Chapter 07

Oscillations

Oscillatory Motion or Vibratory Motion

The to and fro motion of a body about the mean position is called vibratory motion or oscillatory motion.

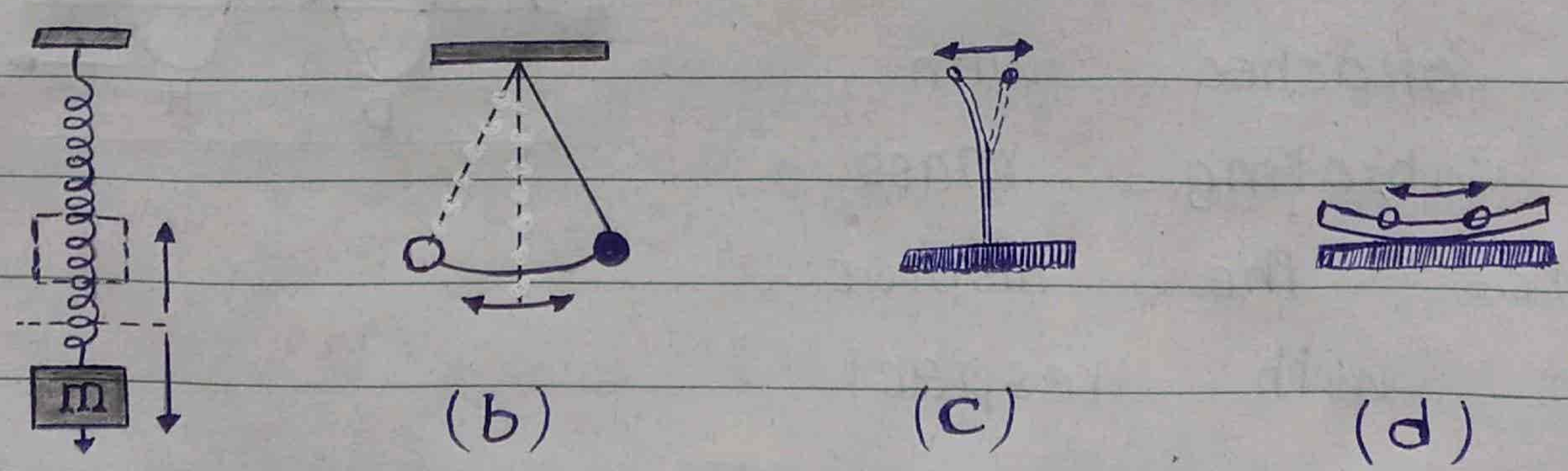
Periodic motion

In this types of vibratory motion the body repeats its motion after equal intervals of time.

Examples of vibratory motion

Motion of

* Mass suspended from a spring.
* Simple pendulum.
* Steel ruler fixed at one end.
* A steel ball roling in a curved dish.



(a) (b) (c) (d)

Vibrating bodies produce wave. For example a vibrating string of a violen produces sound waves.

Displacement

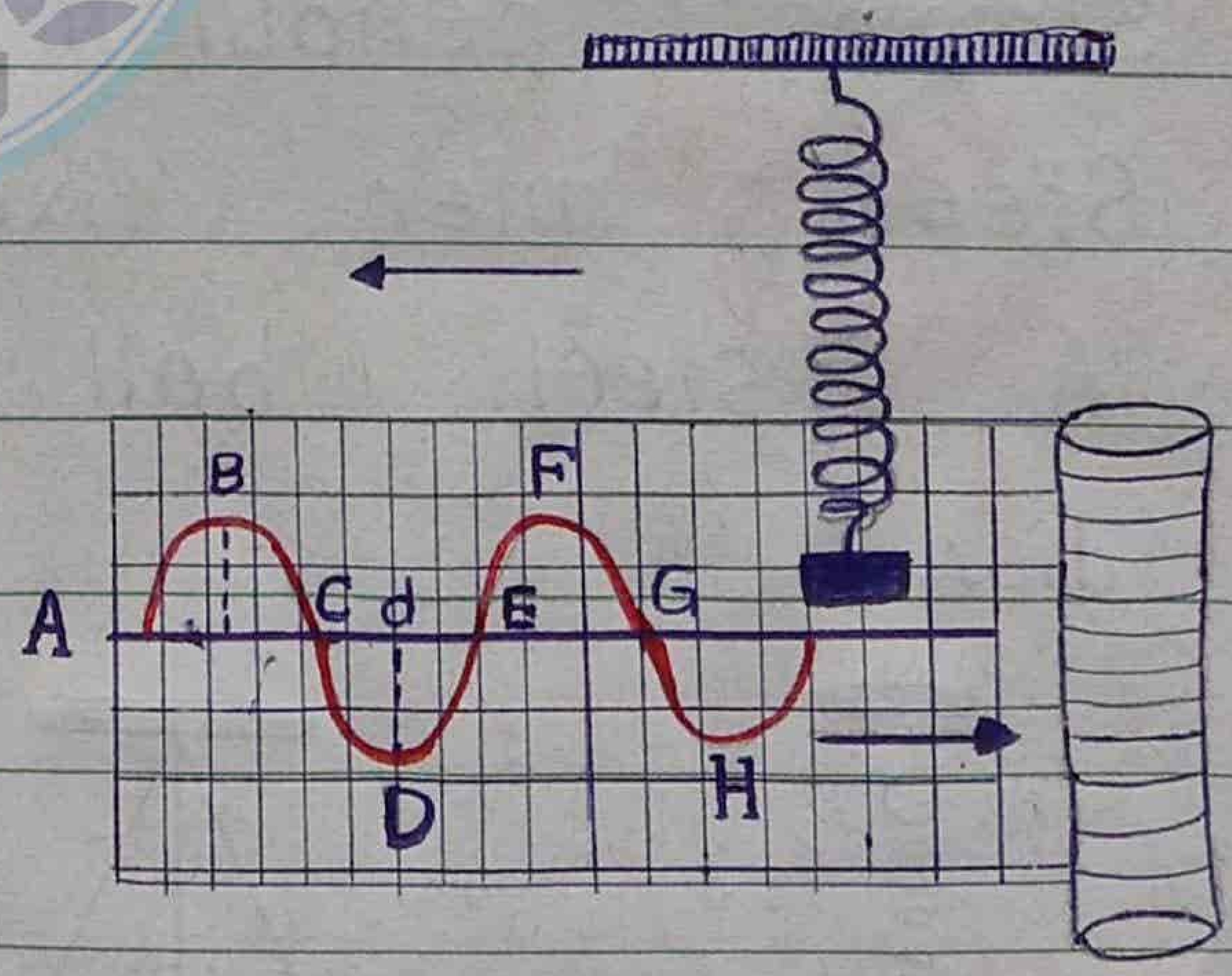
Distance of the vibrating body at any instant from the mean position is called displacement or instantaneous displacement. It is denoted by "X"

Amplitude

Maximum value of displacement on either side of mean position is called amplitude. It is denoted "X_o"

Explanation

Consider the arrangement shown in fig. The spring is vibrating and the strip of paper is moving at constant speed from right to left. A pen attached with the vibrating mass records the displacement with respect



to time. The curve obtained is called sine wave or wave form of S.H.M.

* ; Points "B" and "D" are extreme positions.
* ; Points "A", "C" and "E" show mean position.
* ; Line "ACE" show level of mean position.
* ; bB and dD are the amplitude of vibration.

Vibration

"One complete round trip of the vibrating body about the mean position is called vibration".

Time period

"Time to complete one vibration is called time period". It is denoted by "T".

Frequency

"The number of vibrations completed in one second is called frequency" "F".

Unit of frequency is Hertz.

Hertz

One vibration in one second is called one Hertz or 1 Hz. $\because Hz = s^{-1}$

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f} \quad \text{or} \quad f \times T = 1$$

Angular frequency

"The number of revolutions made by a body in on second is called angular frequency."

As

$$\omega = \frac{\theta}{t}$$

For on revolution
$\theta = 2\pi$
$t = T$ time

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

Relation between frequency and Angular frequency.

$$\omega = \frac{2\pi}{T} = 2\pi \times \frac{1}{T} \qquad \because \frac{1}{T} = f$$

$$\omega = 2\pi f$$

Unit and dimensions of ω.

unit rad s^{-1}
dimensions T^{-1}.

7.1 Simple Harmonic Motion

Motion of a mass attached to a spring.

Definition

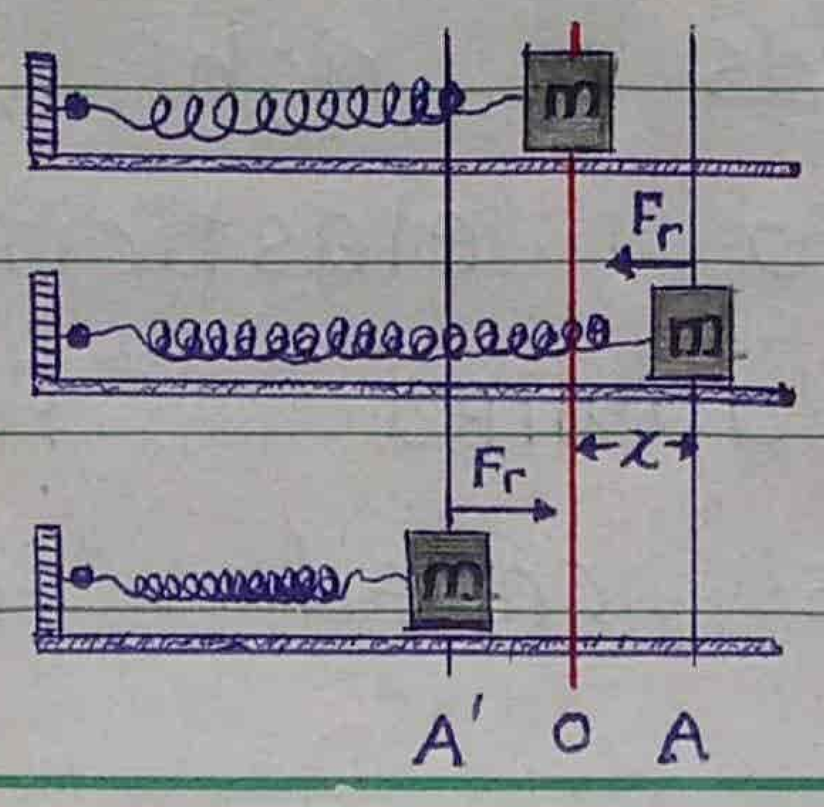
"A vibratory motiom in which acceleration of the vibrating body is directly proportional to the displacement and the direction of acceleration is towads the mean position is called simple harmonic motion."

Examples

* ; Mass spring system.
* ; Simple pendulum.

Explanation

Consider a mass "m" is attached to one end of an elastic spring. It can move freely on a frictionless horizontal surface as shown.



Hooke's Law

"It states that with in elastic limit the applied force is directly proportional to the displacement."

$$F \propto x$$
$$F = Kx$$
$$\frac{F}{x} = K$$

K is spring constant

Unit of k is Nm^{-1}

Dimension of k = MT^{-2}

Definition

" Force needed to produce unite extension is called spring constant"

Restoring force

"Restoring force opposes the applied force and brings the vibrating body towards the mean position."

In fig when the spring is displaced towards right by an applied force. Due to elasticity the spring opposes the applied force $F = Kx$ This force is called elastic restoring force. This is equal and opposite to the applied force

$$\boxed{F_r = -Kx}$$

"The direction of restoring force is always towards the mean position"

Now the mass is released. It begins to vibrate about the mean position.

$$F_r = -Kx$$

$$ma = -Kx$$

$$a = -\frac{K}{m}x \qquad \because \frac{K}{m} = \text{constant}$$

$$a = -(\text{constant})\,x$$

$$a \propto -x$$

negative sign shows that its direction is towards the mean position.

7.2 SHM and Uniform Circular Motion

Consider a mass m suspend from one end of a Spring. It vibrates with time period T, Frequency f and Amplitude x_0.

The motion of the mass is displayed by a pointer P_1 on the line BC with A as mean position Fig (a) B and C are the extreme positions.

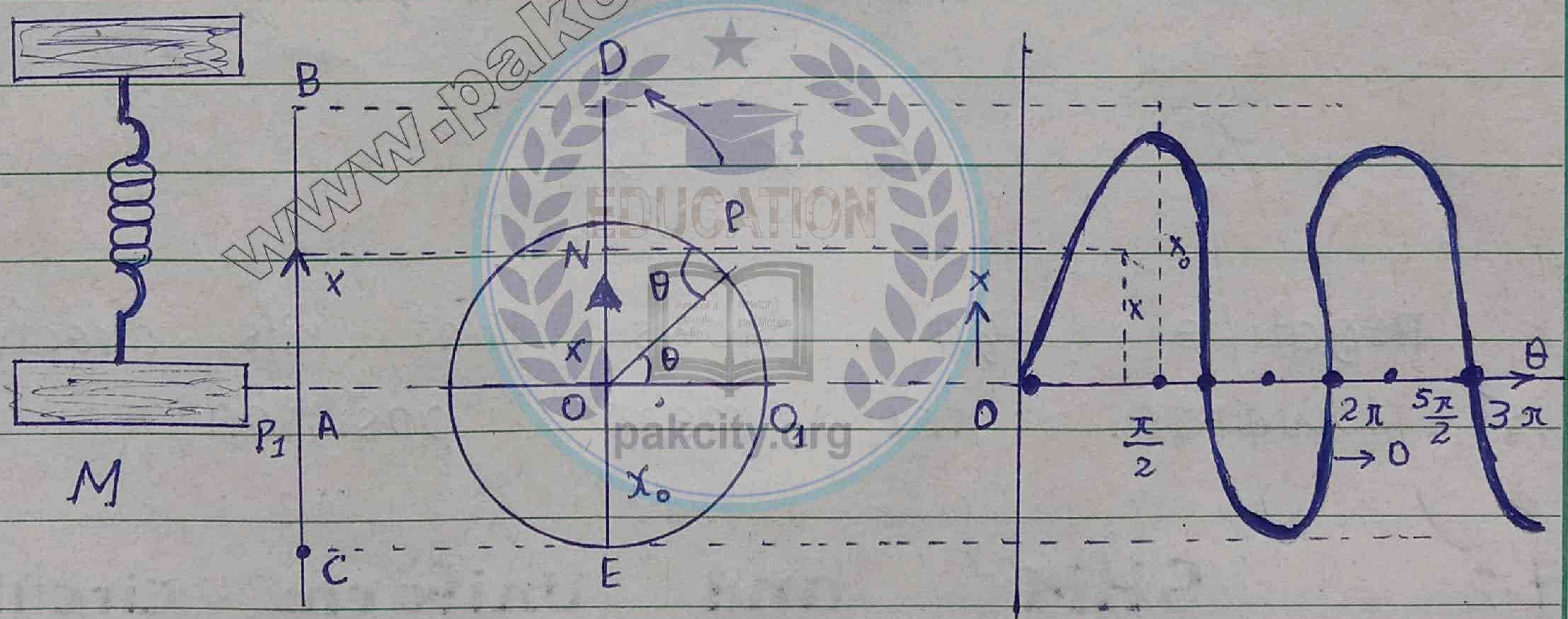
At time $t = 0$ let the pointer is at A moving upwards.

At time $T/4$, $T/2$, $3T/4$, T pointer is at B, AC and A.

This is one complete cycle of vibration.

The amplitude of vibration is

$$x_0 = AB = AC$$



(a) (b) (c)

Now consider the fig (b), a particle P is moving in a circle of radius $r = x_0 = OD$, with constant angular velocity ω.

" N is the projection of P on the diameter DE."

As prevolves in a circle of radius x_0 with uniform angular frequency ω, N vibrates on the diameter DE about the mean position O. Its time period is T.

At time $t = 0$ let the position of P is at O_1.

At time $t = 0, T/4, T/2, 3T/4, T$; the point N will, be at O, D, O, E, O.

So, Motions of the point N and the pointer P_1 is same.

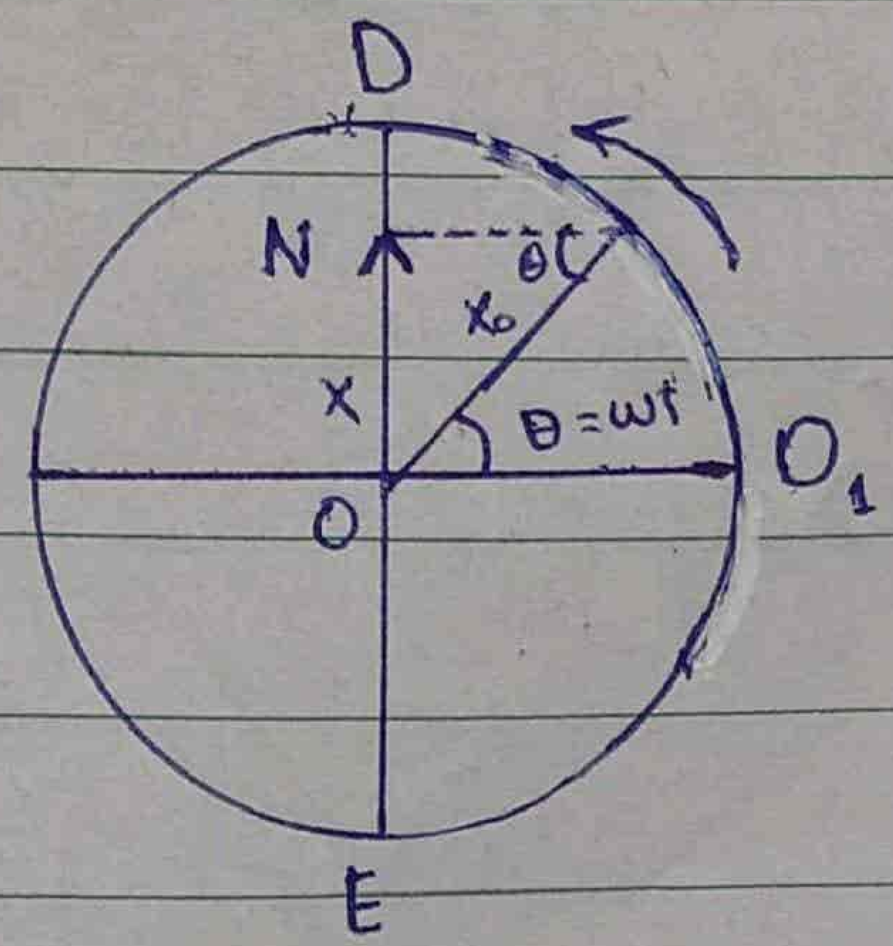
Result :

"When a body moves along a circule with constant speed, its projection (N) vibrates on the diameter and performs SHM."

1- Displacement of Projection N :

At a time $t = 0$ P is at O_1

In a time t = radius OP sweeps an angle $\theta = \omega t$



In ΔONP $\quad \frac{x}{x_0} = \sin\theta$

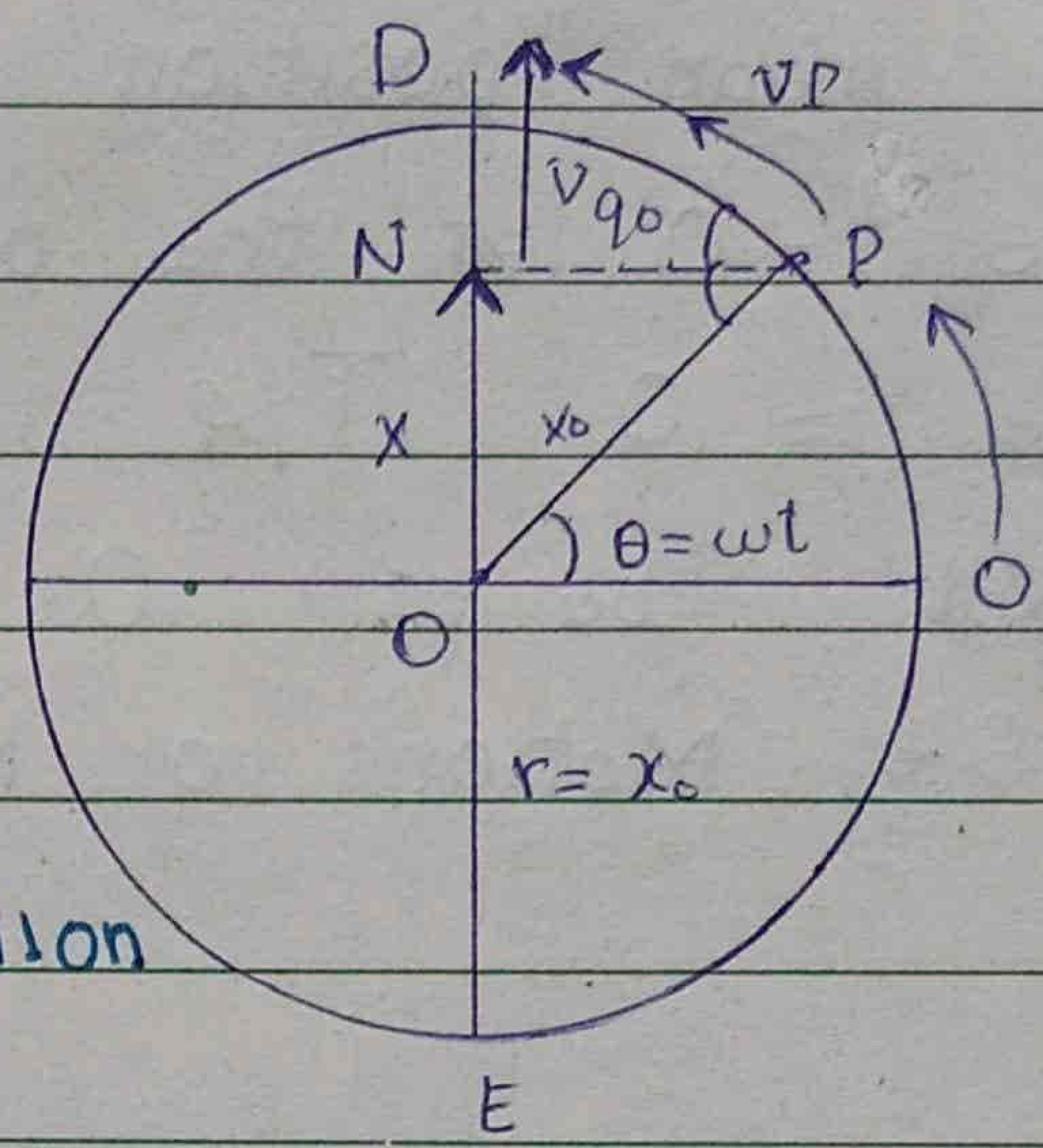
$$x = x_0 \sin\theta$$

$$\boxed{x = \sin \omega t}$$

This is displacement of N at any time t.

2- Intantaneous Velocity of Projection N:

V_p = Velocity of the point P at any instant. It is along the tangent to the circle at P.



V = Velocity of the projection N at any instant.

" V is the compenent of Vp in the direction parallel to the diameter DE "

In fig

$$\frac{V}{V_p} = \sin(90-\theta)$$

$$V = V_p \sin(90-\theta)$$

But

$$\sin(90-\theta) = \cos\theta$$

$$V = V_p$$

As

$$V = r\omega$$

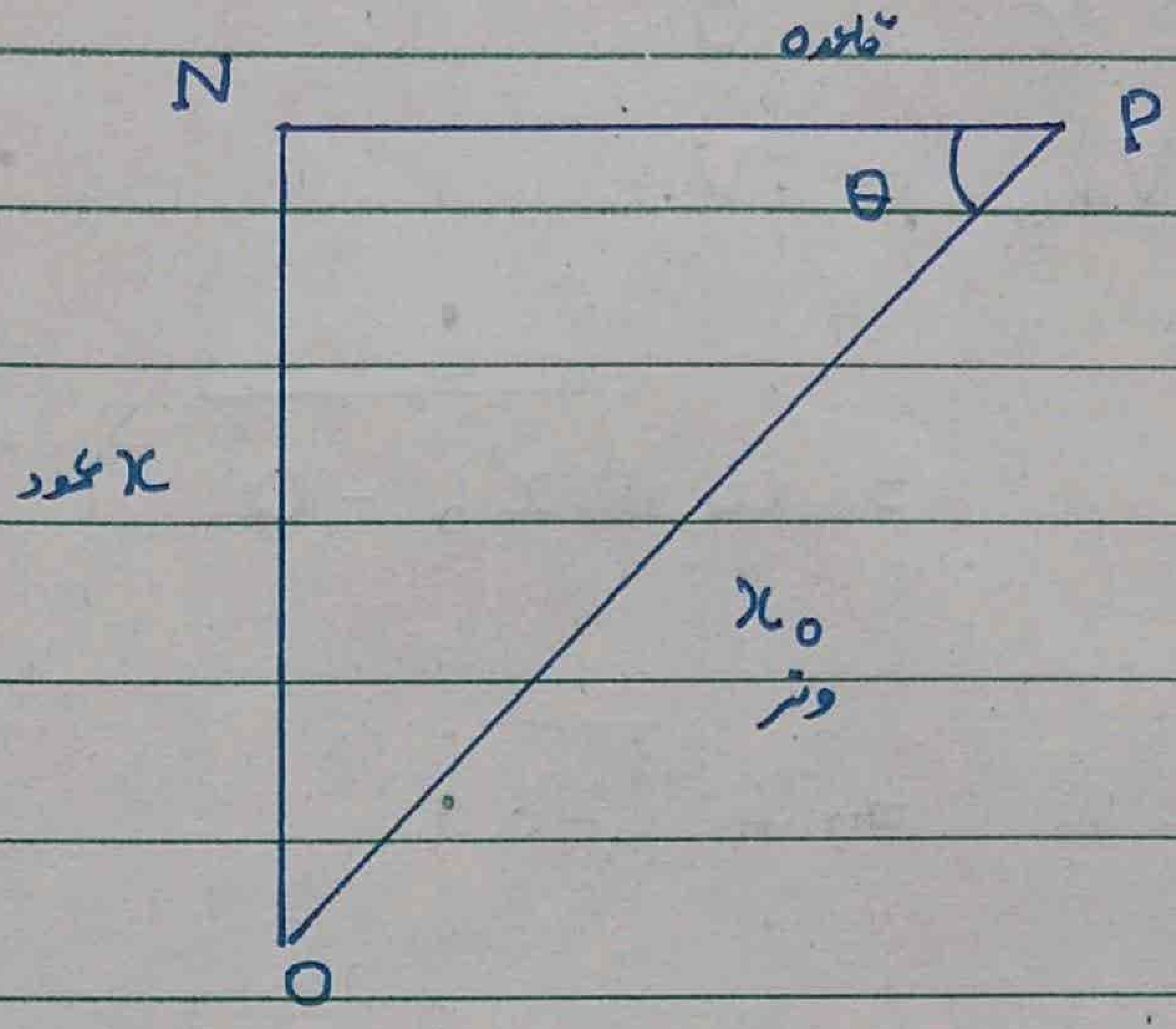
$$V_p = x_0 \omega \qquad \because r = x_0$$

$$\omega = \frac{\theta}{t}$$

$$\theta = \omega t$$

$$V = x_0 \omega \cos\theta \longrightarrow ①$$

$$V = x_0 \omega \cos \omega t$$



In ΔONP

$$x_0 = NP^2 + x^2$$

$$NP^2 = x_0^2 - x^2$$

$$NP = \sqrt{x_0^2 - x^2}$$

In ΔONP

$$\cos\theta = \frac{N}{x_0}$$

$$= \sqrt{\frac{x_0^2 - x^2}{x}}$$

Put in eq ①

$$V = x_0 \, \omega \sqrt{\frac{x_0^2 - x^2}{x_0}}$$

$$\boxed{V = \omega \sqrt{x_0^2 - x^2}}$$

Maximum Velocity:

V_o It is mean position

$$x = 0$$

$$V_{max} = V_o$$

$$= \omega\sqrt{x_o^2 - 0}$$

$$= \omega\sqrt{x_o^2}$$

$$= x_o\,\omega$$

$$\boxed{V_{max} = V_o = x_o\omega}$$

Minimum Velocity:

It is at extreme position

$$x = x_o$$

$$V_{min} = \omega\sqrt{x_o^2 - x_o^2}$$

$$= \omega(0)$$

$$= 0$$

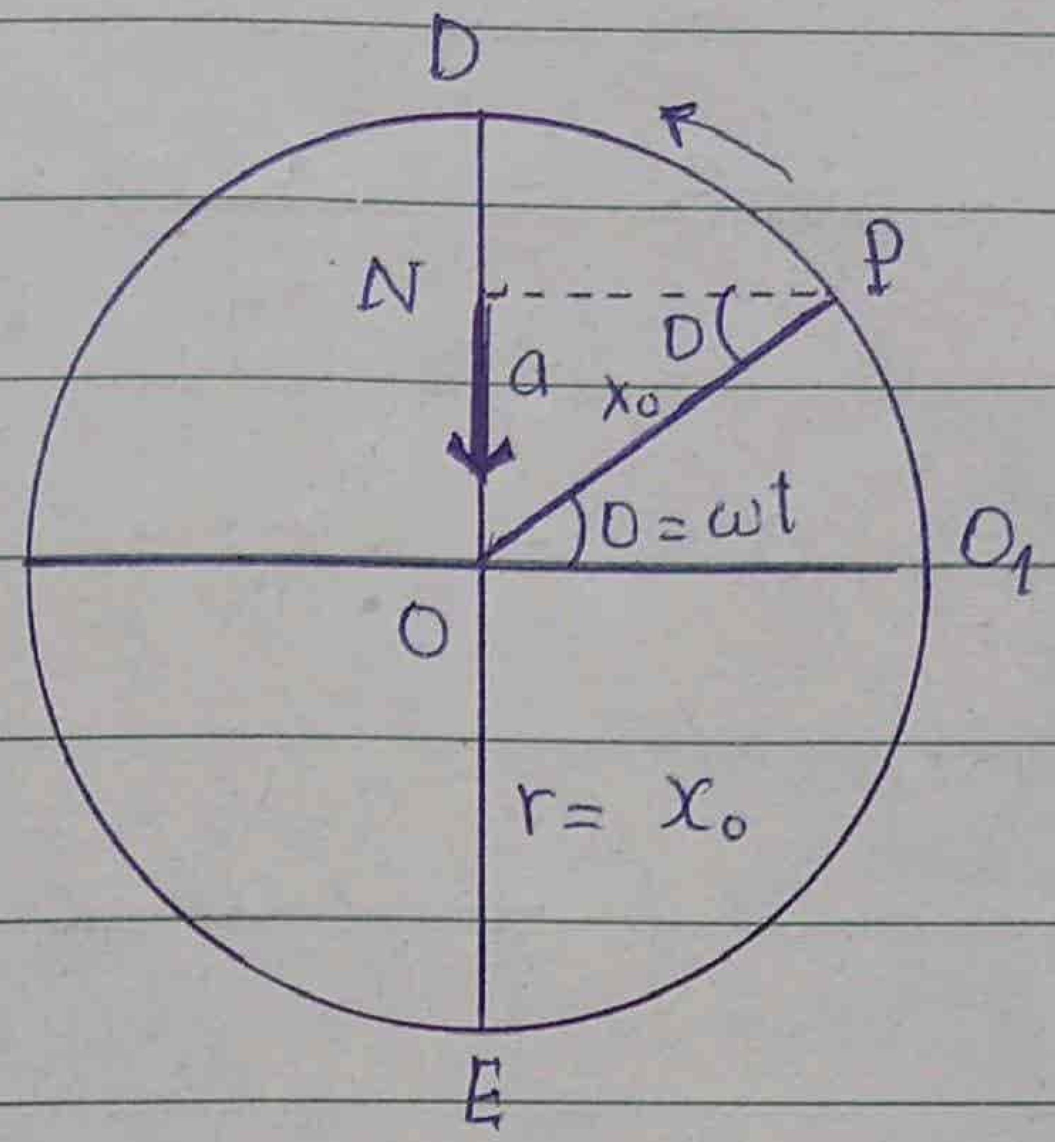
3- Acceleration of the Projection N:

a_p = Acceleration of the point P moving along the circle.

Its direction is towards the center of

of the circle.

a = "Acceleration of the Projection N. It is the component of a_p on the diameter DE."



As

$$a_c = \frac{V^2}{r^2}$$

$r = x_0$

$V = r\omega$

$V_p = x_0 \omega$

$$a_p = \frac{V_p^2}{x_0}$$

$$a_p = \frac{(x_0 \omega)^2}{x_0}$$

$$a_p = \frac{x^2 \omega^2}{x_0}$$

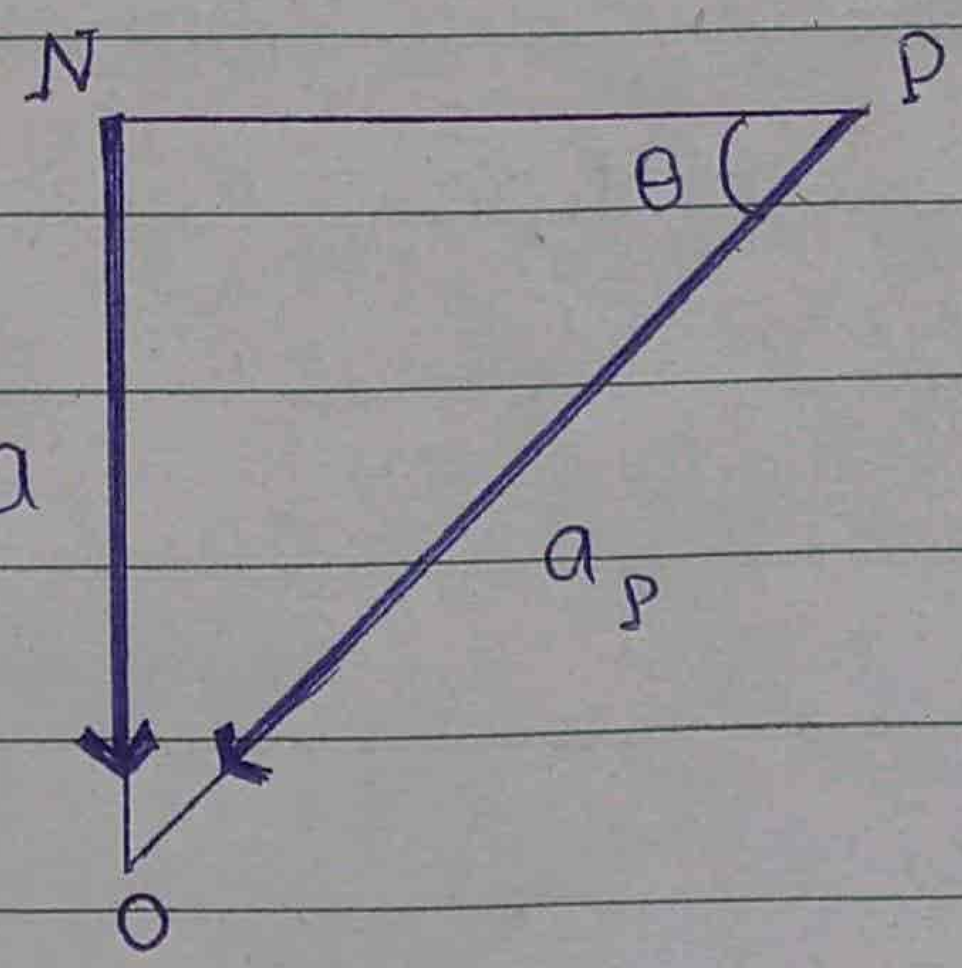
$$a_p = x_0 \omega^2$$

In ΔONP

$$\frac{a}{a_p} = \sin\theta$$

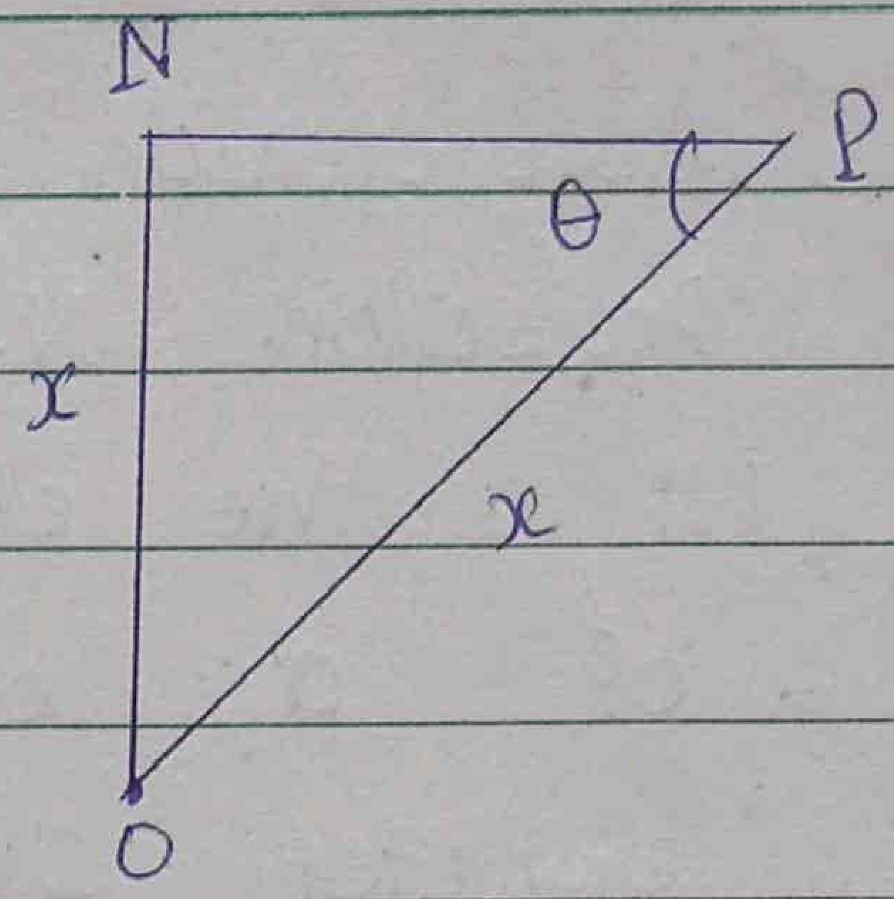
$$a = a_p \sin\theta$$

$$a = x_0 \omega^2 \sin\theta$$



In ΔONP

$$\sin\theta = \frac{x}{x_o}$$



$$a = x_o \omega^2 \left(\frac{x}{x_o}\right)$$

$$a = \omega^2 x$$

As the direction of a and x are opposite.

$$\boxed{a = -\omega^2 x}$$

As ω = constant

$$a = -(\text{constant})\, x$$

$$a \propto -x$$

Result:

" So, the acceleration of the projection N is directly proportional to displacement, -ve sign shows that direction of acceleration is towards the mean poistion, So N performs simple Harmonic Motion :-"

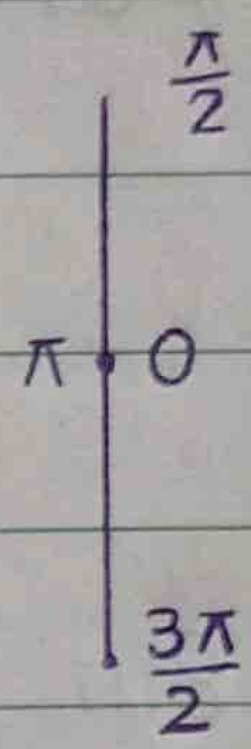
7.3 Phase

Definition

"Angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point performing SHM is known as phase".

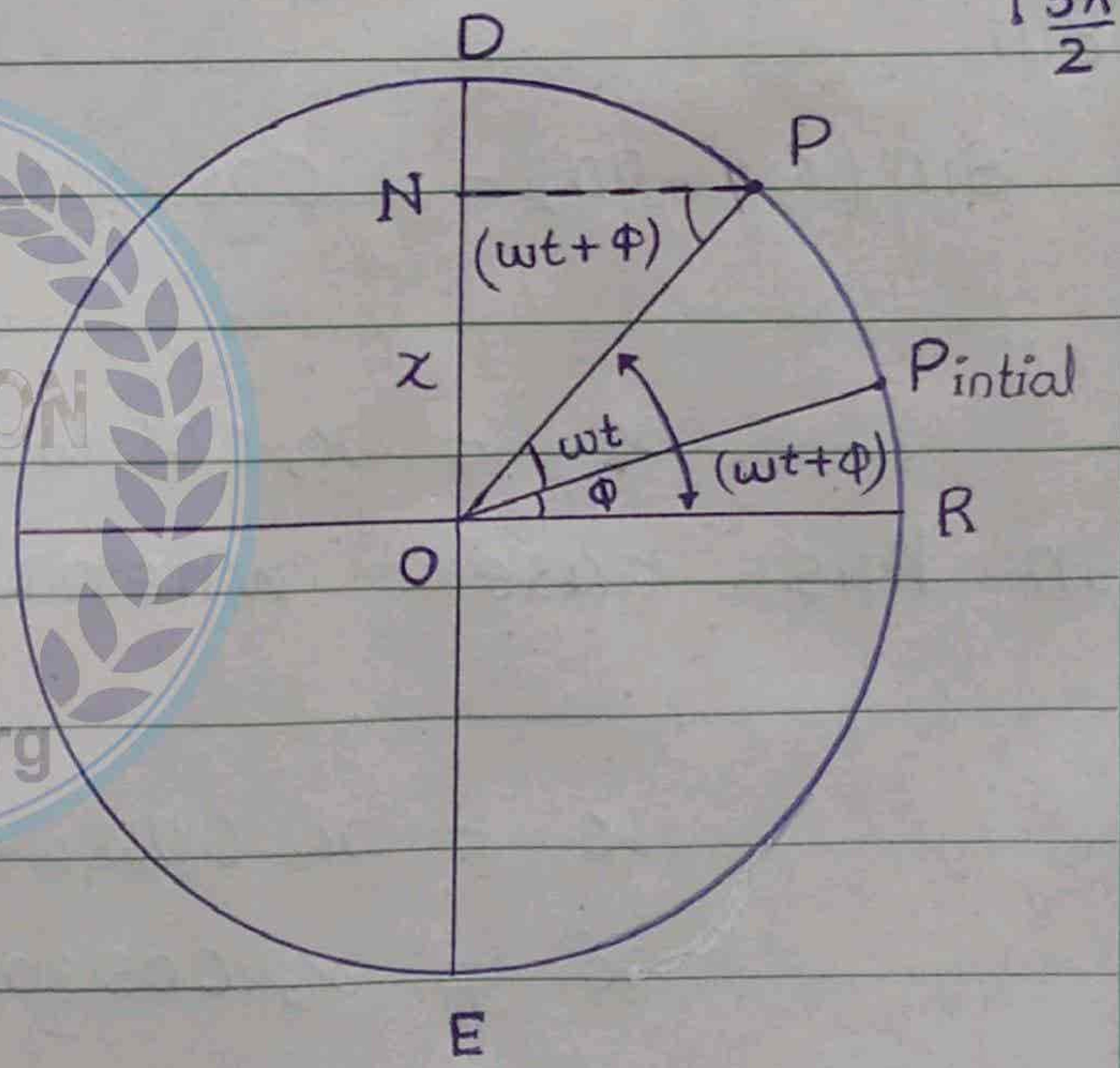
"Phase determines the state of motion of the vibrating point or body".

Consider a point "P" is moving in a circle with constant angular speed ω.



Let

At a time "t" = 0 "OP" makes angle "ϕ" with "OR" after time "t" "OP" makes angle $(\omega t + \phi)$ with "OR"



The displacement "x" = ON at any time "t" is given as.

$$ON = x = OP \sin(\omega t + \phi)$$

or ΔONP $\frac{x}{x_o} = \sin(\omega t + \phi)$

$$x = x_o \sin(\omega t + \phi)$$

Here,

phase angle is

$$\theta = \omega t + \phi$$

At time $t = 0$

$$\theta = \omega(0) + \phi$$

$$\theta = \phi$$

$$\phi = \text{initial phase}$$

Special case

Let us take initial phase

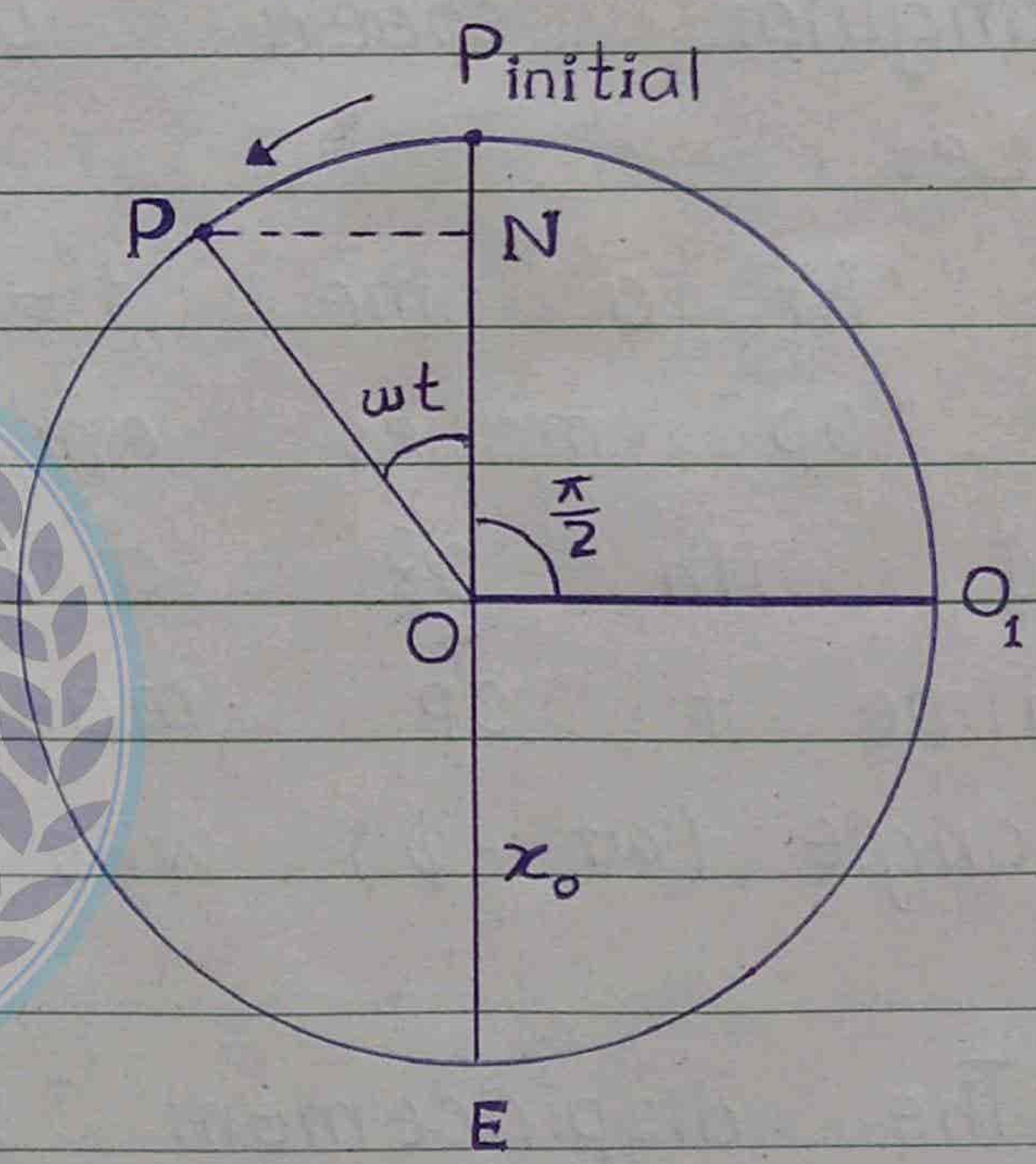
$$\phi = 90\ (\tfrac{\pi}{2})$$

$$x = x_0 \sin(\omega t + 90)$$

But

$$\sin(\omega t + 90) = \cos \omega t$$

$$x = x_0 \cos \omega t$$



In this case when $t = 0$

$$x = x_0 \cos \omega (0)$$

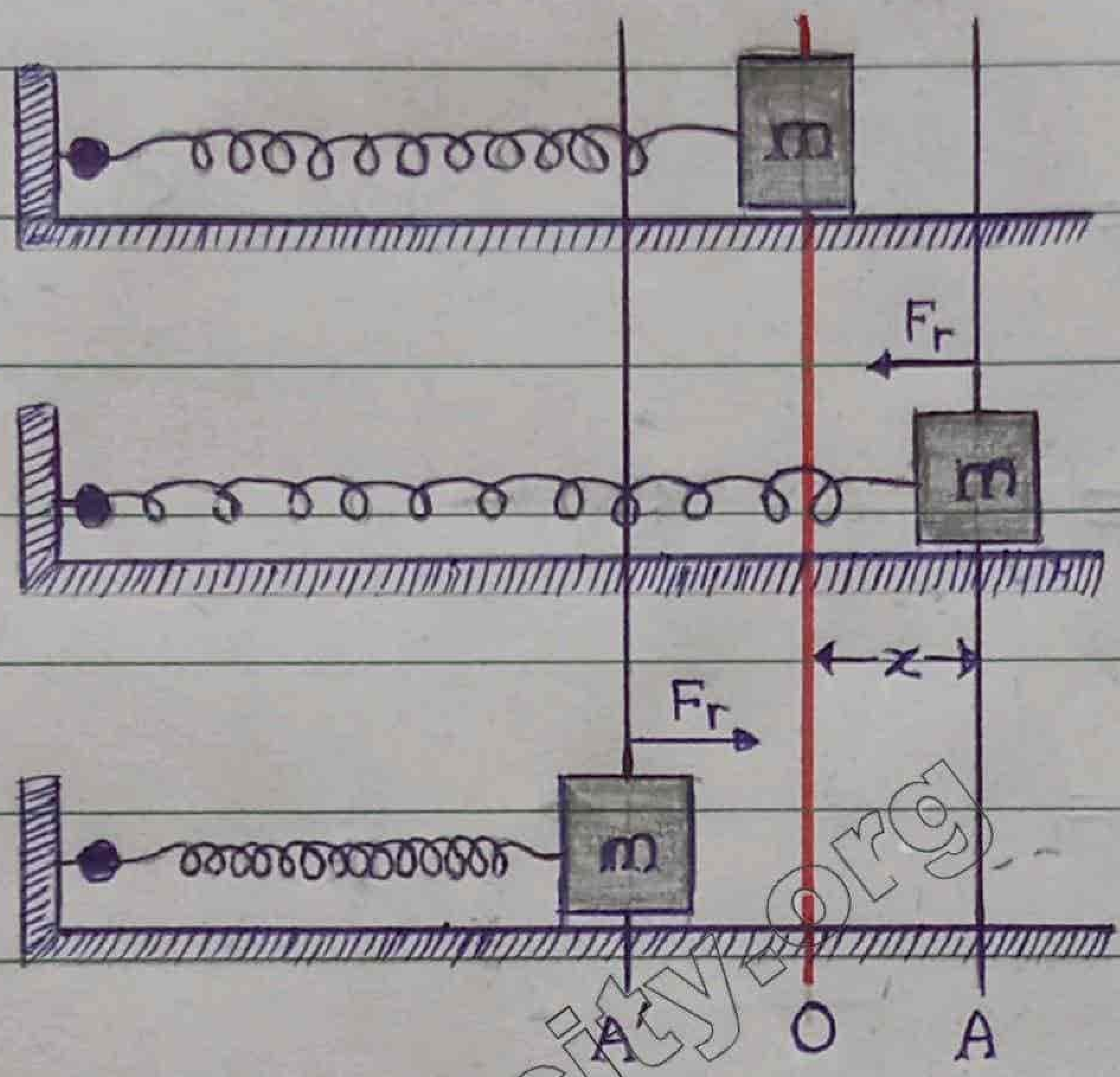
$$x = x_0 \cos(0)$$

$$x = x_0$$

So "N" starts its motion from extreme position "D" instead of mean position.

7-4 A Horizontal Mass Spring System

Consider a mass "m" attached to a spring and vibrating horizontally on friction less surface.



Acceleration at any instant

Restoring force is

$$F_r = -kx \qquad \because F = ma$$

$$ma = -kx$$

$$a = -\frac{k}{m}x \qquad \because \frac{k}{m} = \text{constant}$$

$$a = -(\text{constant})\,x$$

$$a \propto -x$$

Result

This shows that acceleration

is proportional to displacement "X" and its direction is towards the mean position. So mass "m" perform SHM.

Angular frequency

$$a = -\frac{K}{m}$$

$$a = -\omega^2 X \quad \text{for SHM}$$

comparing

$$-\frac{K}{m}X = -\omega^2 X$$

$$\frac{K}{m} = \omega^2$$

$$\omega = \sqrt{\frac{K}{m}}$$

Time period

As

$$\omega = \frac{2\pi}{T}$$

$\because \omega = \frac{\theta}{t}$

If $\theta = 2\pi$

$t = T$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{K}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$\therefore T \propto m$

$T \propto \frac{1}{\sqrt{K}}$

Frequency

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{m}{k}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Instantaneous displacement

As $\quad X = X_o \sin \omega t$

$\because \omega = \sqrt{\frac{k}{m}}$

$$X = X_o \sin\sqrt{\frac{k}{m}}\, t$$

Instantaneous velocity

As $\quad V = \omega\sqrt{X_o^2 - X^2} = \sqrt{\frac{k}{m}}\sqrt{X_o^2 - X^2}$

$$V = \sqrt{\frac{k}{m}}\sqrt{X_o^2 - X_o^2\frac{X^2}{X_o^2}}$$

$$V = \sqrt{\frac{k}{m}}\sqrt{X^2\left(1 - \frac{X_o^2}{X_o^2}\right)}$$

$$V = X_o\sqrt{\frac{k}{m}}\sqrt{1 - \frac{X_o^2}{X_o^2}} \longrightarrow (1)$$

Maximum velocity

Velocity is maximum at mean position $X = 0$, $V = V_o$

$$V = X_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{X_o^2}{X_o^2}} = X_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{0}{X_o^2}}$$

$$V_o = X_o\sqrt{\frac{k}{m}}\sqrt{1-0}$$

$$V_o = X_o\sqrt{\frac{k}{m}} \longrightarrow (2)$$

Put the value equation (2) in equation (1) we get.

$$V = V_o\sqrt{1-\frac{X_o^2}{X_o^2}}$$

Minimum velocity

Velocity is minimum at extreme position $X_o = X$

$$V = X_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{X^2}{X_o^2}} = X_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{X_o^2}{X_o^2}}$$

$$V = X_o\sqrt{\frac{k}{m}}\sqrt{1-1}$$

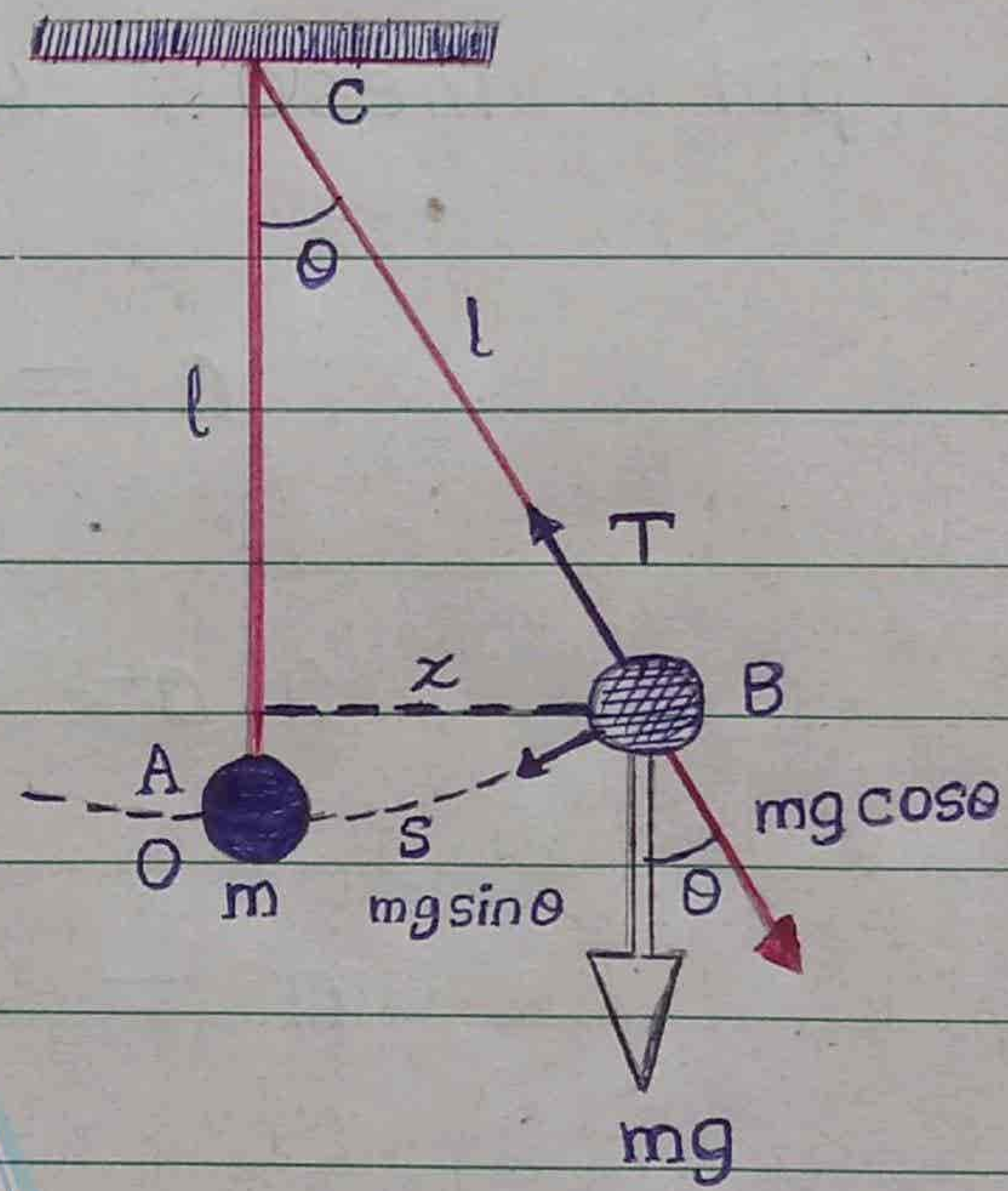
$$V_{min} = 0$$

7.5 Simple Pendulum

Definition:

"A small heavy mass bob attached to a light string which is called a simple pendulum".

When the bob is displaced from its mean position through a small angle "θ" and then released it starts vibrating about the mean position.



Resolve weight into two rectangular components.

* ; $mg\cos\theta$ parallel to string.
* ; $mg\sin\theta$ perpendicular to string.

$$T = mg\cos\theta.$$

Tension "T" is balanced by $mg\cos\theta$. Due to the force $mg\sin\theta$ simple pendulum vibrates. Restoring force acting on the bob is $mg\sin\theta$.

$$F = -mg\sin\theta \qquad \because F = ma$$

$$\cancel{m}a = -\cancel{m}g\sin\theta$$

$$a = -g \sin\theta$$

If θ is small $\sin\theta \approx \theta$, θ in radians

$$a = -g\theta \longrightarrow (1)$$

As

$$S \quad r\theta$$

$$\theta = \frac{s}{r} = \frac{Arc\ AB}{l}$$

$$\theta = \frac{X}{l}$$

θ = small

Arc AB = X

put these values in equation.

$$a = -g\frac{X}{l}$$

$$a = -X\frac{g}{l}$$

As $\frac{g}{l}$ = constant

$$a = -(\text{constant})\,X$$

$$a \propto -X$$

Result

" This shows that acceleration "a" of the bob is directly proportional to the displacement " X " and the negative sign shows that the direction of acceleration is towards the mean position "

Time period

As $a = -\frac{g}{l}X \longrightarrow (3)$

$$a = -\omega^2 X \longrightarrow (4)$$

comparin equation (3) and (4)

$$-\omega^2 X = -\frac{g}{l} X$$

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

Angular frequency of simple pendulum.

As

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{l}}}$$

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \because T \propto \sqrt{l}$$

If length is doubled "T" becom $\sqrt{2}$.

Dependance

Time period depends on the length "l" and "g".

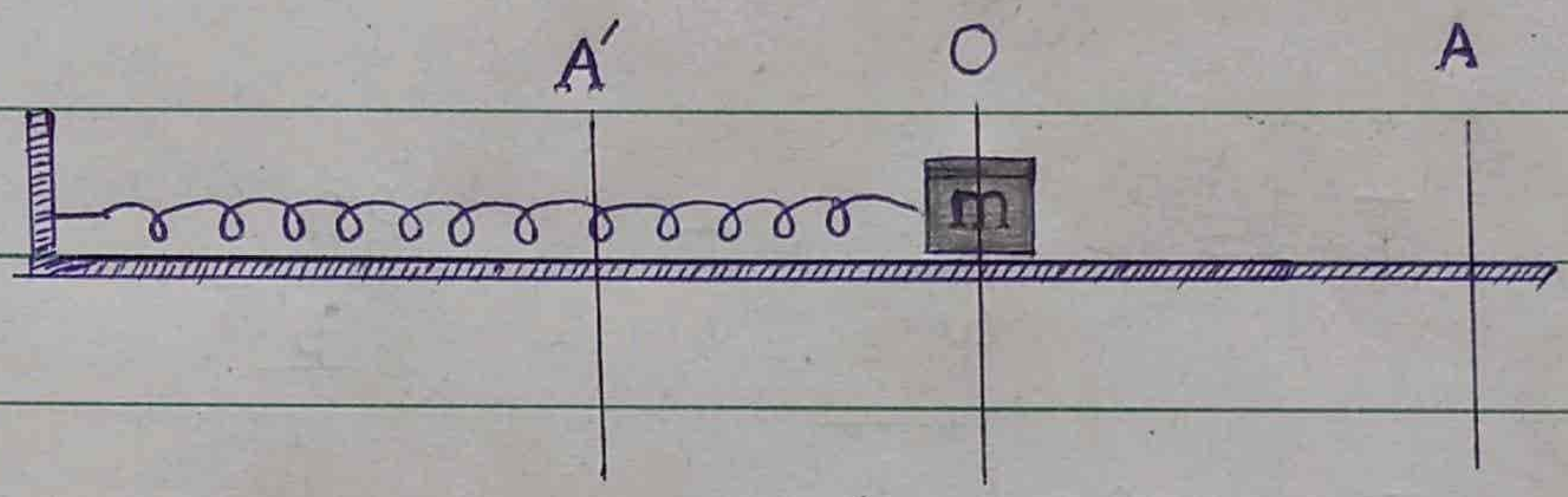
Time period does not depend on the mass of the bob "m" and on the amplitude of motion X_0.

Frequency

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

7.6 Energy Conservation in SHM

Consider a mass spring system. When the mass "m" is pulled slowly, the spring is stretched by x_o aginst the restoring force.



$x = x_o$	$x = 0$	$x = x_o$
$v = 0$	$v = max$	$v = 0$
$a = max$	$a = 0$	$a = max$
$P.E = max$	$P.E = 0$	$P.E = max$
$K.E = 0$	$K.E = max$	$K.E = 0$

The work done in pulling the mass is stored in it the form of elastic P.E. By Hooke's Law

$$F = Kx$$

when $x = 0$ $\quad F = K(0)$

$x = x_o$ $\quad F = Kx_o$

Average force, $F = \dfrac{0 + Kx_o}{2}$

$$F = \frac{1}{2} Kx_o$$

$$W = Fd$$

$$W = \frac{1}{2} K X_o . X_o$$

$$W = \frac{1}{2} K X_o^2$$

Elastic potential energy P.E

$$PE_{max} = \frac{1}{2} K X_o^2$$

It is maximum at extreme position $x = x_o$
This is maximum PE.

Minimum PE

PE is minimum zero at mean position $X = 0$

$$PE = \frac{1}{2} K (0)^2$$

$$PE = 0$$

Potential energy at any instant

When displacement $= X$ general formula for PE is.

$$\boxed{PE = \frac{1}{2} K X^2}$$

Kinetic energy at any instant

$$KE = \frac{1}{2} m v^2$$

put the value of V.

$$KE = \frac{1}{2} m \left[x_0 \sqrt{\frac{k}{m}} \sqrt{1 - \frac{x^2}{x_0^2}} \right]^2$$

$$KE = \frac{1}{2} \cancel{m} x_0^2 \left(\frac{k}{\cancel{m}} \right) \left(1 - \frac{x^2}{x_0^2} \right)$$

$$KE = \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right) \longrightarrow (A)$$

This is general formula for KE at any displacement X.

Maximum KE

KE is maximum at mean position $X = 0$. Put $X = 0$ in equation (A) then we get

$$KE = \frac{1}{2} k x_0^2$$

Minimum KE

KE is minimum at extreme position $X = x_0$. Put $X = x_0$ in equation (A) we get.

$$KE_{min} = 0$$

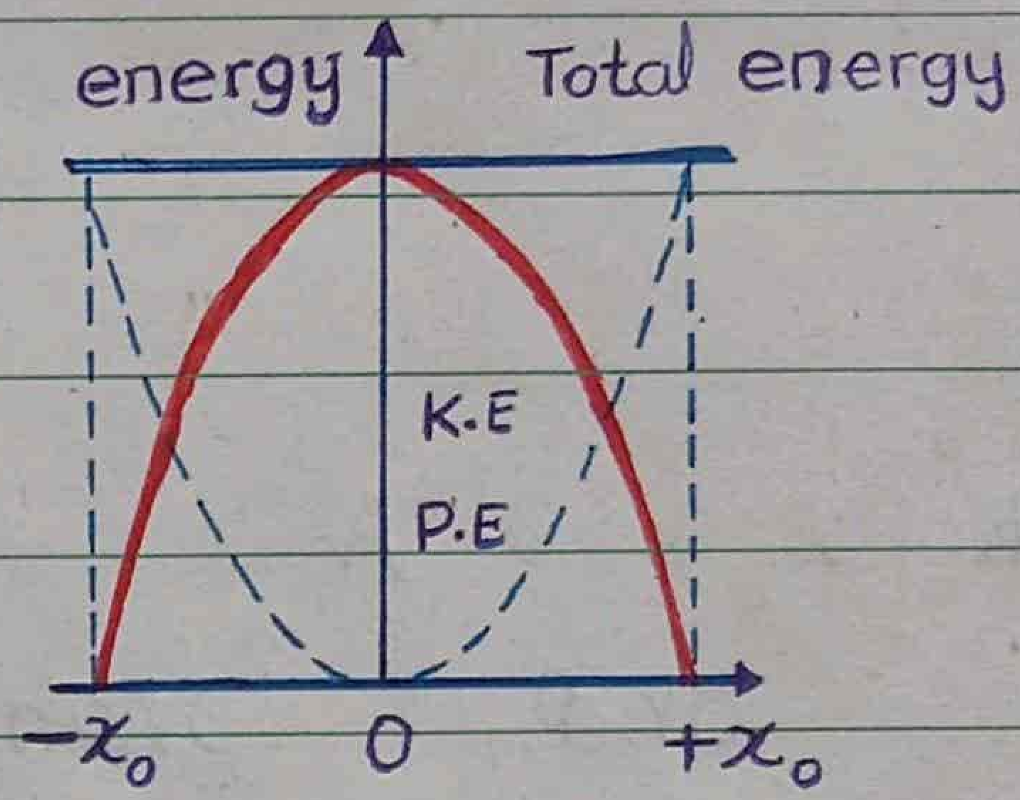
Total energy

$$E_T = PE + KE$$

$$E_T = \frac{1}{2}KX^2 + \frac{1}{2}KX_o^2 \left[1 - \frac{X^2}{X_o^2}\right]$$

$$E_T = \frac{1}{2}KX^2 + \frac{1}{2}KX_o^2 + \frac{1}{2}KX_o^2$$

$$E_T = \frac{1}{2}KX_o^2$$



- Total energy of mass spring system is constant.
- Interchange of "PE" and "KE" occurs continuously as the mass spring system vibrates.
- The exchang of "PE" and "KE" is essential for maintaing oscillations.

7.7 Free and Forced Oscillation

Free Oscillations (OR Vibrations)

Definition

"When a body is oscillating

with its natural frequency without interference of an external force then its oscillations are called free oscillations."

Examples

- ; A simple pendulum of length l is vibrating freely. Its frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

- ; A tuning fork vibrating freely and producing sound waves.
- ; A mass spring system vibrating freely with its natural frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Forced Oscillations

"When a body oscillates under the action of an external force its oscillations are called forced oscillation".

Examples

- ; The vibrations of a factory floor due to the runing of heavy machinery.

* ; The oscillations produced in the parts of runing machinery and in electrical circuits.
* ; Oscillations produced in a swing due to the childs ocassional pushes on the swing.
* ; Oscillations produced in a simple pendulum when bob is struck repeatedly.

Driven harmonic oscillator

"A physical system undergoing forced vibrations is called driven harmonic oscillator."

7.8 Resonance

Definition

"Resonance occurs when the frequency of the applied force is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator."

Or

"The marked increase in amplitude

of a vibrating body under the periodic force whore period is equal to the natural period called resonance."

For example if we blow to puch the pendulum, whenever it comes in front of out mouth, then its amplitude of vibration increases.

Experiment

A horizontal rod "AB" is supported by two strings S_1 and S_2

Three pairs of pendulums aa', bb', cc' are suspended from this rod. Each pair has the same lengths.

Suppose the pendulum "c" is displaced in a direction perpendicular to the plane of paper due to the motion of pendulum "c" the rod "AB" also has small disturbing motion. The time period of "C" is same as that of "C". Hence "C" also starts vibrating with increasing amplitude.

The amplitude of other pendulums remain very small because their lengths and hence their time periods are not the same as that of the disturbing force of the rod "AB".

Result :

"A very small or feeble periodic force produces a large effect."

Applications of resonance

* Resonance can be used to determine the frequency of a given body.
* It is used to find natural frequencies of the different bodies.
* It is used to determine the speed of sound with resonance tube apparatus.
* Mechanical and electrical systems show a good response under

phenomenon of resonance.

Examples of resonance

- ; Swing.
- ; Tuning of a radio set.
- ; March of soldiers on the bridge.
- ; Cooking of food by microwave oven.

7-9 Damped Oscillations

Definition

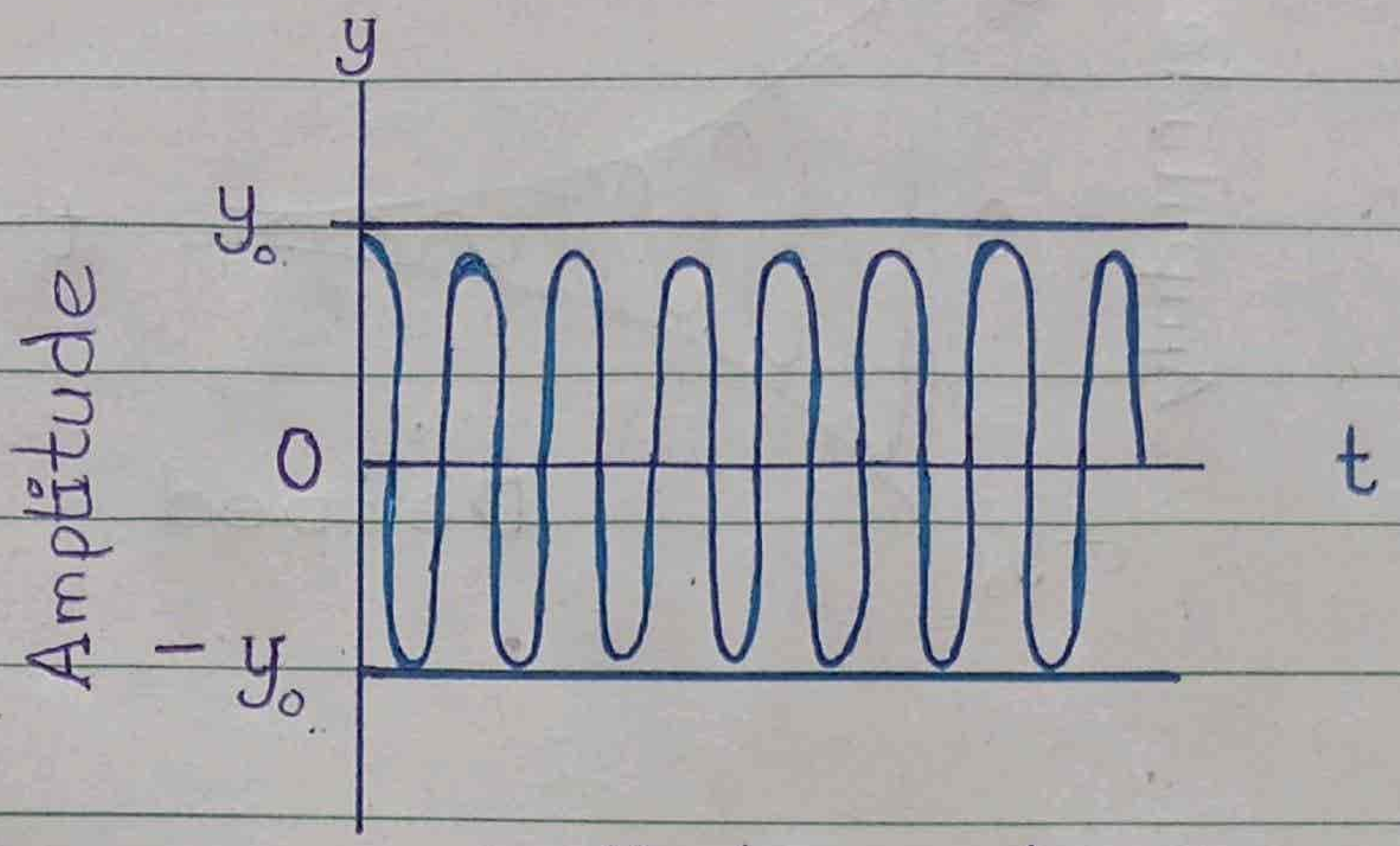
"The oscillations in which the amplitude decreases steadily with time are called damped oscillations".

Damping

"Such a process in which energy is dissipated from the oscillating system is called damed oscillation".

Explanation

Amplitude of vibrations continuously decreases due to damping produced by the air friction. At last pendulum stop vibrating.



Undamped

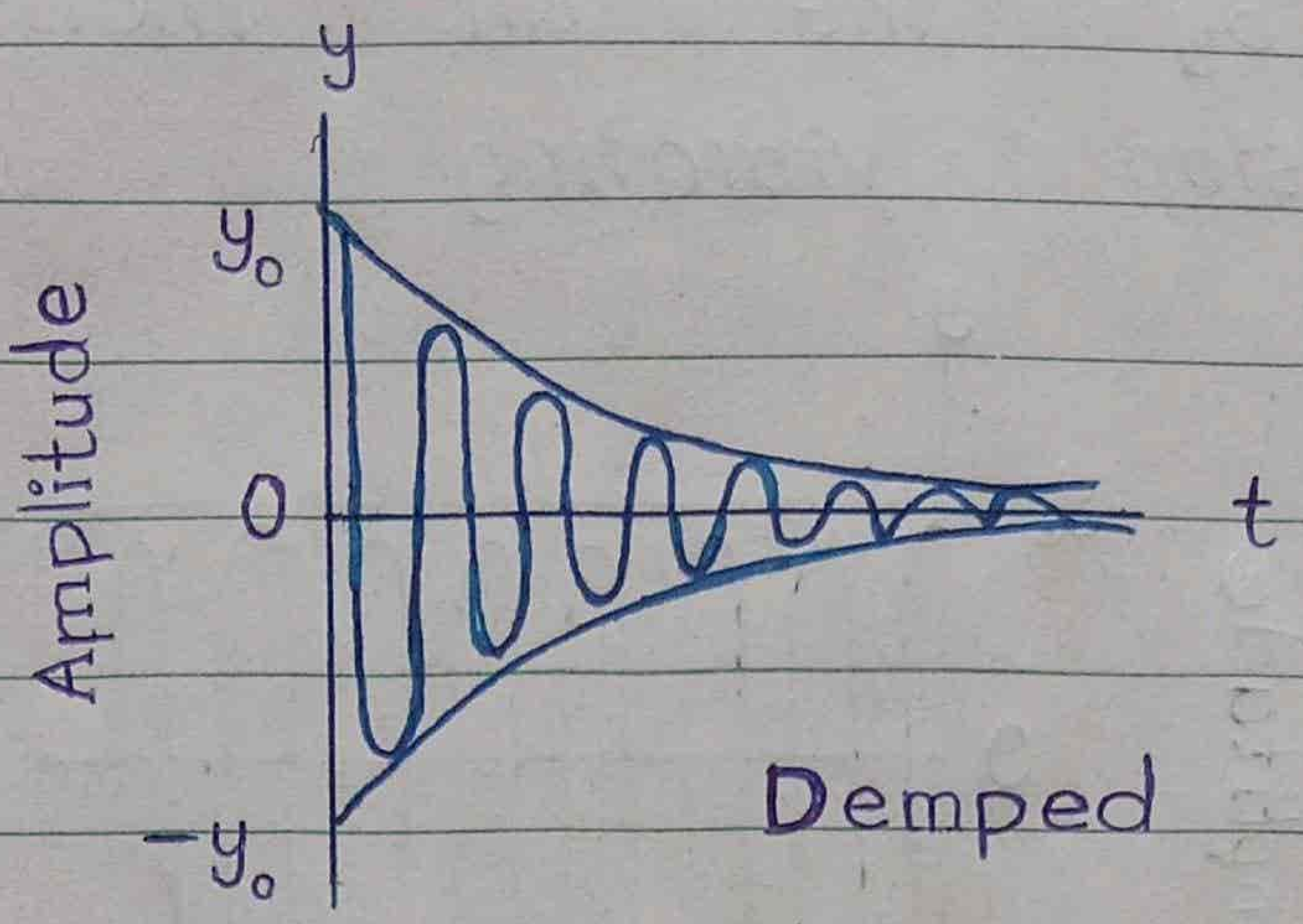
Vibrations of a mass spring system decreases with time gradually due to frictional forces.

Applications

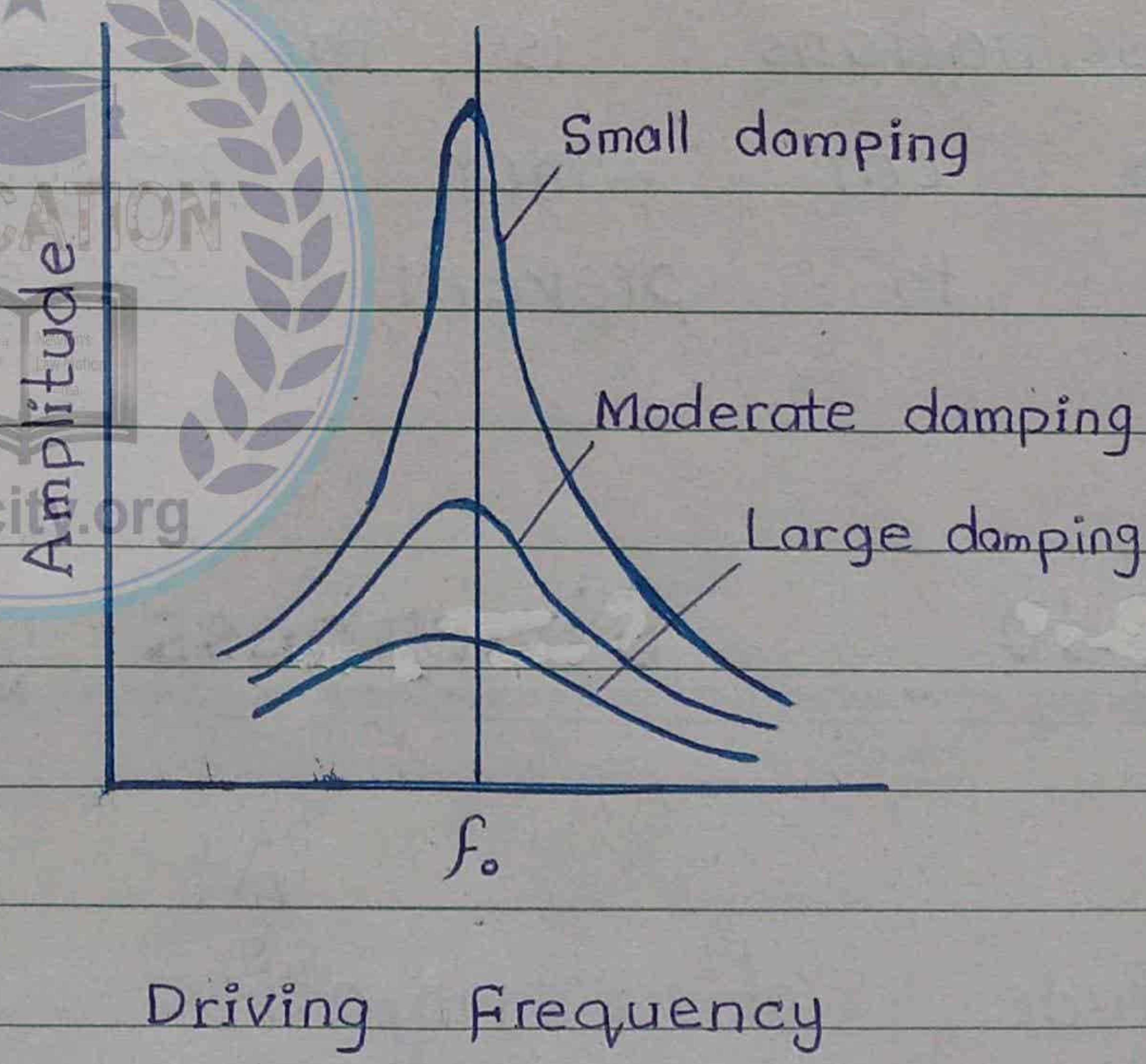
Application of damped oscillations is the Shock Absorber of a car which provides a damping force to prevent excessive oscillations.

7.10 Sharpness of Resonance

At resonance the amplitude of vibration becomes larg when damping is small. Thus damping prevents the amplitude from becoming excessively large.



The amplitude decreases rapidly at a frequency slightly different from the resonant frequancy f_o. A heavily damped system has a fairly flat resonance curve as shown in amplitude frequency graph.



Questions

7.1 Name two characteristics of simple harmonic motion.

7.2 Does frequency depends on amplitude for harmonic oscillators?

7.3 Can we realize an ideal simple pendulum?

7.4 What is the total distance travelled by an object moving with SHM in a time equal to its period, if its amplitude is A?

7.5 What happens to the period of a simple pendulum if its length is doubled? What happens if the suspended mass is doubled? imp

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force? imp 2013 imp

7.8 Under what conditions does the addition of two simple harmonic motions produce a resultant, which is also simple harmonic?

7.9 Show that in SHM the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

7.10 In relation to SHM, explain the equations;

(i) $y = A \sin(\omega t + \varphi)$

(ii) $\mathbf{a} = -\omega^2 \mathbf{x}$

7.11 Explain the relation between total energy, potential energy and kinetic energy for a body oscillating with SHM.

7.12 Describe some common phenomena in which resonance plays an important role.

7.13 If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Chapter 07

Questions

Question 7.1

Answer

$$a \propto -x$$

1- Acceleration is directly proportional to the displacement.

2- Direction of acceleration is always towards the mean position.

Question 7.2

Answer

Frequency of Harmonic Oscillators such as

(i) Mass - Spring System

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

(ii) Simple Pendulm is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

It is clear that frequency does not depend on the amplitude x_o of the oscillator. Frequency depends on other factors.

Question 7.3

Answer

No, we cannot realize a simple pendulum.

An ideal simple pendulum has

(i) a heavy, very small size mass "bob" (point mass)

(ii) a weightless and inextensible string

These conditions cannot be 100% satisfied. So, we cannot realize an ideal simple pendulum.

Question 7.4

Answer

If A = Amplitude, then total distance travelled by the vibrating object in one time period is 4 A

Question 7.5

Answer

Time period of simple pendulum is

(i) $T = 2\pi\sqrt{\frac{l}{g}}$

if $l' = 2l$

$$T' = 2\pi\sqrt{\frac{l'}{g}} = 2\pi\sqrt{\frac{2l}{g}}$$

$$T' = \sqrt{2}\left[2\pi\sqrt{\frac{l}{g}}\right]$$

$$T' = \sqrt{2}\,T$$

$$= 1.414\,T$$

So, Time period becomes $\sqrt{2}$ times or 1.414 times.

(ii) From the formula, it is clear that "T" does not depend on the mass "m" of simple pendulum.

Question 7.6

Answer

Acceleration of Simple Harmonic Oscillatoris

(i) $a = \omega^2 x$ or $a \propto -x$

x = displacement

During SHM displacement, x changes, so acceleration also changes. It does not remain constant.

(ii)

At mean position $x = 0$
So acceleration $a = 0$

At extreme position $(x = x_o)$, acceleration is Max

$$a_{max} = \omega^2 x_o$$

Question 7.7

Answer

Phase Angle : $\theta = \omega t$

(i) " Phase angle specifies the displacement as well as the direction of motion of the point performing simple harmonic motion."

" Phase angle gives the state

of the vibrating body."

ii) It does not define angle between maximum displacement and the driving force.

Question 7.8

Answer

Simple Harmonic motions must

i) have same frequency.

ii) have same initial phase

iii) be parallel

Question 7.9

Answer

Acceleration 'a' and velocity 'v' of SHM are

$$a = -\omega^2 x$$

$$v = \omega\sqrt{x_0^2 - x^2}$$

At mean Position

$x = 0$

$$a = -\omega^2(0)$$

$$= 0$$

$$a = \text{Minimum}$$

$$V = \omega \sqrt{x_o^2 - 0}$$

$$V = \omega x_o$$

$$V = \text{Maximum}$$

Question 7.10

Answer

i) $y = A \sin(\omega t + \phi)$

y = Intantaneous displacement of the body performing SHM.

A = Amplitude

ϕ = Initial Phase

$(\omega t + \phi)$ = Phase angle.

ii) $a = -\omega^2 x$

a = Acceleration of the body performing S.H.M.

ω = Angular frequency

x = Instantaneous displacement

The negative sign shows that the direction of acceleration is always towards the mean position.

Question 7.11

Answer

When frictional forces are not present, the total energy of the body remains constanst. However K.E and P.E are interchanged

$$E_{total} = K.E + P.E = \text{Constant}$$

Question 7.12

Answer

Resonanceplay important roll in the following

i) Tuning of a radio.

ii) Musical instruments

iii) Microwave Oven.

Question 7.13

Answer

Oscillations of the mass - spring system stop due to the presence of frictional force or damping forces.

These resistive forces dissipate energy. So, the motion eventually stops

NUMERICAL PROBLEMS

7.1 A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

(Ans:0.20 kg)

7.2 A load of 15.0g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

[Ans: (i) 1.26s, (ii) 7.35 Nm^{-1}, (iii) 49.0 cm s^{-1}]

7.3 An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find

(i) Period

(ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

[Ans: (i) 1.3 s, (ii) 3.0 ms^{-2}, 1.4 ms^{-1}, 7.6 J, 1.44 J]

7.4 A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant $k = 1960\ Nm^{-1}$, Find the maximum distance through which the spring will be compressed. imp

(Ans: 0.18 m)

7.5 A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where $g = 9.8\ ms^{-2}$? imp, 2012 2014

(Ans: 0.70 Hz)

7.6 A block of mass 1.6 kg is attached to a spring with spring constant $1000\ Nm^{-1}$, as shown in Fig. 7.14. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, $x = 0$, if the surface is frictionless.

(Ans: $0.50\ ms^{-1}$)

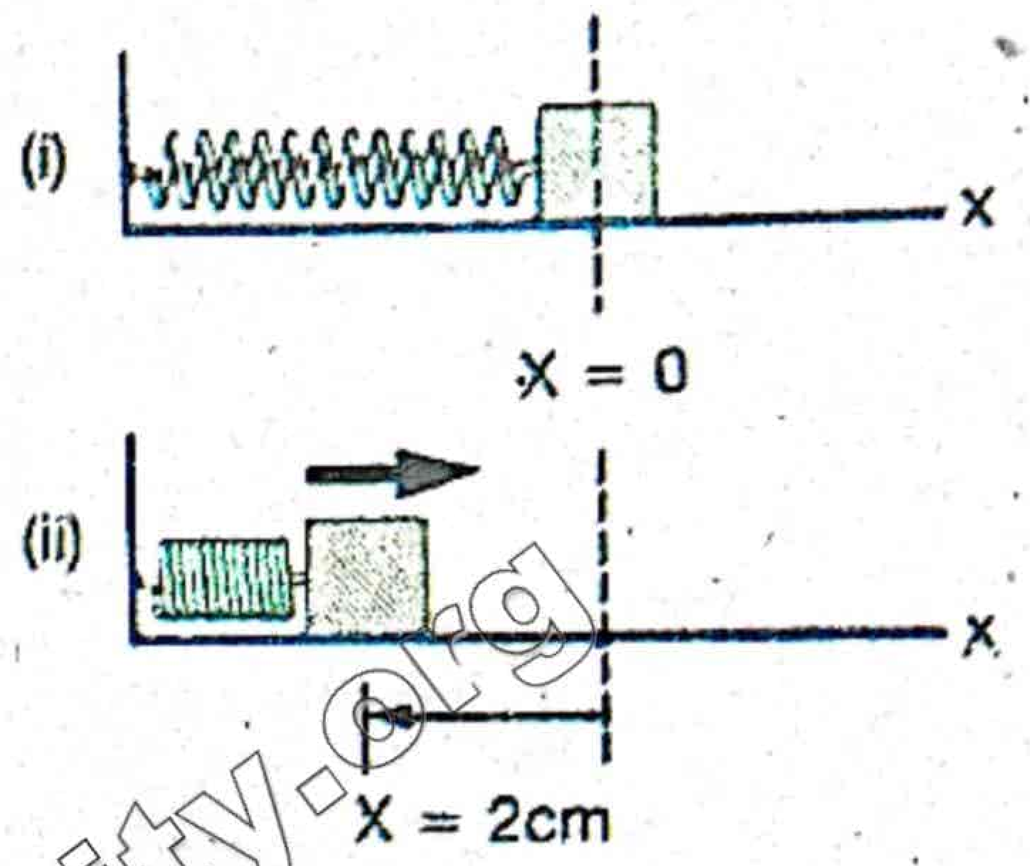


Fig. 7.14

7.7 A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant $20,000\ Nm^{-1}$. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

(Ans: 1.18 Hz)

7.8 Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is imp

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t$$

What is the displacement of the object after 2.0 s?

(Ans: 0.25 m, $\frac{1}{16}$ Hz, 16 s, $x = 0.18$ m)

Chapter 07

Problems

Problem 7.1

Solution

$$M = 100\,g$$

$$M = \frac{100}{100}\,kg$$

$$M = 0.1\,kg$$

$$x = 4.0\,cm$$

$$x = \frac{4}{100}\,m$$

$$x = 0.04\,m$$

$$\begin{cases} T = 0.568\,s \\ m = ? \end{cases}$$

$\because W = mg$

$$F = kx$$

$$k = \frac{F}{x}$$

$$K = \frac{W}{x}$$

$$K = \frac{Mg}{x}$$

$$K = \frac{0.1 \times 9.8}{0.04}$$

$$k = 24.5 \ Nm^{-1}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Squaring

$$T^2 = 4\pi^2 \frac{m}{k}$$

$$m = \frac{KT^2}{4\pi^2}$$

$$m = \frac{24.5 \times (0.568)^2}{4 \times (3.14)^2}$$

$$m = 0.2 \ kg$$

$$m = 0.2 \times 1000 \ g$$

$$\boxed{m = 200 \ g}$$

Problem 7.2

Solution

$M = 15.0\ g$

$M = \frac{15}{1000}\ kg$

$M = 0.015\ kg$

$x = 2.0\ cm$

$x = \frac{2}{100}\ m$

$x = 0.02\ m$

$m = 294 g$

$m = \frac{294}{1000}\ kg$

$m = 0.294\ kg$

$x_o = 10.0\ cm$

$x_o = \frac{10}{100}\ m$

$x_o = 0.1\ m$

i) $T = ?$

ii) $k = ?$

iii) $V_{max} = V_o = ?$

i)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2 \times 3.14 \sqrt{\frac{0.294}{7.35}}$$

$$\boxed{T = 1.26 \text{ s}}$$

ii)

$$F = kx \qquad \because W = mg$$

$$k = \frac{F}{x}$$

$$k = \frac{W}{x}$$

$$k = \frac{Mg}{x}$$

$$k = \frac{0.015 \times 9.8}{0.02}$$

$$\boxed{k = 7.35 \text{ Nm}^{-1}}$$

iii)

$$V_{max} = V_o$$

$$V_o = x_o \sqrt{\frac{k}{m}}$$

$$V_o = 0.1 \sqrt{\frac{7.35}{0.294}}$$

$V_o = 0.5\ ms^{-1}$

$V_o = 0.5 \times 100\ cm s^{-1}$

$$\boxed{V_o = 50.0\ cms^{-1}.}$$

Problem 7.3

Solution

$m = 8.0\ kg$

$x_o = 30\ cm$

$x_o = \frac{30}{100}\ m$

$x_o = 0.30\ m$

$F = 60\ N$

(i)

$T = ?$ when $x = 30\ cm$

$x = 0.30\ m$

(ii)

$a = ?$

$V = ?$

$\left.\begin{array}{l} KE = ? \\ PE = ? \end{array}\right\}$ when $x = 12cm$

$x = 0.12cm$

(i)

$F = kx$

$K = \frac{F}{x}$

$K = \frac{60}{0.3}$

$K = 200\ Nm^{-1}$

$T = 2\pi\sqrt{\frac{m}{K}}$

$T = 2 \times 3.14\sqrt{\frac{8}{200}}$

$\boxed{T = 1.256\ s.}$

(ii)

$a = -\frac{K}{m}x$

$a = -\frac{200}{8} \times 0.12$

$$a = -3.0\ ms^{-2}$$

Magnitude of 'a' is

$$\boxed{a = 3.0\ ms^{-2}}$$

iii)

$$V = \omega\sqrt{x_o^2 - x^2}$$

$$V = \frac{2\pi}{T}\sqrt{x^o - x^2}$$

$$V = \frac{2\times 3.14}{1.256}\sqrt{(0.3)^2 - (0.12)^2}$$

$$V = 1.37\ ms^{-1} = 1.4\ ms^{-1}$$

$$KE = \frac{1}{2}Kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$KE = \frac{1}{2}(200)(0.3)^2\left[1 - \frac{(0.12)^2}{(0.3)}\right]$$

$$\boxed{KE = 7.6\ J}$$

$$PE = \frac{1}{2}Kx^2$$

$$PE = \frac{1}{2}\times 200\ (0.12)^2$$

$$\boxed{PE = 1.44 \text{ J}}$$

Problem 7.4

Solution

$$PE_g = mgh$$

$m = 4.0 \text{ kg}$

$h = 0.80 \text{ m}$

$k = 1960 \text{ Nm}^{-1}$

$x_o = ?$

Loss of Gravitational PE = Gain in Elastic PE

$$mgh = \frac{1}{2} k x_o^2$$

$$x_o^2 = \frac{2mgh}{k}$$

$$x_o = \sqrt{\frac{2mgh}{k}}$$

$$x_o = \sqrt{\frac{2 \times 4 \times 9.8 \times 0.80}{1960}}$$

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$$x_o = 0.1788 \text{ m}$$

$$\boxed{x_o = 0.18 \text{ m}}$$

Problem 7.5

Solution

$$L = 50 \text{ cm}$$

$$L = \frac{50}{100} \text{ m}$$

$$L = 0.50 \text{ m}$$

$$g = 9.8 \text{ ms}^{-2}$$

$$f = ?$$

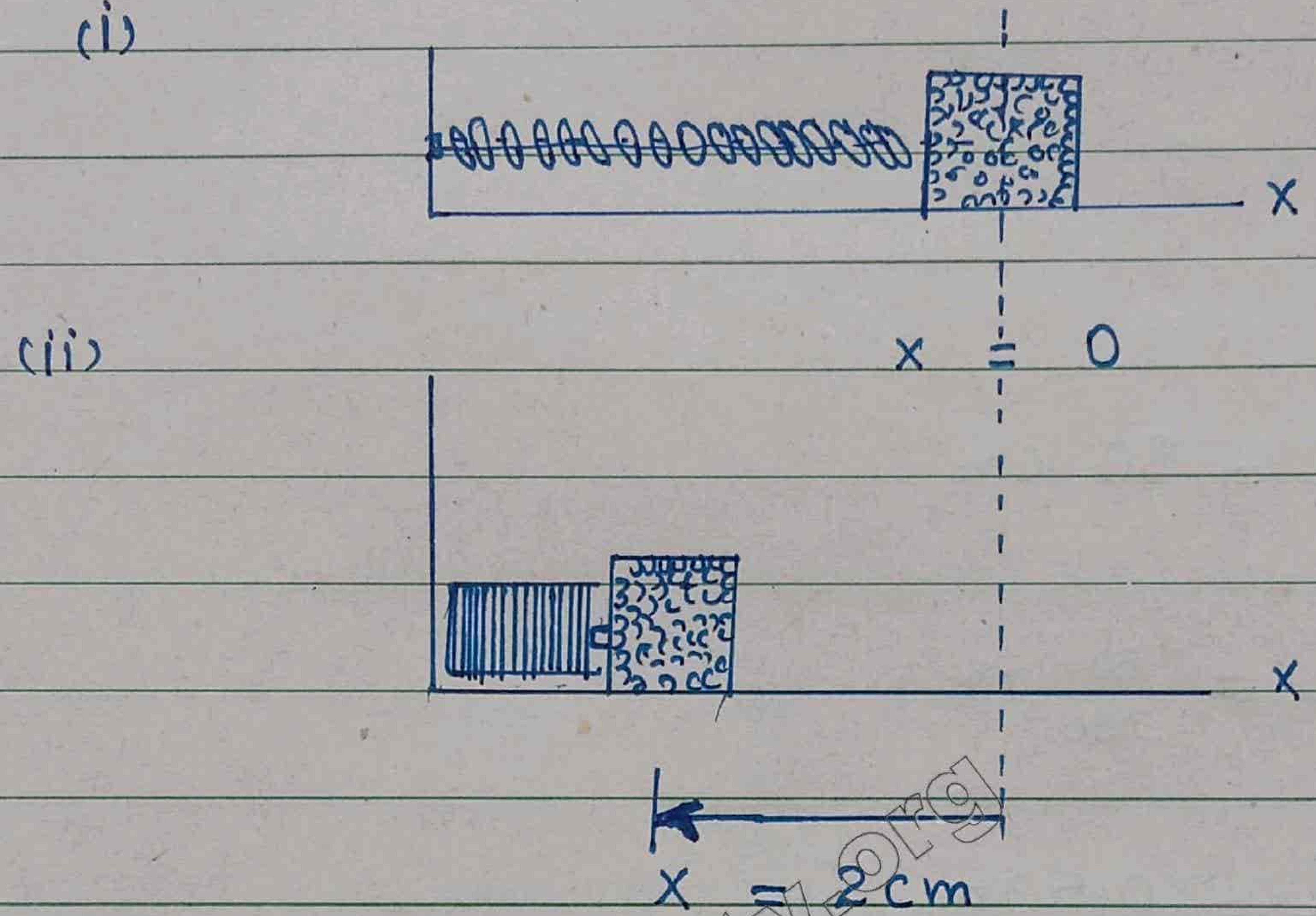
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2 \times 3.14}\sqrt{\frac{9.8}{0.50}}$$

$$\boxed{f = 0.71 \text{ Hz}}$$

Problem 7.6

Solution



V = ?

$m = 1.6\ kg$

$K = 1000\ N m^{-1}$

$x_o = 2cm$

$x_o = \frac{2}{100}\ m$

$x_o = 0.02\ m$

Velocity at equilibrium position is maximum $= V_0$

$$V_0 = ?$$

$$V_0 = x_0\sqrt{\frac{k}{m}}$$

$$V_0 = 0.02\sqrt{\frac{1000}{1.6}}$$

$$\boxed{V_0 = 0.5\ ms^{-1}}$$

Problem 7.7

Solution

$$m_1 = 1300\ kg$$

$$m_2 = 160\ kg$$

$$f = ?$$

For 4-springs

$$k = 4 \times 2000$$

$$k = 80000\ Nm^{-1}$$

Total mass $= m$

$$m = m_1 + m_2$$

$$m = 1300 + 160$$

$$m = 1460 \text{ kg}$$

As $$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2(3.14}\sqrt{\frac{80000}{1460}}$$

$$f = 1.178 \text{ Hz}$$

$$\boxed{f \simeq 1.18 \text{ Hz.}}$$

Problem 7.8

Solution

$f = ?$

$T = ?$ $x_o = ?$

$x = ?$ at $t = 2.0S$

$$x = x_o \cos \omega t \longrightarrow ①$$

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t \longrightarrow ②$$

Comparing eq ① and eq ②

$$\boxed{x_0 = 0.25 \text{ m}}$$

$$\omega = \frac{\pi}{8}$$

$$2\cancel{\pi} f = \frac{\cancel{\pi}}{8} \qquad \omega = 2\pi f$$

$$2f = \frac{1}{8}$$

$$\boxed{f = \frac{1}{16}}$$

As $f = \frac{1}{T}$

$$\frac{1}{16} = \frac{1}{T}$$

$$T = 16 \text{ s.}$$

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t \qquad \because t = 2\text{s}$$

$$x = 0.25 \cos\left(\frac{\pi}{\cancel{8}_4}\right)(\cancel{2})$$

$$x = 0.25 \cos \frac{\pi}{4}$$

$$x = 0.25 \cos 45^\circ$$

$$x = 0.25\ (0.707$$

$$x = 0.25\ (0.707)$$

$$\boxed{x = 0.18\ m}$$

Chapter 07

Examples

Example 7.1

Solution

$M = 4.0\text{ kg}$

$x = 0.16\text{ m}$

$T = ?$

$F = kx$

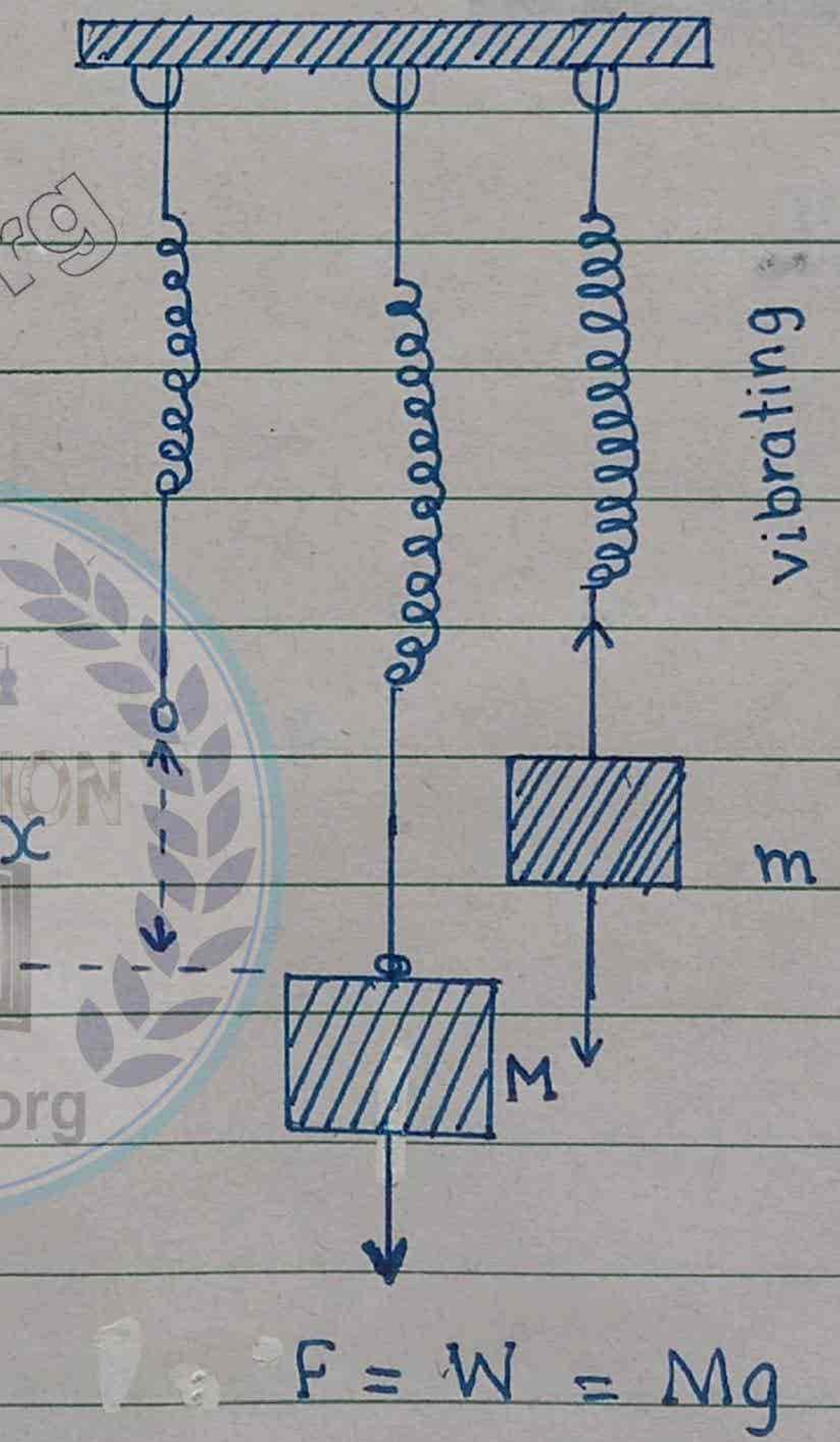
$k = \frac{F}{x}$

$k = \frac{W}{x}$

$k = \frac{Mg}{x}$

$k = \frac{4 \times 9.8}{0.16}$

$k = 245\text{ Nm}^{-1}$



As

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2 \times 3.14 \cdot \sqrt{\frac{0.5}{245}}$$

$$\boxed{T = 0.28\ s.}$$

Example 7.2

Solution

$T = 1s$

$g = 9.8\ ms^{-2}$

$l = ?$

$f = ?$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Squaring

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$L = \frac{gT^2}{4\pi^2}$$

$$L = \frac{9.8 \times (1)^2}{4 \times (3.14}$$

$$L = 0.25 \text{ m}$$

$$F = \frac{1}{T}$$

$$F = \frac{1}{1.0}$$

$$\boxed{F = 1 \text{ Hz}}$$

Example 7.3

Solution

$K = 80.0 \text{ N m}^{-1}$

$m = 1.0 \text{ kg}$

$V_{max} = V_o$

$= 1.0 \text{ ms}^{-1}$

$$V_o = x_o \sqrt{\frac{k}{m}}$$

$$x_o = V_o \sqrt{\frac{m}{k}}$$

$$x_o = 1.0\sqrt{\frac{1}{80}}$$

$$\boxed{x_o = 0.11 \text{ m}}$$