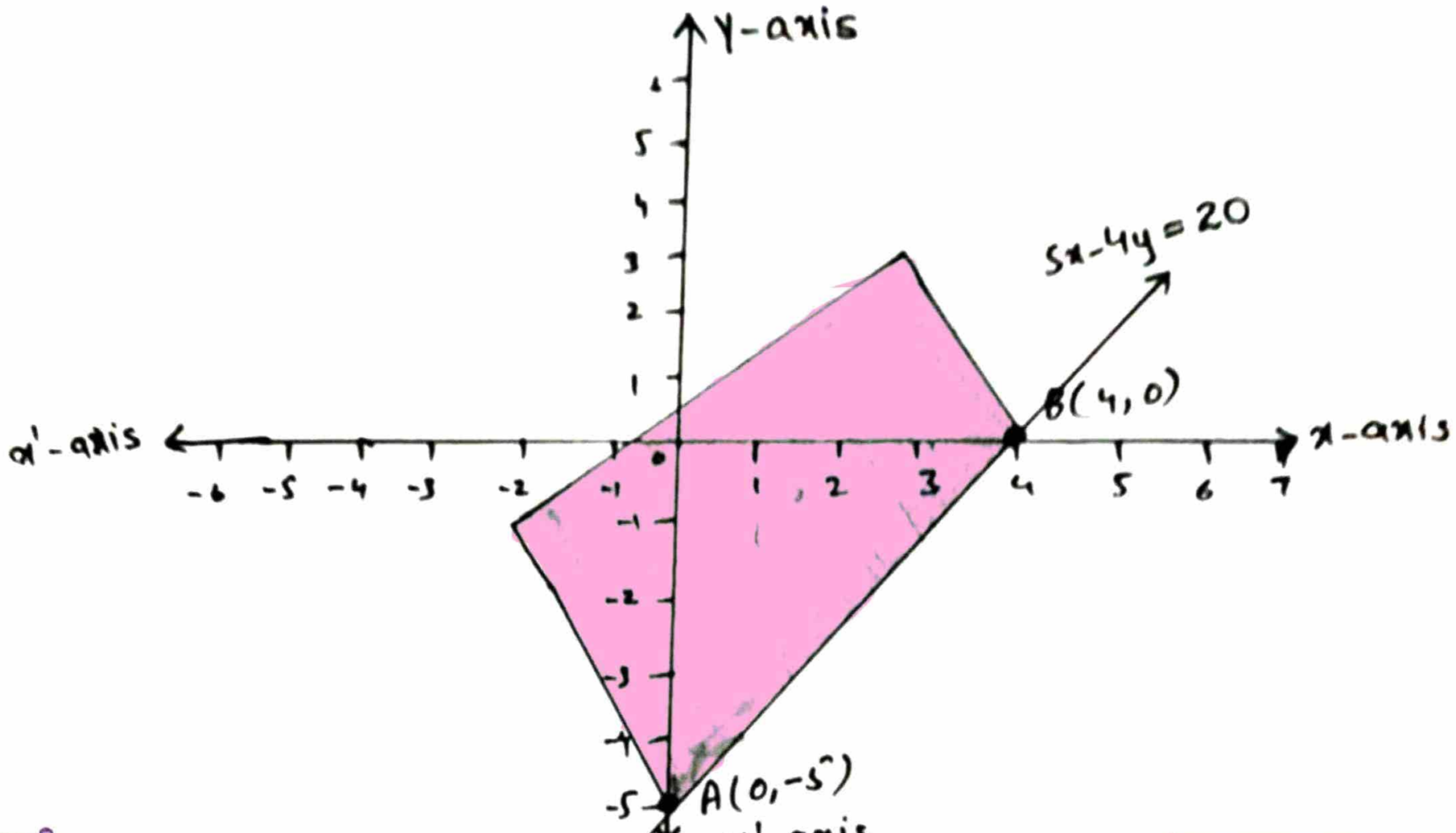
$B(4, 0)$.

Point test: O(0,0)

$5x - 4y < 20$

$5(0) - 4(0) < 20$

$0 < 20$ (True)



$2x+1 \geq 0$ (LHR - 2014, 15)

$2x+1 \geq 0$

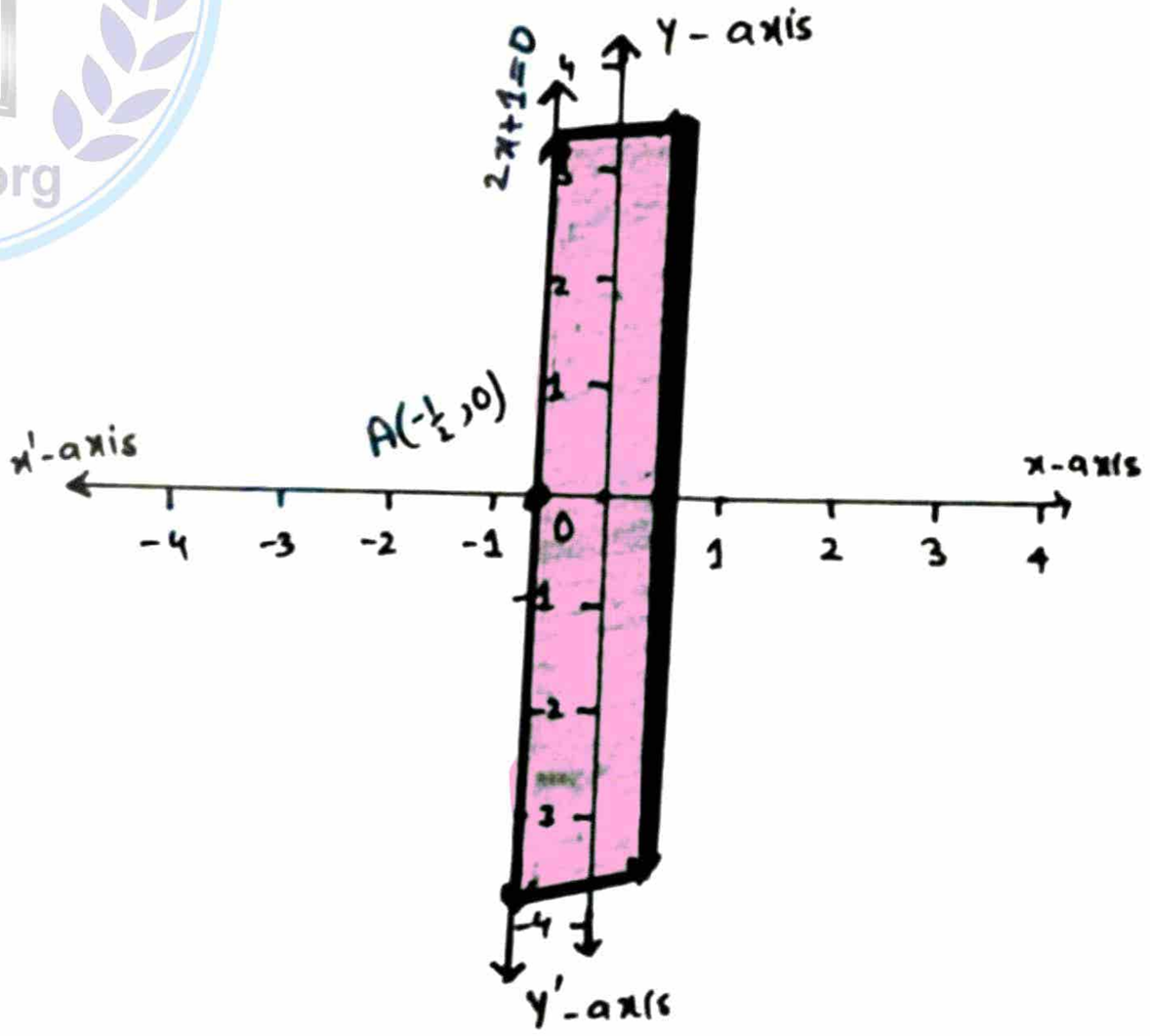
Its associated Equation:

$2x+1=0$ ——— (i)

$2x=-1$

$x=-\frac{1}{2}$

$A(-\frac{1}{2}, 0)$



Point test: O(0,0).

$2x+1 > 0$

$2(0)+1 > 0$

$1 > 0$ (True)

$$3y - 4 \leq 0$$

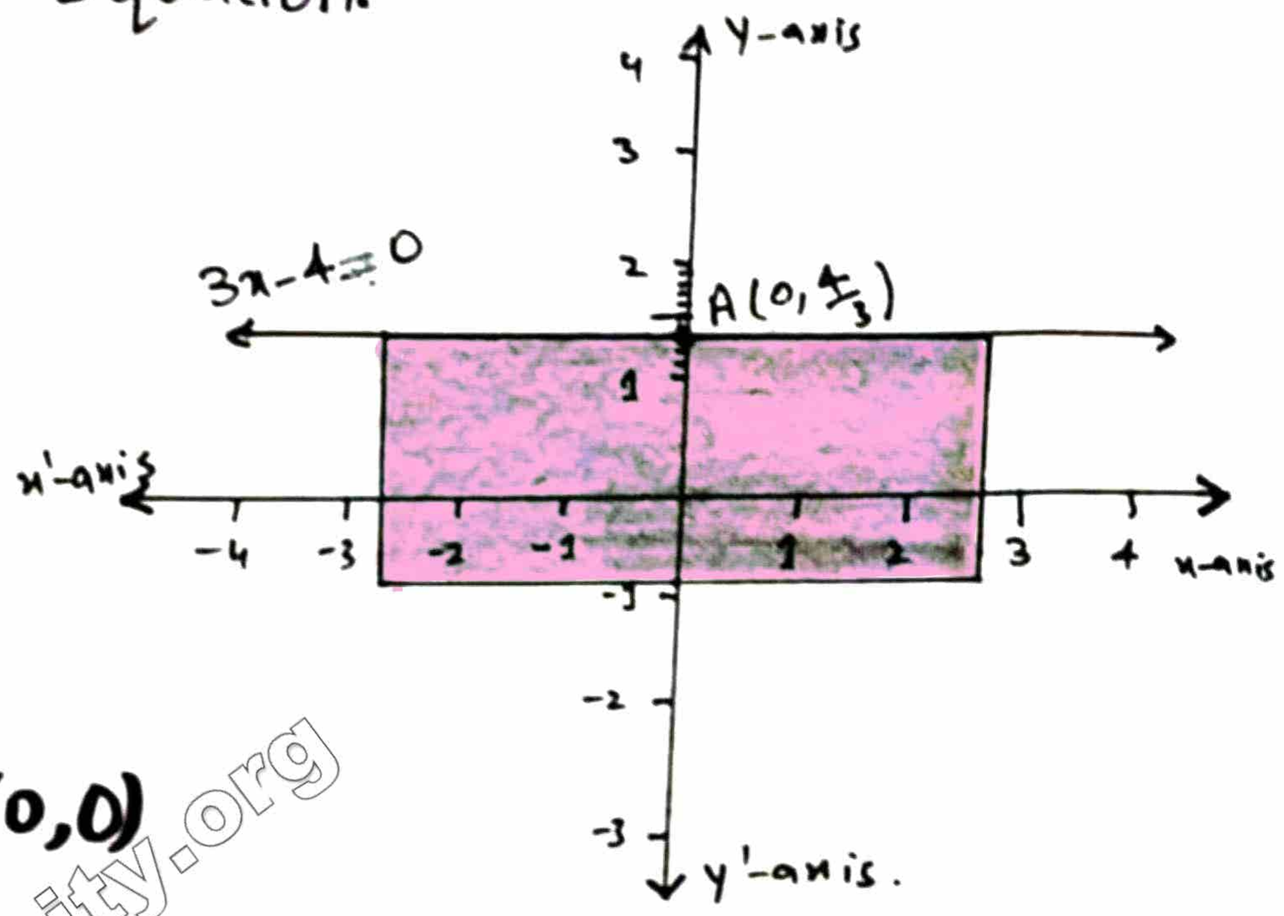
Its associated Equation:

$$3y - 4 = 0$$

$$3y = 4$$

$$y = 4/3$$

$A(0, 4/3)$.



Point test: O(0,0)

$$3y - 4 < 0$$

$$3(0) - 4 < 0$$

$-4 < 0$ (True).

Question: 2

Indicate the solution set of the following systems of linear inequalities by shading:

$$2x - 3y \leq 6$$

$$2x + 3y \leq 12$$

Its associated Equation

$2x - 3y = 6$ — (i)

Put $x = 0$ in (i)

$2(0) - 3y = 6$

$y = -6/3 \Rightarrow y = -2$

A(0, -2).

Put $y = 0$ in (i)

$2x - 3(0) = 6$

$x = 6/2 \Rightarrow x = 3$

B(3, 0)

Point test: O(0,0)

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$ (True).

Its associated Equation.

$2x + 3y = 12 \rightarrow$ (ii)

Put $x = 0$ in (ii)

$2(0) + 3y = 12$

$y = 12/3 \Rightarrow y = 4$

C(0, 4)

Put $y = 0$ in (ii)

$2x + 3(0) = 12$

$x = \frac{12}{2} \Rightarrow x = 6$

D(6, 0)

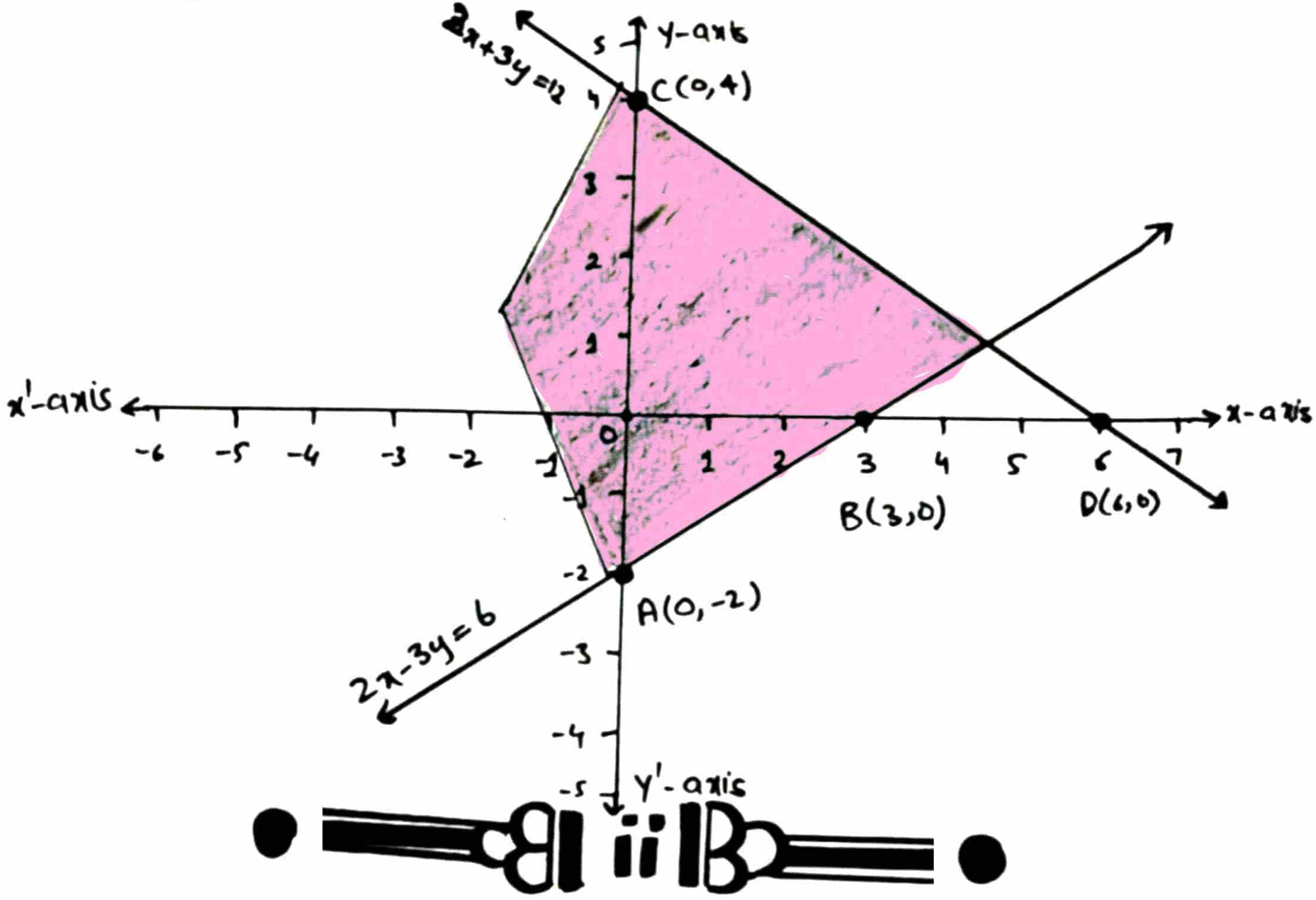
Point test: O(0,0)

$2x + 3y < 12$

$2(0) + 3(0) < 12$

$0 < 12$ (True).

So, The Combined graph:



$x+y \geq 5$

$-y+x \leq 1$

Its associated Equation:

$x+y = 5$ ——— (i)

Put $x=0$ in (i)

$0+y = 5 \Rightarrow y=5$

A(0, 5)

Put $y=0$ in (i)

$x+0=5 \Rightarrow x=5$

B(5, 0).

Its associated Eq.

$x-y=1 \rightarrow$ (ii)

Put $x=0$ in (ii)

$0-y=1$

$y=-1$

C(0, -1)

Put $y=0$ in (ii)

$x-0=1 \rightarrow x=1$

D (1, 0)

Point test: O(0,0)

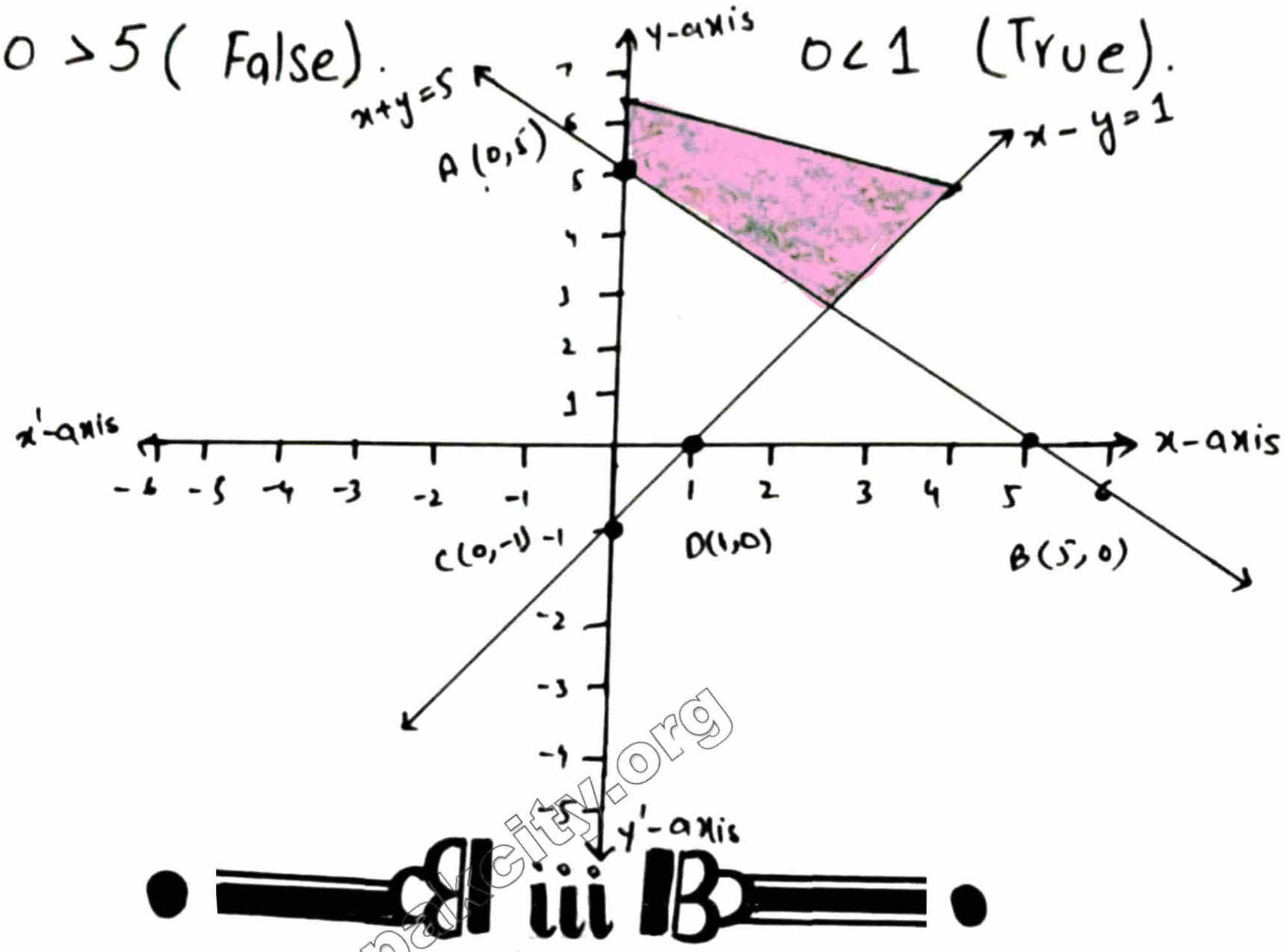
$x+y>5 \Rightarrow 0+0>5$

$0>5$ (False).

Point test: O(0,0)

$x-y<1 \Rightarrow 0-0<1$

$0<1$ (True).



iii B

$3x+7y \geq 21$

$x-y \leq 2$

Its associated Eq.

$3x+7y=21$ — i)

Put $x=0$ in i)

$3(0)+7y=21$

$y=\frac{21}{7} \Rightarrow y=3$

A(0,3)

Put $y=0$ in ii)

$3x+7(0)=21$

$x=\frac{21}{3} \Rightarrow x=7$

Its associated Eq.

$x-y=2$ — ii)

Put $x=0$ in ii)

$0-y=2$

$y=-2$

C(0,-2)

Put $y=0$ in ii)

$x+0=2 \Rightarrow x=2$

D(2,0)

B(7, 0)

Point test: O(0,0)

$3x+7y>21$

$3(0)+7(0)>21$

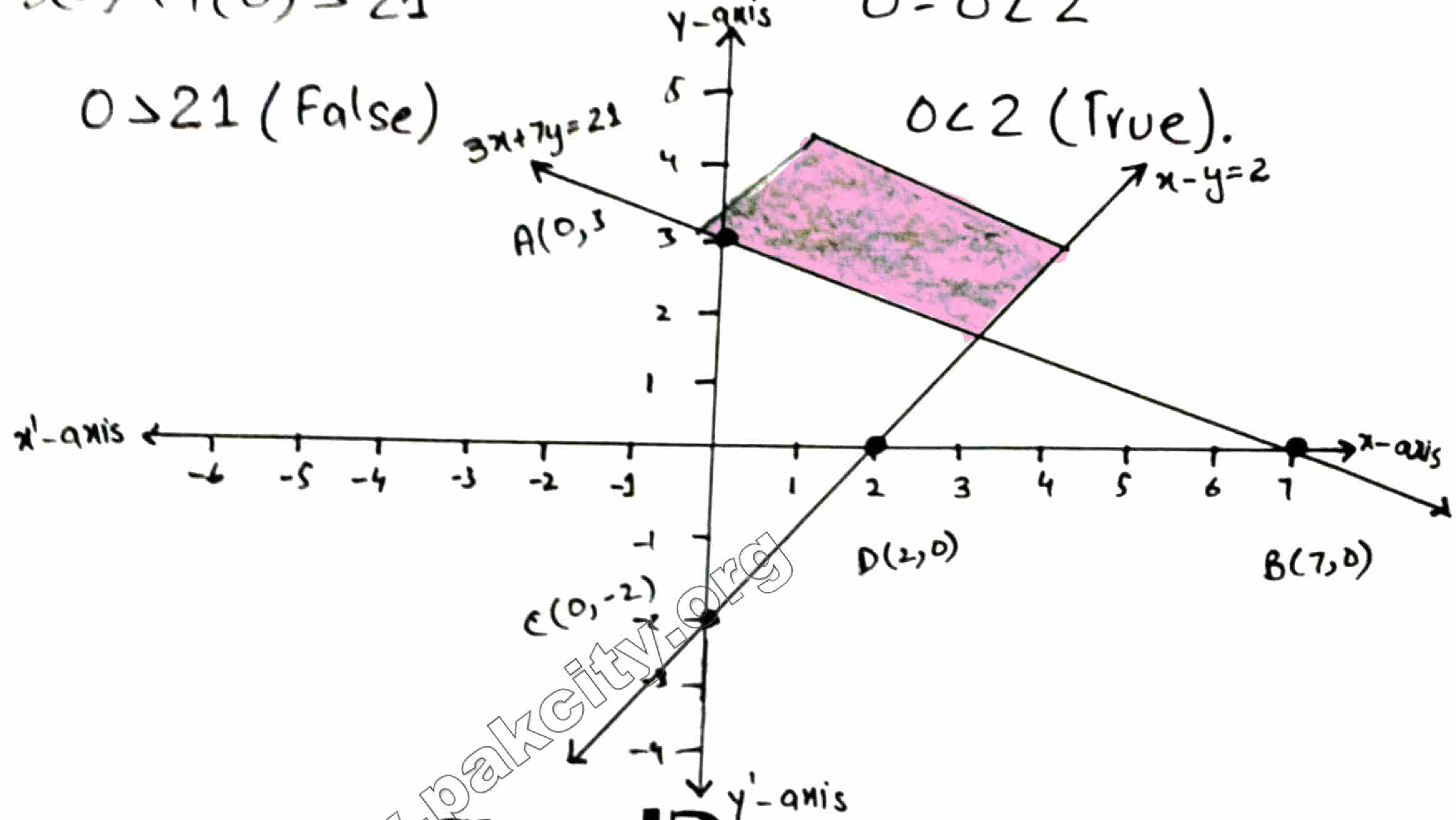
$0>21$ (False)

Point test: O(0,0)

$x-y<2$

$0-0<2$

$0<2$ (True).



iv

$4x-3y\le 12$ **(LHR - 2011)**

$x\ge -\frac{3}{2}$

$4x-3y\le 12$

Its associated Eq.

$4x-3y=12$ — (i)

Put $x=0$ in (i)

$4(0)-3y=12$

$y=-12/3$

Its associated Eq.

$x=\frac{-3}{2}$

$C(-3/2, 0)$

Point test: O(0,0)

$0>-\frac{3}{2}$ (False).

$y = -4$

A(0, -4)

Put $y = 0$ in (i)

$$4x - 3(0) = 12$$

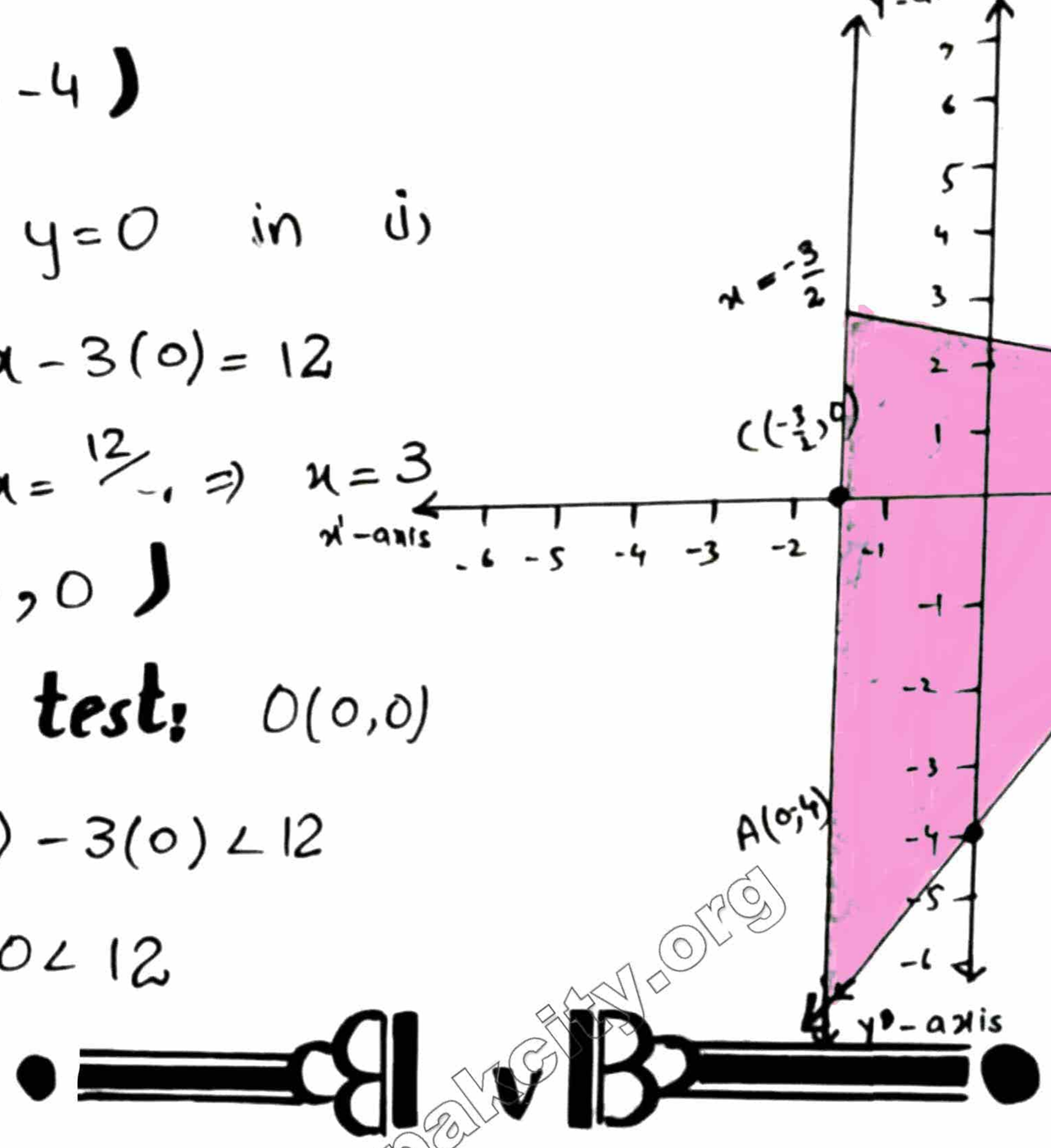
$$x = \frac{12}{4} \Rightarrow x = 3$$

B(3, 0)

Point test: O(0,0)

$$4(0) - 3(0) < 12$$

$$0 < 12$$



$3x + 7y \geq 21$

$y \leq 4$

$3x + 7y \geq 21$

Its associated Eq.

$$3x + 7y = 21 \quad \text{— (i)}$$

Put $x = 0$ in (i)

$$3(0) + 7y = 21$$

$$y = 3$$

A(0, 3)

$y \leq 4$

Its associated Eq.

$$y = 4$$

C(0, 4)

Point test: O(0,0)

$$y < 4$$

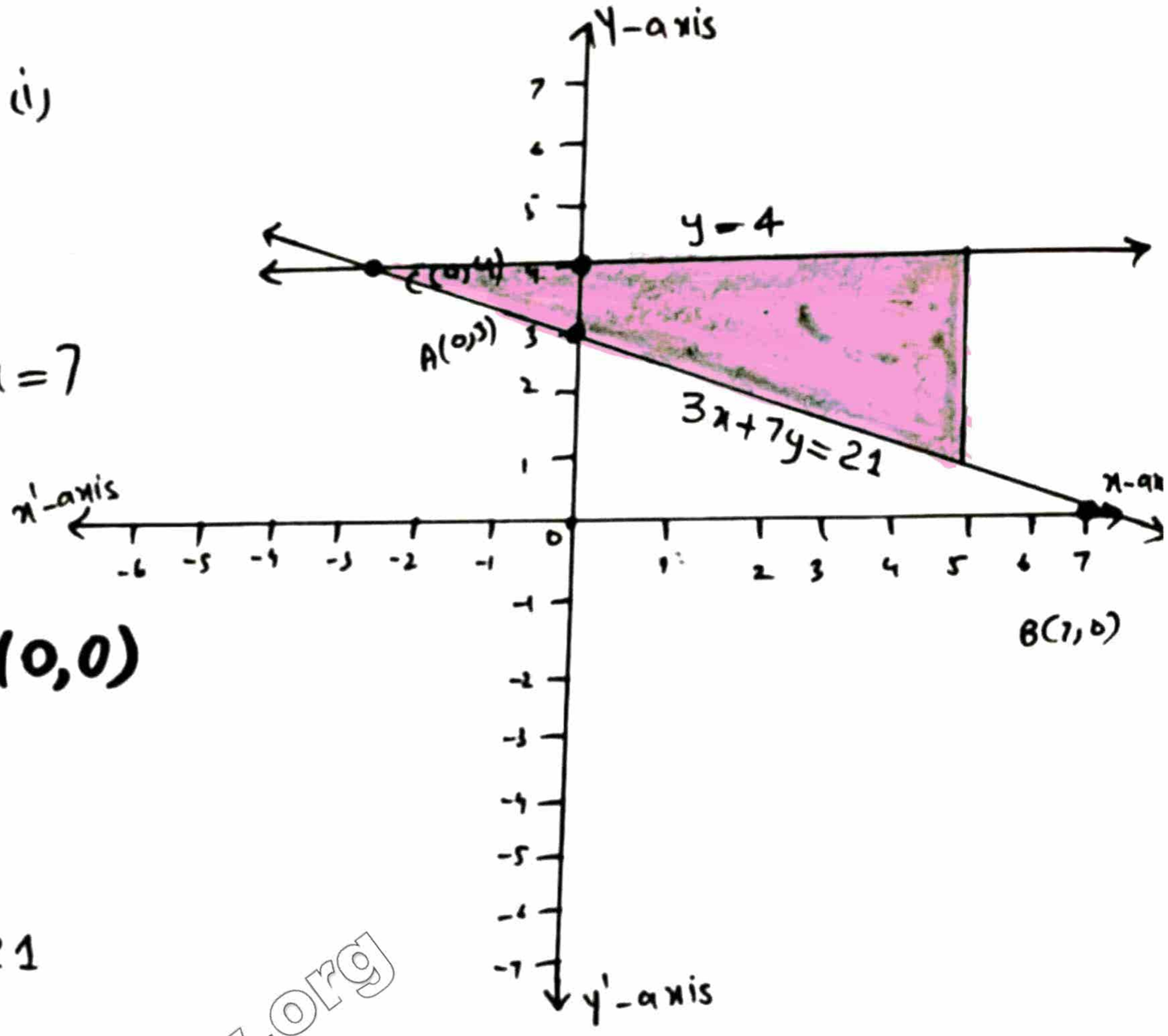
$$0 < 4 \text{ (True).}$$

Put $y=0$ in (i)

$$3x+7(0)=21$$

$$x=\frac{21}{3} \Rightarrow x=7$$

B(7, 0)



Point test: O(0,0)

$$3x+7y > 21$$

$$3(0)+7(0) > 21$$

$$0 > 21 \text{ (False)}.$$

Question: 3

Indicate the solution region of the following systems of linear inequalities by shading:

i

$$2x-3y \leq 6$$

$$2x+3y \leq 12$$

$$y \geq 0$$

$2x-3y \leq 6$

Its associated Equation:

$2x-3y=6$ — (i)

Put $x=0$ in (i)

$2(0)-3y=6$

$y=-2$

A(0,-2)

Put $y=0$ in (i)

$2x-3(0)=6$

$x=3$

B(3,0)

Point test: O(0,0)

$2x-3y<6$

$2(0)-3(0)<6$

$0<6$ (True).

$2x+3y \leq 12$

Its associated Eq.

$2x+3y=12$ → (ii)

Put $x=0$ in (ii)

$2(0)+3y=12$

$y=4$

C(0,4)

Put $y=0$ in (ii)

$2x+3(0)=12$

$x=6$

D(6,0)

Point test: O(0,0).

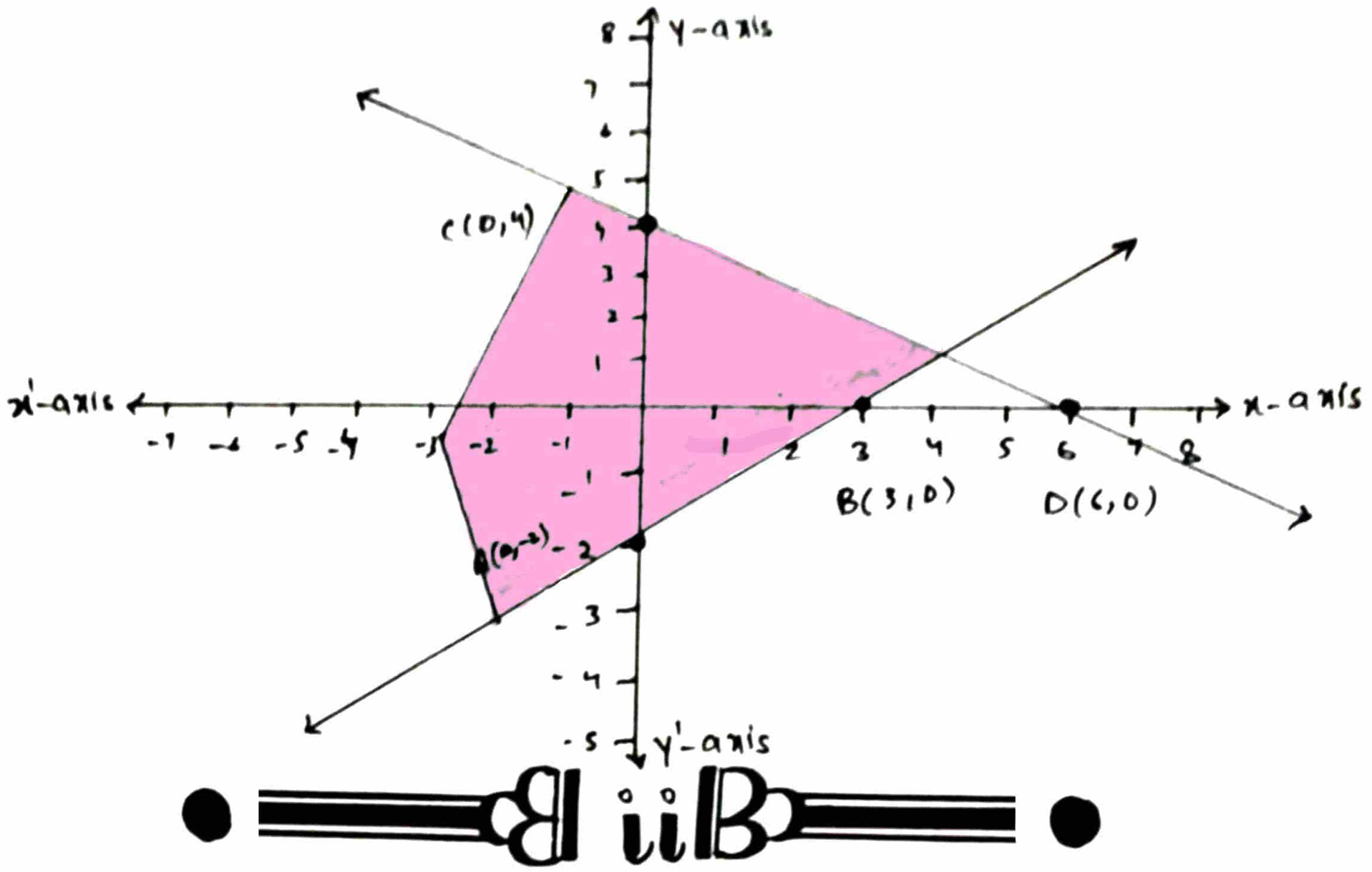
$2x+3y<12$

$2(0)+3(0)<12$

$0<12$ (True).

And: $y \geq 0$

Its associated Eq: $y=0$



$$x+y \leq 5$$

$$y-2x \leq 2$$

$$x \geq 0$$

$x+y \leq 5$

Its associated Eq.

$x+y = 5$ — (i)

Put $x=0$ in (i)

$0+y=5$

$y=5$

A(0,5)

Put $y=0$ in (i)

$x+0=5$

$x=5$

B(5,0)

$y-2x \leq 2$

Its associated Eq.

$y-2x = 2$ — (ii)

Put $x=0$ in (ii).

$y-2(0)=2$

$y=2$

C(0,2)

Put $y=0$ in (ii)

$0-2x=2$

$x=-1$

D(-1,0)

B(5,0)

Point test: O(0,0).

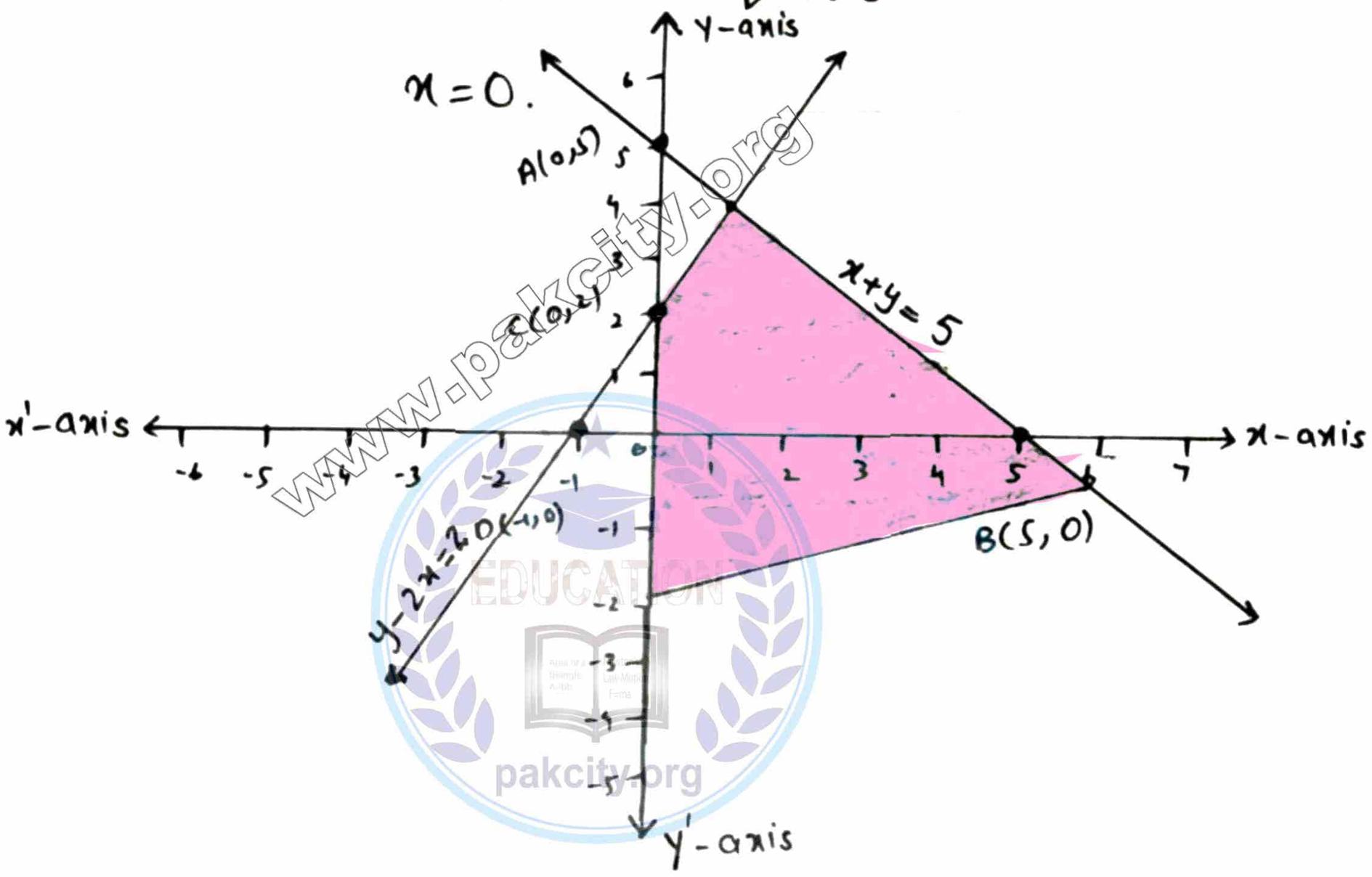
$x+y < 5$

$0+0 < 5$

$0 < 5$ (True).

As; $x \geq 0$

Its associated Equation:



D(-1,0)

Point test: O(0,0)

$y-2x < 2$

$0-2(0) < 2$

$0 < 2$ (True).

$x+y \geq 5$

$x-y \geq 1$

$y \geq 0$

$x+y > 5$

Its associated Eq.

$x+y=5$ — (i)

Put $x=0$ in (i)

$0+y=5$

$y=5$

A(0,5)

Put $y=0$ in (i)

$x+0=5$

$x=5$

B(5,0)

Point test: O(0,0).

$x+y>5$

$0+0>5$

$0>5$ (False).

$x-y \geq 1$

Its associated Eq.

$x-y=1$ — (ii)

Put $x=0$ in (ii)

$0-y=1$

$y=-1$

C(0,-1).

Put $y=0$ in (ii)

$x+0=1$

$x=1$

D(1,0)

:O(0,0)

$x-y>1$

$0-0>1$

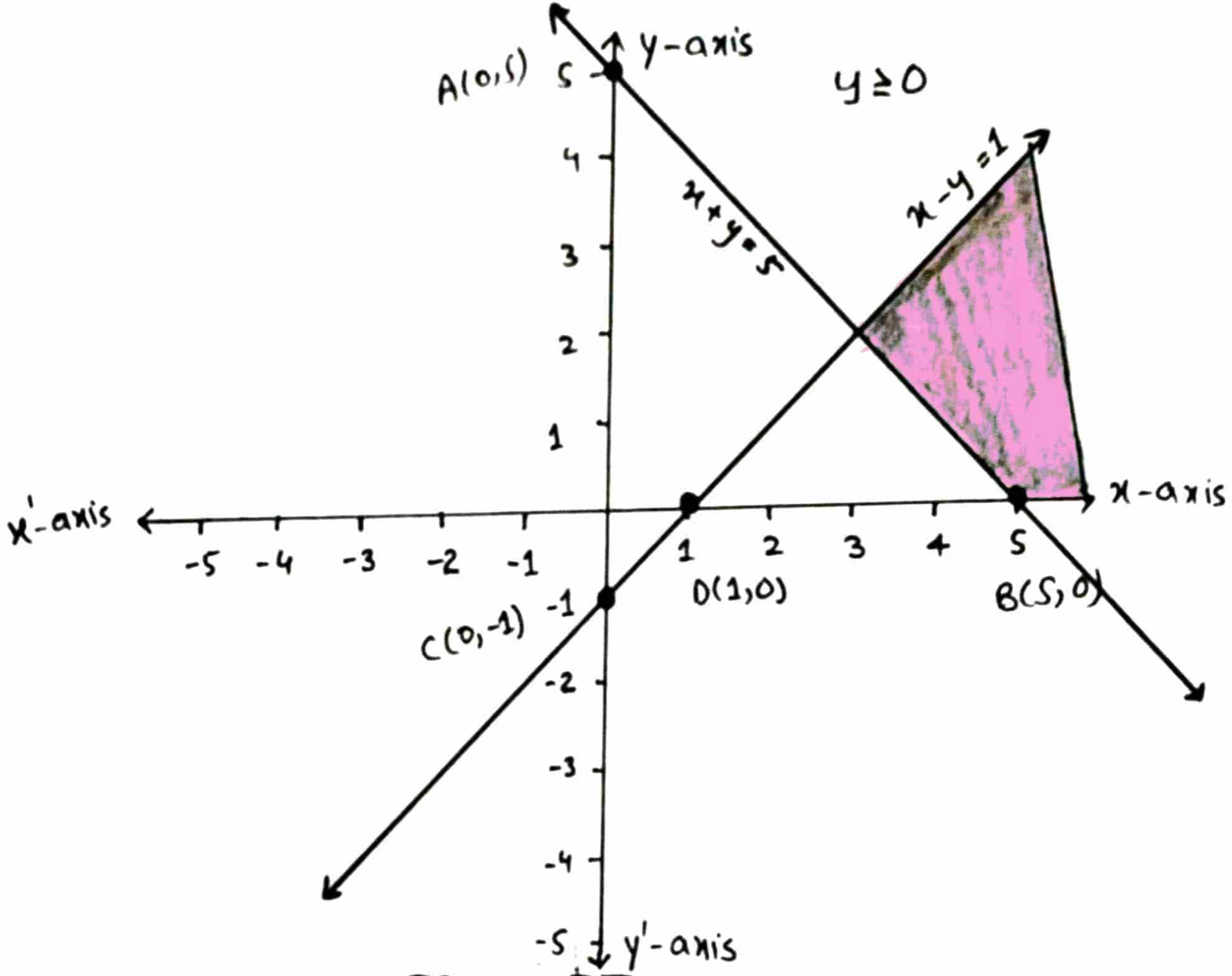
$0>1$ (False).

And;

$$y \geq 0$$

Its associated Equation.

$y=0$



iv

$3x + 7y \leq 21$

$x - y \leq 2$

$x \geq 0$

$3x + 7y \leq 21$

Its associated Equation:

$3x + 7y = 21$ —(i)

Put $x = 0$ in (i)

$3(0) + 7y = 21$

$y = 3$

$A(0,3)$

Put $y = 0$ in (i)

$3x + 7(0) = 21$

$x = 7$

$x - y \leq 2$

Its associated Equation

$x - y = 2$ —(ii)

Put $x = 0$ in (ii).

$0 - y = 2$

$y = -2$

$C(0,-2)$

Put $y = 0$ in (ii)

$x - 0 = 2$

$x = 2$

B(7,0)

Point test: O(0,0)

$3x+7y < 21$

$3(0)+7(0) < 21$

$0 < 21$ (True)

D(2,0)

Point test: O(0,0)

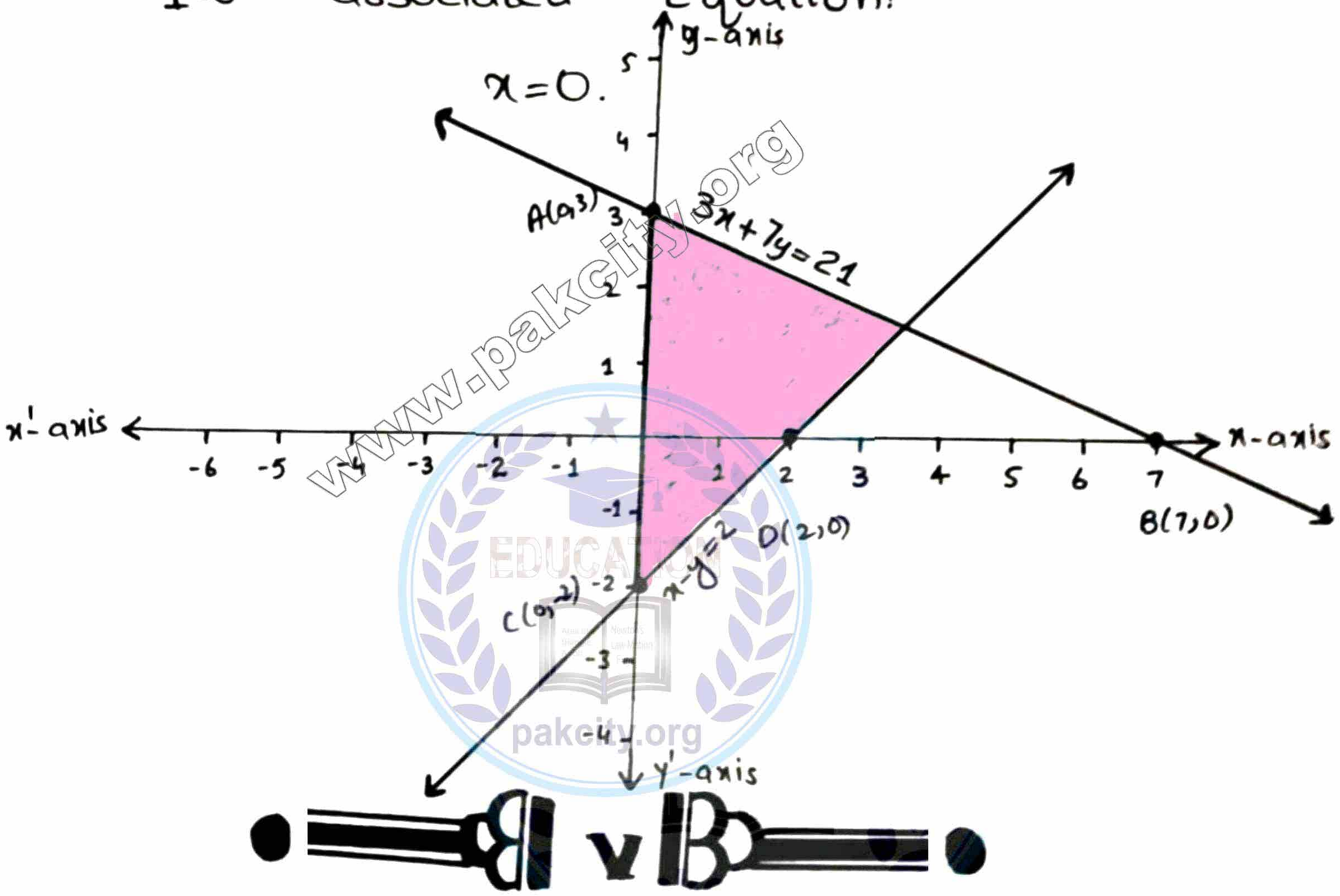
$x-y < 2$

$0-0 < 2$

$0 < 2$ (True).

As; $x \geq 0$

Its associated Equation:



$3x+7y \leq 21$

$x-y \leq 2$

$y \geq 0$

$3x+7y \leq 21$

Its associated Equation:

$3x+7y = 21$ ——(i)

Put $x=0$ in (i)

$3(0)+7y = 21$

$y = 3$

$A(0,3)$

Put $y=0$ in (i)

$3x+7(0) = 21$

$x = 7$

$B(7,0)$

Point test: $O(0,0)$

$3x+7y < 21$

$3(0)+7(0) < 21$

$0 < 21$ (True).

$x-y \leq 2$

Its associated Eq.

$x-y = 2$ —(ii)

Put $x=0$ in (ii).

$0-y = 2$

$y = -2$

$C(0,-2)$

Put $y=0$ in (ii)

$x-0 = 2$

$x = 2$

$D(2,0)$

Point test: $O(0,0)$.

$x-y < 2$

$0-0 < 2$

$0 < 2$ (True).

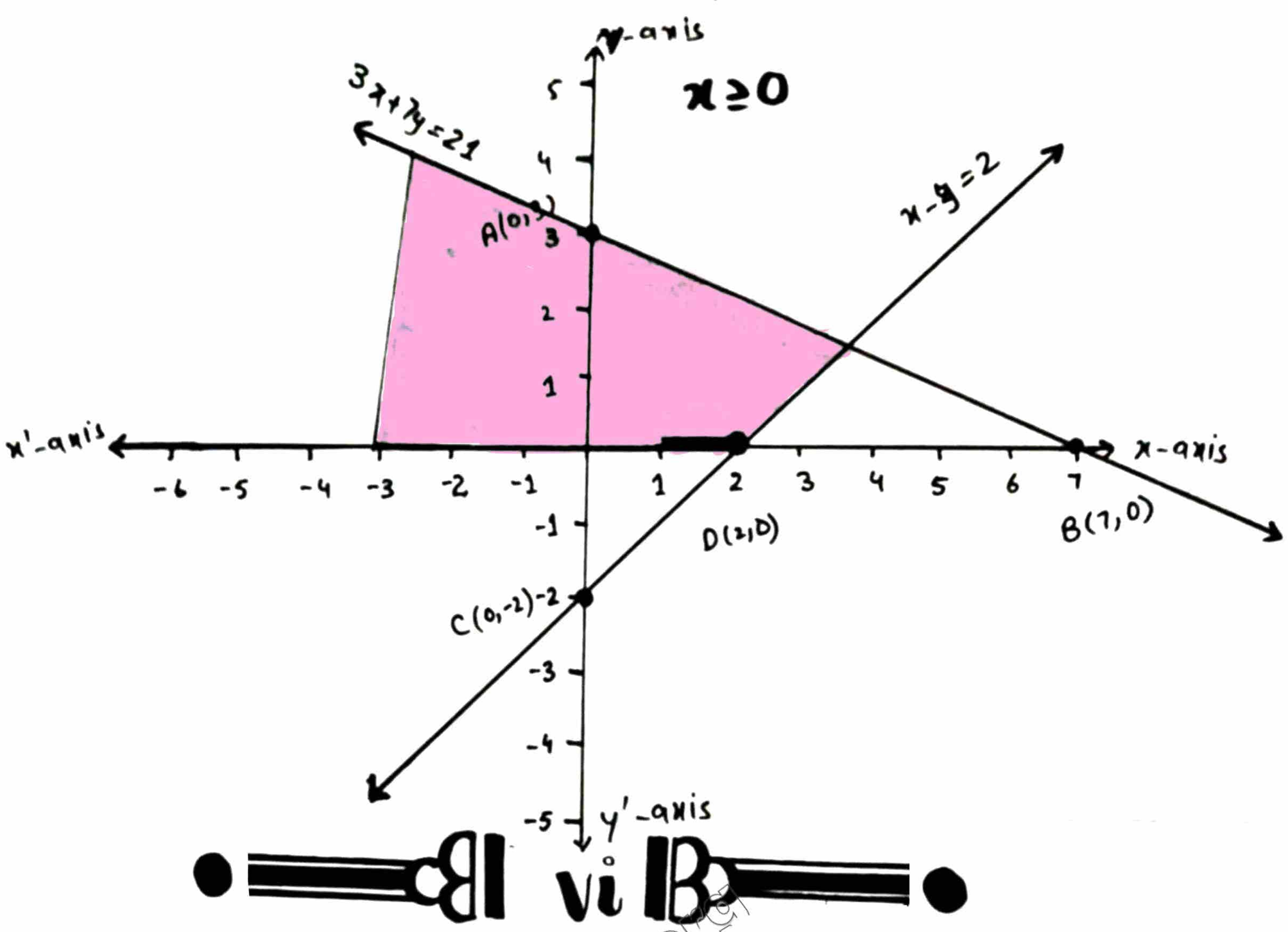
And $y \geq 0$

Its associated Equation

$y = 0$

So,

the combined graph:



$3x+7y \leq 21$

$2x-y \geq -3$

$x \geq 0$

$3x+7y \leq 21$

Its associated equation,

$3x+7y = 21$ ——— (i)

Put $x=0$ in (i)

$3(0)+7y = 21$

$7y = 3$

$A(0,3)$

Put $y=0$ in (i)

$3x+7(0) = 21$

$x = 7$

$2x-y \geq -3$

Its associated equation

$2x-y = -3$

Put $x=0$ in (i)

$2(0)-y = -3$

$y = 3$

$C(0,3)$

Put $y=0$ in (i).

$2x-0 = -3$

$x = -3/2$

B(7, 0)

D($-\frac{3}{2}$, 0).

Point test: O(0,0)

$3x + 7y < 21$

$3(0) + 7(0) < 21$

$0 < 21$ (True)

Point test: O(0,0)

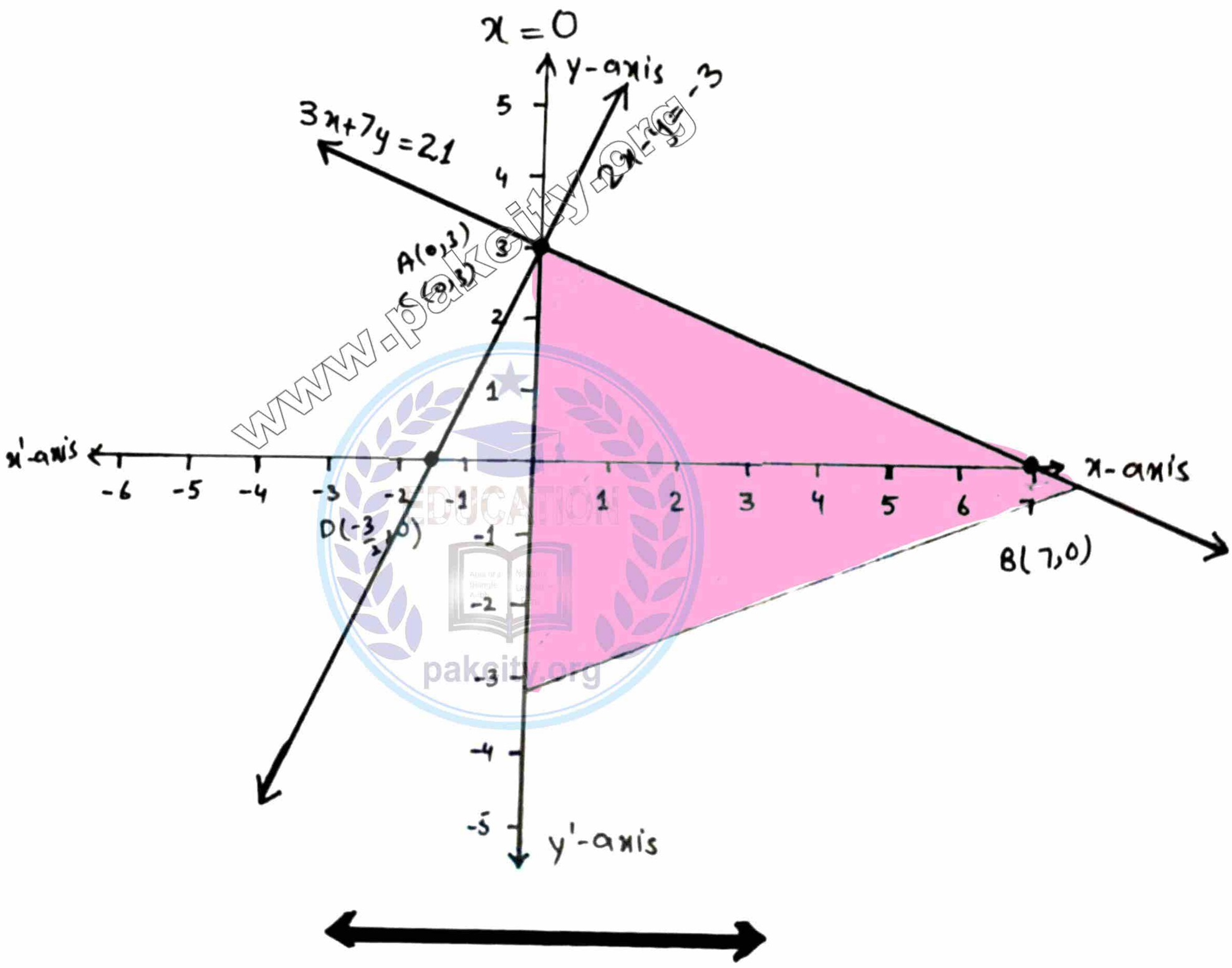
$2x - y > -3$

$2(0) - 0 > -3$

$0 > -3$ (True).

And: $x \geq 0$

Its associated equation.



Question: 5

Graph the solution region of the following system of linear inequalities by shading:

$$3x - 4y \leq 12$$

$$3x + 2y \geq 3$$

$$x + 2y \leq 9$$

Its associated eq.

$$3x - 4y = 12 \quad \text{--- (i)}$$

Put $x = 0$ in (i)

$$3(0) - 4y = 12$$

$$y = -3$$

A(0, -3)

Put $y = 0$ in (i)

$$3x - 4(0) = 12$$

$$x = 4$$

B(4, 0).

Its associated eq.

$$3x + 2y = 3 \quad \text{--- (ii)}$$

Put $x = 0$ in (ii)

$$3(0) + 2y = 3$$

$$y = 3/2$$

C(0, 3/2)

Put $y = 0$ in (ii)

$$3x + 2(0) = 3$$

$$x = 1$$

D(1, 0)

Its associated eq.

$$x + 2y = 9 \quad \text{--- (iii)}$$

Put $x = 0$ in (iii)

$$0 + 2y = 9/2$$

$$y = 9/2$$

E(0, 9/2)

Put $y = 0$ in (iii).

$$x + 2(0) = 9$$

$$x = 9$$

F(9, 0)

Point test: O(0,0)

$3x - 4y < 12$

$3(0) - 4(0) < 12$

$0 < 12$ (True).

Point test: O(0,0)

$3x + 2y > 3$

$3(0) + 2(0) > 3$

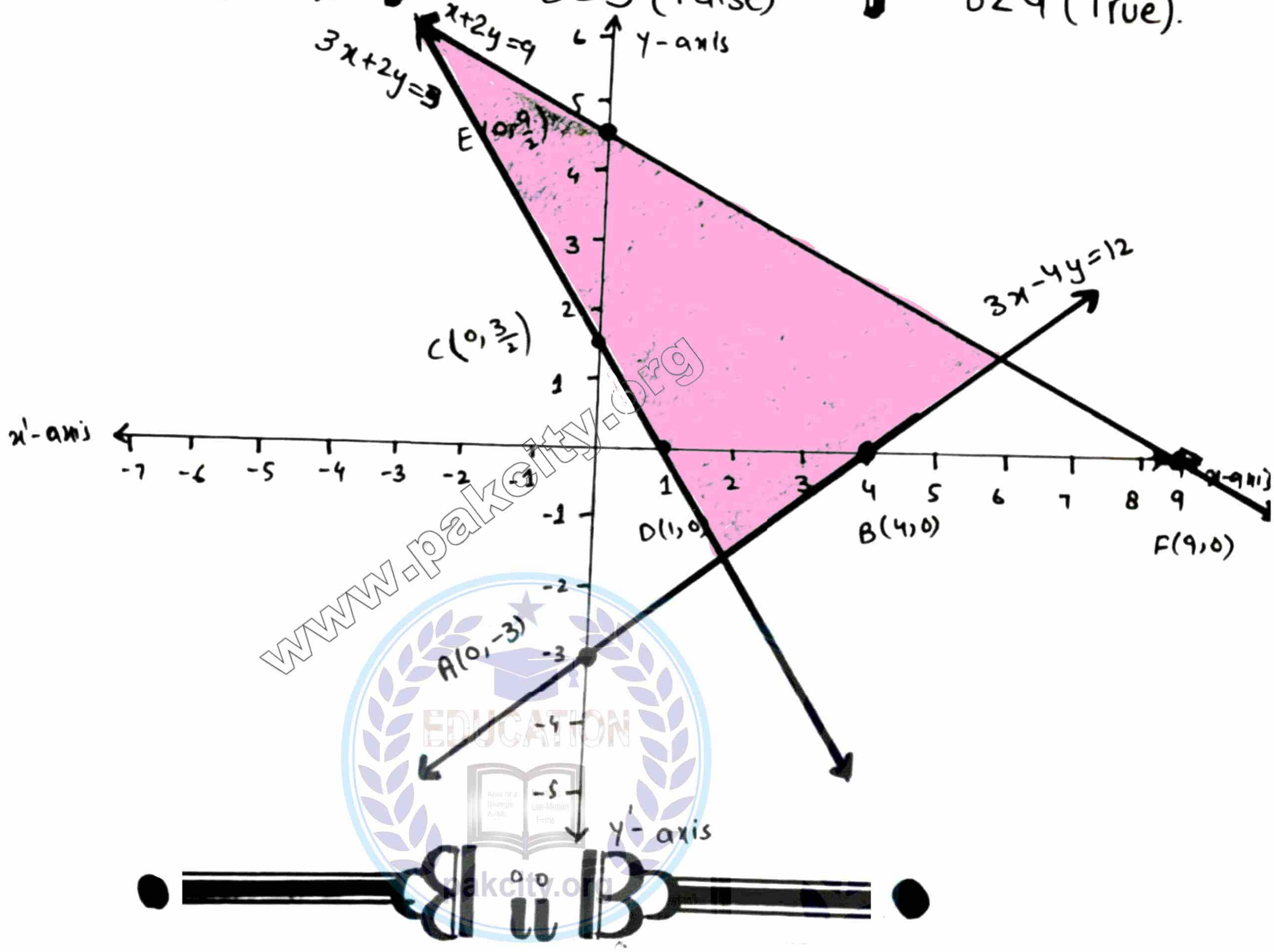
$0 > 3$ (False)

Point test: O(0,0)

$x + 2y < 9$

$0 + 2(0) < 9$

$0 < 9$ (True).



$3x - 4y \leq 12$

$x + 2y \leq 6$

$x + 1 \geq 1$

$3x - 4y \leq 12$

Its associated eq.

$x + 2y \leq 6$

Its associated eq

$x + y \geq 1$

Its associated eq.

$3x-4y=12$ —(i)

Put $x=0$ in (i)

$3(0)-4y=12$

$y=-3$

$A(0,-3)$

Put $y=0$ in (i)

$3x-4(0)=12$

$x=4$

$B(4,0)$

Point test: $O(0,0)$

$3x-4y<12$

$3(0)-4(0)<12$

$0<12$ (True)

$x+2y=6$ —(ii)

Put $x=0$ in (ii)

$0+2y=6$

$y=3$

$C(0,3)$

Put $y=0$ in (ii)

$x+2(0)=6$

$x=6$

$D(6,0)$

Point test:

$x+2y<6$

$0+2(0)<6$

$0<6$ (True)

$x+y=1$ —(iii)

Put $x=0$ in (iii)

$0+y=1$

$y=1$

$E(0,1)$

Put $y=0$ in (iii)

$x+0=1$

$x=1$

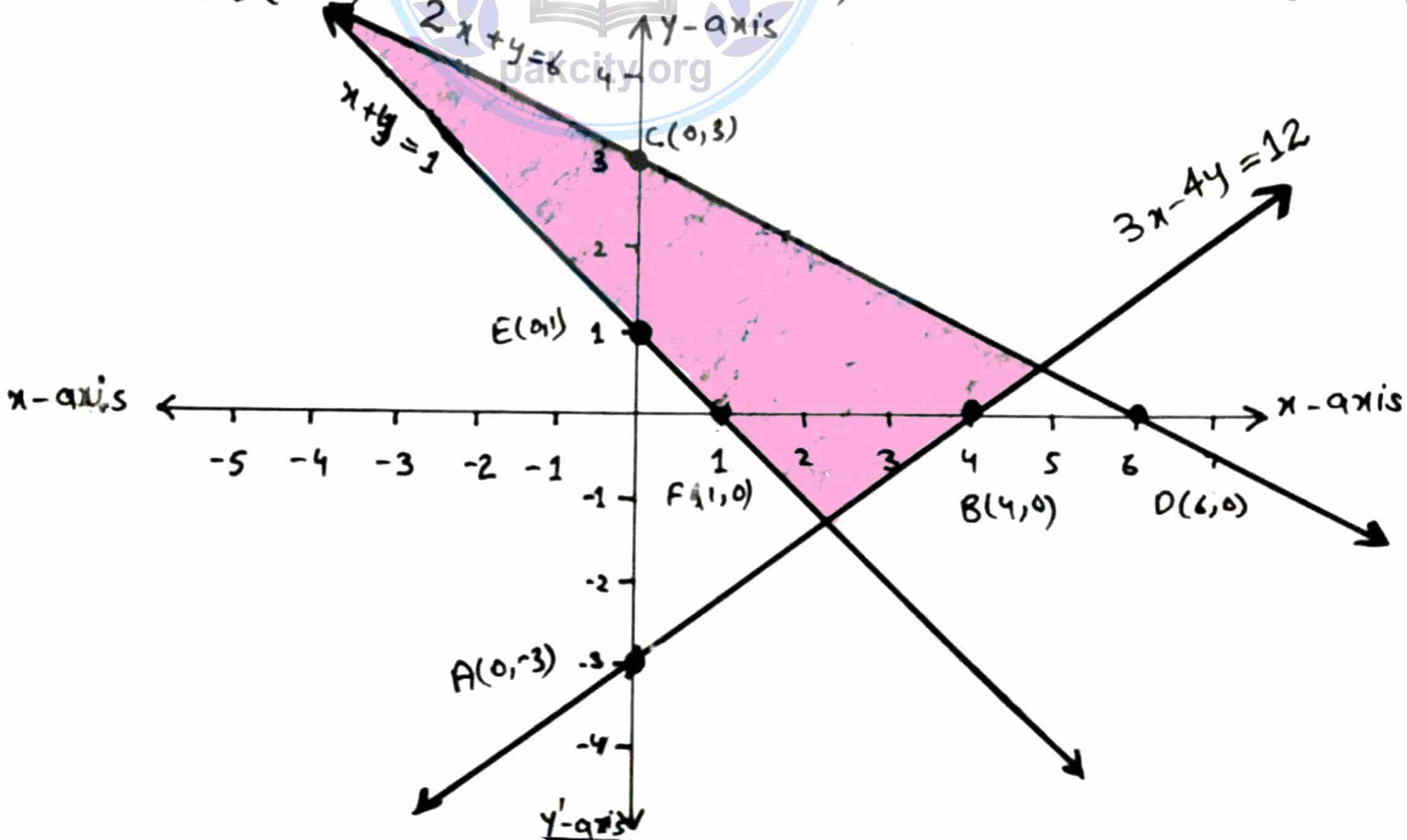
$F(1,0)$.

Point test:

$x+y>1$

$0+0>1$

$0>1$ (False).



$2x+y \leq 4$

$2x-3y \geq 12$

$x+2y \leq 6$

$2x+y \leq 4$

Its associated eq.

$2x+y=4$ ___ (i)

Put $x=0$ in (i).

$2(0)+y=4$

$y=4$

A$(0,4)$

Put $y=0$ in (i).

$2x+0=4$

$x=2$

B$(2,0)$

Point test: $O(0,0)$

$2x+y < 4$

$2(0)+0 < 4$

$0 < 4$ (True).

$2x-3y \geq 12$

Its associated eq.

$2x-3y=12$ ___ (ii)

Put $x=0$ in (ii).

$2(0)-3y=12$

$y=-4$

C$(0,-4)$

Put $y=0$ in (ii).

$2x-3(0)=12$

$x=\frac{12}{2}$

$x=6$

D$(6,0)$

Point test: $O(0,0)$.

$2x-3y > 12$

$2(0)-3(0) > 12$

$0 > 12$ (False).

$x+2y \leq 6$

Its associated eq.

$x+2y=6$ ___ (iii)

Put $x=0$ in (iii).

$0+2y=6$

$y=3$

E$(0,3)$

Put $y=0$ in (iii).

$x+2(0)=6$

$x=6$

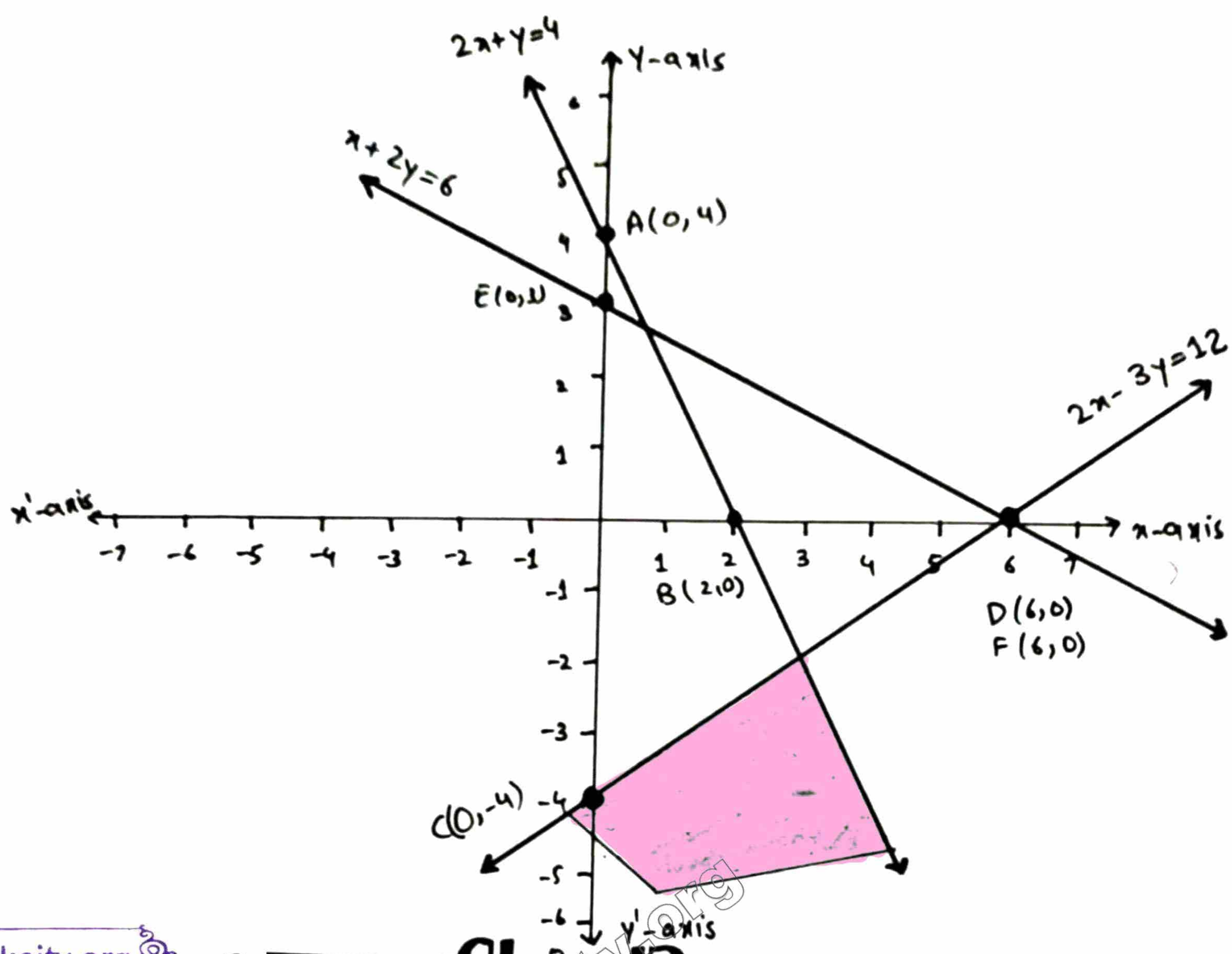
F$(6,0)$

Point test: $O(0,0)$

$x+2y < 6$

$0+2(0) < 6$

$0 < 6$ (True)



BI IVB

$2x+y \leq 10$

$x+y \leq 7$

$-2x+y \leq 4$

Its associated eq.

$2x+y=10$ — (i)

Put $x=0$ in (i)

$2(0)+y=10$

$y=10$

$A(0,10)$

Put $y=0$ in (i)

Its associated eq.

$x+y=7$ — (ii)

Put $x=0$ in (ii)

$0+y=7$

$y=7$

$C(0,7)$

Put $y=0$ in (ii)

Its associated eq.

$-2x+y=4$ — (iii)

Put $x=0$ in (iii)

$-2(0)+y=4$

$y=4$

$E(0,4)$

Put $y=0$ in (iii)

$2x+0=10$

$x=5$

$B(5,0)$

Point test:

$2x+y<10$

$2(0)+(0)<10$

$0<10$ (True)

$x+0=7$

$x=7$

$D(7,0)$

Point test:

$x+y<7$

$0+0<7$

$0<7$ (True).

$-2x+0=4$

$x=-2$

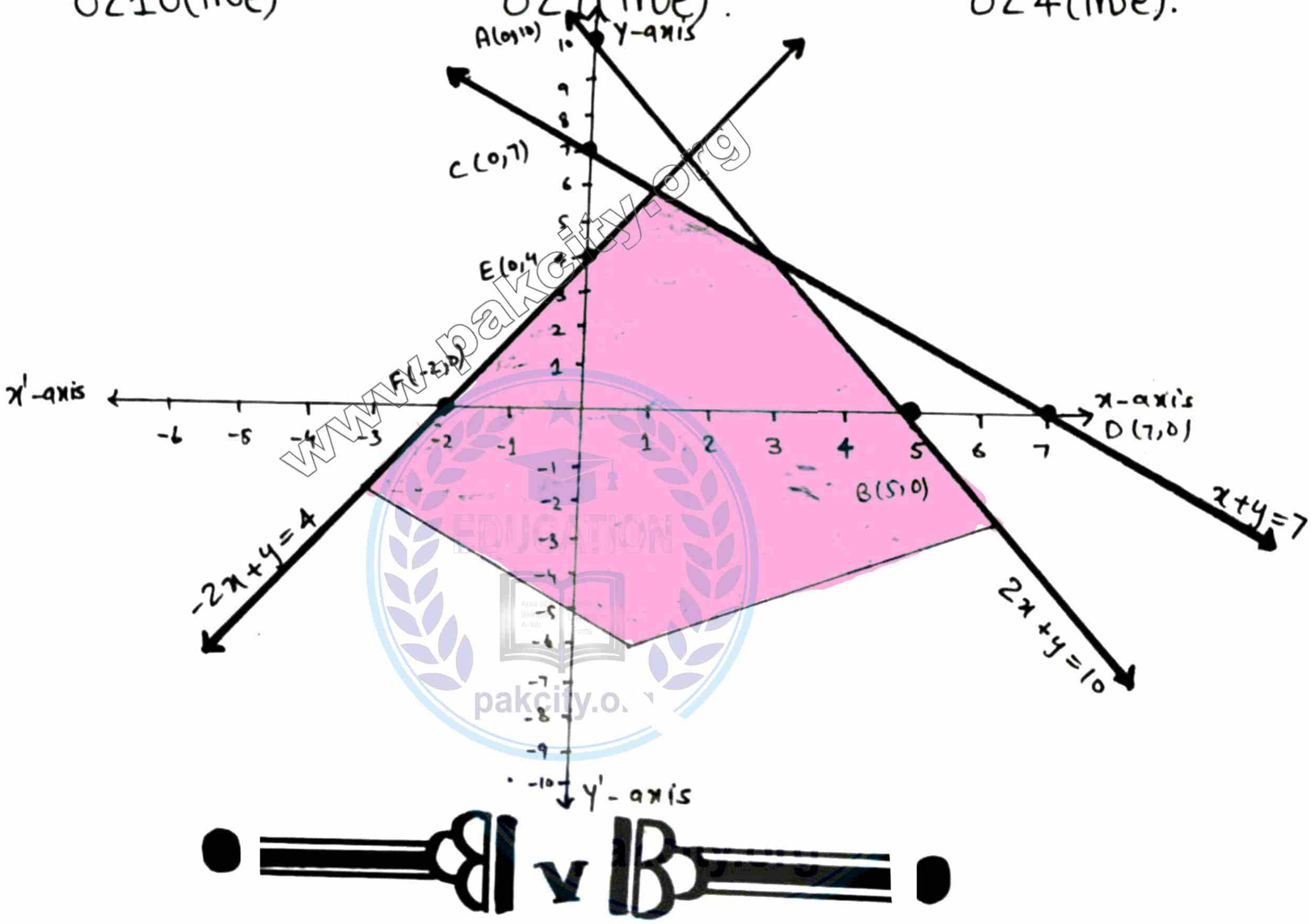
$F(-2,0)$.

Point test:

$-2x+y<4$

$-2(0)+0<4$

$0<4$ (True).



$2x+3y\leq 18$

$2x+y\leq 10$

$-2x+y\leq 2$

$2x+3y \leq 18$

Its associated eq

$2x+3y=18$ — (i)

Put $x=0$ in (i)

$2(0)+3y=18$

$y=6$

A(0, 6)

Put $y=0$ in (i)

$2x+3(0)=18$

$x=9$

B(9, 0)

Point test: O(0,0)

$2x+3y<18$

$2(0)+3(0)<18$

$0<18$ (True).

$2x+y \leq 10$

Its associated eq.

$2x+y=10$ — (ii)

Put $x=0$ in (ii).

$2(0)+y=10$

$y=10$

C(0, 10)

Put $y=0$ in (ii)

$2x+0=10$

$x=5$

D(5, 0)

Point test: O(0,0)

$2x+y<10$

$2(0)+0<10$

$0<10$ (True).

$-2x+y \leq 2$

Its associated eq.

$-2x+y=2$ — (iii)

Put $x=0$ in (iii).

$-2(0)+y=2$

$y=2$

E(0, 2)

Put $y=0$ in (iii)

$-2x+0=2$

$x=-1$

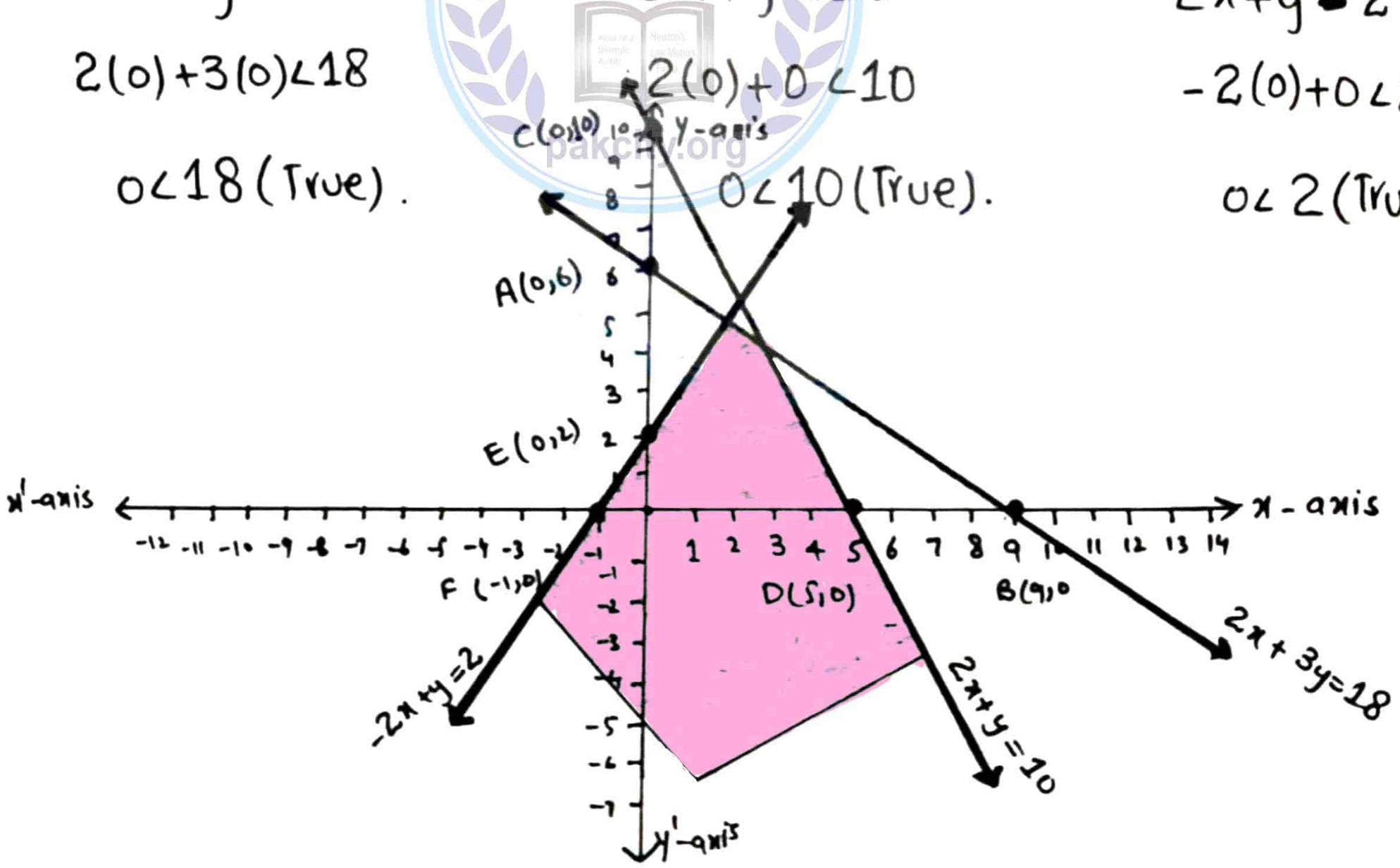
F(-1, 0)

Point test: O(0,0)

$-2x+y<2$

$-2(0)+0<2$

$0<2$ (True).



$3x-2y \geq 3$

$x+4y \leq 12$

$3x+y \leq 12$

$3x-2y \geq 3$

Its associated eq.

$3x-2y=3$ — (i)

Put $x=0$ in (i).

$3(0)-2y=3$

$y=-3/2$

A(0, -3/2)

Put $y=0$ in (i)

$3x-2(0) \geq 3$

$3x=3$

$x=1$

B(1, 0)

Point test:

$3x-2y>3$

$3(0)-2(0)>3$

$0>3$ (False).

$x+4y \leq 12$

Its associated eq.

$x+4y=12$ — (ii)

Put $x=0$ in (ii).

$0+4y=12$

$y=3$

C(0, 3)

Put $y=0$ in (ii).

$x+4(0)=12$

$x=12$

D(12, 0)

Point test:

$x+4y<12$

$0+4(0)<12$

$0<12$ (True).

$3x+y \leq 12$

Its associated eq.

$3x+y=12$ — (iii)

Put $x=0$ in (iii)

$3(0)+y=12$

$y=12$

E(0, 12)

Put $y=0$ in (iii)

$3x+0=12$

$x=4$

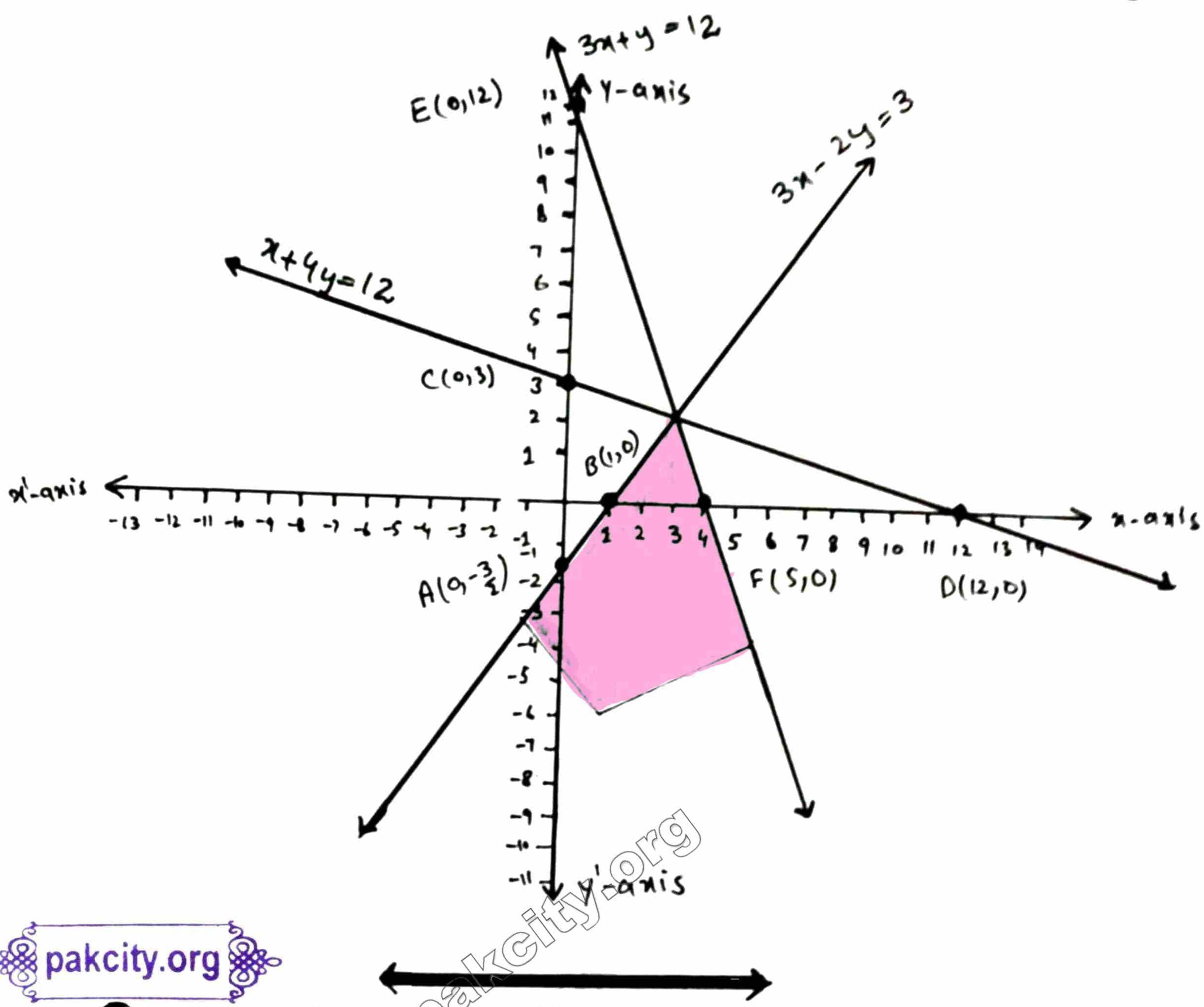
F(4, 0).

Point test:

$3x+y<12$

$3(0)+(0)<12$

$0<12$ (True).



Question: 4

BiB

$$2x-3y\leq 6$$

$$2x+3y\leq 12$$

$$x\geq 0$$

$2x-3y\leq 6$

Its associated eq.

$2x-3y=6$ — (i)

Put $x=0$ in (i)

$2x+3y\leq 12$

Its associated eq.

$2x+3y=12$ — (ii)

Put $x=0$ in (ii)

$2(0) - 3y = 6$

$y = -2$

$A(0, -2)$

Put $y=0$ in (i)

$2x - 3(0) = 6$

$x = 3$

$B(3,0)$

Point test: $O(0,0)$

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$ (True)

$2(0) + 3(y) = 12$

$y = 4$

$C(0,2)$

Put $y=0$ in (ii)

$2x + 3(0) = 12$

$x = 6$

$D(6,0)$

Point test: $O(0,0)$

$2x + 3y < 12$

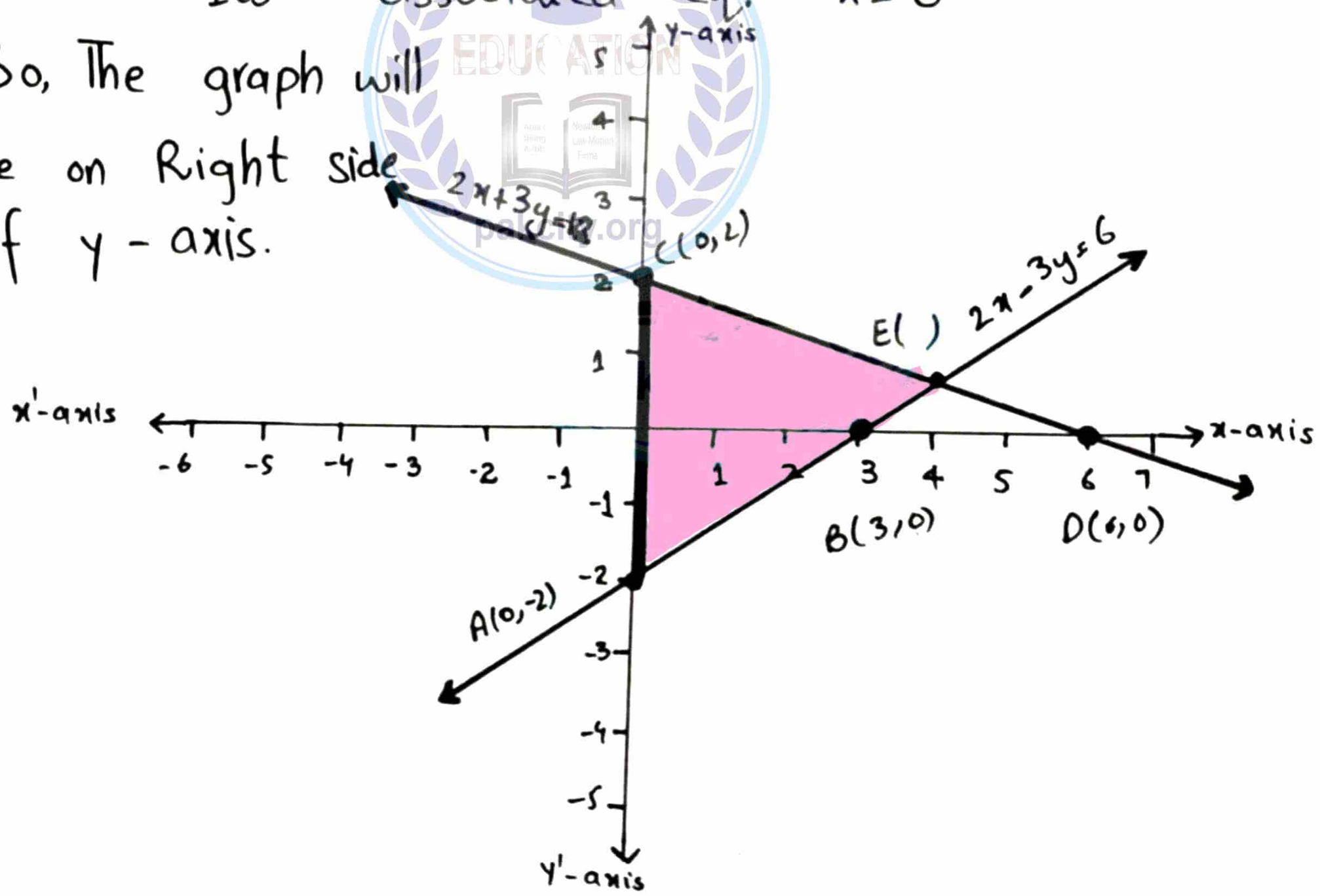
$2(0) + 3(0) < 12$

$0 < 12$ (True).

$x \geq 0$

Its associated eq. $x = 0$

So, The graph will be on Right side of y - axis.



Corner point:

O(0,0) ,A(0,-2), B(3,0), C(0,4), E( ).

For E:

$$2x - 3y = 6 \quad \text{---(1)}$$

$$2x + 3y = 12 \quad \text{---(2)}$$

By adding eq (3) and (4).

$$2x - 3y = 6$$

$$2x + 3y = 12$$

$$4x = 18$$

$$\boxed{x = 9/2}$$

Put $x = 9/2$ in eq (3).

$$2(9/2) - 3y = 6$$

$$9 - 3y = 6 \quad \Rightarrow \quad -3y = 6 - 9$$

$$-3y = -3$$

$$\boxed{y = 1}$$

So, $E(9/2, 1)$

$$x + y \leq 5$$

$$-2x + y \leq 2$$

$$y \geq 0$$

$x + y \leq 5$

Its associated Eq.

$x + y = 5$ ---(i)

$-2x + y \leq 2$

Its associated Eq.

$-2x + y = 2$ ---(ii)

Put $x=0$ in i)

$$0+y=5$$

$$y=5$$

$A(0,5)$

Put $y=0$ in i)

$$x+0=5$$

$$x=5$$

$B(5,0)$

Point test: $O(0,0)$

$$x+y<5$$

$$0+0<5$$

$0<5$ (True).

Put $x=0$ in ii).

$$-2(0)+y=2$$

$$y=2$$

$C(0,2)$

Put $y=0$ in ii)

$$-2x+0=2$$

$$x=-1$$

$D(-1,0)$.

Point test: $O(0,0)$

$$-2x+y<2$$

$$-2(0)+0<2$$

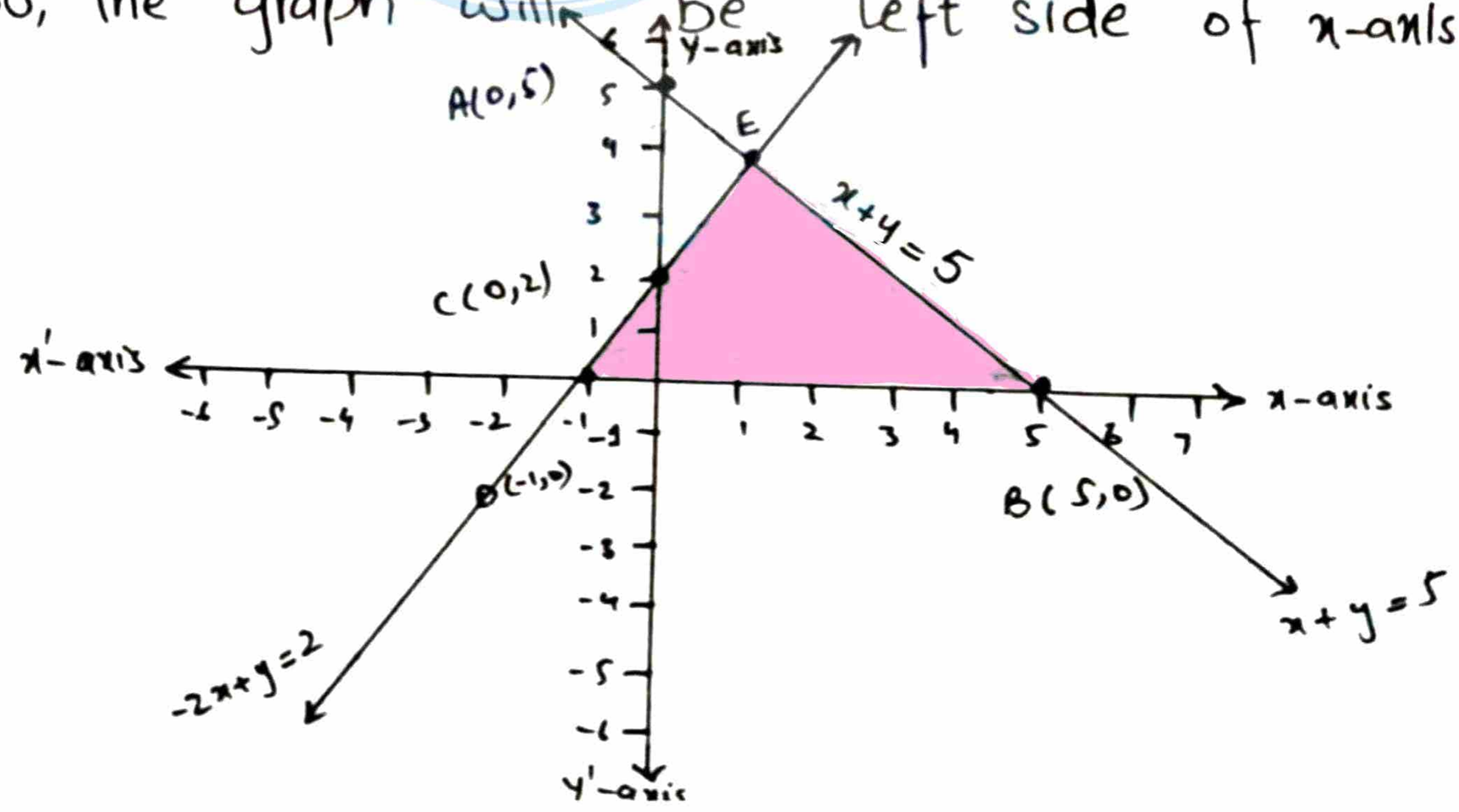
$0<2$ (True).

$$y \geq 0$$

Its associated Equation.

$$y=0$$

So, the graph will be left side of x-axis.



Corner point:

B(5, 0), C(0, 2), D(-1, 0), E(1, 4).

For E:

$$x + y = 5 \longrightarrow (3)$$

$$-2x + y = 2 \longrightarrow (4)$$

Subtracting (3) and (4).

$$-2x + y = 2$$
$$-x \pm y = 5$$
$$-3x = -3$$
$$\boxed{x = 1}$$

Put $x = 1$ in eq (3).

$$1 + y = 5$$
$$y = 5 - 1$$
$$\boxed{y = 4}$$

So, E(1, 4)

$3x + 7y \leq 21$

$2x - y \leq -3$

$y \geq 0$

$3x + 7y \leq 21$

Its associated eq.

$3x + 7y = 21$ — (i)

(LHR-2012)

$2x - y \leq -3$

Its associated eq.

$2x - y = -3$

Put $x=0$ in (i)

$3(0)+7y=21$

$y=3 \Rightarrow A(0,3)$

Put $y=0$ in (i)

$3x+7(0)=21$

$x=7$

$B(7,0)$

Point test: $O(0,0)$

$3x+7y<21$

$3(0)+7(0)<21$

$0<21$ (True)

Put $y=0$ in (ii)

$2x-0=-3$

$x=-3/2$

$D(-3/2, 0)$

Put $x=0$ in (ii)

$2(0)-y=-3$

$y=3$

$C(0,3)$

Point test: $O(0,0)$

$2x-y<3$

$2(0)-0<3$

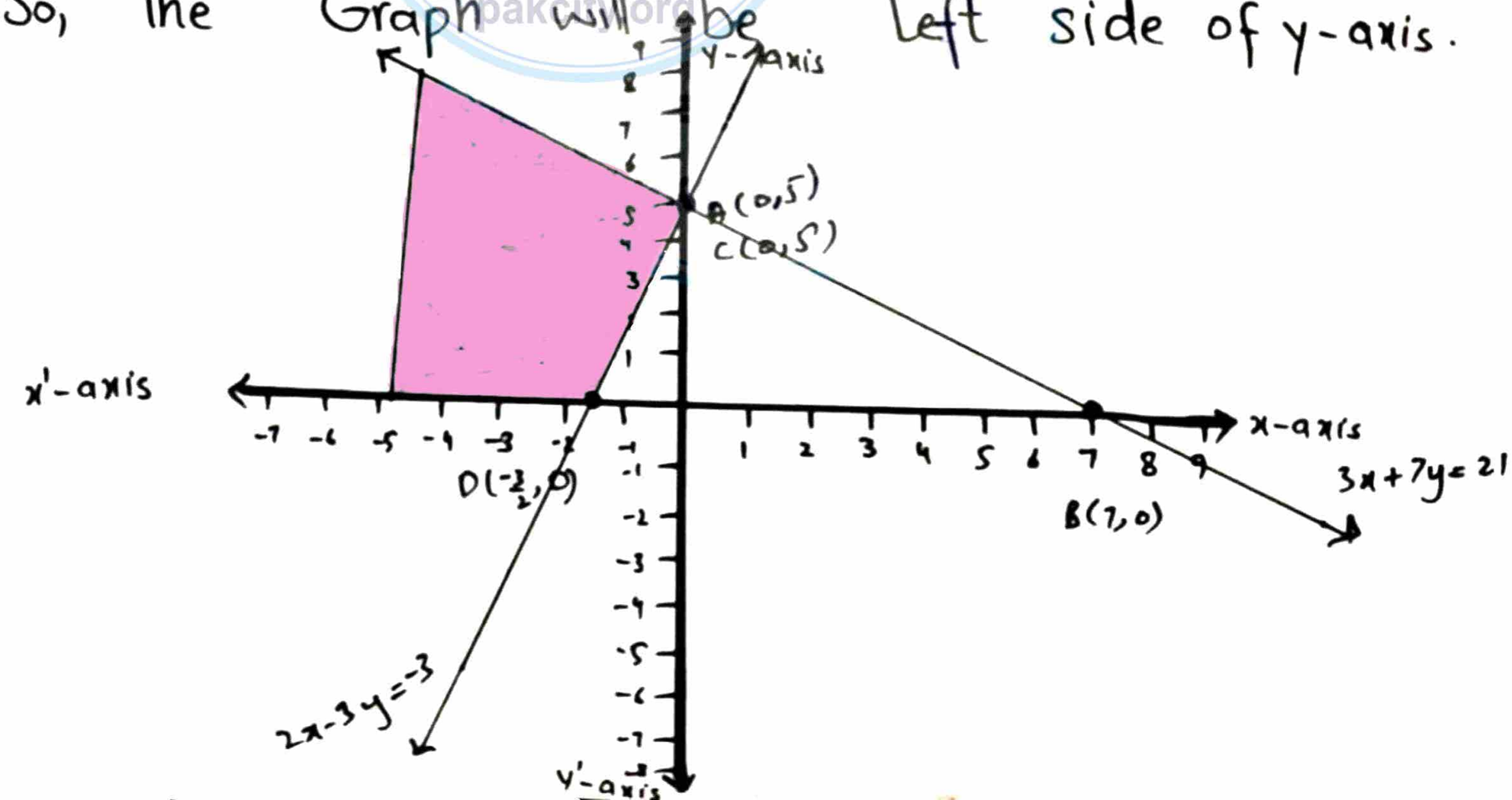
$0<3$ (True)

$y \geq 0$

Its associated equation.

$y=0$

So, The Graph will be Left side of y-axis.



Corner points:

A(0,3), C(0,3), D($-\frac{3}{2}$,0)

$3x+2y \geq 6$

$x+3y \leq 6$

$y \geq 0$

$3x+2y \geq 6$

Its associated equation.

$3x+2y=6$ — (i)

Put $x=0$ in (i)

$3(0)+2y=6$

$y=3$

$A(0,3)$

Put $y=0$ in (i)

$3x+2(0)=6$

$x=2$

$B(2,0)$

Point test: $O(0,0)$

$3x+2y > 6$

$3(0)+2(0) > 6$

$0 > 6$ (False).

$x+3y \leq 6$

Its associated Equation:

$x+3y=6$ — (ii)

Put $x=0$ in (ii)

$0+3y=6$

$y=2$

$C(0,2)$

Put $y=0$ in (ii)

$x+3(0)=6$

$x=6$

$D(6,0)$

Point test: $O(0,0)$.

$x+3y < 6$

$0+3(0) < 6$

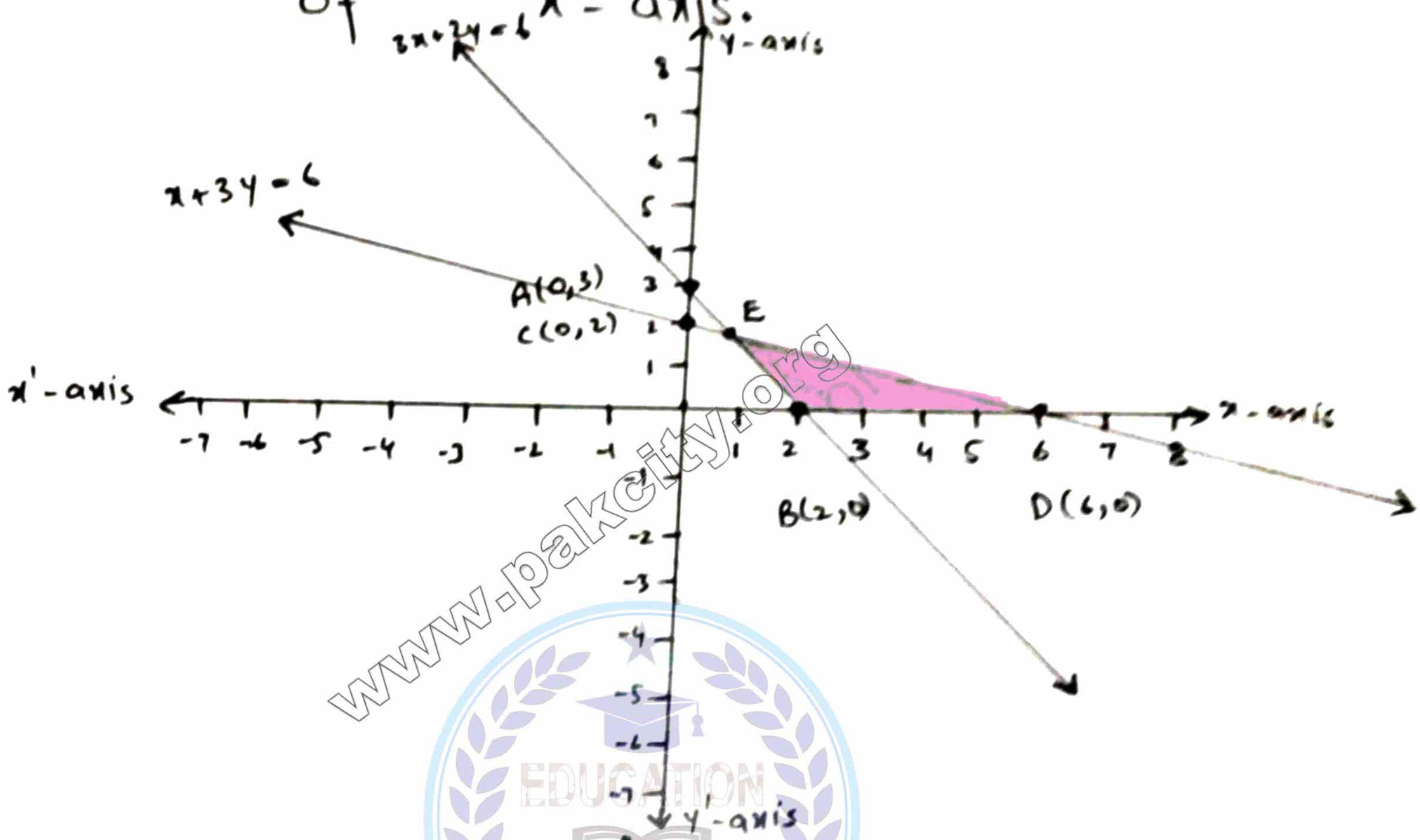
$0 < 6$ (True).

$$y = 0$$

Its associated Equation: $y = 0$

So,

The Graph will be left side of x - axis.



Corner points:

$B(2,0), D(6,0), E(\frac{6}{7}, \frac{12}{7})$

For E

$$3x + 2y = 6 \longrightarrow (2)$$

$$x + 3y = 6 \longrightarrow (3)$$

Multiplying eq (3) by 3.

$$3x + 9y = 18 \longrightarrow (4)$$

Subtracting eq (4) and (2).

$$3x + 2y = 6$$
$$-3x \pm 9y = -18$$
$$-7y = -12$$

$$\boxed{y = 12/7}$$

Put $y = 12/7$ in eq(3).

$$x + 3\left(\frac{12}{7}\right) = 6$$

$$x + \frac{36}{7} = 6 \quad \Rightarrow x = 6 - \frac{36}{7}$$

$$x = \frac{42 - 36}{7} \quad \Rightarrow$$

$$\boxed{x = \frac{6}{7}}$$

So, $E\left(\frac{6}{7}, \frac{12}{7}\right)$

$$5x + 7y \leq 35$$
$$-x + 3y \leq 3$$
$$x \geq 0$$

$5x + 7y \leq 35$

Its associated eq.

$$5x + 7y = 35 \quad \text{—(i)}$$

Put $x = 0$ in (i).

$$5(0) + 7y = 35$$

$-x + 3y \leq 3$

Its associated eq.

$$-x + 3y = 3 \quad \text{—(ii)}$$

Put $x = 0$ in (ii)

$$(0) + 3y = 3$$

$7y = 35$

$y = 5$

$A(0, 5)$

Put $y=0$ in (i)

$5x + 7(0) = 35$

$x = 7$

$B(7, 0)$

Point test: $O(0,0)$

$5x + 7y < 35$

$5(0) + 7(0) < 35$

$0 < 35$

$3y = 3$

$y = 1$

$C(0, 1)$

Put $y=0$ in (i).

$-x + 3(0) = 3$

$x = -3$

$D(-3, 0)$

Point test:

$-x + 3y < 3$

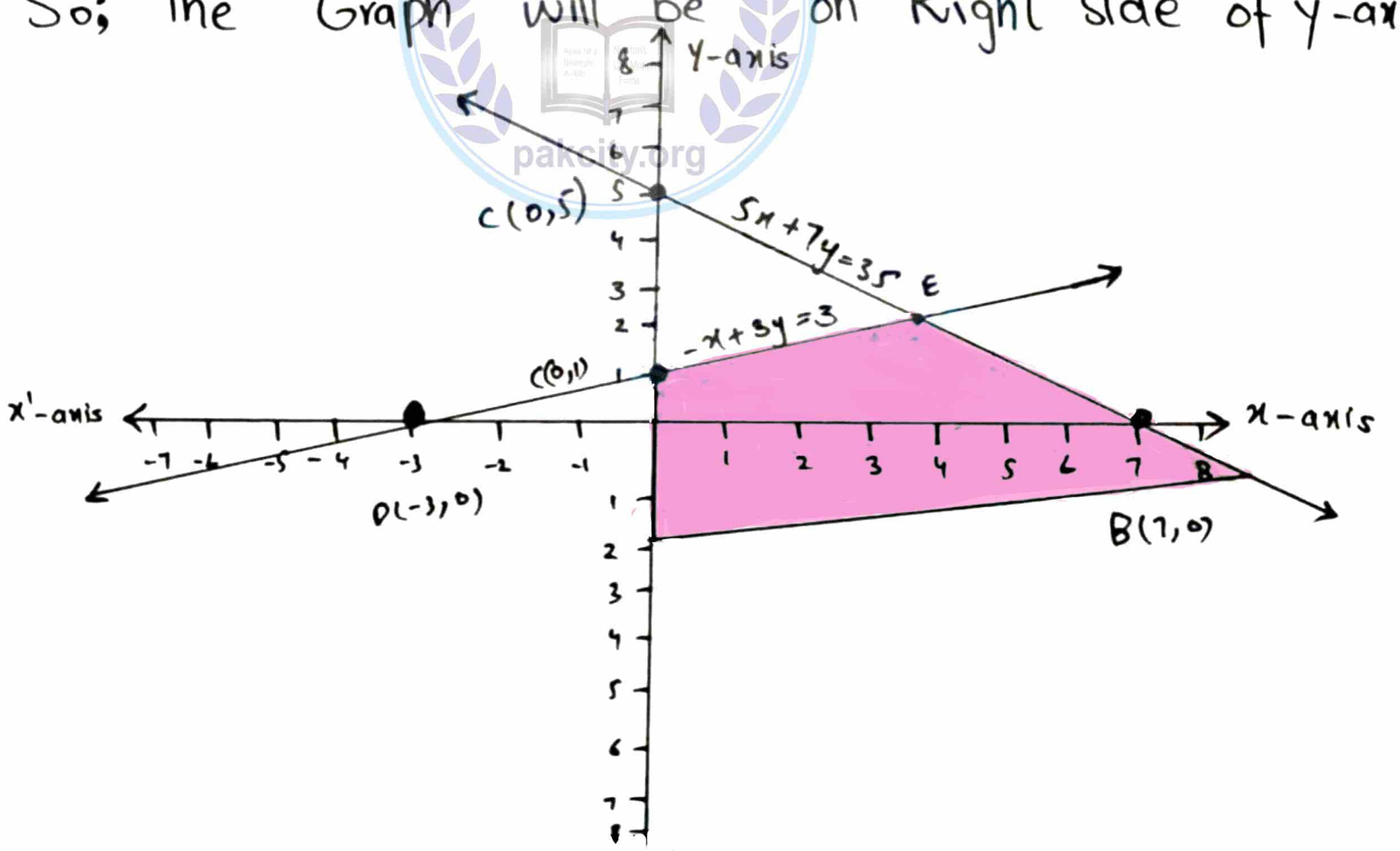
$-0 + 3(0) < 3$

$0 < 3$ (True).

and; $x \geq 0$

Its associated equation: $x = 0$

So; The Graph will be on Right side of y-axis.



Corner points:-

$O(0,0)$, $B(7,0)$, $C(0,1)$, $E\left(\frac{42}{11}, \frac{25}{11}\right)$.

For E,

$$5x + 7y = 35 \rightarrow (1)$$

$$-x + 3y = 3 \rightarrow (2)$$

Multiplying eq (2) by 5.

$$-5x + 15y = 15 \rightarrow (3).$$

Adding eq (1) and (3).

$$5x + 7y = 35$$

$$-5x + 15y = 15$$

$$22y = 50$$

$$y = \frac{50}{22} \Rightarrow \boxed{y = \frac{25}{11}}$$

Put $y = \frac{25}{11}$ in (i).

$$-x + 3\left(\frac{25}{11}\right) = 3$$

$$-x + \frac{75}{11} = 3 \Rightarrow -x = 3 - \frac{75}{11}$$

$$-x = \frac{33 - 75}{11}$$

$$-x = +\frac{42}{11} \Rightarrow \boxed{x = \frac{42}{11}}$$

$E\left(\frac{42}{11}, \frac{25}{11}\right)$.

$$5x+7y \leq 35$$

$$x-2y \leq 4$$

$$x \geq 0$$

$5x+7y \leq 35$

Its associated Equation:

$5x+7y = 35$ — (i).

Put $x=0$ in (i).

$5(0)+7y = 35$

$y = 5$

$A(0,5)$

Put $y=0$ in (i).

$5x+7(0) = 35$

$x = 7$

$B(7,0)$.

Point test: $O(0,0)$

$5x+7y < 35$

$5(0)+7(0) < 35$

$0 < 35$ (True)

$x-2y \leq 4$

Its associated Equation.

$x-2y = 4$ — (ii)

Put $x=0$ in (ii)

$0-2y = 4$

$y = -2$

$C(0,-2)$

Put $y=0$ in (ii).

$x-2(0) = 4$

$x = 4$

$D(4,0)$.

Point test: $O(0,0)$.

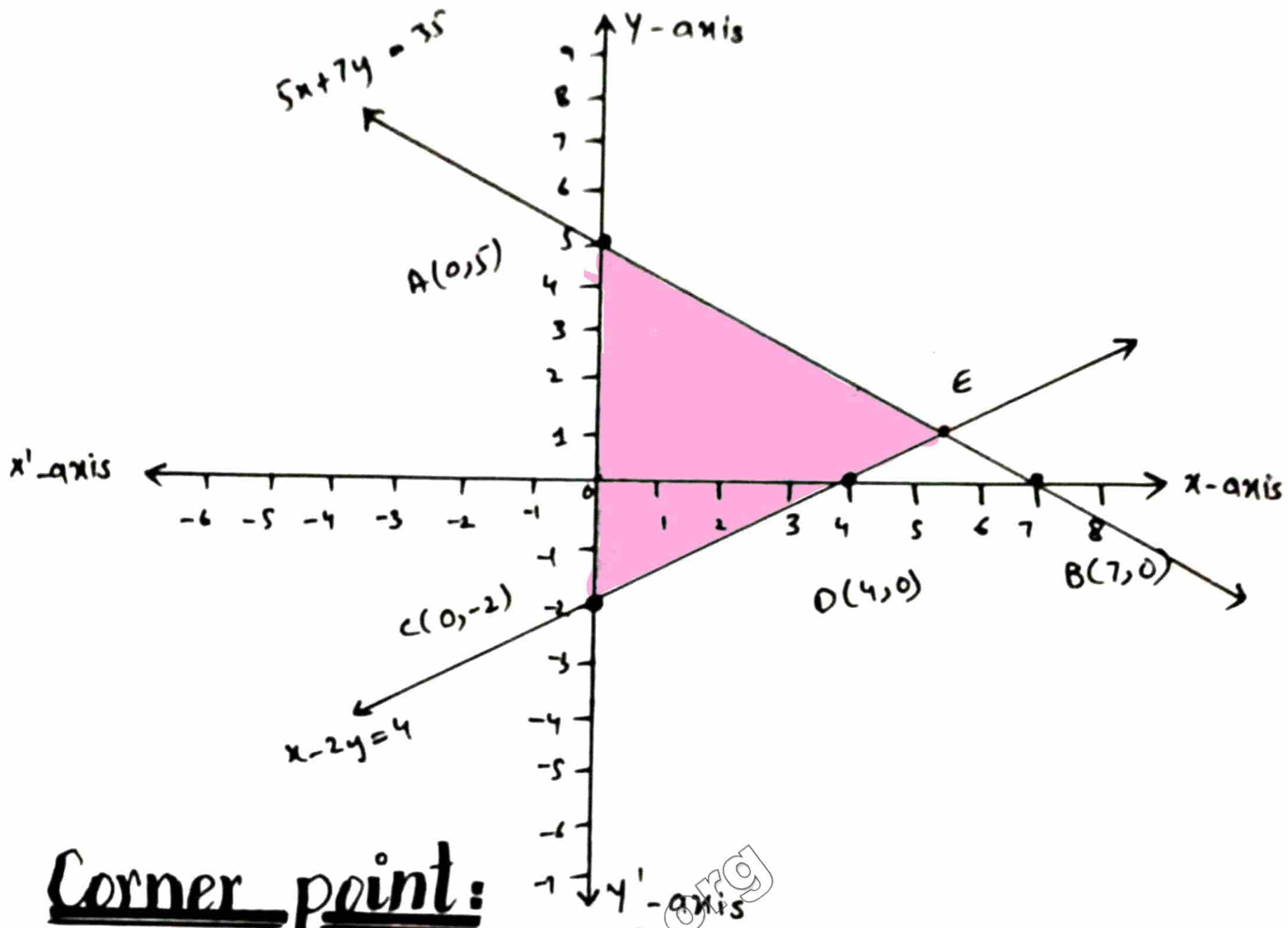
$x-2y < 4$

$0-2(0) < 4$

$0 < 4$ (True).

Its associated Equation.

So, The graph will be on Right side of y-axis.



Corner point:

A(0,5), C(0,-2), D(4,0), E( )

For E:

$$5x + 7y = 35 \longrightarrow (1)$$

$$x - 2y = 4 \longrightarrow (2)$$

Multiplying eq(2) by 5.

$$5x - 10y = 20 \longrightarrow (3)$$

Subtracting eq (1) and (3).

$$5x + 7y = 35$$

$$-5x + 10y = -20$$

$$17y =$$

Problem Constraints

(LHR-13,15)

"The system of linear inequalities involved in the problem concerned are called problem constraints."

Non-negative Constraints:

"The variables used in the system of linear inequalities relating to the problem of every day life are non-negative and are called non-negative constraints."

Feasible region:

"A region (which is restricted to the first quadrant) is referred to as a feasible region for the set of given constraints."

Decision variables:

"The non-negative constraints plays an important role for taking decision. So, these non-negative variables are called decision variables."

feasible solution: (LHR-2014)

"In feasible region, each pant of region is called a feasible solution of the system of linear inequalities."

feasible solution set: (LHR-2013)

"A set consisting of all the feasible solutions of the system of linear inequalities is called feasible solution set."

Convex:

If the line segment obtained by joining any two points of a region lies entirely within the region, then the region is called convex.

Example: 3

(a) $2x - 3y \leq 6$

$2x + y \geq 2$

$x \geq 0 \ , \ y \geq 2$

(LHR-2013,14,17)

$2x - 3y \leq 6$

Its associated equation:

$2x - 3y = 6 \rightarrow$ (i)

Put $x = 0$ in eq (i)

$2(0) - 3y = 6$

$y = -2$

$A(0, -2)$

Put $y = 0$ in eq (i).

$2x - 3(0) = 6$

$x = 3$

$B(3, 0)$

Point test: $O(0,0)$

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$ (True).

$2x + y \geq 2$

Its associated equation.

$2x + y = 2$ — (ii)

Put $x = 0$ in (ii)

$2(0) + y = 2$

$y = 2$

$C(0, 2)$

Put $y = 0$ in (ii).

$2x + 0 = 2$

$x = 1$

$D(1, 0)$.

Point test: $O(0,0)$.

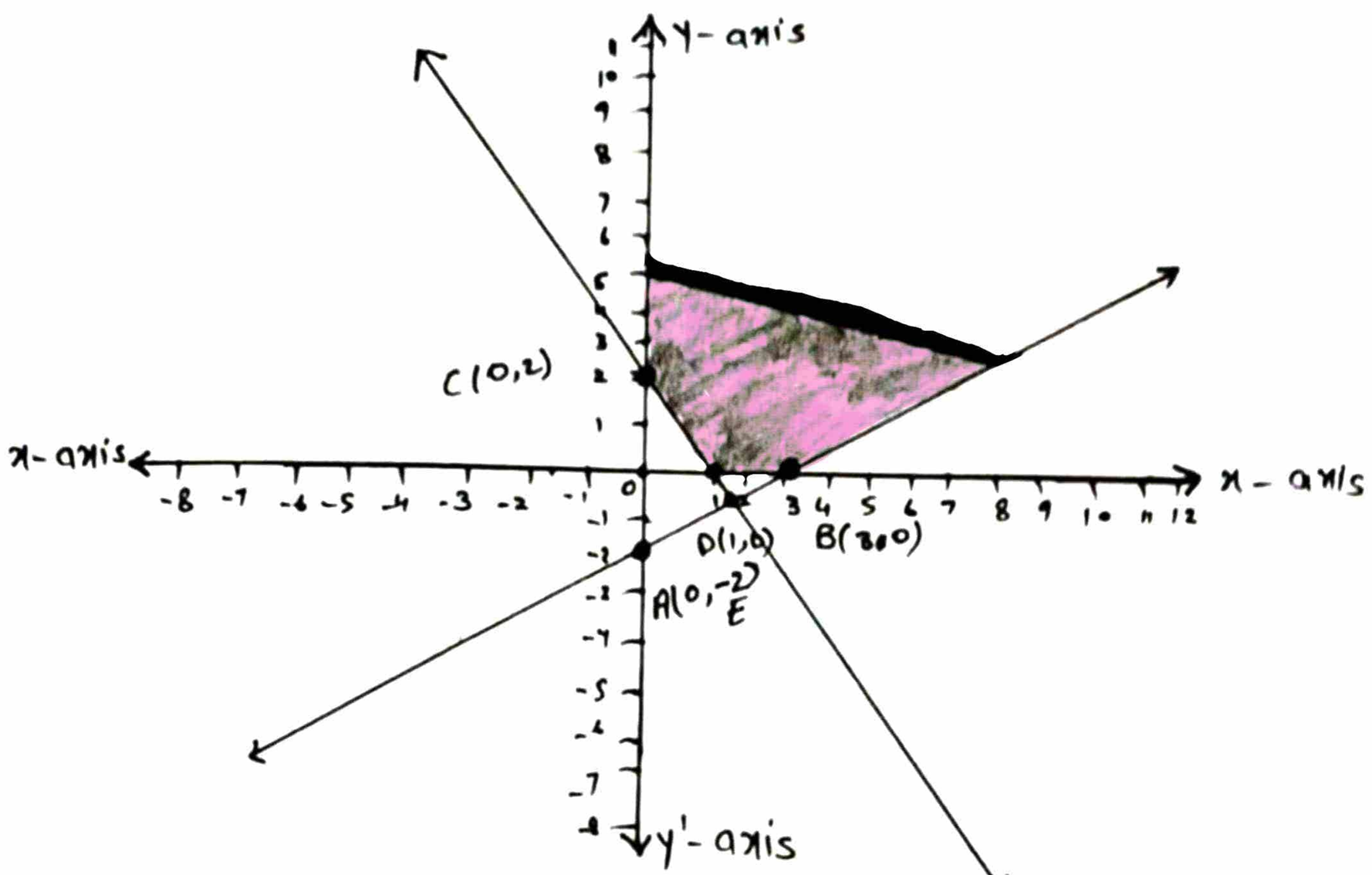
$2x + y \geq 2$

$2(0) + 0 \geq 2$

$0 \geq 2$ (False).

and: $x \geq 0 \ , \ y \geq 2$

The associated Eq: $x = 0 \ , \ y = 2$



Coner point:

B(3,0), C(0,2), D(1,0), E( ).

For E:

$$2x - 3y = 6 \longrightarrow (1)$$

$$2x + y = 2 \longrightarrow (2)$$

Subtracting eq (1) and (2).

$$2x - 3y = 6$$

$$\underline{-2x \pm 3y = -2}$$

$$-4y = 4$$

$$y = -1$$

Putting $y = -1$ in eq(2).

$$2x - 1 = 2$$

$$2x = 2 + 1 \Rightarrow 2x = 3$$

$$x = 3/2$$

$$E\left(\frac{3}{2}, -1\right).$$

Exercise: 5.2

Question: 1

Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i)

$$2x - 3y \leq 6$$

$$2x + 3y \leq 12$$

(LHR - 2017)

$$x \geq 0, \quad y \geq 0$$

$2x - 3y \leq 6$

Its associated equation.

$2x - 3y = 6$ — (i)

Put $x = 0$ in (i)

$2(0) - 3y = 6$

$y = -2$

$A(0, -2)$

Put $y = 0$ in (i)

$2x + 3y \leq 12$

Its associated equation.

$2x + 3y = 12$ — (ii)

Put $x = 0$ in (ii).

$2(0) + 3y = 12$

$y = 4$

$C(0, 4)$

Put $y = 0$ in (ii)

$2x + 3(0) = 6$

$x = 3$

$B(3,0)$

Point test: $O(0,0)$.

$2x + 3y < 6$

$2(0) + 3(0) < 6$

$0 < 6$ (True).

$2x + 3(0) = 12$

$x = 6$

$D(6,0)$.

Point test: $O(0,0)$

$2x + 3y < 12$

$2(0) + 3(0) < 12$

$0 < 12$ (True).

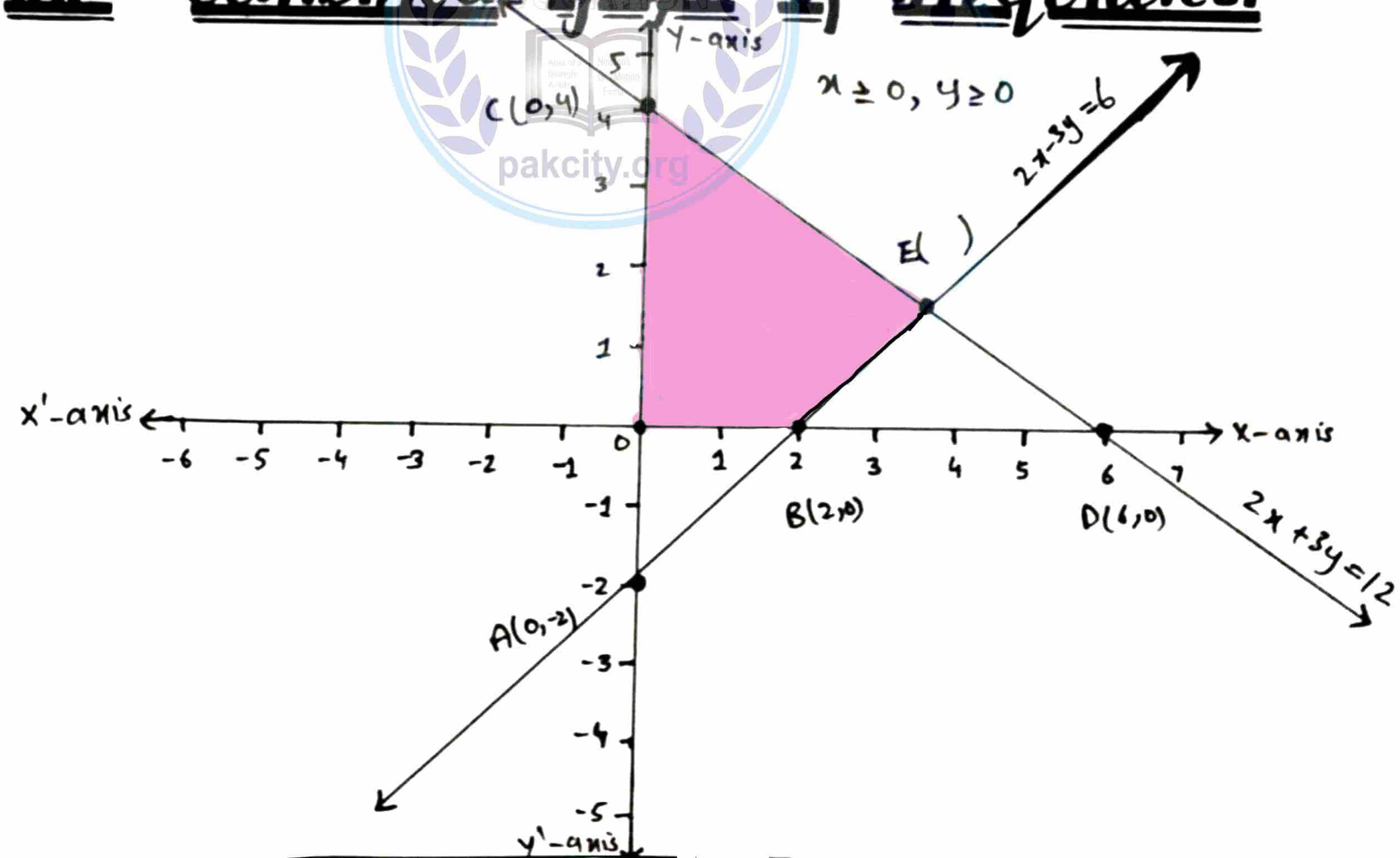
And: $x \geq 0$, $y \geq 0$

The associated Equation:

$x = 0$, $y = 0$

The Graph will be in 1st Quad.

The Combined graph of inequlities:



Corner point:

$O(0,0)$, $B(2,0)$, $C(0,4)$, $E\left(\frac{9}{2},1\right)$.

For E:-

$$2x + 3y = 12 \quad \text{--- (iii)}$$
$$2x - 3y = 6 \quad \text{--- (iv)}$$

By adding eq (iii) and (iv).

$$2x + 3y = 12$$
$$2x - 3y = 6$$
$$4x = 18$$
$$\boxed{x = 9/2}$$

Put $x = 9/2$ in (iv).

$$2(9/2) - 3y = 6$$
$$9 - 3y = 6$$
$$-3y = 6 - 9 \Rightarrow +3y = +3$$
$$\boxed{y = 1}$$

So; $E\left[\frac{9}{2}, 1\right]$

ii

$x+y \leq 5$ (S.Q : LHR-2011, 15)

$-2x+y \leq 2$ (L.Q: LHR.-2019)

$x \geq 0, y \geq 0$

$x+y \leq 5$

Its associated Equation:

$-2x + y \leq 2$

Its associated eq.

$x + y = 5$ — (i)

Put $x = 0$ in (i)

$0 + y = 5$

$y = 5$

$A(0, 5)$

Put $y = 0$ in (i)

$x + 0 = 5$

$x = 5$

$B(5, 0)$

$-2x + y = 2$ — (ii)

Put $x = 0$ in (ii)

$-2(0) + y = 2$

$y = 2$

$C(0, 2)$

Put $y = 0$ in (ii).

$-2x + 0 = 2$

$x = -1$

$D(-1, 0)$.

Point test: $O(0,0)$.

$x + y \angle 5$

$0 + 0 \angle 5$

$0 \angle 5$ (True).

Point test: $O(0,0)$

$-2x + y \angle 2$

$-2(0) + (0) \angle 2$

$0 \angle 2$ (True).

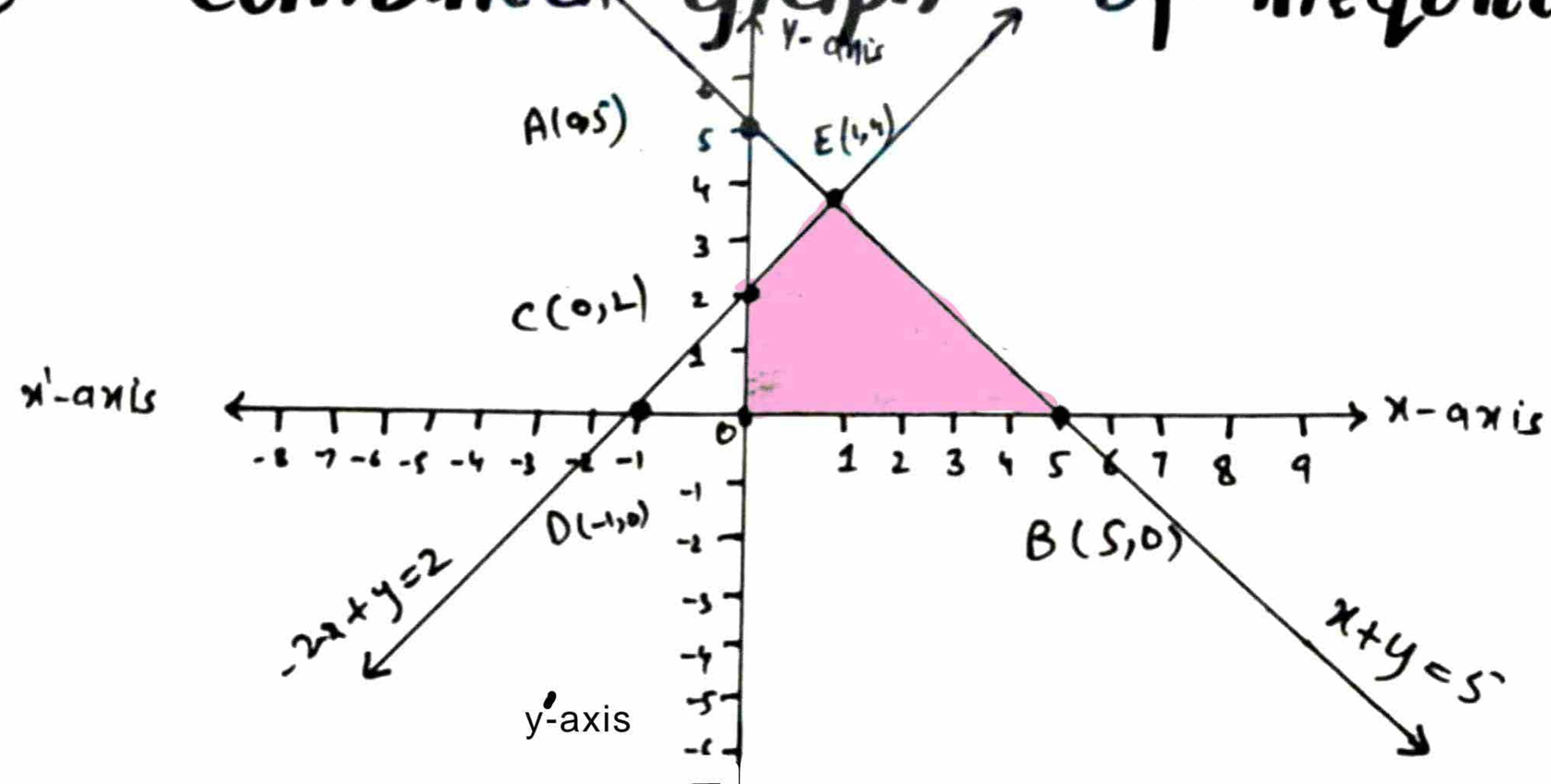
And: $x \geq 0$, $y \geq 0$

The associated Equation:

$x = 0$, $y = 0$

So: The graph will be in 1st Quad:

The combined graph of inequlities:



Corner points:

O(0,0), B(5,0), C(0,2), E(1,4)

For E:

$$x + y = 5 \longrightarrow (1)$$

$$-2x + y = 2 \longrightarrow (2)$$

Subtracting eq (1) and (2).

$$x + y = 5$$
$$-2x + y = 2$$
$$3x = 3 \Rightarrow x = 1$$

Put $x = 1$ in eq (1)

$$1 + y = 5 \Rightarrow y = 5 - 1$$
$$y = 4$$

E(1, 4)

$$x + y \leq 5$$
$$-2x + y \geq 2$$
$$x \geq 0$$

$x + y \leq 5$

Its associated Equation:

$$x + y = 5 \quad \text{--- (i)}$$

Put $x = 0$ in (i)

$$0 + y = 5 \Rightarrow y = 5$$

$-2x + y \geq 2$

Its associated eq.

$$-2x + y = 2 \quad \text{--- (ii)}$$

Put $x = 0$ in (ii)

$$-2(0) + y = 2 \Rightarrow y = 2$$

A(0,5)

Put $y=0$ in (i)

$x+0=5 \Rightarrow x=5$

C(5,0)

Point test: O(0,0)

$x+y \leq 5$

$\Rightarrow 0+0 < 5$

$0 < 5$ (True)

C(0,2)

Put $y=0$ in (ii)

$-2x+(0)=2 \Rightarrow x=-1$

D(-1,0)

Point test: O(0,0)

$-2x+y>2$

$-2(0)+0>2 \Rightarrow 0>2$ (False)

As; $x \geq 0$

Its associated Equation:

$x=0$

The Graph will be right side of yaxis

The combined Graph of inequlities:

Corner points:

$A(0,5), C(0,2), E(1,4)$

For E:

$x+y=5$ —(1) , $-2x+y=2$ →(2)

Subtracting eq (i) and (2).

$$x+y=5$$
$$\mp 2x \pm y = \_2$$
$$3x = 3 \quad \Rightarrow \quad x=1$$

Put $x=1$ in eq (1).

$$1+y=5 \quad \Rightarrow \quad y=5-1$$
$$y=4$$

E(1,4)

$3x+7y \leq 21$

$x-y \leq 3$

$x \geq 0 , y \geq 0$

$3x+7y \leq 21$

Its associated Eq.

$3x+7y=21$ —(i)

Put $x=0$ in (i)

$3(0)+7y=21 \Rightarrow y=3$

$x-y \leq 3$

Its associated eq.

$x-y=3$ —(ii)

Put $x=0$ in (ii)

$0-y=3 \Rightarrow y=-3$

A(0,3)

Put $y=0$ in (i)

$3x+7(0)=21 \Rightarrow x=7$

B(7,0)

Point test: O(0,0)

$3x+7y<21$

$3(0)+7(0)<21$

$0<21$ (True)

C(0,-3)

Put $y=0$ in (ii)

$x-0=3 \Rightarrow x=3$

D(3,0).

Point test: O(0,0)

$x-y<3$

$0-0<3$

$0<3$ (True).

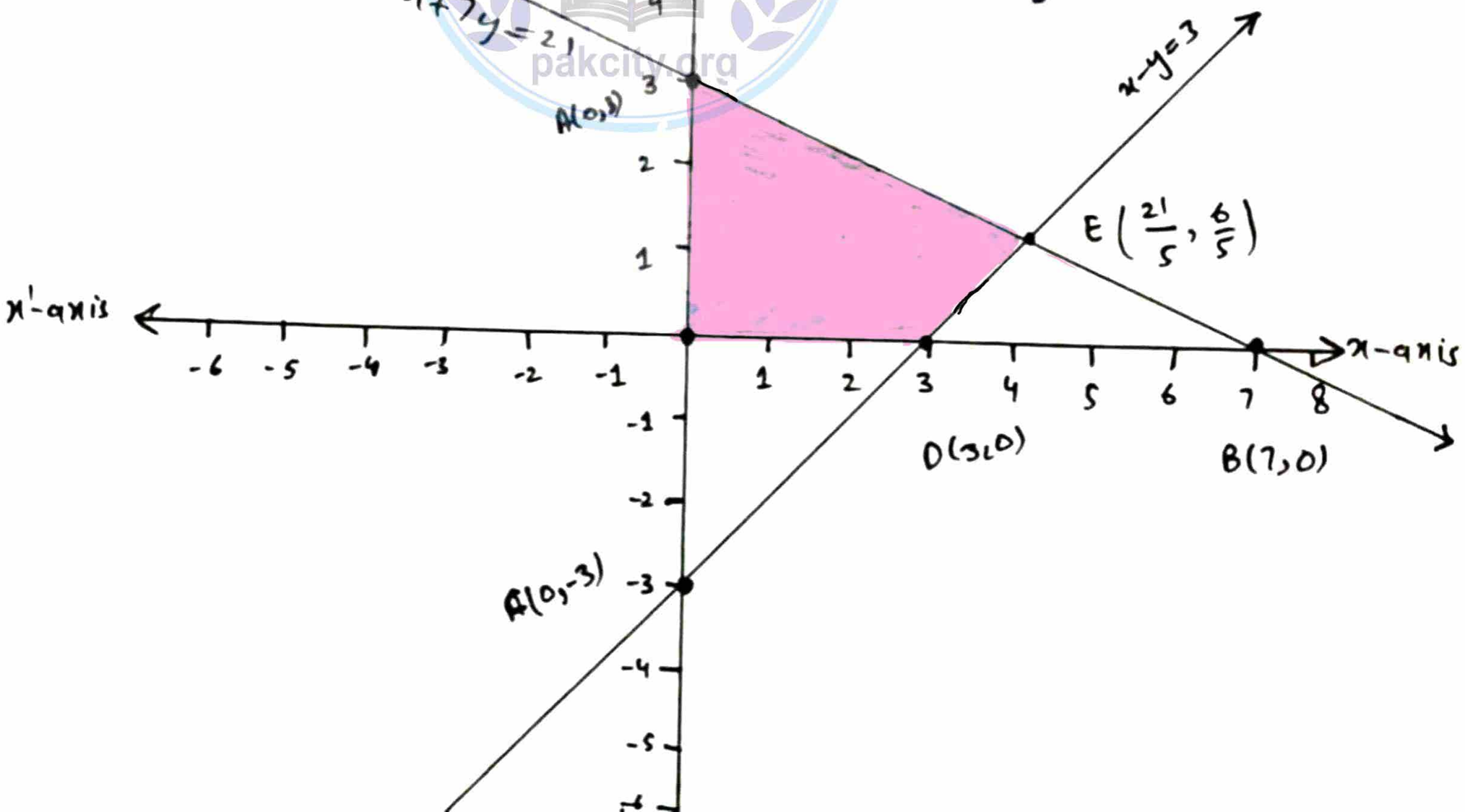
As $x \geq 0, \quad y \geq 0$

The associated Eq.

$x=0, \quad y=0$

So, the Graph will be in first Quad.

The Combined graph of inequlities:



Corner points:

$O(0,0)$, $A(0,3)$, $D(3,0)$, $E\left(\frac{21}{5}, \frac{6}{5}\right)$

For E:

$3x+7y=21$ —(1) , $x-y=3$ —(2)

Multiplying eq (2) by 7.

$7x-7y=21$ → (3)

Adding eq (1) and (3).

$$3x+7y=21$$
$$7x-7y=21$$
$$10x=42 \Rightarrow \boxed{x=\frac{42}{10}}$$
$$x=\frac{21}{5}$$

Put $x=21/5$ in (2).

$$\frac{21}{5}-y=3 \Rightarrow \frac{21}{5}-3=y$$
$$y=\frac{21-15}{5} \Rightarrow \boxed{y=\frac{6}{5}}$$

$E\left(\frac{21}{5}, \frac{6}{5}\right)$

$3x+2y \geq 6$

$x+y \leq 4$

$x \geq 0, y \geq 0$

$3x + 2y \geq 6$

Its associated eq.

$3x + 2y = 6$ — (i)

Put $x = 0$ in (i)

$3(0) + 2y = 6$

$y = 3$

$A(0, 3)$

Put $y = 0$ in (i)

$3x + 2(0) = 6$

$x = 2$

$B(2, 0)$

Point test: $O(0, 0)$

$3x + 2y > 6$

$3(0) + 2(0) > 6$

$0 > 6$ (False)

$x + y \leq 4$

Its associated eq.

$x + y = 4$ — (ii).

Put $x = 0$ in (ii)

$0 + y = 4$

$y = 4$

$C(0, 4)$

Put $y = 0$ in (ii)

$x + 0 = 4$

$x = 4$

$D(4, 0)$

Point test:

$x + y < 4$

$0 + 0 < 4$

$0 < 4$ (True).

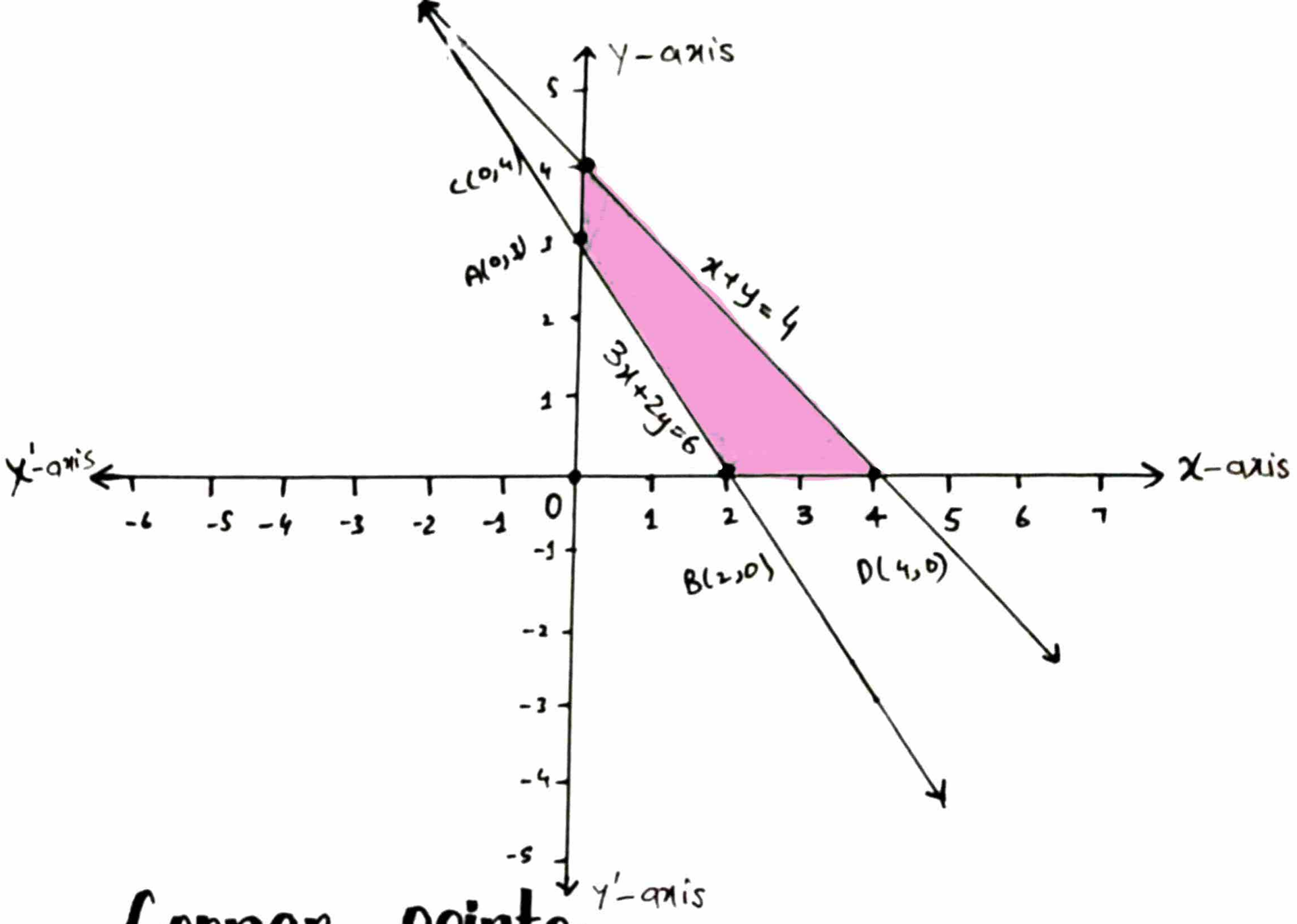
As: $x \geq 0, \quad y \geq 0$

The associated Equation:

$x = 0, \quad y = 0$

So; The Graph will be in first Quad.

The Combined graph of inequbties:



Corner points:

A(0,3), B(2,0), C(0,4), D(4,0).

$5x + 7y \leq 35$

$x - 2y \leq 4$

$x \geq 0, \quad y \geq 0$

$5x + 7y \leq 35$

Its associated Eq.

$5x + 7y = 35$ — (i)

Put $x = 0$ in (i)

$5(0) + 7y = 35$

$y = 5$

$A(0, 5)$

Put $y = 0$ in (i)

$5x + 7(0) = 35$

$x - 2y \leq 4$

Its associated eq.

$x - 2y = 4$ — (ii)

Put $x = 0$ in (ii)

$0 - 2y = 4$

$y = -2$

$C(0, -2)$

Put $y = 0$ in (ii)

$x - 2(0) = 4$

$x = 7$

$B(7, 0)$

Point test: $O(0,0)$

$5x + 7y < 35$

$5(0) + 7(0) < 35$

$0 < 35$ (True)

$x = 4$

$D(4, 0)$

Point test: $O(0,0)$

$x - 2y < 4$

$0 - 2(0) < 4$

$0 < 4$ (True).

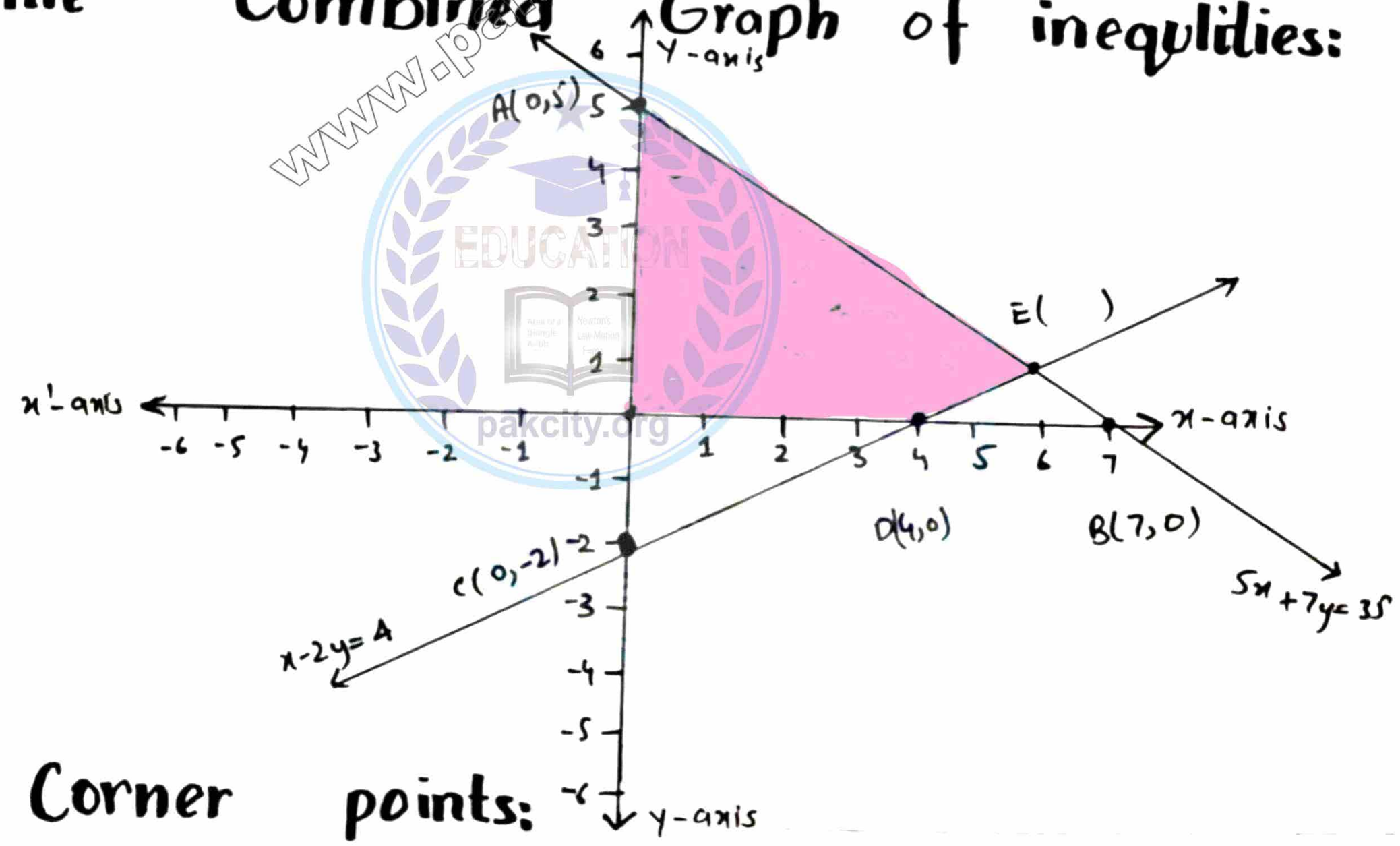
As; $x \geq 0$, $y \geq 0$

The associated equation:

$x = 0$, $y = 0$

So, the graph will be in first Quad.

The Combined Graph of inequlities:



Corner points:

$O(0,0)$, $A(0,5)$, $D(4,0)$, $E\left(\frac{98}{17}, \frac{15}{17}\right)$

For E:

$5x + 7y = 35 \longrightarrow (1)$, $x - 2y = 4 \longrightarrow (2)$.

Multiplying eq (2) by 5.

$$5x - 10y = 20 \rightarrow (3)$$

Subtracting eq (3) and (4).

$$5x + 7y = 35$$

$$\underline{-5x \mp 10y = -20}$$

$$17y = 15 \quad \Rightarrow \boxed{y = \frac{15}{17}}$$

Putting $y = 15/17$ in (ii).

$$x - 2(15/17) = 4$$

$$x - 30/17 = 4 \quad \Rightarrow \quad x = 4 + 30/17$$

$$x = \frac{68 + 30}{17} \quad \Rightarrow \boxed{x = \frac{98}{17}}$$

$$E\left(\frac{98}{17}, \frac{15}{17}\right).$$

Question: 2

Bilib.

$$2x + y \leq 10$$

$$x + 4y \leq 12$$

$$x + 2y \leq 10$$

$$x \geq 0, \quad y \geq 0$$

$2x+y \leq 10$

Its associated Eq.

$2x+y=10$ — (i)

Put $x=0$ in (i)

$2(0)+y=10$

$y=10$

$A(0,10)$

Put $y=0$ in (i)

$2x+0=10$

$x=5$

$B(5,0)$

Point test: $O(0,0)$

$2x+y < 10$

$2(0)+0 < 10$

$0 < 10$ (True)

$x+4y \leq 12$

Its associated eq.

$x+4y=12$ — (ii)

Put $x=0$ in (ii)

$0+4y=12$

$y=3$

$C(0,3)$

Put $y=0$ in (ii).

$x+4(0)=12$

$x=12$

$D(12,0)$

Point test: $O(0,0)$

$x+4y < 12$

$0+4(0) < 12$

$0 < 12$ (True).

$x+2y \leq 10$

Its associated eq.

$x+2y=10$ — (iii).

Put $x=0$ in (iii).

$0+2y=10$

$y=5$

$E(0,5)$

Put $y=0$ in (iii).

$x+2(0)=10$

$x=10$

$F(10,0)$

Point test: $O(0,0)$

$x+2y < 10$

$0+2(0) < 10$

$0 < 10$ (True).

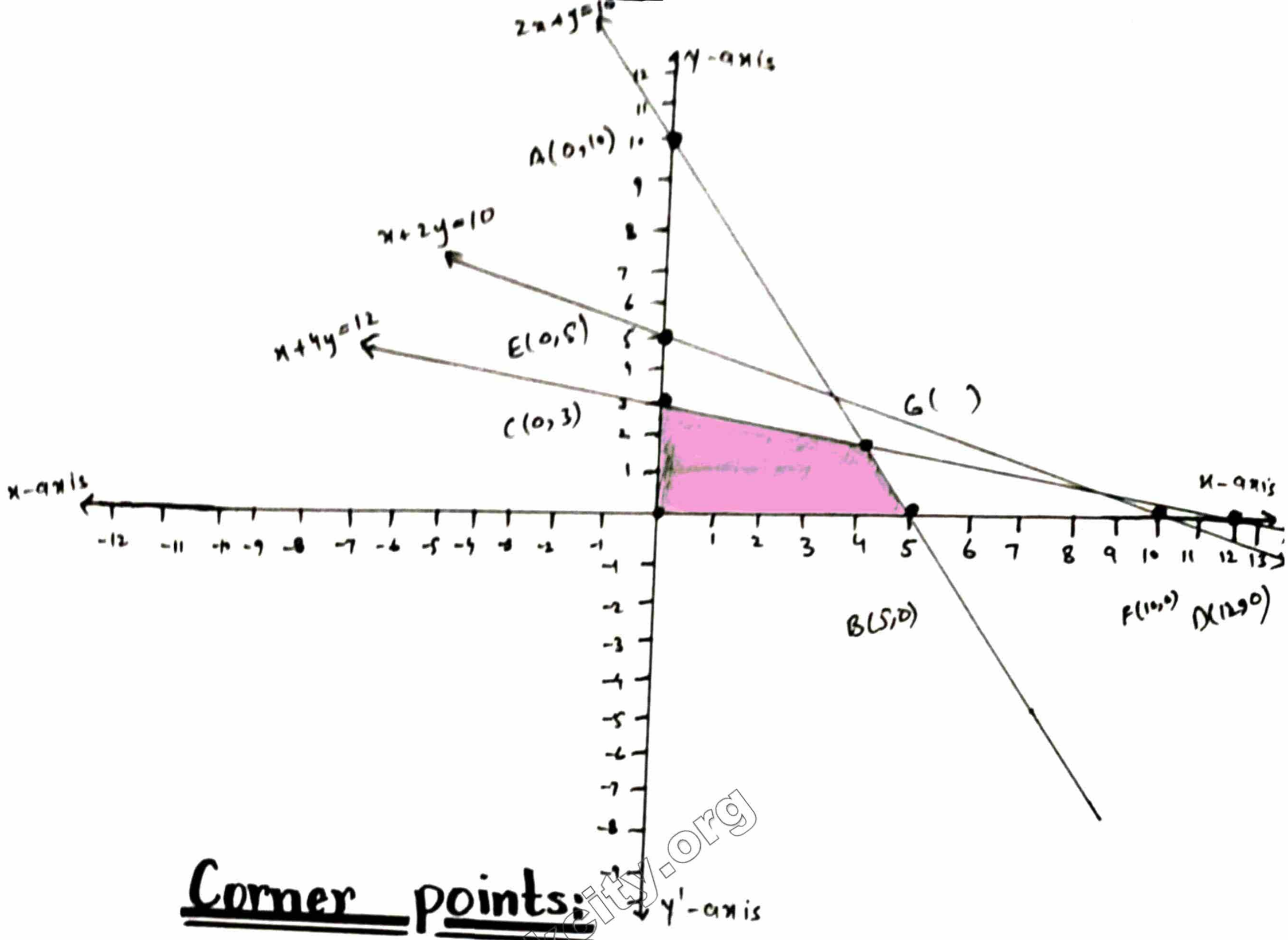
As; $x \geq 0, \quad y \geq 0$

The associated Equation:

$x=0, \quad y=0,$

So, the Graph will be in first Quad.

The Combined graph of inequlities:



Corner points:

O(0,0), B(5,0), C(0,3), G(4,2).

For G:

$x + 4y = 12$ ____ (1), $2x + y = 10 \rightarrow (2)$.

Multiplying eq (1) by 2.

$2x + 8y = 24 \rightarrow (3)$.

Subtracting eq (2) and (3).

$$2x + y = 10$$
$$-2x \pm 8y = \pm 24$$

$$\pm 7y = \pm 14 \Rightarrow \boxed{y = 2}$$

Put $y = 2$ in (1).

$$x + 4(2) = 12 \Rightarrow x = 12 - 8$$

$\boxed{x = 4}$ $\Rightarrow$ So G(4,2).

$$2x+3y \leq 18$$
$$2x+y \leq 10$$
$$x+4y \leq 12$$
$$x \geq 0 \ , \ y \geq 0$$

$2x+3y \leq 18$

Its associated Eq.

$2x+3y=18$ — i)

Put $x=0$ in i)

$2(0)+3y=18$

$y=6 \Rightarrow A(0,6)$

Put $y=0$ in i).

$2x+3(0)=18$

$x=9$

$B(9,0)$

Point test: $O(0,0)$

$2x+3y<18$

$2(0)+3(0)<18$

$0<18$ (True).

$2x+y \leq 10$

Its associated Eq.

$2x+y=10$ — ii)

Put $x=0$ in ii).

$2(0)+y=10$

$y=10 \Rightarrow C(0,10)$

Put $y=0$ in ii).

$2x+0=10$

$x=5$

$D(5,0)$

Point test: $O(0,0)$

$2x+y<10$

$2(0)+0<10$

$0<10$ (True)

$x+4y \leq 12$

Its associated eq.

$x+4y=12$ — iii)

Put $x=0$ in iii).

$0+4y=12$

$y=3 \Rightarrow E(0,3)$

Put $y=0$ in iii).

$x+4(0)=12$

$x=12$

$F(12,0)$.

Point test: $O(0,0)$

$x+4y<12$

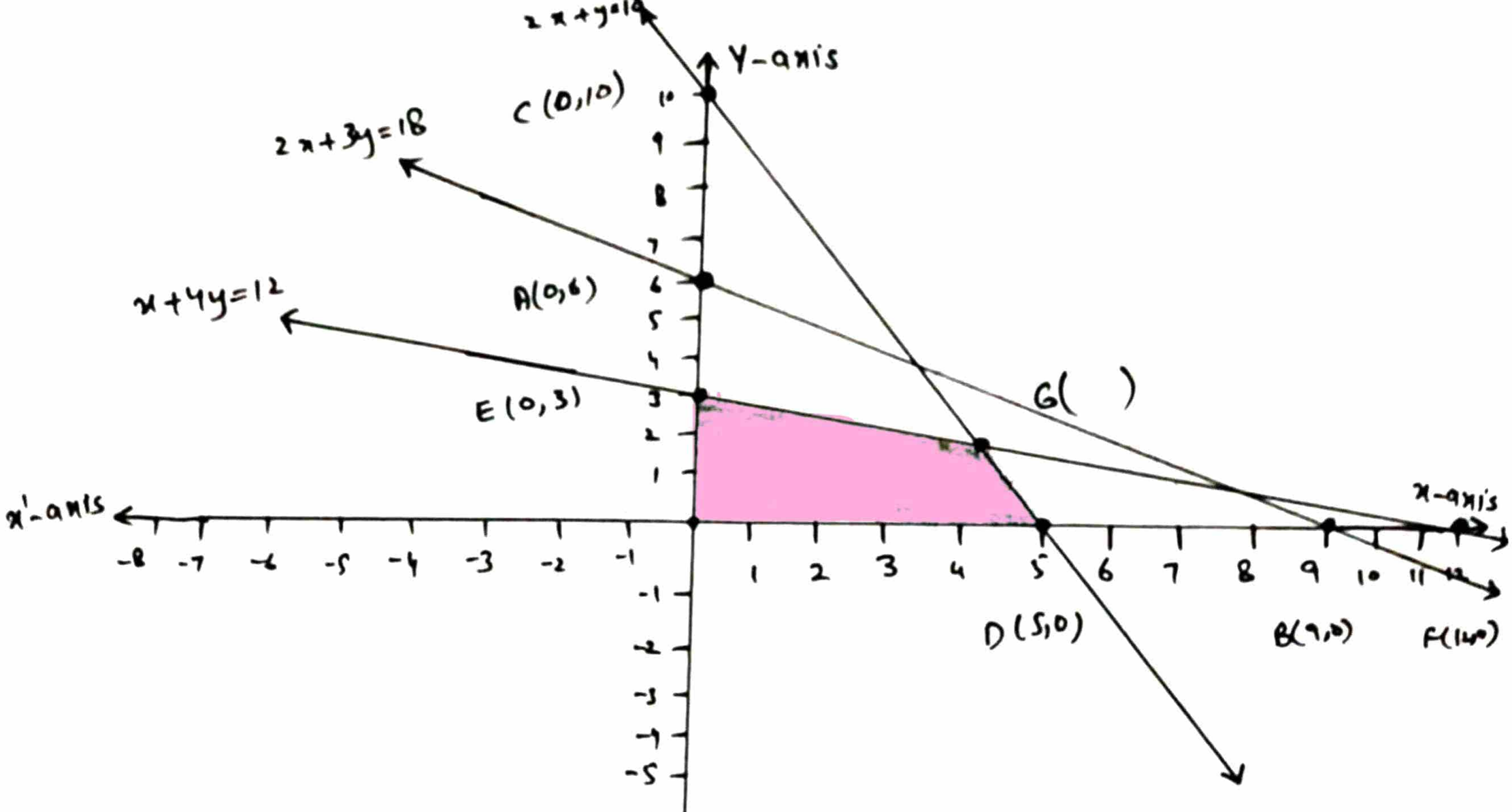
$0+4(0)<12$

$0<12$ (True)

As; $x \geq 0, \ y \geq 0$

The associated Equation: $x=0, \ y=0$

The graph will be in first Quad.



Corner points:

O(0,0), D(5,0), E(0,3), G(4,2).

For G:

$2x+y=10 \longrightarrow (1)$, $x+4y=12 \longrightarrow (2)$

Multiplying eq (2) by (2).

$2x+8y=24 \longrightarrow (3)$.

Subtracting eq (2) and (3).

$$2x+y=10$$
$$-2x\pm 8y=-24$$
$$\pm 7y=\pm 14$$

$$\boxed{y=2}$$

Put $y=2$ in (1).

$$2x+2=10 \Rightarrow 2x=10-2$$
$$2x=8 \Rightarrow \boxed{x=4}$$

G(4,2).

$2x+3y \leq 18$

$x+4y \leq 12$

$3x+y \leq 12$

$x \geq 0, \quad y \geq 0$

(LHR - 2018)

$2x+3y \leq 18$

Its associated Eq.

$2x+3y=18$ —(i)

Put $x=0$ in (i)

$2(0)+3y=18$

$y=6 \Rightarrow A(0,6)$

Put $y=0$ in (i)

$2x+3(0)=18$

$x=9$

$B(9,0)$

Point test: $O(0,0)$

$2x+3y<18$

$2(0)+3(0)<18$

$0<18$ (True)

$x+4y \leq 12$

Its associated Eq.

$x+4y=12$ —(ii)

Put $x=0$ in (ii)

$0+4y=12$

$y=3 \Rightarrow B(0,3)$

Put $y=0$ in (ii).

$x+4(0)=12$

$x=12$

$D(12,0)$

Point test: $O(0,0)$

$x+4y<12$

$0+4(0)<12$

$0<12$ (True)

$3x+y \leq 12$

Its associated Eq.

$3x+y=12$ —(iii).

Put $x=0$ in (iii).

$3(0)+y=12$

$y=12 \Rightarrow E(0,12)$

Put $y=0$ in (iii).

$3x+(0)=12$

$x=4$

$F(4,0)$.

Point test: $O(0,0)$

$3x+y<12$

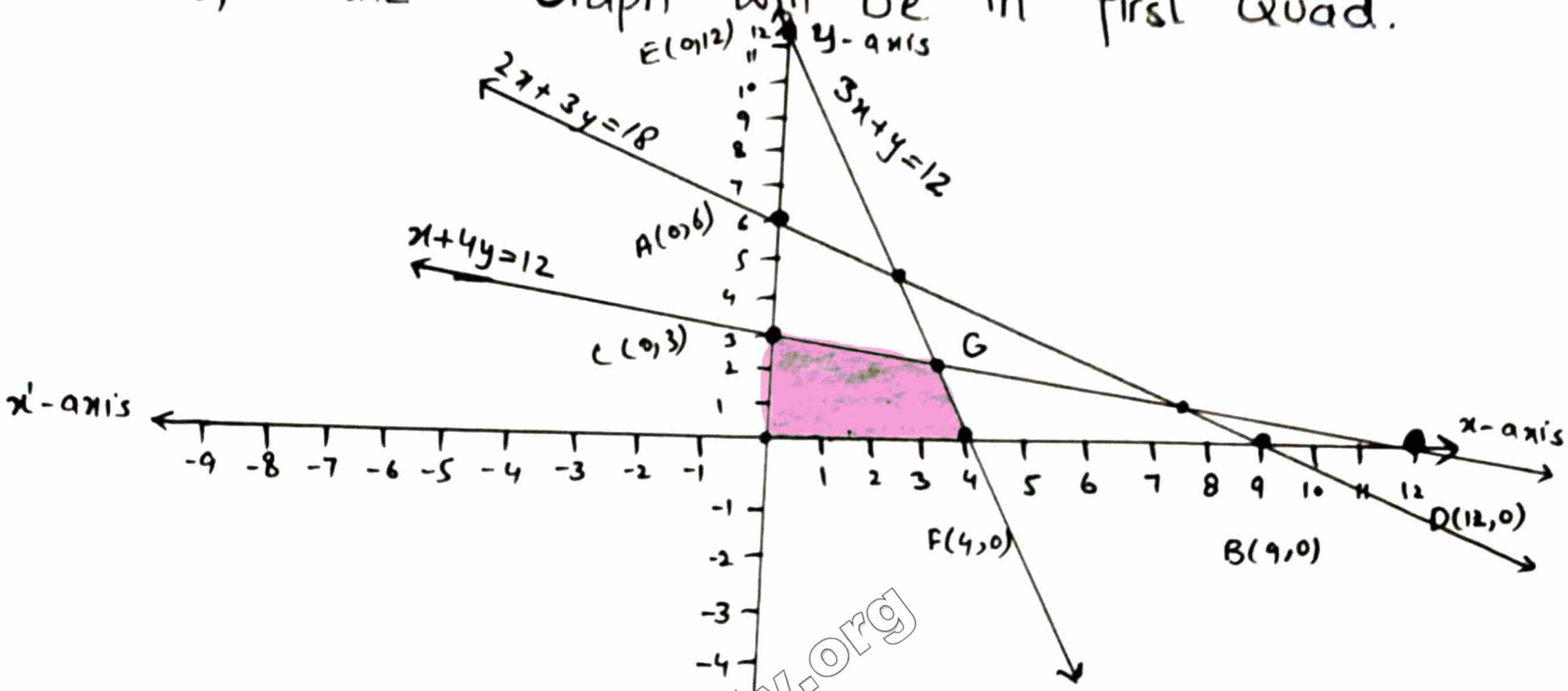
$3(0)+(0)<12$

$0<12$ (True).

As; $x \geq 0, \quad y \geq 0$

The associated Eq. $x = 0$, $y = 0$

So, the Graph will be in first Quad.



Corner points:

$O(0,0)$, $C(0,3)$, $F(4,0)$, $G\left(\frac{36}{11}, \frac{24}{11}\right)$

For G:

$3x + y = 12 \longrightarrow (1)$, $x + 4y = 12 \longrightarrow (2)$.

Multiplying eq (2) by 3.

$3x + 12y = 36$

Subtracting eq (1) and (3),

$$3x + y = 12$$
$$-3x \pm 12y = -36$$
$$\pm 11y = \pm 24$$
$$\boxed{y = \frac{24}{11}}$$

Put $y = 24/11$ in (1).

$3x + (24/11) = 12$

$3x = 12 - \frac{24}{11}$

$3x = \frac{132 - 24}{11}$

$3x = \frac{108}{11} \Rightarrow x = \frac{36}{11}$

$G\left(\frac{36}{11}, \frac{24}{11}\right)$.

$x+2y \leq 14$

$3x+4y \leq 36$

$2x+y \leq 10$

$x \geq 0$, $y \geq 0$

$x+2y \leq 14$

Its associated Eq.

$x+2y=14$ ___ (i)

Put $x=0$ in (i)

$0+2y=14$

$y=7$

$A(0,7)$

Put $y=0$ in (i).

$x+2(0)=14$

$x=14$

$B(14,0)$

Point test: $O(0,0)$

$x+2y<14$

$0+2(0)<14$

$0<14$ (True)

$3x+4y \leq 36$

Its associated Eq.

$3x+4y=36$ ___ (ii)

Put $x=0$ in (ii).

$3(0)+4y=36$

$y=9$

$C(0,9)$

Put $y=0$ in (ii).

$3x+4(0)=36$

$x=12$

$D(12,0)$

Point test:

$3x+4y<36$

$3(0)+4(0)<36$

$0<36$ (True)

$2x+y \leq 10$

Its associatedeq.

$2x+y=10$ ___ (iii).

Put $x=0$ in (iii).

$2(0)+y=10$

$y=10$

$E(0,10)$

Put $y=0$ in (iii).

$2x+0=10$

$x=5$

$F(5,0)$

Point test:

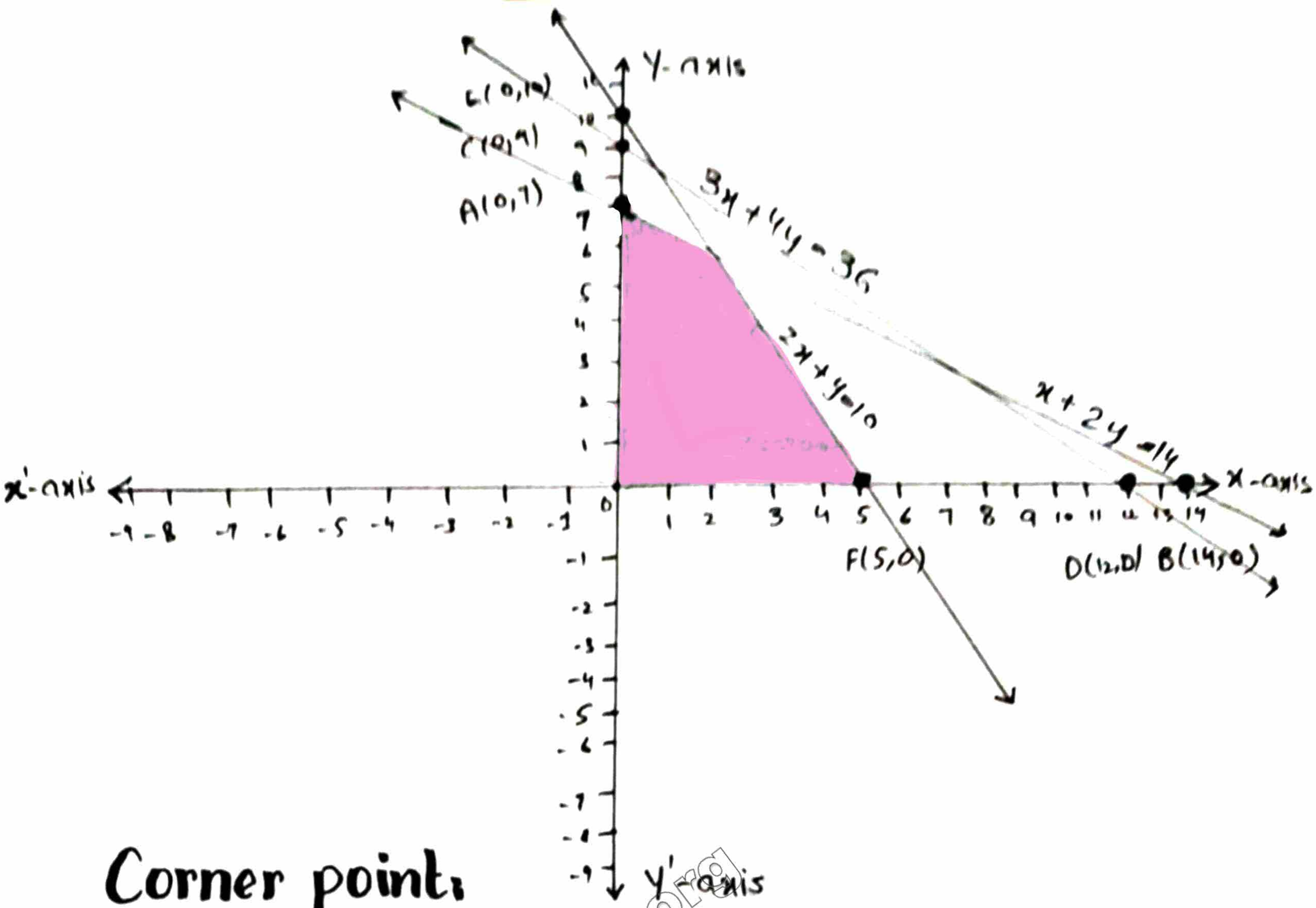
$2x+y<10$

$2(0)+0<10$

$0<10$ (True).

And; $x \geq 0$, $y \geq 0$

The associated Eq. $x=0$, $y=0$



Corner point:

$O(0,0)$, $A(0,7)$, $F(5,0)$, $G(2,6)$.

For G:

$x+2y=14 \longrightarrow (1)$, $2x+y=10 \longrightarrow (2)$.

Multiplying eq (2) by 2.

$4x+2y=20 \longrightarrow (3)$.

Subtracting eq (3) and (1).

$$x+2y=14$$
$$-4x+2y=-20$$

$$-3x=-6$$

$$\boxed{x=2}$$

Put $x=2$ in (1)

$$2+2y=14 \Rightarrow 2y=14-2$$
$$2y=12 \Rightarrow \boxed{y=6}$$
$$G(2,6)$$

pakcity.org

$x+3y \leq 15$

$2x+y \leq 12$

$4x+3y \leq 24$

$x \geq 0, \quad y \geq 0$

$x+3y \leq 15$

Its associated Eq.

$x+3y=15$ — (i)

Put $x=0$ in (i)

$0+3y=15$

$y=5$

$A(0,5)$

Put $y=0$ in (i)

$x+3(0)=15$

$x=15$

$B(15,0)$

Point test: $O(0,0)$

$x+3y<15$

$0+3(0)<15$

$0<15$ (True)

$2x+y \leq 12$

Its associated Eq.

$2x+y=12$ — (ii)

Put $x=0$ in (ii).

$2(0)+y=12$

$y=12$

$C(0,12)$

Put $y=0$ in (ii)

$2x+0=12$

$x=6$

$D(6,0)$

Point test: $O(0,0)$

$2x+y<12$

$2(0)+0<12$

$0<12$ (True)

$4x+3y \leq 24$

Its associatedeq.

$4x+3y=24$ — (iii).

Put $x=0$ in (iii).

$4(0)+3y=24$

$y=8$

$E(0,8)$

Put $y=0$ in (iii).

$4x+3(0)=24$

$x=6$

$F(6,0)$

Point test: $O(0,0)$

$4x+3y<24$

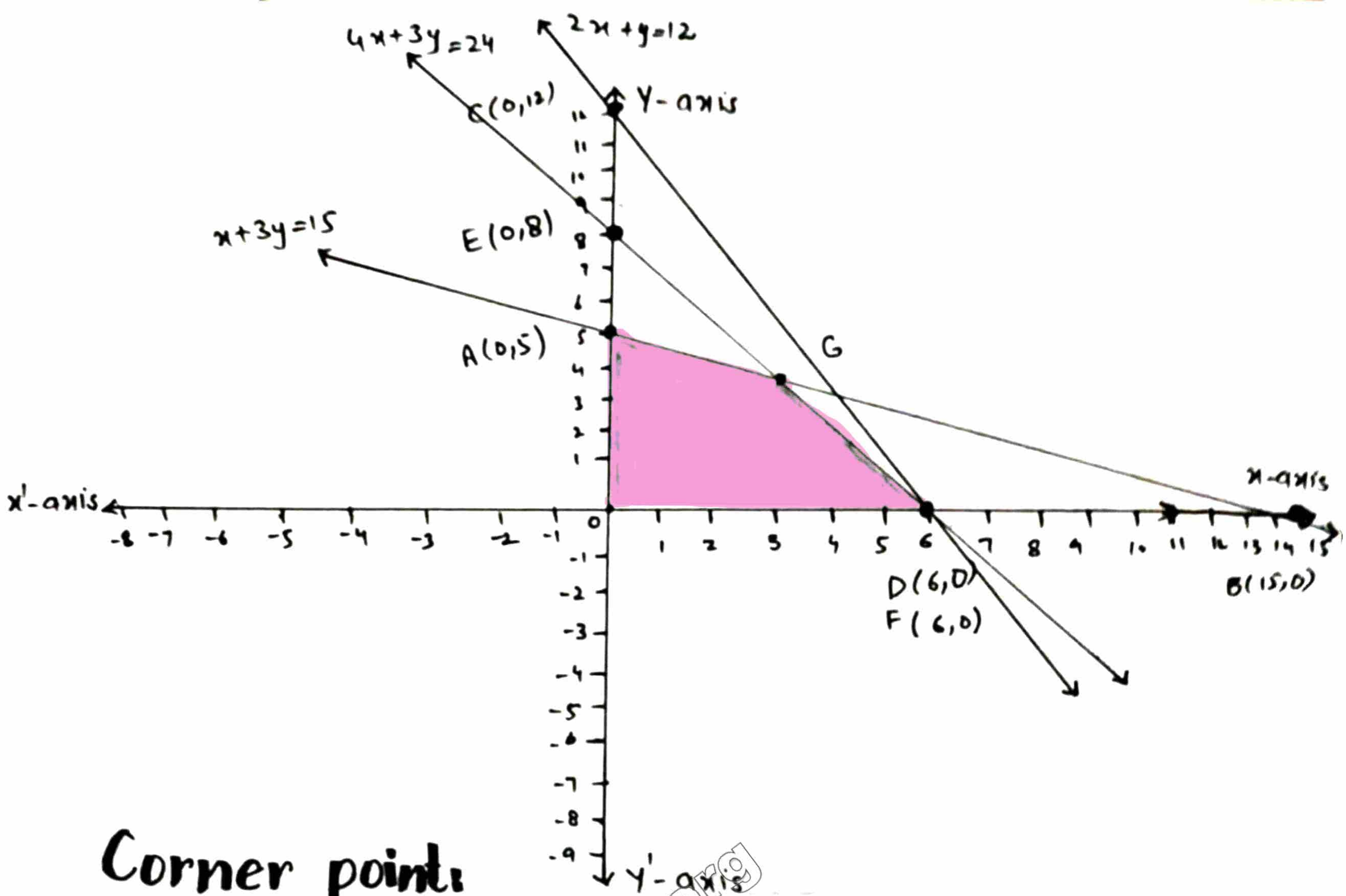
$4(0)+3(0)<24$

$0<24$.

And; $x \geq 0, \quad y \geq 0$

The associated Eq. $x=0$, $y=0$,

The Graph will be in first Quad.



Corner points:

O(0,0), A(0,5), D(6,0), F(6,0), G(3,4).

For G:

$x+3y=15 \rightarrow (1)$, $4x+3y=24 \rightarrow (2)$

Subtracting (1) and (2)

$$x+3y=15$$
$$-4x\pm 3y=\pm 24$$
$$\pm 3x=\pm 9$$

$$\boxed{x=3}$$

Put $x=3$ in (1).

$$3+3y=15 \Rightarrow 3y=15-3$$

$$3y=12 \Rightarrow \boxed{y=4}$$

G(3,4)

$2x+y \leq 20$

$8x+15y \leq 120$

$x+y \leq 11$

$x \geq 0, \quad y \geq 0$

$2x+y \leq 20$

Its associated Eq.

$2x+y = 20$ — (i)

Put $x=0$ in (i)

$2(0)+y = 20$

$y = 20$

$A(0,20)$

Put $y=0$ in (i)

$2x+0 = 20$

$x = 10$

$B(10,0)$

Point test: $O(0,0)$

$2x+y \leq 20$

$2(0)+0 \leq 20$

$0 \leq 20$ (True)

$8x+15y \leq 120$

Its associated Eq.

$8x+15y = 120$ — (ii)

Put $x=0$ (ii)

$8(0)+15y = 120$

$y = \frac{120}{15} \Rightarrow y = 8$

$C(0,8)$

Put $y=0$ in (ii).

$8x+15(0) = 120$

$x = 15$

$D(15,0)$

Point test: $O(0,0)$

$8x+15y \leq 120$

$8(0)+15(0) \leq 120$

$0 \leq 120$ (True)

$x+y \leq 11$

Its associated eq.

$x+y = 11$ — (iii).

Put $x=0$ in (iii).

$0+y = 11$

$y = 11$

$E(0,11)$

Put $y=0$ in (iii).

$x+0 = 11$

$x = 11$

$F(11,0)$.

Point test: $O(0,0)$

$x+y \leq 11$

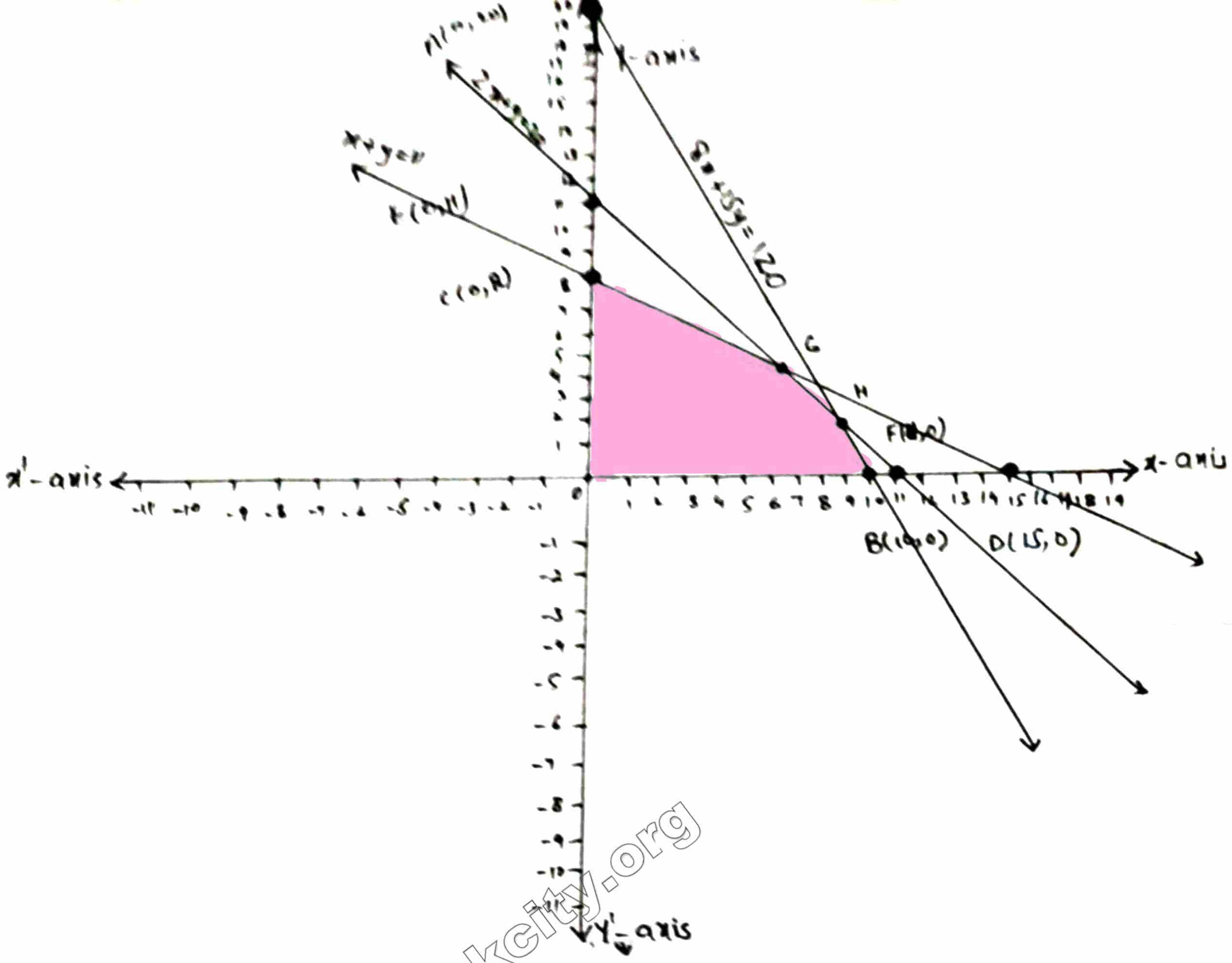
$0+0 \leq 11$

$0 \leq 11$ (True).

Ans: $x \geq 0, \quad y \geq 0$

The associated Eq:- $x=0, \quad y=0$

The graph will be in first Quad.



$O(0,0)$, $B(11,0)$, $C(0,8)$, $G\left(\frac{45}{7}, \frac{32}{7}\right)$, $H(9,2)$.

For G:

$x+y=11 \rightarrow (1)$, $\quad 8x+15y=120 \rightarrow (2)$

Multiplying eq (i) by 15.

$$15x+15y=165 \quad -(3)$$

Subtracting (iii) and (ii).

$$8x+15y=120$$
$$-15x \pm 15y = -165$$
$$+7x = +45 \Rightarrow x = \frac{45}{7}$$

Put $x = \frac{45}{7}$ (i),

$$\frac{45}{7} + y = 11 \Rightarrow y = -\frac{45}{} + 11$$
$$y \Rightarrow \frac{77-45}{7} \Rightarrow y = \frac{32}{7}$$

$G\left(\frac{45}{7}, \frac{32}{7}\right)$

For H:

$2x+y=20 \quad -(4)$

$x+y=11 \rightarrow (5)$

Subtracting (4) and (5)

$$2x+y=20$$
$$-x \pm y = -11$$
$$x = 9$$

Put $x=9$ in (5)

$$9+y=11$$
$$y=11-9$$
$$y=2$$

$H(9,2)$

Linear Programming

Objective function: (LHR-2011)

"A function which is to be maximized or minimized is called an objective function."

Optimal solution:

"The feasible solution which maximizes or minimizes the objective function is called optimal Solution."

(LHR-2014)

Theorem of linear programming

The theorem of linear programming states:

"The maximum and minimum values of the objective function occur at corner points of the feasible solution."

Exercise: 5.3

Q 1

Maximize $f(x,y) = 2x+5y$

$2y - x \leq 8$

$x - y \leq 4$

$x \geq 0 , y \geq 0$

(LHR-2013, 21)

$2y - x \leq 8$

Its associated Eq.

$2y - x = 8$ ___ i)

Put $x=0$ in i)

$2y - 0 = 8$

$y = 4$

$A(0, 4)$

Put $y=0$ in i).

$2(0) - x = 8$

$x = -8$

$B(-8, 0)$

Point test:

$2(0) - 0 < 8$

$0 < 8$ (True)

$x - y \leq 4$

Its associated Eq.

$x - y = 4$ ___ ii)

Put $x=0$ in ii)

$0 - y = 4$

$y = -4$

$C(0, -4)$

Put $y=0$ in ii).

$x - 0 = 4$

$x = 4$

$D(4, 0)$

Point test:

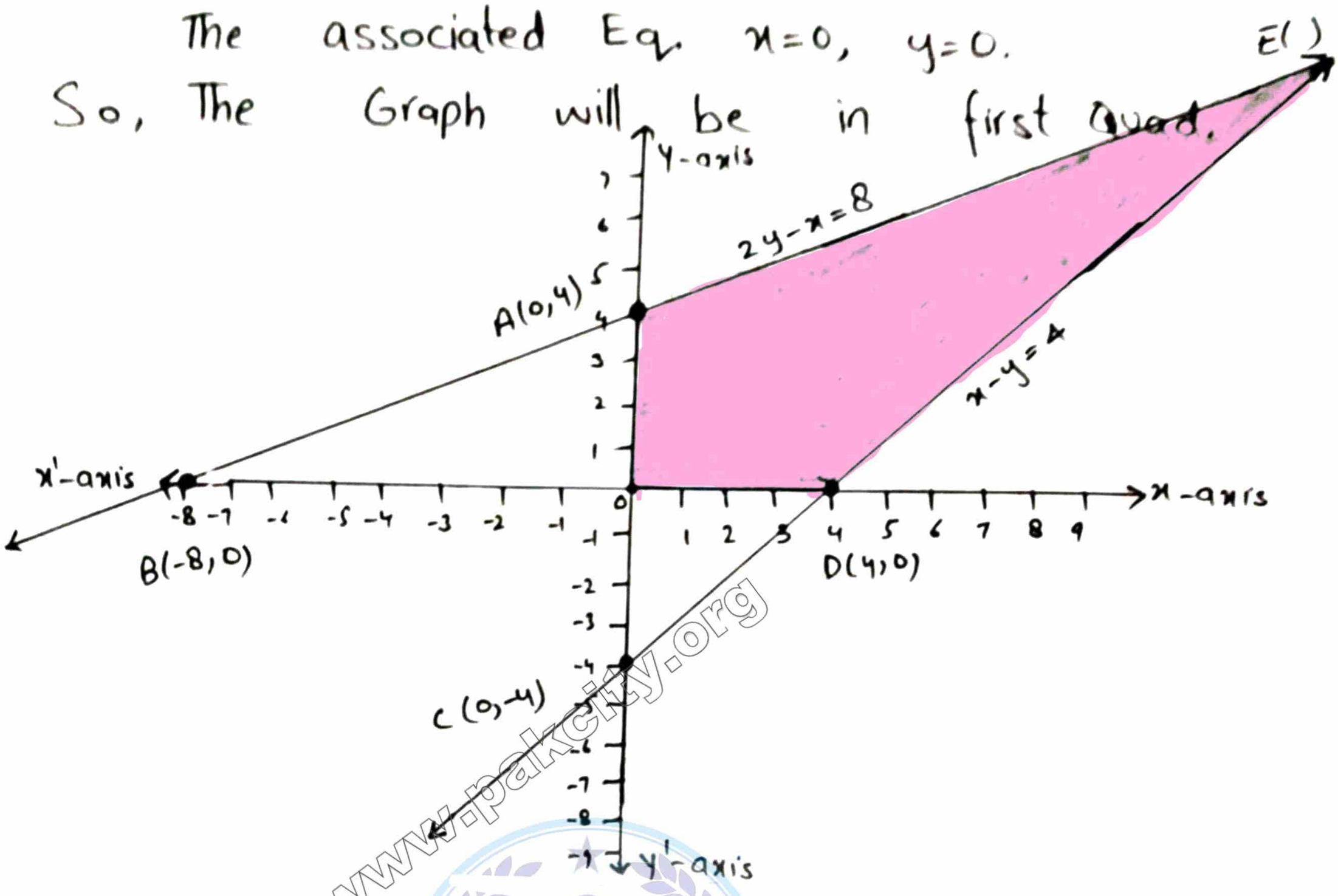
$0 - 0 < 4$

$0 < 4$ (True)

And; $x \geq 0, \quad y \geq 0$

The associated Eq. $x = 0, \quad y = 0.$

So, The Graph will be in first Quad.



Corner point:

$O(0,0)$, $A(0,4)$, $D(4,0)$, $E(16,12)$

For E:

$-x + 2y = 8$ — (1) , $x - y = 4$ — (2)

Adding (1) and (2)

$$\begin{array}{r} -x + 2y = 8 \\ x - y = 4 \\ \hline \boxed{y = 12} \end{array}$$

$x = 4 + 12$

$\boxed{x = 16}$

$E\ (16, 12)$

Put $y = 12$ in (2)

$x - 12 = 4$

Now; **Corner points** | **$f(x,y)=2x+5y$**

$O(0,0)$ | $f(x,y)=2(0)+5(0)=0$

$A(0,4)$ | $f(x,y)=2(0)+5(4)=20$

$D(4,0)$ | $f(x,y)=2(4)+5(0)=8$

$E(16,12)$ | $f(x,y)=2(16)+5(12)=92$

So, function is maximum at corner point $E(16,12)$.

Maximize $f(x,y)=x+3y$

$2x+5y\leq 30$

$5x+4y\leq 20$

$x\geq 0,\ y\geq 0$

(LHR-2016,17)

$2x+5y\leq 30$

Its associated Eq.

$2x+5y=30$ ___(i)

Put $x=0$ in (i).

$2(0)+5y=30$

$y=6$

$A(0,6)$

Put $y=0$ in (i)

$2x+5(0)=30$

$x=15$

$B(15,0)$

$5x+4y\leq 20$

Its associated Eq.

$5x+4y=20$ ___(ii)

Put $x=0$ in (ii)

$5(0)+4y=120.$

$y=5$

$C(0,5)$

Put $y=0$ in (ii)

$5x+4(0)=20$

$x=4$

$D(4,0).$

Point test: $O(0,0)$

$$2x + 5y < 30$$

$$2(0) + 5(0) < 30$$

$$0 < 30 \text{ (True)}.$$

Point test: $O(0,0)$.

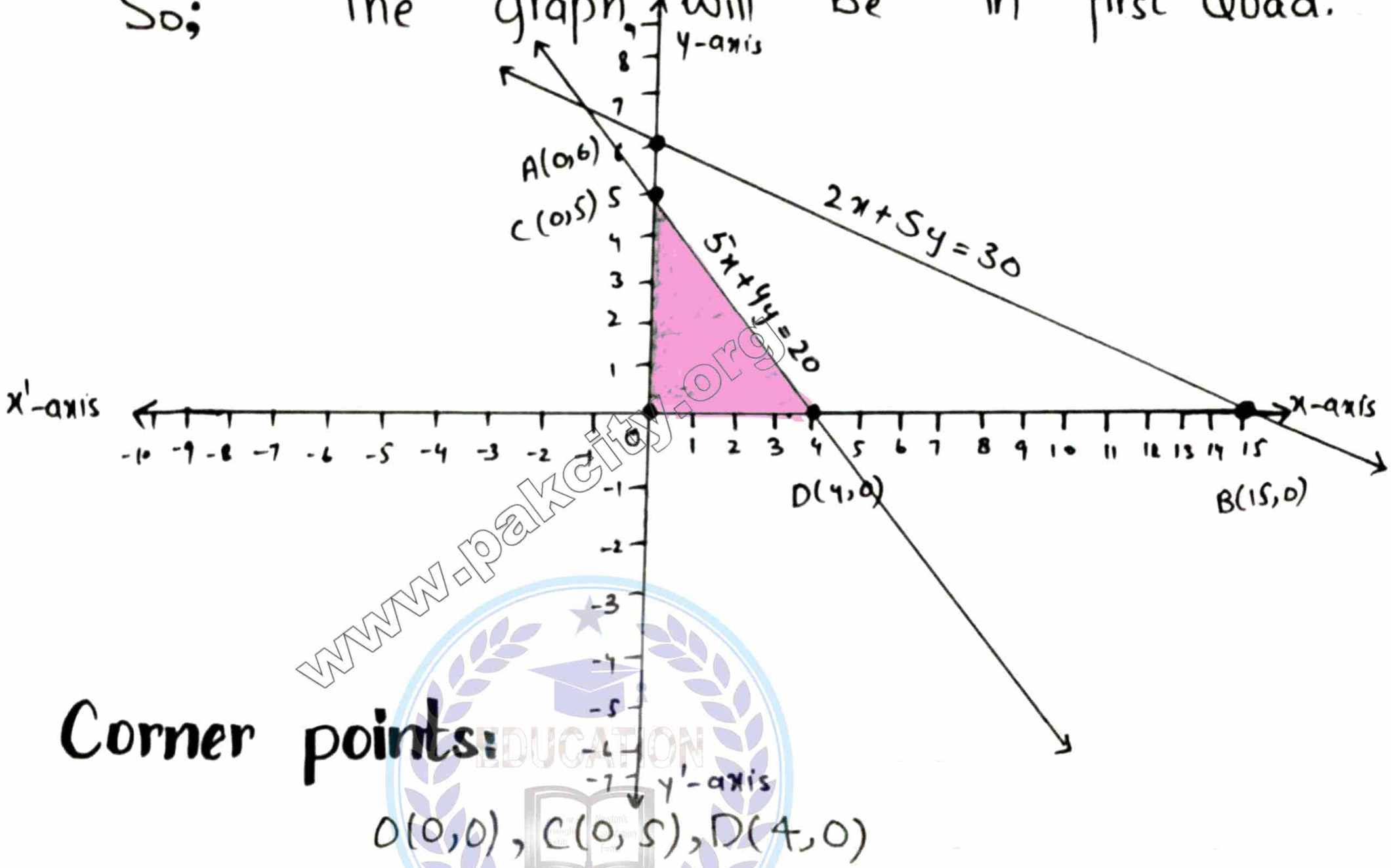
$$5x + 4y < 20$$

$$5(0) + 4(0) < 20$$

$$0 < 20 \text{ (True)}.$$

And; $x \geq 0,\ y \geq 0$

So; the graph will be in first Quad.



Corner points:

$O(0,0)$, $C(0,5)$, $D(4,0)$

Now;

Corner point	$f(x,y) = x + 3y$
$O(0,0)$	$f(x,y) = 0 + 3(0) = 0$
$C(0,5)$	$f(x,y) = 0 + 3(5) = 15$
$D(4,0)$	$f(x,y) = 4 + 3(0) = 4$

So,

function is maximum at corner point $C(0,5)$.

Maximize $z = 2x + 3y$

$3x + 4y \leq 12$

$2x + y \leq 4$

$4x - y \leq 4$

$x \geq 0$, $y \geq 0$

(LHR- 2014, 18, 21)

$3x - 4y \leq 12$

Its associated Eq.

$3x - 4y = 12 \rightarrow (1)$.

Put $x = 0$ in (1).

$3(0) - 4y = 12$

$y = -3$

$A(0, -3)$

Put $y = 0$ in (1).

$3x - 4(0) = 12$

$x = 4$

$B(4, 0)$

Point test: $O(0,0)$

$3x + 4y < 12$

$3(0) + 4(0) < 12$

$0 < 12$ (True).

$2x + y \leq 4$

Its associated Eq.

$2x + y = 4 \rightarrow (2)$

Put $x = 0$ in (2).

$2(0) + y = 4$

$y = 4$

$C(0, 4)$

Put $y = 0$ in (2).

$2x + 0 = 4$

$x = 2$

$D(2, 0)$

Point test: $O(0,0)$

$2x + 4y < 4$

$2(0) + 0 < 4$

$0 < 4$ (True).

$4x - y \leq 4$

Its associated Eq.

$4x - y = 4 \rightarrow (3)$

Put $x = 0$ in (3)

$4(0) - y = 4$

$y = -4$

$E(0, -4)$

Put $y = 0$ in (3).

$4x - 0 = 4$

$x = 1$

$F(1, 0)$

Point test: $O(0,0)$

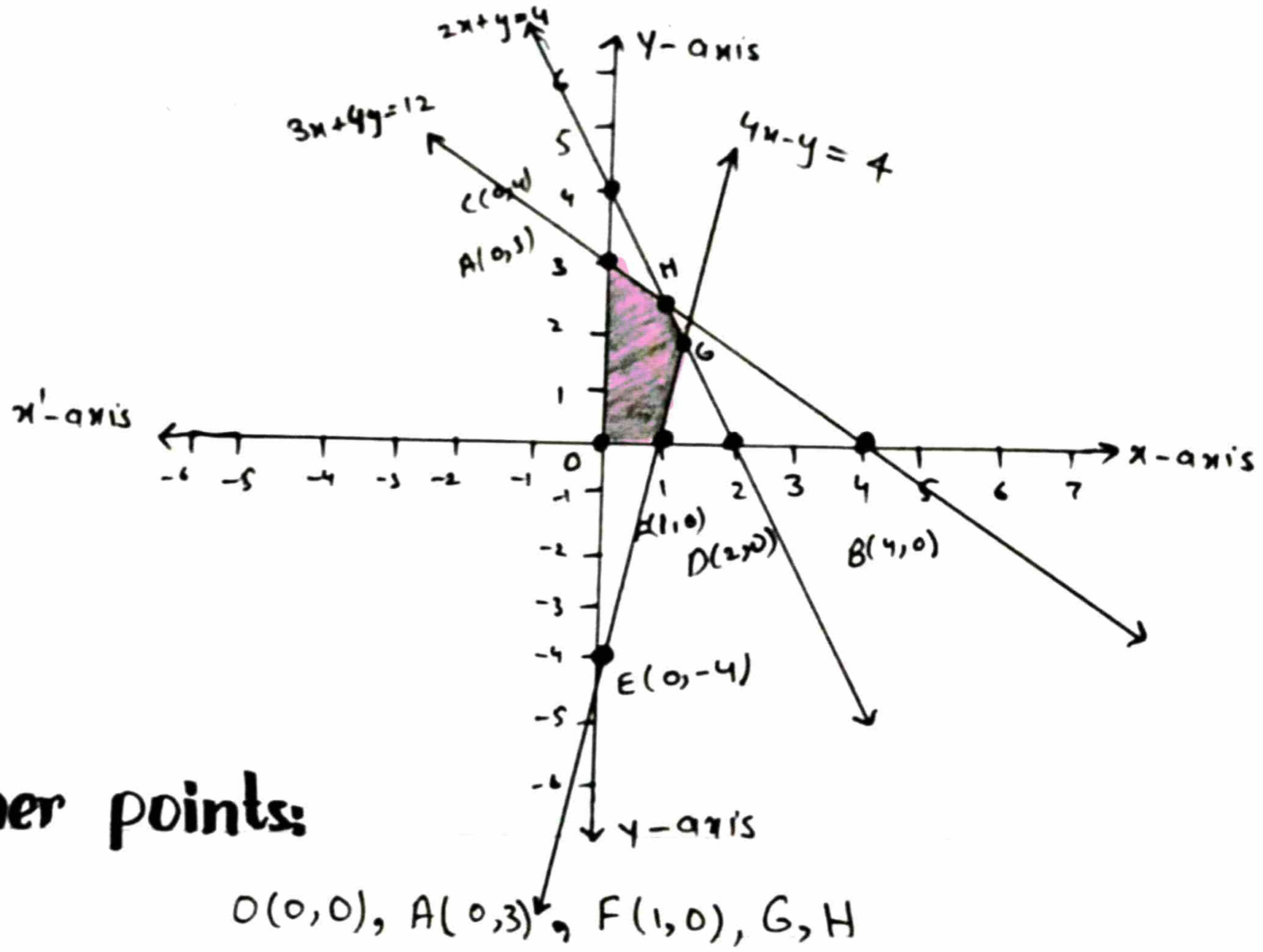
$4x - y < 4$

$4(0) - 0 < 4$

$0 < 4$ (True)

And; $x \geq 0$, $y \geq 0$

The associated eq: $x = 0$, $y = 0$.



Corner points:

O(0,0), A(0,3), F(1,0), G, H

For G:

$2x+y=4 \rightarrow (4)$, $4x-y=4 \rightarrow (5)$

Adding eq (4) and (5).

$$2x+y=4$$
$$\underline{4x-y=4}$$
$$6x=8 \Rightarrow x=4/3$$

Put $x=4/3$ in (5)

$$4(4/3)-y=4 \Rightarrow -y=4-16/3$$
$$-y=\frac{12-16}{3} \Rightarrow -y=-4/3$$
$$y=4/3$$

$$G\left(\frac{4}{3},\frac{4}{3}\right)$$

For H:

$3x+4y=12 \rightarrow (6)$, $2x+y=4 \rightarrow (7)$

Multiplying eq (7) by 4.

$$8x+4y=16 \rightarrow (8)$$

Subtracting eq (8) and (6)

$$8x + 4y = 16$$
$$-3x \pm 4y = -12$$
$$5x = 4 \quad \Rightarrow \quad x = 4/5$$

Put $x = 4/5$ in (7).

$$2\left(\frac{4}{5}\right) + y = 4 \quad \Rightarrow \quad y = 4 - 8/5$$

$$y = \frac{20-8}{5} \quad \Rightarrow \quad y = \frac{12}{5}$$

$$H\left(\frac{4}{5}, \frac{12}{5}\right)$$

Now: **Corner points:**

$$z = 2x + 3y$$

$O(0,0)$ $\quad z = 2(0) + 3(0) = 0$

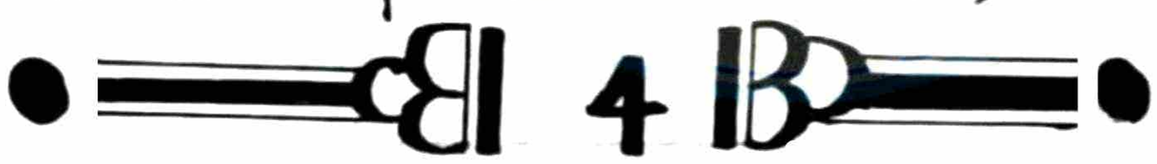
$A(0,3)$ $\quad z = 2(0) + 3(3) = 9$

$B(1,0)$ $\quad z = 2(1) + 3(0) = 2$

$G\left(\frac{4}{3}, \frac{4}{3}\right)$ $\quad z = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right) = \frac{8}{3} + \frac{12}{3} = 6.8$

$H\left(\frac{4}{5}, \frac{12}{5}\right)$ $\quad z = 2\left(\frac{4}{5}\right) + 3\left(\frac{12}{5}\right) = 8$

So, function is maximum at corner point $A(0,3)$.



Minimize $z = 2x + y$

$x + y \geq 3$

$7x + 5y \leq 35$ (LHR-2014,18,21)

$x \geq 0, \quad y \geq 0$

$x+y \geq 3$

Its associated Eq.

$x+y=3$ — (i)

Put $x=0$ in (i).

$0+y=3$

$y=3$

$A(0,3)$

Put $y=0$ in (i).

$x+0=3$

$x=3$

$B(3,0)$

Point test: $O(0,0)$

$x+y>3$

$0+0>3$

$0>3$ (False)

$7x+5y \leq 35$

Its associated Eq.

$7x+5y=35$ — (ii).

Put $x=0$ in (ii).

$7(0)+5y=35$

$y=7$

$C(0,7)$

Put $y=0$ in (ii).

$7x+5(0)=35$

$x=5$

$D(5,0)$.

Point test: $O(0,0)$

$7x+5y<35$

$7(0)+5(0)<35$

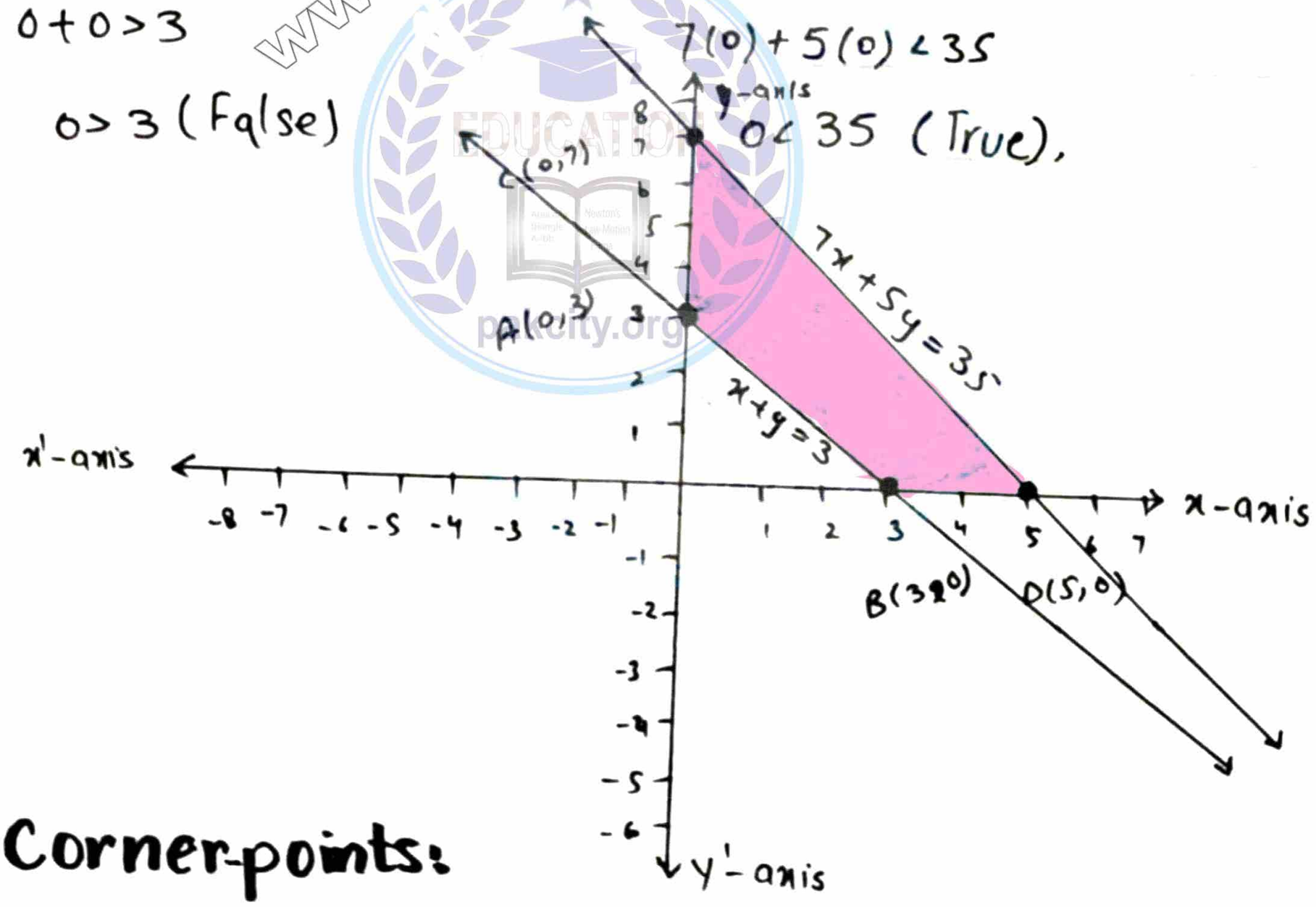
$0<35$ (True).

$x \geq 0$, $y \geq 0$

The associated eq. $x=0$, $y=0$

So,

The graph will be in 1st Quad.



Corner-points:

$A(0,3)$, $B(3,0)$, $D(5,0)$, $C(0,7)$

Corner point	$z = 2x+y$
A (0,3)	$z = 2(0)+3 = 3$
B(3,0)	$z = 2(3)+0 = 6$
D(5,0)	$z = 2(5)+0 = 10$
C(0,7)	$z = 2(0)+7 = 7$

So.

function is minimum at corner point A(0,3).

Q16

$3x+5y \geq 15$

$x+6y \geq 9$

$x \geq 0, \quad y \geq 0$

(LHR - 2011)

$3x+5y \geq 15$

Its associated Eq.

$3x+5y = 15 \rightarrow (1)$.

Put $x=0$ in (1).

$3(0)+5y = 15$

$y = 3$

$A(0,3)$

Put $y=0$ in (1).

$3x+5(0) = 15$

$x = 5$

$B(5,0)$

$x+6y \geq 9$

Its associated Eq.

$x+6y = 9$ —(ii)

Put $x=0$ in (ii).

$0+6y = 9$

$y = 3/2$

$C(0, 3/2)$.

Put $y=0$ in (ii).

$x+6(0) = 9$

$x = 9$

$D(9,0)$

$x \geq 0, y \geq 0$

The associate Eq.

$x=0, y=0$

So, the graph will be in first Quad.

Point test: $O(0,0)$

$3x+5y \geq 15$

$3(0)+5(0) \geq 15$

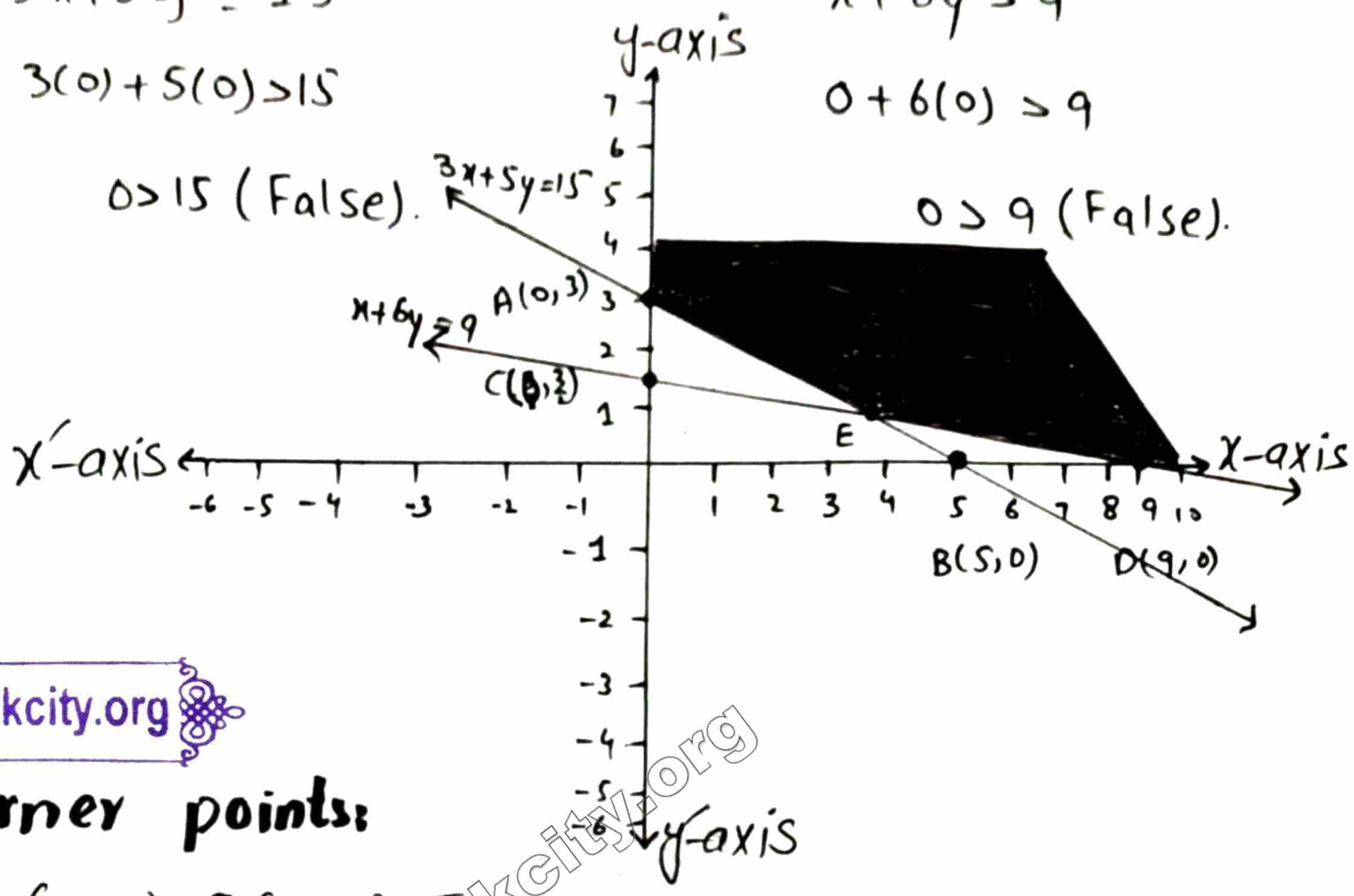
$0 \geq 15$ (False).

Point test: $O(0,0)$

$x+6y \geq 9$

$0+6(0) \geq 9$

$0 \geq 9$ (False).



Corner points:

A(0,3), D(9,0), E

For Corner Point E:-

$x+6y=9 \rightarrow (3)$

$3x+6y=15 \rightarrow (4)$

$3 \times Eq.(3) - Eq.(4)$

$3x+18y=27$

$3x+5y=15$

$13y=12$

$y=\frac{12}{13}$

$Eq.(3) \Rightarrow x+6\left(\frac{12}{13}\right)=9$

$x=9-\frac{72}{13}$

$x=\frac{117-72}{13}=\frac{45}{13}$

Thus Point of Intersection is $E\left(\frac{45}{13},\frac{12}{13}\right)$.

Now Value of z at Corner Points.

Corner Points	$z=3x+y$
A(0,3)	$z=3(0)+3=3$
D(9,0)	$z=3(9)+0=27$
$E\left(\frac{45}{13},\frac{12}{13}\right)$	$z=3\left(\frac{45}{13}\right)+\frac{12}{13}=\frac{147}{13}=11.3$

So z is Minimum at A(0,3).