

Chapter-1:

Measurements

1.1 Introduction to Physics

Physics:

"Physics deal with the study of matter and energy and the retationship between them."

Physics is the most fundamental of all sciences and provides other branches of science, basic principals and fundamental laws. Physics gave new branches of science such as physical chemistry, biophysics astrophysics, health physics engineering and electronics etc.

Areas of Physics:

are shown in table.

Areas of Physics
Mechanics
Heat and Thermodynamics
Electromagnetism
Optics

Nuclear Physics:

It deals with atomic nuclei.

Particle Physics:

This branch of physics deals with the properties and behaviour of elementary particles.

Relativistic Mechanics:

It deals with velocities approaching to the speed of light ($c = 3 \times 10^8\ ms^{-1}$).

Solid State Physics:

This branch of physics deal with the structure and properties of solids.

Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic Physics
Molecular physics
Nuclear physics
Solid-state physics
Particle physics
Superconductivity
Super Fluidity
Plasma physics
Magneto hydrodynamics
Space physics

Interdisciplinary areas of physics

Astrophysics
Biophysics
Chemical physics
Engineering physics
Geophysics
Medical physics
Physical oceanography
Physics music

1.2 Physical Quantities

The quantities which can be measured physically or by their physical effect are called physical quantities.

Examples:

Length, mass, time, force, speed etc.

Types of Physical Quantities:

1. Base Quantities

The quantities which are not defined in terms of other physical quantities are called base quantities.

Base quantities are the minimum number of those physical quantities in terms of which other physical quantities can be defined.

Examples:

Length, mass, time etc.

Measurement of Base Quantity:

It is based on two steps.

(1) Choice of standard.

(2) To set up a procedure for comparing the quantity with the standard.

Characteristics of an ideal standard:

An ideal standard has two main characteristics.

(i) It is accessible.

(ii) It is invariable.

2. Derived Quantities:

Derived quantities are those whose definitions are based on other physical quantities.

Example:

Velocity, acceleration, Force etc.

1.3 International System of Units

In 1960 an international committee agreed on a set of definitions and standard to describe the physical quantities. The system that was established is called the System International (SI).

1. Base Units:

There are seven base units are given in table -1.

Table-1

Physical Quantity	SI Unit	Symbol
Length	metre	m
Mass	Kilogram	kg
Time	Second	S
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Intensity of light	candela	cd
Amount of Substance	mole	mol

2. Supplementary Units:

Two supplementary units are shown in table-2.

Table-2

Physical Quantity	SI Unit	Symbol
plane angle	radian	rad
Solid angle	Steradian	Sr

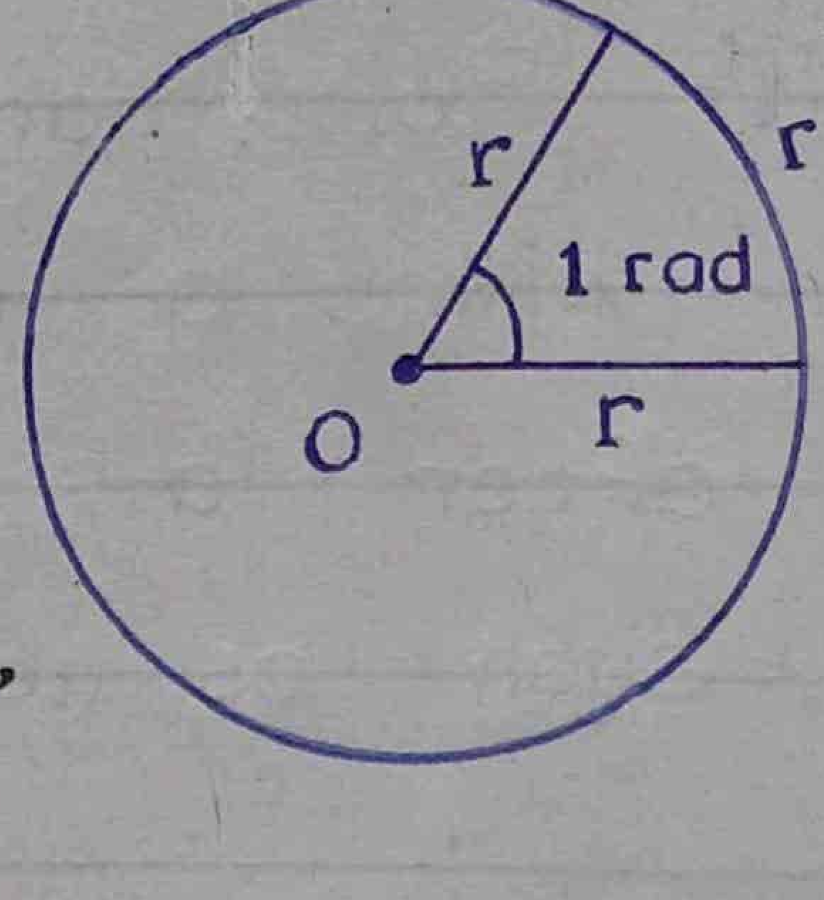
3. Derived Units:

These are derived from base units and supplementary units. Some derived units are given in table-3.

Table-3

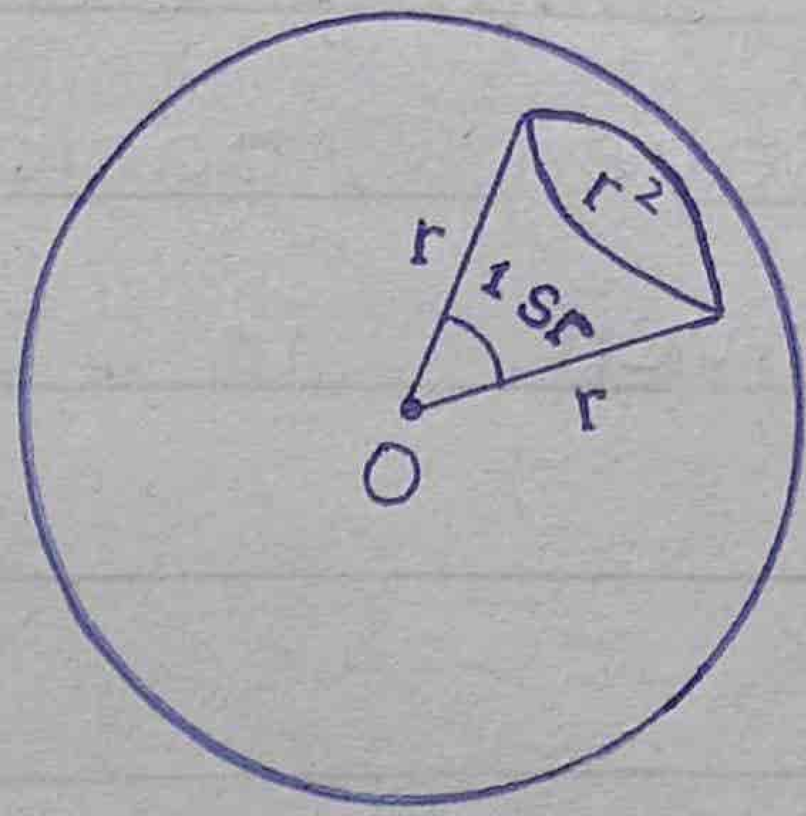
Physical quantity	Unit	Symbol	In terms of base units
Force	newton	N	$Kg\,m\,s^{-2}$
Work	joule	J	$Nm = kg\,m^2 s^{-2}$
power	watt	W	$Js^{-1} = kg\,m^2 s^{-3}$
pressure	pascal	Pa	$Nm^{-2} = kg\,m^{-1} s^{-2}$
Electric charge	Coulomb	C	As

Radian:



" The radian is the plane angle between two radii of a circle which cut off the circumference, an arc equal in length to the radius as shown in figure. It symbol is "rad".

Steradian:



"It is the solid angle (three dimensional angle) made at the centre of a sphere by an area of its surface equal to the square of radius of the sphere."
Its symbol is "sr".

Scientific Notation:

Numbers are represented in standard form called scientific notation which uses powers of ten.

In the internationally accepted form there should be only non-zero digit left of decimal.

Example:

The number 134.7 is written as 1.347×10^{2}.

The number 0.0023 is written as 2.3×10^{-3}.

Conventions For Indicating Units:

(i) Full name of the unit does not begin with a capital letter even if named after a scientist.
e.g. newton.

(ii) The symbol of unit named after a scientist has initial capital letter such as N for newton.

(iii) The prefix should be written before the unit without any space, such as 1×10^{-3} m is written as 1 mm.

(iv) Compound prefixes are not allowed. For example 1 μμF should be written as 1pF.

(v) A combination of base units is written each with one space apart. For example, newton meter is written as N m.

(vi) A number such as 5.0×10^{4} cm may be expressed in scientific notation as 5.0×10^{2} m.

(vii) When a multiple of base unit is is raised to a power, the power applies to the whole multiple and not the base unit alone.

Thus $1\,km^2 = 1\,(km)^2 = 1 \times 10^6\,m^2$.

(viii) Measurement in practical work must be recorded in the most suitable unit.

1.4 Errors and Uncertainties

All physical measurements are uncertain بے یقینی

(درست نہ ہونا) (کسی حد تک)
or imprecise to some extent.

ختم کرنا
All the errors cannot be completely eliminated.

Reason for Errors:

(i) Negligence or inexperience of a person.
(ii) The faulty apparatus.
(iii) Inappropriate method or techinique.

Types of Errors:

Two main types of errors.

i) Random Error:

Random error occurs when repeated measurements of the quantity, give different values of the quantity, under the same conditions.

Reason: It is due to some unknown reason.

Removal of Random Error:

Repeating the measurements many times and taking an average can reduce the effect of random error.

ii) Systematic Error:

It affects all the measurements of a particular quantity equally. It produces the same difference in readings.

Reson: (i) Zero error of the instrument.
(ii) Incorrect markings.

Removal of systematic error:

It can be reduced by comparing the instrument with other more accurate instrument and a correction factor can be applied.

1.5 Significant Figures

In any measurement the accurately known digits and the first doubtful digit are called significant figures.

Rules for finding out number of Significant Figures

(i) All digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are significant.

(ii) A zero between two significant figures is significant. e.g. 3201, 57.09.

(iii) Zeros to the left of significant figures are not significant. In 0.04 or 02.59 none of the zeros is significant.

(iv) Zeros to the right of a significant figure are significant. For example all zeros in 3.570 or 7.4000 are significant.

(v) Zeros obtained through actual measurements are significant. e.g 490 has three significant figures, if L.C of measuring scale is 1kg.

(vi) When a measurement is recorded in scientific notation, the figures other than powers of 10 are significant. For example in 8.70×10^4, there are three significant figures.

(vii) In multiplying and dividing numbers, the number of significant figures in the product or quotient is equal to the least number of significant figures of the quantities to be multiplied or divided. For example

$$5.47 \times 19.89 = 108.7983 = 109$$

$$19.89 \div 5.47 = 3.63619 = 3.64$$

Rounding off Data

Rules:

(i) If the digit to be dropped is less than 5, the last significant digit will remain the same.

(ii) If the digit to be dropped is greater than 5, the last significant digit is increased by one.

(iii) If the digit to be dropped is 5, the last significant digit is increased by one if it is odd and will remain the same if it is even.

Examples:

The following numbers are rounded off to three significant figures. The digits are deleted one by one.

43.75 is rounded off as 43.8

56.8546 is rounded off as 56.9

73.650 is rounded off as 73.6

64.350 is rounded off as 64.4

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.45768982 \times 10^{3}$$

As 3.64×10^4 is least significant, so after rounding off the answer is 1.46×10^3.

In Adding or Subtracting numbers the number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted.

In this case the number of significant figures is not important. It is the position of decimal that matters. For example adding quantities in meters.

(i)	(ii)
72.1	2.7543
3.42	4.10
+ 0.003	+ 1.273
75.523	8.1273
Correct Answer = 75.5 m	8.13 m

Meter rod:

$$L.C = 1\,mm = \frac{1}{10}\,cm = 0.1\,cm$$

Vernier Callipers:

$$L.C = \frac{1}{100}\,mm = 0.01\,mm \quad \because \frac{1}{1000}\,cm = 0.001\,cm$$

Screw Gauge:

$$L.C = \frac{1}{100}\,mm = 0.01\,mm \quad \therefore \frac{1}{1000}\,cm = 0.001\,cm$$

Smaller L.C means more precise measurement.

1.6 Precision and Accuracy

Precision:

"The precision of a measurement is determined by the instrument or device being used."

Precise Measurement:

A precise measurement

is that which has smaller value of absolute uncertainty or smaller value of least count (L.C).

Absolute Uncertainty:

"Absolute uncertainty is equal to the least count of the measuring device." It is also called precision of the instrument.

$$\text{precision} = \text{Absolute uncertainty} = \text{Least count}$$

Fractional Uncertainty:

"It is the ratio of least count of the measuring device to the measured value."

$$\text{Fractional Uncertainty} = \frac{\text{Least Count}}{\text{measured value}}$$

Percentage Uncertainty:

$$\text{Percentage Uncertainty} = \frac{\text{Least Count}}{\text{measured value}} \times 100\%$$

Accuracy:

"Accuracy depends upon the fractional or percentage uncertainty in the measurement."

Accurate Measurement:

"An accurate measurement is that which has less Fractional uncertainty or less percentage uncertainty."

Explanation with Examples:

Example-1:

Suppose the length of an object is measured by a meter rod as 25.5 cm. Least count of the meter rod is 0.1 cm (L.C = 1 mm).

Absolute uncertainty = Least count = Precision = ±0.1 cm

$$L = 25.5 \pm 0.1 \text{ cm}$$

$$\text{Fractional uncertainty} = \frac{0.1}{25.5} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1}{25.5} \times 100\% = 0.4\%$$

Example-2:

Consider another measurement made by vernier callipers as 0.45 cm. Its least count is 0.01 cm ($\frac{1}{10}$ millimeter).

Precision or absolute uncertainty (LC) = ±0.01 cm

$$\ell = 0.45 \pm 0.01 \text{ cm}$$

$$\text{Fractional Uncertainty} = \frac{0.01}{0.45} = 0.02$$

$$\text{Percentage Uncertainty} = \frac{0.01}{0.45} \times 100\% = 2.0\%$$

Conclusion:

(i) The reading by a meter rod (25.5 cm) is although less precise, but it is more accurate due to smaller value of percentage uncertainty. i-e 0.4%

(ii) The reading 0.45 cm taken by the vernier callipers is more precise, but it is less accurate due to larger value of percentage uncertainty. i.e 2.0%

In fact, it is the relative measurement which is important. For a smaller physical quantity, more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such a micrometer screw gauge, with least count 0.001 cm should have been used.

1.7 Assessment of total uncertainty in the Final result

To find total uncertainty it is necessary to find the uncertainties in all the factors involved in calculation.

1. For Addition and Subtraction:

Absolute Uncertainties are added:

Suppose the distance x is found by the difference between two seperate position measurements

$$x_1 = 10.5 \pm 0.1\ \text{cm}$$

and

$$x_2 = 26.8 \pm 0.1\ \text{cm}$$

Then

$$x = x_2 - x_1 = (26.8 - 10.5) \pm 0.2\,cm$$

$$= 16.3 \pm 0.2\,cm$$

2. For Multiplication and division:

Percentage Uncertainties are added:

If measurements are multiplied or divided, then their percentage uncertainties are added to find the total uncertainty in the final result.

Example:

If $V = 5.2 \pm 0.1\,V$

$I = 0.84 \pm 0.05\,A$

$R = \frac{V}{I}$ is found as

%age uncertainty for $V = \frac{0.1}{5.2} \times 100\% = 2\%$

%age uncertainty for $I = \frac{0.05}{0.84} \times 100\% = 6\%$

Total %age uncertainty $= 2\% + 6\% = 8\%$

$$R = \frac{5.2}{0.84} = 6.2 \pm 8\%\ \text{ohms}$$

$$\left(8\% \text{ of } 6.2 = 6.2 \times \frac{8}{100} = 0.5\right)$$

$$R = (6.2 \pm 0.5)\ \text{ohms}$$

3. For Power Factor:

"Multiply the percentage uncertainty by that power."

For example in

$$V = \frac{4}{3}\pi r^3$$

% age uncertainty in V = 3 x %age uncertainty in radius r.

Example:

Consider the radius of a small sphere is measured as 2.25cm by a vernier calliper with least count 0.01 cm, then

$$r = 2.25 \pm 0.01\,cm$$

% age uncertainty in radius $r = \frac{0.01}{2.25} \times 100\% = 0.4\%$

Total percentage uncertainty in

$$V = 3 \times 0.4\% = 1.2\%$$

As

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (2.25)^3$$

$V = 47.689\,cm^3$ with 1.2% uncertainty.

$$V = 47.7\,cm^3 \pm 1.2\%$$

$\left(1.2\% \text{ of } 47.7 = 44.7 \times \frac{1.2}{100} = 0.6\right)$

$$V = 47.7 \pm 0.6\,cm^3$$

4. For uncertainty in average value:

(i) Find the average value of measured values.

(ii) Find the difference (deviation) between each measured value and average value.

(iii) The mean difference is the uncertainty in the average value.

For example

1.20, 1.22, 1.23, 1.19, 1.22, 1.21

are measured values in mm.

$$\text{The average} = \frac{1.20+1.22+1.23+1.19+1.22+1.21}{6} = 1.21 \text{ mm}$$

The difference between each measured value and average value neglecting the sign is

0.01, 0.01, 0.02, 0.02, 0.01, 0.

$$\text{Mean difference} = \frac{0.01+0.01+0.02+0.02+0.01+0}{6} = 0.01 \text{ mm}$$

Thus uncertainty in the average value is $= 1.21 \pm 0.01$ mm

5. For the uncertainty in a timing experiment

"The uncertainty in the time period of a vibrating body is found by dividing the least count of the timing device by the number of vibrations."

Uncertainty in the time period = $\frac{\text{Least Count}}{\text{No of vibrations}}$

For example

The time taken to complete 30 vibrations of a simple pendulum is measured 54.6 s by a stop watch whose least count is 0.1 s, then

Uncertainty in time period $= \frac{0.1}{30} = 0.003\,s$

So,

Time period $T = \frac{54.6}{30} = 1.82 \pm 0.003\,s$

Hence, it is advisable to count number of swings (vibrations) to reduce the timing uncertainty.

Example 1.1:

The length, breath and thickness of a sheet are 3.233 m, 2.105 m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.

Solution: $l = 3.233\,m$, $b = 2.105\,m$

$h = 1.05\,cm = 1.05 \times 10^{-2}\,m$

Volume $V = l \times b \times h$

$V = 3.233 \times 2.105 \times 1.05 \times 10^{-2}$

$V = 7.14573825 \times 10^{-2}\,m^3$

As the quantity 1.05 cm has the minimum number of significant figures equal to three. So volume is recorded upto 3 significant figures,

Hence $V = 7.15 \times 10^{-2}\,m^3$.

Example-1.2:

The mass of a metal box measured by a lever balance is 2.2Kg. Two silver coins of masses 10.01g and 10.02g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision.

Solution:

$$m_1 = 2.2\,Kg$$

$$m_2 = 10.01g = \frac{10.01}{1000}\,kg = 0.01001\,Kg$$

$$m_3 = 10.02g = \frac{10.02}{1000}\,kg = 0.01002\,kg$$

$$m = m_1 + m_2 + m_3 = 2.2kg + 0.01001kg + 0.01002\,Kg$$

$$m = 2.22003\,Kg$$

As the least precise is 2.2kg.

So $m = 2.2kg$ by rounding off.

Exercise-1.3:

The diameter and length of a metal cylinder measured with the help of vernier callipers of least count 0.01cm are 1.22cm and 5.35cm. Calculate the volume V of the cylinder and uncertainty in it.

Solution:

$$d = 1.22 \pm 0.01\text{ cm}$$

$$L.C = 0.01\text{ cm}$$

$$l = 5.35 \pm 0.01\text{ cm}$$

%age uncertainty in d $= \frac{0.01}{1.22} \times 100\% = 0.8\%$

%age uncertainty in l $= \frac{0.01}{5.35} \times 100\% = 0.2\%$

Volume is

$$V = \pi r^2 l = \pi \left(\frac{d}{2}\right)^2 l = \frac{\pi d^2 l}{4}$$

Total uncertainty in V $= 2 \times 0.8\% + 0.2\%$

Total uncertainty in V $= 1.6\% + 0.2\% = 1.8\%$

Then

$$V = \frac{3.14 \times (1.22) \times 5.35}{4}$$

$$= 6.2509079 \text{ cm}^3 \pm 1.8\%$$

$$V = 6.25 \text{ cm}^3 \pm 1.8\%$$

$(1.8\% \text{ of } 6.25 = 6.25 \times \frac{1.8}{100} = 0.1)$

Thus

$$V = (6.25 \pm 0.1) \text{ cm}^3$$

1.8 Dimensions of Physical Quantities

Each base quantity is considered a dimension denoted by a capital letter written within a square bracket [ ]. It stands for the qualitative nature of the physical quantity.

For example:

Dimension of mass is [M].

Dimension of Length is $[L]$

Dimension of time is $[T]$

Other physical quantities may be the combination of these dimensions.

For example:

$\therefore v = \frac{s}{t}$

$$\text{dimensions of speed} = \frac{\text{Dimension of length}}{\text{Dimension of time}}$$

$$[v] = \frac{[L]}{[T]} = [L][T^{-1}] = [LT^{-1}]$$

Similarly

Dimension of acceleration $\therefore \left(a = \frac{v}{t}\right)$

$$[a] = \frac{[LT^{-1}]}{T^{-1}} = [LT^{-2}]$$

Dimensional Analysis:

Using the method of dimensions called the dimensional analysis, we can check the correctness of a given formula or an equation and can also derive it.

Uses:

(i) Checking Homogeneity of physical equation

To check the correctness of an equation, the dimension of the quantities on both sides of the equation must be same. This is called the Principal of Homogeneity of dimensions.

(ii) Deriving a possible Formula

To derive a relation for a physical quantity, the correct guessing of various factors on which the physical quantity depends is important.

Example-1.4:

Check the correctness of the relation $v = \sqrt{\frac{F \times l}{m}}$ where v is the speed of transverse wave on the stretched string of tension F, length l and mass m.

Solution:

$$v = \sqrt{\frac{F \times l}{m}} = (F \times l\, m^{-1})^{1/2}$$

L.H.S

$$[v] = [LT^{-1}]$$

R.H.S

$$= \left([F] \times [l] \times [m^{-1}]\right)^{\frac{1}{2}}$$

$$= \left([MLT^{-2}][L][M^{-1}]\right)^{\frac{1}{2}}$$

$$= \left([M^{1-1}\, L^{1+1}\, T^{-2}]\right)^{\frac{1}{2}}$$

$$= \left([M^{0}\, L^{2}\, T^{-2}]\right)^{\frac{1}{2}}$$

$$= [1 .\, L^{2 \times \frac{1}{2}}\, T^{-2 \times \frac{1}{2}}]$$

$$R.H.S = [LT^{-1}]$$

So

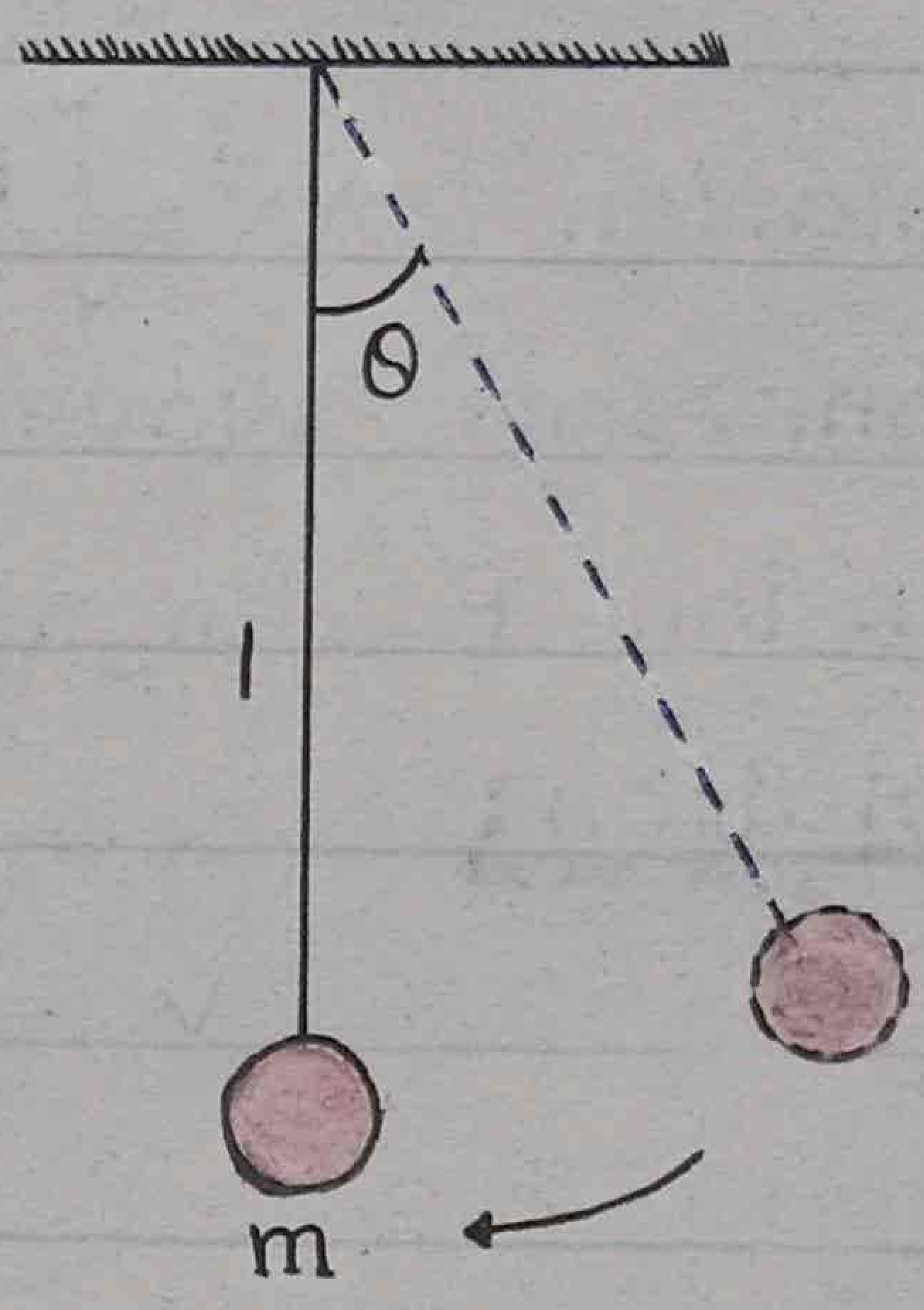
$$L.H.S = R.H.S$$

Hence the equation is dimensionally corret.

Example 1.5:

Derive a relation for the time period of a simple pendulum shown in figure using dimensional analysis. The various possible factors on which the time period T may depend are:

(i) Length of the pendulum (l)
(ii) Mass of the bob (m)
(iii) Angle θ which the thread makes with the vertical
(iv) Acceleration due to gravity (g)



Solution:

$$T \propto m^a . l^b . \theta^c . g^d$$

$$T = \text{constant} \ m^a . l^b . \theta^c . g^d$$

We have to find the values of the powers a, b, c, d.

$$[T] = [M]^a [L]^b [LL^{-1}]^c [LT^{-2}]^d$$

$$[M]^0 [L]^0 [T] = [M]^a [L^{b+d+c-c}] [T]^{-2d}$$

Equating the dimensions on both sides

$$[M]^0 = [M]^a$$

$$[L]^0 = [L]^{b+d+c-c}$$

$$[T]^1 = [T]^{-2d}$$

Equating the powers on both sides

$$a = 0$$

$$-2d = 1 \Rightarrow d = -\frac{1}{2}$$

$$b + d + \cancel{c} - \cancel{c} = 0 \Rightarrow b + d = 0$$

$$b = -d \Rightarrow b = -\left(-\frac{1}{2}\right)$$

$$b = \frac{1}{2}$$

$$\theta = [LL^{-1}]^c = [L^0]^c = 1.$$

$$T = \text{constant} \times m^0 \times l^{\frac{1}{2}} \times 1 \times g^{-\frac{1}{2}}$$

$$= \text{constant} \times 1 \times l^{\frac{1}{2}} \times g^{-\frac{1}{2}}$$

$$= \text{constant} \left(\frac{l}{g}\right)^{\frac{1}{2}}$$

$$T = \text{constant} \sqrt{\frac{l}{g}}$$

Note:

The numerical value of constant cannot be determined by dimensional analysis, however it can be found by experiments.

Example 1.6:

Find the dimensions and hence, the SI units of coefficient of viscosity η in the relation of Stokes' law for the drag force F for a spherical object of radius r moving with velocity v given as

$$F = 6\pi\eta r v$$

Solution:

6π is a number having no dimensions.

$$[F] = [\eta r v]$$

$$[\eta] = \frac{[F]}{[r][v]}$$

$$= \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

$$[\eta] = [ML^{1-1-1}T^{-1}]$$

$$[\eta] = [ML^{1}T^{-1}]$$

So, SI unit of $\eta = kg\,m^{-1}s^{-1}$.

$$F = 6\pi\eta r v$$

$$\eta = \frac{F}{6\pi r v}$$

units of η

$$\eta = \frac{N}{m.\,ms^{-1}}$$

$$\eta = Nsm^{-2}$$

Questions

1.1 Name several repetitive phenomenon occuring in nature which could serve as reasonoble time standards.

Answer:

(i) Rotation of earth about its own axis and around the sun.

(ii) Rotation of moon around the Earth.

(iii) The heart beat or human pulse rate.

(iv) Lattice vibrations in a crystal.

1.2 Give the drawbacks to use the period of a pendulum as a time standard.

Answer:

Time period of the simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$ depends on the value of "l" and "g". Value of g is not same at all the places of earth. So the simple pendulum cannot be taken as a time standard.

Morever we cannot realize an ideal simple pendulum.

1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

Answer:

Kilogram and mole are the two units for the amount of substance.

Kilogram is used when we have to measure the amount of substance without considering the number of atoms/molecules. Kilogram is used at Macroscopic level.

Mole is used at microscopic level where the number of atoms/molecules is important.

1.4 Three students measured the length of a needle with a scale on which minimum division is 1mm and recorded as (i) 0.2145 m (ii) 0.21 m (iii) 0.214 m. Which record is correct and why?

Answer:

The least count of the scale is LC = 1mm = 0.001 m.

So, the lengh can be measured up to that accuracy (three decimal places). Hence the measurement 0.214 m is correct.

1.5 An old saying is that "A chain is only as strong as its weakest link". What analogous statement can you make regarding experimental data used in a computation?

Answer:

The old saying "A chain is only as strong as its weakest link" is analogous to the statement.

"In computation of experimental data the result is as much accurate as its least accurate reading in the data."

1.6 The period of simple pendulum is measured by a stop watch. What type of errors are possible in the time period?

Answer:

Following errors are possible in measuring the time period of a simple pendulum by a stop watch.

(i) Zero Error in the stop watch.

(ii) Measuring skill of the observer (personal error).

(iii) Least Count of stop watch.

1.7 Does a dimensional analysis give any information on constant of proportionality

that may appear in an algebraic expression? Explain.

Answer: No.

The dimensional analysis does not give any information about the constant of proportionality. It can be measured experimentally.

In

$$T = \text{constant}\sqrt{\frac{l}{g}}$$

The constant can be measured experimently.

1.8 Write the dimensions of

(i) Pressure (ii) Density

Answer:

(i) As

$$P = \frac{F}{A}$$

$[F] = [MLT^{-2}]$

$[A] = [L^2]$

$$[P] = \frac{[MLT^{-2}]}{[L^2]}$$

$$[P] = [ML^{1-2}T^{-2}]$$

$$[P] = [ML^{-1}T^{-2}]$$

(ii) As

$$\rho = \frac{m}{V}$$

$[m] = [M]$

$[V] = [L^3]$

$$[\rho] = \frac{[m]}{[V]}$$

$$[\rho] = \frac{[M]}{[L^3]}$$

$$[\rho] = [ML^{-3}]$$

1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that

$$[\lambda] = [L], \quad [v] = [LT^{-1}] \quad \text{and} \quad [f] = [T^{-1}]$$

Decide which of the following is correct.

$$f = v\lambda \quad \text{or} \quad f = \frac{v}{\lambda}$$

Answer: Using dimensional analysis

$$f = v\lambda$$

$$[T^{-1}] = [LT^{-1}][L]$$

$$[T^{-1}] = [L^2T^{-1}]$$

LHS ≠ RHS

So $f = v\lambda$ is not correct

$$f = \frac{v}{\lambda}$$

$$[T^{-1}] = \frac{[LT^{-1}]}{[L]}$$

$$[T^{-1}] = [T^{-1}]$$

LHS = RHS

So $f = \frac{v}{\lambda}$ is dimensionally correct.

Light Year:

One light year is the distance travelled by Light in one year. One light year $= 9.46 \times 10^{15}$ m. Light Year is a unit of Distance.

NUMERICAL PROBLEMS

1.1 A light year is the distance light travels in one year. How many metres are there in one light year: (speed of light = 3.0×10^8 ms^{-1}).

(Ans: 9.5×10^{15} m)

1.2 a) How many seconds are there in 1 year?

b) How many nanoseconds in 1 year?

c) How many years in 1 second?

[Ans. (a) 3.1536×10^7 s, (b) 3.1536×10^{16} ns, (c) 3.1×10^{-8} yr]

1.3 The Length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

(Ans: 196 cm^2)

1.4 Add the following messes given in kg up to appropriate precision. 2.189, 0.089, 11.8 and 5.32.

(Ans: 19.4 kg)

1.5 Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment

Length of a simple pendulum = 100 cm

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

(Ans: 9.76 ± 0.06 ms^{-2})

1.6 What are the dimensions and units of gravitational constant G in the formula

$F = G\,\frac{m_1 m_2}{r^2}$

(Ans: [$M^{-1} L^3 T^{-2}$], $Nm^2\,kg^{-2}$)

1.7 Show that the expression $v_f = v_i + at$ is a dimensionally correct, where v_i is the velocity at t = 0, a is the acceleration and v_f is the velocity at time t.

1.8 The speed of v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. deduced by the method of dimensions, the formula for the speed of sound.

(Ans: v = Constant $\sqrt{\frac{E}{\rho}}$)

1.9 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

1.10 Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v ?

(Ans: n = -1 , m= 2)

NUMERICAL PROBLEMS

P-1.1

$v = c = 3.0 \times 10^{8}\ ms^{-1}$

$t = 1$ Year

$t = 1 \times 365 \times 24 \times 60 \times 60\ s$

$t = 31536000\ s$

$S = ?$

$S = vt$

$S = 3 \times 10^{8} \times 31536000$

$S = 9.46 \times 10^{15}\ m$

$S = 9.5 \times 10^{15}\ m$

One light Year $= 9.5 \times 10^{15}\ m$

P-1.2

(a) $t = 1$ Year

$t = 1 \times 365 \times 24 \times 60 \times 60\ s$

$t = 3.1536 \times 10^{7}\ s$

(b)

$10^{-9} = n = \text{nano}$

$t = 3.1536 \times 10^{7} \times 10^{+9} \times 10^{-9}\ s$

$t = 3.1536 \times 10^{7} \times 10^{9}\ ns$

$t = 3.1536 \times 10^{16}\ ns$

(c)

$1\ \text{Year} = 3.1536 \times 10^{7}\ s$

$$\frac{1\ \text{Year}}{3.1536 \times 10^{7}} = 1\ s$$

$1\ s = 3.1 \times 10^{-8}$ Year

P-1.3

$l = 15.3\ cm$

$W = 12.80\ cm$

$A = ?$

$A = l \times W$

$A = 15.3 \times 12.80$

$A = 195.84\ cm^{2}$

$A = 196\ cm^{2}$

P-1.4

$$\begin{array}{r} 5.32 \\ 2.189 \\ 0.089 \\ +\ 11.8 \\ \hline 19.398 \end{array}$$

In the quantity 11.8, there is only one term after the decimal position. So, the answer is rounded off as 19.4 kg.

P-1.5:

$l = 100\,cm = 1.0\ m$

Time For 20 vibrations = 40.2 s

Time period $T = \frac{40.2}{20}$

$T = 2.01\,s$

Least Count of meter rod is

$L.C = 1\,mm$

$L.C = 0.1\,cm$

Least Count of stop watch is

$= 0.1\,s$

Uncertainty in time period

is $= \frac{0.1}{20}\,s$

$= 0.005\,s$

$g = ?$

As

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2\frac{l}{g}$$

$$g = 4\pi^2\frac{l}{T^2}$$

$$= 4(3.14)^2\frac{1}{(2.01)^2}$$

$$g = 9.76\ ms^{-2}$$

$l = 100\ cm \pm 0.1\ cm$

$T = 2.01\ s \pm 0.005\ s$

%age uncertainty in l

$$= \frac{0.1}{100}\times 100\% = 0.1\%$$

%age uncertainty in T

$$= \frac{0.005}{2.01}\times 100\% = 0.248\%$$

$$= 0.25\%$$

Total uncertainty in g

$= 0.1\% + 2(0.25\%)$

$= 0.1\% + 0.5\%$

$= 0.6\%$

$g = 9.76\ ms^{-2} \pm 0.6\%$

(0.6% of $9.76 = 9.76 \times \frac{0.6}{100}$

$= 0.05856$

$= 0.06$)

$g = 9.76 \pm 0.06\ ms^{-2}$

P-1.6:

$$F = G\frac{m_1 m_2}{r^2}$$

Dimensions and units of

$G = ?$

$$G = \frac{F \times r^2}{m_1\ m_2}$$

$$G = \frac{[F][r^2]}{[m_1][m_2]}$$

$$G = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

$$G = [M^{1-1-1} L^{1+2} T^{-2}]$$

$$G = [M^{-1} L^3 T^{-2}]$$

units of G is

$$G = \frac{N m^2}{kg \times kg}$$

$$G = N m^2 kg^{-2}$$

P-1.7:

$$v_f = v_i + at$$

$$[v_f] = [v_i] + [a][t]$$

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2+1}]$$

$$[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

$$[LT^{-1}] = [LT^{-1}]$$

L.H.S = R.H.S

So, the equation is dimensionally correct.

P-1.8:

v depends on ρ and E

Formula for the speed of Sound = v = ?

$$v \propto \rho^a E^b$$

$$v = \text{constant}\ \rho^a E^b \qquad (1)$$

$$[v] = \text{constant}\ [\rho^a][E^b]$$

$$[\rho] = \frac{[Mass]}{[Volume]} = \frac{[M]}{[L^3]}$$

$$[\rho] = [ML^{-3}]$$

$$[E] = \frac{Stress}{Strain}$$

$$= \frac{[Force]}{[Area] \times [Strain]}$$

$$[E] = \frac{[MLT^{-2}]}{[L^2] \times 1}$$

$[E] = [ML^{-1}T^{-2}]$

$[V] = [LT^{-1}]$

So

$[LT^{-1}] = \text{constant} \times [ML^{-3}]^{a}[ML^{-1}T^{-2}]^{b}$

$[LT^{-1}] = \text{constant}\,[M^{a}L^{-3a}][M^{b}L^{-b}T^{-2b}]$

$[LT^{-1}] = \text{constant}\,[M^{a+b}L^{-3a-b}T^{-2b}]$

$[M^{0}L^{1}T^{-1}] = \text{constant}\,[M^{a+b}L^{-3a-b}T^{-2b}]$

Comparing dimensions on both sides.

$[M^{0}] = [M^{a+b}]$

$[L^{1}] = [L^{-3a-b}]$

$[T^{-1}] = [T^{-2b}]$

Equating powers on both sides;

$a + b = 0$

$-3a - b = -1$

$-2b = -1$

$b = \frac{1}{2}$

$a + \frac{1}{2} = 0$

$a = -\frac{1}{2}$

Put the values of a and b in (1)

$V = \text{constant}\ \rho^{-\frac{1}{2}} \times E^{\frac{1}{2}}$

$V = \text{constant}\ \frac{E^{\frac{1}{2}}}{\rho^{\frac{1}{2}}}$

$V = \text{constant}\ \left(\frac{E}{\rho}\right)^{\frac{1}{2}}$

$V = \text{constant}\ \sqrt{\frac{E}{\rho}}$

P-1.9:

$E = mc^{2}$

$[E] = [m][c^{2}]$

$[ML^{2}T^{-2}] = [M][LT^{-1}]^{2}$

$[ML^{2}T^{-2}] = [M][L^{2}T^{-2}]$

$[ML^{2}T^{-2}] = [ML^{2}T^{-2}]$

L.H.S = R.H.S

So, the equation is dimensionally correct.

P-1.10:

$a \propto r^n v^m$

$n = ?$

$m = ?$

$a = \text{constant} \times r^n \times v^m$ —— (1)

$[a] = \text{constant}\ [r^n][v^m]$

$[LT^{-2}] = \text{constant}\ [L]^n [LT^{-1}]^m$

$[LT^{-2}] = \text{constant}\ [L^n][L^m T^{-m}]$

$[LT^{-2}] = \text{constant}\ [L^{n+m}][T^{-m}]$

$[L]^1 [T]^{-2} = \text{constant} [L]^{n+m} [T]^{-m}$

Comparing the dimensions

$[L]^1 = [L]^{n+m}$

$[T]^{-2} = [T]^{-m}$

Comparing powers on both sides

$n + m = 1$

$-m = -2$

$m = 2$

$m = 2$

$n + 2 = 1$

$n = 1 - 2$

$n = -1$

Hence

$m = 2$

$n = -1$