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Chapter - 4:

Work and Energy

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(4.1) Work done by a constant force

Work:

"The scalar product of force and displacement is called work."

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

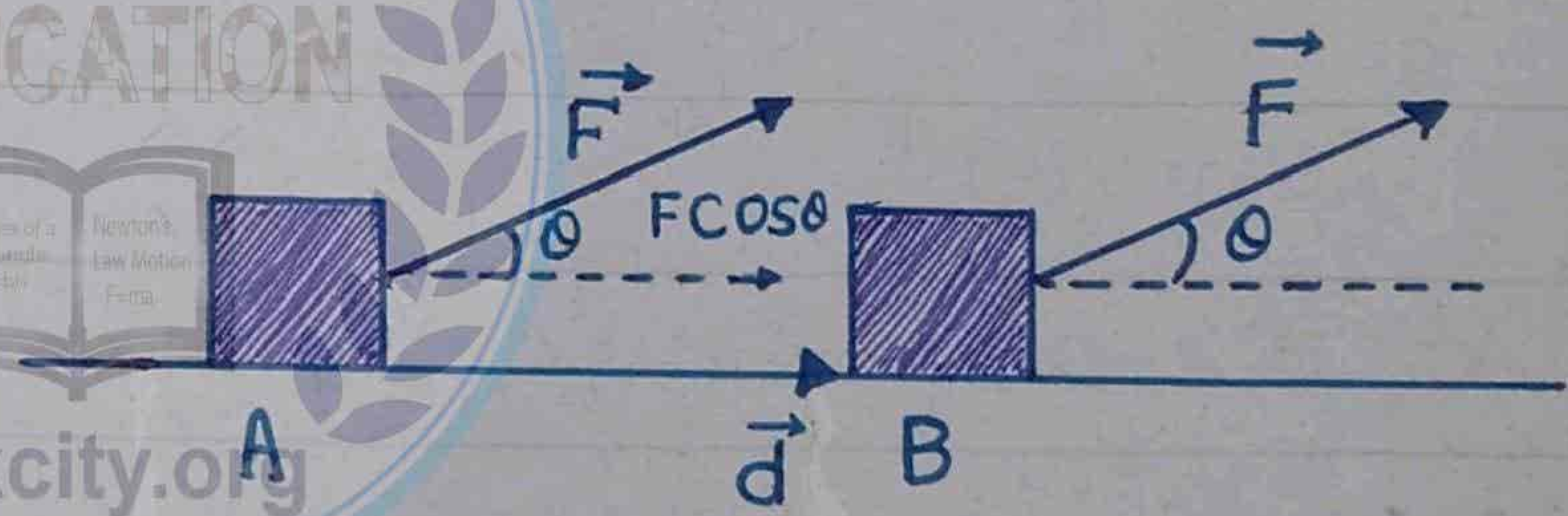
Explanation:

Consider a constant force $\vec{F}$ acts on a body. $\vec{F}$ makes an angle θ with the direction of displacement $\vec{d}$. The force moves the body from point "A" to "B".

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd\cos\theta$$

$$W = (F\cos\theta)(d)$$



W = (Component of force in the direction of displacement). (distance moved)

Definition:

"Work is the product of magnitude of component of force in the direction of displacement and the magnitude of displacement."

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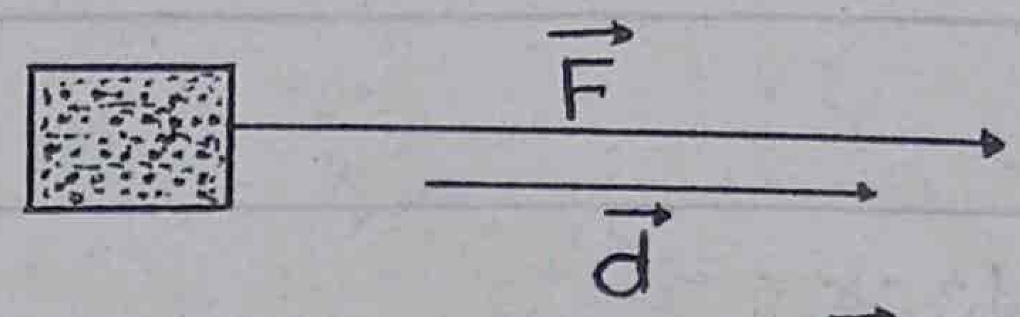
Work is a scalar quantity.

Positive Work:

If $\theta < 90°$, work is positive.

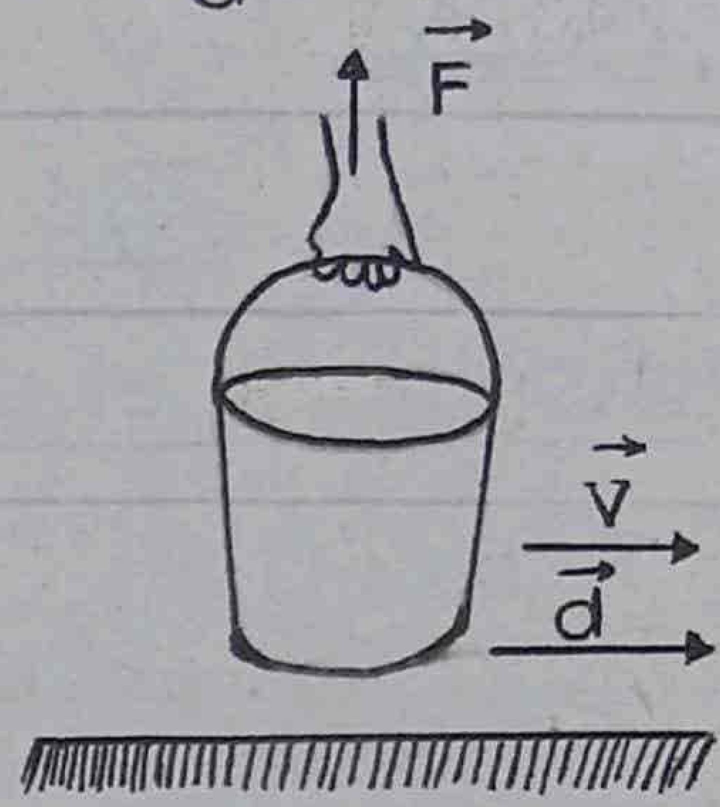
e.g. Let $\theta = 0°$



$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$W = Fd\cos 0°$

$W = Fd\,(1)$



$W = Fd$ (At $\theta = 0°$ Maximum work is done)

Zero Work:

If $\theta = 90°$, work is zero.

$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$W = Fd\cos 0° = 0$

$W = 0$

Example: "Work done by Centripetal Force is zero."

Negative Work:

If $\theta > 90°$, work is negative.

Let $\theta = 180°$

$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$= Fd\cos 180° = Fd(-1) = -Fd$

$W = -Fd$

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Example:

Work done by Frictional Force is negative. It act opposite to the direction of motion.

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Dimensions:

$$W = Fd \qquad \because F = ma$$

$$W = mad$$

$$W = M.LT^{-2}.L$$

$$[W] = [ML^2T^{-2}]$$

Unit: SI unit of work is **Nm** known as Joule(J).

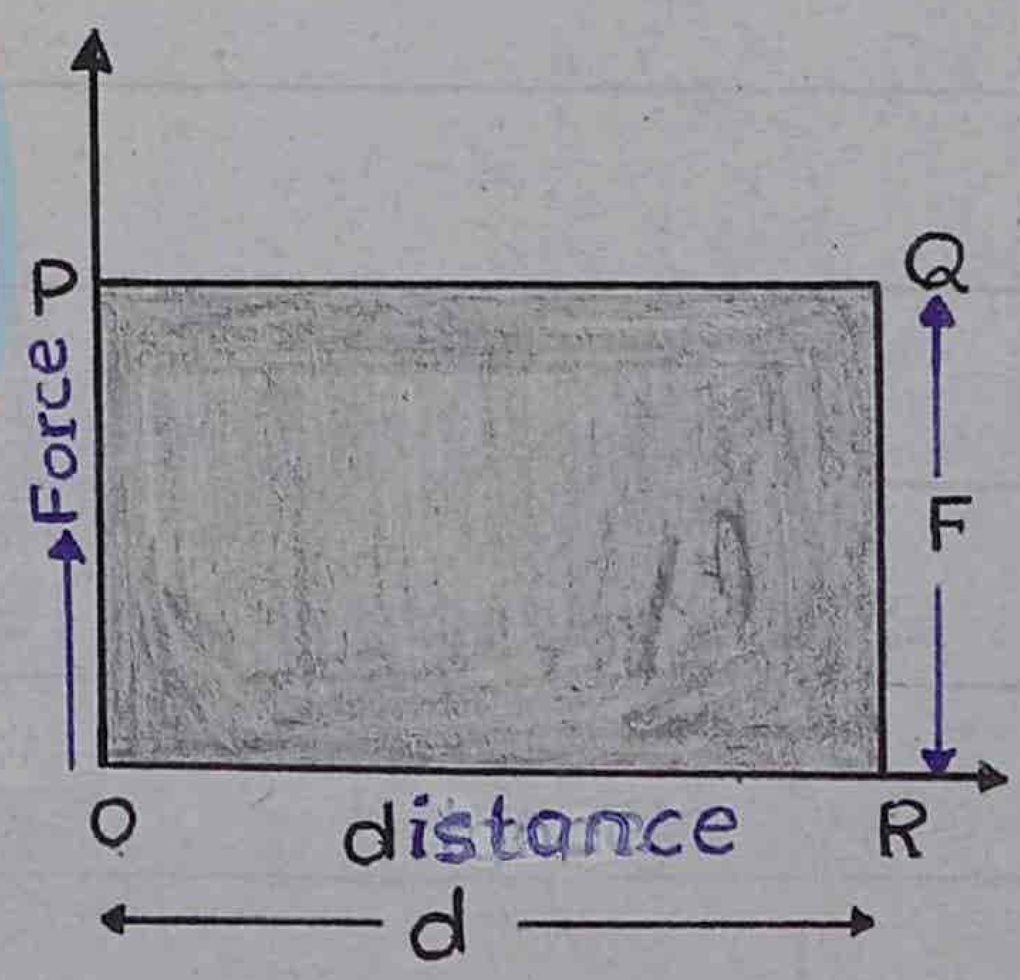
Joule:

"Work is one joule when a force of one newton displace (ہٹانا) a body through a distance of one meter along the direction of force."

Erg is smaller unit of work. $1\,\text{Joule} = 10^7\,\text{erg}$

To Find the work From Force-displacement graph:

Consider a constant force "F" acts on a body through a distance "d". Graph is plotted between "F" and "d". This is called Force-displacement graph.



"F" is along y-axis and "d" is along x-axis.

"For a constant force, graph is a straight line."

If force $\vec{F}$ (Newton) and displacement d(meters) are parallel ($\theta = 0°$).

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Work done $= \vec{F}\cdot\vec{d} = Fd\cos 0^\circ = Fd$ (Joules) —(I)

But

Area under F–d graph = Fd —(II)

By I and II

$$\boxed{\text{Work done} = \text{Area under the force-displacement graph}}$$

Note:

If force $\vec{F}$ and displacement $\vec{d}$ are not parallel, then we draw a graph between "$F\cos\theta$" and "d".

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(4.2) Work done by a Variable Force

In many cases Force is not constant.

For example

(i) When a spring is stretched, force changes with displacement;

$$F \propto x$$

$$F = kx \qquad \text{Hook's law}$$

(ii) When a rocket moves away from the earth, work is done against the force of gravity. This force is inversely proportional to square of distance.

$$F \propto \frac{1}{r^2}$$

$$\left(F = G\frac{m_1 m_2}{r^2}\right)$$

Explanation:

Consider a variable force takes

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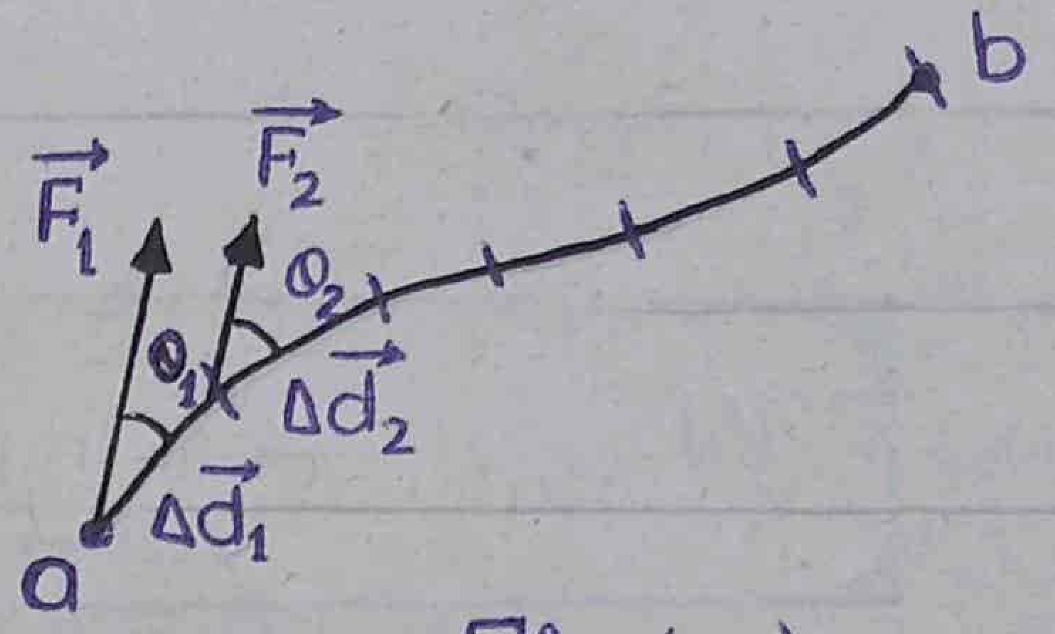
a particle from "a" to "b" as shown in Fig(I). The path is divided into "n" small intervals of displacements

$\Delta\vec{d}_1$, $\Delta\vec{d}_2$, ..., Δd_n

and

F_1, F_2, ..., F_n

are forces during these displacements.



Fig(I)

During each displacement Force is approximately constant.

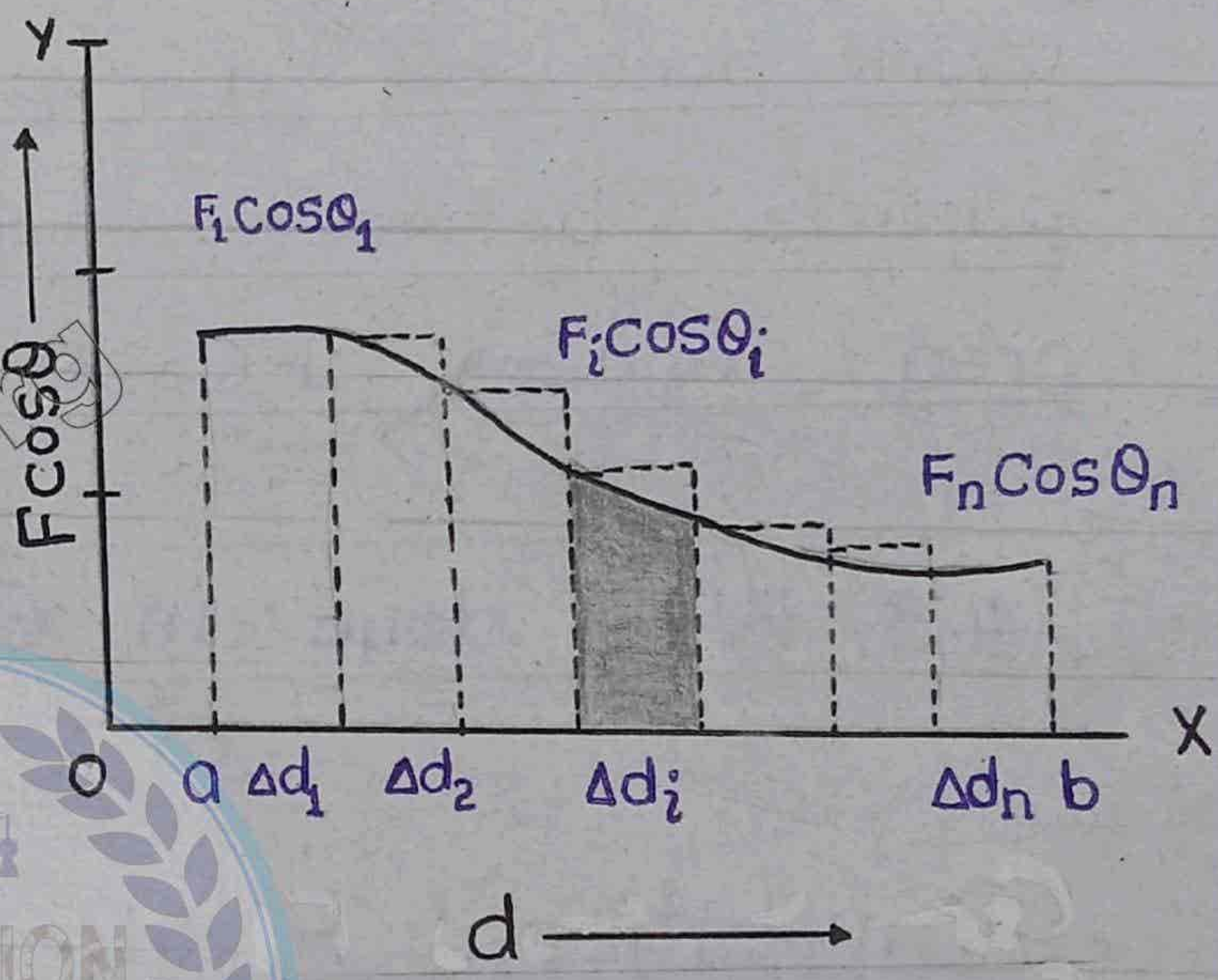
$$W_1 = \vec{F}_1 \cdot \Delta\vec{d}_1$$

$$W_1 = F_1 \Delta d_1 \cos\theta_1$$

$$W_1 = F_1 \cos\theta_1 \Delta d_1$$

$$W_2 = F_2 \cos\theta_2 \Delta d_2$$

$$W_n = F_n \cos\theta_n \Delta d_n$$



Fig(b)

$$W_{total} = W_1 + W_2 + \cdots + W_n$$

$$= F_1\cos\theta_1 \Delta d_1 + F_2\cos\theta_2 \Delta d_2 + \cdots + F_n\cos\theta_n \Delta d_n$$

$$\boxed{W_{total} = \sum_{i=1}^{n} F_i \cos\theta_i \cdot \Delta d_i}$$

This work done is approximately equal to the sum of areas of all the rectangles, as shown in Fig(b).

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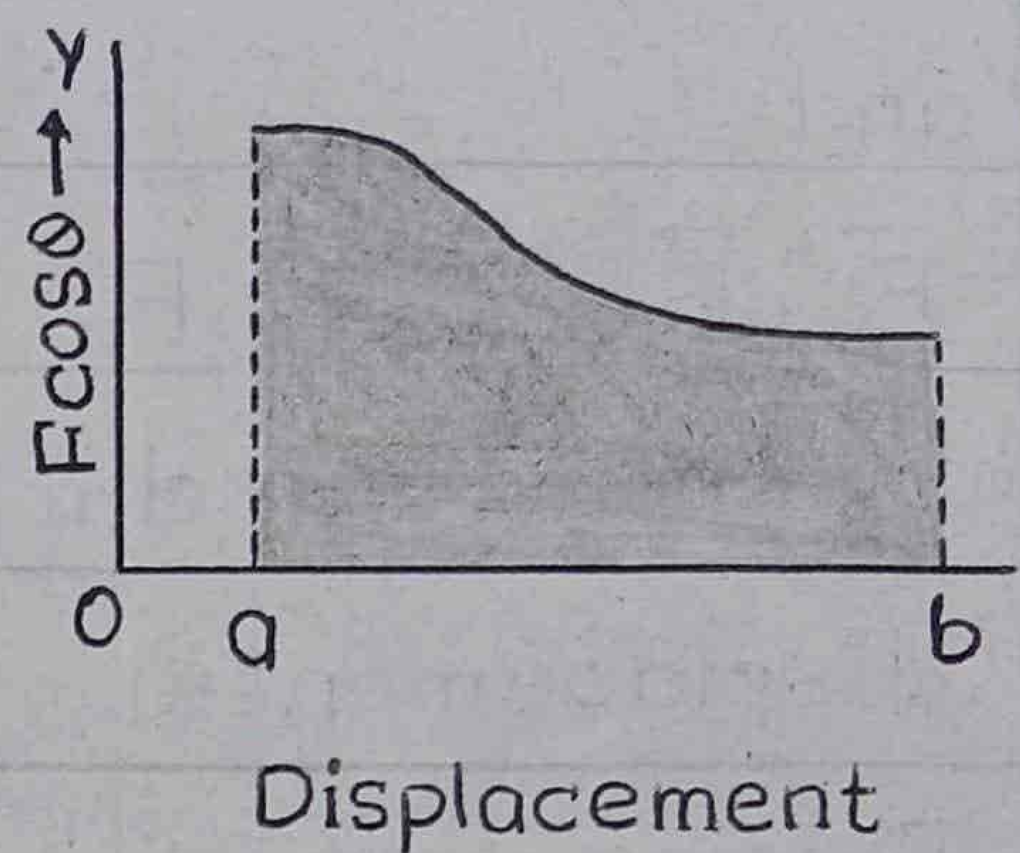
To get more accurate result, we divide the total displacement into very large number of intervals such that

$$\Delta d \longrightarrow 0$$

$$\boxed{W_{Total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \, \Delta d_i}$$



So

"Work done by a variable Force is moving a particle between two points is equal to area under "FCosθ" and "d" graph."

4.3 Work done in Gravitational Field

Gravitational Field:

"The space or region around the earth in which its gravitational Force acts on a body is called gravitational Field."

Conservative Field:

(i) "The Field in which the work done on a body does not depend on the path Followed by the body."

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OR (ii)

"The field in which the work done in a closed path is zero is called conservative field."

Examples: (i) Gravitational Field

(ii) Electric Field

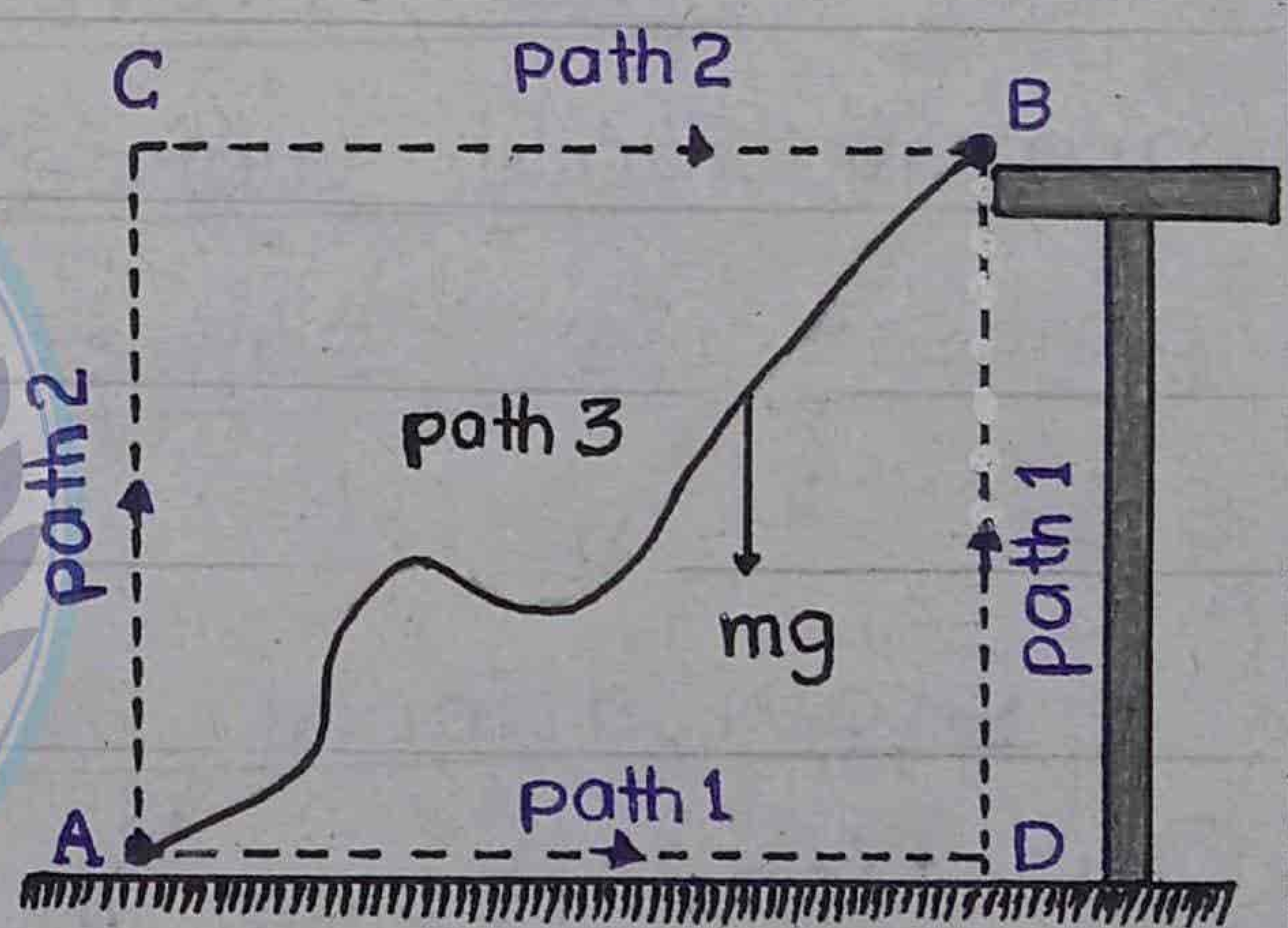
(iii) Magnetic Field

Consider a body of mass m is displaced with constant velocity from A to B along different paths in the gravitational field.

Here, the gravitational force is equal to the weight "mg" of the body.

$$F = W = mg$$

Path -1:



$$W_{ADB} = W_{A \to D} + W_{D \to B}$$

$$W_{ADB} = mg(AD)\cos 90^\circ + mgh\cos 180^\circ$$

$$= 0 + mgh(-1)$$

$$W_{ADB} = -mgh \quad \text{——— (i)}$$

Path - 2:

$$W_{ACB} = W_{A \to C} + W_{C \to B}$$

$$= mgh\cos 180^\circ + mg(CB)\cos 90^\circ$$

$$= mgh(-1) + 0$$

$$W_{ACB} = -mgh \quad \text{——— (ii)}$$

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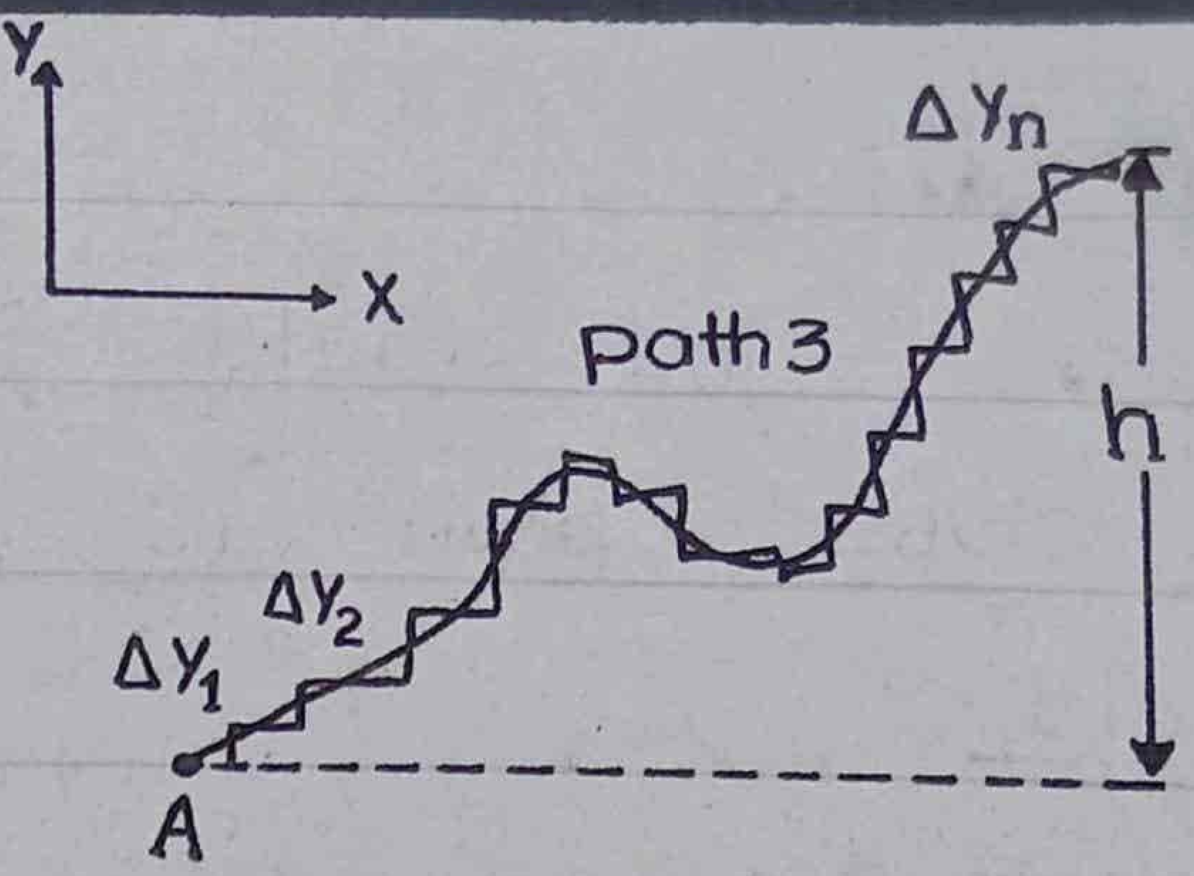
Path - 3:



Curved path - 3 is broken into a series of horizontal and verticle steps.

(i) Work done in horizontal steps is zero.

(ii) Work done in verticle steps $\Delta Y_1, \Delta Y_2, \cdots, \Delta Y_n$. is

$$W_{AB} = -mg\Delta y_1 - mg\Delta y_2 - \cdots\cdots - mg\Delta y_n$$

$$W_{AB} = -mg(\Delta y_1 + \Delta y_1 + \cdots\cdots + \Delta y_n)$$

$$\therefore \quad \Delta y_1 + \Delta y_2 + \cdots + \Delta y_n = h$$

$$W_{AB} = -mgh \qquad \text{(iii)}$$

Result:

This proves that work done by the gravitational force from A to B in different paths is same (- mgh).

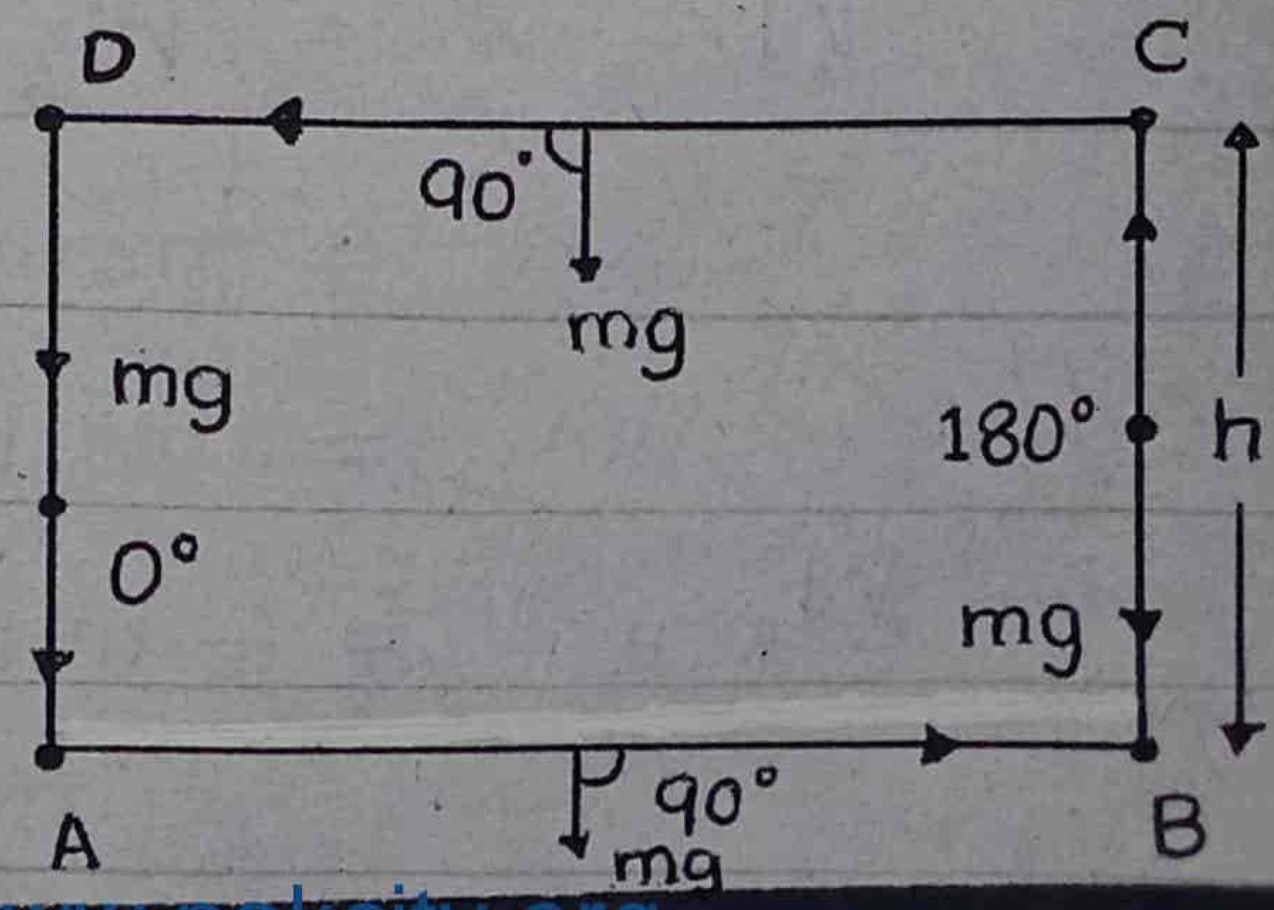
So,

"Gravitational Field is a conservative field."

Question: Prove that

Work done in Gravitational Field around a closed path:

Consider a closed path ABCDA in the gravitational field.



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Consider a body of mass m is taken along this closed path.
In the gravitational field the force acting on the body is equal to its weight

$$F = W = mg$$

$$W_{\text{Closed Path}} = W_{ABCDA}$$

$$W_{ABCDA} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= mg(AB)\cos 90^\circ + mgh\cos 180^\circ + mg(CD)\cos 90^\circ + mgh\cos 0^\circ$$

$$= 0 + mgh(-1) + 0 + mgh(1)$$

$$= -\cancel{mgh} + \cancel{mgh}$$

$$W_{ABCDA} = 0$$

Result:

The total work done in the gravitational field along the closed path is zero. So, the gravitational field is a conservative field.

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Non-Conservative Field:

"A field in which work done in moving a body between two points depends upon the path followed by the body."

e.g. Frictional force is a non-conservative force.

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Work done against the Frictional Force depends upon the path Followed by the body.

4.4 power

Defination:

"Rate of doing work is called power."

$$P = \frac{W}{t} \qquad \text{power} = \frac{\text{Work}}{\text{time}} = \frac{\text{Energy}}{\text{time}}$$

Average Power:

"Total work done "ΔW" divided by total time taken "Δt" is called average power."

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power:

"The power during a very small interval of time ($\Delta t \rightarrow 0$) is called instantaneous power."

$$P_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}$$

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Power and Velocity:

Suppose a constant Force $\vec{F}$ acts on a body and the body moves with constant velocity $\vec{v}$.

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Instantaneous Power is

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \qquad \text{But } \Delta W = \vec{F} \cdot \Delta \vec{d}$$

$$P = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}$$

$$P = \vec{F} \cdot \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} \qquad \text{But } \lim \frac{\Delta \vec{d}}{\Delta t} = \vec{V}$$

$$P = \vec{F} \cdot \vec{V}$$

power = Scalar Product of Force and Velocity.

SI unit of Power is Watt.

Note:

Power is a Scalar quantity.

Watt: defined as

"One joule of work done in one second."

One kilowatt hour is the work done in one hour by an agency whose power is one kilowatt.

$$1\,KWh = 1000\,W \times 3600\,s$$

or

$$1\,KWh = 3.6 \times 10^6\,J$$

$$1\,KWh = 3.6\,MJ$$

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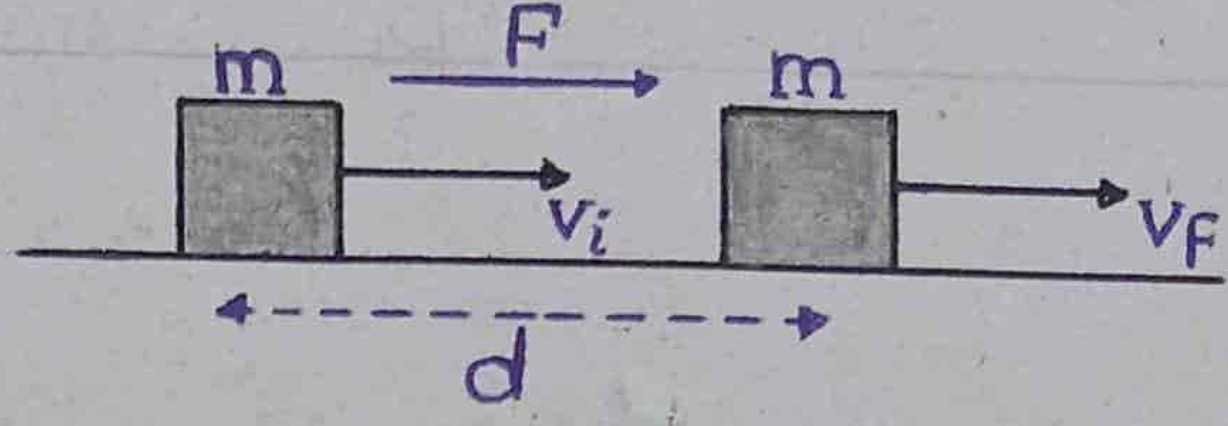
Work Energy Principle

Work done on a body = Change in K.E of body

Proof:



Consider a body of mass "m" moving with velocity "v_i". A force "F" acts on the body over a distance "d". The Final velocity becomes "v_F".

$$2aS = v_F^2 - v_i^2$$

$$2ad = v_F^2 - v_i^2$$

$$d = \frac{v_F^2 - v_i^2}{2a}$$

Work done by the force is

$$W = Fd$$

$$W = mad$$

$$W = m\cancel{a}\left(\frac{v_F^2 - v_i^2}{2\cancel{a}}\right)$$

$$W = \frac{1}{2}m(v_F^2 - v_i^2)$$

$$\boxed{Fd = \frac{1}{2}mv_F^2 - \frac{1}{2}mv_i^2}$$

$$\text{Work} = KE_F - KE_i$$

$$\text{Work} = \Delta KE$$

$$\boxed{\text{Work done} = \text{Change in KE}}$$

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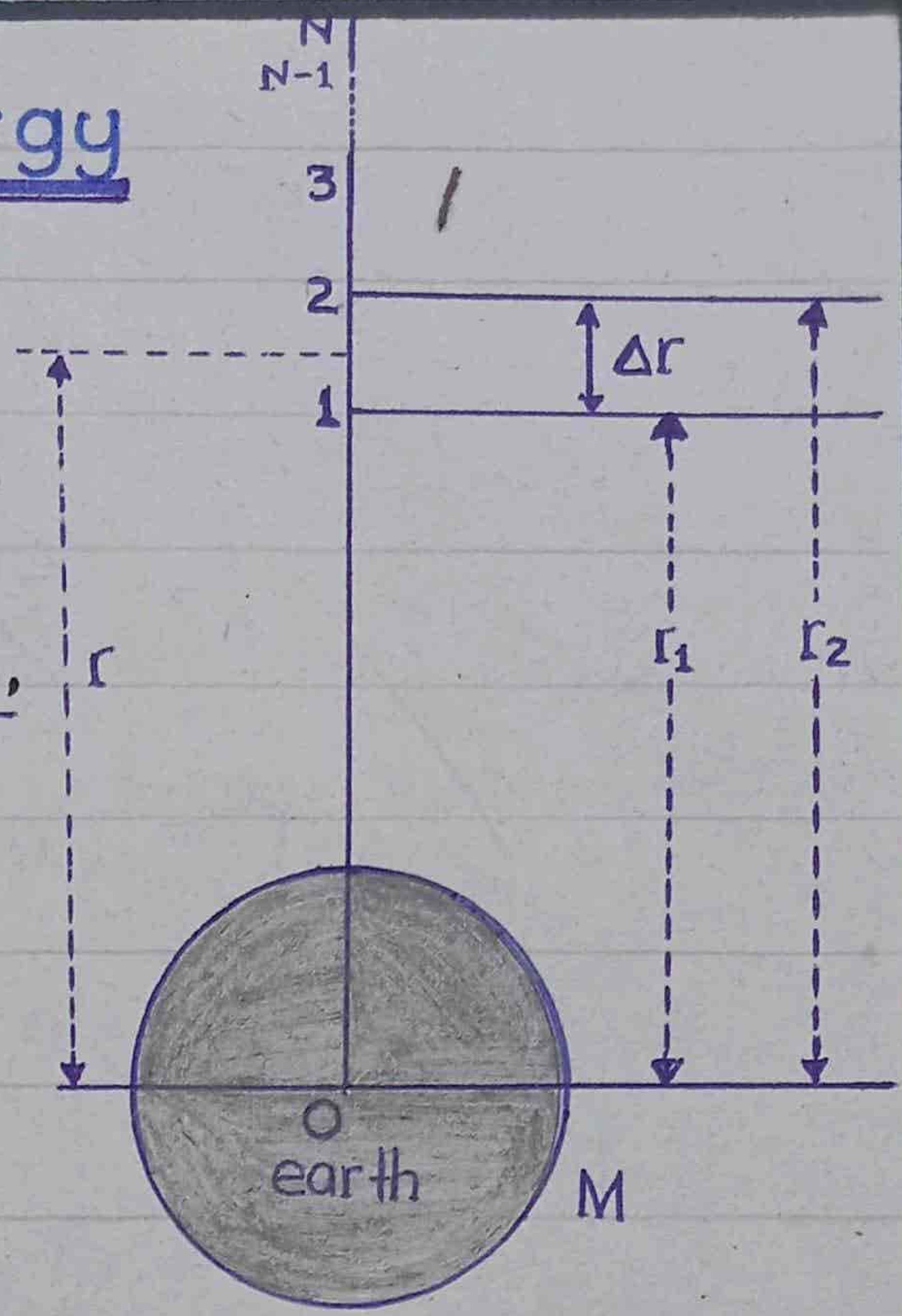
Absolute Potential Energy

"It is the amout of work done in moving a body from the surface of the earth to infinity, where the force of gravity becomes zero."

Absolute PE $= -\frac{GMm}{R}$



Proof:

Consider a body of mass "m" is lifted from point 1 to a far off point N in the gravitational field.

As the gravitational force $F \propto \frac{1}{r^2}$, so it does not remain constant.

Hence the distance from 1 to N is divided into small steps of length Δr.

Δr is so small that gravitational force remains constant during Δr.

Let

r = distance of the centre of the step Δr to centre of earth.

$$\Delta r = r_2 - r_1$$

$$r_2 = \Delta r + r_1$$

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$$r = \frac{r_1 + r_2}{2}$$

$$= \frac{r_1 + r_1 + \Delta r}{2} = \frac{2r_1 + \Delta r}{2}$$

$$= \frac{\cancel{2}r_1}{\cancel{2}} + \frac{\Delta r}{2} = r_1 + \frac{\Delta r}{2}$$

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$= r_1^2 + \frac{\Delta r^2}{4} + \cancel{2} \cdot r_1 \cdot \frac{\Delta r}{\cancel{2}}$$

$$= r_1^2 + \frac{\Delta r^2}{4} + r_1 \Delta r$$

$\Delta r << r_1$; $\frac{\Delta r^2}{4}$ is neglected

$$r^2 = r_1^2 + r_1 \Delta r$$

$$= r_1^2 + r_1(r_2 - r_1)$$

$$= r_1^2 + r_1 r_2 - r_1 \cdot r_1$$

$$= \cancel{r_1^2} + r_1 r_2 - \cancel{r_1^2}$$

$$r^2 = r_1 r_2$$

Gravitational Force at the centre of the step is:

$$F = G\frac{Mm}{r^2}$$

$$F = G\frac{Mm}{r_1 r_2}$$

$$W_{1\to 2} = \vec{F} \cdot \vec{\Delta r} = F \Delta r \cos 180^\circ$$

$$W_{1\to 2} = F \Delta r (-1)$$

$$W_{1\to 2} = -F\Delta r$$

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$$W_{1\to2} = -F\Delta r$$

$$W_{1\to2} = -G\frac{Mm}{r_1 r_2}(r_2 - r_1)$$

$$= -GMm\left(\frac{r_2 - r_1}{r_1\ r_2}\right)$$

$$= -GMm\left(\frac{\cancel{r_2}}{r_1\cancel{r_2}} - \frac{\cancel{r_1}}{\cancel{r_1}r_2}\right)$$

$$W_{1\to2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly;

$$W_{2\to3} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$W_{Total} = W_{1\to2} + W_{2\to3} + \cdots\cdots + W_{N-1\to N}$$

$$W_{total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right) - \cdots\cdots - GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$W_{Total} = -GMm\left[\left(\frac{1}{r_1} - \cancel{\frac{1}{r_2}}\right) + \left(\cancel{\frac{1}{r_2}} - \cancel{\frac{1}{r_3}}\right) + \cancel{\cdots\cdots} + \left(\cancel{\frac{1}{r_{N-1}}} - \frac{1}{r_N}\right)\right]$$

$$W_{Total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

As N is at infinity $r_N = \infty$

$$\frac{1}{r_N} = \frac{1}{\infty} = 0$$

$$W_{Total} = -GMm\left(\frac{1}{r_1} - 0\right)$$

$$W_{Total} = -\frac{GMm}{r_1}$$

In general $r_1 = r$

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$$W_{Total} = -\frac{GMm}{r}$$

This work is gravitational P.E = U

$$U = -\frac{GMm}{r}$$

At the surface of earth r = R

Absolute Potential Energy = $U_g = -\frac{GMm}{R}$

Escape Velocity:

"The minimum initial velocity of an object with which its goes out of the gravitational Field."

$$V_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Proof:

We want to lift a body from the surface of the earth to infinity. For this purpose energy equal to Absolute PE must be supplied to the body in the form of Kinetic Energy.

Initial KE = Absolute PE

$$\frac{1}{2}\cancel{m}V_{esc}^2 = \frac{GM\cancel{m}}{R}$$

$$V_{esc}^2 = \frac{2GM}{R}$$

Information

$G = 6.67\times10^{-11}\,Nm^2kg^{-2}$

$M = 6\times10^{24}\,kg$

$R = 6.4\times10^6\,m$

$V_{esc} = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2\times6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^6}}$

$V_{esc} = 11.2\,km\,s^{-1}$

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$$V_{esc} = \sqrt{\frac{2GM}{R}} \quad \text{———} \quad (1)$$

For a body of mass "m" on the surface of earth.

$$F = \frac{GMm}{R^2}$$

$$\therefore \quad F = mg$$

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

x by R

$$gR = \frac{GM}{R}$$

$$V_{esc} = \sqrt{2gR}$$

$$V_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^4}$$

$$= 1.12 \times 10^4 \ ms^{-1}$$

$$V_{esc} = 11.2 \times 10^3 \ ms^{-1}$$

$$V_{esc} = 11.2 \ Kms^{-1}$$

$g = 9.8 \ ms^{-2}$

$R = 6400 \ Km$

$R = 6400 \times 10 \ m$

$R = 64 \times 10^5 \ m$

$R = 6.4 \times 10^6 \ m$

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4.6 Interconservation of potential Energy and Kinetic Energy

Case-1 When no Air Friction:

Kinetic energy can be converted into Potential energy and PE can be converted into K.E.

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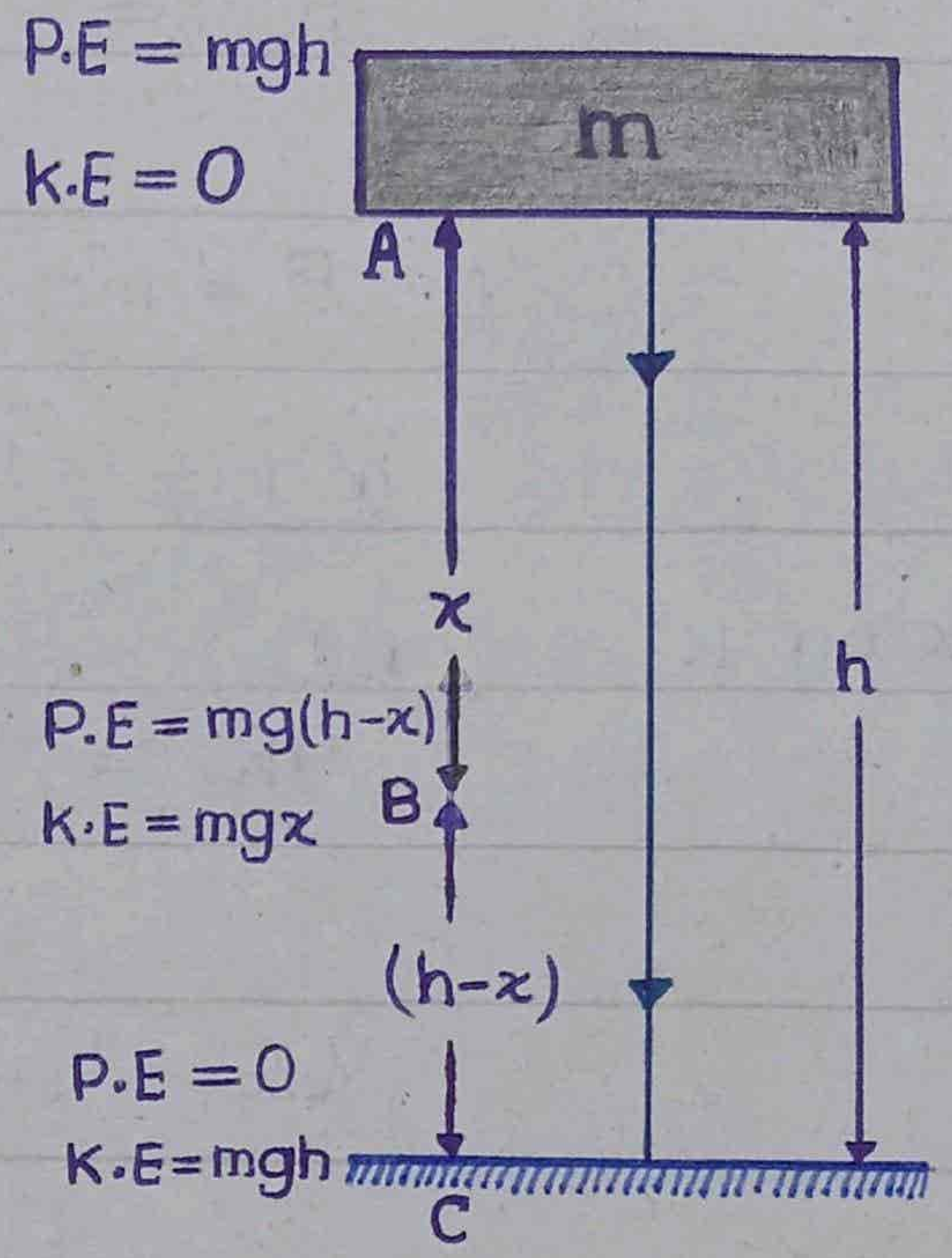
Consider a body of mass "m" at rest, at a height "h" above the ground. When it falls freely downward, its PE goes on chainging into KE.



At Point - A:

$P.E = mgh \quad ; \quad KE = 0$

$$\text{Total Energy} = K.E + P.E = 0 + mgh$$

$$\text{Total Energy} = mgh \quad \text{———} \quad (1)$$

At Point - B:

The body falls a distance "x".

$$P.E = mg(h - x)$$

$$V_f^2 - V_i^2 = 2aS \qquad \because \; V_i = 0,\; S = x,\; a = g$$

$$V_B^2 - 0 = 2gx$$

$$V_B^2 = 2gx$$

$$K.E = \frac{1}{2} m V_B^2 = \frac{1}{2} m(2gx)$$

$$K.E = mgx$$

$$\text{Total Energy} = K.E + P.E$$

$$= mgx + mg(h - x)$$

$$= \cancel{mgx} + mgh - \cancel{mgx}$$

$$\text{Total Energy} = mgh \quad \text{———} \quad (2)$$

At Point C:

$$P.E = 0$$

$$\text{Loss of } P.E \text{ at } C = mgh - 0 = mgh$$

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$$K.E = \frac{1}{2} m v_c^2 = \frac{1}{\cancel{2}} m(\cancel{2}gh)$$

$\because V_f^2 - V_i^2 = 2as$

$V_c^2 - 0 = 2gh$

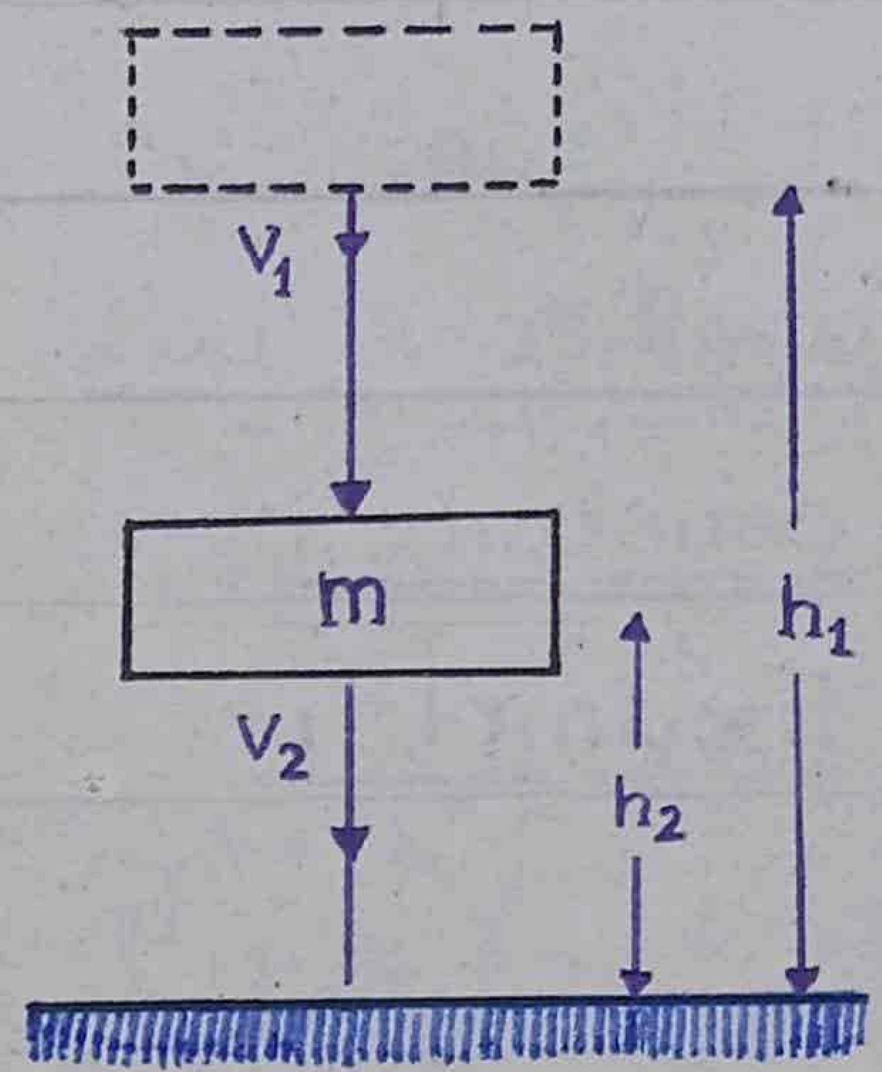
$V_c^2 = 2gh$

$$K.E = mgh$$

Total Energy = K.E + P.E

= mgh + 0

Total Energy = mgh ——— (3)



So, Gain in KE = mgh

Hence

$$\boxed{\text{Loss of P.E = Gain in K.E}}$$

From Fig.

$$mgh_1 - mgh_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m V_1^2$$

$$mg(h_1 - h_2) = \frac{1}{2} m(V_2^2 - V_1^2)$$

V_1 and V_2 are velocities at heights "h_1" and "h_2".

Case-2 When Frictional Force is present:

When a Frictional Force "F" is present during the downward motion. Then a part of P.E is used in doing work against Friction and it is equal to "Fh".

The remaining P.E = mgh − Fh is converted into K.E.

Remaining P.E = Gain in K.E

$$mgh - Fh = \frac{1}{2} mv^2$$

$$mgh = \frac{1}{2} mv^2 + Fh$$

$$\boxed{\text{Loss of P.E = Gain in K.E + Work done against Friction}}$$

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4.7 Law of Conservation of Energy

It states that

"Energy cannot be destroyed, it can be transformed from one kind to another, but the total amount of energy remains constant."

Example:

In case of freely falling body

$$\boxed{\text{Total mechanical energy} = P.E + K.E = \text{Constant}}$$

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QUESTIONS

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4.1: A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

4.2: Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m.

4.3: A force F acts through a distance L. The force is then increased to 3 F, and then acts through a further distance of 2 L. Draw the work diagram to scale.

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Answers of Short Questions

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Q-4.1:

Man and car are stationary displacement $\vec{d} = 0$.

So, work done in both the cases is zero.

Hence, the situations are similar.

$$W = \vec{F}\cdot\vec{d} = \vec{F}\cdot(0) = 0$$

Q-4.2:

$m = 10\,kg$ $\quad$ $h = 10\,m$

Work done against gravity is $W = mgh$

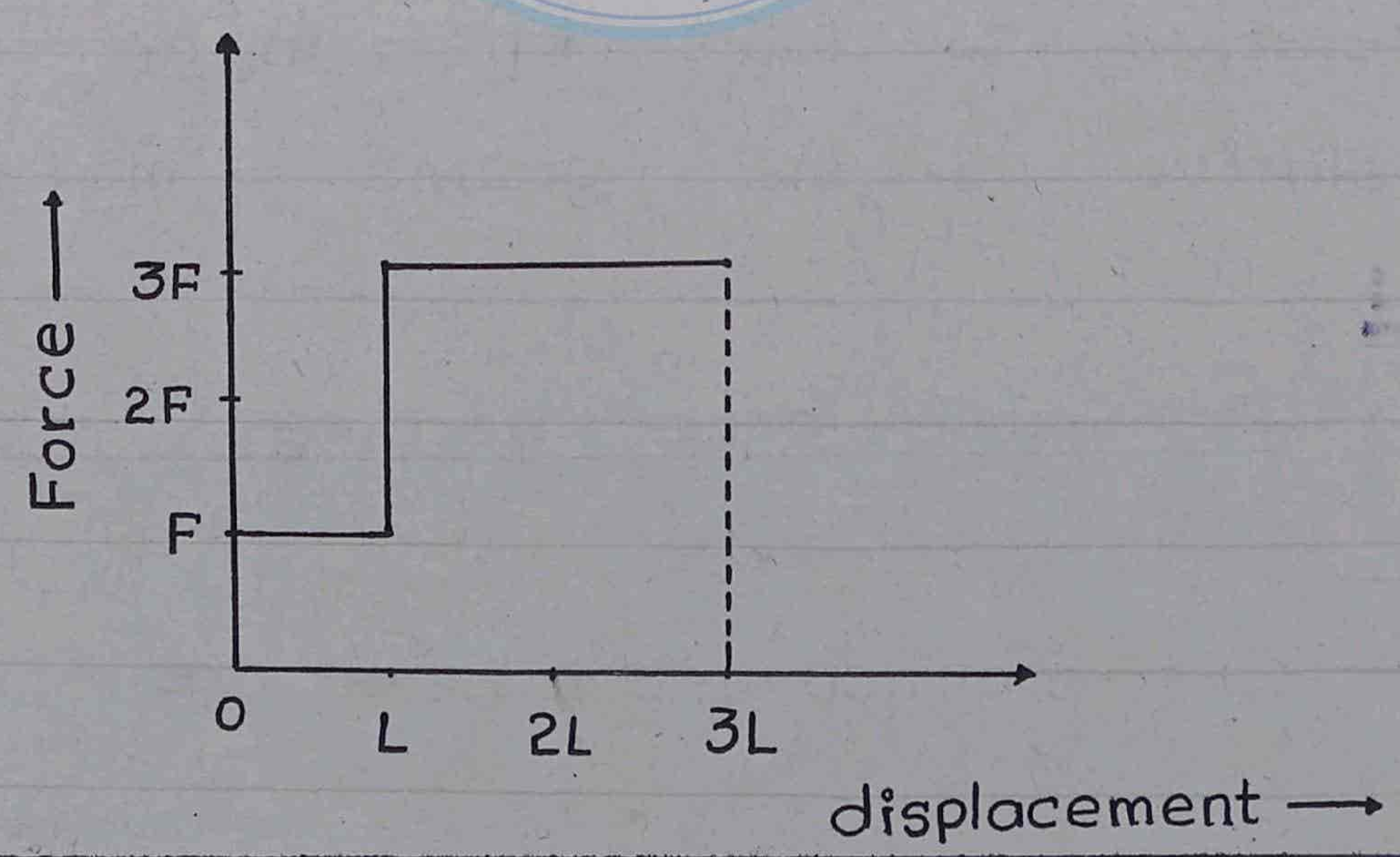
$$W = 10 \times 9.8 \times 10 = 980J$$

$$W = \frac{980}{1000}\,KJ$$

$$W = 0.98\,KJ$$

Q-4.3:

Work diagram



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Q-4.4:

$m = 50\,kg$ $h = 50\,cm = 0.5\,m$

$F = 50\,N$ $d = 2\,m$

In First Case:

$$W_1 = mgh = 50 \times 9.8 \times 0.5$$

$$W_1 = 245\,J$$

In Second Case:

$$W_2 = Fd = 50 \times 2 = 100\,J$$

$$W_1 > W_2$$

In First case more work is done.

Q-4.5:

An object having 1 Joule P.E means that it has capacity to do 1 Joule of work. The Form of P.E may be elastic P.E or gravitational P.E.

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Q-4.6:

Both statements are correct.

P.E relative to table top $= mgh_1$

P.E relative to the ground $= mg(h_1 + h_2)$

Q-4.7:

The nose cone becomes hot due to Friction.

In atomosphere there are dust

particles, water vapours and molecules of air etc. Some K.E of the rocket is dissipated (ضائع ہونا) as heat due to friction offered by the particles.

Q-4.8:

(a) In a compressed spring there is Elastic P.E.
(b) Water in a high dam has gravitational P.E.
(c) A moving car has Kinetic energy.

Q-4.9:

The gravitational P.E of the cup is converted into K.E when it reaches the ground. On striking the ground, some of its K.E is used upto break the cup into pieces, Some of it goes into the K.E of broken pieces, some of it goes into Heat energy and some of it is converted into sound energy.

Q-4.10:

Elastic P.E of the catapult (غلیل) is transferred to stone as K.E.
When the stone strikes the greenhouse window, K.E of the stone is converted into
(i) Work done in breaking the window.

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(ii) heat energy

(iii) Sound energy

(iv) Kinetic energy of the broken pieces.

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Numerical Problems

P-4.1:

$F = 40N$

$\theta = 20°$

$d = 20\,m$

$W = ?$

$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$W = (40)(20)\cos 20°$

$= 800 \times 0.94$

$W = 7.5 \times 10^2 J$

P-4.2:

$m = 3.35 \times 10^{-5}\ kg$

$h = 100\ m$

(a) Work done by gravity

$W = ?$

$W = mgh$

$W = 3.35 \times 10^{-5} \times 9.8 \times 100$

$W = 0.0328\ J$

$W = 3.28 \times 10^{-2}\ J$

(b) Work done by Friction

$W = ?$

Work done by Friction is Same but it is negative because Force of Friction and displacement are in opposite direction.

$W = -3.28 \times 10^{-2}\ J$

P-4.3:

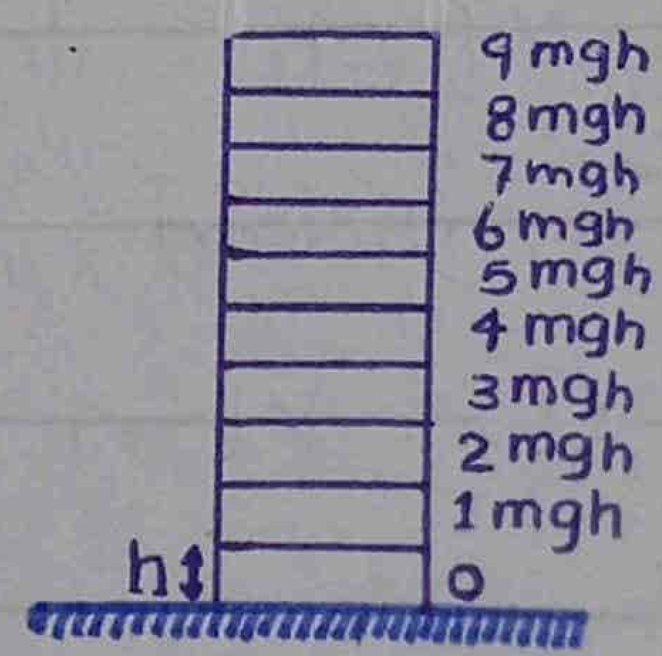
$h = 6\,cm$

$h = \frac{6}{100}\,m$, $h = 0.06\,m$

$m = 1.5\,kg$

No work done for First brick.

$W = 0 + mgh + 2mgh$
$+ 3mgh + 4mgh$
$+ 5mgh + 6mgh$
$+ 7mgh + 8mgh$
$+ 9mgh$



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$W = 45\,mgh$

$W = 45 \times 1.5 \times 9.8 \times 0.06$

$W = 39.69\,J$

$W \cong 40\ J$ □

P- 4.4:

$m = 800\,kg$

$v_i = 54\,kmh^{-1} = \frac{54 \times 1000}{3600}\,ms^{-1}$

$v_i = 15\ ms^{-1}$

$v_f = 0$

$d = 60\ m$

$F = ?$ (retarding force)

By Work-Energy Principle

Work done = Change in K.E

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$W = \frac{1}{2}m(v_f^2 - v_i^2)$

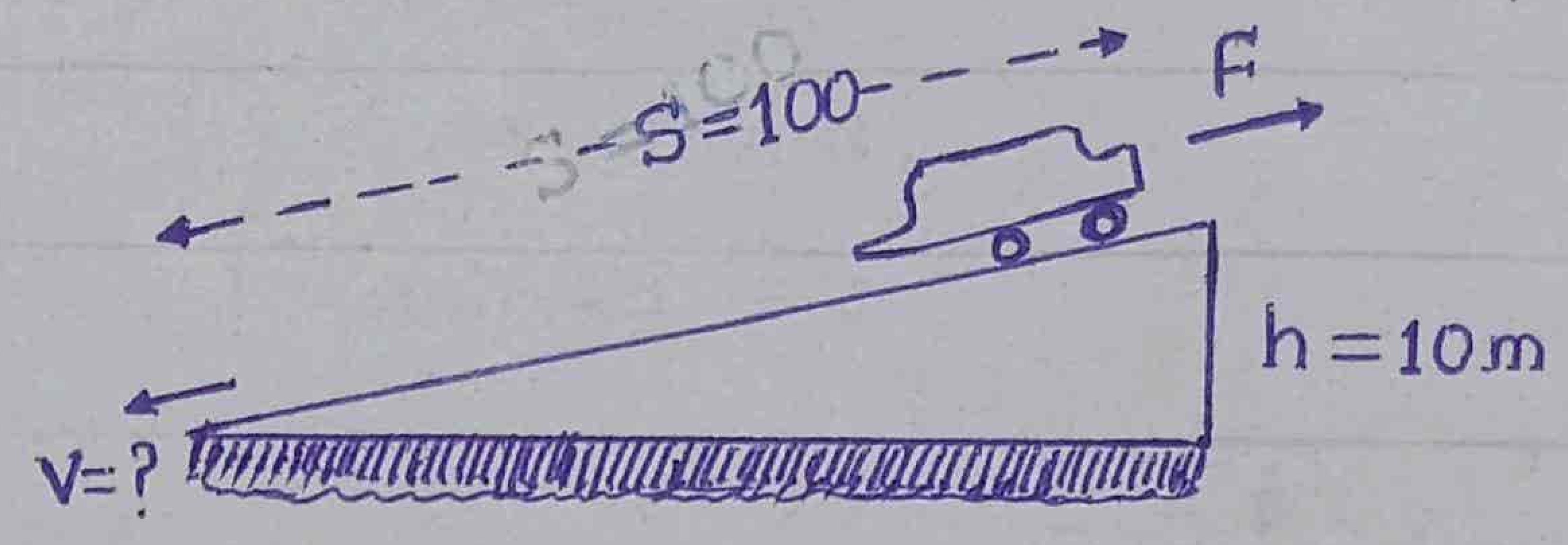
$Fd = \frac{1}{2}m(v_f^2 - v_i^2)$

$F = \frac{1}{2d}m(v_f^2 - v_i^2)$

$F = \frac{1 \times 800}{2 \times 600}(0 - (15)^2)$

$F = -1500\ N$ □

P- 4.5:



$m = 1000\ kg$

$h = 10\ m$, $F = 480N$

$S = d = 100m$

$v = ?$

By Conservation of energy When retarding force is present.

Loss of PE = gain in K.E + Work against Friction

$mgh = \frac{1}{2}mv^2 + F \times h$

$1000 \times 9.8 \times 10 = \frac{1}{2} \times 1000\,v^2 + 480 \times 100$

$98000 = 500v^2 + 48000$

$500v^2 = 98000 - 48000$

$500v^2 = 50000$

$v^2 = \frac{50000}{500}$

$v^2 = 100$

$v = 10\ ms^{-1}$ □

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P-4.6:

$V = 100\ m^3$

$h = 10\ m$

$t = 20\ min$

$t = 20 \times 60\ s$

$t = 1200\ s$

$\rho = 1000\ kg\ m^{-3}$

(a) Increase in P.E = ?

(b) power $P = ?$

(a) $\rho = \frac{m}{V}$

$m = \rho \times V$

$m = 1000 \times 100$

$m = 1 \times 10^5\ kg$

Increase in P.E = mgh

$P.E = 1 \times 10^5 \times 9.8 \times 10$

$P.E = 9.8 \times 10^6\ J$

(b) $P = \frac{W}{t} = \frac{mgh}{t}$

$P = \frac{1 \times 10^5 \times 9.8 \times 10}{1200}$

$P = \frac{9.8 \times 10^6}{1200}$

$P = 8166.6\ W$

$P = 8.166 \times 10^3\ W$

$P \simeq 8.2\ kW$

P-4.7:

$F = 400\ N$

$V = 80\ Kmh^{-1}$

$V = \frac{80 \times 1000}{3600}\ ms^{-1}$

$V = 22.2\ ms^{-1}$

$P = ?$

$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$

$P = (400)(22.2)\cos 0^\circ$

$P = 8880$ Watt

$P = \frac{8880}{1000}\ KW$

$P = 8.88\ KW$

$P = 8.9\ KW$

P-4.8:

$F = ?$

$m = 9.1 \times 10^{-31}\ kg$

$v_i = 0$

$v_f = 2 \times 10^7\ ms^{-1}$

$d = 5cm = \frac{5}{100}\ m$

$d = 0.05\ m$

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By Work-Energy Principle

Work done = Change in K.E

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$Fd = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$F = \frac{m}{2d} (v_f^2 - v_i^2)$$

$$F = \frac{9.1 \times 10^{-31}}{2 \times 0.05} \left[(2 \times 10^7)^2 - 0 \right]$$

$$F = 3.6 \times 10^{-15} \text{ N}$$

P-4.9:

$h_1 = 10$ m

$W = 750$ N

$h_2 = 5$ m

$v = ?$

Loss of P.E = Gain in K.E

$$\cancel{m} g(h_1 - h_2) = \frac{1}{2} \cancel{m} v^2$$

$$g(h_1 - h_2) = \frac{1}{2} v^2$$

$$v^2 = 2g(h_1 - h_2)$$

$$v = \sqrt{2g(h_1 - h_2)}$$

$$v = \sqrt{2 \times 9.8 (10 - 5)}$$

$$v = 9.9 \text{ ms}^{-1}$$

P-4.10: $h = 4$ m

(a) $v = ?$

Loss of P.E = Gain in K.E

$$\cancel{m} gh = \frac{1}{2} \cancel{m} v^2$$

$$gh = \frac{1}{2} v^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 4}$$

$$v = 8.8 \text{ ms}^{-1}$$

(b) $v' = 6 \text{ ms}^{-1}$

%age loss of energy = ?

$$\%\text{age loss in Energy} = \frac{K.E - K.E'}{KE} \times 100\%$$

$$= \frac{\frac{1}{2} mv^2 - \frac{1}{2} mv'^2}{\frac{1}{2} mv^2} \times 100\%$$

$$= \frac{\cancel{\frac{1}{2}} \cancel{m} (v^2 - v'^2)}{\cancel{\frac{1}{2}} \cancel{m} v^2} \times 100\%$$

$$= \frac{v^2 - v'^2}{v^2} \times 100\%$$

$$= \frac{(8.8)^2 - (6)^2}{(8.8)^2} \times 100\%$$

$$= 54\%$$

%age loss of Energy = 54%

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QUESTIONS

4.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m. 2015 short, imp

4.3 A force F acts through a distance L. The force is then increased to 3 F, and then acts through a further distance of 2 L. Draw the work diagram to scale. imp 2015

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4.10 A boy uses a catapult to throw a stone which accidentally smashes a green house window. List the possible energy changes. imp, imp

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NUMERICAL PROBLEMS

4.1 A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

(Ans: 7.5×10^2 J)

4.2 A rain drop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100 m, how much work is done by (a) gravity and (b) friction.

[Ans: (a) 0.0328 J (b) – 0.0328 J]

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4.3 Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of another?

(Ans: 40 J)

4.4 A car of mass 800 kg travelling at 54 kmh^{-1} is brought to rest in 60 metres. Find the average retarding force on the car. What has happened to original kinetic energy?

(Ans: 1500 N)

4.5 A 1000 kg automobile at the top of an incline 10 metre high and 100 m long is released and rolls down the hill. What is its speed at the bottom of the incline if the average retarding force due to friction is 480 N?

(Ans: 10 ms^{-1})

4.6 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is 1000 kg m^{-3}, find imp

(a) the increase in P.E.

(b) the power delivered by the pump.

[Ans: (a) 9.8×10^6 J (b) 8.2 kW]

4.7 A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 kmh^{-1}. What power (kW) must the engine develop?

imp

(Ans: 8.9 kW)

4.8 How large a force is required to accelerate an electron ($m = 9.1 \times 10^{-31}$ kg) from rest to a speed of $2.0 \times 10^7 ms^{-1}$ through a distance of 5.0 cm? imp 2021 imp imp 2010 2014 2012

(Ans: 3.6×10^{-15} N)

4.9 A diver weighing 750 N dives from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting air friction.

(Ans: 9.9 ms^{-1})

4.10. A child starts from rest at the top of a slide of height 4.0 m. (a) What is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of 6 ms^{-1}, what percentage of his total energy at the top of the slide is lost as a result of friction?

[Ans: (a) 8.8 ms^{-1} (b) 54%]

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