Chapter 08

Waves

Waves

Definition

"Wave is a mechanism by which energy is transported from one point to another".

Transportation of energy is carried out disturbances which sprea out from the source.

"Waves carry energy second momentum but not matter".

Explanation

Waves are produced by oscillations in a vibrating boby and propagate by means of oscillations

Examples

* ; Water waves are produced when a stone is dropped in a pond of water.
* ; Sound waves are produced

when we pluck a string of a guitar.

* ; A vibrating charge produces electromagnetic waves.

For the propagation of mechanical waves two things are required. Inertia and Elesticity of the medium. Due to inertia the medium stores energy and due to elasticity the medium has ability to return to the orignal condition after being disturbed.

Types of waves

* ; Mechanical waves.
* ; Electromagnetic waves.
* ; Matter waves.

Mechanical waves

"Wave which require a medium for their propagation are called mechanical waves".

Examples

* ; Sound waves.
* ; Water waves.

* ; Waves in the coils of a spring.
* ; waves in the string of a musical instrument.

Electromagnetic wave

"Waves which do not require a medium for their propagation are called eletromagnetic waves."

Examples

* ; X-rays.
* ; Heat waes.
* ; Ultraviolet rays.
* ; Light waves

Matter waves

"Waves associated with matter in motion are called matter waves".

Example

Fast moving electrons behave like waves not like particles. Such waves ar matter waves.

8.1 Progressive Waves

Definition

"The waves which transfer energy in moving away from the source of disturbance are called progressive waves or travelling waves".

Explanation

Drop a pebble into a pond of water. Ripples are produced and they spread out across the water. These ripples are progressive waves because they carry energy across the water surface.

Types of progressive waves

(i) Transverse waves
(ii) Longitudinal waves.

(i) Transverse waves

"Such waves in which particles of the medium vibrate along a line perpendicular to the direction of propagation of waves are called transverse waves".

In transverse waves particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves.

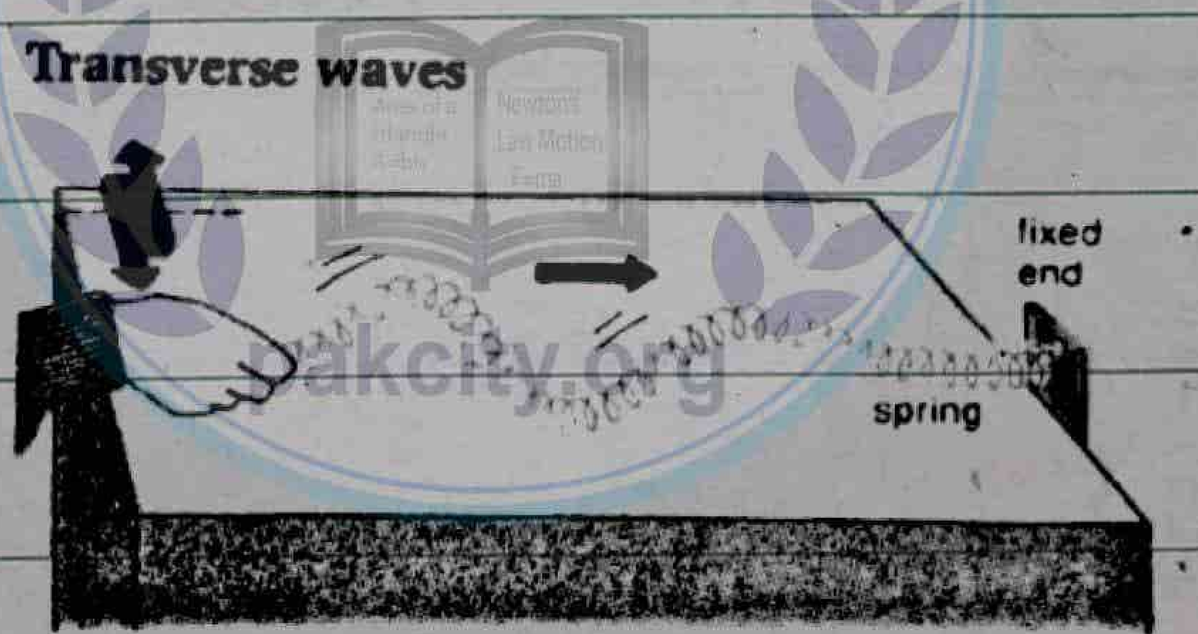
Examples

★ ; water waves.

★ ; All of the electromagnetic waves such as light waves are transverse waves.

Explanation

Consider a slinky spring is lying on a smooth table. Its one end is fixed. When the free end is moved from side to side waves



are produced in the spring and they move along the spring. These waves are transverse waves. Transverse waves consist of crests and troghs.

ii) Longitudinal waves

"Such waves in which particles of the medium vibrate parallel to the direction of propagation of the waves are called Longitudinal or compressional waves".

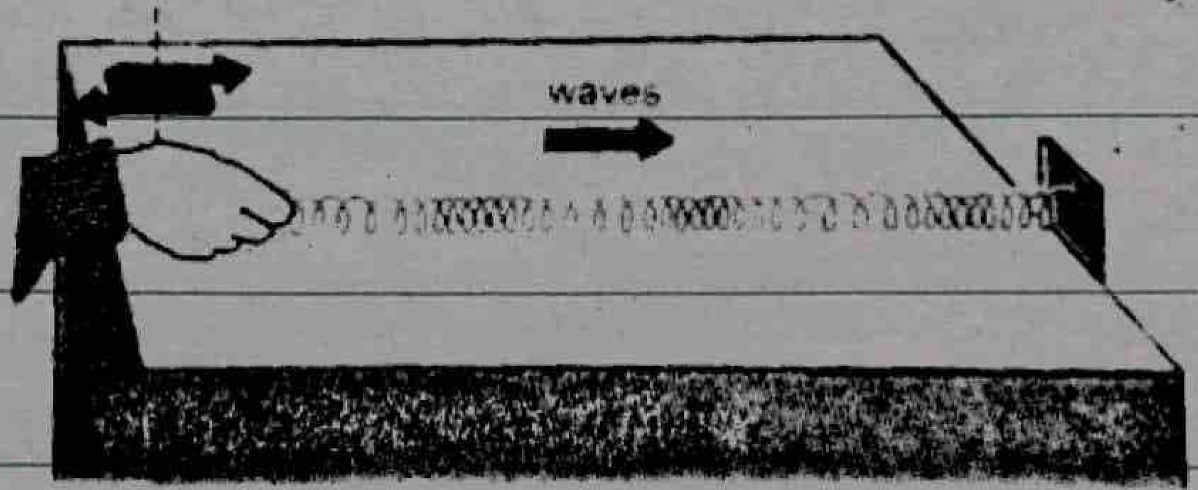
Example

Sound waves and longitudinal waves.

Explanation

Consider a slinky spring lying on a smooth table. Its one end is fixed, and the other free end is moved back and forth along the direction of the spring back and forth displacements will move along the spring. These are longitudinal waves. Longitudinal wave consists of compressions and rerefactions.

Longitudinal waves



8.2 Periodic Waves

Definition

"The periodic vibrations of a source produce continuous, regular and rythamic disturbances in a medium. These are called periodic waves"

Example

An oscillating mass spring system is an example of periodic vibrator which can produce periodic waves.

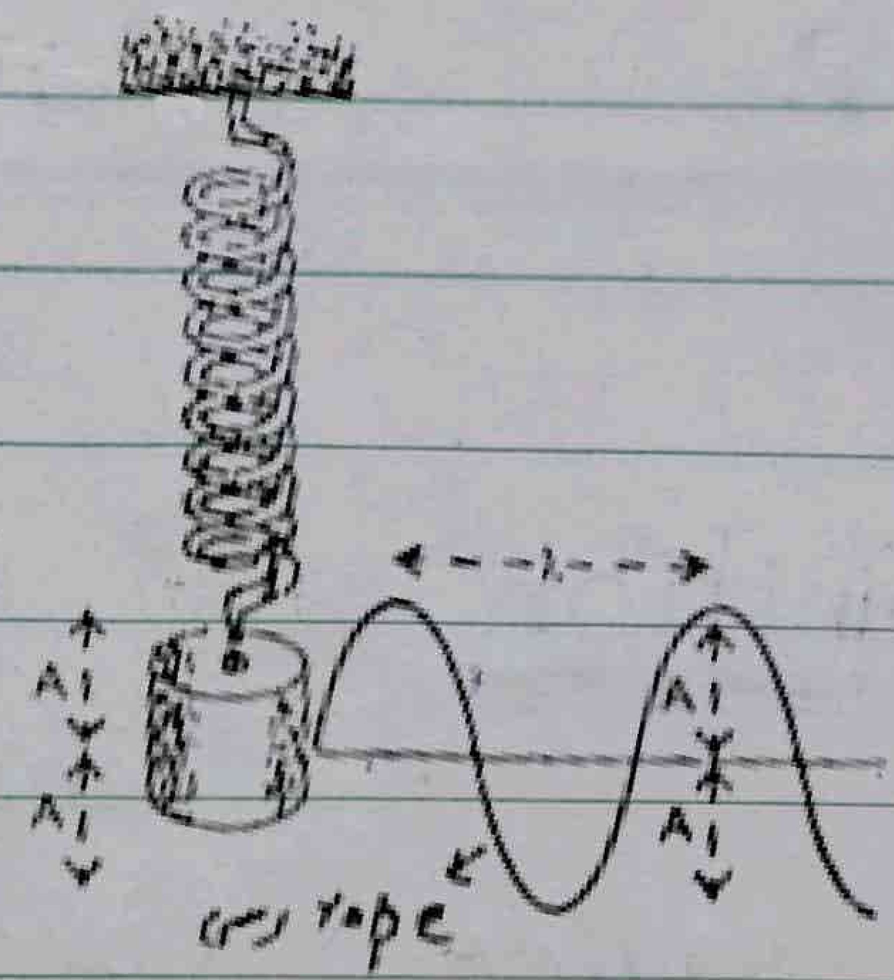
Types of periodic waves

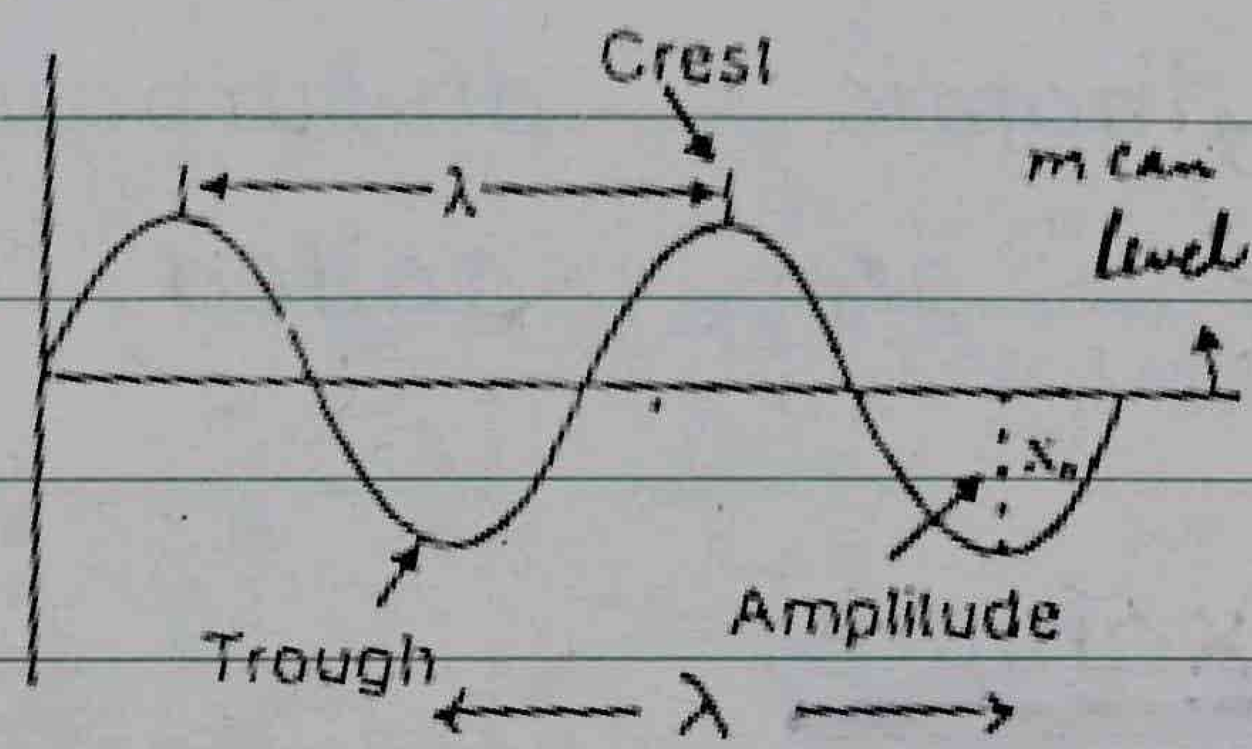
i) Transvers periodic waves.
ii) Longitudinal periodic waves.

i) Transvers periodic waves

Consider a verticle mass spring system. A rope is attached to the mass as shown in fig. When the mass spring system oscillates up and down transeve waves are produced which travel along the length of the rope horizontally.

However each particle of the rope vibrate vertically perpendicular to the direction of propagation of the waves. These waves are "transverse periodic waves".



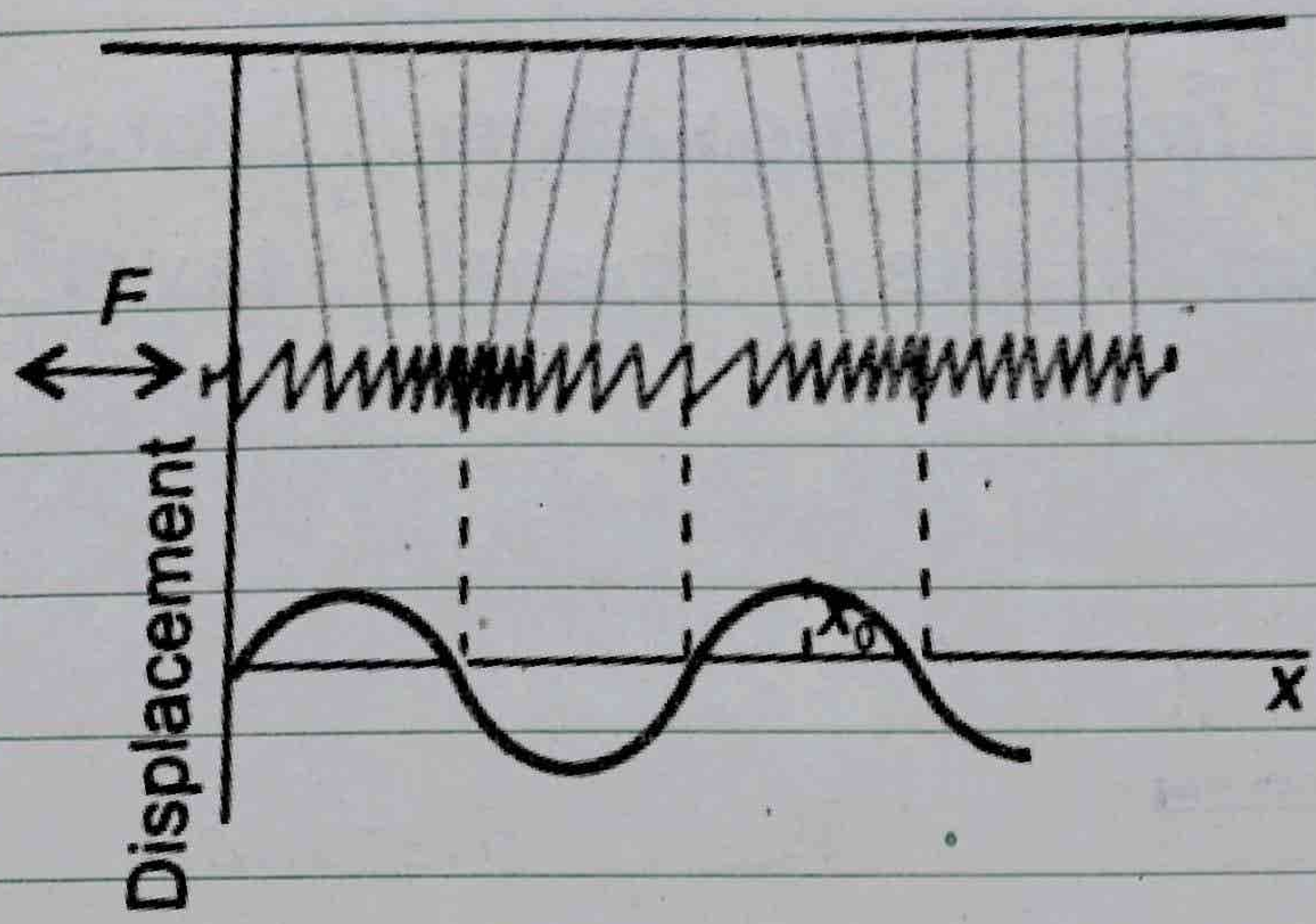


ii) Longitudinal periodic waves

"Such periodic waves in which medium particles vibrate in a direction parallel to the direction of propagation of the waves are called longitudinal periodic waves".

Explanation

Consider a coil spring is suspended horizontally by a number of vertical threads. When an oscillating force "F" is applied to its end at left as shown in fig, a series of waves is produced along the spring. Compressions and rarefactions are produced in the spring. A wave move towards right. Each particle of the spring vibrate parallel to the wave motion.



Crest

"Portion of the wave above the mean level is called crest".

Trough

"Portion of the wave below the level is called trough".

Compression

"In longitudinal waves, the portion of the medium at which density of particles and the pressure is maximum is called compression".

Rarefaction

"In longitudinal waves, the portion of the medium at which density of particles and the pressure is minimum ".

Amplitude

"The maximum value of displacement from the mean level is called amplitude", "X_0".

Wave length λ

"The distance between any two consective crests or any two consecutive troughs is called one wave-length".

Time period

"Time taken by the wave to cover a distance equal to one wavelength is called time period", "T".

Frequency

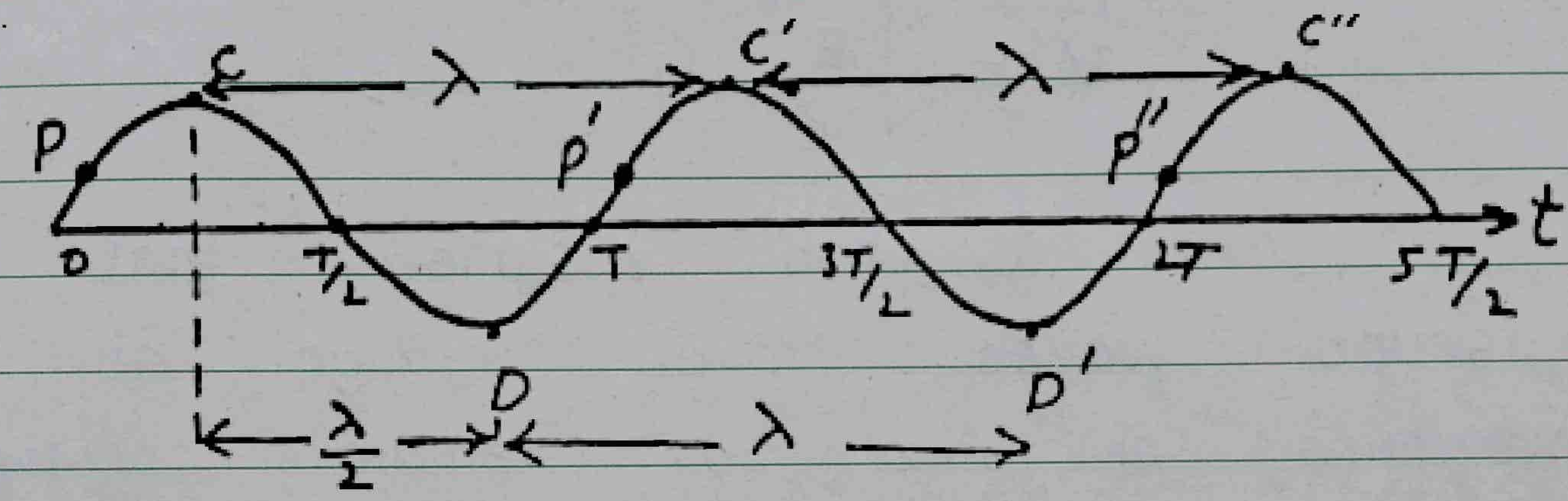
"The number of wave pulses passing through a certain point in one second is called frequency". Its unit is Hertz, "Hz".

Frequency is one Hz if one wave pulse pass through a point in one second. The relation between speed "V", frequency f and wavelength λ.

$$V = f\lambda$$

Sine wave or sinusoidal wave :

The periodic waves produced by a source performimg S.H.M is represented by a sine wave or sinusoidal wave. It is shown in fig.



In phase

" The points which are in the same state of vibration are called in phase". They have same displacements and same velocity.

Out of phase

" The points which are exactly out of step are called out of phase".

8.3 Speed of sound in air

Newton's formula for the Speed of sound in air.

Speed of sound in air depends upon two factors.

* : Modulus of elasticity, "E".
* : Density of the medium air, "ρ".

$$V = \sqrt{\frac{E}{\rho}} \longrightarrow (1)$$

Newton assumed that when sound waves travel through air, the temperature of the air remains constant during compressions and rarefactions. It is an isothermal process.

It a compression the pressure of air changes, from

P to $P + \Delta P$

and volume changes from

V to $V - \Delta V$

Using Boyles law, Temperature = constant

$$PV = \text{constant}$$

$$P_1 V_1 = P_2 V_2$$

$$PV = P + \Delta P \cdot V - \Delta V$$

$$PV = PV - P\Delta V + \Delta P V - \Delta P \Delta V$$

As ΔP, ΔV are very small $\Delta P \Delta V$ is neglect

$$\cancel{PV} = \cancel{PV} - P\Delta V + \Delta P V$$

$$P\Delta V = \Delta P V$$

$$P\frac{\Delta V}{V} = \Delta P \qquad \because P = 1.013\times10^{5}\,Nm^{-2}$$

$$\rho = 1.293\,Kgm^{-3}$$

$$P = \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{\text{stress}}{\text{strain}} = E$$

So $P = E$ Put in eq (1)

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.013\times10^{5}}{1.293}}$$

$$\boxed{V = 280\ ms^{-1}}$$

This value $280\ ms^{-1}$ is 16% less then the actual value $332\ ms^{-1}$ of speed of sound.

Laplace Correction

"According to Laplace temperature does not remain constant when sound waves travel in a medium."

i) : The process is not Isothermal.

ii) : The process is Adiabatic.

Boyle's law is not applicable.

1) : At compression, temperature increases.

2) : At rarefaction, temperature decreases.

As, air is a bad conductor of heat so, at compression air cannot lose heat, at rarefaction air cannot gain heat so. heat remains constant.

For adiabatic process

$$PV^{\gamma} = \text{constant}$$

$$\gamma = \frac{C_p}{C_v} = \frac{\text{Molar specific heat at constant pressure.}}{\text{Molar specific heat at constant volume.}}$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$PV^{\gamma} = (P+\Delta P)\,(V-\Delta V)^{\gamma}$$

$$PV^{\gamma} = (P+\Delta P)\left[\left(V - V\frac{\Delta V}{V}\right)\right]^{\gamma}$$

$$P\cancel{V^{\gamma}} = (P+\Delta P)\,\cancel{V^{\gamma}}\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

And Binomial theorem neglecting highr terms.

$$\because \left(1 - \frac{\Delta V}{V}\right)^{\gamma} = 1 - \gamma\frac{\Delta V}{V}$$

$$P = (P+\Delta P)\left(1 - \gamma\frac{\Delta V}{V}\right)$$

$$P = P - \gamma P\frac{\Delta V}{V} + \Delta P - \gamma\frac{\Delta P\Delta V}{V}$$

neglect small term $\gamma\frac{\Delta P\Delta V}{V}$, arranged,

$$\Delta P = \cancel{P} - \cancel{P} - \gamma P\left(\frac{\Delta V}{V}\right)$$

$$\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = \gamma P = \frac{\text{stress}}{\text{strain}} = E$$

$$\gamma P = E \longrightarrow (2)$$

put eq (2) in eq (1). $\because \gamma = 1.4$

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma} \cdot \sqrt{\frac{P}{\rho}} = \sqrt{1.4 \times 280\, ms^{-1}}$$

$$\boxed{V = 333\ ms^{-1}}$$

This value is also to experimental value $332\ ms^{-1}$

Effect of variation of Pressure, Density and temperature on the speed of sound in a gas

1) Effect of pressure

As $V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$

The density of a gas is directly proportional to pressure. The ratio $\frac{P}{\rho}$ remains constant. "So variation of pressure has no effect on the speed of sound".

2) Effect of dansity

At the same temperature and pressure for the gases having same $V \propto \frac{1}{\sqrt{\rho}}$ value of γ speed of sound is inversly proportional to the squre root of density.

"Speed of sound in oxygen is 4-times less then that in Hydrogen."

$$\boxed{V_{O_2} < V_{H_2}}$$

$\because S_{O_2} : S_{H_2}$ $V_{H_2} : V_{O_2}$

32 : 2   4 : 1

16 : 1

3) Effect of temperature

Prove that $V_t = V_o + 0.61t$

Prove

When gas is heated at constant pressure, its volume increases and hence its density decreases.

As

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{or} \quad V \propto \frac{1}{\sqrt{\rho}}$$

So speed of sound increases with rise in temperature.

Since,

V_o = Speed of sound at $0^\circ C$.

ρ_o = Density of gas at $0^\circ C$.

V_t = Speed of sound at $t^\circ C$.

ρ_t = Density of gas at $t^\circ C$.

$$V_o = \sqrt{\frac{\gamma P}{\rho_o}} \quad \text{and} \quad \sqrt{\frac{\gamma P}{\rho_t}} = V_t$$

$$\frac{V_t}{V_o} = \frac{\sqrt{\frac{\gamma P}{\rho_t}}}{\sqrt{\frac{\gamma P}{\rho_o}}} = \sqrt{\frac{\gamma P}{\rho_t}} \cdot \sqrt{\frac{\rho_o}{\gamma P}}$$

$$\frac{V_t}{V_o} = \sqrt{\frac{\rho_o}{\rho_t}} \longrightarrow (3)$$

Let, V_o = volume of gas at $0°c$.

V_t = volume of gas at $t°c$.

For volume expansion of a gas.

$$V_t = V_o(1 + \beta t) \qquad \because \beta = \text{coefficient volume expansion}$$

$$V_t = V_o\left(1 + \frac{t}{273}\right)$$

As density = $\frac{\text{mass}}{\text{volume}}$

$$\because V_t = \frac{m}{\rho_t} \qquad V_o = \frac{m}{\rho_o}$$

$$\frac{\cancel{m}}{\rho_t} = \frac{\cancel{m}}{\rho_o}\left(1 + \frac{t}{273}\right)$$

$$\frac{\rho_o}{\rho_t} = \left(1 + \frac{t}{273}\right)$$

$$\sqrt{\frac{\rho_o}{\rho_t}} = \sqrt{1 + \frac{t}{273}}$$

Put these value in eq (3)

$$\frac{V_t}{V_o} = \sqrt{1 + \frac{t}{273}} = \sqrt{\frac{273 + t}{273}} \longrightarrow (4)$$

$\because 273 + t = T$

$273 = T_o$

$$\frac{V_t}{V_o} = \sqrt{\frac{T}{T_o}}$$

$$v \propto \sqrt{T}$$

Hence ; T and T_o are absolute temperatures corresponding to $t°C$ and $0°C$ respectively.

Result

Speed of sound is directly proportional to the squre root of absolute temperature of the gas.

From equation (4).

$$\frac{V_t}{V_o} = \left(1 + \frac{t}{273}\right)^{1/2}$$

$$\frac{V_t}{V_o} = 1 + \frac{1}{2}\left(\frac{t}{273}\right)$$

$$V_t = V_o\left(1 + \frac{1}{546}t\right)$$

$$V_t = V_o + \frac{V_o}{546}t \qquad \because V_o = 332\,ms^{-1}$$

$$V_t = V_o + \frac{332}{546}t$$

$$\boxed{V_t = V_o + 0.61\,t}$$

Result

It means that "$1°C$ rise in temperature inceases speed of sound by $0.61\ ms^{-1}$". or $61\,cms^{-1}$ or 2 foot / sec.

8.4 Priciple of superposition

Statement

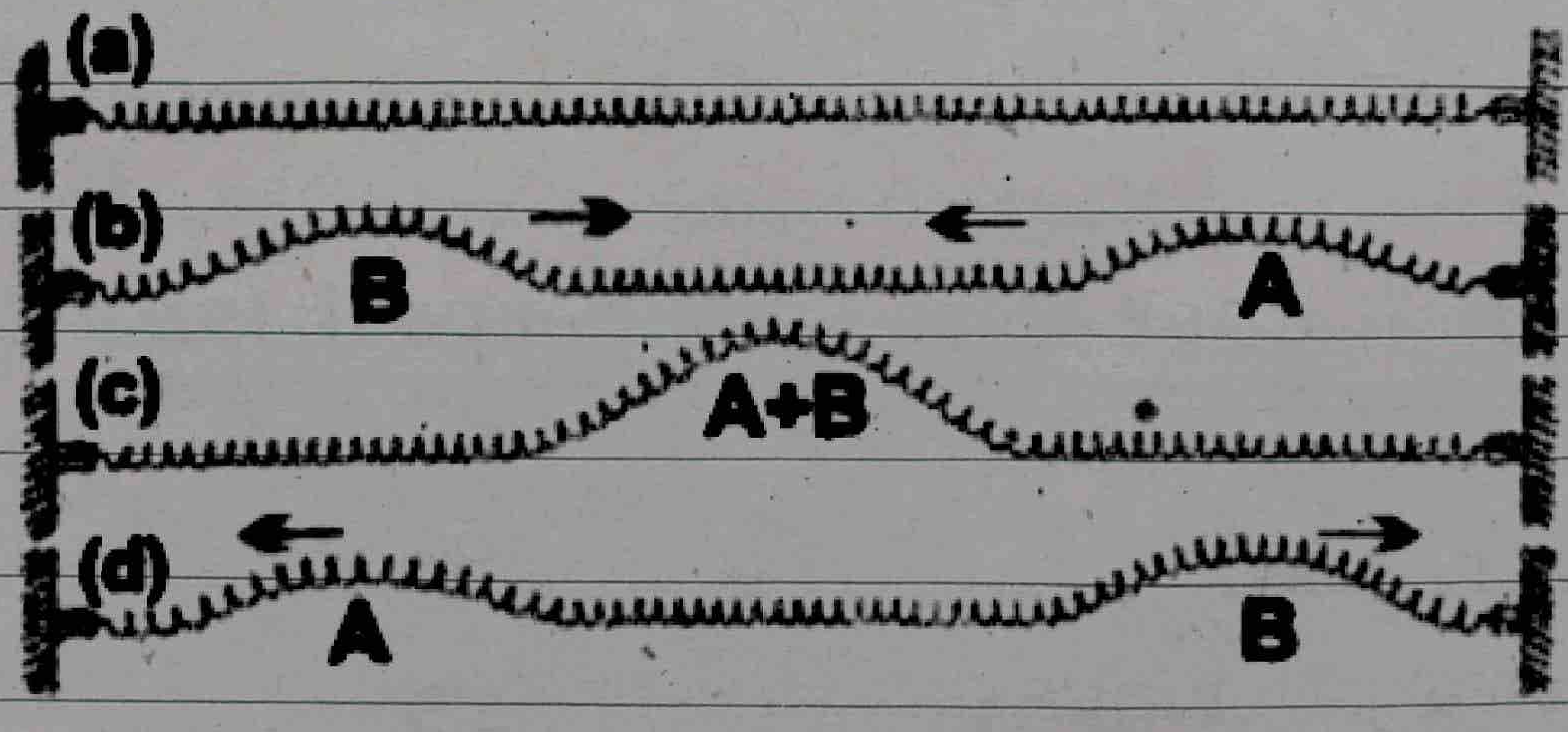
"When a particle of a medium is simultaneously actd upon by two or more waves, the resultant displacement of the particle is the vector sum of their individual displacements."

Or

"The combined effect of two or more waves travelling in the same medium is called principle of superposition."

Explanation

Consider two waves having displacements A and B approach each other on a coil spring as shown in fig In opposite direction. When they overlap their displacemets are added and the net displacemet is A + B. Both waves

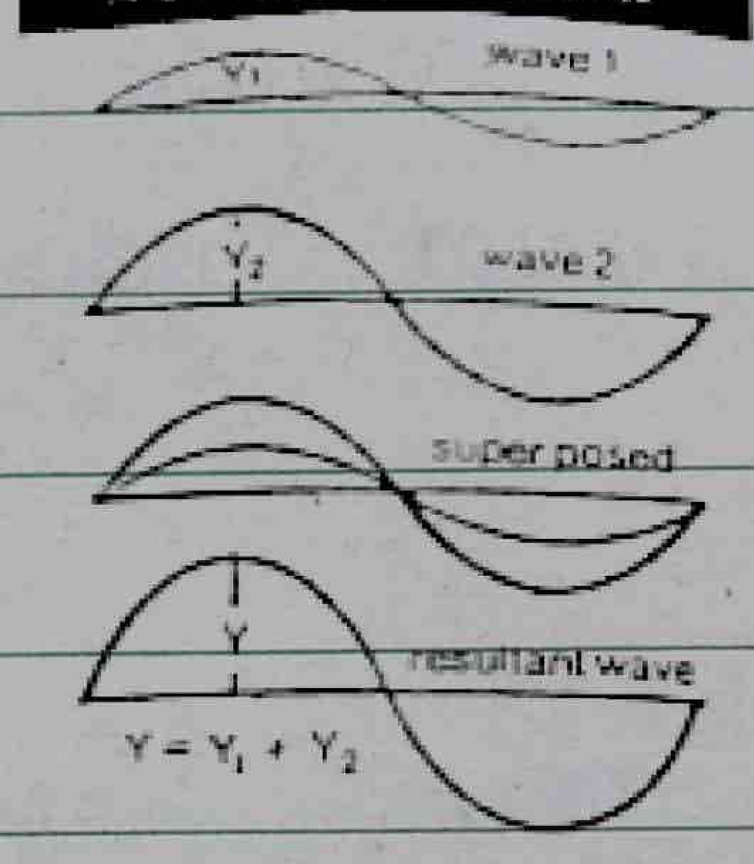


pass through each other without any change as show in fig.

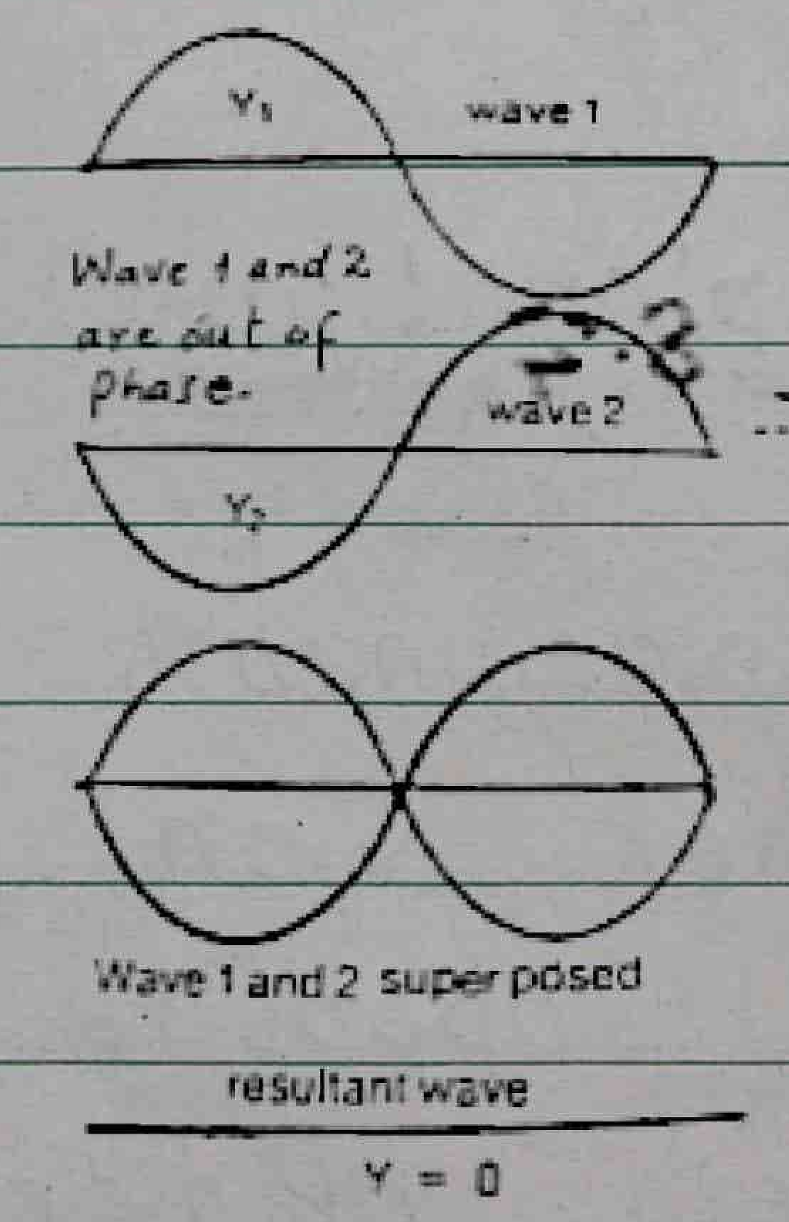
* ; When two waves having displacements y_1 and y_2 superpose, resultant effect at time of overlaping is

$$Y = y_1 + y_2$$

For Your Information



Superposition of two waves of the same frequency which are exactly in phase.



Superposition of two waves of the same frequency which are exactly out of phase

* ; For n wave having displacements.

$$y_1 , y_2 , y_3 \ldots\ldots y_n$$

The resultant effect at the time of overlaping is

$$Y = y_1 + y_2 + y_3 \ldots y_n$$

* ; When two waves which are out of phase cross eachother resultant displacement is

$$Y = y_1 - y_2 \qquad \text{if} \quad y_1 = y_2 \quad Y = 0$$

Important case of superposition.

* ; Interference. * ; Beats.
* ; Stationary waves.

8-5 Interference

Statement

"Superposition of two waves having the same frequency and travelling in the same direction is called interference."

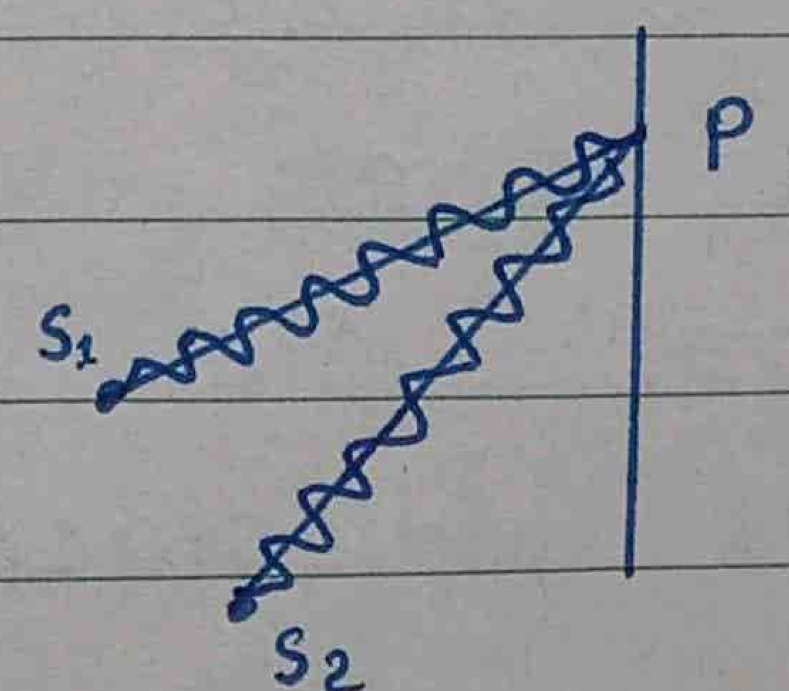
Types of interference

Their are two types.

*; Constrctive interference.
*; Destructive interference.

Constructive interference

When two sound waves of same frequency from two sources "S_1" and "S_2" reach at a point "P" in phase i.e. compression of one falls on the rarefaction of the other and the rarefaction of on falls on the rarefation of the other, then their displacements are added up and a large resultant displacement is produced. so a loud sound is heard at "P".



Condition for constructive interference

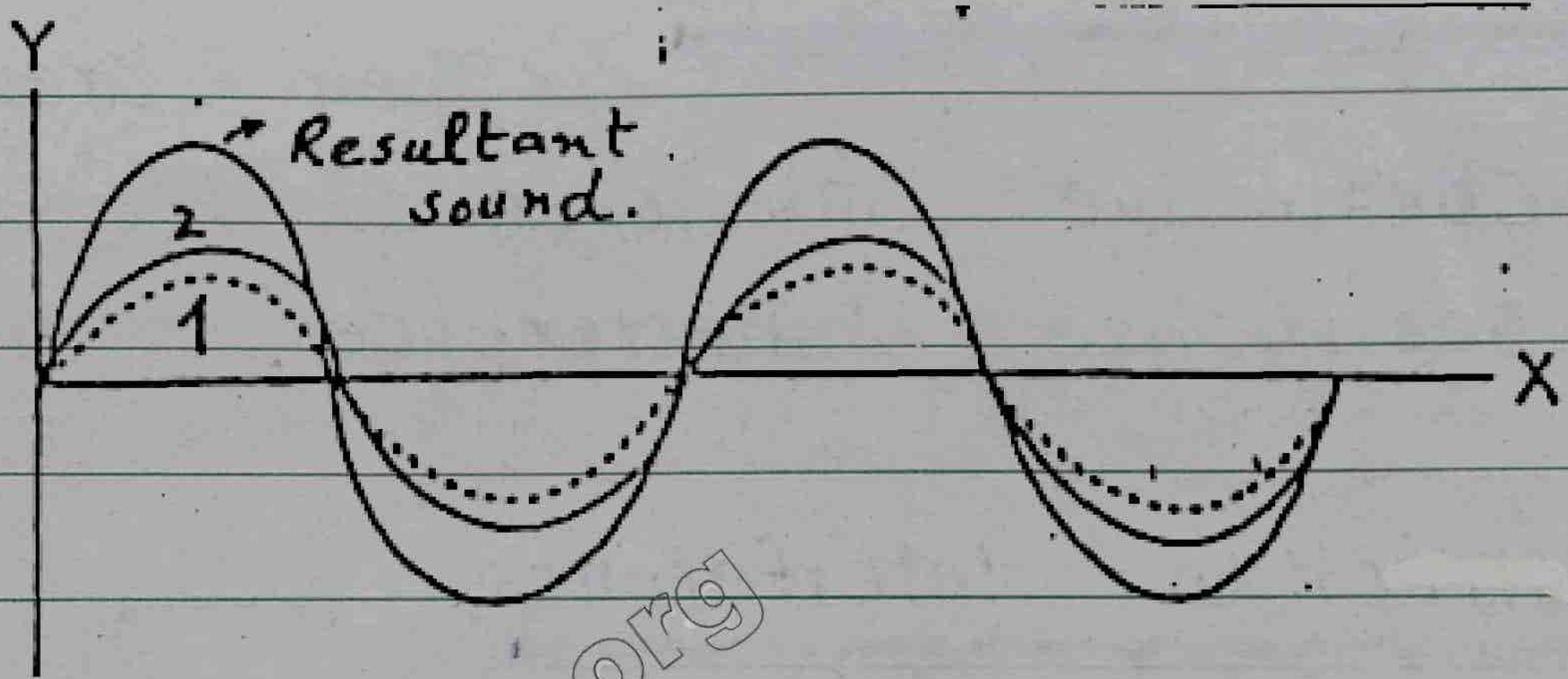
Path difference between two waves is given by.

$$\text{Path difference} = 0, 1\lambda, 2\lambda \dots m\lambda$$

$$\text{Path difference} = m\lambda$$

where $m = 0, 1, 2, 3 \dots$

"Path difference between two waves is an integral multiple of wavelength λ."



2) Destructive interference

When two waves reach at the point "P" from sources "S_1" and "S_2" out of phase i.e. compression of one falls on the rarefaction of the other and the rarefaction of one falls on the compression of the other wave. The two displacements cancel the effect of each other. Resultant displacement is decreased. So a low sound is heard.

Condition for destructive interference

sensitive detector of sound waves. To observe interference microphone is placed at different points.

*; At points P_1, P_3, P_5 CRO gives a large signal a waveform is seen as shown in fig. Compression of one wave meets with compression of the other and rarefaction of one meets with the rarefaction of the other wave. Resultant sound is louder.

The path difference ΔS between the waves at is

$$\Delta S = S_2P_1 - S_1P_1$$

$$\Delta S = 4\frac{1}{2}\lambda - 3\frac{1}{2}\lambda$$

$$\boxed{\Delta S = \lambda}$$

Similarly at points P_3 and P_5 path difference is zero and $-\lambda$ respectively. So condition, condition for path difference.

$$\Delta S = n\lambda$$

where $n = 0, \pm 1, \pm 2 \ldots$

"Path difference = Integral multiple of λ"

*; At points P_2 and P_4 CRO gives

Path difference $= 1\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots\ldots 2m+1\frac{\lambda}{2}$

Path difference $= 2m+1\frac{\lambda}{2}$

where $m = 0, 1, 2 \ldots$

"Path difference betwee two waves must be an odd integral multiple of half the wavelength $\frac{\lambda}{2}$".

Experimental set up to explain interferenc

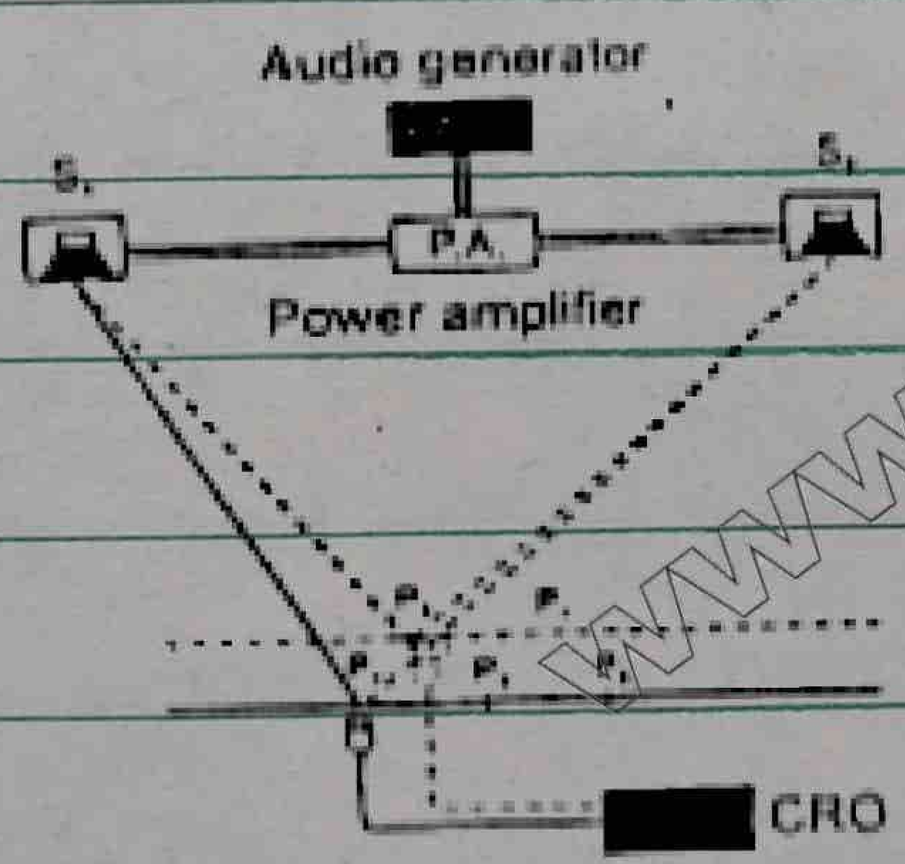


Fig (a)

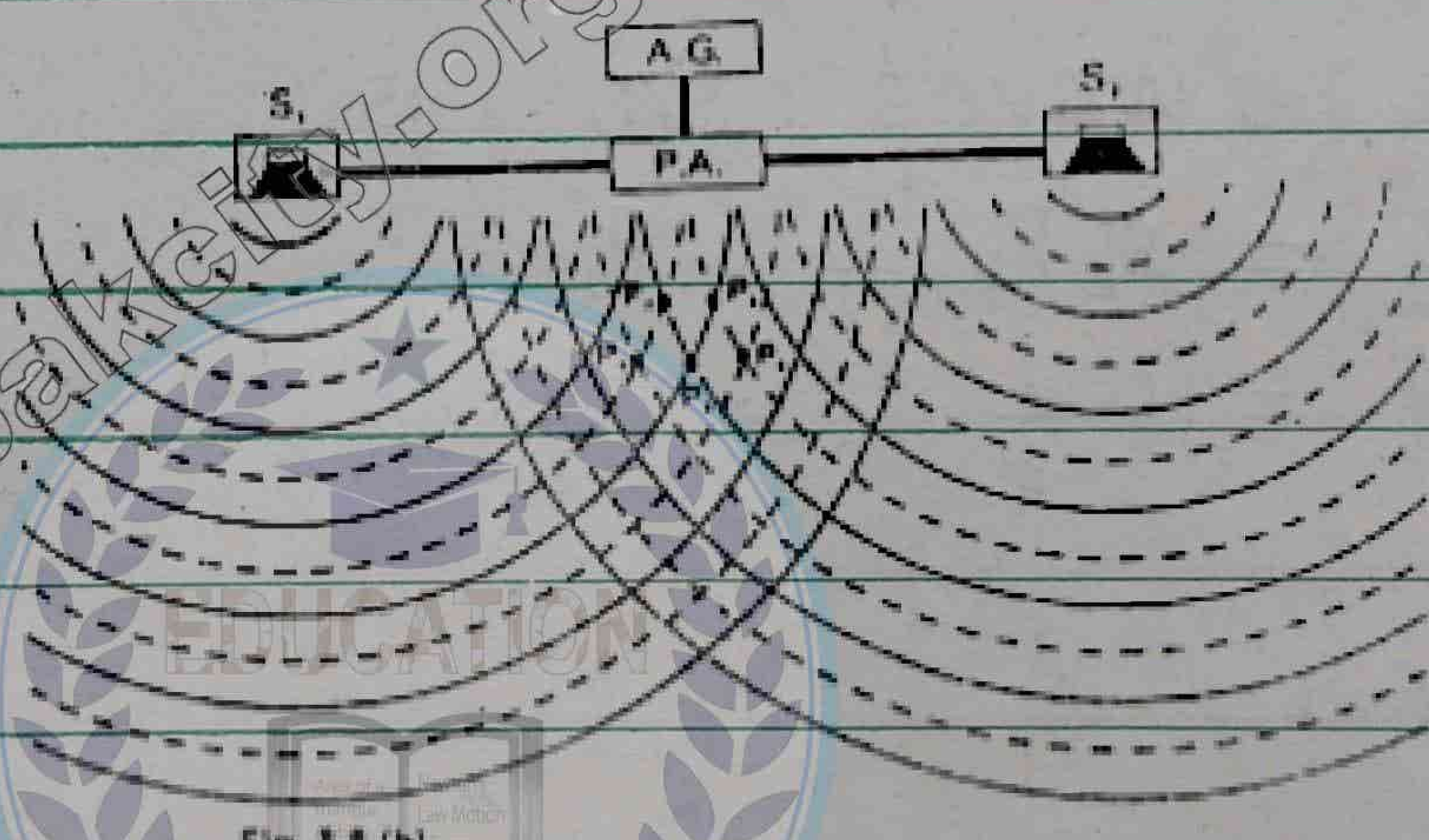


Fig. 8.8 (b)
Interference of sound waves.
Points P_1, P_2, P_3 are points of constructive interference.
Points P_4 and P_5 are points of destructive interference.

Fig: b)

In fig two loud speakers S_1 and S_2 act as two sources of sound waves of same frequency produced by an Audio Generator (A.G).

A microphone atted to a cathode ray oscilloscope (CRO) acts as a no signal. It is shown in fig. At these points compression of one wave meets the rarefaction of the other. Resultant sound is decreased.

The path difference ΔS between the waves at points P_2 and P_4 is.

i) For point P_2

$$\Delta S = S_2 P_2 - S_1 P_2$$

$$\Delta S = 4\lambda - 3\frac{\lambda}{2}$$

$$\Delta S = \frac{1}{2}\lambda$$

ii) For point P_4

$$\Delta S = -\frac{1}{2}\lambda$$

In general condition for destructive interference.

$$\Delta S = (2n+1)\frac{\lambda}{2}$$

where $n = 0, \pm 1, \pm 2 \ldots$

"Path difference is equal to an odd integral multiple of $\frac{\lambda}{2}$ half welenth".

8.6 Beats

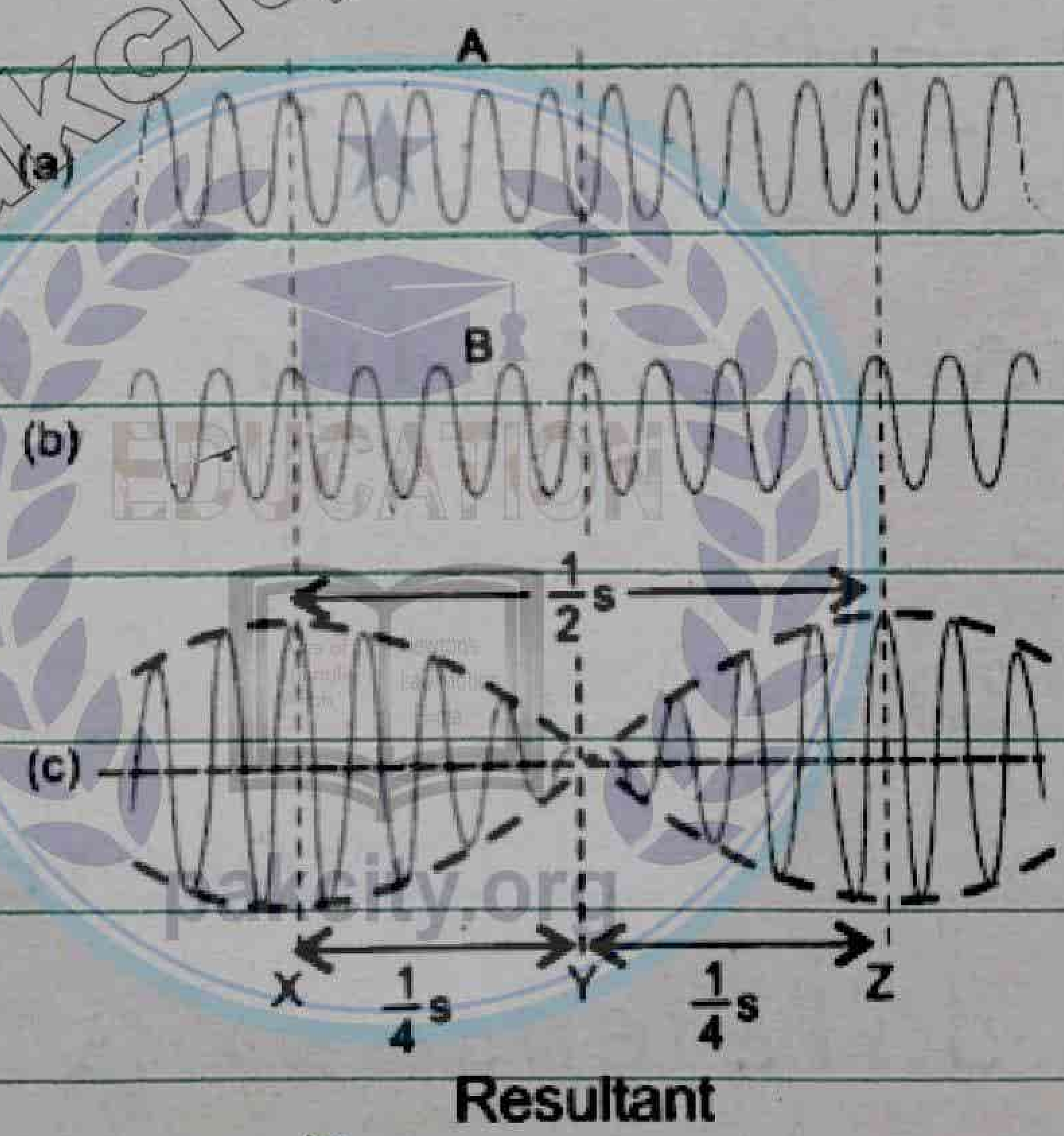
Defination

"Periodic variations of sound between maximum and minimum loudness is called beats".

The production of beats is a special type of interference. "It is the interference between two sound waves of slightly different frequencies".

Explanation

When two tuning forks "A" an "B" having the same frequency are sounded together. The two sounds cannot be recognized seperately. It means that note of "B".



When two tuning forks "A" and B having frequencies 256 Hz and 252 Hz are sounded together notes of sound beats are heard clearly.

Fig and show wavform emitted by the two tuning forks "256 Hz" and "252 Hz" their resultant waveform is shown in Fig.

★ ; At some instant "X" the two waves meet in phase. The resultant wave is large. A large sound is heard.

★ ; After $\frac{1}{8}$ sec. at point "Y" the two waves meet out of phase. Resultant wave is decreased. A faint sound is heard.

★ ; After another $\frac{1}{8}$ sec., The two waves meet in phase at Z. The resultant wave is increased, so loud sound is again heard. An so on. "four rises and four falls in loudness are heard in one second".

The difference in frequencies is.

$$f_A - f_B = 256\,Hz - 252\,Hz$$

$$f_A - f_B = 4 \text{ beats per second}$$

$$f_A - f_B = \frac{\text{No. of beats}}{\text{second}}$$

i, : One Beat ; "one rise and one fall in loudness is called one beat".

ii, : When the difference between the frequencies of the two sounds is more then 10 Hz, the it difficult to recognize beats.

Uses

* ; To tune a string musical instrument such as piano or violin.
* ; To find the unknown frequency of a vibrating body.
* ; To produce variety in musical sound.

8.7 Reflection of Waves

Definition

"When a wave falls on the boundory of two media a part of it is reflected back. This bouncing back of the wave is called reflection of waves."

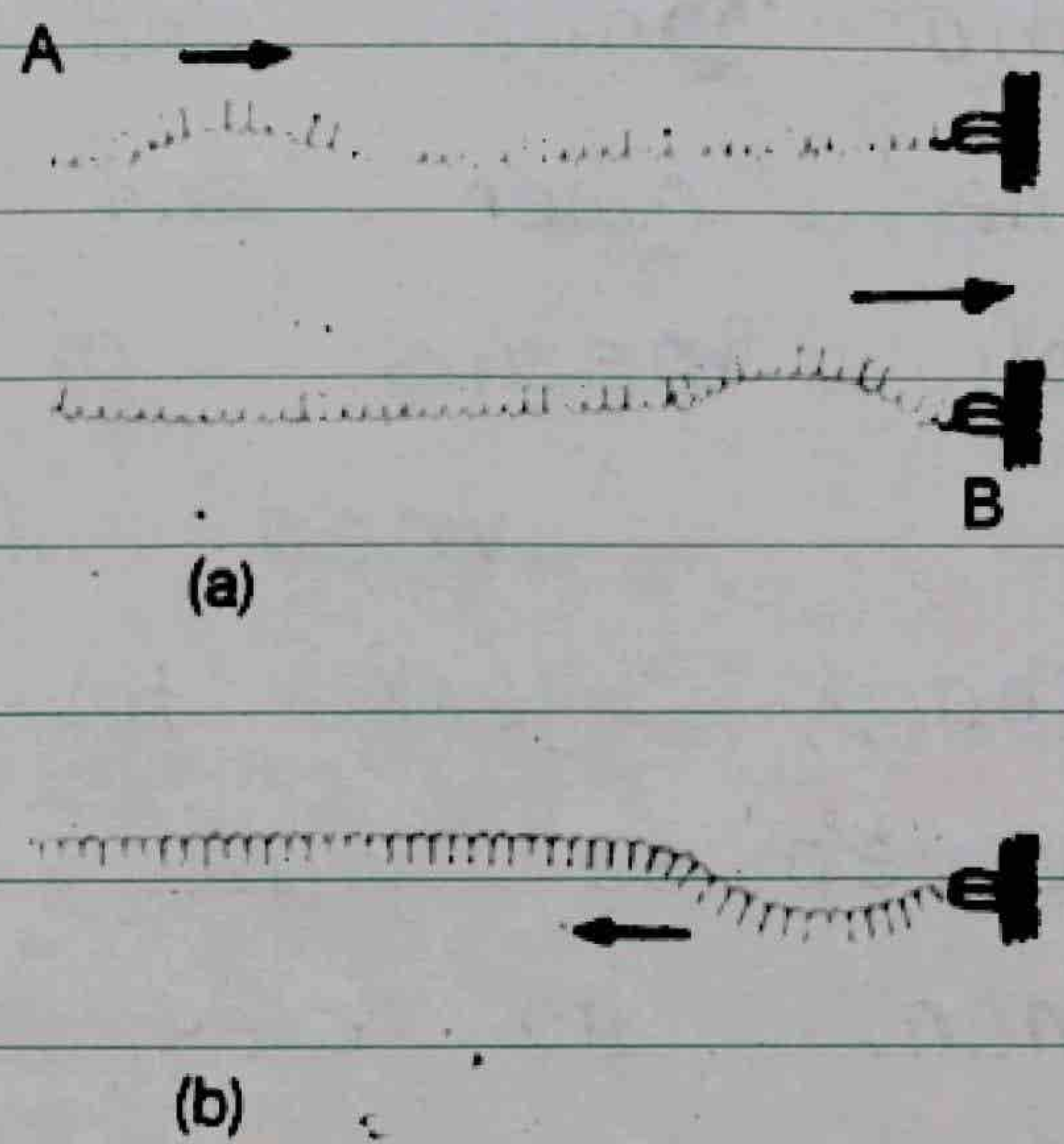
i) ; The reflected wave has the same frequency and same wavelenth.

ii) ; The phase of reflected wave depends on the nature of the boundary.

Two cases of reflection.

1) Reflection from Denser medium

Consider slinky spring (a long and lose coil spring) lying on a smooth table surface. Its one end is fixed at 'B' and the other end "A" is free.



When the free end A is given a sharp jerk, a wavepulse with its displacement up (crest) is produced. It travels towards right along the spring. When it reaches near hook is rigid and fixed it acts as a denser medium.

It exerts a reaction force in opposite direction i.e. towards down travels back in the same medium.

So, the crest is reflected as trough.

Result

"When a transverse wave travelling in a raver medium is incident on a denser medium it is reflected with a phase change of $180^\circ (\pi)$".

"Crest reflected as Trough and vice verse".

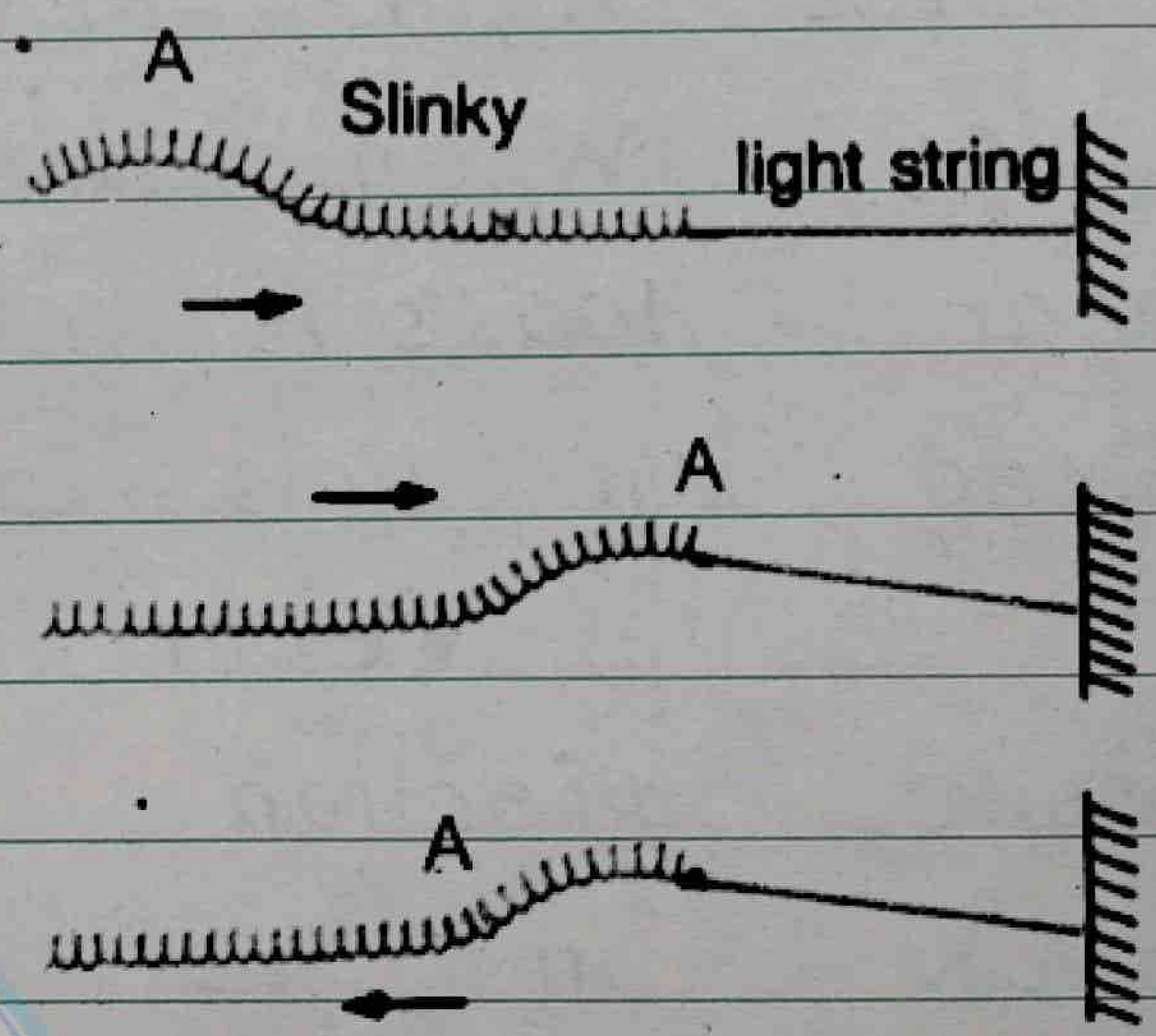
phase change produced $= 180^\circ \pi$

Path change produced $= \frac{\lambda}{2}$.

2) Reflection from a Rare medium

Consider that one end of slinky is free while the other end is attaced to the hook by means of a "Light thread".

When its free end is given a sharp jerk towards 'A' a wavepulse with its displacement up (crest) travels along the spring. It moves to the right.



'In this case spring behaves as denser medium while the thread is a rare medium.'

When the wave reaches "B" (rare boundary), it will pull the thread up. The thread does not oppose the motion of the spring, the spring behaves as if it has been plucked up.

The reflected wave travels back in the spring with its displacement up i.e. crest as crest, as shown in Fig.

Phase difference produced = zero.

Conclusions

★ ; If a transverse wave travelling in a rarer medium is incident. It is reflected such that it undergoes a phase change of 180°.

★ ; If a transverse wave travelling in a denser medium is incident on a rarer medium, it is reflected without any change in phase.

8.8 Stationary Waves

Definition

"When two identical waves having the same frequency, time period wavelength and amplitude travelling in opposite direction superpose each other as a result stationary waves are produced."

These are called stationary waves as there is no flow of energy along the waves. The medium vibrate as a whole in opposite directions. These are also called stationary waves because some of the particles of the medium remain permanantly standing or stationary. These points are called nodes.

or

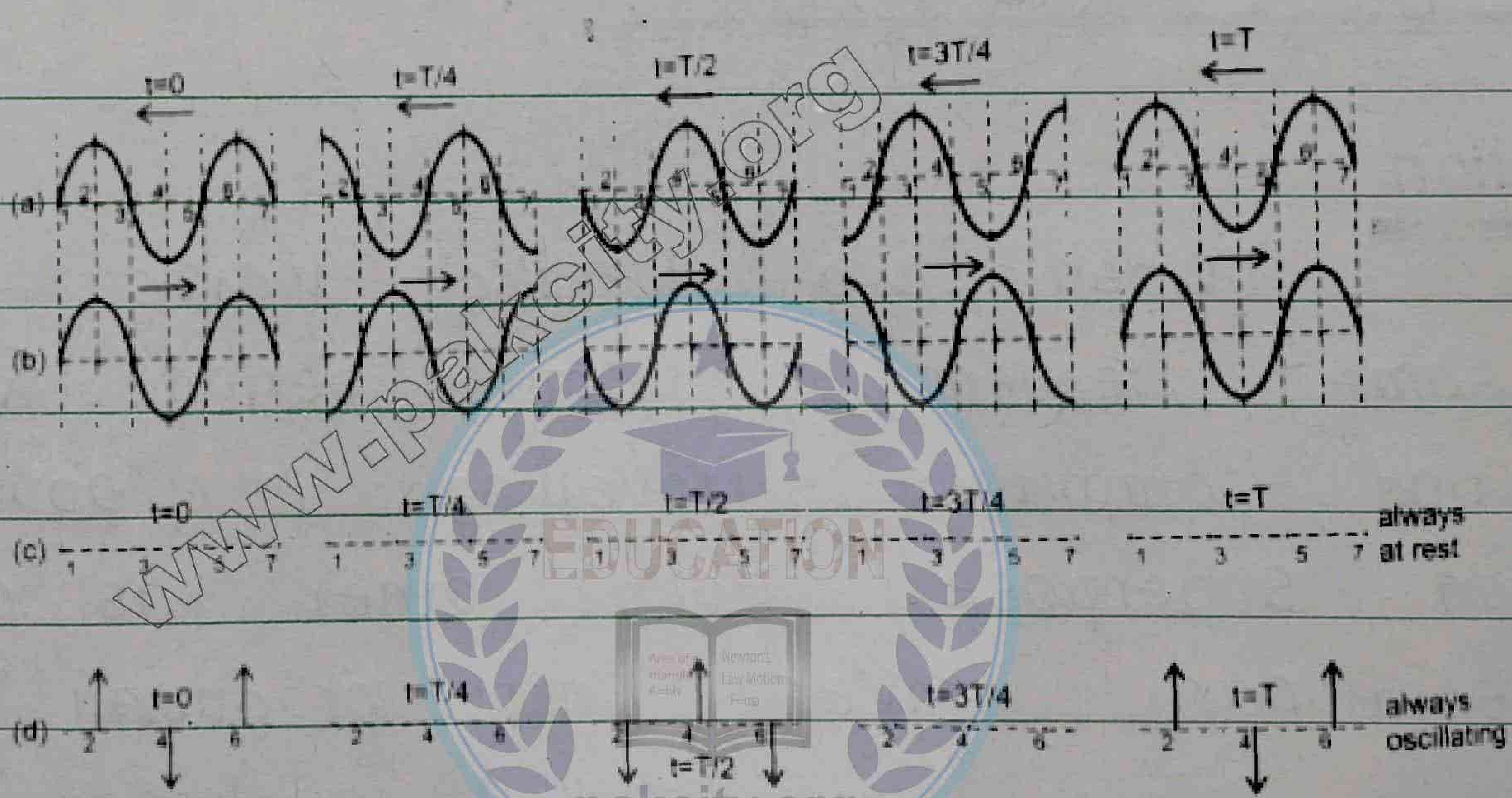
Nodes

"These points at which the amplitude of the zero are called Nodes".

Antinodes

"Those points at which the amplitude of the wave is maximum are called Antinodes".

Explanation



Consider the superposition of two waves moving in opposite directions along a string. Their displacements are show in fig, at points 1, 2, 3, 4, 5, 6, and 7. Distance between each point and its next point is $\frac{\lambda}{4}$. Where λ is the wavelength

Fig shows the profile of the two waves at instants $t = 0, \frac{T}{4}, \frac{T}{2}, 3\frac{T}{4}$ and T. Hence T is the time period of the wave. Fig shows resultant displacements of the points, 1, 3, 5 and 7 a the instants $t = 0, \frac{T}{4}, \frac{T}{2}, 3\frac{T}{4}$ and T. It is clear from the fig (c) that the resultant displacement of these points is always zero.

The distance between two consecutive nodes is $= \frac{\lambda}{2}$

Fig (d) shows the resultant displacement of the points 2, 4 and 6 at the instants $t = 0, \frac{T}{4}, \frac{T}{2}, 3\frac{T}{4}$

The figure shows that these points are moving with an amplitude which is the sum of the amplitudes of the two waves "a" and "b" these points are Know as antinode. At antinodes displacement is maximum. Antinodes are situated midway between the nodes.

The distance betwee two consecutive antinodes. $= \frac{\lambda}{2}$

The distance between a node and next antinode is $\frac{\lambda}{4}$.

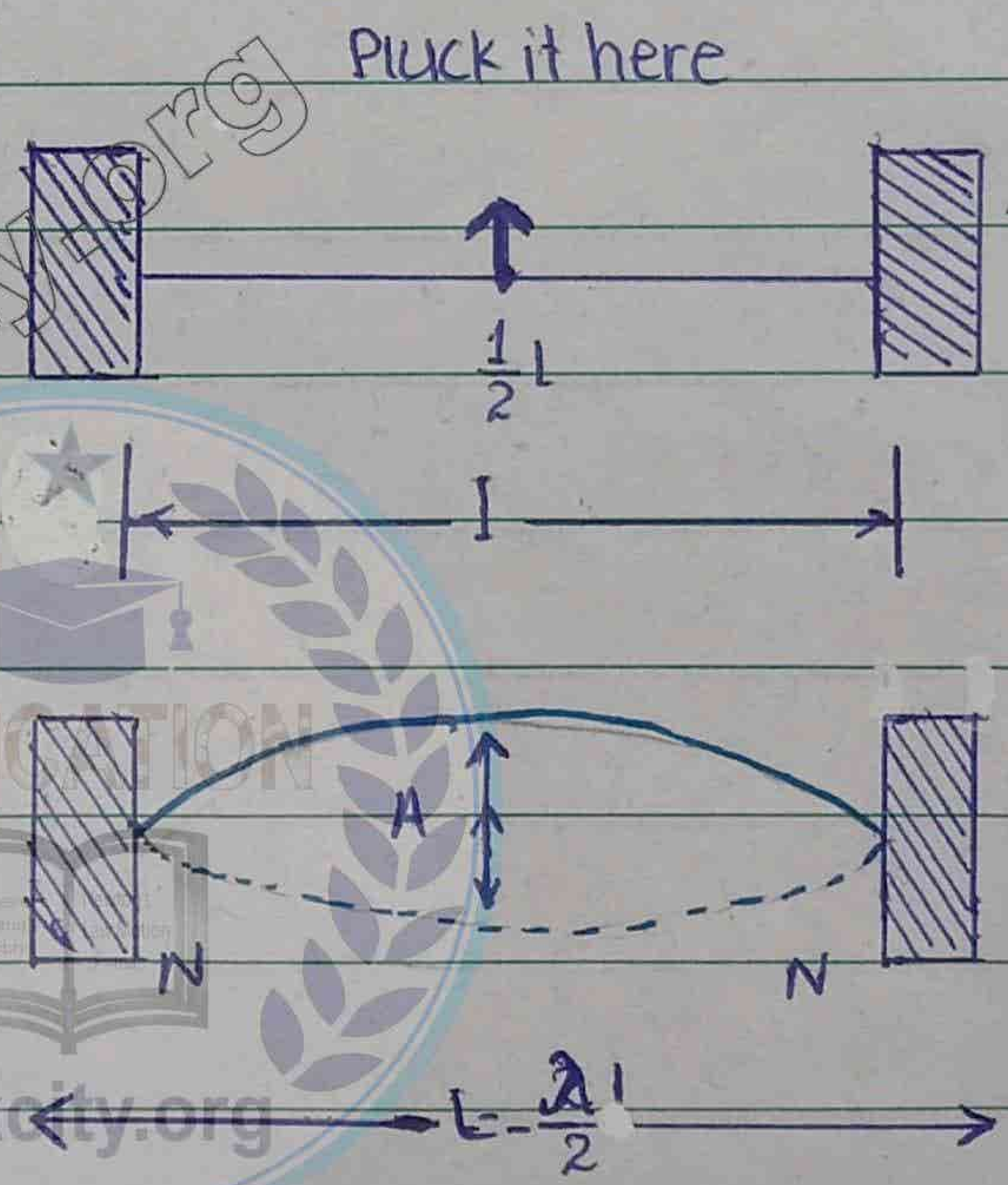
Characteristics of stationary waves

* No energy is transfered from particle to particle in a stationary wave.
* All particles except nodes, perform S.H.M with the same period as the component waves.
* Between two consecutive nodes, these must be an antinode and and between two consecutive antinodes there must be a node.

8.9 Transverse Stationary waves in A Strectched String.

Consider a string of length $= L$
It is stretched between two clamps. $\left[m = \frac{mass}{Length} \right]$
Tension in the string $= F$
Mass per unit length of string $= m$
Speed of waves in the string $= V$

Frequency of 1st - Mode of Vibration :- f_1



We pluck the string at the centre, two transevers waves will move in opposite directions. One towards right, the other towards left. On reaching the clamps they are reflected back. They super impose each other producing stationary or standing waves. The string vibration in one loop. Position of the nodes and antinodes are shown in fig

$V = f\lambda$

$V_1 = f_1 \lambda_1$

$$f_1 = \frac{V}{\lambda_1}$$

$$\because \lambda_1 = \frac{2L}{1}$$

$$\boxed{f_1 = \frac{V}{2L}} \longrightarrow ①$$

Speed of waves is given by

$$\boxed{V = \sqrt{\frac{F}{m}}}$$

$$f_1 = \frac{1}{2L} \times V$$

$$\boxed{f_1 = \frac{1}{2L}\sqrt{\frac{F}{m}}}$$

$V \propto \sqrt{F}$

If Tension 'F' is made 4-times speed of wave increases by 2-time i,e, V is doubled

Frequency of 2nd Mode of Vibration : f_2.

When the string is plucked at one quarter of its length $(\frac{1}{4})L$, it vibration in two loops.

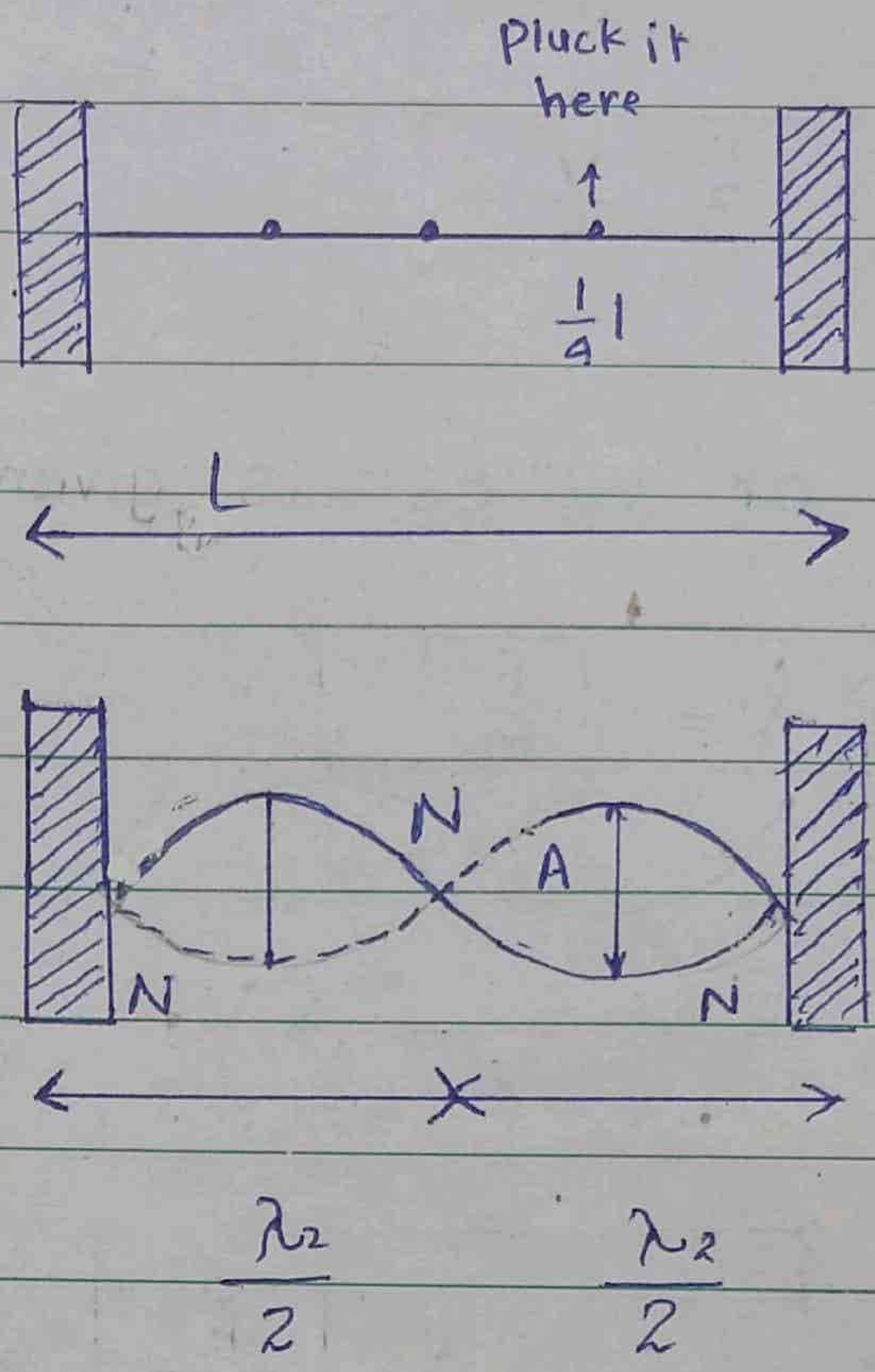
V = Speed of the waves is same, but frequency and wavelength one diffrent.

$$V = f_2 \lambda_2$$

$$f_2 = \frac{V}{\lambda_2}$$

So $f_2 = \frac{V}{L}$

From Fig



$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

$$L = \lambda_2$$

$$\lambda_2 = L$$

or

$$\lambda_2 = \frac{2L}{2}$$

$\times \div$ by 2

$$f_2 = \frac{2V}{2L}$$

$$= 2\left(\frac{V}{2L}\right) \text{ by eq } ①$$

$$\boxed{f_2 = 2f_1}$$

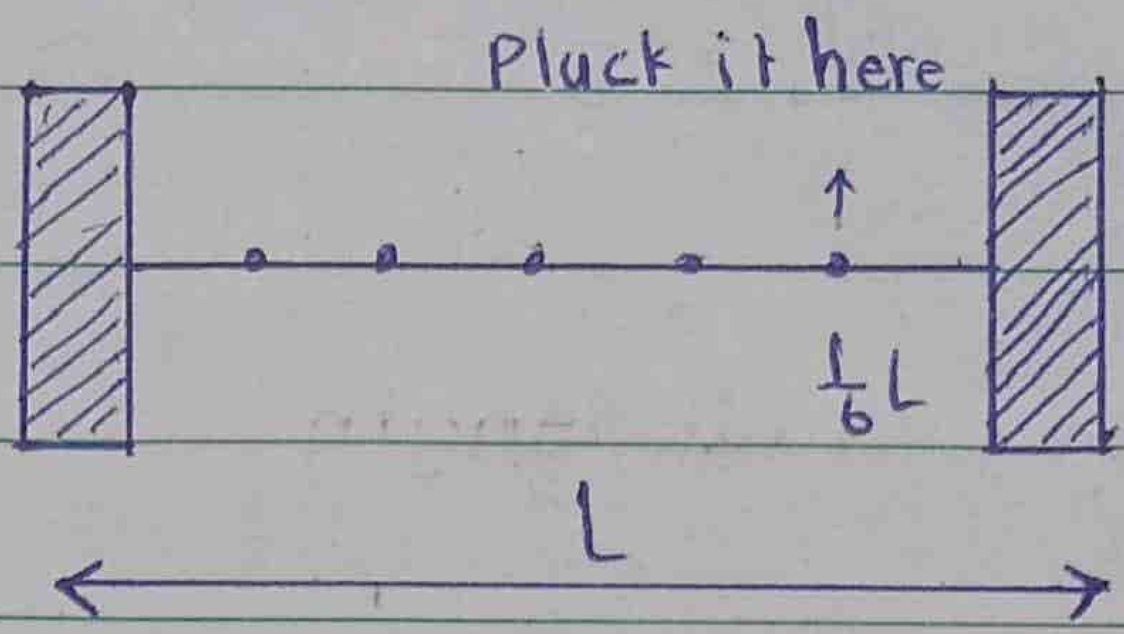
Frequency of 3RD Mode of Vibration :- f_3

When the string is plucked at $\frac{1}{6}$th its length, it vibrates in three loops

$$V = f_3 \lambda_3$$

$$F_3 = \frac{V}{\lambda_3}$$

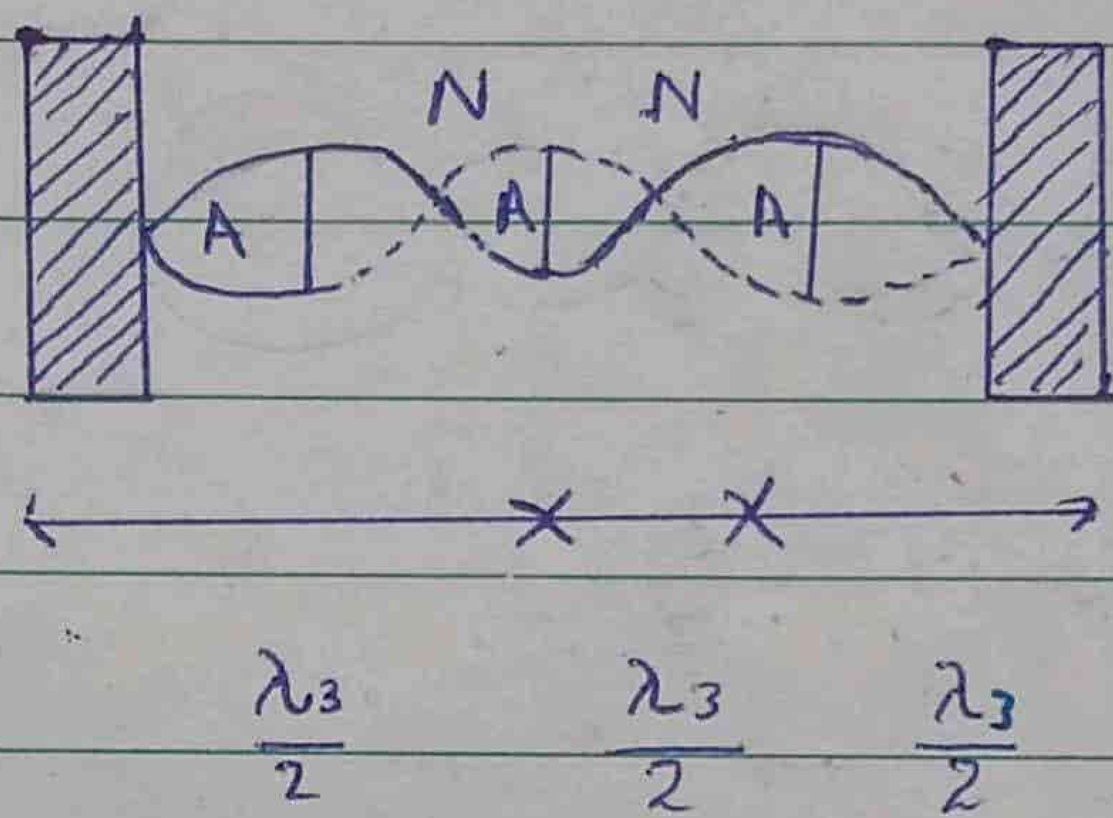
From fig



$$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$L = \frac{3\lambda_3}{2}$$

$$L = 3\frac{\lambda_3}{2}$$



$$\boxed{\lambda_3 = \frac{2L}{3}}$$

$$F_3 = \frac{V}{\frac{2l}{3}}$$

$$F_3 = 3\left(\frac{V}{2L}\right)$$

$$F_3 = 3F_1$$

by eq. 1

Similarly

$$F_4 = 4F_1$$

$$\boxed{F_n = nF_1}$$

$n = 1, 2, 3, \ldots\ldots$

or $f_n = n\left(\frac{v}{2L}\right)$

$f_n = 1, 2, 3, \ldots\ldots$

For wave length

$\lambda_1 = \frac{2L}{1}$

$\lambda_2 = \frac{2L}{2}$

$\lambda_3 = \frac{2L}{3}$

$\vdots$

$\lambda_n = \frac{2L}{n}$ $\quad n = 1, 2, 3, \ldots\ldots$

When the string vibrates in more loops, (n increases) its frequency increases and wavelength decreases.

A discrete set of frequencies $f_1, 2f_1, 3f_1, \ldots\ldots, nf_1$ can be set up. It is known as harmonic series.

f_1 = Fundamental Frequency

f_2 = 1st harmonic

f_3 = 2nd harmonic and so on.

stationary waves whose frequency is not in the above harmonic series cannot be produced in the string.

Sapeed of waves only depend upon

$v = \sqrt{\frac{F}{m}}$

(i) F = Tension in the string.
(ii) m = mass per unit length of string.
Sapeed = V does not depend upon the number of loops.

ORGAN PIPE :

It is long pipe in which air is forced from its one end. The air column in the pipe vibrates and sound is produced. The waves are stationary. These are produced due to resonance.

Types of Organ Pipe

Closed Pipe :-

If one end of the organ pipe is closed, then it is called closed pipe.

Open Pipe :-

If both ends of organ pipe are open then, it is called open pipe.

The closed end always behaves as a 'Node' and open end as 'Antinode

In an open pipe the fundamental frequency is

$\left(\frac{V}{2L}\right)$ and all harmonics are present

$$f_n = n\left(\frac{V}{2L}\right) \qquad n = 1, 2, 3, \ldots$$

In closed pipe, the fundamental frequency is

$\left(\frac{V}{4L}\right)$ and only odd harmonics are present

$$f_n = n\left(\frac{V}{4L}\right) \qquad n = 1, 3, 5, 7, \ldots$$

8.10 Stationary Waves in Air Columns

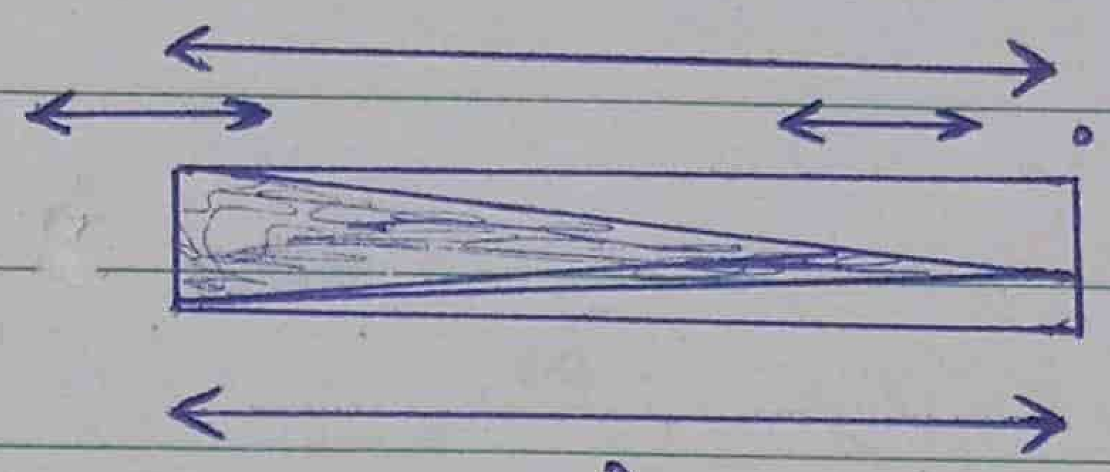
(a) Modes of Vibration in A Closed Pipe:

Consider a vibrating a vibrating tuning fork is held at the open pipe of lenght L. The closed end is Node end the open end is antinode.

(i) First Mode of Vibration:-

Let f_1 is the frequency and λ_1 is the wavelength of longitudinal waves in this mode. This is fundamental mode of vibration.

From fig



$$l = \frac{\lambda_1}{4}$$

$$L = \frac{\lambda_1}{4}$$

OR

$$\lambda_1 = 4L$$

$$V = f_1 \lambda_1$$

OR

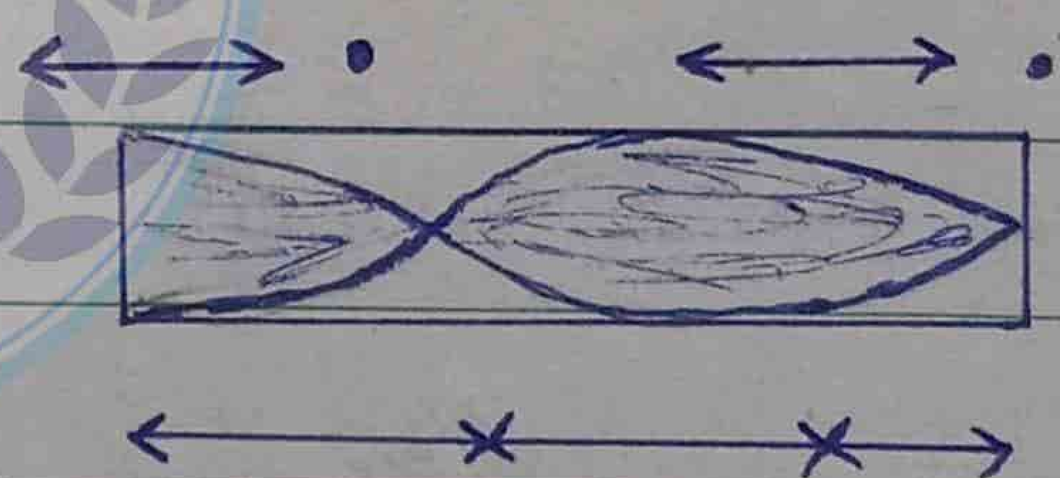
$$\lambda_1 = \frac{4L}{1}$$

$$f_1 = \frac{V}{\lambda_1}$$

$$\boxed{f_1 = \frac{V}{4L}} \longrightarrow (1)$$

(ii) Second Mode of Vibration :-

From fig



$$\frac{\lambda_3}{4} \quad \frac{\lambda_3}{4} \quad \frac{\lambda_3}{4}$$

$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{4} + \frac{\lambda_3}{4}$$

$$L = \frac{3\lambda_3}{4}$$

$$\lambda_3 = \frac{4L}{3}$$

$$\left[\begin{array}{l} \lambda_1 = \frac{4L}{1} \\ \lambda_3 = \frac{4L}{3} \\ \lambda_5 = \frac{4L}{5} \\ \vdots \\ \boxed{\lambda_n = \frac{4L}{n}} \end{array}\right. \quad n = 1, 3, 5 \ldots$$

$$V = F_3 \lambda_3$$

$$F_3 = \frac{V}{\lambda_3}$$

$$F_3 = \frac{V}{\frac{4L}{3}}$$

$$F_3 = 3\left(\frac{V}{4L}\right)$$

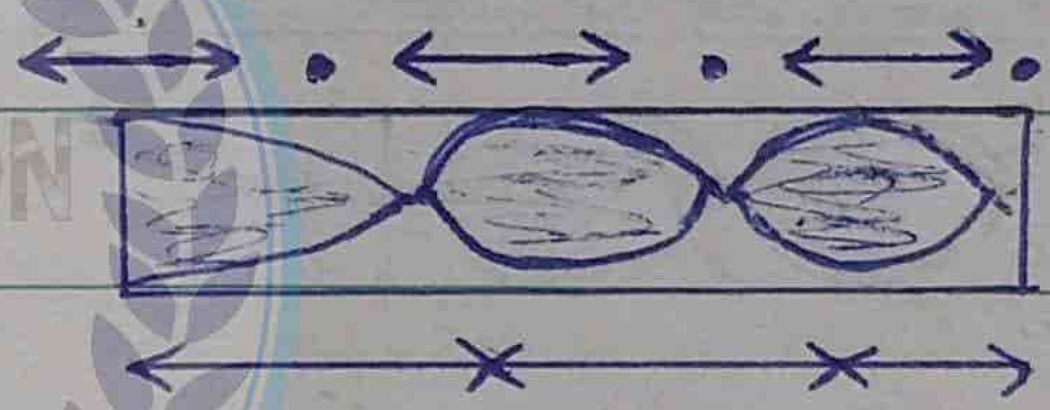
$$F_3 = 3(F_1) \quad \text{by eq 1}$$

$$\boxed{F_3 = 3F_1}$$

(iii) Third Mode of Vibration :-

From Fig

$$L = \frac{\lambda_5}{4} + \frac{\lambda_5}{2} + \frac{\lambda_5}{2}$$



$$L = \frac{\lambda_5 + 2\lambda_5 + 2\lambda_5}{4}$$

$$L = \frac{5\lambda_5}{4}$$

$$\lambda_5 = \frac{4L}{5}$$

$$V = F_5 \lambda_5$$

$$F_5 = \frac{V}{\lambda_5}$$

$$f_5 = \frac{V}{\frac{4L}{5}}$$

$$f_5 = 5\left(\frac{V}{4L}\right)$$

$$f_5 = 5f_1$$

$$\boxed{f_n = nf_1}$$

$f_5 = 5f_1$

$f_7 = 7f_1$

$f_9 = 9f_1$

In a pipe closed at one end only odd harmonics can be produced.

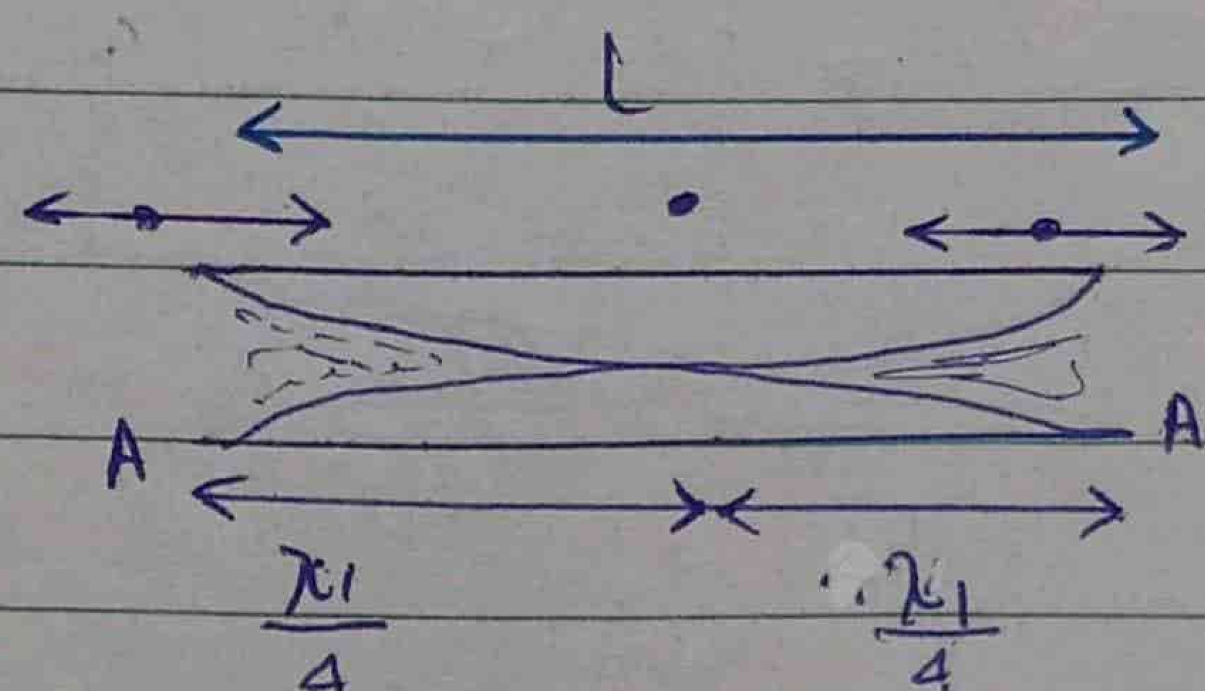
$$\boxed{f_n = n\left(\frac{V}{4L}\right)}$$

(b) Modes of Vibration IN An Open Pipe:-

Consider a vibrating tunning fork is held at the open of an open pipe. When the pipe is open at both ends, then its ends behave as antinodes.

1st - Mode of Vibration:-

Let f_1 is the frequency and λ_1 is the wavelength of longitudinal waves



from Fig

$$L = \frac{\lambda_1}{4} + \frac{\lambda_1}{4}$$

$$L = \frac{\lambda_1 + \lambda_1}{4}$$

$$L = \frac{2\lambda_1}{4}$$

$$L = \frac{\lambda_1}{2}$$

$$\boxed{\lambda_1 = 2L}$$

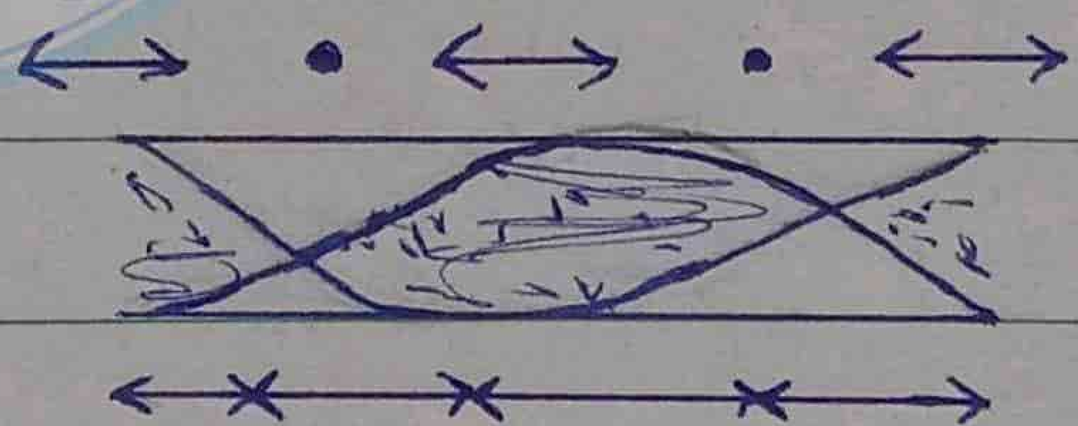
$$V = f_1 \lambda_1$$

$$f_1 = \frac{V}{\lambda_1}$$

$$\boxed{f_1 = \frac{V}{2L}}$$

2nd - Mode of Vibration :-

from Fig



$\frac{\lambda_2}{4}$ $\frac{\lambda_2}{2}$ $\frac{\lambda_2}{4}$

$$L = \frac{\lambda_2}{4} \; \frac{\lambda_2}{2} \; \frac{\lambda_2}{4}$$

$$L = \frac{\lambda_2 + 2\lambda_2 + \lambda_2}{4}$$

$$L = \frac{4\lambda_2}{4}$$

$$L = \lambda_2$$

$$\lambda_2 = L$$

OR

$$\lambda_2 = \frac{2L}{2}$$

$$v = f_2 \lambda_2$$

$$f_2 = \frac{v}{\lambda_2}$$

$$f_2 = \frac{v}{L}$$

$$f_2 = \frac{2v}{2L}$$

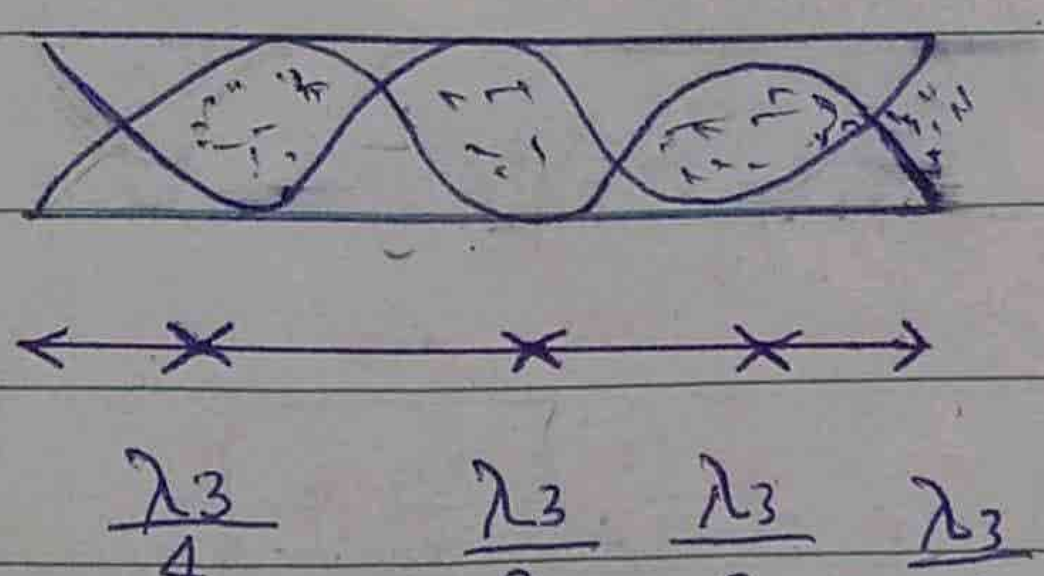
$$f_2 = 2\left(\frac{v}{2L}\right)$$

$$f_2 = 2f_1 \quad \text{by eq } (1)$$

$$\boxed{f_2 = 2f_1}$$

3rd Mode of vibration :-

from fig



$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4}$$

$$L = \frac{\lambda_3 + 2\lambda_3 + 2\lambda_3 + \lambda_3}{4}$$

$$L = \frac{6\lambda_3}{4}$$

$$L = \frac{3\lambda_3}{2}$$

$$L = 3\frac{\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

$$V = f_3 \lambda_3$$

$$f_3 = \frac{V}{\lambda_3}$$

$$f_3 = \frac{V}{\left(\frac{2L}{3}\right)}$$

$$f_3 = 3\left(\frac{V}{2L}\right)$$

$$f_3 = 3f_1$$

$$\lambda_1 = \frac{2L}{1}$$

$$\lambda_2 = \frac{2L}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

$\vdots$

$$\lambda_n = \frac{2L}{n}$$

$n = 1, 2, 3, 4, 5, \ldots$

Similarly

$$f_4 = 4f_1$$

$$f_5 = 5f_1$$

$\vdots$

$$\boxed{f_n = nf_1}$$

OR

$$\boxed{f_n = n\left(\frac{V}{2L}\right)}$$

8.11 Doppler Effect

Statement

"The apparent change in the frequency of sound due to the relative motion between the source of sound and observer is called source of sound and observer is called Dopper Effect."

Example :-

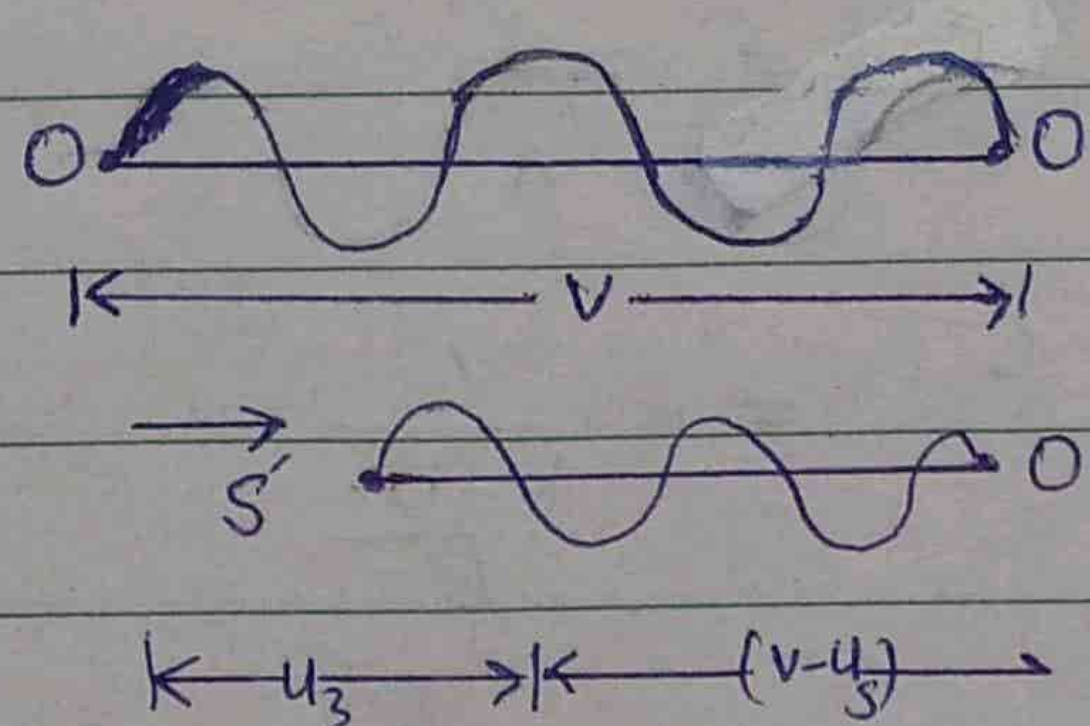
Suppose a person is standing on a locomotive (train) approaches the person, the frequency heard by the person increases. But when the Locomotive moves away from the person, frequency of sound waves heared by him decreases.

(i) When the source moves towards an observer at rest

Let $V =$ Velocity soundwaves

$f =$ Frequency of Sound Waves

$\lambda =$ wave length of Soundwaves



When both the Sources and observer O and are at rest, the number of waves received by the observer in one second are f

$$\boxed{f = \frac{v}{\lambda}}$$

$$\left[\begin{array}{l} v = f\lambda \\ f = \frac{v}{\lambda} \end{array}\right.$$

In one second the source will move from S to S' towards the observer O, with velocity u_s

$(v - u_s)$ = distance from the source to the observer.

Now

'f' waves are contained in a distance $= (v - u_s)$

(waves are compressed)

So, the new wavelength is λ'

$$\lambda' = \frac{v - u_s}{f}$$

$$f\lambda = v$$

$$f'\lambda' = v$$

$$f' = \frac{v}{\lambda'}$$

$$f' = \frac{v}{\frac{v - u_s}{f}}$$

$$f' = \left(\frac{v}{v - u_s}\right)$$

$$\boxed{f' = \left(\frac{v}{v-u_s}\right) f}$$

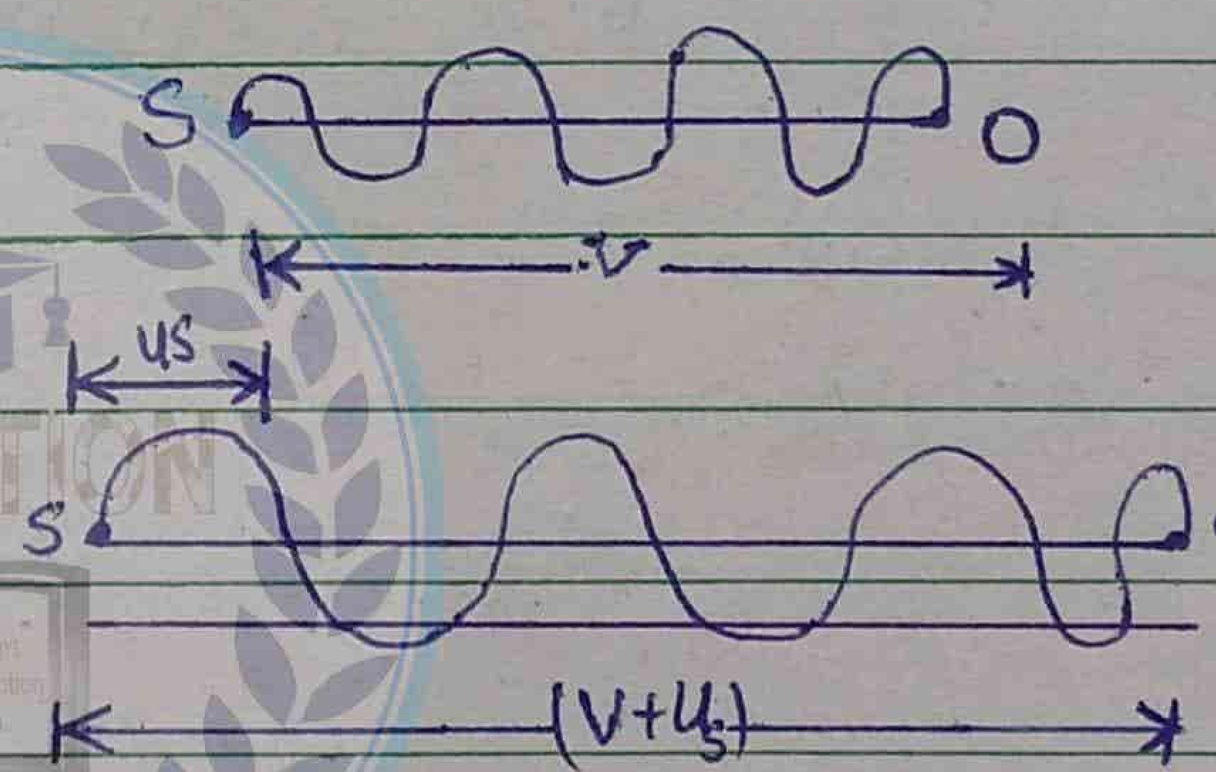
$$= f' > f$$

Result:-

Frequency increases.

(II) When the source moves away from an observer at rest

When both the sources and observer O are at rest, the number of waves received by the observer in one second are f



$$f = \frac{v}{\lambda}$$

$$\left[\begin{array}{l} v = f\lambda \\ f = \frac{v}{f} \end{array}\right]$$

In one second the source will moves from S to S' away from the observer O, with velocity us.

$(v + u_s)$ = distance from the source to the observer.

Now f waves are contained in a distance = $(V + u_s)$

The wavelength is λ''

$$\lambda'' = \frac{V + u_s}{f}$$

$$V = f\lambda$$

$$V = f''\lambda''$$

$$f'' = \frac{V}{\lambda''}$$

$$f'' = \frac{V}{\frac{V + u_s}{f}}$$

$$f'' = \left(\frac{V}{V + u_s}\right) f$$

$$\boxed{f'' = \left(\frac{V}{V + u_s}\right) f} \qquad f'' < f$$

Result :-

Frequency decreases.

(III) When the observer moves towards a source at rest:-

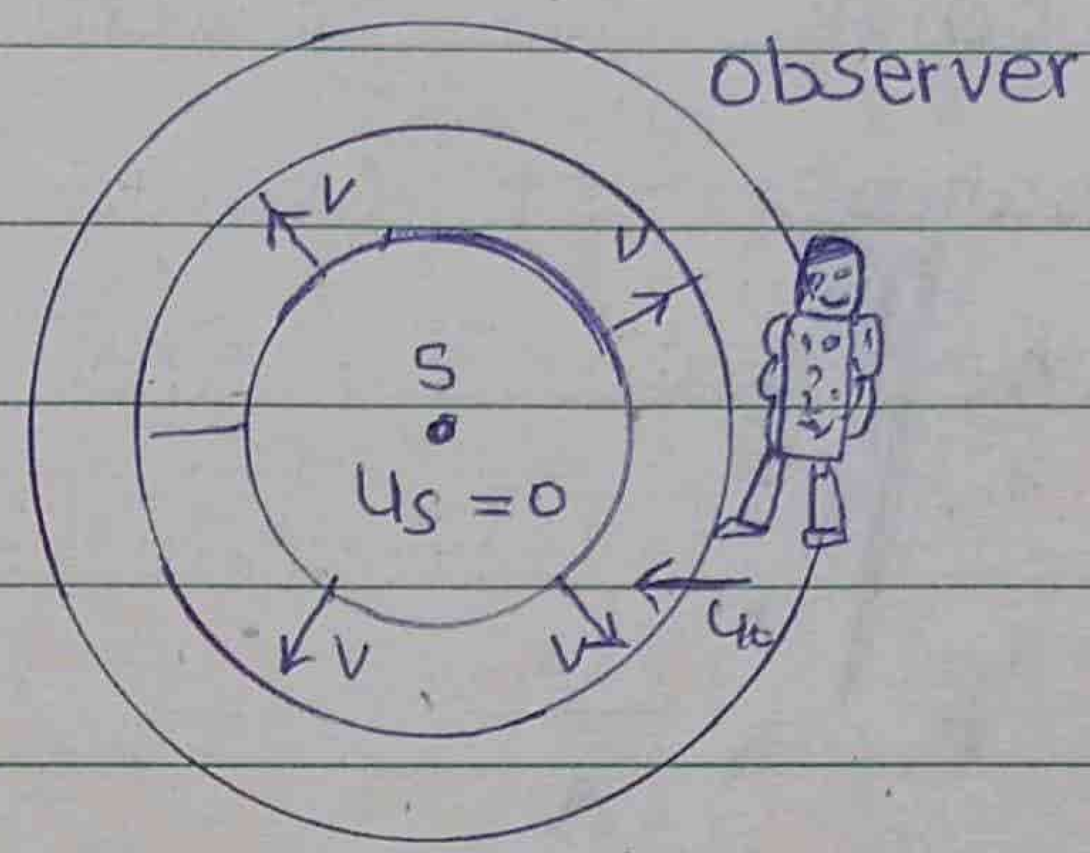
When both the observer are at rest

then the number of waves received by the observer in onesecond are

$$f = \frac{v}{\lambda}$$

When the observer moves with velocity u_o towards the source at rest, then the relative velocity of the sound waves and observer increases to $= (v + u_o)$.

$$\left[\begin{array}{l} v = f\lambda \\ f = \frac{v}{\lambda} \end{array} \right]$$



Now the number of waves received by observer in one second are

$$f''' = \frac{v + u_o}{\lambda}$$

But $v = f\lambda$

$$\lambda = \frac{v}{f}$$

$$f''' = \frac{v + u_o}{\frac{v}{f}}$$

$$\boxed{f''' = \left(\frac{v + u_o}{v}\right) f}$$

$$f'' > f$$

Result:-

Frequency increases.

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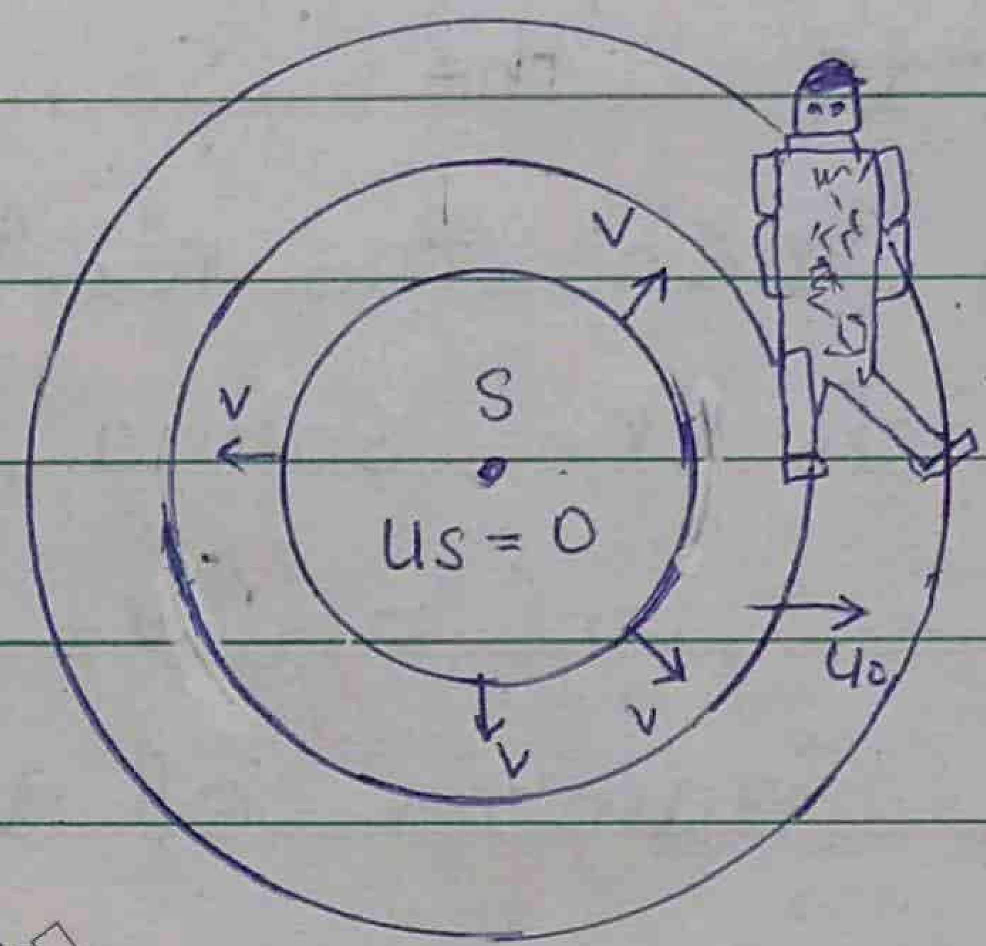
(IV) When observer moves away from a source at rest:-

When both the source and observer are at rest, the number of waves received by the observer in one second are

$$f = \frac{v}{\lambda}$$



When the observer moves with velocity u_o away from the source at rest, then the relative velocity sound waves and observer decreases to $= (v - u_o)$

The number of waves received in one second by the observer are

$$f^{iv} = \frac{v - u_o}{\lambda}$$

But $v = f\lambda$

$$\lambda = \frac{v}{f}$$

$$f^{iv} = \frac{v - u_o}{\frac{v}{f}}$$

$$\boxed{f^{iv} = \left[\frac{v - u_o}{v}\right] f}$$

$$f^{iv} < f$$

Result :-

The frequency dereases.

Applications of Dopplers Effect.

1- For Electromagnetic Waves :-

Doppler's Effect is notonly applicable for sound waves which are mechanical waves, but also applicable for Electromagnetic Waves.

2- Radar System :-

Radio waves sent out from rader are reflected from the aeroplanes. The frequency of reflected waves is decreased if the plane is moving away from the radar. The frequency increases if plane is moving towards the plane.

3. Sonar :-

We can observe the motion of objects

under water. When sound waves (called sonar) under water are reflected from a moving submarine. the change in frequency of the reflected waves helps to find the speed and position of the submarine. (sonar = Sound Navigation and Ranging.)

4- Earth satellites:-

Velocity of Earth Satellites are found by the change in frequency of radio waves emitted by the satellites.

5. Stars:-

Astronomers find the speed of the stars and glaxies by using Dopper's shift in the star light.

1- A star moving towards earth shows Blue Shift.
2- A star moving away from earth shows Red Shift.

In case of a star approaching earth frequency of light increases and wavelength decreases.

6. Speed of Automobiles:-

Traffic police measures the speed of Automobiles using dopplers effect.

For this purpose a radar gun is fixed on a police car.

The frequency change is measured and hence the speed of Automobile is calculated by computer programme.

Radar gun Microwaves.

QUESTIONS

imp imp

8.1 What features do longitudinal waves have in common with transverse waves?

8.2 The five possible waveforms obtained, when the output from a microphone is fed into the Y-input of cathode ray oscilloscope, with the time base on, are shown in Fig.8.23. These waveforms are obtained under the same adjustment of the cathode ray oscilloscope controls. Indicate the waveform

a) which trace represents the loudest note?

b) which trace represents the highest frequency?

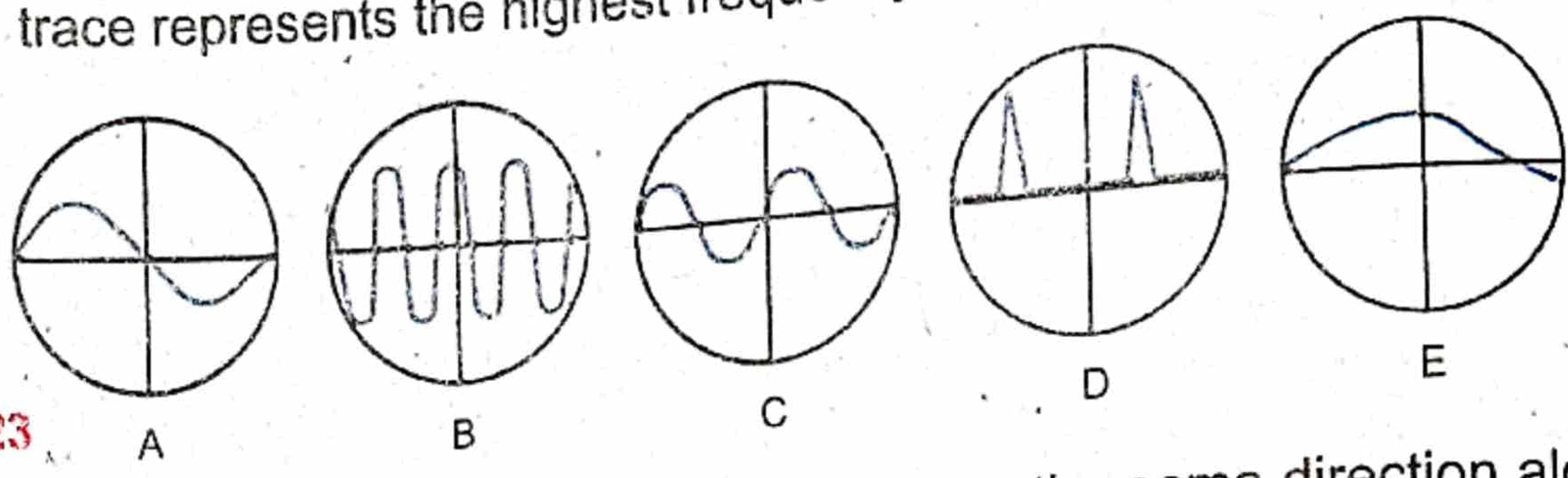


Fig. 8.23

8.3 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave? imp 2015 imp

8.4 A wave is produced along a stretched string but some of its particles permanently show zero displacement. What type of wave is it?

8.5 Explain the terms crest, trough, node and antinode. imp

8.6 Why does sound travel faster in solids than in gases? imp, imp imp

8.7 How are beats useful in tuning musical instruments? imp imp

8.8 When two notes of frequencies f_1 and f_2 are sounded together, beats are formed. If $f_1 > f_2$, what will be the frequency of beats?

i) $f_1 + f_2$ ii) $\frac{1}{2}(f_1 + f_2)$

iii) $f_1 - f_2$ iv) $\frac{1}{2}(f_1 - f_2)$

8.9 As a result of a distant explosion, an observer senses a ground tremor and then hears the explosion. Explain the time difference.

8.10 Explain why sound travels faster in warm air than in cold air. imp

8.11 How should a sound source move with respect to an observer so that the frequency of its sound does not change? 2015 imp

Chapter 08

Questions

Question 8.1

Answer

i) Both waves produce disturbances in the medium through which they pass.

ii) Both transport energy from one place to another but not matter.

iii) Both satisfy the equation $v = f\lambda$.

v = speed of waves;

f = frequency ;

λ = wavelength ;

Question 8.2

Answer

(a) Wave D has loudest sound because it has maximum amplitude.

(b) Wave B shows maximum frequency because it has maximum number of waves

Question 8.3

Answer

No It is not possible for two identical waves travilling in the same direction along astring to produce stationay waves.

Two identical waves travilling in opposite direction, superpose each other and stationary wave are produced.

Question 8.4

Answer

This is a stationary wave.
At Nodes, displacement is permanently zero.

Question 8.5

Answer

CRest

In a transverse wave, it is the portion of medium above its mean level.

Trough

In a transver wave, it is the portion of medium below its mean level.

Node

A node is a point in a vibrating body where the strain is maximum and amplitude is zero.

Antinode

A antinode is a point in a vibrating body where the strain is minimum and the amplitude is maximum.

Question 8.6

Answer

The sppeed of Sound in a medium is $V = \sqrt{\frac{E}{\rho}}$

E = modulus of elasticty

ρ = density of medium

Although the density of solids is greater then the density of gasses, but modulus of eleticity E for solids is much greater as compared to that for gases. Hence Elastic effect 'E' wins over the denisty effect 'P' Therefore, speed of sound in gases

$V_{solids} > V_{gases}$.

Question 8.7

Answer

Beats are useful to tune a string instrument, such as piano or violen by beating (producing sound) a note. against a note of known frequency. The string can be adjusted to the desired frequency by tightening or losening it until no beats are heard

Question 8.8

Answer

$f_1 - f_2$ is correct

$$f_1 - f_2 = \frac{\text{Nuber of beats}}{\text{Second}}$$

Question 8.9

Answer

Sound travels faster in solids than in gases. Due to this fact an observer senses a ground tremor first throug the ground which is solid and then hears the explosion later through air. This causes the time difference.

Question 8.10

Answer

As $v \propto \sqrt{T}$ Velocity of sound is directly proportional to the square root of absolute temperature of the gas (medium) Hence speed of sound increases as temperature increases. Also, we know that

$V_t = V_0 + 0.61t$.

$1^\circ C$ rise in temprature increases speed of sound by $0.61\ ms^{-1}$.

Also from $v = \sqrt{\dfrac{E}{\rho}}$

$$v \propto \frac{1}{\sqrt{\rho}}$$

$\rho_{\text{warm air}} < \rho_{\text{cold air}}$

warm air cold air

So

$$V_{\text{warm air}} > V_{\text{cold air}}$$

"So speed of sound in warm air is greater than that in cold air."

Question 8.11

Answer

Doppler's Effects due to relative motion between source of sound and observer. When source and observer both move in the same direction with same speed. Their relative velocity is Zero. So frequency of sound waves will not change.

Chapter 08

Examples

Example 8.1

Solution

$$T_o = (10^{\circ}C + 273)\ k$$

$$T_o = 283k$$

$$T = ?$$

$$V_t = 2V_o$$

$$\frac{V_t}{V_o} = \sqrt{\frac{T}{T_o}}$$

$$\frac{2V_o}{V_o} = \sqrt{\frac{T}{283k}}$$

$$2 = \sqrt{\frac{T}{283k}}$$

Squaring

$$4 = \frac{T}{283k}$$

$$T = 4 \times 283\ k$$

$T = 1132\ K$

$t^{\circ}C = 1132 - 273$

$\boxed{t^{\circ}C = 859^{\circ}C}$

Example 8.2

Solution

Beat frequency = 4
Frequency of A = 320 Hz.
Frequency of B = 320 + 4
= 324 Hz

By loading B its frequency will decrease. If 324 Hz is original frequency of the beat frequency will reduce. which is not the case.

If 316 Hz is the original frequency of B, the beat frequency will increase, which is the case (beat frequency increases from 4 to 6)

So the original frequency of B is = 316 Hz

when loaded frequency of

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$B = 314 \text{ Hz}$

Since

$320^{Hz} - 314^{Hz} = 6 \text{ beats/s}$

Example 8.3

Solution

$W = F$

$= \text{Tension}$

$= 80 \text{ N}$

$D = 0.5 \text{ mm}$

$= 0.5 \times 10^{-3} \text{ m}$

$r = \frac{D}{2}$

$= \frac{0.5}{2} \times 10^{-3}$

$= 0.25 \times 10^{-3} \text{ m}$

$l = 1.5 \text{ m}$

$\rho = 7.8 \times 10^{3} \text{ kg m}^{-3}$

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mass = Denisty x Volume (wire)

mass = $\rho \times \pi r^2 l$

$$m = \frac{\text{mass}}{\text{length}}$$

m = mass per unitlength

$$m = \frac{\rho \times \pi r^2 l}{l}$$

$$= \rho \times \pi r^2$$

F_1 = ?

$$F_1 = \frac{1}{2L}\sqrt{\frac{F}{m}}$$

$$F_1 = \frac{1}{2L}\sqrt{\frac{F}{\rho \times \pi r^2}}$$

$$F_1 = \frac{1}{2 \times 1.5}\sqrt{\frac{80}{7.8 \times 10^3 \times 3.14 \times (.25 \times 10^{-3})^2}}$$

$$\boxed{F_1 = 76\ Hz}$$

Example 8.4

Solution

(a)

$$f_n = \frac{nv}{2L}$$

$$V = 3\ 0\ ms^{-1}$$

$$L = 1m$$

$$f_n = n\left(\frac{V}{2L}\right)n$$

$$f_1 = \frac{1 \times 340\ ms^{-1}}{2 \times 1m}$$

$$f_1 = 170\ s^{-1}$$

$$f_1 = 170\ Hz$$

$$f_2 = 2f_1$$

$$f_2 = 2 \times 170\ Hz$$

$$f_2 = 340\ Hz$$

$$f_3 = 3f_1$$

$$f_3 = 3 \times 170 \text{ Hz}$$

$$\boxed{f_3 = 510 \text{ Hz}}$$

(b)

$$f_n = \frac{nv}{4l}$$

OR

$$f_n = n\left(\frac{v}{4l}\right)$$

$$n = 1, 3, 5, 7, \ldots$$

In this case only odd harmonics are present,

$$f_1 = \frac{1 \times 340}{4 \times 1}$$

$$f_1 = 85 \text{ Hz}$$

$$f_3 = 3f_1$$

$$f_3 = 3 \times 85 \text{ Hz}$$

$$f_3 = 255 \text{ Hz}$$

$$f_5 = 5f_1$$

$$f_5 = 5 \times 85 \text{ Hz}$$

$$\boxed{f_5 = 425 \text{ Hz}}$$

Example 8.5

Solution

$$f = 1000 \text{ Hz}$$

$$u_s = 90 \text{ km h}^{-1}$$

$$u_s = \frac{90 \times 1000 \text{ m}}{60 \times 60 \text{ s}}$$

$$u_s = 25 \text{ m/s}$$

$$V = 340 \text{ m/s}$$

$$f' = ?$$

1, 3, 5, ...

$$f'' = ?$$

$$f' = \left(\frac{V}{V - u_s}\right) f$$

$$f_1 = \left(\frac{340}{340 - 25}\right) \times 1000$$

$$f' = 1079.4 \text{ Hz}$$

$$f'' = \left(\frac{V}{V + U_s}\right) f$$

$$f'' = \left(\frac{340}{340 - 25}\right) \times 1000$$

$$\boxed{f'' = 931.5 \text{ Hz}}$$

NUMERICAL PROBLEMS

8.1 The wavelength of the signals from a radio transmitter is 1500 m and the frequency is 200 kHz. What is the wavelength for a transmitter operating at 1000 kHz and with what speed the radio waves travel?

(Ans: 300 m, 3×10^8 ms^{-1})

8.2 Two speakers are arranged as shown in Fig. 8.24. The distance between them is 3 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.00 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the centre of the line and directly opposite each speakers. Calculate the speed of sound.

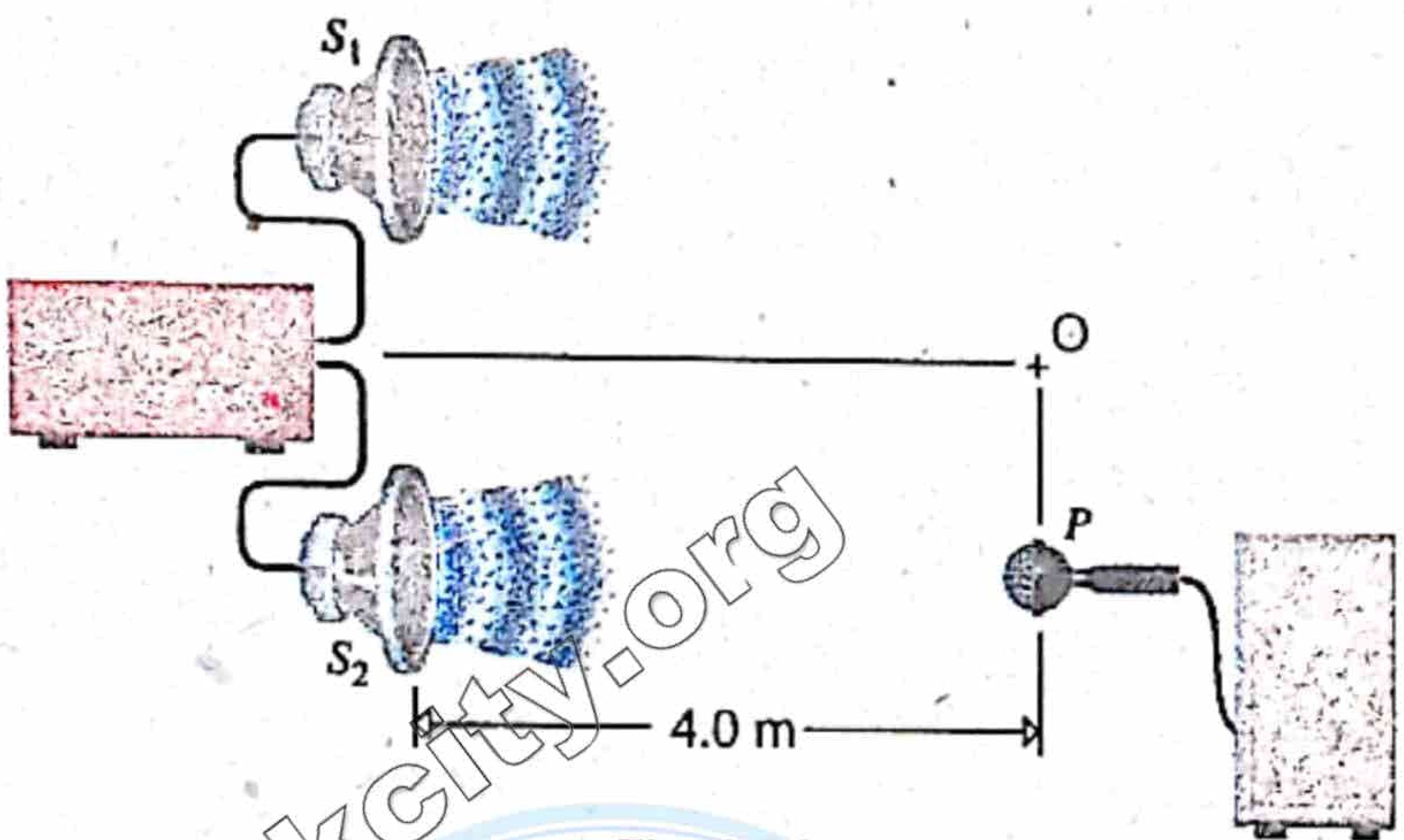

Fig. 8.24

(Ans: 344 ms^{-1})

8.3 A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments, at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency? 11 P 2023

(Ans: 0.6 m, 30 Hz)

8.4 The frequency of the note emitted by a stretched string is 300 Hz. What will be the frequency of this note when;

(a) the length of the wave is reduced by one-third without changing the tension.

(b) the tension is increased by one-third without changing the length of the wire.

(Ans: 450 Hz, 346 Hz)

8.5 An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is

(a) open at both ends.

(b) closed at one end.

(Speed of sound = 350 ms^{-1})

[Ans: (a) 350 Hz, 700 Hz, (b) 175 Hz, 525 Hz]

8.6 A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes.

(Speed of sound = 340 ms^{-1}) imp

(Ans: 21 Hz to 2833 Hz)

8.7 Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prong. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

(Ans: 253 Hz)

8.8 Two cars P and Q are travelling along a motorway in the same direction. The leading car P travels at a steady speed of 12 ms^{-1}; the other car Q, travelling at a steady speed of 20 ms^{-1}, sound its horn to emit a steady note which P's driver estimates, has a frequency of 830 Hz. What frequency does Q's own driver hear?

(Speed of sound = 340 ms^{-1})

(Ans: 810 Hz)

8.9 A train sounds its horn before it sets off from the station and an observer waiting on the plateform estimates its frequency at 1200 Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the plateform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50 s?

(Speed of sound = 340 ms^{-1})

(Ans: 17.9 ms^{-1}, 448 m)

8.10 The absorption spectrum of faint galaxy is measured and the wavelength of one of the lines identified as the Calcium α line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in a laboratory.

a) Is the galaxy moving towards or away from the Earth?

b) Calculate the speed of the galaxy relative to Earth.

(Speed of light = 3.0×10^{8} ms^{-1})

[Ans: (a) away from the Earth, (b) 6.1×10^{7} ms^{-1}]

Chapter 08

Problems

Problem 8.1

Solution

$\lambda_1 = 1500\ m$

$f_1 = 200\ k\ Hz$

$f_1 = 200 \times 10^3\ Hz$

$v = ?$

$\lambda_2 = ?$ at

$f_2 = 1000\ k\ Hz$

$f_2 = 1000 \times 10^3\ Hz$

$v = f\lambda$

$v = f_1 \lambda_1$

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$$V = 200 \times 10^3 \times 1500$$

$$V = 200 \times 10^3 \times 1500$$

$$\boxed{V = 3 \times 10^8 \ ms^{-1}}$$

Speed of radio waves $= 3 \times 10^8 \ m/s$

Now

$$V = f_2 \lambda_2$$

$[Hz = s^{-1}]$

$$\lambda_2 = \frac{V}{f_2}$$

$$\lambda_2 = \frac{3 \times 10^8}{1000 \times 10^2}$$

$$\lambda_2 = \frac{3 \times 10^8}{1000 \times 10^3}$$

$$\lambda_2 = 3 \times 10^2 \ m$$

$$\boxed{\lambda_2 = 300m}$$

Problem 8.2

Solution

$$f = 344 \ Hz$$

Distance between speakers $= 3\ m$

Distance between Speakers and line of motion $= 4\ m$

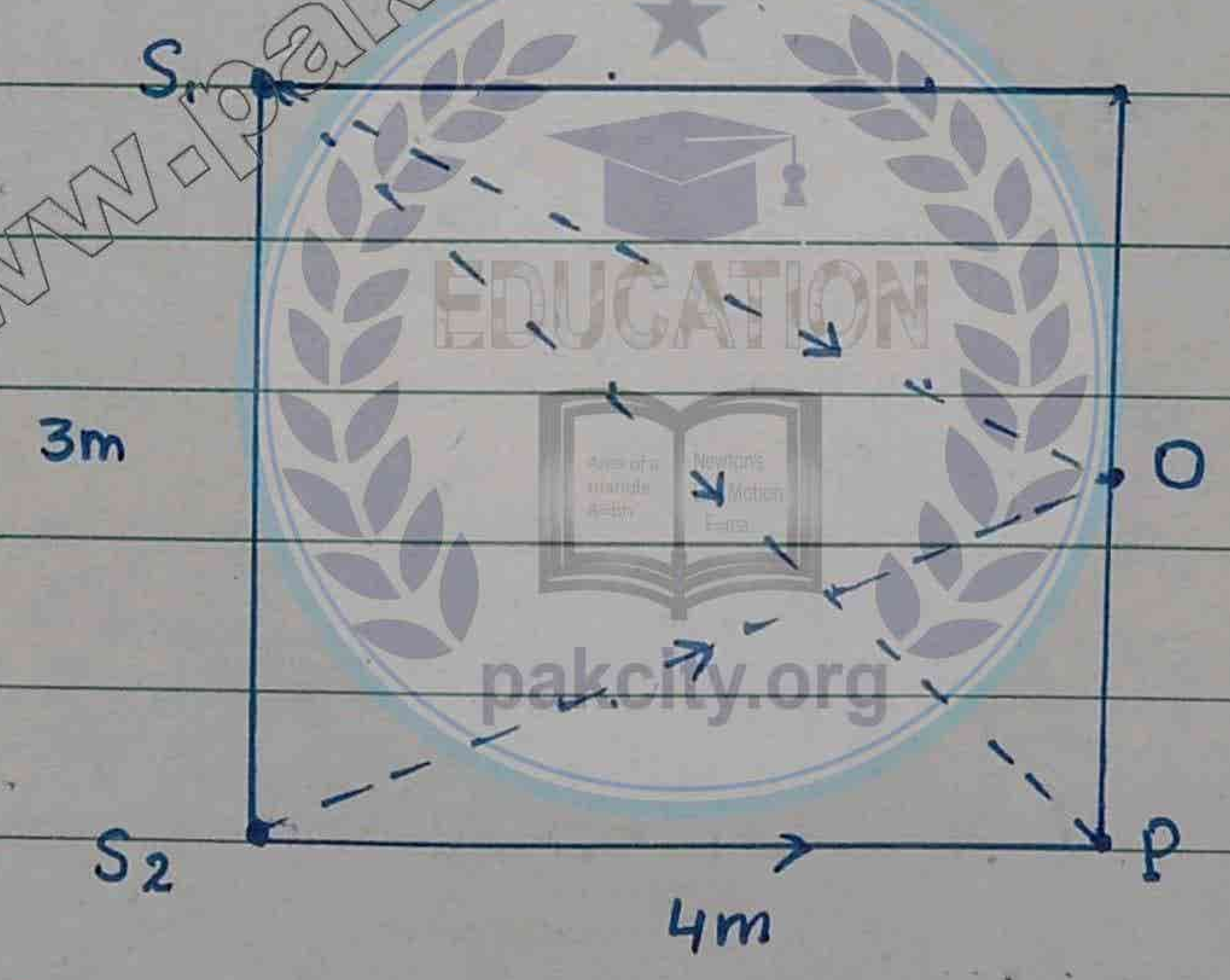
$V = ?$ (Speed of Sound)

Condition for maximum loudness or constructive interferences is

Path difference $= 0, 1\lambda, 2\lambda, \ldots m\lambda$

For point O $S_1O = S_2O$

So path difference $=$ Zero



For point P

Path difference $= 1\lambda$

$$S_1P - S_2P = 1\lambda \rightarrow (1)$$

In $\Delta S_1 S_2 P$

$$S_1P^2 = S_2P^2 + S_1S^2$$

$$S_1P^2 = 4^2 + 3^2$$

$$S_1P^2 = 16 + 9$$

$$S_1P^2 = 25$$

$$S_1P = \sqrt{25}$$

$$S_1P = 5\ m$$ Put in eq ①

$$S_1P - S_2P = \lambda$$

$$5\ m - 4\ m = \lambda$$

$$1\ m = \lambda \qquad \therefore \lambda = 1m$$

$$v = f\lambda$$

$$= 344\ Hz \times 1\ m$$

$$v = 344\ s^{-1} \times 1m$$

$$\boxed{v = 344\ ms^{-1}}$$

Problem 8.3

Solution

$$L = 120\ cm$$

$$L = \frac{120}{100}\ m$$

$$L = 1.2$$

$$n = 4$$

$$f_4 = 120\ Hz$$

$$\lambda = ?$$

$$f_1 = ?$$

$$\lambda_n = \frac{2L}{n}$$

$$\lambda_4 = \frac{2L}{4}$$

$$\lambda_4 = \frac{2 \times 1.2}{4}$$

$$\lambda_4 = 0.60\ m$$

$$\boxed{\lambda_4 = 0.6m}$$

$$f_n = nf_1$$

$$f_4 = 4f_1$$

OR

$$f_4 = f_1 = \frac{f_1}{4}$$

$$f_1 = \frac{120}{4}$$

$$\boxed{f_1 = 30\ Hz}$$

Problem 8.4

Solution

$$f = 300\ Hz$$

(a) $f' = ?$

F = Tension is kept constant

$$\lambda' = \lambda - \frac{\lambda}{3}$$

$$\lambda' = \frac{3\lambda - \lambda}{3}$$

$$\lambda' = \frac{2\lambda}{3}$$

When the tension is kept constant the speed remains same.

$$v = v'$$

$$f\lambda = f'\lambda'$$

$$f' = \frac{f\lambda}{\lambda'}$$

$$f' = \frac{300\,Hz \times \lambda}{\frac{2\lambda}{3}}$$

$$f' = 300\,Hz \times \frac{3}{2}$$

$$\boxed{f' = 450\,Hz}$$

(b) $f' = ?$

$$f = 300\,Hz$$

$$f' = F + \frac{F}{3}$$

$$f' = \frac{3f + f}{3}$$

$$f' = \frac{4f}{3}$$

$$f = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

$$f' = \frac{1}{2L}\sqrt{\frac{F'}{m}}$$

$$f' = \frac{1}{2l}\sqrt{\frac{\frac{4F}{3}}{m}}$$

$$f' = \frac{1}{2l}\sqrt{\frac{4F}{3m}}$$

$$f' = \sqrt{\frac{4}{3}} \times \frac{1}{2l}\sqrt{\frac{F}{m}}$$

$$f' = \sqrt{\frac{4}{3}} \times f$$

$$f' = 1.153 \times 300\ Hz$$

$$\boxed{f' = 346\ Hz}$$

Problem 8.5

solution

$$L = 50cm$$

$$L = \frac{50}{100}\ m$$

$$L = 0.50\ m$$

$$\boxed{F_1 = 350\ ms^{-1}}$$

(a)

$$F_1 = ?$$

$$F_2 = ?$$

$$F_n = \frac{nv}{2L}$$

OR $$F_n = n\left(\frac{v}{2L}\right)$$

$$F_1 = \frac{1 \times v}{2L}$$

$$F_1 = \frac{1 \times 350}{2 \times .50}$$

$$F_1 = 350\ Hz$$

$$F_2 = 2F_1$$

$$F_2 = 2 \times 350\ Hz$$

$$\boxed{F_2 = 700\ Hz}$$

(b) $$F_n = \frac{nv}{4L}$$ $\because F_n = n\left(\frac{v}{4L}\right)$

only odd harmonics are present $\because n = 1, 3, 5 \ldots\ldots$

$$F_1 = \frac{1 \times V}{4L}$$

$$F_1 = \frac{1 \times 350}{4 \times 0.5}$$

$$F_1 = 175 \text{ Hz}$$

$$F_3 = 3F_1$$

$$F_3 = 3 \times 175 \text{ Hz}$$

$$\boxed{F_3 = 525 \text{ Hz}}$$

Problem 8.6

Solution

$$L = 30\,mm$$

$$L = 30 \times 10^{-3}\,m$$

$$L' = 4\,m$$

$$V = 340\,ms^{-1}$$

$$F_1 = ?$$

$$f_1' = ?$$

$$f_n = \frac{nv}{4L}$$

OR

$$f_n = n\left(\frac{V}{4L}\right)$$

$$f_1 = \frac{1 \times V}{4L}$$

$$f_1 = \frac{1 \times 340}{4 \times 30 \times 10^{-3}}$$

$$\boxed{f_1 = 2833 \text{ Hz}}$$

$$f_n' = \frac{nv}{4L'}$$

$$f_1' = \frac{1 \times v}{4L'}$$

$$f_1' = \frac{1 \times 340}{4 \times 4}$$

$$\boxed{f_1' = 21 \text{ Hz}}$$

Frequency Range . 21 Hz $\rightarrow$ 2833 Hz.

Problem 8.7

Solution

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Number of beats per second = 3

Frequency of one fork = 256 Hz

Frequency of second fork = (256 ± 3) Hz

= 259 Hz or

= 253 Hz

When fork one is loaded, then number of beats per second becomes 1.

After loading fork one, its frequency will decrease.

If the frequency of second fork is taken 259 Hz, then the number of beats will increase and will not be equal to 1.

So the frequency of the second fork should be 253 Hz.

Problem 8.8

Solution

$$V_p = 12\ ms^{-1}$$

$$V_a = 20\,ms^{-1}$$

$$f' = f_p$$

$$f' = 830\,Hz$$

$$V = 340\,ms^{-1}$$

Original frequency $f = ?$

Speed of Source $= V_s$

$$V_s = V_a - V_p$$

$$V_s = 20\,ms^{-1} - 12\,ms^{-1}$$

$$V_s = 8\,ms^{-1}$$

$$f' = \left(\frac{V}{V - V_s}\right) f$$

$$f = f' \times \left(\frac{V - V_s}{V}\right)$$

$$f = \left(\frac{340\,ms^{-1} - 8}{340}\right) \times 830\,Hz$$

$$f = \frac{332}{340} \times 830\,Hz$$

Problem 8.9

Solution

$f = 1200\ Hz$

$f' = 1140\ Hz$

$v = 340\ ms^{-1}$

$t = 50\ s.$

$v_s = ?$ (Speed of source) train

$S = ?$

$$f' = \left(\frac{v}{v + v_s}\right) f$$

$$v + v_s = v\ \frac{f}{f'}$$

$$v_s = v\ \frac{f}{f'} - v$$

$$v_s = 340\ ms^{-1} \times \frac{1200}{1140} - 340\ ms^{-1}$$

$$v_s = 17.89\ ms^{-1}$$

$$\boxed{v_s = 17.9\ ms^{-1}}$$

$$S = V_{avg} \times t$$

$$S = \left(\frac{0 + V_s}{2}\right) \times t$$

$$S = \frac{0 + 17.9}{2} \times 50$$

$$\boxed{S = 447.5 \text{ m}}$$

Problem 8.10

Solution

Apparent wave length $\lambda' = 478$ nm

$\lambda' = 478 \times 10^{-9}$ m

Original wave length $\lambda = 397$ nm

$\lambda = 397 \times 10^{-9}$ m

$\lambda = 3.0 \times 10^{8}$ ms^{-1}

$$f'\lambda' = v$$

$$f' = \frac{v}{\lambda'}$$

$$f' = \frac{3 \times 10^{8}}{478 \times 10^{-9}}$$

$$f' = 6.28 \times 10^{14}\ s^{-1}$$

$$f' = 6.28 \times 10^{14}\ Hz$$

$$f\lambda = v$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{3 \times 10^{8}}{397 \times 10^{-9}}$$

$$f = 7.56 \times 10^{14}\ Hz$$

$$f' < f$$

So glaxy is moving away from the Earth

For Source moving away from observer

$$f' = \left(\frac{v}{v + v_s} \right) f$$

$$(v + v_s) f' = vf$$

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$$V + V_s = V \frac{f}{f'}$$

$$V_s = V \frac{f}{f'} - V$$

$$V_s = 3 \times 10^8 \ ms^{-1} \times \frac{7.56 \times 10^{14}}{6.28 \times 10^{14}} - 3 \times 10^8 \ ms^{-1}$$

$$\boxed{V_s = 6.11 \times 10^7 \ ms^{-1}}$$