Chapter - 3

Motion and Force

Displacement:

"Shortest directed distance between two points is called displacement."

OR

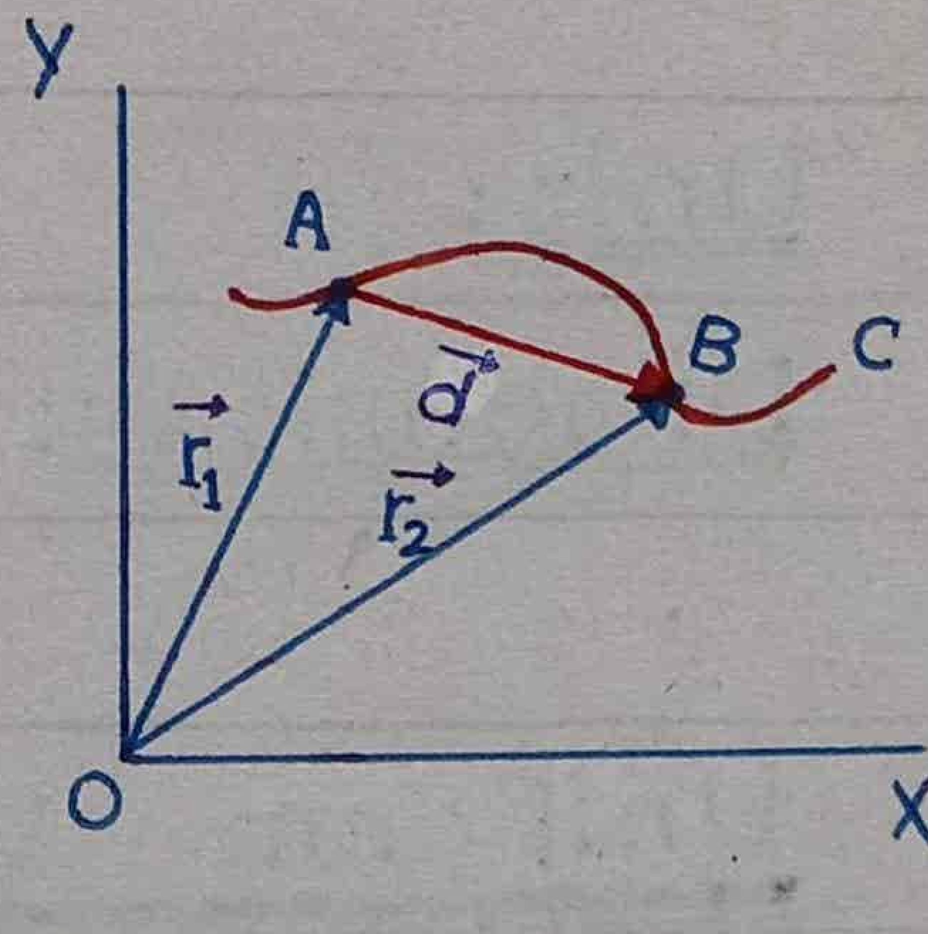
"When a body moves from one position to another the change in the position of the body from its initial position to its final position is called displacement."

It is a vector quantity.

The tail of the displacement vector is at the initial position and its arrowhead at the final position.

Explanation:

Consider a body is moving from initial position "A" to final position "B" along a curve.



Fig(a)

Displacement $\vec{d} = \vec{AB}$

$\vec{r_1}$ = position vector of A

$\vec{r_2}$ = position vector of B

By Head to Tail rule

$$\vec{r_1} + \vec{d} = \vec{r_2}$$

$$\boxed{\vec{d} = \vec{r_2} - \vec{r_1}}$$

In this case displacement is different from the path of motion. fig (a)

When the body moves in a straight line, the displacement coincides with the path of motion. fig (b)

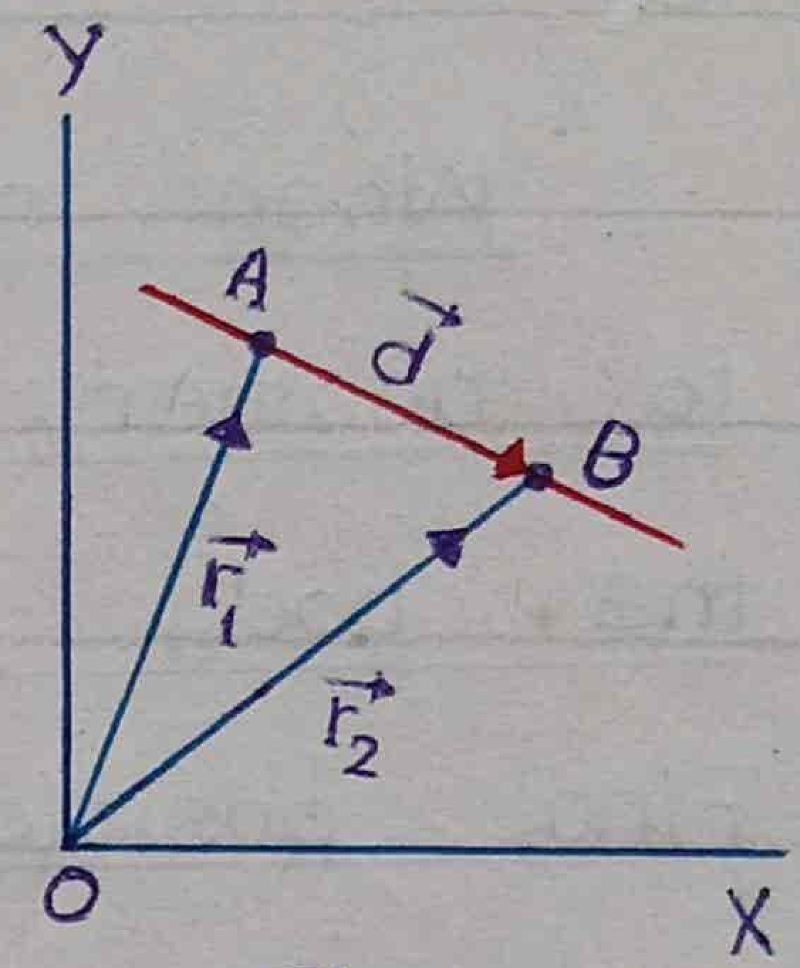


fig (b)

Velocity:

"Time rate of change of displacement is called velocity."

$$\vec{v} = \frac{\vec{d}}{t}$$

Unit: ms^{-1}

Dimensions:

$$[LT^{-1}]$$

Uniform Velocity:

"When a body covers equal displacements in equal intervals of time, its velocity is uniform."

Variable Velocity:

"When a body covers unequal displacements in equal intervals of time, its velocity is variable."

Average Velocity:

"Total displacement of a body divided by total time taken is average velocity."

$$\vec{V}_{avg} = \frac{\Delta \vec{d}}{\Delta t}$$

Instantaneous Velocity:

"The velocity of a body at a particular instant of time is called instantaneous velocity."

OR

It is defined as the limiting value of $\frac{\Delta \vec{d}}{\Delta t}$ as the time interval Δt following the time "t" approaches to zero, $(\Delta t \to 0)$.

$$\boxed{\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}}$$

Note: For a body moving with uniform velocity.

$$\boxed{\vec{V}_{avg} = \vec{V}_{inst}}$$

Acceleration:

"Time rate of change of velocity is called acceleration."

$$\boxed{\vec{a} = \frac{\Delta\vec{V}}{\Delta t}}$$

Unit:

ms^{-2}

Dimensions: $[LT^{-2}]$

Note: The change in velocity may be due to the change in its magnitude or change in direction or in both.

Positive Acceleration:

"When the velocity of a body is increasing, it has positive acceleration."

Negative Acceleration:

When the velocity of a body is decreasing, it has negative acceleration. It is also called "retardation" or "deceleration".

Average Acceleration:

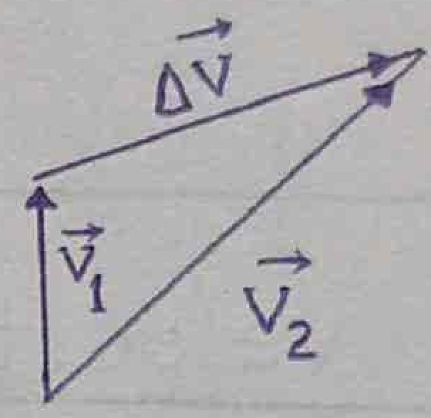
"Total change in velocity

divided by total time taken is called average acceleration."

$$a_{avg} = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V_2} - \vec{V_1}}{\Delta t}$$

$$\vec{V_1} + \Delta\vec{V} = \vec{V_2}$$

$$\Delta\vec{V} = \vec{V_2} - \vec{V_1}$$



Instantaneous Acceleration:

"The acceleration of a body at a particular instant of time is called instantaneous acceleration."

OR

"Limiting value of $\frac{\Delta\vec{V}}{\Delta t}$ as time interval $\Delta t \rightarrow 0$ is called instantaneous acceleration."

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{V}}{\Delta t}$$

Uniform Acceleration:

"When the change in velocity is equal in equal intervals of time, then acceleration is uniform."

Note:

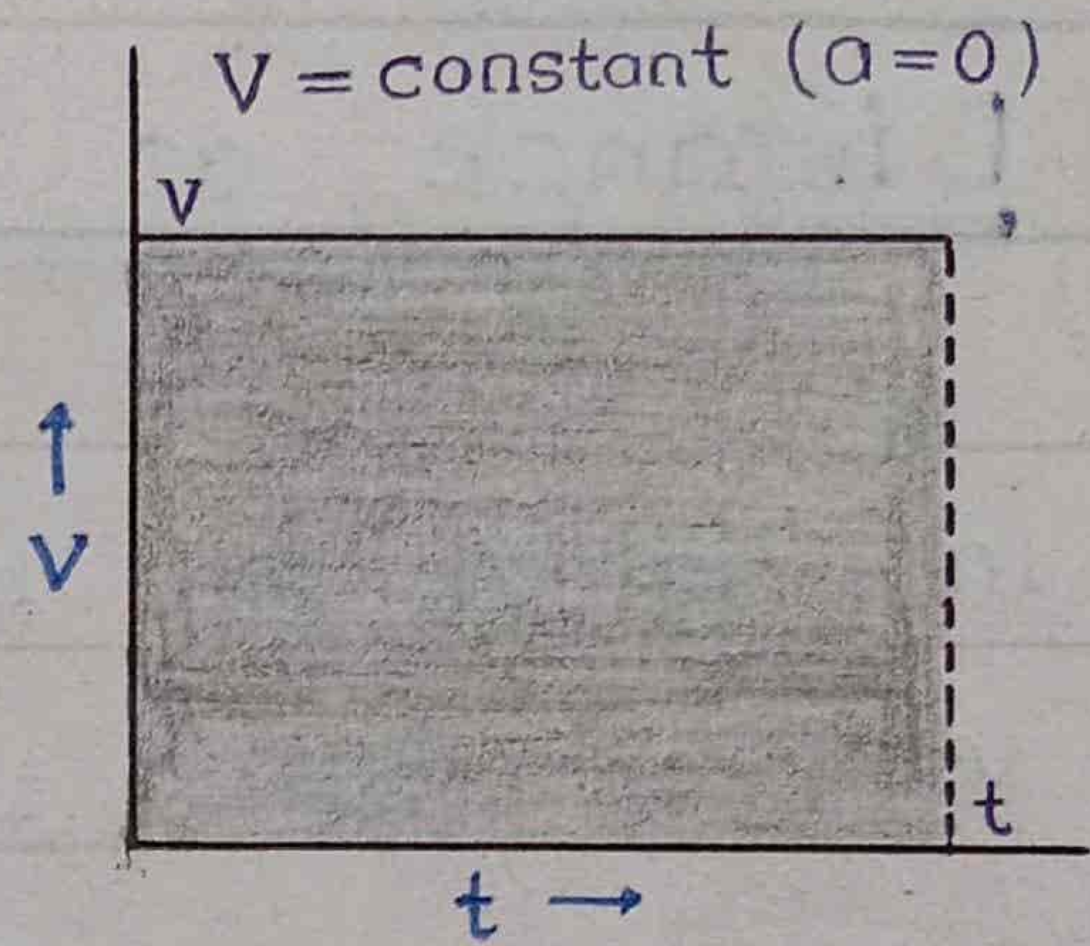
For a body moving with uniform acceleration.

$$\vec{a}_{inst} = a_{avg}$$

Velocity Time Graph:

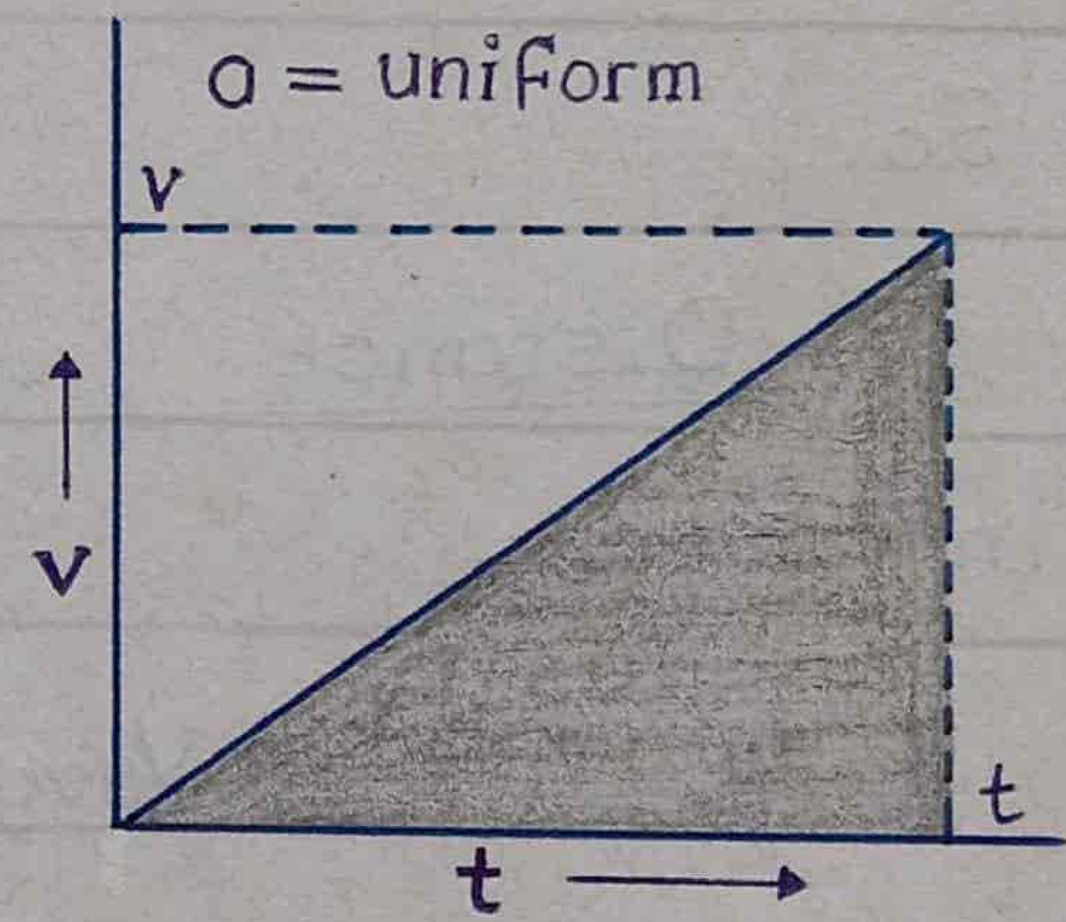
Graph plotted between the veocity of a body and time is called velocity-time graph. (vt - graph)

1- When a car moves with constant velocity the velocity-time graph is horizontal straight line. Fig(a).



Fig(a)

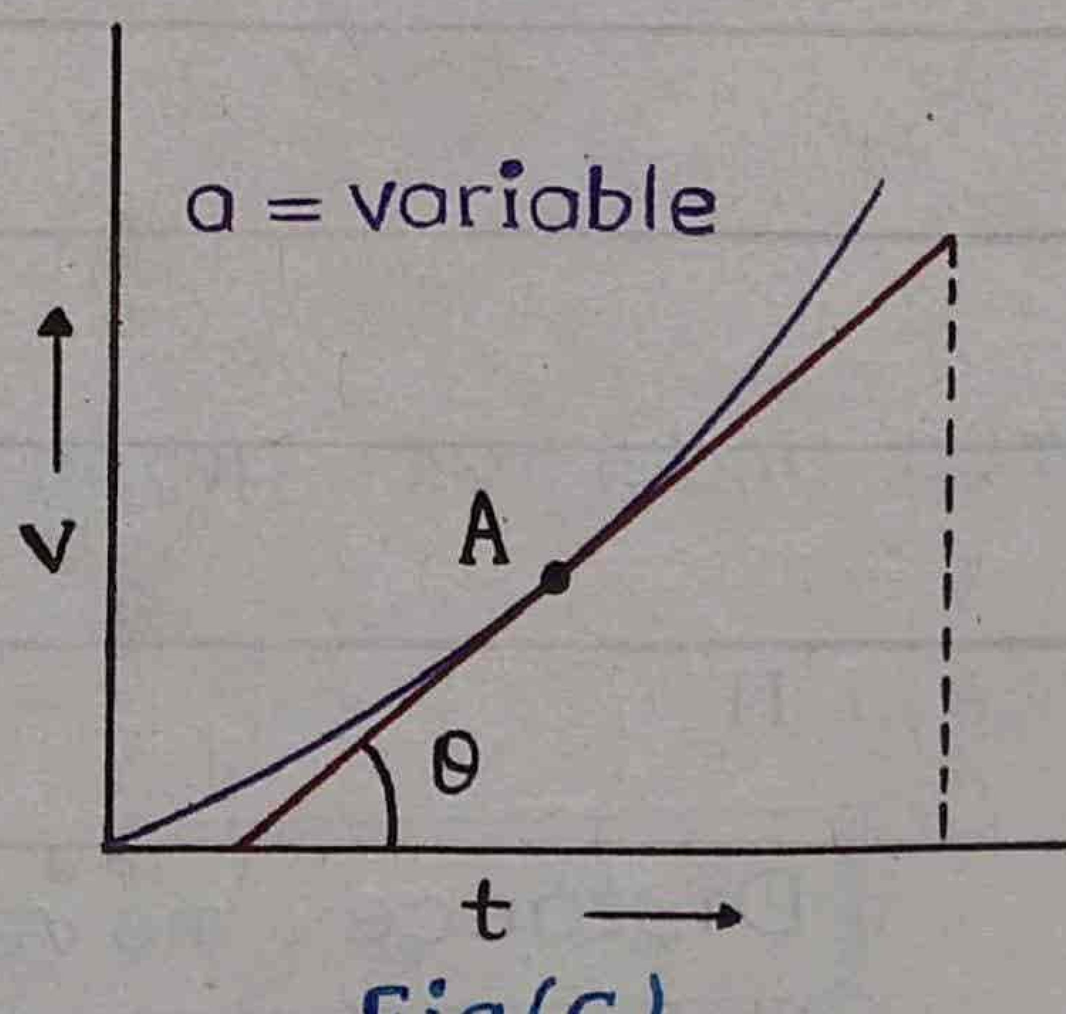
2- When the car moves with constant acceleration, the velocity-time graph is a straight line, which rises the same height for equal intervals of time. Fig(b).



Fig(b)

Note: The Slope of velocity-time graph is equal to the acceleration.

3- When the car moves with increasing acceleration velocity-time graph is a curve. Fig (c).



Fig(c)

The instantaneous acceleration of a body at a point "A" on the velocity-time graph is equal to the slope of tangent at the point "A".

Distance as Area under Velocity-time graph:

In Fig.(a) the body moves with constant velocity "v" for time "t".

$$\text{Distance} = S = vt \quad \text{——— (i)}$$

$$\text{Area under vt-graph} = vt \quad \text{——— (ii)}$$

So,

Distance moved = Area under vt-graph.

In Fig (b)

$$V_{avg} = \frac{0+V}{2} = \frac{1}{2}V$$

$$\text{Distance} = S = V_{avg} \times t = \frac{1}{2} V \times t \quad \text{——— (i)}$$

$$\text{Area under vt-graph} = \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2} t \times V$$

$$\text{Area under vt-graph} = \frac{1}{2} vt \quad \text{——— (ii)}$$

From I, II

$$\boxed{\text{Distance moved} = \text{Area under vt-graph}}$$

3.5 Equations of Uniformly Accelerated motion:

$$V_f = V_i + at$$

$$S = V_{avg} \times t = \left(\frac{V_i + V_f}{2}\right) \times t$$

$$S = V_i t + \frac{1}{2} a t^2$$

$$2aS = V_f^2 - V_i^2$$

For freely falling body $a = g$ $g = 9.8\ ms^{-2}$

Note: In all these equations "a = constant".

3.6 Newton's Laws of Motion:

Newton's laws are applicable when speed of bodies is small as compared to the speed of light "c" $(v << c)$.

$(c = 3 \times 10^8\ ms^{-1} = 300{,}000\ Kms^{-1})$.

For very fast moving objects such as atomic particles "Relativistic Mechanics" developed by Einstein is applicable. (Here Newton's Laws Fail).

Newtons First Law of Motion:

Law of inertia

"A body at rest will remain at rest and a body moving with uniform velocity will continue to do so, unless acted on by some unbalanced external force."

Inertia:

"The property of a body tending to maintain the state of rest or state of uniform motion is called its inertia".

Note:

"The mass of an object is a quantitative measure of its inertia."

The Frame of reference in which Newton's First Law of motion holds is known as inertial frame of reference.

A Frame of reference stationed on Earth is approximately an inertial Frame of reference.

Newton's Second Law of Motion

"Whenever a net force acts on a body, it produced acceleration in its own direction. This acceleration produced is directly proportional to the force and is inversely proportional to the mass of body."

$$\boxed{\vec{F} = m\vec{a}}$$

Newton $= kg \times \frac{m}{s^2}$

$F = ma$

$F = M.LT^{-2}$

$[F] = [MLT^{-2}]$

Unit of force is Newton (N).

Dimensions of force is $[MLT^{-2}]$

Newton's Third Law of Motion

"Action and reaction are equal but opposite in direction."

Note:

Action and reaction force never act on the same body, they act on two different interacting bodies.

Examples: (i) Flying of birds.

(ii) Motion of a rocket.

(iii) Rowing of a boat.

3.7 Momentum

"The product of mass "m" and linear velocity $\vec{v}$ of a body is called momentum or linear momentum."

It is denoted by "P".

$$\boxed{\vec{P} = m\vec{v}}$$

Momentum is a vector quantity. Its direction is same as that of velocity.

SI Unit:

(i) $kg \times ms^{-1}$

(ii) $N \times s$

$$kg \times \frac{m}{s} = kg \times \frac{m}{s} \times \frac{s}{s}$$

$$kg \times \frac{m}{s} = kg \times \frac{m}{s^2} \times s$$

$$kg \times \frac{m}{s} = N \times s$$

Dimensions:

$[MLT^{-1}]$

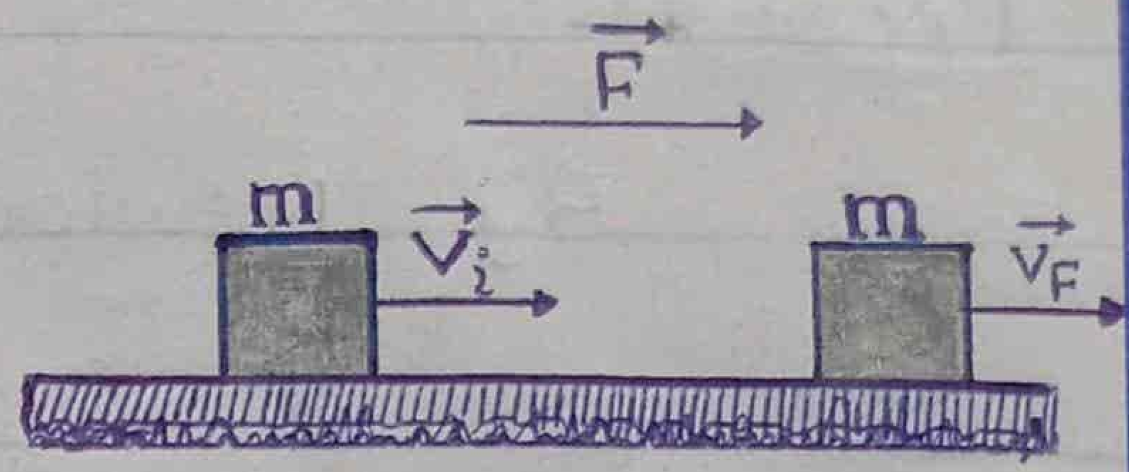
"Quantity of Motion of a body is called Momentum."

Newton's 2nd Law of Motion in terms of Momentum

Consider a body of mass "m" moving with initial velocity $\vec{v_i}$. A force $\vec{F}$ acts on the body for a time "t" and changes the velocity to $\vec{v_f}$.

The acceleration $\vec{a}$ produced is

$$\vec{a} = \frac{\Delta\vec{V}}{\Delta t} = \left(\frac{\vec{V}_f - \vec{V}_i}{t}\right)$$



$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\left(\frac{\vec{V}_f - \vec{V}_i}{t}\right)$$

$$\vec{F} = \frac{m\vec{V}_f - m\vec{V}_i}{t}$$

$m\vec{V}_i$ = Initial Momentum

$m\vec{V}_f$ = Final Momentum

$m\vec{V}_f - m\vec{V}_i$ = Change in Momentum

$m\vec{V}_f - m\vec{V}_i = \Delta\vec{P}$

$$\boxed{\vec{F} = \frac{\Delta\vec{P}}{\Delta t}}$$

Force = Rate of change of Momentum

Statement:

"Time rate of change of momentum of a body is equal to the applied Force."

Impulse:

"Product of Force and time is called impulse."

$$\boxed{\text{Impulse} = \vec{F} \times t}$$

Unit:

(i) N × S

(ii) $kg\,m\,s^{-1}$

Dimensions:

$$[MLT^{-1}]$$

Proof:

By Newton 2^{nd} Law

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t}$$

$$\vec{F} \times t = m\vec{v}_f - m\vec{v}_i$$

$$\boxed{\text{Impulse} = \text{Change in Momentum}}$$

Note:

Impulse and momentum have same units and same dimensions.

Law of Conservation of Momentum

Statement:

"Total linear momentum of an isolated system remains constant."

Isolated System:

"Isolated system is that on which no external force acts."

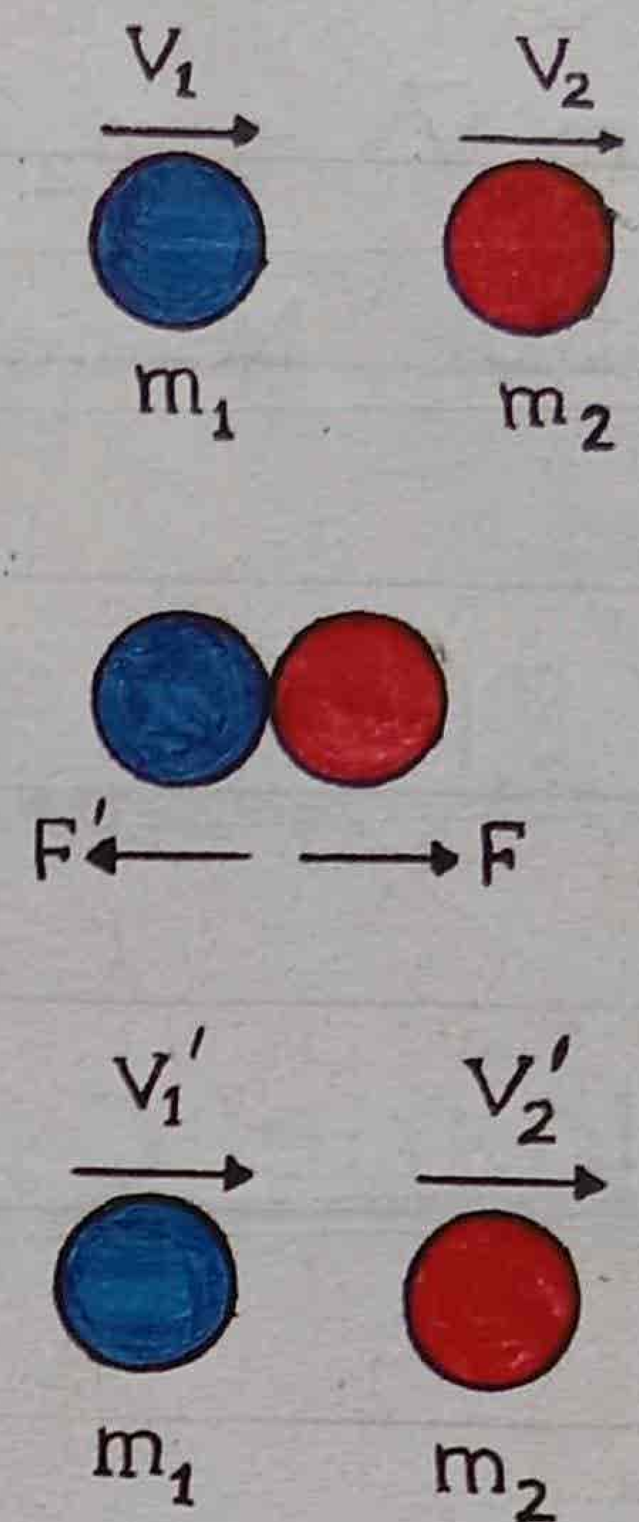
Explanation:

Consider an isolated system of two interacting balls moving in the same direction.

$\vec{V_1}$ and $\vec{V_2}$ are the velocities of the balls of masses m_1 and m_2 before collision. $\vec{V_1'}$, $\vec{V_2'}$ after collision.

Let

$$\vec{V_1} > \vec{V_2}$$



After collision they move along the same direction.

By Newton's 2nd Law

$$\vec{F} = \frac{m\vec{v_f} - m\vec{v_i}}{t}$$

$$\vec{F} \times t = m\vec{v_f} - m\vec{v_i}$$

Impulse = Change in momentum

1- Change in momentum for first ball

$$\vec{F} \times t = m_1\vec{V_1'} - m_1\vec{V_1} \quad \text{———} \quad (i)$$

2- Change in momentum for second ball

$$\vec{F'} \times t = m_2\vec{V_2'} - m_2\vec{V_2} \quad \text{———} \quad (ii)$$

Add (i) and (ii), we have

$$\vec{F} \times t + \vec{F'} \times t = (m_1\vec{V_1'} - m_1\vec{V_1}) + (m_2\vec{V_2'} - m_2\vec{V_2})$$

$$(\vec{F} + \vec{F'})t = m_1\vec{V_1'} - m_1\vec{V_1} + m_2\vec{V_2'} - m_2\vec{V_2}$$

As $\vec{F'} = -\vec{F}$ (By Third Low)

$$[\cancel{\vec{F}} + (\cancel{-\vec{F}})]\,t = m_1\vec{v}_1' - m_1\vec{v}_1 + m_2\vec{v}_2' - m_2\vec{v}_2$$

$$(0)\,t = m_1\vec{v}_1' - m_1\vec{v}_1 + m_2\vec{v}_2' - m_2\vec{v}_2$$

$$0 = m_1\vec{v}_1' - m_1\vec{v}_1 + m_2\vec{v}_2' - m_2\vec{v}_2$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\boxed{\text{Initial Momentum} = \text{Final Momentum}}$$

3.8 Elastic and Inelastic Collisions

Elastic Collision:

"A collision in which K.E of the system is conserved is called elastic collision."

Here, No K.E is lost.

Inelastic Collision:

"A collision in which K.E of the system is not conserved is called inelastic collision."

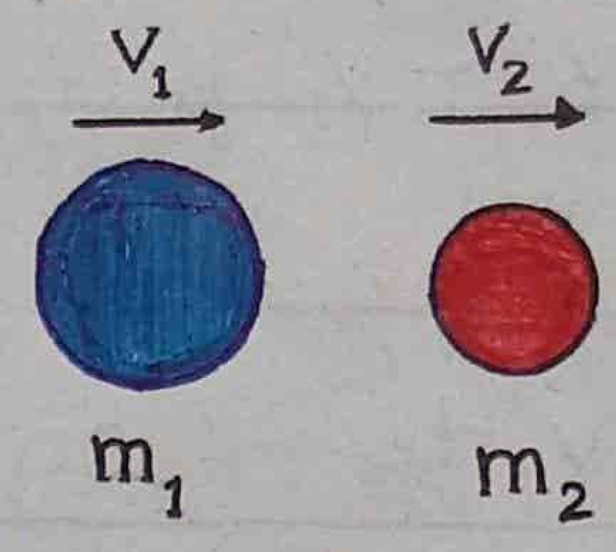
Here

Some K.E is lost and is conserved into other forms of energy.

Note:

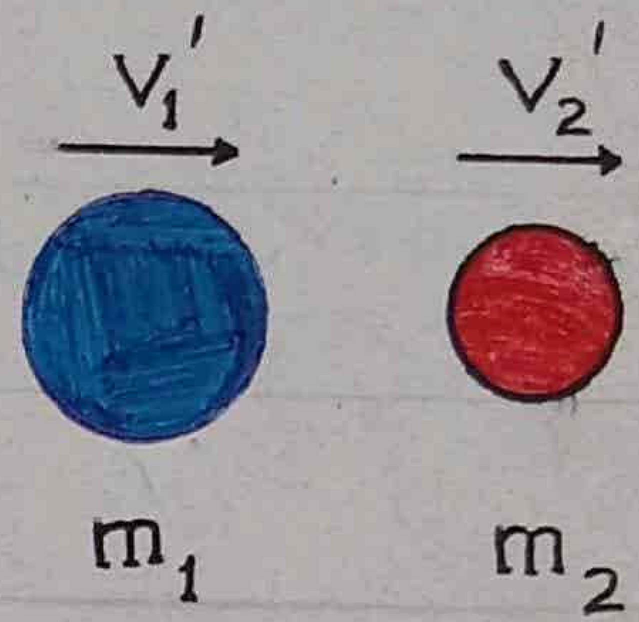
"Momentum and total energy is conserved in both types of collision."

Elastic Collision in one Dimension



Before collision

Consider two non-rotating, smooth balls of masses m_1 and m_2 moving with initial velocities v_1 and v_2 in the same direction. $(v_1 > v_2)$.



After collision

After collision they move with velocities v_1' and v_2' along the same straight line.

Using Law of conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2) \quad \text{———— (i)}$$

Using Conservation of kinetic Energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$\frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_1 v_1'^2 = \frac{1}{2} m_2 v_2'^2 - \frac{1}{2} m_2 v_2^2$$

$$\cancel{\frac{1}{2}} m_1 (v_1^2 - v_1'^2) = \cancel{\frac{1}{2}} m_2 (v_2'^2 - v_2^2)$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2) \quad \therefore a^2 - b^2 = (a-b)(a+b)$$

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2' + v_2) \quad \text{———— (ii)}$$

Divide equation (ii) by (i),

$$\frac{\cancel{m_1}\cancel{(v_1 - v_1')}(v_1 + v_1')}{\cancel{m_1}\cancel{(v_1 - v_1')}} = \frac{\cancel{m_2}\cancel{(v_2' - v_2)}(v_2' + v_2)}{\cancel{m_2}\cancel{(v_2' - v_2)}}$$

$$v_1 + v_1' = v_2' + v_2$$

$$v_1 - v_2 = v_2' - v_1'$$

$$v_1 - v_2 = -(v_1' - v_2')$$

$(v_1 - v_2)$ = Relative velocity of balls before collision

$(v_1' - v_2')$ = Relative velocity of balls after collision

Hence

"The relative velocities of the balls before and after collision are equal but opposite in direction."

Solving equation (i) and (ii), we get v_1' and v_2'.

$$\boxed{v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2} \quad \text{———— (A)}$$

$$\boxed{v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2} \quad \text{———— (B)}$$

Special Cases

Case - I

When masses of two balls are equal.

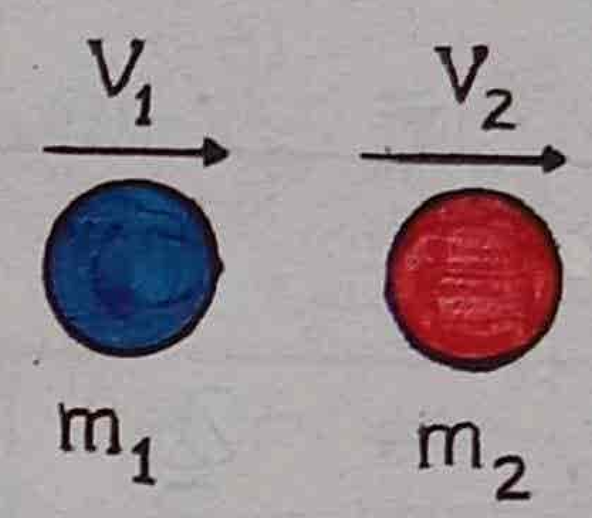
$$m_1 = m_2 = m$$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 + \left(\frac{2m_2}{m_1 + m_2}\right)V_2$$

$$V_1' = \left(\frac{\cancel{m} - \cancel{m}}{m + m}\right)V_1 + \left(\frac{2m}{m + m}\right)V_2$$

$$V_1' = (0)V_1 + \left(\frac{\cancel{2m}}{\cancel{2m}}\right)V_2$$

$$\boxed{V_1' = V_2}$$



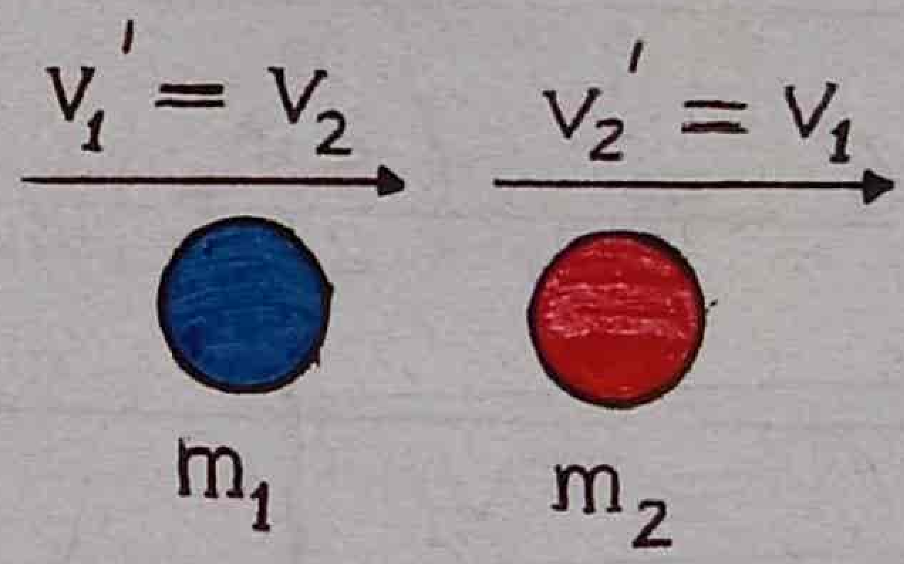
Before collision

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)V_2$$

$$V_2' = \left(\frac{2m}{m + m}\right)V_1 + \left(\frac{\cancel{m} - \cancel{m}}{m + m}\right)V_2$$

$$V_2' = \left(\frac{\cancel{2m}}{\cancel{2m}}\right)V_1 + (0)V_2$$

$$\boxed{V_2' = V_1}$$



After collision

Result:

"After collision velocities of the balls are interchaged."

Case-2: When first ball collide with the second ball of same mass at rest:

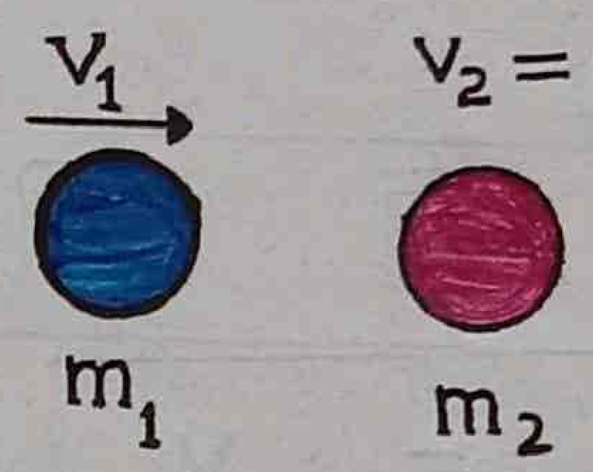
$$m_1 = m_2 = m \quad ; \quad v_2 = 0$$

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$v_1' = \left(\frac{\cancel{m} - \cancel{m}}{m + m}\right)v_1 + \left(\frac{2m}{m + m}\right)v_2$$

$$v_1' = (0)\,v_1 + \left(\frac{\cancel{2m}}{\cancel{2m}}\right) \times 0$$

$$\boxed{v_1' = 0}$$



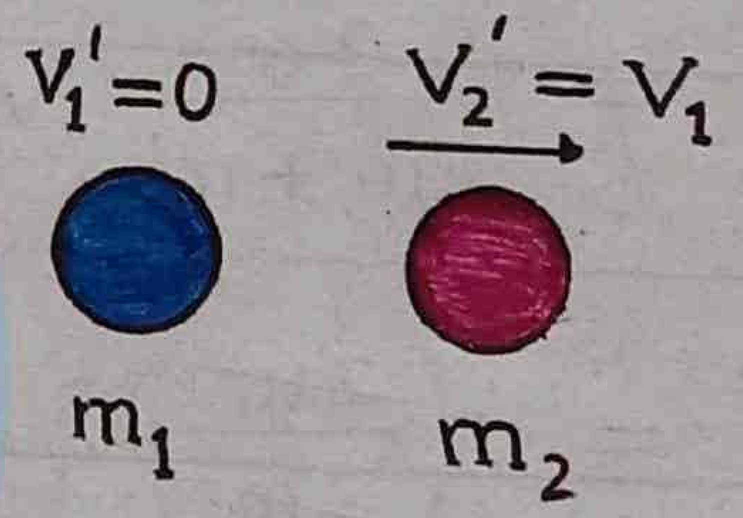
Before collision

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2$$

$$v_2' = \left(\frac{2m}{m + m}\right)v_1 + \left(\frac{m - m}{m + m}\right) \times 0$$

$$v_2' = \left(\frac{\cancel{2m}}{\cancel{2m}}\right)v_1 + 0$$

$$\boxed{v_2' = v_1}$$



After collision

Result:

"After collision 1^{st} ball comes to rest and the 2^{nd} ball moves with the velocity of the 1^{st} ball v_1."

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Case-3:

When lighter ball collides with a very heavy ball at rest:

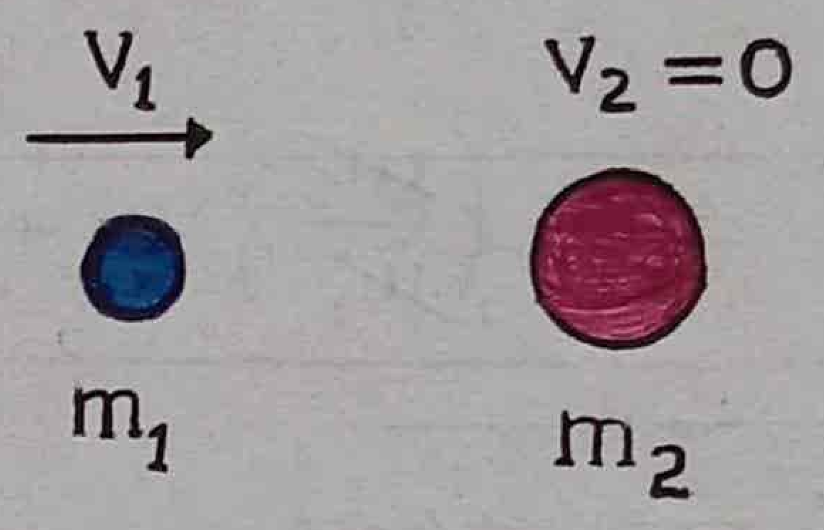
$$m_1 << m_2 \quad ; \quad m_1 \simeq 0 \quad ; \quad V_2 = 0$$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 + \left(\frac{2m_2}{m_1 + m_2}\right)V_2$$

$$= \left(\frac{0 - m_2}{0 + m_2}\right)V_1 + \left(\frac{2m_2}{0 + m_2}\right)\times 0$$

$$= \left(-\frac{\cancel{m_2}}{\cancel{m_2}}\right)V_1 + 0$$

$$\boxed{V_1' = -V_1}$$



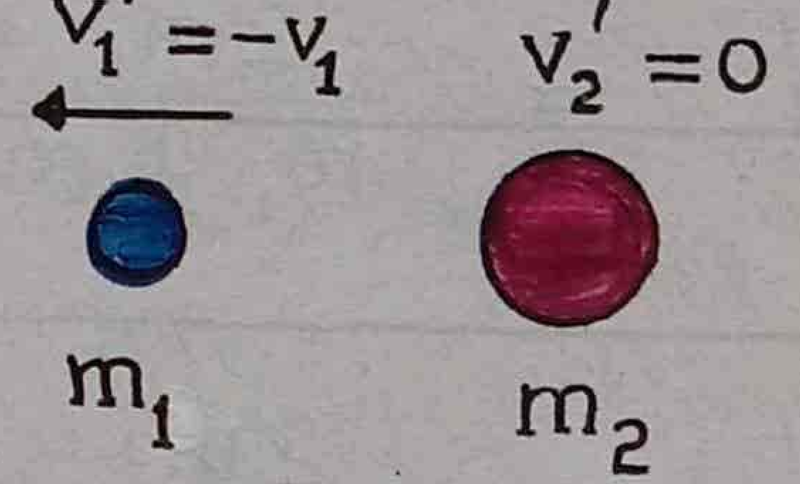
Before collision

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)V_2$$

$$V_2' = \left(\frac{2(0)}{0 + m_2}\right)V_1 + \left(\frac{m_2 - 0}{0 + m_2}\right)\times 0$$

$$V_2' = 0 + 0$$

$$\boxed{V_2' = 0}$$



After collision

Result:

"The lighter ball moves back with the same initial speed and the second ball remains at rest."

Case-4:

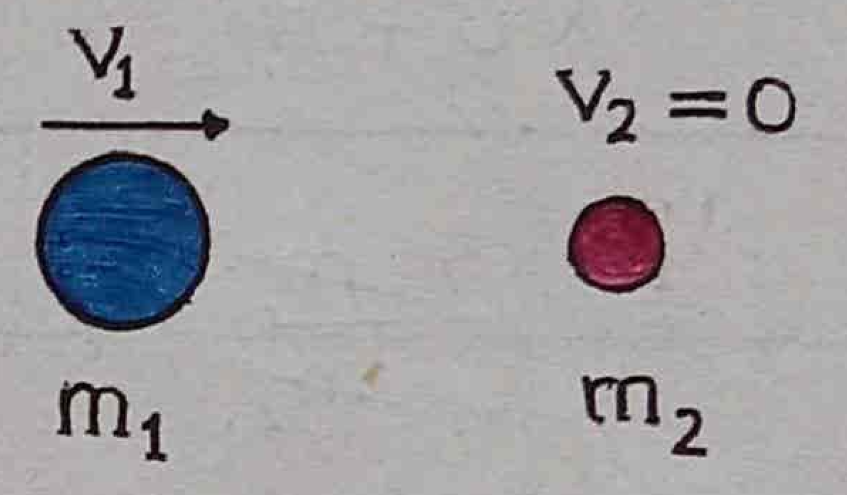
When a very heavy ball collides with a lighter ball at rest:

$$m_1 >> m_2 \quad ; \quad m_2 \simeq 0 \quad ; \quad V_2 = 0$$

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 + \left(\frac{2m_2}{m_1 + m_2}\right)V_2$$

$$V_1' = \left(\frac{m_1 - 0}{m_1 + 0}\right)V_1 + \left(\frac{2(0)}{m_1 + 0}\right) \times 0$$

$$V_1' = \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right)V_1 + 0 \times 0$$



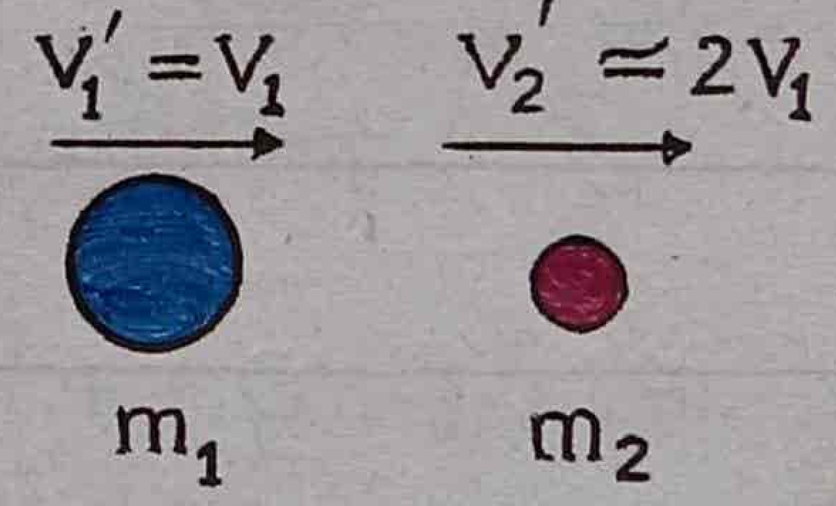
Before Collision

$$\boxed{V_1' = V_1}$$

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right)V_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)V_2$$

$$V_2' = \left(\frac{2m_1}{m_1 + 0}\right)V_1 + \left(\frac{0 - m_1}{m_1 + 0}\right) \times 0$$

$$V_2' = \left(\frac{2\cancel{m_1}}{\cancel{m_1}}\right)V_1 + 0 \times 0$$



After Collision

$$\boxed{V_2' = 2V_1}$$

Result:

"The heavy ball moves with the same initial velocity and the lighter ball moves double the initial velocity of the first ball."

3.9 Force due to water Flow

When water from a horizontal pipe strikes a wall normally, it exerts a force on the wall.

Suppose initial velocity of water $= v_i = v$

water comes to rest after striking the wall $v_f = 0$.

Let "m" mass of water strikes the wall in time "t".

Apply second law of motion

$$F = \frac{mv_f - mv_i}{t} = \frac{m(v_f - v_i)}{t}$$

$$F = \frac{m}{t}(v_f - v_i) = \frac{m}{t}(0 - v_i) \qquad \therefore v_i = v$$

$$F = -\frac{m}{t}v$$

By Newton's third law, reaction force exerted by the water on the wall is equal and opposite.

$$F = -\left(-\frac{m}{t}v\right)$$

$$F = \left(\frac{m}{t}\right)v$$

$\frac{m}{t}$ = mass per second

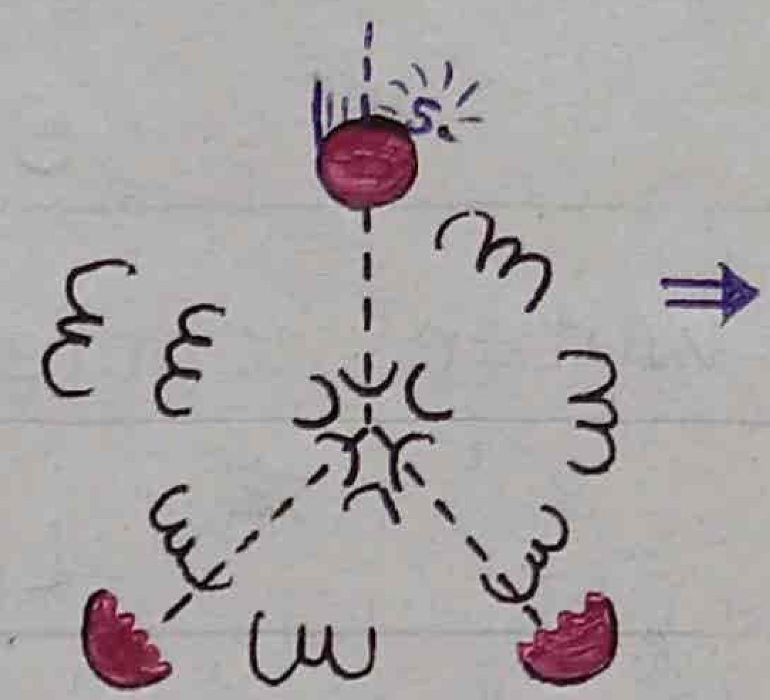
v = change in velocity

$$\boxed{\text{Force} = (\text{mass per second}) \times \text{change in velocity}}$$

3.10 Momentum and Explosive Forces

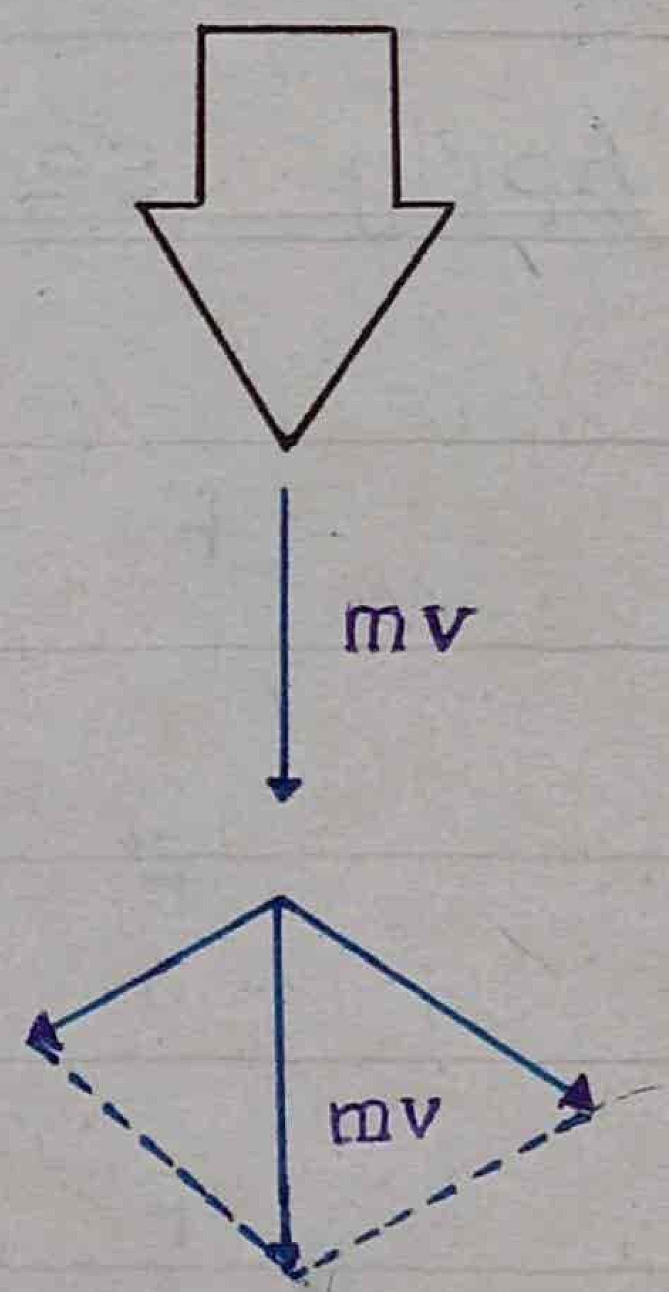
There are many examples in which momentum changes are produced by explosive forces within an isolated system.

Example-1:



When a shell explodes in a mid air, parts fly off in different directions. The total momentum of all of its parts is equal to the initial momentum of the shell.

Suppose a flying bomb breaks up into two parts. It is shown in fig.



(Momentum before explosion mv) = (Sum of momentum of two parts after explosion)

Example-2:

Consider a bullet of mass "m" is fired from a gun of mass "M". The initial velocity of bullet and gun = zero.
The final velocity of bullet is V and that of gun is V'.

Using Law of Conservation of Momentum

Initial Momentum = Final Momentum

$$0 + 0 = mv + Mv'$$

$$Mv' = -mv$$

This shows that momentum of gun is equal and opposite to that of bullet.

$$\frac{v'}{v} = -\frac{m}{M}$$

$$\boxed{v' = -\frac{mv}{M}}$$

Velocities are inversely proportional to the masses:

Since mass of gun is much greater than the mass of bullet, so the gun moves back (recoils جھٹکا لگنا) with very small velocity as compared to the velocity of the bullet.

3.11 Rocket Propulsion:

"The motion of a rocket is an application of law of conservation of momentum and the third law of motion."

Rocket moves up by ejecting hot gases at the tail of the rocket with very high velocity in the downward direction. The rocket gains momentum equal to the momentum of the gas ejected from the tail of the rocket but in opposite direction. (By third Law of Motion)

A rocket carries its own fuel in the form of a liquid or solid Hydrogen and Oxygen.

A typical rocket consumes about $10000\ kg\,s^{-1}$ ($1\times10^4\ kg\,s^{-1}$) of fuel and ejects gases at speed more than $4000\ ms^{-1}$ ($4\ Kms^{-1}$). More than 80% of total mass of rocket is the mass of fuel.

In multi-stage rockets, when fuel of one rocket is burnt, it is discarded (علیحدہ کرنا). So, the remaining rocket moves with greater speed.

m = mass of gases ejected per second.

$\vec{v}$ = Velocity of gases relative to the rocket.

$m\vec{v}$ = Change of momentum of gases per second.

OR $m\vec{V}$ = Upward thrus (Force) on the rocket.

M = Mass of the rocket.

$\vec{a}$ = Acceleration of the rocket is

$\because F = ma$

$a = \frac{F}{m}$

$$\text{Acceleration} = \frac{\text{Force}}{\text{mass}}$$

$$\vec{a} = \frac{m\vec{V}}{M}$$

"With the passage of time mass 'M" of the rocket decreases, so acceleration $\vec{a}$ of the rocket increases."

Motion in one dimension:

The motion of a body in a straight line is called motion in one dimension. Only one coordinate x-coordinate or y-coordinate or z-coordinate is required to describe the motion.

Motion in two dimension:

Motion in a plane such as xy-plane is called motion in two dimension. Two coordinates (x, y) are required describe the motion.

3.12 Projectile Motion

"Projectile motion is two dimensional motion under constant acceleration due to gravity."

Projectile:

"When a body is thrown with some angle θ with the horizontal and moves freely under the action of gravity is called a projectile."

The path followed by a projectile is called its 'trajectory'.

Path of a projectile is a parabola

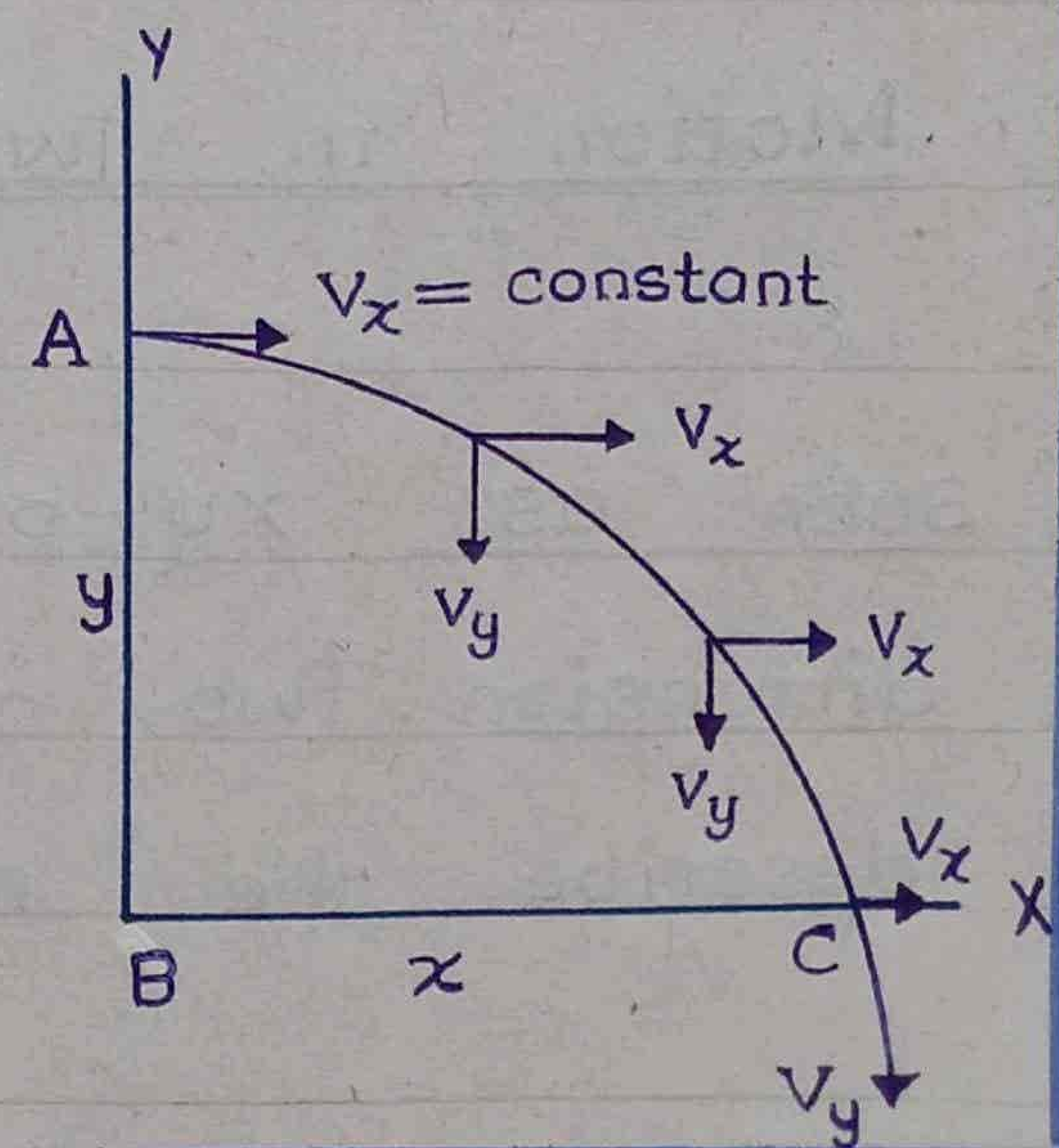
Examples:

(i) A Football kicked off by a player.

(ii) A ball thrown by a crickter.

(iii) A bullet fired from a gun.

1- Horizontal Motion:



Consider a ball is thrown from a point "A" with some horizontal component of velocity V_x.

Assume air resistance is zero. No Force acting along x-axis ($a_x = 0$).

So

Horizontal component of velocity "V_x" = constant

$$S = v_{ix}t + \frac{1}{2}a_x t^2$$

$a_x = 0$

$v_{ix} = V_x$

$S = x$

$$\boxed{x = v_x t}$$

2- Vertical Motion:

The initial component of velocity $v_{iy} = 0$, $a_y = -g$, $S = -y$

Take:

Upward direction = +ve

Downward direction = −ve

$$S = v_{iy}t + \frac{1}{2}a_y t^2$$

$$y = 0 + \frac{1}{2}(-g)t^2$$

$$\boxed{y = -\frac{1}{2}gt^2}$$

Now consider a projectile is thrown with initial velocity 'v_i' making an angle 'θ' with the horizontal.

Its motion is composed of two independent motions.

1- Horizontal motion with constant velocity = V_x

$$a_x = 0$$

2- Vertical motion with constant acceleration due to gravity

$$a_y = -g$$

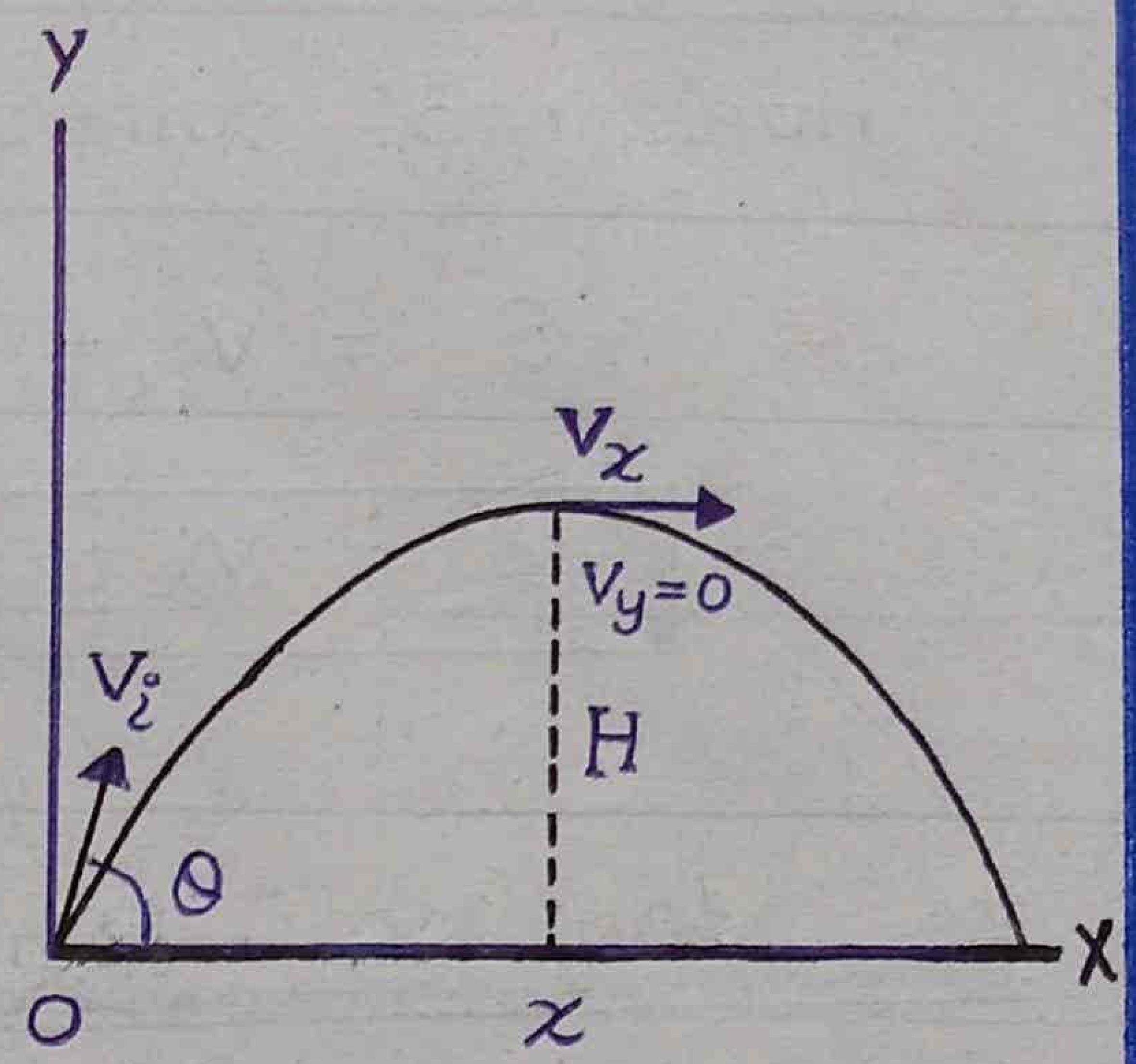
$$v_f = v_i + at$$

$$v_{fx} = v_{ix} + a_x t \qquad (a_x = 0)$$

$$v_{fx} = v_{ix} + (0)t \qquad (v_{ix} = v_i \cos\theta)$$

$$v_{fx} = v_{ix}$$

$$\boxed{v_{fx} = v_i \cos\theta}$$



$$v_f = v_i + at$$

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = v_i \sin\theta + (-g)t \qquad (a_y = -g)$$

$$(v_{iy} = v_i \sin\theta)$$

$$\boxed{v_{fy} = v_i \sin\theta - gt}$$

(i) Magnitude of velocity at any instant

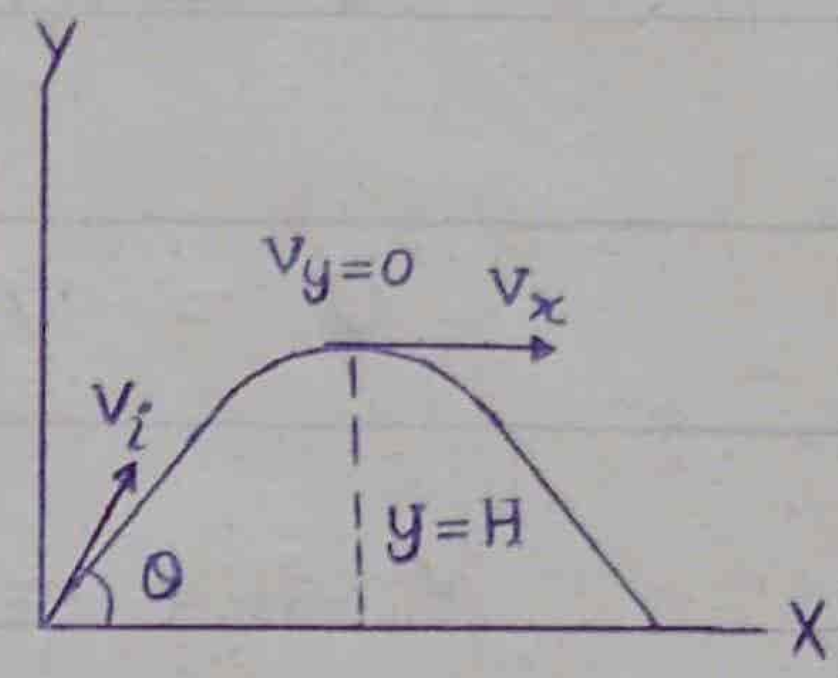
$$\boxed{v = \sqrt{v_x^2 + v_y^2}}$$

(ii) If ϕ be the angle made by velocity v at any instant with x-axis.

$$\boxed{\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)}$$

Maximum Height of the Projectile:

"At maximum height vertical component of velocity is zero."



$a_y = -g$

$V_{iy} = V_i \sin\theta$

$$2aS = V_f^2 - V_i^2$$

$$2a_y H = V_{fy}^2 - V_{iy}^2$$

$$2(-g)H = 0 - (V_i \sin\theta)^2$$

$$-2gH = -V_i^2 \sin^2\theta$$

$$\boxed{H = \frac{V_i^2 \sin^2\theta}{2g}}$$

Note:

At maximum height the only component of velocity is $V_x = V_i \cos\theta$ and here $V_y = 0$.

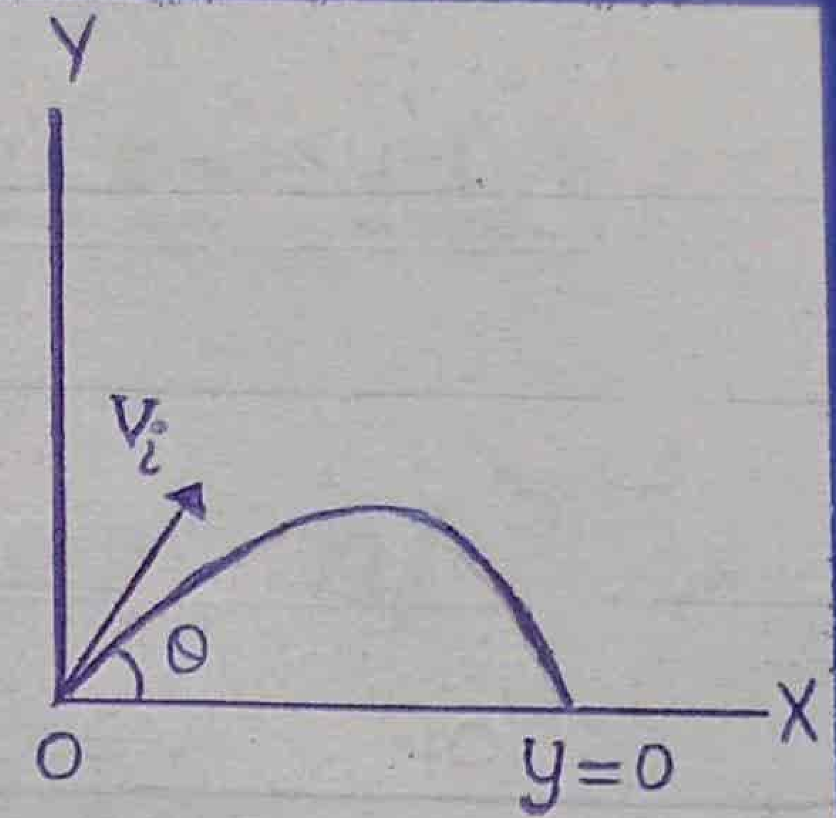
Time of Flight:

Since the projectile returns to the same level, the vertical distance covered by the projectile is zero.

$$S = V_i t + \frac{1}{2} a t^2$$

$$y = V_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = v_{iy} t + \frac{1}{2}(-g)t^2$$



$$0 = v_i \sin\theta \cdot t - \frac{1}{2} g t^2$$

$$0 = \left(v_i \sin\theta - \frac{1}{2} g t\right) t$$

$$0 = v_i \sin\theta - \frac{1}{2} g t \quad , \quad t \neq 0$$

$$v_i \sin\theta = \frac{1}{2} g t$$

or

$$\frac{1}{2} g t = v_i \sin\theta$$

$$g t = 2 v_i \sin\theta$$

$$\boxed{t = \frac{2 v_i \sin\theta}{g}}$$

Horizontal Range = R

"Horizontal distance covered by the projectile is called horizontal range. ($x = R$)."

$$S = v_i t + \frac{1}{2} a t^2$$

$$x = v_{ix} t + \frac{1}{2} a_x t^2 \qquad (a_x = 0)$$

$$x = v_{ix} t + \frac{1}{2}(0) t^2 \qquad \left(t = \frac{2 v_i \sin\theta}{g}\right)$$

$$x = v_{ix} \cdot t \qquad (x = R)$$

$$x = v_{ix} \cdot t$$

$$R = v_i \cos\theta \cdot \left(\frac{2 v_i \sin\theta}{g}\right)$$

$$R = \frac{v_i^2 \cdot (2\sin\theta\cos\theta)}{g}$$ $\because 2\sin\theta\cos\theta = \sin 2\theta$

$$\boxed{R = \frac{v_i^2 \sin 2\theta}{g}}$$

Maximum Range = R_{max}

"Range is maximum when $\sin 2\theta$ has maximum value = 1"

$$\sin 2\theta = 1$$

But $\sin 90^\circ = 1$

Comparing

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

Note:

So, range of projectile is maximum at $\theta = 45^\circ$.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R_{max} = \frac{v_i^2 \sin 2\times 45^\circ}{g}$$

$$R_{max} = \frac{v_i^2 \sin 90^\circ}{g}$$ $(\sin 90^\circ = 1)$

$$\boxed{R_{max} = \frac{v_i^2}{g}}$$

Note:

For angles which exceed (بڑھ جانا) or fall short (کم ہو جانا) of 45° by equal amounts, the ranges are equal.

Let

$$\theta = 45^\circ - 15^\circ = 30^\circ$$

$$\theta = 45^\circ + 15^\circ = 60^\circ$$

At angles $(30^\circ, 60^\circ)$ ranges are equal.

Similarly, we can make pairs such as $(40^\circ, 50^\circ)$, $(35^\circ, 55^\circ)$, $(20^\circ, 70^\circ)$ etc.

Note:

At $\theta = 76^\circ$

Height and Range of a projectile are equal.

$$H = R$$

Note:

$$R = R_{max} \sin 2\theta$$

Application to Ballistic Missiles

"An un-powered and un-guided missile is called a ballistic missile and the path followed by it is called ballistic trajectory."

"A ballistic flight is that in which a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity."

A ballistic missile moves as a result of superposition of two independent motions.

1- A straight line inertial flight in the direction of launch.

2- A vertical gravity fall.

At high speeds and for long trajectories, the air friction is not neglible and some times the force of air friction is more than gravity. It effects both the horizontal and vertical motions.

Shooting a missile on a selected distont (دور) spot is a major element of warfawe (جنگ).

The actual flights of missiles are found to high degree of pression. The modified equations of trajectory are very complicated.

Note:

"The ballistic missiles are useful for short ranges."

"For long ranges and greater precession powered and remote control guided missiles are used."

QUESTIONS

3.1 What is the difference between uniform and variable velocity? From the explanation of variable velocity, define acceleration. Give SI units of velocity and acceleration. imp

3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

3.3 Can the velocity of an object reverse the direction when acceleration is constant? If so, give an example. imp

3.4 Specify the correct statements:

a. An object can have a constant velocity even its speed is changing.

b. An object can have a constant speed even its velocity is changing.

c. An object can have a zero velocity even its acceleration is not zero.

d. An object subjected to a constant acceleration can reverse its velocity.

3.5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have larger speed when it strikes the ground? Ignore air friction.

3.6 Explain the circumstances in which the velocity **v** and acceleration **a** of a car are

(i) Parallel (ii) Anti-parallel (iii) Perpendicular to one another

(iv) **v** is zero but **a** is not zero (v) **a** is zero but **v** is not zero

3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

3.9 Define impulse and show that how it is related to linear momentum?

3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

3.12 Explain what is meant by projectile motion. Derive expressions for

a. the time of flight b. the range of projectile.

Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

3.13 imp At what point or points in its path does a projectile have its minimum speed, its maximum speed?

3.14 Each of the following questions is followed by four answers, one of which is correct answer. Identify that answer.

i. What is meant by a ballistic trajectory?

a. The paths followed by an un-powered and unguided projectile.

b. The path followed by the powered and unguided projectile.

c. The path followed by un-powered but guided projectile.

d. The path followed by powered and guided projectile.

ii. What happens when a system of two bodies undergoes an elastic collision?

a. The momentum of the system changes.

b. The momentum of the system does not change

c. The bodies come to rest after collision.

d. The energy conservation law is violated.

Q-3.1: Uniform Velocity:

"When a body covers equal displacements in equal intervals of time, its velocity is uniform."

Variable Velocity:

"When a body covers unequal displacements in equal intervals of time velocity is variable."

Acceleration:

"Rate of change of velocity is called acceleration."

(i) Units of velocity are (ms^{-1}).

(ii) Units acceleration are (ms^{-2}).

Q-3.2: When an object is thrown verically upwards, its

(i) Upward velocity is taken as positive.

(ii) downward acceleration due to gravity is taken as negative.

They have opposite signs because their directions are opposite.

When the objects moves down, its acceleration and velocity both have same signs, because the have same direction.

$V = +ve$ $a = +g$

Q-3.3: Yes.

When an object is thrown vertically upwards its

(i) Velocity is upward.

(ii) acceleration due to gravity is downward

This object comes to rest at the heighest point, then it moves down. Now its velocity is reversed (downward). during its flight, the magnitude of acceleration due to gravity remains constant.

Q-3.4:

Correct Statements are (b), (c), (d). Statement (a) is not correct.

Q-3.5:

Both the balls will have same speed on reaching the ground.

Reason: When a body is thrown vertically upward with a certain initial velocity, it will come down at the same point with the same speed.

Q-3.6:

(i) When the velocity of the car is increasing its velocity $\vec{v}$ and acceleration $\vec{a}$ are parallel.

(ii) When the velocity of the car is decreasing, its velocity $\vec{v}$ and acceleration $\vec{a}$ are anti-parallel.

(iii) When the car moves along a circular path or curved path its velocity $\vec{v}$ is along the tangent and acceleration $\vec{a}$ is along the radius, so $\vec{a}$ and $\vec{v}$ are perpendicular.

(iv) When brakes are applied to stop a moving car it comes to rest. $\vec{v} = 0$ but acceleration $\vec{a}$ is not zero at that moment.

(v) When the car is moving with uniform velocity, $\vec{a}$ is zero but $\vec{v}$ is not zero.

Q-3.7: Yes.

When a body moves with constant

velocity its acceleration is zero, which remains constant. So, uniform velocity is a special case of motion with constant acceleration.

Q-3.8:

Consider a force $\vec{F}$ changes the velocity of a body from $\vec{v_i}$ to $\vec{v_f}$ in a time "t".

An acceleration $\vec{a}$ is produced in the body whose mass is "m".

$$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t}$$

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\left(\frac{\vec{v_f} - \vec{v_i}}{t}\right)$$

$$\vec{F} = \frac{m\vec{v_f} - m\vec{v_i}}{t}$$

$$\vec{F} \times t = m\vec{v_f} - m\vec{v_i}$$

$$\vec{F} \times t = \text{Change in Momentum}$$

$$\vec{F} = \frac{m\vec{v_f} - m\vec{v_i}}{t} = \frac{\text{change in momentum}}{\text{time}}$$

$$\boxed{\text{Force} = \text{Time rate of change of momentum}}$$

Statement:

"Force applied on a body is equal to the rate of change of momentum of the body."

$$\boxed{\vec{F} = \frac{\Delta\vec{P}}{\Delta t}}$$

This is Newton second law of motion.

Q-3.9:

Impulse:

"Product of force "$\vec{F}$" and and time "t" is called impulse."

$$\text{Impulse} = \vec{F} \times t$$

Relation between impulse and linear momentum

According to the 2^{nd} law of motion:

Force = Time rate of change of momentum

$$\vec{F} = \frac{m\vec{v_f} - m\vec{v_i}}{t}$$

$$\vec{F} \times t = m\vec{v_f} - m\vec{v_i}$$

$$\boxed{\text{Impulse} = \text{Change in Momentum}}$$

Q-3.10: Law of Conservation of Linear Momentum

Statement:

"It states that the total linear momentum of an isolated system remains constant."

Isolated System:

"A system on which no external force acts."

When the system is not isolated and external forces act on it. Then, if the resultant of external forces is zero, law of conservation of momentum holds good.

Also, if the external forces are neglible as compared to the interacting forces within the system, the law of conservation of momentum is still applicable.

For example, for a system of gas molecules inside a vessel, we apply the law of conservation of momentum, even force of gravity (which is an external force) is acting on the molecules. Here the force of gravity is neglected.

Q-3.11:

Elastic Collision:

"In this collision K.E of the system remains conserved."

e.g. In molecules of gas, there is elastic collision.

Inelastic Collision:

"In this collision, K.E of the system does not remain conserved (Some K.E is lost into other forms of energy)."

(i) In elastic collision the bouncing ball comes back to the same height.
i.e. no K.E is lost.

(ii) In inelastic collision, the bouncing ball does not come back to the same height. because some K.E is lost in the form of Height energy, Sound energy etc.

Non-conservation of K.E:

In most of the cases, K.E is not conserved because some of it is dissipated in doing work against friction or deforming (شکل بگاڑنا) the colliding body.

Q-3.12:

Projectile Motion

When a body is thrown with some initial velocity making a certain angle $\theta < 90°$ with horizontal direction. This two dimensional motion under gravity is called projectile motion.

Time of Flight:

"Time taken by the body to complete its projectile motion is called Time of Flight."

$$S = v_i t + \frac{1}{2} a t^2$$

$$y = v_{iy} t + \frac{1}{2} a_y t^2$$

$a_y = -g$

$y = 0$

$t \neq 0$

$$0 = v_{iy} \cdot t + \frac{1}{2}(-g) t^2$$

$$0 = v_{iy} \cdot t - \frac{1}{2} g t^2$$

$$0 = \left(v_{iy} - \frac{1}{2}gt\right)\cdot t$$

$$\left(v_{iy} - \frac{1}{2}gt\right) = 0$$

$$v_i \sin\theta - \frac{1}{2}\cdot gt = 0$$

$$\frac{1}{2}gt = v_i \sin\theta$$

$$gt = 2v_i \sin\theta$$

$$\boxed{t = \frac{2v_i \sin\theta}{g}}$$

Horizontal Range: R

"Horizontal distance covered by the projectile is called horizontal range."

As $$S = v_i t + \frac{1}{2}at^2$$

$$x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$x = v_{ix}t + \frac{1}{2}(0)t^2$$

$$x = v_{ix}t$$

$$R = v_i\cos\theta \times t$$

$$R = v_i\cos\theta \times \frac{2v_i\sin\theta}{g}$$

$a_x = 0$

$x = R$

$t = \frac{2v_i \sin\theta}{g}$

$2\sin\theta\cos\theta = \sin 2\theta$

$$R = \frac{v_i^2(2\sin\theta\cos\theta)}{g}$$

$$\boxed{R = \frac{v_i^2\sin 2\theta}{g}}$$

Range of Projectile is maximum at $\theta = 45°$

Range is

$$R = \frac{v_i^2\sin 2\theta}{g}$$

Range will be maximum if $\sin 2\theta$ has maximum value.

Maximum value of $\sin 2\theta = 1$

But $\sin 90° = 1$

Comparing

$$2\theta = 90°$$

$$\theta = \frac{90°}{2}$$

$$\theta = 45°$$

So, Range is maximum at $\theta = 45°$.

$$R_{max} = \frac{v_i^2\sin 2\times 45°}{g} = \frac{v_i^2\sin 90°}{g} = \frac{v_i^2(1)}{g}$$

$$\boxed{R_{max} = \frac{v_i^2}{g}}$$

Q-3.13:

(i) At the highest point velocity of the projectile is Maximum.

(ii) At the point of projection and point of landing on the ground velocity of the projectile is maximum.

Proof:

Velocity of the projectile at any instant is

$$v = \sqrt{V_x^2 + V_y^2}$$

(i) At the heighest point (maximum height) $V_y = 0$

$$v = \sqrt{V_x^2 + (0)^2} = \sqrt{V_x^2}$$

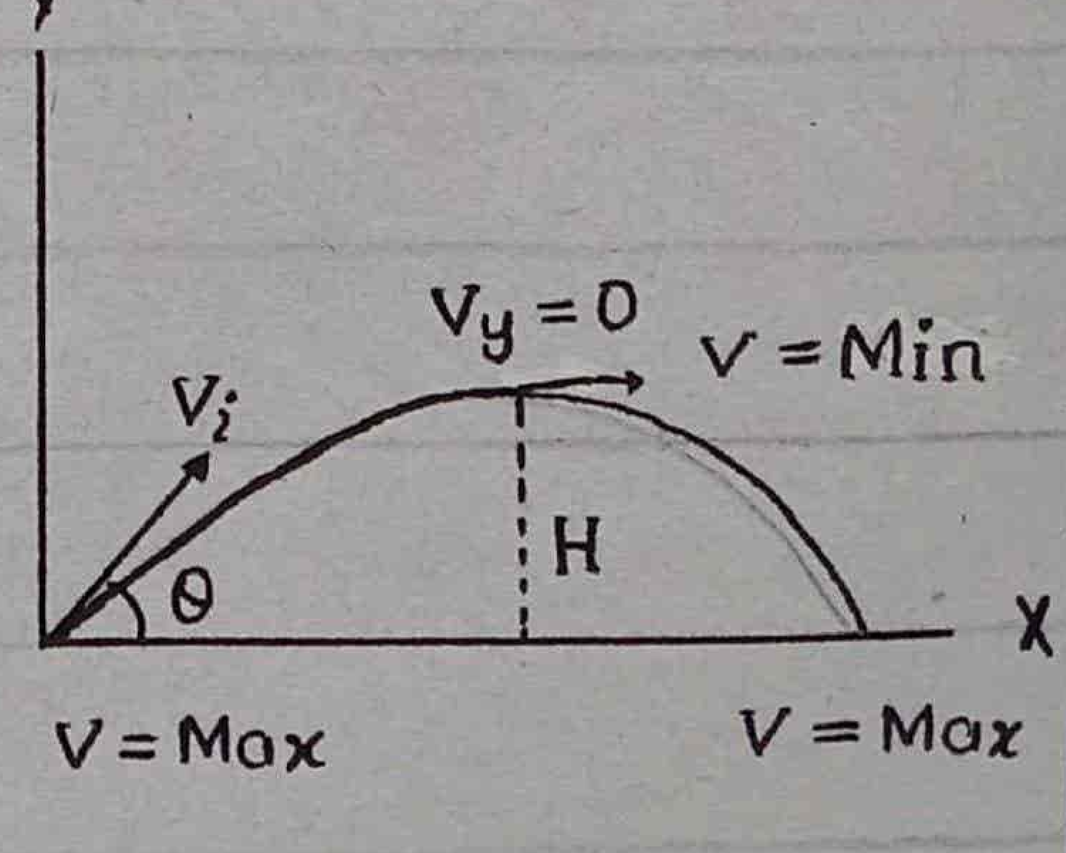
$$v = V_x$$

(ii) At the point of projection and point of landing V_y = Maximum

$$v = \sqrt{V_x^2 + V_y^2} = \text{Maximum value}$$

Note:

V_x remains constant during projectile motion.



Q-3.14:

Correct answer in (i) is (a).

Correct answer in (ii) is (b).