Chapter = 11

Heat and Thermodynamics

Thermodynamics:

"Thermodynamics deals with various phenomena of a energy and related properties of matter, especially transformation of Heat into other forms of energy."

11.1 Kinetic Theory of Gases

Kinetic Theory of gases explain the behaviour of gases on microscopic approach.

Evidences in favour of this theory are:

(i) diffusion of gases.

(ii) Brownian motion of smoke particles etc.

Postulates:

(i) A finite volume of gas conists of very Large number of molecules.

(ii) The size of the molecules is

smaller than the seperation between the molecules.

(iii) The gas molecules are in random motion and may change their direction of motion after every collision.

(iv) Collision between the gas molecules themselves and the walls of the container are perfectly elastic.

(v) Molecules do not exert force on each other except during collision (PE = 0 for ideal gas molecules.)

Pressure of the gas:

Pressure exerted by a gas is the momentum transferred to the walls of the container per second per unit area due to continous collision of gass molecules

Pressure = Force per unit area

OR

$$\left[P = \frac{F}{A} \right]$$

$$P = \frac{F}{A} = \frac{\Delta p \Delta t}{A}$$

$$P = \frac{\Delta P}{\Delta t \times A}$$

$$P = \frac{\text{momentum}}{\text{Sec} \times \text{Area}}$$

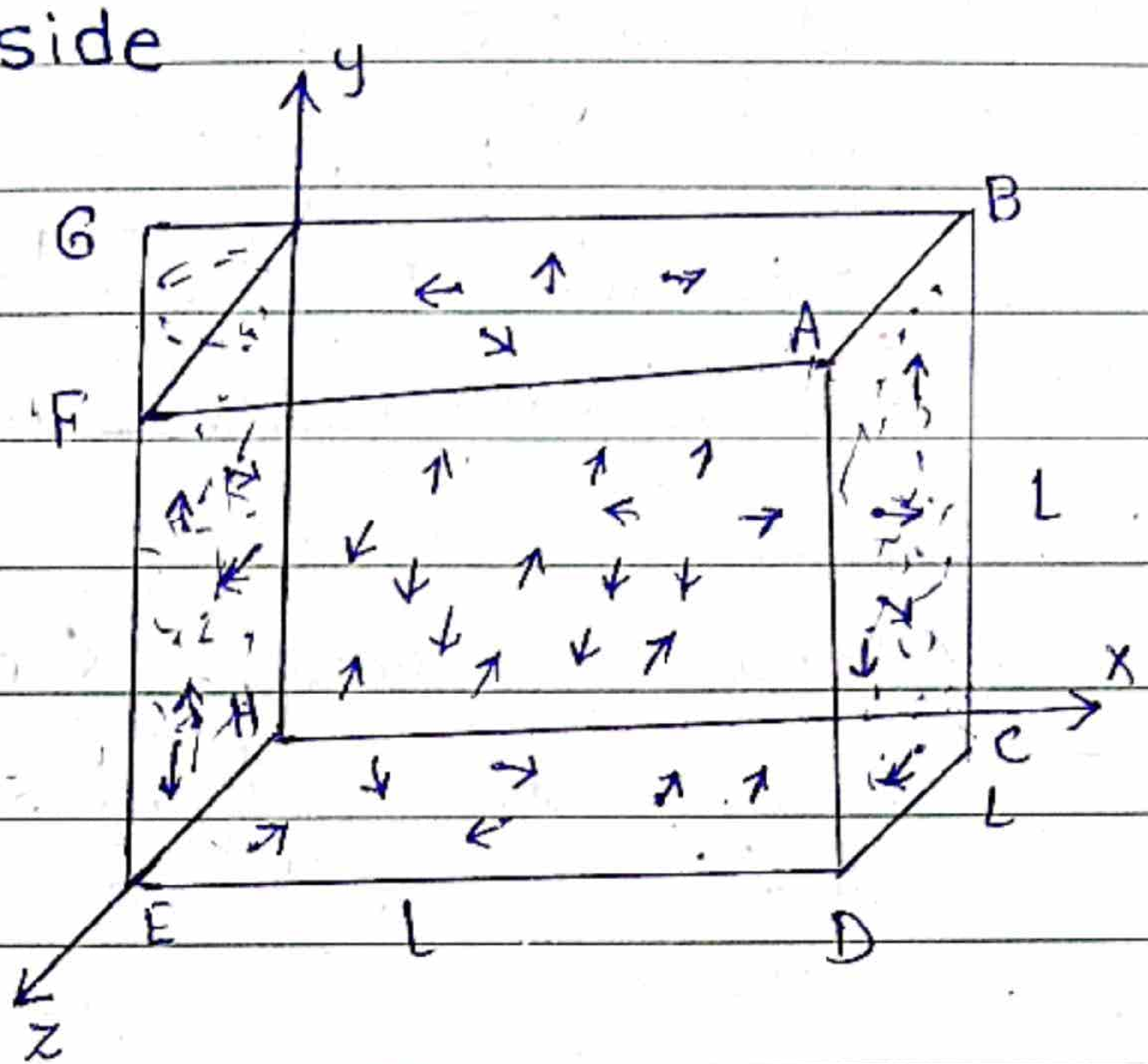
Relation for the Pressure of th Gas:

Let

N= Total number of gas molecules in the container

L = length of each side of the cubical container

V_1 = Velocity of any one of the molecules in a ceirtain direction



Resolve V_1 into r ctangular components V_{1x}, V_{1y}, V_{1z}.

Initial momentum of the molecule striking the face ABCDA = $mV_{1}x$

Final momentum = $-mV_{1}x$

Change in momentum =

$$\Delta p = -mV_{1x} - mV_{1x}$$

$$\boxed{\Delta p = -2mV_{1x}}$$

After recoil, the molecule will strike the face E,F,G,H,E. From this face, it rebounds and travels back to face ABCDA.

So distance covered

$$S = L + L = 2L$$

Time between two successive collision $= \Delta t$

As

$$S = vt$$

$$t = \frac{S}{v}$$

$$\boxed{\Delta t = \frac{2l}{V_{1x}}}$$

Rate of change of momentum $= \frac{\Delta P}{\Delta t}$

$$= \frac{-2mV_{1x}}{\frac{2l}{V_{1x}}}$$

$$\frac{\Delta P}{\Delta t} = -\frac{\cancel{2}mV_{1x} \times V_{1x}}{\cancel{2}l}$$

$$\frac{\Delta P}{\Delta t} = \frac{-mV_{1x}}{L}$$

Rate of change of momentum = Force applied by the wall on the molecule

$$= \frac{-mV_{1x}^2}{L}$$

By Newton's 3RD Law:

The force applied by the molecule on the wall $= F_{1x} = -\left(\frac{-mV_{1x}^2}{L}\right)$

$$\boxed{F_{1x} = \frac{mV_{1x}^2}{L}}$$

Now the total x-directed force is

$$F_x = F_{1x} + F_{2x} + F_{3x} + \cdots\cdots + F_{nx}$$

$$F_x = \frac{mV_{1x}^2}{L} + \frac{mV_{2x}^2}{L} + \frac{mV_{3x}^2}{L} + \cdots\cdots \frac{mV_{nx}^2}{L}$$

As

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Area of the force $= l \times l = l^2$

$$P_x = \frac{F_x}{l^2}$$

$$P_x = \frac{1}{l^2}\left[\frac{mV_{1x}^2}{L} + \frac{mV_{2x}^2}{l} + \frac{mV_{3x}^2}{L} + \cdots \frac{mV_{nx}^2}{L}\right]$$

$$P_x = \frac{1}{l^2} \cdot \frac{m}{l}\left[V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \cdots\cdots + V_{Nx}^2\right]$$

$$P_x = \frac{m}{l^3}\left[V_{1x}^2 + V_{2x}^2 + V_{3x}^3 + \cdots\cdots + V_{nx}^2\right] \longrightarrow (1)$$

As

m = mass of one molecule

Nm = mass of N-molecules

density $\rho = \dfrac{\text{mass}}{\text{Volume}}$

$$\rho = \frac{Nm}{L^3}$$

$$\rho = \frac{Nm}{L^3}$$

$$\frac{m}{L^3} = \frac{\rho}{N}$$ Put in eq (1)

$$P_x = \frac{\rho}{N}\left[V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \cdots\cdots V_{nx}^2\right]$$

$$P = \rho\left[\frac{V_{1x}^2 + V_{2x}^2 + V_{3x}^3 + \cdots\cdots V_{nx}^2}{N}\right]$$

Here,

$$\left[\frac{V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \cdots\cdots + V_{Nx}^2}{N}\right] = \text{Mean square velocity}$$
$$= \langle V_x^2 \rangle$$

So

$$\boxed{P_x = \rho\langle V_x^2 \rangle} \longrightarrow (2)$$

Similarly

$$P_y = \rho\langle V_y^2 \rangle$$

$$P_z = \rho < V_z^2 >$$

Since all the compontents of Velocitiesare equally possible

$$< V_x^2 > = < V_y^2 > = < V_z^2 >$$

Root mean square Volecity

$$_xV_{rms} = < V_x^2 >$$

$$V_{xrms} = \sqrt{\frac{V_{1x}^2 + V_{2x}^2 + \cdots V_{nx}^2}{N}}$$

By the rule of vector addition

$$< V^2 > = < V_x^2 > + < V_y > + < V_z^2 >$$

$$< V^2 > = < V_x^2 > + < V_x^2 > + < V_x^2 >$$

$$< V^2 > = 3 < V_x^2 >$$

$\frac{1}{3} < V^2 > = < V_x^2 >$ Put in eq (2)

$$P_x = \rho \frac{1}{3} < V^2 >$$

$$P_x = \frac{\rho}{3} < V^2 >$$

By Pascals Law,

Pressure on each side of the container and every where in the container is Same, if the density of the gas is uniform. So,

$$P_x = P_y = P_z = \frac{\rho}{3} \langle v^2 \rangle$$

In general,

$$\boxed{P = \frac{1}{3} \rho \langle v^2 \rangle}$$

As

density $\rho = \frac{\text{mass}}{\text{Volume}}$

$$= \frac{Nm}{V}$$

Put $\rho = \frac{N}{V}$ in the above equation.

$$P = \frac{1}{3} \frac{mN}{V} \langle v^2 \rangle$$

$x \div y$ 2

$$\boxed{P = \frac{2}{3} \frac{N}{V} \langle \frac{1}{2} m v^2 \rangle}$$

$$P = \frac{2}{3} N_o \langle \frac{1}{2} m v^2 \rangle$$

$N_o = \frac{N}{N}$ = Number of gas molecules per unit volume

$$P = \text{Constant} \langle \frac{1}{2} m v^2 \rangle$$

$\frac{2}{3} N_o$ = constant

$$P \propto \langle \frac{1}{2} m v^2 \rangle$$

$$\boxed{P \propto \langle K.E. \rangle}$$

So,

" Pressure is directly proportional to the average translational Kinetic Energy of the gas molecules."

While derving the above relation we have ignored the rational and vibrational motion of the gas molecules.

Interpretation of Temperature:

Ideal gas law is

$$\boxed{PV = nRT}$$

n = Number of moles of the gas in Volume V.
T = Absolute Temperature.
R = 8.314 J/mol.K

Let.

N_A = Avogadros Number

Number of gas molecules in one-mole = N_A
" " " n-moles = $n N_A$

$$N = n N_A$$

$$= \frac{N}{N_A}$$

$$PV = nRT$$

$$PV = \frac{N}{N_A} RT$$

$$PV = N \frac{R}{NA} T$$

Here,

$$\frac{R}{NA} = K$$

K = Boltzman constant

$$K = 1.38 \times 10^{-23}\ J\ K$$

K = gas constant per molecule

Unit of PV

$$P.V = \frac{F}{A} . V$$

$$P.V = \frac{N}{m^2} \times m^3$$

$$P.V = N \times M$$

$P.V$ = Joule

Unit of R

$$P.V = nRT$$

$$R = \frac{PV}{nT}$$

$$R = \frac{\text{Joule}}{mol.k}$$

$$= J mol\, k^{-1}$$

$$\boxed{PV = NKT} \longrightarrow (i)$$ Ideal gas Law.

As,

$$P = \frac{2}{3} \frac{N}{V} \langle \frac{1}{2} mv^2 \rangle$$

$$P = \frac{2}{3} N \langle \frac{1}{2} mv^2 \rangle \longrightarrow (ii)$$

Comparing eq (i) and (ii)

$$\cancel{N}KT = \frac{2}{3} \cancel{N} \langle \frac{1}{2} mv^2 \rangle$$

$$\boxed{T = \frac{2}{3k} \langle \frac{1}{2} mv^2 \rangle}$$

$$\frac{2}{3k} = \text{constant}$$

$$T = \frac{2}{3} \langle K.E \rangle$$

$$T = \text{constant} \langle \frac{1}{2} mv^2 \rangle$$

$$T \propto \langle \frac{1}{2} mv^2 \rangle$$

$$\boxed{T \propto \langle K.E \rangle}$$

So,

"Absolute Temperature of an ideal gas is directly proportional to the averge translational Kinetic Energy of gas molecules."

Result:

"Averge Translational K.E. of gas molecules shows itself macroscopically in the Form of temperature."

Derivation of Gas Laws

(i) Boyles Law:

From the kinetic Theory of gases

$$P = \frac{2}{3} \frac{N}{V} \langle \frac{1}{2} mv^2 \rangle$$

$$PV = \frac{2}{3} N \langle \frac{1}{2} mv^2 \rangle$$

If we keep the temprature = constant

$$T \propto \langle \frac{1}{2} m v^2 \rangle$$

$$\text{Avg K.E} = \langle \frac{1}{2} mv^2 \rangle = \text{constant}$$

also

$$\frac{2}{3} N = \text{constant}$$

$$PV = \text{constan}$$

$$V = \frac{\text{constant}}{P}$$

OR

$$\boxed{V \propto \frac{1}{P}}$$

Thus

"Volume of a gas is inversely proportional to the pressure at constant temperature, which is Boyle's Law."

(ii) Charle's Law:

As

$$P = \frac{2}{3} \frac{N}{V} \langle \frac{1}{2} mv^2 \rangle$$

$$V = \frac{2}{3}\frac{N}{P}\left\langle \frac{1}{2}mv^2 \right\rangle$$

If pressure is kept constant $\frac{2}{3}\frac{N}{P}$ = constant

$$V = \text{constan} \left\langle \frac{1}{2}mv^2 \right\rangle$$

$$V \propto \left\langle \frac{1}{2}mv^2 \right\rangle$$

As $\left\langle \frac{1}{2}mv^2 \right\rangle \propto T$

so,

$$\boxed{V \propto T}$$

Hence,

"Volume V is directly proportional to the absolute Temperature is kept constant. whic is charles Law."

11.2 Internal Energy

Defination:

"The sum of all forms of molecular energies (ie. K.E and P.E.) of a substance is called internal energy."

In thermodynamics we consider an ideal gas as a working substance. The molecules of an ideal gas are nearly mass points which exert no forces on one another. So the internal energy on an ideal gas system is generally the sum of translational K.E of its molecules.

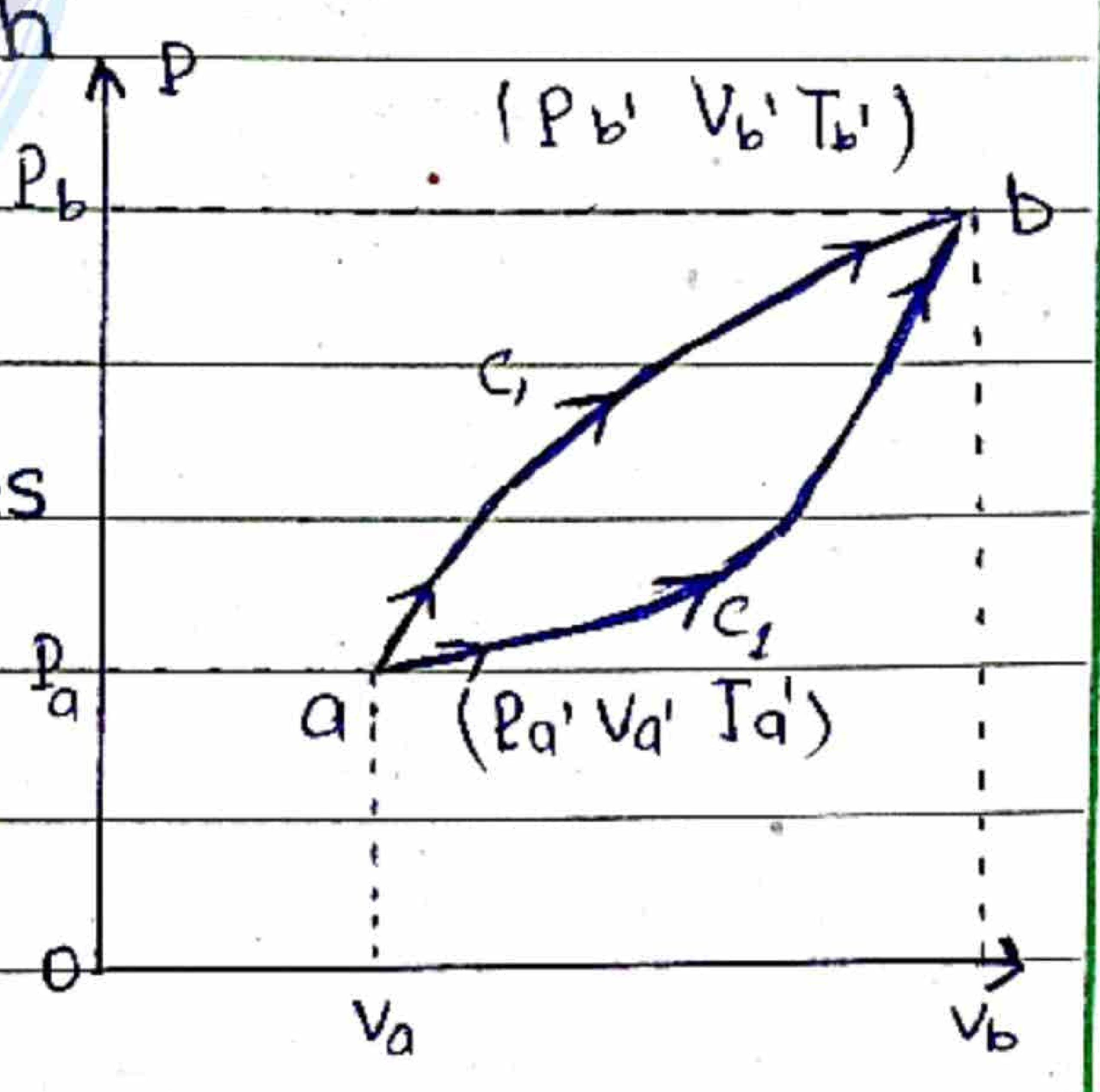
Temprature :

"The temperature of a system is defined as the averge K.E of its molecules." So, for ideal gas system, the internal energy is directly proportional to its temperature.

$$\boxed{U \propto T}$$

When a substance is heated, K.E of its molecules increases i.e heat is converted into internal energy.

It is imortant to note that energy can be added to a system even though no heat transfer takes place. For example when objects are rubbed together, their increases because of mechanical work. The increases in the temperture of an object is an indication of increases in the internal energy. Similarly when an



object slides down a surface, mechanical work done on or by the system is partially converted into internal energy.

In thrmodynamics, internal energy is a function of state. It does not depend upon the path but depends upon initial and final states of the system as shown in fig.

Consider a system in which intial pressure P_a and volume V_a changes to final Pressure P_b and volume V_b. The change in internal energy is always the same and is indipendent of paths C_1 and C_2 as shown in fig.

So internal energy is similar to gravitational Just like potential Energy, the change in the internal energy is importent, not the absolute value of internal energy.

11.3 Work and Heat

We know that both heat and work help in transferring energy by some means. This idea was first applied to the steam engine where heat is given and work is taken out.

$+Q$ = When heat Q is added to a system, it is taken as +ive.

$-Q$ = When heat Q is taken out of a system, it is taken as -ve.

Work done by the system is +ive $= +W$

Work done on the system is -ve $= -W$

If an amount of heat Q enters the system, it is used either to increase the internal energy or work done by th system.

Consider a gas enclosed in a cylinder having a moveable frictionless piston of

Area $= A$

Volume of the System $= V$

Pressure on the Piston $= P$

Force on the

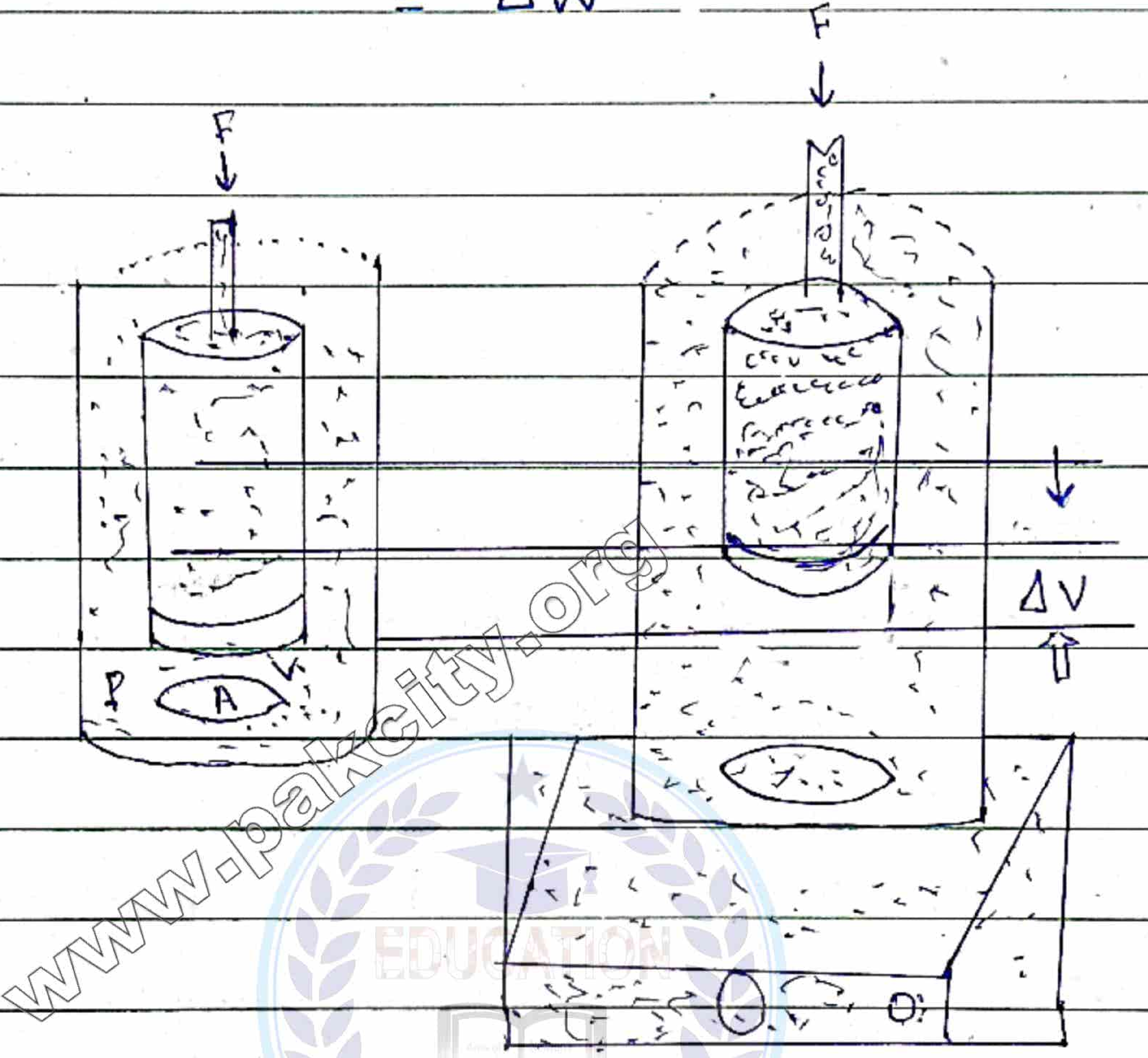
$$\boxed{\begin{array}{l} P = \frac{F}{A} \\ F = PA \end{array}}$$

Piston = F = PA

change in th Volume = ΔV

Distance moved by Piston = Δy

Work done by the gas = ΔW



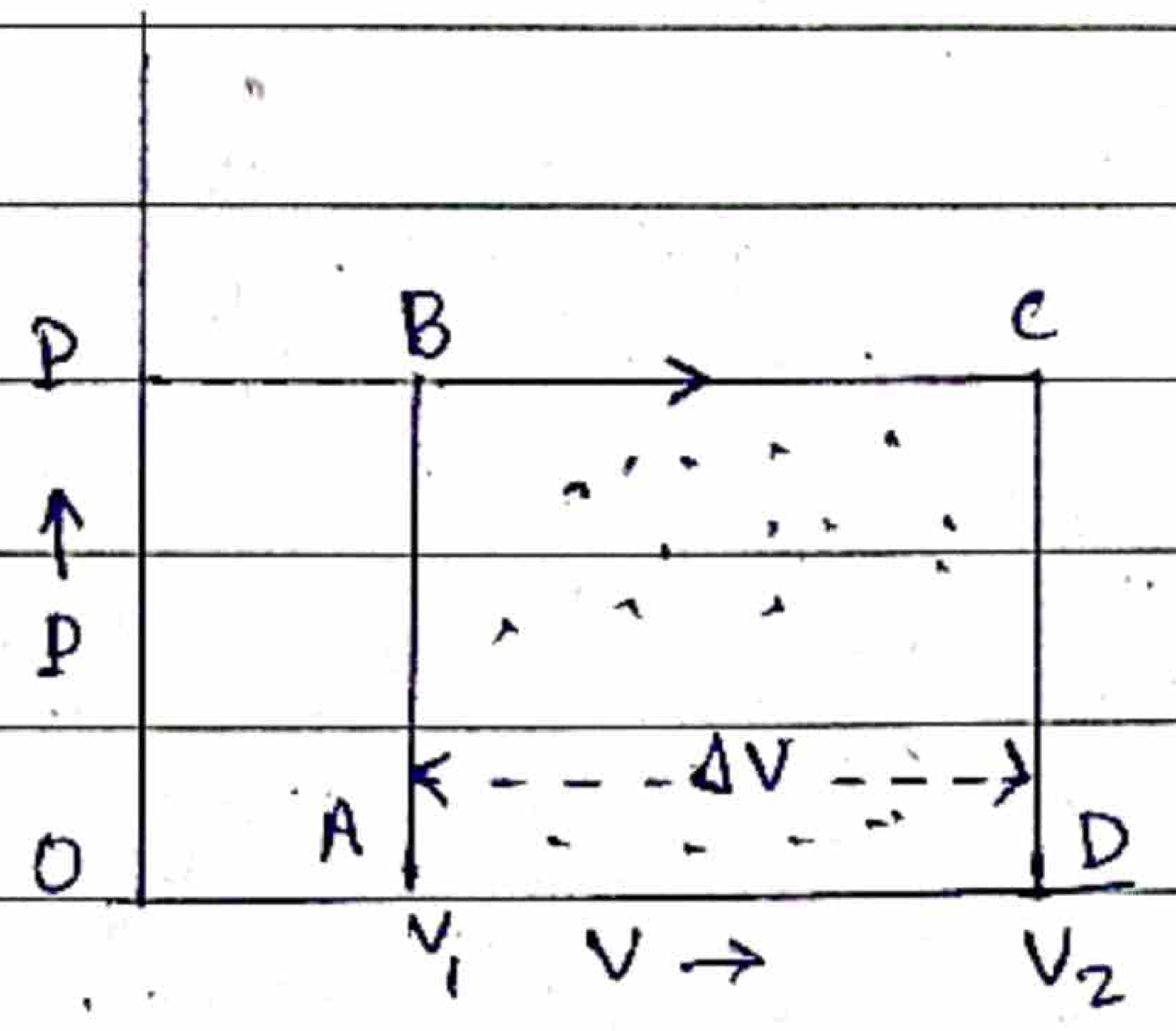
Work = Force x distance

$\Delta W = F\Delta y$

$\Delta W = PA.\Delta y$

$\Delta W = P(A\Delta y)$

$$\boxed{\Delta W = P\Delta V}$$



$A\Delta y = \Delta V$

PV – Graph

11.4 First Law of Thermodynamics

Statement:

"In any Thermodynamic process, when heat Q is added to a system, this energy appears as an increase in a internal energy ΔU stored in the system plus the work done by the system on its surrounding."

Explantion:

When heat added to a system there is rise of temperatur, an increase in pressure or chnge in the state of the system.

If the system is allowed to expand, it will do work on its surroundings. So heat Q is used to increase the internal energy from U_1 to U_2 and to do work.

$$Q = (U_2 + U_1) + W$$

$$\boxed{Q = \Delta U + W}$$

This is 1st Law of Thermodynamics.

change in the internal energy ΔU is

$$\boxed{\Delta U = Q - W}$$

It means that

The change in the internal energy of the system is equal to the energy flowing in as heat energy minus the energy flowing out as work.

Examples:

(i) When we pump on the handle of bicycle pump rapidly, it becomes hot. The work done on the gas (air) increases its internal energy and hence increasing the temprerature.

(ii) We block the outlet of a bicyle pump and connect a thermocouple to it. We push the piston rapidly. Thermometer will show rise in temperature. It is due to increase internal energy.

(iii) Human beings and other animals do work when they walk, run or more heavy objects. Energy is consumed. Energy is also needed for the growth of new cells and to replace old cells. This process of energy transformation within an organism is called Metabolism.

Work done by the human body decreases its internal energy and the temperature which is maintained by the food we eat.

By the first law of thermodynamics

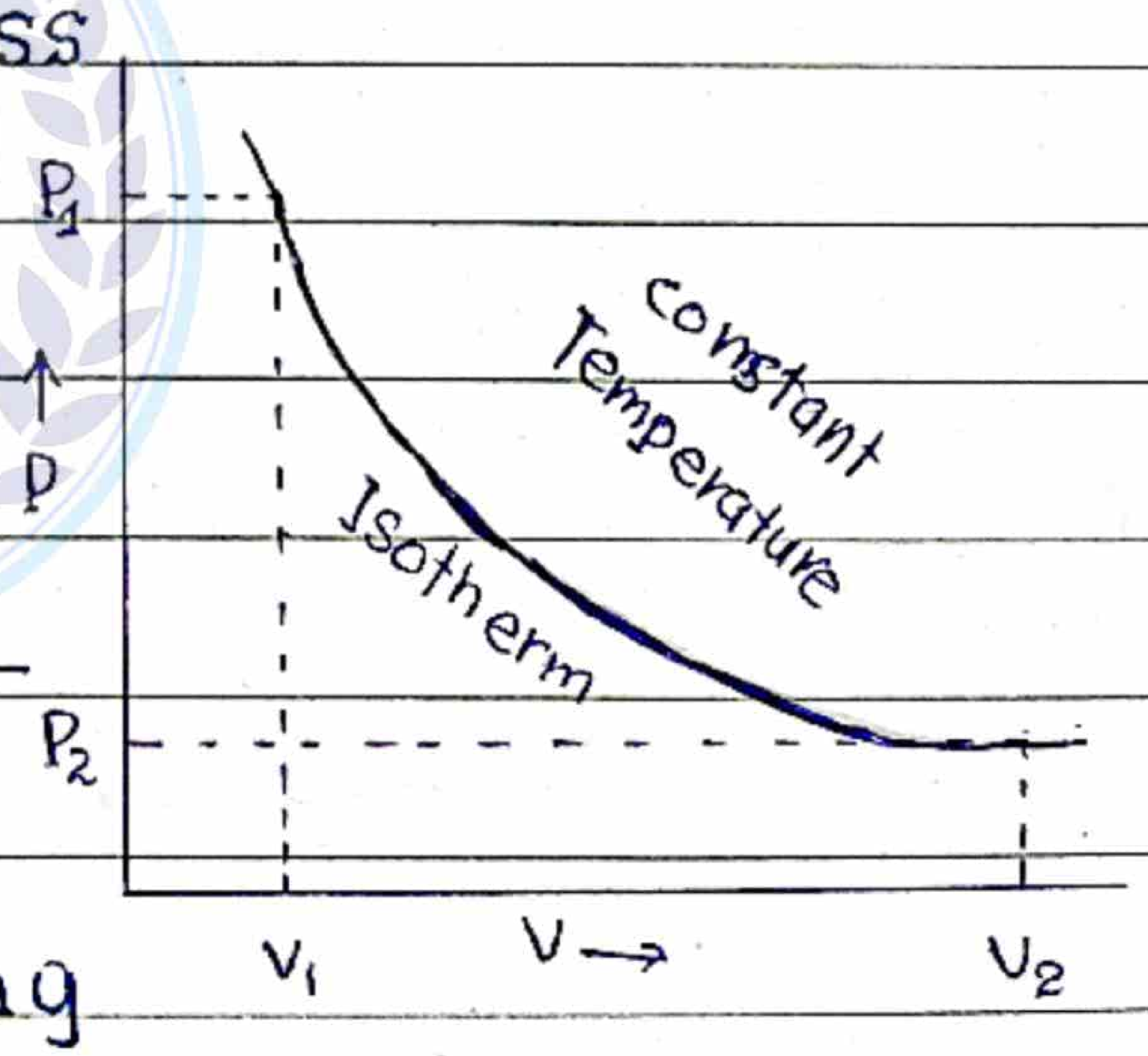
$$\boxed{\Delta U = Q - W}$$

Isothermal Process:

"It is a process in which the temperature of the system remains constant"

$\Delta U = 0$ $T = \text{constant}$

Explanation:



Isothermal process fulfills the condition of Boyle's Law (T = constant).

When a gas expands or is compressed isothermally, the product of its pressure and volume during the process remains constant.

Consider the pressure and volume of a system changes from P_1, V_1, to P_2, V_2 during an isothermal process then

$$\boxed{P_1 V_1 = P_2 V_2 = \text{Constant}}$$

OR

$$\boxed{PV = \text{Constant}}$$

For an ideal gas P.E of gas molecules = 0 (zero)
So, the internal energy of an ideal gas depends upon its tempreature, which is constant in this case

Hence

Internal Energy = Constant

$$\Delta U = 0 \qquad \{\text{As Temp} = \text{constant}$$

By 1st Law of thermodynamics

$$Q = \Delta U + W$$

$$Q = 0 + W$$

So

$$\boxed{Q = W}$$

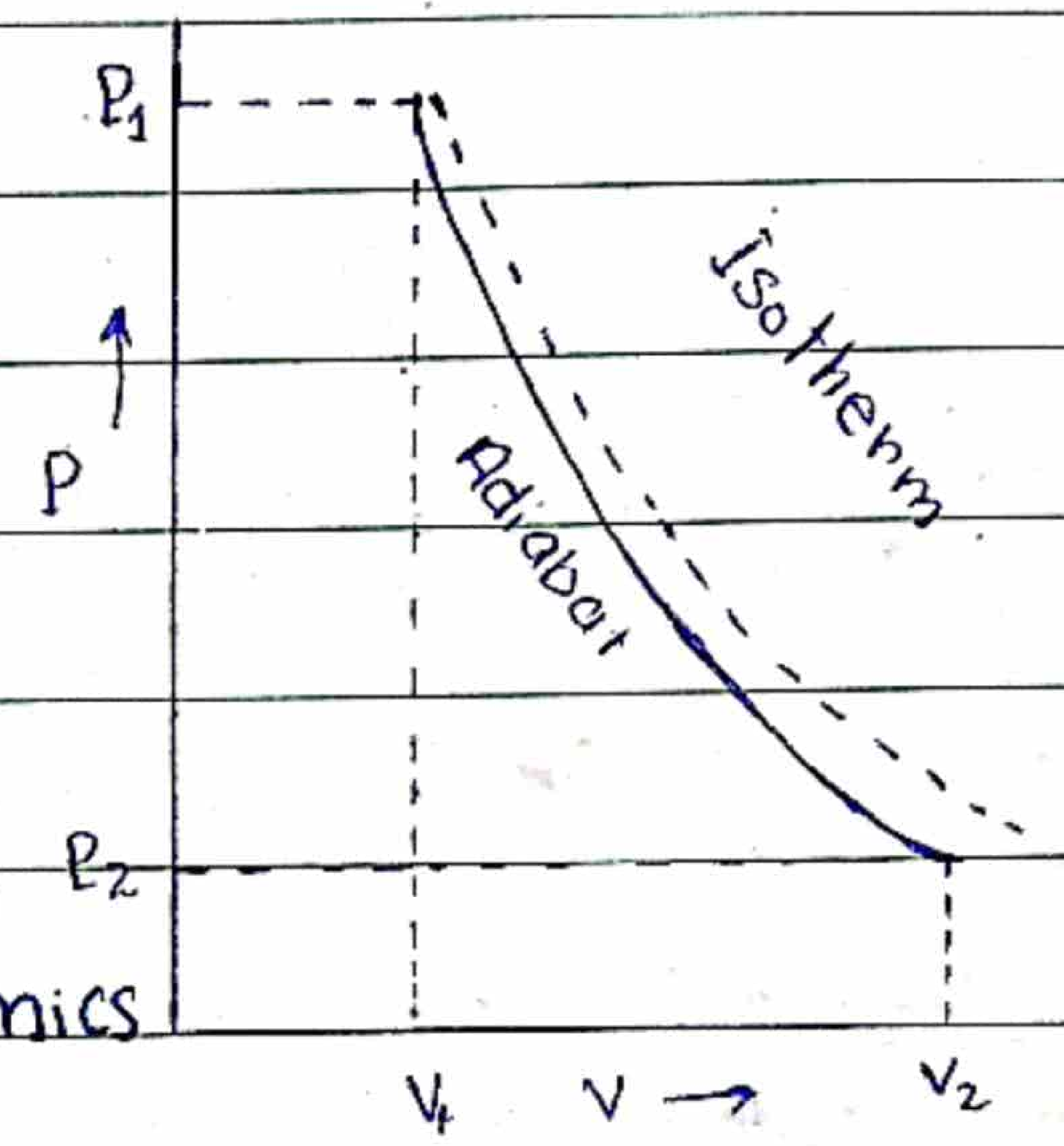
Q = W shows that total energy supplied is used to do external work.

To keep the temperature constant the expansion or compression must take place very slowly.

The curve of an isothermal process is called Isotherm.

A Diabatic Process:

" In this process, no heat enters or leaves the system."



Explanation:

$$\Delta Q = 0$$

By 1st Law of thermodynamics

$$\Delta Q = \Delta U + W$$

$$0 = \Delta U + W$$

Adiabatic Expansion:

$$0 = \Delta U + W$$

$$\boxed{+W = -\Delta U}$$

+W shows that work is done by the gas at the cost of internal energy. Internal energy decreases $(-\Delta U)$ and hence temperature.

also decreases.

Adiabatic Compression:

$$0 = \Delta U + W$$

$$\boxed{-W = \Delta U}$$

In a adiabatic compression work done on the gas (-W) increases its internal energy (+ΔU) and hence its temperature also rises.

Examples:

Rapid escape of air from a burst tyre.
The rapid expanision and comperison of air, when a small wave passes through it.
Formation of clouds in the atmosphere.

For an adiabatic process

$$\boxed{PV^{r} = \text{Constant}} \qquad r = \frac{C_p}{C_v}$$

11.5 Molar Specific Heats of A Gas

Molar Specific Heats of A Gas:

Definition:

"Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through 1K." It is denoted by "C".

Mathematical form:

$$\boxed{Q = nC_m \Delta T}$$

Unit:

Its SI unit is $j\,Mole^{-1}\ K^{-1}$.

- ★ ; In case of liquids and solid the change of volume and hence work done against external pressure during heating is negligibly small.
- ★ ; In case of gases because they suffer changes in pressure as well as in volme with the rise in temperature.

Using first Law of thermo

$$Q_v = \Delta U + W$$

From eq (1)

$$C_v \Delta T = \Delta U + 0$$

$$\boxed{\Delta U = C_v \Delta T}$$

At constant volume $\Delta V = 0$

$\because W = p\Delta V = p(0)$

$W = 0$

Where ΔU is the increasc in internal energy of the gas.

ii) Molar specific heat at constant pressure

Definition:

"It is defined as the amount of heat required to rasie the temperature of one mole of a gas through 1K at constant pressure." It is denoted by C_p.

Mathematical form:

$$Q = C_p \Delta T$$

For "n" moles of gas.

$$\boxed{Q = nC_p \Delta T}$$

Unit:

Its IS units $j\ mole^{-1}\ K^{-1}$.

* ; Gasses can be heated either at constant pressure or at constant volume.

Types of molar specific heat of a gas:

Molar specific heat of a gas in two following ways.

i, : Molar specific heat at constant volume

ii, : Molar specific heat at constant pressure.

i, Molar specific heat at constant volume:

Definition:

"It is defined as the amount of heat reqired to raise the temperature of one mole of gas through 1K at constant volume." It is denoted by C_v.

Mathematical form:

$$\boxed{Q = C_v \Delta T} \longrightarrow (1)$$

Unit

Its SI units $j\ mole^{-1}\ K^{-1}$.

By ideal gas Law.

$$PV = nRT \qquad \because n = 1$$

$$PV = RT$$

$$P\Delta V = R\Delta T$$

put in equation (4)

$$C_p \Delta T = C_v \Delta T + R\Delta T$$

$$C_p \cancel{\Delta T} = \cancel{\Delta T}\,(C_v + R)$$

$$C_p = C_v + R$$

$$\boxed{R = C_p - C_v}$$

So. $C_p > C_v$ by an amount equal to R universal gas constat.

11.6 Reversible and Irreversible Processes.

Reversible Process:

Definition:

"The process which can be re-traced in exactly reverse order without produicing any change in the surroundings is called reversible process".

In the reverse process the working substance passes through the same

Prove that $\boxed{C_p - C_v = R}$

At constant volume:

Consider one mole of a gas is heated at constant volume. The rise of temperature ΔT. The heat required is given by

$$\Delta U = C_v \Delta T$$

At constant pressure:

At constant pressure the heat required to raise the temperature of one mole of a gas through ΔT.

$$\boxed{Q_p = C_p \Delta T}$$

By first Law of Thermodynamics

$$Q_p = \Delta U + W \longrightarrow (2)$$

$$C_p \Delta T = C_v \Delta T + W \longrightarrow (3)$$

As, $\because P = \frac{F}{A}$

$F = PA$

$$W = Fd$$

$$W = PAd = P(Ad)$$

$$W = P\Delta V$$

from eq (3)

$$C_p \Delta T = C_v \Delta T + P\Delta V \longrightarrow (4)$$

stages as in the direct process but the thermal and mechanical effect at each stage are exactly reversed. If heat is absorbed in the direct process it will be given out in the reverse process. If work is done by the system in the direct process, the work will be done on the system in the reverse process, so the working substance comes to its initial state.

Examples:

i) : No cange is completely reversible. But liquification and evaporation of a substance, performed slowly, are practically reversible.

ii) : Slow compression is reversible because a compression can be changed into expansion by slowly decreasing the pressure on the gas.

Cycle:

"A succession of events which bring the system back to its initial condition is called a cycle. In a reversible cycle all the changes are reversible".

Irreversible process:

"A process which cannot be retraced in the backward direction by reversing the controlling factors is called an irreversible process".

Examples:

i) : All changes which occur suddenly or which involve friction or loss of energy through conduction convection or radiation are irreversible.

ii) : Explosion is a highly irreversible process.

iii) : All the heat engines are practically irreversible.

11.7 Heat Engine

Definition:

"A device which converts heat energy into mechanical work is called Heat engine."

Principle:

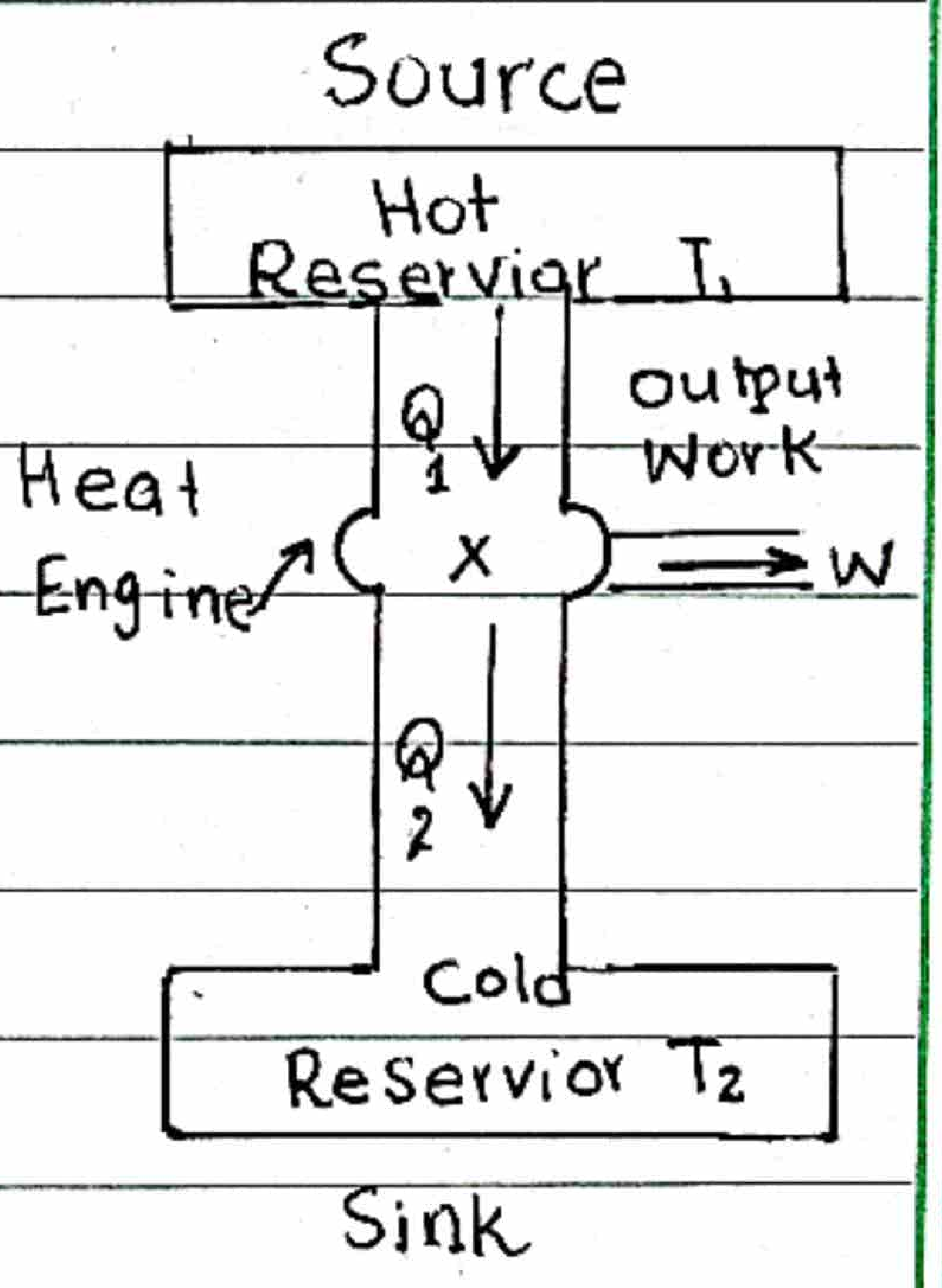
Evey engine absorbs some quantity of heat energy from hot body called source", converts a part of heat into work and rejects the remaining part of heat to cold body called sink. It means that two bodies at different temperatures are necessary for the operation of a heat engine.

Construction:

It consists of a hot reservoir at a higher temperature "T_1" and cold reservoir at lower temperature "T_2" called sink.

Working:

A heat engine absorbs heat "Q_1" from a hot reservoir (source) convert a part of this energy into useful work W and rejects the remaining heat "Q_2" to the cold reservoir (sink)



A heat engine works in a cycle to produce continuous supply of work because by conversion of heat into work temperature of working substance decreases.

$$W = Q_1 - Q_2$$

Efficiency of heat engine:

"The ratio of work done by the engine to the quantity of heat absorbed from the hot reservoir or source". It is denoted by η (eta)

As,

Heat absorbed by the engine = Q_1

Work done by the engine = $W = Q_1 - Q_2$

$$\text{Efficiency } \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\boxed{\eta = 1 - \frac{Q_2}{Q_1}} \longrightarrow (1)$$

Hear Q_2 is the heat rejected in a cycle, and heat engines operate in a cycle.

11.8 Second Law of Thermodynamics

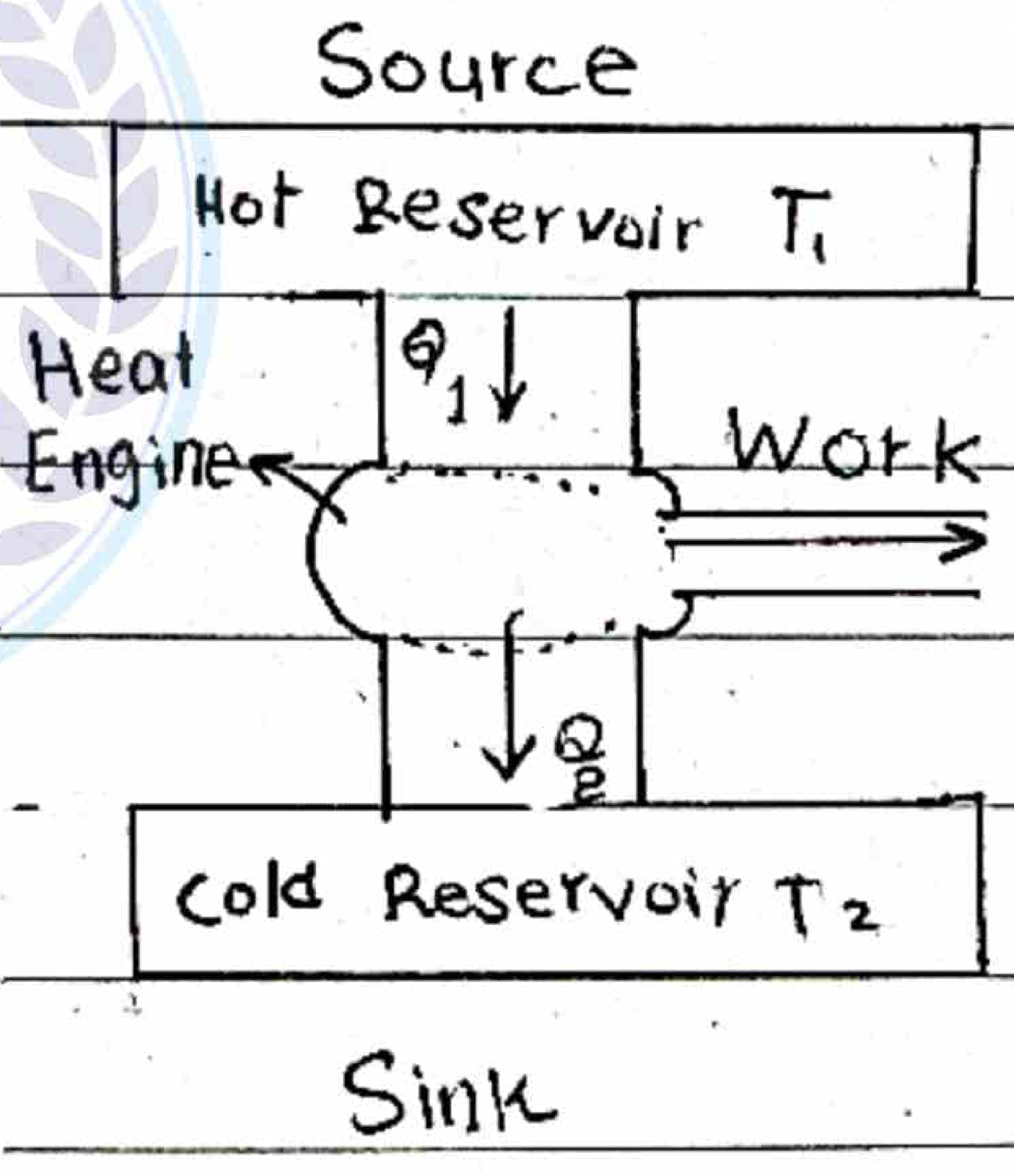
First Law of Thermodynamics tells us that heat energy can be converted into work. The second of concerned with the circumstances in which heat can be converted into work and the direction of flow of heat.

Kelvin's Statement:

"It is impossible to devise a process or heat engine which can extract heat from a single source and convert all of it into useful work without rejecting a part of it to a cold reservoir".

Explantion:

Second Law tells that a single heat reservoir cannot act as a heat engine. For the conversion of Heat energy into useful work there must be two different heat reservoirs at different temperatures. One at a high



temperatures, and the second at a lower temperature. This is why heat content of oceans and atmosphere cannot be utilized because there is no reservoir at a temperature lower than any one of the two.

Consider a heat engine which absorbs a quantity of heat Q_1 from a hot reservoir at temperature T_1. It does work W and expells heat Q_2 to the low temperature reservoir (or cold reservoir) at a low temperature T_2.

As working substance goes through cyclic process, the change in internal energy is zero

$$\Delta U = 0$$

Using First Law,

$$\Delta Q = \Delta U + W$$

$$Q_1 - Q_2 = 0 + W$$

$$\boxed{W = Q_1 - Q_2}$$

So, net work done is equal to net heat absorbed.

Examples:

i) : The petrol engine of a motor car extracts heat from the burning fuel convert a part of it into work and expells the remaing heat into the atmosphere.

ii) : A petrol engine converts obout 25% and a diesel engine 35% to 40% available heat energy into work.

11.9 Carnot Engine and Carnot's Theorem

Carnot engine:

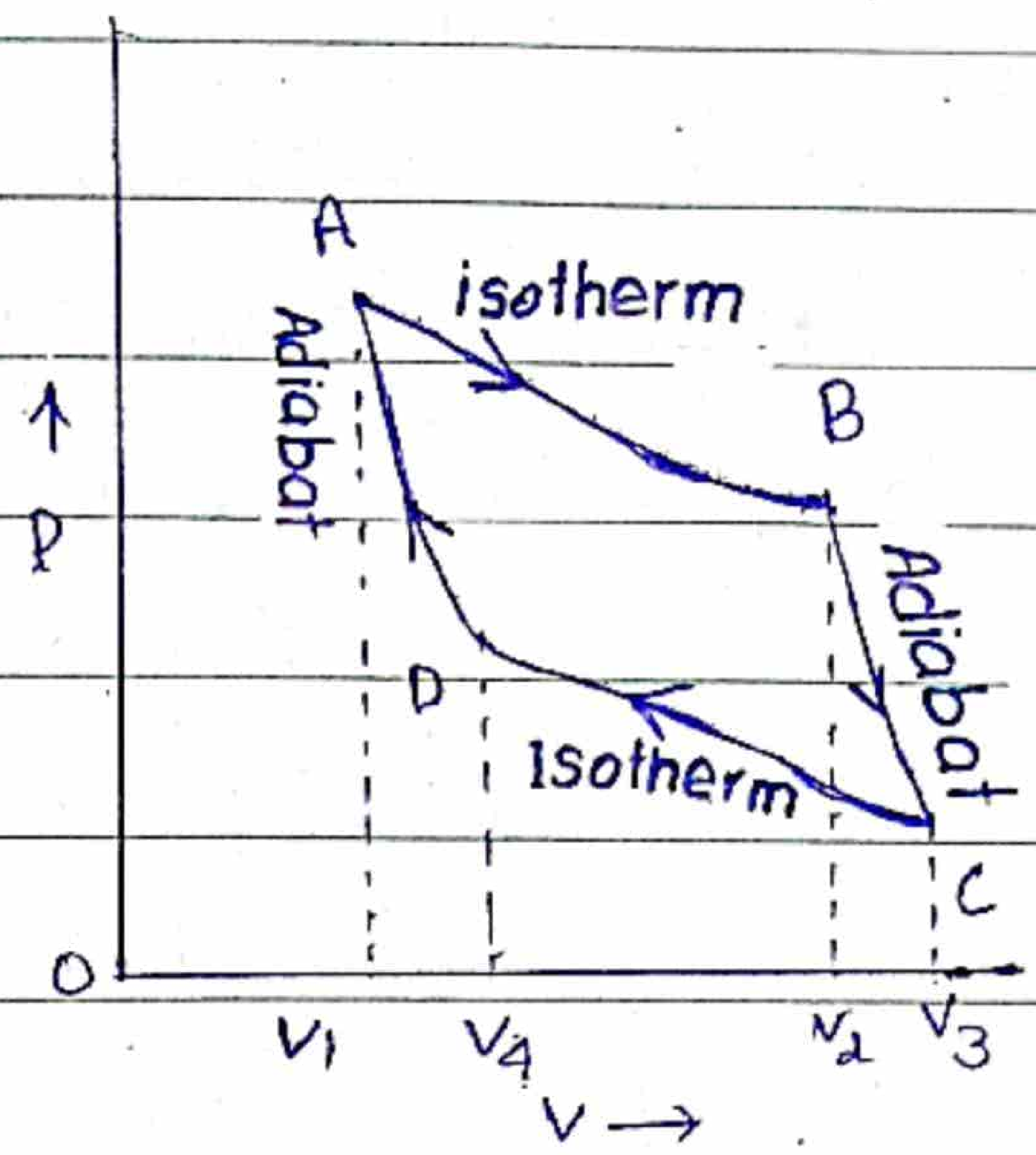
Sadi carnot in 1840 described an ideal engine with maximum possible effeciency (however not 100%), it uses only isothermal and adiabatic processes. It operates in an ideal eversible cycle called "Carnot cycle" between two reserviors at different temperatures. It uses an ideal gas as a working substance.

4-Steps of a Carnot cycle:

A carnot cycle is shown in PV-diagram.

i) Isothermal expansion:

The gas is allowed to expand isothermally at temperature T_1 absorbing heat Q_1 from the hot reservior. The process is reprsented by curve "AB".



ii) Adiabatic Exponsion:

The gas is then allowed to expand adiabatically unit its temperature drops to T_2. The process is represented by cure "BC".

iii) Isothermal Compression:

The gas is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservior. The process is represented by curve "CD".

iv) Adiabatic Compression:

Finally the gas is compressed adiabatically to restore its initial state at temperature T_1. The process is represented by curve "DA".

During the complete cycle there is no change in the internal energy i.e.

$$\Delta U = 0$$

The net work done during one cycle is equal to the area enclosed by the path "ABCDA" in the PV-diagram.

Heat absorbed $= Q_1$

Net heat absorbed $= \Delta Q = Q_1 - Q_2$

By First Law

$$\Delta Q = \Delta U + \Delta W$$

$$Q_1 - Q_2 = 0 + \Delta W$$

$$\boxed{\Delta W = Q_1 - Q_2}$$

The efficiency is defined as

$$\eta = \frac{\text{Output work}}{\text{Input work}} = \frac{\Delta W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

so.

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

∵ Heat Q_1 and Q_2 are proportional to the temperature T_1 and T_2.

Percentage Efficiency

$$\boxed{\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100}$$

The efficiency depends upon the temperatures T_1 and T_2 and not on the working substance.

At absolut zero $T_2 = 0\,K$ from above equation.

$$\boxed{\eta = 1}$$

But, there is no cold reservoir at $T_2 = 0K$, so. efficiency cannot be 100 %. It is always less then 100 %.

Carnot's Theorem:

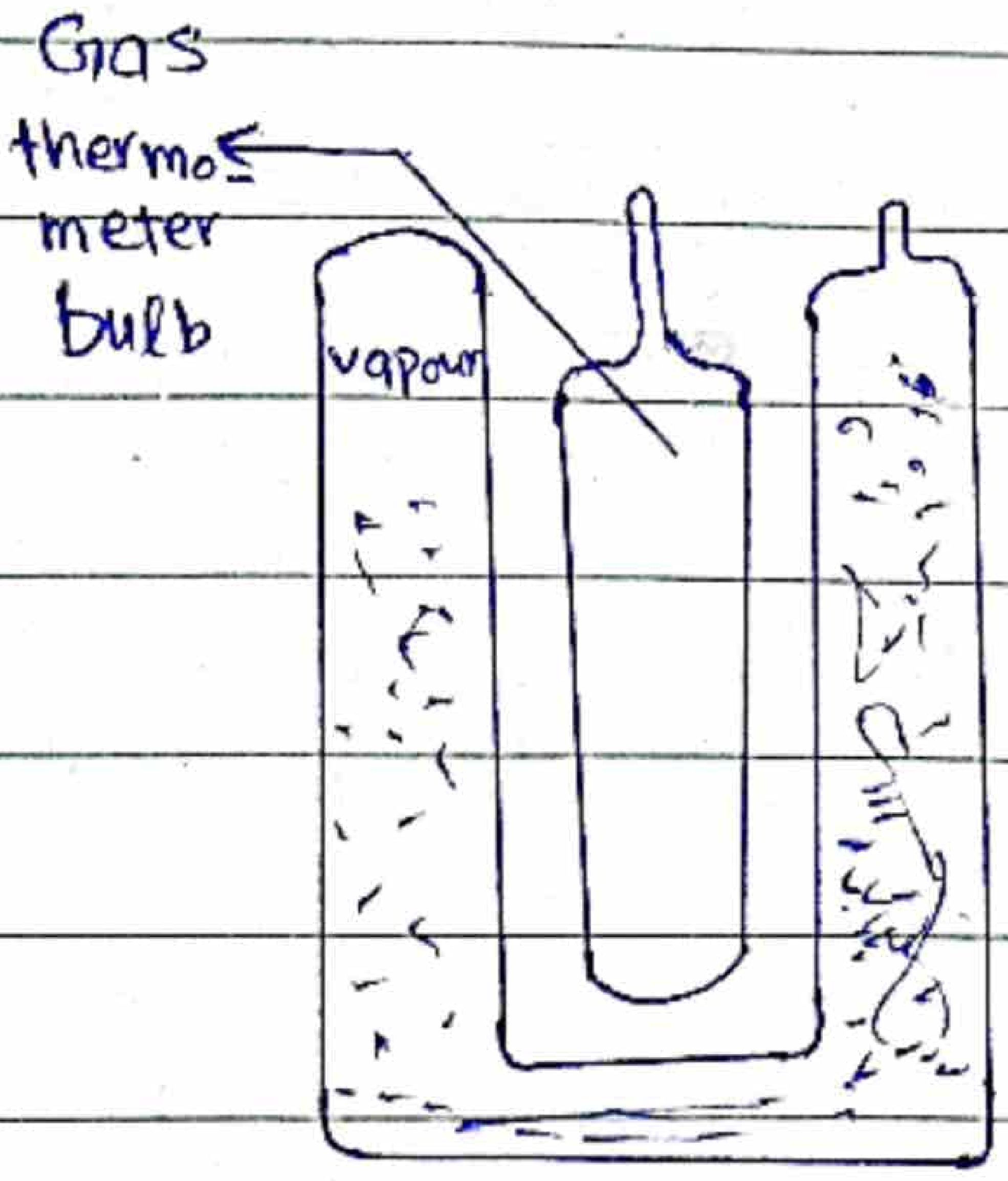
"No heat engine can be more efficient then a carnot engine operating between the same two temperatures".

11.10 Thermodynamic Scale of Temperature

Triple point:

"The point at which the ice water and vapour phases of a substance are in equilibrium."

The carnot cycle provides us basis to define a temperature scale that is independent of material properties. According to it, the ratio $\frac{Q_2}{Q_1}$ depends only on the temperature of two heat reserviors.



The triple point occurs for a unique set of values of temperature, pressure and volume.

The ratio of two temperatures of $\frac{T_2}{T_1}$ can be found by operating a reversible carnot cycle between these two temperatures and measuring the heat transfers Q_2 and Q_1.

The "Thermodynamic scale of temperature" is define by choosing 273.16 K as the obsolute fixed point and absolute zero, as the other fixed point.

"The unit of Thermodynamic scale is Kelvin".

1 Kelvin:

1 Kelvin is defined as $\frac{1}{273.16}$ of the Thermodynamic temperature of the triple point of water.

If heat Q_1 is absorbed or rejected by the system at corresponding temperature "T" When the system is taken through a carnot cycle and Q_3 is the heat absorbed or rejected by the system when it is at the temperature "T" in Kelvin is given by

$$T = 273.16 \frac{Q}{Q_3}$$

Since this scale is independent of the property of the working substance, it can be applied at very low temperature.

11.11 Petrol Engine

Petrol engine converts heat energy into machanical energy (work). Different engines are different in their construction but they are based on the principle of carnot cycle.

A typical four stroke petrol engine consists of four successive processes in each cycle.

i) Intake stroke:

Piston moves outward and petrol air mixture is drawn through an inlet value.

ii) Compression stroke:

The inlet value is closed and the mixture is compressed by inward movement of the piston.

iii) Power stroke:

A spark fires the mixture causing a rapid increase in pressure and temperature. The burning mixture expands adiabatically and forces the piston to move outward. This stoke provides power to the engine.

iv) Exhaust stroke:

The outlet volve opens. The residual gasses are expelled and piston moves inward.

The efficiency of a petrol engine is not more than 25% to 30% because of friction and other heat losses.

Diesel engine:

No spark plug is needed in a disel engine.

Diesel is sprayed into

the cylinder at maximum compression. Because air is at very high temperature immediately after compression, the fuel mixture ignites on contact with the air in the cylinder and pushes the piston outword.

The effeciency of a diesel engine is about 35% to 40%.

11.12 Entropy

Definition:

"Entropy is the measure of disorder of a system". It is denoted by S.

Explanation:

In 1856 Rudolph Clausius gave the concept of entropy. Entropy is a state variable used to describe the state of a system like other state variables temperature, Pressure, Volume and Internal energy.

Consider a reversible process in which a quantity of heat "ΔQ" is added to a system at absolute temperature "T", the increase in the entropy of the system is given by

$$\boxed{\Delta S = \frac{\Delta Q}{T}}$$

Unit of entropy $= \frac{J}{K}$

i) : When heat is added to a system (ΔQ + ive). The entropy of the system increases. ΔS is also + ive.

ii) : When heat is removed from a system ΔQ is − ive, then ΔS is also − ive. Entropy of the system decreases.

Like internal energy change in entropy of a system is important not its absolute value.

Suppose an amount of heat Q flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 and $T_1 > T_2$.

$\frac{Q}{T_1}$ = decrease in the entropy of reservoir at temperature T_1.

$\frac{Q}{T_2}$ = Increase in the entropy of reservoir at temperature T_2.

As

$$T_1 > T_2 \quad \text{or} \quad T_2 < T_1$$

Then

$$\frac{Q}{T_2} > \frac{Q}{T_1}$$

So that, Net change in entropy is positive. It means that net entropy is increased.

$$\text{Entropy} = \frac{Q}{T_2} - \frac{Q}{T_1}$$

Second Law of Thermodynamics in terms of entropy:

Statement:

"In all natural processes where heat flows from one system to another, there is always a net increase in entropy".

It is observed that a natural process proceed toward a state of greater disorder. So, there is a relation between entropy and molecular disorder.

Law of increase of entropy:

It states that

"The entropy of a system increases in all irreversible process."

For example, an irreversible heat flow from a hot to a cold subtance of a

system increases disorder because of increase in average molecular speeds and therefore will creat disorder of molecular motion.
The process for which entropy remains constant is a reversible process.
All natural process continue towards a state of greater disorder. The entropy increases. The increase in entropy means degradation of energy or unavailability of energy to do useful work. when the entropy of universe will reach its maximum value, everything will be at the same temperature. Then there will be no way convert heat energy into work. This is called heat Death.

11.13 Environmental Crisis As Entropy Crisis:

The Second Law of Thermodynamics given the idea for both understanding our enviromental crisis and understanding how we must deal with crsis.

From a human point of view the enviromental crisis results from out attempts to order nature for our comforts.

From a physical point

of view, however, the enviromental crisis is an entropy or disorder crisis, resulting from our invain efforts to ignore the second Law of thermodynamics.

The energy process we are not very afficient. As a result most of the energy is lost as heat to the environment. Although we can improve the afficiency, but second Law of Thermodynamics imposes an upper limit on efficiency improvement.

Thermal pollution is a direct result of 2^{ND} Law of Thermodynamics and the heat is the ultimate death of any form of energy. The means increase in the thermal pollution of the enviroment means increase in entropy.

In addition to thermal pollution the most energy transformation process such as heat engine used for transportation and for power generation causes air pollution.

In effect all forms of energy production have some bad effects and in some cases all problems cannot be fore seen.

Chapter = 11

Questions

Question : 11.1

Answer:

In random motion of gas molecules that number of molecules moving towards right with velocity $+V_x$ is equal to the number of molecules moving towards left with velocity $-V_x$. So averge velocity is zero

$$V_{x\,avg} = \frac{\cancel{+V_x} + (-\cancel{V_x})}{2}$$

$$= \text{Zero}$$

The average of squar of velocities is

$$(V_x^2)_{avg} = \frac{(+V_x)^2 + (-V_x)^2}{2}$$

$$= \frac{V_x^2 + V_x^2}{2}$$

$$= \text{zero}$$

Hence, the averge of the square of velocity is not zero.

Question : 11.2

Answer:

Friction between the tyre and the road heats up the gas in the tyre and its temperature increases. So, the average K.E of gas molecules increases.

$$T \propto \left\langle \frac{1}{2} mv^2 \right\rangle$$

As

$$P = \frac{2}{3} \frac{N}{V} \left\langle \frac{1}{2} mv^2 \right\rangle$$

$$P \propto \left\langle \frac{1}{2} mv^2 \right\rangle$$

$$\boxed{P \propto \langle K.E \rangle}$$

As < K.E > increases, So pressure of the gas increases.

Question: 11.3

Answer:

Temp = Constant

As

$$\boxed{T \propto \langle K.E \rangle}$$

So $\langle K.E \rangle$ = Constant

Hence Internal energy = Constant

Therefore, no change in the Internal Energy.

Question : 11.4

Answer:

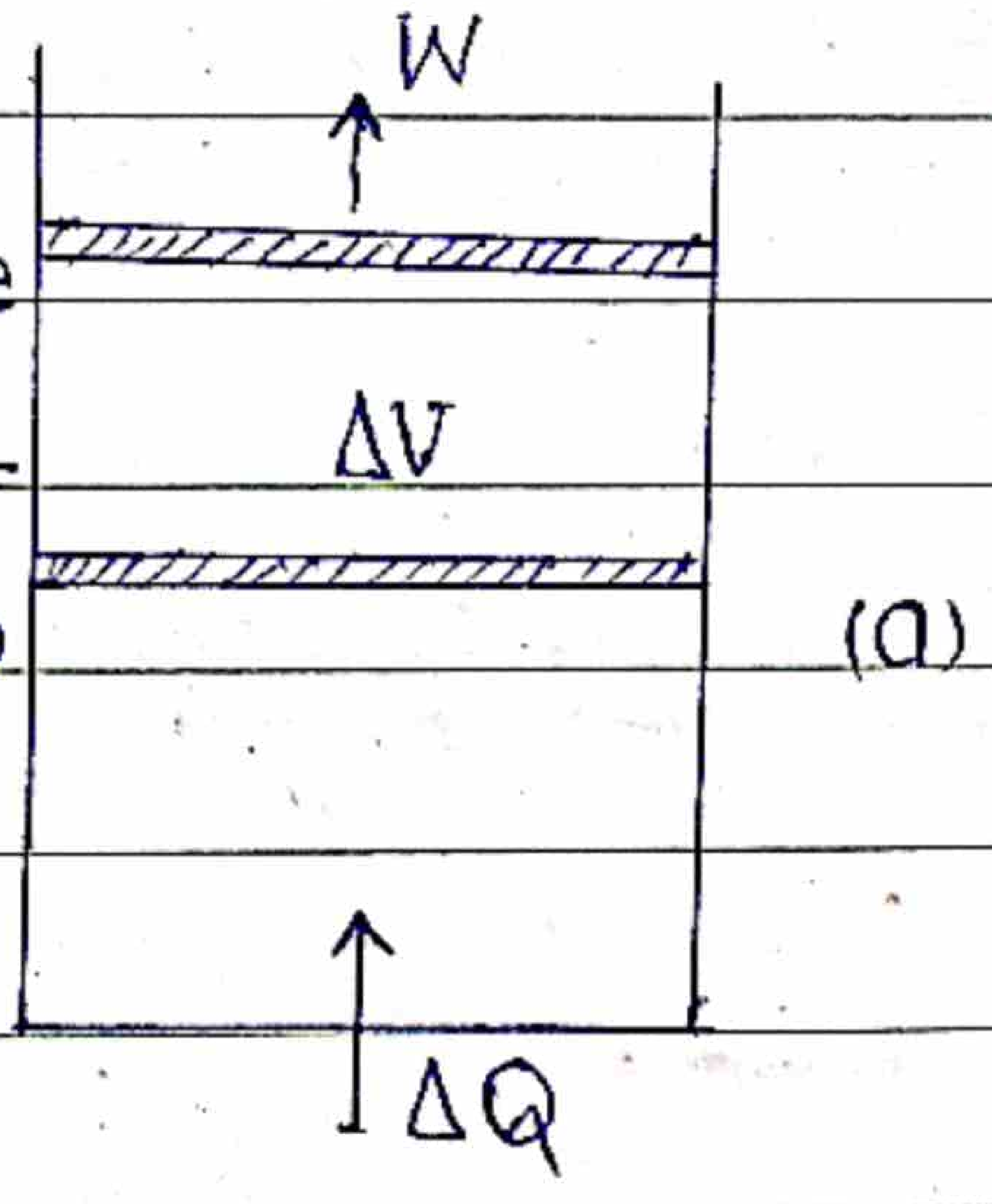
In all the three paths, the System returns to the initial state. So, there is no change in the internal energy.

Question : 11.5

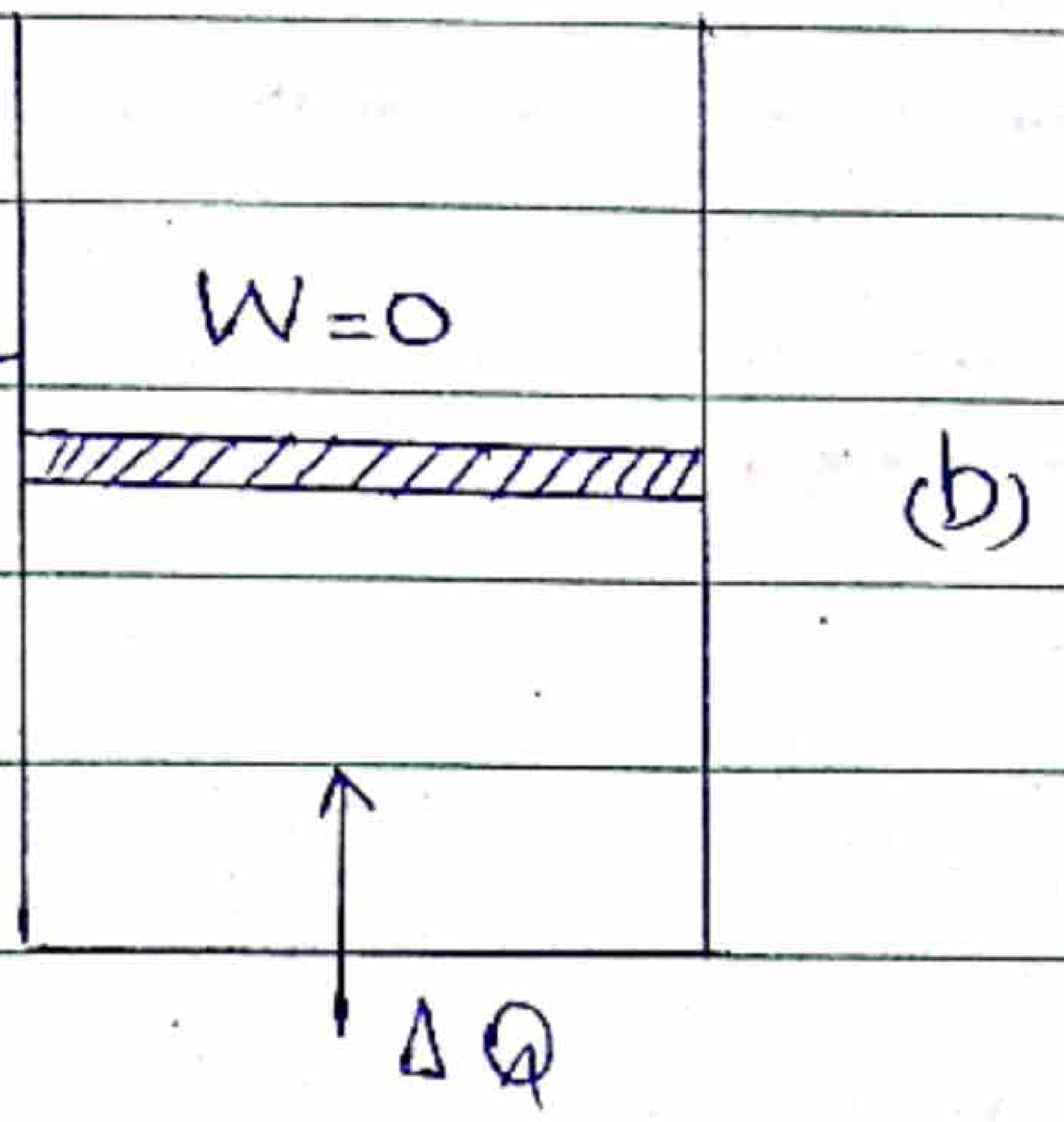
Answer:

At Constant Pressure

(i) heat is required not only to increase the temperature and internal energy, but also to do work as gas expands in fig. a



(ii) At Constant Volume entire heat is used to increase the temperature and internal energy. No heat is used to do work. in Fig.



Hence more heat is required to heat a gas at constant pressure than at constant volume for the same rise of temperature.

Hence $\boxed{C_p > C_v}$

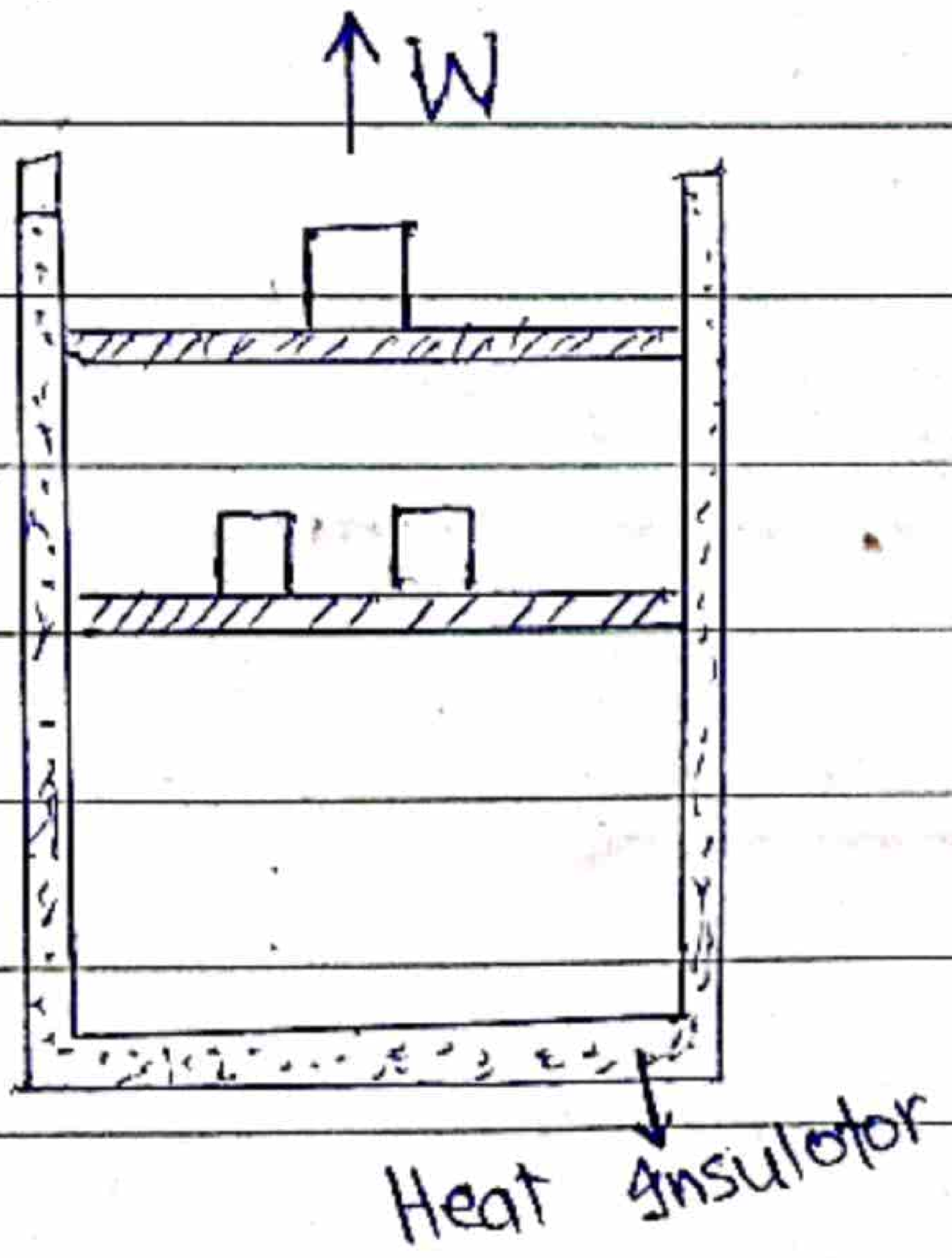
Specific heat of a gas at constant pressure is greater than specific heat of the gas at constant volume.

Question : 11.6

Answer:

In Adiabatic Process

In adiabatic expansion, work is due at the cost of internal energy of the system. So the temperature of the system decreases.



Question : 11.7

Answer:

Yes, it is possible to convert internal energy into mechanical energy.

Example:

Inadiabatic expansion, work is done at the cost of internal energy of the system.

Question : 11.8

Answer:

No, it is not possible. By 2nd Law of thermodynamics, it is not possible to construct a heat engine without a cold body.

When an engine works it expless a part of heat energy into cold body or atmosphere.

Question: 11.9

Answer:

Yes, when milk in a thermoflask is shaken rapidly work is done in shaking it. This work is converted into K.E of molecules of milk.

As

$$\boxed{T \propto \langle K.E \rangle}$$

So, T. Temperature of milk increases.

Question : 11.10

Answer :

Yes, mechanical energy can be converted completely into heat energy.

Example :

When mechanical energy i.e work is done in compressing the gas in the adiabatic process, the increase in the internal energy of the gas is equal to the work done on it.

Question : 11.12

Answer :

According to the Law of Increases of Entropy, the entropy of a system increases in all irreversible process. As work done by friction is an irreversible process. So, the entropy of the system Increases.

Question : 11.13

Answer:

When ice melts into water it absorbs heat ΔQ

So

$$\boxed{\Delta S = \frac{\Delta Q}{T}}$$

ΔS is +ive , so entropy increases.

Question : 11.14

Answer:

Statement (a) is correct

Question 11.15

Answer:

Statement (d) is correct

Question : 11.16

Answer:

statement (b) is Correct

Chapter = 11

Examples

Example : 11.1

Solution:

$$T = \frac{2}{3k} \langle K.E \rangle$$

$$\langle K.E \rangle = \frac{3kT}{2}$$

$$T = 27^\circ C$$

$$= (27 + 273) K$$

$$T = 300 K$$

$$k = 1.38 \times 10^{-23} \text{ J}K^{-1}$$

$$\langle K.E \rangle = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$\boxed{\langle K.E \rangle = 6.21 \times 10^{-21} \text{ J}}$$

Example : 11.2

Solution :

AT STP

$T = 0^\circ C$

$= 0 + 273 K$

$T = 273 K$

$m = \frac{\text{molecular mass}}{N_A}$

$m = \frac{M}{N_A} = \frac{32 g}{6.022 \times 10^{26}}$

$$\boxed{m = \frac{32 kg}{6.022 \times 10^{26}}}$$

$\therefore T = \frac{2}{3k} \langle \frac{1}{2} mv^2 \rangle$

$T = \frac{m}{3k} \langle v^2 \rangle$

$\langle v^2 \rangle = \frac{3kT}{m}$

$$\langle V^2 \rangle = \frac{3 \times 1.38 \times 10^{-23} \times 273}{\frac{32}{6.022 \times 10^{26}}}$$

$$\langle V^2 \rangle = \frac{3 \times 1.38 \times 10^{-23} \times 273 \times 6.022 \times 10^{26}}{32}$$

$$\langle V^2 \rangle = 212693 \ m^2/s^2$$

$$\boxed{\langle V \rangle = 461 \ m/s}$$

Example : 11.3

Solution:

$A = 0.10 \ m^2$

$P = 8000 \ Nm^{-2}$

$\Delta y = 4cm$

$\Delta y = .04m$

$Q = 42 \ J$

$\Delta U = ?$

$W = P.\Delta V = P.A\,\Delta y$

$W = 8000 \times 0.10 \times 0.4$

$W \quad 32\,J$

By 1st Law of thermodynamics

$Q = \Delta U + W$

$\Delta U = Q - W$

$\Delta U = 42\,J - 32\,J$

$\boxed{\Delta U = 10\,J}$

Example : 11.4

Solution:

$T_1 = 427°C$

$= (427 + 273)K$

$T_1 = 700K$

$T_2 = 77^\circ C$

$= (77 + 273) K$

$T_2 = 350 K$

$\eta = ?$

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\eta = \frac{700k - 350K}{700k}$$

$$\eta = \frac{350}{700k}$$

$$\eta = \frac{1}{2}$$

$\eta = 0.5$

$$\eta\% = \frac{1}{2} \times 100$$

$$\boxed{\eta\% = 50\%}$$

Example : 11.5

Solution:

$m = 1kg$

$T = 0°C$

$T = 273K$

$L_f = 3.36 \times 10^5 \ J \ kg^{-1}$

$$\Delta S = \frac{\Delta Q}{T}$$

$\Delta Q = m L_f$

$$\Delta S = \frac{m L_f}{T}$$

$$\Delta S = \frac{1kg \times 3.36 \times 10^5 J \ kg^{-1}}{273K}$$

$$\boxed{\Delta S = 1.23 \times 10^3 \ J K^{-1}}$$

Chapter = 11

Problems

Problem : 11.1

Solution:

At stp

$T = 0^\circ C = 273K$

$\langle V \rangle = ?$

$m = \frac{\text{molecular mass}}{N_A}$

for N_2

$$m = \frac{28g}{6.022 \times 10^{23}}$$

$$m = \frac{28 \times \overset{\text{kilo}\downarrow}{10^{3}} g}{6.02 \times 10^{23} \times 10^{3}}$$

$$m = \frac{28\ kg}{6.022 \times 10^{26}}$$

$$K = 1.38 \times 10^{-23}\ JK^{-1}$$

$$T = \frac{\cancel{2}}{3k} \left\langle \frac{1}{\cancel{2}} mv^2 \right\rangle$$

$$T = \frac{m}{3k} \langle v^2 \rangle$$

$$\langle v^2 \rangle = \frac{3kT}{m}$$

$$\langle v^2 \rangle = \frac{3 \times 1.3 \times 10^{-2} \times 273}{\frac{28}{6.022 \times 10^{26}}}$$

$$\langle v^2 \rangle = \frac{3 \times 1.38 \times 10^{-23} \times 273 \times 6.022 \times 10^{26}}{28}$$

$$\langle v^2 \rangle = 24.31 \times 10^4\ m^2/s^2$$

$$\boxed{\langle v \rangle = 493\ m/s}$$

Problem : 11.2

Solution :

$\langle v^2 \rangle$ = means Square Speed

$\sqrt{\langle v^2 \rangle}$ = root - means square speed

As

$$T = \frac{2}{3k} \left\langle \frac{1}{2} m v^2 \right\rangle$$

For first gas

$$T = \frac{2}{3k} \left\langle \frac{1}{2} m_1 v_1^2 \right\rangle$$

For 2nd gas

$$T = \frac{2}{3k} \left\langle \frac{1}{2} m_2 v_2^2 \right\rangle$$

Temperature is same

Comparing Eq (1) and (2)

$$\cancel{\frac{2}{3k}} \left\langle \cancel{\frac{1}{2}} m_1^2 v_1^2 \right\rangle = \cancel{\frac{2}{3k}} \left\langle \cancel{\frac{1}{2}} m_2 v_2^2 \right\rangle$$

$$\langle m_1^2 \; v_1^2 \rangle = \langle m_2^2 v_2^2 \rangle$$

$$m_1 \langle v_1^2 \rangle = m_2 \langle v_2^2 \rangle$$

$$\frac{\langle v_1^2 \rangle}{\langle v_2^2 \rangle} = \frac{m_2}{m_1}$$

Taking square-root of both side

$$\frac{\sqrt{\langle v^2 \rangle}}{\sqrt{\langle v_2^2 \rangle}} = \sqrt{\frac{m_2}{m_1}}$$

Problem : 11.3

Solution:

Let initial Volume $= V$

Final Volume $= \frac{V}{2} = ?$

$$P = 1.25 \times 10^5 \, Nm^{-2}$$

$$W = 100 \, J$$

$$W = P \Delta V$$

$$\Delta V = \frac{W}{P}$$

$$= \frac{100}{1.25 \times 10^5}$$

$$\Delta V = 8 \times 10^{-4} \, m^3$$

Initial Volume − Final volume

$$V - \frac{V}{2} = \Delta V$$

$$\frac{2V - V}{2} = \Delta V$$

Final volume $= \frac{v}{2} = \Delta V = 8\times10^{-4}\,m^3$

$$\boxed{\Delta V = 8\times10^{-4}\,m^3}$$

Problem : 11.4

Solution:

$\Delta U = -300\,J$

-ive sign for decrease in internal Energy

$W = -120\,J$

-ive sign for work done on the system.

By 1st Law

$Q = ?$

$Q = \Delta U + W$

$Q = -300\,J - 120\,J$

$$\boxed{Q = -420\,J}$$

-ive sign for heat Lost.

Problem : 11.5

Solution:

$T_1 = 227°C$

$= 227 + 273$

$T_1 = 500\,K$

$T_2 = 127°C$

$= 127 + 273$

$T_2 = 400K$

$\eta = ?$

$Q_1 = ?$

$Q_2 = ?$

$W = 10{,}000\,J$

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\eta = \frac{500K - 400K}{500K}$$

$$\eta = \frac{100\,K}{500K}$$

$$\eta = \frac{1}{5}$$

$$\eta = 0.2$$

$$\eta\% = \frac{1}{5} \times 100$$

$$\eta\% = 20\%$$

$$\eta = \frac{\text{out put}}{\text{In put}}$$

$$\eta = \frac{W}{Q_1}$$

$$Q_1 = \frac{W}{\eta}$$

$$Q_1 = \frac{10000}{0.2}$$

$$Q_1 = 50000\ J$$

As

$$W = Q_1 - Q_2$$

$$Q_2 = Q_1 - W$$

$$Q_2 = 50000\ J - 10{,}000\ J$$

$$\boxed{Q_2 = 40{,}000\ J}$$

Problem : 11.6

Solution:

$$T_1 - T_2 = 100°C$$

OR

$$T_1 - T_2 = 100k$$

$$Q_1 = 746\ J$$

$$Q_2 = 546\ J$$

$T_1 = ?$ (for source)

$T_2 = ?$ (for sink)

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$T_1 - T_2 = \frac{Q_1 - Q_2}{Q_1} \times T_1$$

$$(T_1 - T_2) \times \frac{Q_1}{Q_1 - Q_2} = T_1$$

$$100K \times \frac{746\ J}{746 - 546 J} = T_1$$

$$100\ K \times \frac{746 \cancel{J}}{200 \cancel{J}} = T_1$$

$$\frac{746}{2} K = T_1$$

$$T_1 = 373 K$$

$$T_1 = (373 - 273)°C$$

$$\boxed{T = 100°C}$$

$$T_1 - T_2 = 100 K$$

$$T_1 - 100 K = T_2$$

$$373 k - 100 K = T_2$$

$$T_2 = 273 K$$

$$T_2 = (273 - 273)°C$$

$$\boxed{T_2 = 0°C}$$

Problem : 11.7

Solution:

$T_1 = 327°C$

$= 327 + 273$

$T_1 = 600 K$

$T_2 = 27°C$

$= 27 + 273$

$T_2 = 300 K$

$\eta \% = 52\%$

$\eta = ?$

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\eta = \frac{600K - 300K}{600K}$$

$$\eta = \frac{300\cancel{K}}{600\cancel{K}}$$

$$\eta = \frac{1}{2}$$

$$\eta = 0.5$$

$$\eta\% = \frac{1}{2} \times 100$$

$$\eta\% = 50\%$$

The efficiency is 50% not 52%. So claim is not correct.

Problem : 11.8

Solution:

$$W = 100\,J$$

$$Q_2 = 400\,J$$

$$\eta = ?$$

$$W = Q_1 \quad Q_2$$

$$Q_1 = W + Q_2$$

$$Q_1 = 100\,J + 400\,J$$
$$= 500\,J$$

$$\eta = \frac{\text{output}}{\text{Input}}$$

$$\eta = \frac{W}{Q_1}$$

$$\eta = \frac{100\,J}{500\,J}$$

$$\eta = \frac{1}{5}$$

$$\eta = 0.2$$

$$\eta\% = \frac{1}{5} \times 100$$

$$\boxed{\eta\% = 20\%}$$

Problem : 11.9

Solution:

$$\eta = 1 - \frac{T_2}{T_1} \quad , \quad \Delta T = ?$$

$$\eta = \frac{1}{2}$$

$$\frac{1}{2} = 1 - \frac{280}{T_1}$$

$$\frac{1}{2} = \frac{T_1 - 280}{T_1}$$

$T_2 = 7°C + 273$

$= 280K$

$\eta = 50\% \quad \frac{50}{100}$

$= \frac{1}{2}$

$\eta = 70\% = \frac{70}{100}$

$= \frac{7}{10}$

$$2(T_1 - 280) = T_1$$

$$2T_1 - 560 = T_1$$

$$2T_1 - T_1 = 560$$

$$\boxed{T_1 = 560K}$$

when,

$$\eta' = \frac{7}{10}$$

$$\eta' = 1 - \frac{T_2}{T_1}$$

$$\frac{7}{10} = \frac{T_1' - 280}{T_1'}$$

$$10(T_1' - 280) = 7T_1$$

$$10T_1' - 2800 = 7T_1'$$

$$10T_1' - 7T_1' = 2800$$

$$3T_1' = 2800$$

$$T_1' = \frac{2800}{3}$$

$$T_1' = 933.3K$$

Increase in temperature of source

$$\Delta T = T_1' - T_1$$

$$\Delta T = 933 - 560$$

$$\Delta T = 373K$$

$$\boxed{\Delta T = 373K}$$

Problem : 11.10

Solution :

$$T_1 = 450K$$

$$T_2 = 300K$$

$$\eta = ?$$

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\eta = \frac{450K - 300K}{450K}$$

$$\eta = \frac{150\cancel{K}}{450\cancel{K}}$$

$$\eta = \frac{1}{3}$$

$$\eta = 0.333$$

$$\eta_{\%} = \frac{1}{3} \times 100$$

$$= 33.3\%$$

$$\boxed{\eta\% = 33\%}$$

Problem : 11.11

Solution :

Latent heat of fusion of ice $L_f = 336\ J/g$

$$L_f = 336\ Jg^{-1}$$

$$\Delta S = ?$$

$$m = 30g$$

$T = 0°C$

$= 0 + 273\,K$

$T = 273\,K$

$\Delta Q = mL_f$

$= 30g \times 336\,\frac{J}{g}$

$\Delta Q = 10080\,J$

As heat is taken out so ΔQ is -ive

$\Delta S = \frac{\Delta Q}{T}$

$\Delta S = \frac{-10080\,J}{273\,K}$

$\boxed{\Delta S = -36.8\,J/K}$