(Short Questions)

(i)

Find θ when $l = 1.5cm$, $r = 2.5cm$. (LHR-2011+14.)

Answer: $\theta = ?$, $l = 1.5cm$, $r = 2.5cm$.

Formula: $l = r\theta$, $\theta = \frac{l}{r}$

$$\theta = \frac{1.5cm}{2.5cm} \Rightarrow 0.6 rad.$$

(ii)

Verify $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = 3/\sqrt{2}$ (LHR-2011+12+14+21)

Answer: $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = 3/\sqrt{2}$

$L.H.S = 2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(\sqrt{2}) \Rightarrow \sqrt{2}\cdot\cancel{\sqrt{2}}\left(\frac{1}{\cancel{\sqrt{2}}}\right) + \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\cdot\sqrt{2}}$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} \Rightarrow \frac{2+1}{\sqrt{2}} \Rightarrow 3/\sqrt{2}$$

(iii)

Show that $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

Answer: (LHR-2011+12)

$L.H.S = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) \Rightarrow \sec^2\theta - \tan^2\theta$

$$= 1 \quad \because \sec^2\theta - \tan^2\theta = 1$$

(iv)

Find all values of trigonometric functions of 420°. (LHR-2012+14)

Answer:- 420°

$\therefore \quad 420^\circ = 60^\circ + 360^\circ \quad \because \ k = 1$

As we know that:

$\theta = \theta + 2\pi k$ as $k \in \mathbb{Z}$

$\sin 420^\circ = \sin 60^\circ = \sqrt{3}/2$

$\cos 420^\circ = \cos 60^\circ = 1/2$

$\tan 420^\circ = \tan 60^\circ = \sqrt{3}$

$\operatorname{cosec} 420^\circ = \operatorname{cosec} 60^\circ = 2/\sqrt{3}$

$\sec 420^\circ = \sec 60^\circ = 2$

$\cot 420^\circ = \cot 60^\circ = 1/\sqrt{3}$

(v)

Define radian (LHR-2012+15)

Answer:-

Radian is the measure of the angle subtanded at the center of the circle by an arc whose length is equal to the radius of the circle.

(vi)

Prove that $\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$

(LHR-2013)

Answer:- $\sin^3\theta+\cos^3\theta=(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)$.

$L.H.S = \sin^3\theta+\cos^3\theta$ $\quad \because a^3+b^3=(a+b)(a^2+b^2-ab)$.

$= (\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)$.

$= (\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)$.

$\because \sin^2\theta+\cos^2\theta=1$.

So, $L.H.S = R.H.S$.

(vii)

Express in radian $75^\circ 6' 30''$. (LHR-2013)

Answer: $75^\circ 6' 30''$

$= 75^\circ+\left(\frac{6}{60}\right)^\circ+\left(\frac{30}{3600}\right)^\circ \Rightarrow \left(75+\frac{6}{60}+\frac{30}{3600}\right)^\circ$

$= \left(\frac{270000+360+30}{3600}\right)^\circ \Rightarrow \left(\frac{270390}{3600}\right)^\circ \Rightarrow \left(\frac{9013}{120}\right)^\circ$

$= \left(\frac{9013}{120}\times\frac{\pi}{180}\right)\text{rad} \Rightarrow \frac{9013\pi}{21600}\text{ rad}$.

(viii)

Prove that $\cos\theta+\tan\theta\sin\theta=\sec\theta$. (LHR-2013)

Answer: $\cos\theta+\tan\theta\sin\theta=\sec\theta$

L.H.S = $\cos\theta+\tan\theta\sin\theta$.

$= \cos\theta+\frac{\sin\theta}{\cos\theta}\sin\theta \Rightarrow \cos\theta+\frac{\sin^2\theta}{\cos\theta} \Rightarrow \frac{\cos^2\theta+\sin^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} \Rightarrow \sec\theta$ So, L.H.S = R.H.S.

(ix)

What is the length of arc intercepted on a circle of radius **14cm** by the arm of central angle **45°**.

Answer: (LHR-2014)

$r = 14cm$, $\theta = 45°$, $l = ?$

$\theta = 45°$ change in radian.

$\theta = \left(45 \times \frac{\pi}{180}\right) rad$

$\theta = \frac{\pi}{4} rad$

Formula:

$l = r\theta$

$l = (14cm)\left(\frac{\pi}{4}\right)$

$l = 10.9cm$

(x)

Prove $\sec\theta\,\mathrm{cosec}\theta\,\sin\theta\cos\theta = 1$ (LHR-2015)

Answer: $\sec\theta\,\mathrm{cosec}\theta\,\sin\theta\cos\theta = 1$

L.H.S $= \sec\theta\,\mathrm{cosec}\theta\,\sin\theta\cos\theta$

$= \frac{1}{\cancel{\cos\theta}} \cdot \frac{1}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} \cdot \cancel{\cos\theta}$

$= 1$ So, L.H.S = R.H.S.

(xi)

Convert $\frac{2\pi}{3}$ **radian into degree.** (LHR-2015)

Answer: $\frac{2\pi}{3} \Rightarrow \left(\frac{2\cancel{\pi}}{3} \times \frac{180}{\cancel{\pi}}\right)^{\circ} \Rightarrow 120°$.

(xii)

Convert 54°45′ into radian (LHR-2016+21)

Answer: $54°45'$

$$= \left(54+\frac{45}{60}\right)^{\circ} \Rightarrow \left(\frac{3240+45}{60}\right)^{\circ} \Rightarrow \left(\frac{3285}{60}\right)^{\circ} \Rightarrow \left(\frac{657}{12}\right)^{\circ}$$

$$= \left(\frac{219}{4}\right)^{\circ} \Rightarrow \left(\frac{219}{4}\times\frac{\pi}{180}\right)\text{rad} \Rightarrow \frac{219\pi}{720} \Rightarrow 0.958\,\text{rad}.$$

(xiii)

Prove that $\frac{\sin\theta}{1+\cos\theta}+\cot\theta=\csc\theta$ (LHR-2016+17.)

Answer: $\frac{\sin\theta}{1+\cos\theta}+\cot\theta=\text{cosec}\,\theta.$

$$L.H.S = \frac{\sin\theta}{1+\cos\theta}+\cot\theta \Rightarrow \frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta} \Rightarrow \frac{\sin^2\theta+\cos\theta(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{\sin^2\theta+\cos\theta+\cos^2\theta}{(\sin\theta)(1+\cos\theta)} \Rightarrow \frac{\sin^2\theta+\cos^2\theta+\cos\theta}{(\sin\theta)(1+\cos\theta)} \Rightarrow \frac{\cancel{1+\cos\theta}}{\sin\theta\cancel{(1+\cos\theta)}}$$

$$= \frac{1}{\sin\theta} \Rightarrow \text{cosec}\,\theta. \quad \because \cot\theta=\frac{\cos\theta}{\sin\theta}, \quad \because \sin^2\theta+\cos^2\theta=1.$$

(xiv)

Prove that $\cos^2\theta-\sin^2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$. (LHR-2013+16+19)

Answer: $\cos^2\theta-\sin^2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$R.H.S = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow 1-\frac{\sin^2\theta}{\cos^2\theta} \Big/ 1+\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cancel{\cos^2\theta}}}{\frac{\cos^2\theta + \sin^2\theta}{\cancel{\cos^2\theta}}} \quad \because \sin^2\theta + \cos^2\theta = 1.$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1} \Rightarrow \cos^2\theta - \sin^2\theta.$$

(xv)

Verify $\cos 2\theta = \cos^2\theta - \sin^2\theta$. when $\theta = 30°, 45°$ (LHR-2016+17.)

Answer: $\cos 2\theta = \cos^2\theta - \sin^2\theta$

When $\theta = 30°$

$$\cos 2(30°) = \cos^2 30° - \sin^2 30°$$
$$\cos 60° = (\cos 30)^2 - (\sin 30)^2$$
$$\cos 60° = (\cos 30)^2 - (\sin 30)^2$$
$$\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$
$$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$$
$$\frac{1}{2} = \frac{3-1}{4}$$
$$\frac{1}{2} = 2/4$$
$$1/2 = 1/2$$

When $\theta = 45°$.

$$\cos 2(45) = \cos^2 45° - \sin^2 45°$$
$$\cos 90° = (\cos 45°)^2 - (\sin 45°)^2$$
$$0 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$0 = \frac{1}{2} - \frac{1}{2}$$
$$\therefore 0 = \frac{\cancel{1} - \cancel{1}}{2}$$
$$0 = 0/2$$
$$0 = 0.$$

(xvi)

Prove that $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$ (LHR-2016.)

Answer:- $(\tan\theta+\cot\theta)^2 = \sec^2\theta\,\mathrm{cosec}^2\theta$.

L.H.S:- $(\tan\theta+\cot\theta)^2 \Rightarrow \left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{\sin^2\theta+\cos^2\theta}{(\sin\theta)(\cos\theta)}\right)^2 = \frac{1}{\sin^2\theta\cos^2\theta} = \frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta}$

$= \sec^2\theta\,\mathrm{cosec}^2\theta$

(xvii)

find l when $\theta = \pi$ rad, $r = 6$cm. (LHR-2017.)

Answer:- formula: $l = r\theta$.

Given:- $\theta = \pi$ rad, $r = 6$cm.

$l = r\theta$, $l = (6)(\pi) \Rightarrow l = 18.84$cm

(xviii)

Verify: $\cos 2\theta = 2\cos^2\theta - 1$ when $\theta = 30°, 45°$ (LHR-2017.)

Answer:- $\cos 2\theta = 2\cos^2\theta - 1$.

When $\theta = 30°$

$\cos 2(30°) = 2\cos^2(30°) - 1$

$\cos 60° = 2(\cos 30°)^2 - 1$

$1/2 = 2\left(\sqrt{3}/2\right)^2 - 1 \Rightarrow 3/2 - 1$

$1/2 = \frac{3-2}{2} \Rightarrow 1/2$

When $\theta = 45°$.

$\cos 2(45°) = 2\cos^2(45°) - 1$

$\cos 90° = 2(\cos 45°)^2 - 1$.

$0 = 2(1/\sqrt{2})^2 - 1 \Rightarrow 2\left(\frac{1}{2}\right) - 1$

$0 = 1 - 1 = 0$

(xlviii)

Prove that $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$ (LHR-2017+19.)

Answer: $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

$$R.H.S = \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta} \Rightarrow \frac{(\cos\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta - \sin\theta\cos\theta}{1-\sin^2\theta} \Rightarrow \frac{\cos\theta - \sin\theta\cos\theta}{\cos^2\theta} \Rightarrow \frac{\cos\theta}{\cos^2\theta} - \frac{\sin\theta\cos\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{1-\sin\theta}{\cos\theta}$$

(xlix)

find l when $\theta = 65°20'$, $r = 18mm$ (LHR-2017+19.)

Answer: Given:- $r = 18mm$.

$$\theta = 65°20' \Rightarrow 65 + \frac{20}{60} \Rightarrow 65 + \frac{1}{3} \Rightarrow \frac{195+1}{3} \Rightarrow \left(\frac{196}{3}\right)°$$

$$\theta = \left(\frac{196}{3} \times \frac{\pi}{180}\right) rad \Rightarrow 1.14 \, rad$$

formula:- $l = r\theta$

$$l = (18)(1.14) \Rightarrow 20.25mm.$$

(xlviii)

9) $\tan^2 45 - \cos^2 60 = x \sin 45° \cos 45° \tan 60°$ find x.

(LHR-2017+18)

Answer:- $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

$$(\tan 45^\circ)^2 - (\cos 60^\circ)^2 = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$$

$$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3}).$$

$$1 - \frac{1}{4} = x \frac{\sqrt{3}}{2}$$

$$\frac{4-1}{4} = \frac{\sqrt{3}}{2} x$$

$$\frac{3}{4} \times \frac{2}{\sqrt{3}} = x$$

$$\frac{\sqrt{3}\,\cancel{\sqrt{3}}}{2\,\cancel{\sqrt{3}}} = x$$

$$\frac{\sqrt{3}}{2} = x.$$

(xvxii)

An arc subtended an angle of **70°** at the center of the circle and its length is **132mm**. Find the radius of the circle? (LHR-2018)

Answer:-

Given:-

$$l = 132mm.$$

$$\theta = 70^\circ \Rightarrow \left(70 \times \frac{\pi}{180}\right) rad \Rightarrow \frac{11}{9} rad.$$

formula:- $l = r\theta$, $r = l/\theta$

$$r = \frac{132}{11/9} \Rightarrow 132 \times \frac{9}{11} \Rightarrow 108mm.$$

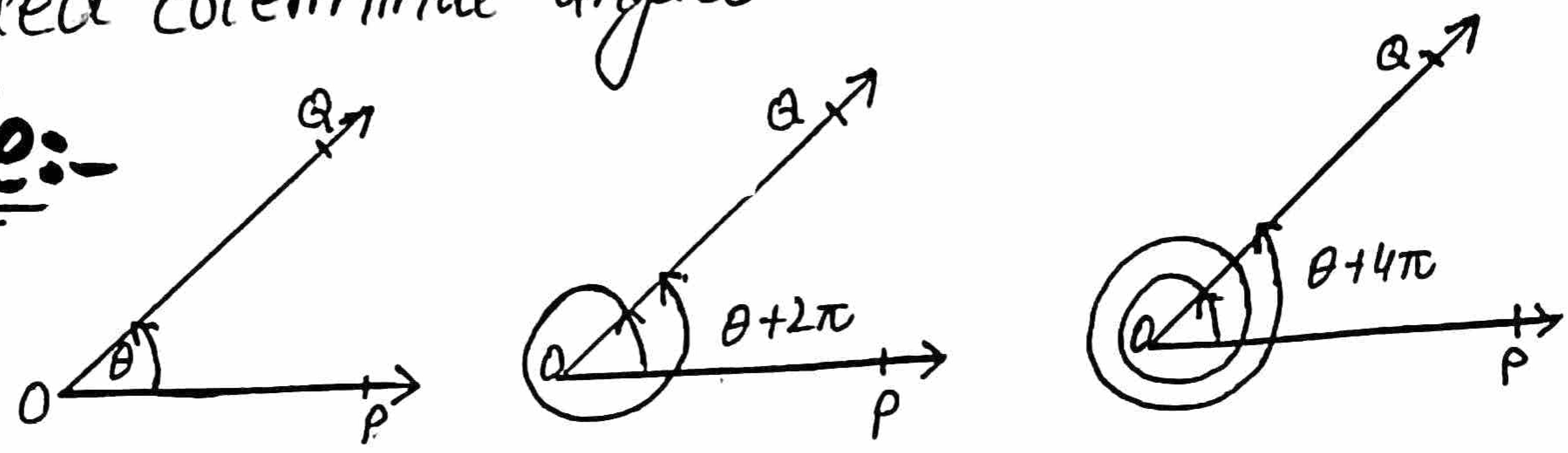
(xvxiii)

Define coterminal angle. (LHR-2018)

Answer:-

There can be many angles with the same initial and terminal sides. These angles are called coterminal angles.

Example:-



(xlvxix)

Verify $\sin^2\frac{\pi}{6}+\sin^2\frac{\pi}{3}+\tan^2\frac{\pi}{4}=2$ (LHR-2010+18+19)

Answer:- $\sin^2\frac{\pi}{6}+\sin^2\frac{\pi}{3}+\tan^2\frac{\pi}{4}=2$

L.H.S $= \sin^2\frac{\pi}{6}+\sin^2\frac{\pi}{3}+\tan^2\frac{\pi}{4}$

$= \left(\sin\frac{\pi}{6}\right)^2+\left(\sin\frac{\pi}{3}\right)^2+\left(\tan\frac{\pi}{4}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2+(1)^2$

$= \frac{1}{4}+\frac{3}{4}+1 \Rightarrow \frac{1+3+4}{4} \Rightarrow \frac{8}{4} \Rightarrow 2.$

So, L.H.S = R.H.S.

(xlvxv)

Define angle and angle in standard position? (LHR-2011) with figure define angle in standard position? (LHR-2018.)

Answer:

Angle:

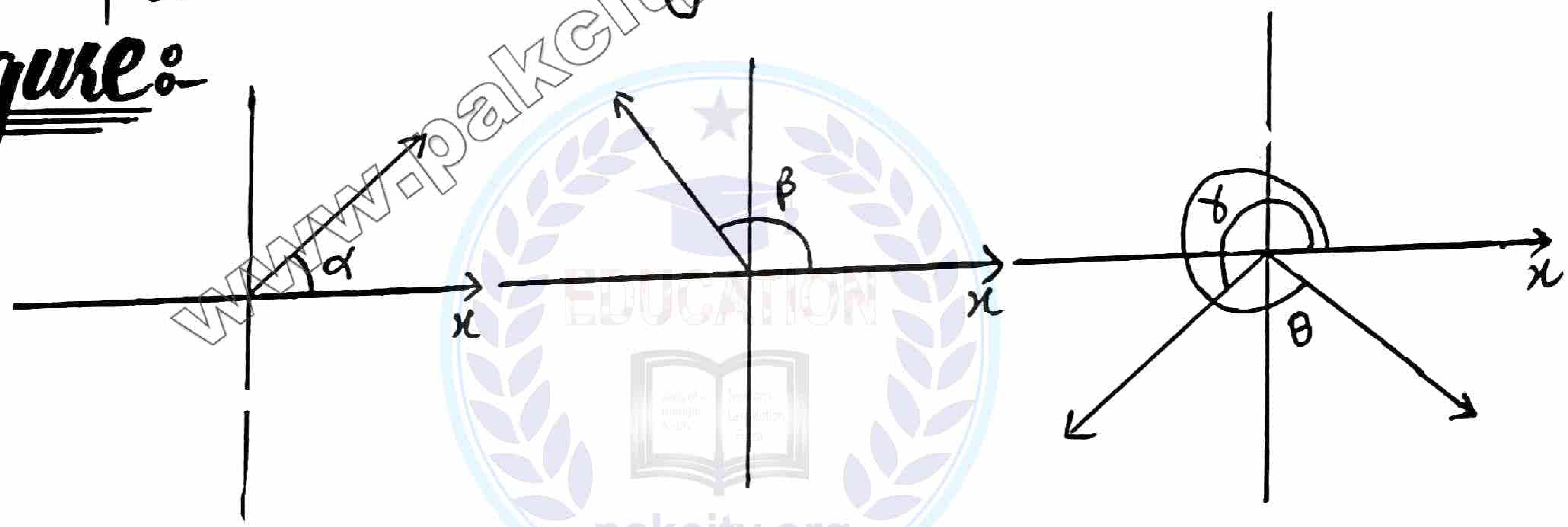
Two rays with common starting point form an angle One of the ray of the angle is called initial side and the other as a terminal side of an angle.

Angle in standard position:-

Angle in standard position is defined as:

If the vertex of the angle lies the origin of a rectangular coordinate system and its initial side along the positive x-axis.

Figure:



(xuxvi)

Prove that $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$ (LHR-2017+18)

Answer:- $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$= \frac{1-\cancel{\sin\theta}+1+\cancel{\sin\theta}}{(1-\sin^2\theta)} \Rightarrow \frac{2}{\cos^2\theta} \Rightarrow 2\sec^2\theta$$

(xlvxvii)

Verify: $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4.$ (L.HR-2019.)

Answer:- $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4.$

$L.H.S = \sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2}$

$= \left(\sin\frac{\pi}{6}\right)^2 : \left(\sin\frac{\pi}{4}\right)^2 : \left(\sin\frac{\pi}{3}\right)^2 : \left(\sin\frac{\pi}{2}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2$

$= \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$ By Multiplying 4 we get

$= 1 : 2 : 3 : 4.$ So, L.H.S = R.H.S.

(xlvxviii)

find r when $l = 56cm$, $\theta = 45°$. (LHR-2019.)

Answer:- Given:- $l = 56cm$, $\theta = 45° \Rightarrow \left(45 \times \frac{\pi}{180}\right)^{rad}$

$\theta = 0.78539\ rad.$ formula:- $l = r\theta$, $r = l/\theta$

$= \frac{56}{0.78539} \Rightarrow 71.302.$

(xlvxvix)

find values of all trigonomatric functions of -15π (LHR-2012+14).

Answer:-

$-15\pi = \pi - 16\pi$

$-15\pi = \pi + (-16)(\pi) \quad \because k = -16.$

As we know that:

$\theta + 2\pi k = \theta$ and $k \in \mathbb{Z}$

$\sin -15\pi = \sin \pi = 0 \quad , \quad \operatorname{cosec} -15\pi = \operatorname{cosec} \pi = \infty$

$\cos -15\pi = \cos \pi = -1 \quad , \quad \sec -15\pi = \sec \pi = -1$

$\tan -15\pi = \tan \pi = 0 \quad , \quad \cot -15\pi = \cot \pi = \infty.$

(xvxvx)

find r when $l = 5cm$, $\theta = \frac{1}{2}$ rad (LHR-2010+22.)

Answer:- Given:- $l = 5cm$, $\theta = \frac{1}{2}$ rad.

formula:- $l = r\theta$, $r = l/\theta$

$r = 5/\tfrac{1}{2} \Rightarrow 5 \times 2 \Rightarrow 10cm$

(xvxvxi)

Verify that: $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$. (LHR-2022.)

Answer:- $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ.$

$L.H.S = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \Rightarrow \frac{3}{4} - \frac{1}{4} \Rightarrow \frac{3-1}{4} \Rightarrow \frac{2}{4}$

$= \frac{1}{2} = \sin 30^\circ$ So, L.H.S = R.H.S.

Question xxvxxvxii

Prove that $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

(LHR-2022.)

Answer:- $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$.

L.H.S $= \cos^4\theta - \sin^4\theta$

using $a^4 - b^4 = (a^2+b^2)(a^2-b^2)$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$.

$= 1(\cos^2\theta - \sin^2\theta)$ $\because \cos^2\theta + \sin^2\theta = 1$.

$= \cos^2\theta - \sin^2\theta$

Long Questions

(i)

Prove that $\dfrac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$.

Answer: (LHR – 2011+21)

L.H.S: $\dfrac{1+\cos\theta}{1-\cos\theta} \Rightarrow \dfrac{1+\cos\theta}{1-\cos\theta} \times \dfrac{1+\cos\theta}{1+\cos\theta}$

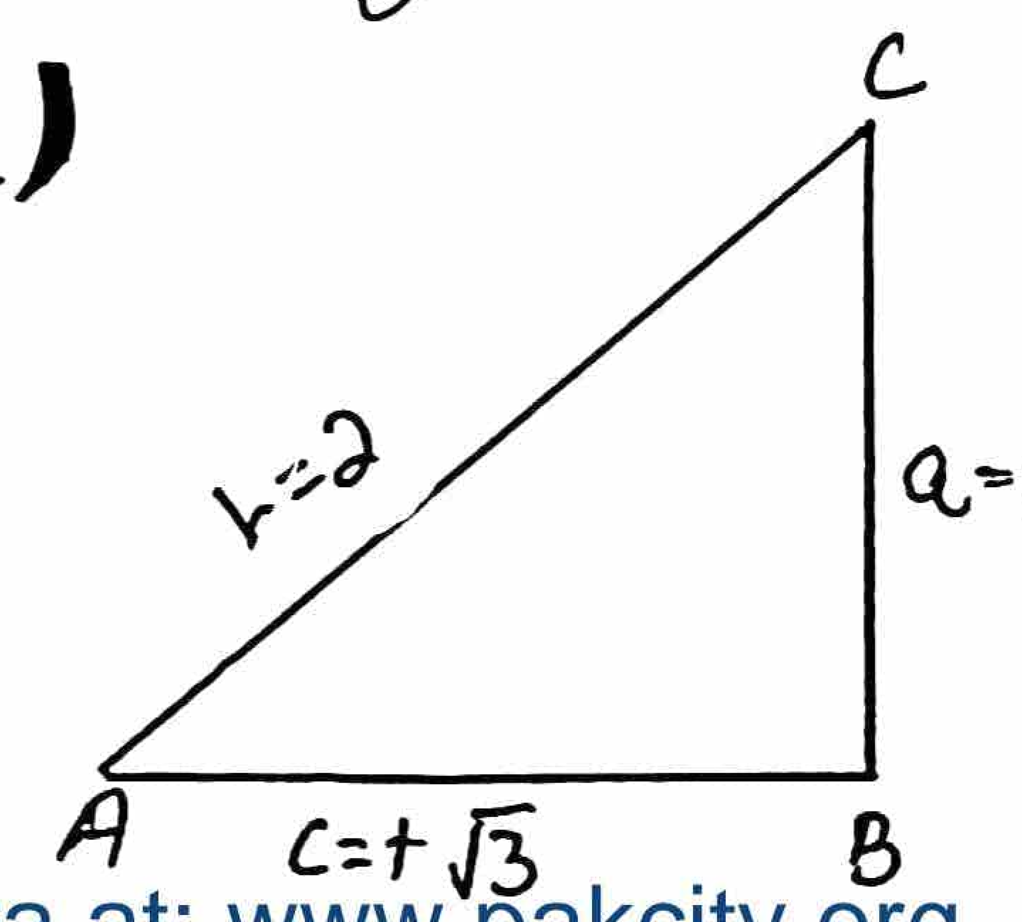
$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \Rightarrow \frac{(1+\cos\theta)^2}{(1-\cos^2\theta)} \Rightarrow \frac{(1+\cos\theta)^2}{\sin^2\theta}$$

$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2 \Rightarrow \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 \Rightarrow (\operatorname{cosec}\theta + \cot\theta)^2$$

So, L.H.S = R.H.S.

(ii)

If $\cos\theta = -\dfrac{\sqrt{3}}{2}$ and $\dfrac{\pi}{2} < \theta < \pi$ find the values of trignometric functions. (LHR–2011.)



Answer: $\cos\theta = -\dfrac{\sqrt{3}}{2}$

According to pythagorass theorm:

$$b^2 = a^2 + c^2$$

$2^2 = a^2 + (\sqrt{3})^2$

$4 - 3 = a^2$

$1 = a^2$

$1 = a$

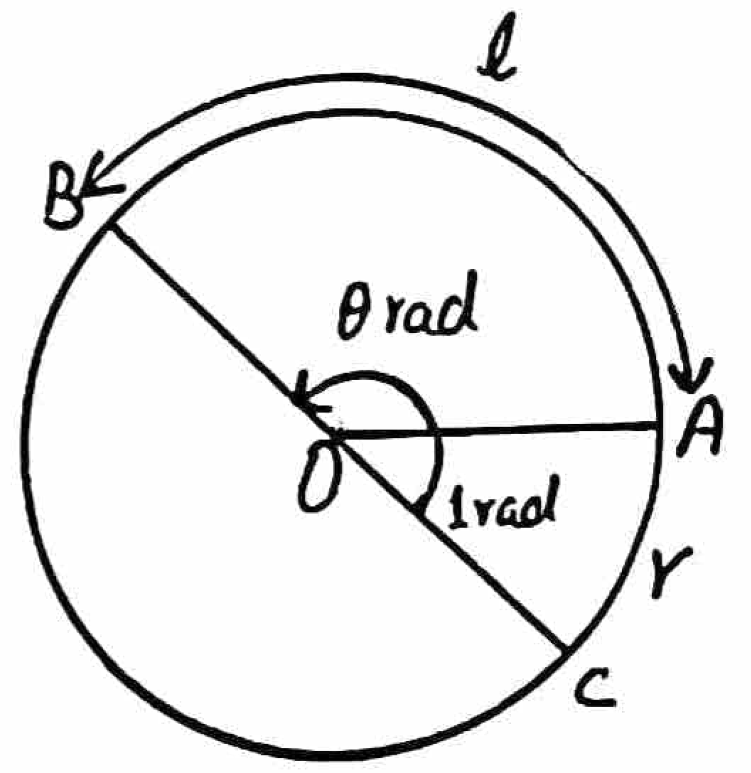
$\sin\theta = \frac{1}{2}$ $\quad$ $\operatorname{cosec}\theta = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad$ $\sec\theta = \frac{2}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\quad$ $\cot\theta = \sqrt{3}$

(iii)

If l is arc length of the circle central angle of an arc is θ radian and r radius of the circle. (Relation b/w arc length and angle) prove that $l = r\theta$.



Answer:-

Proof:

Consider arc arc $\overset{\frown}{AB}$ of a circle with centre O and radius r

Such that

$m\overset{\frown}{AB} = l$, $m\angle AOB = \theta$ rad.

also let $m\overset{\frown}{AC} = r$, $m\angle AOC = 1$ rad

We know that:

length of the arc of the circle are directly proportion to the measure of their central angle.

$$m\overset{\frown}{AB} : m\overset{\frown}{AC} = m\angle AOB : m\angle AOC$$

$$l : r = \theta : 1$$

$$\frac{l}{r} = \frac{\theta}{1}$$

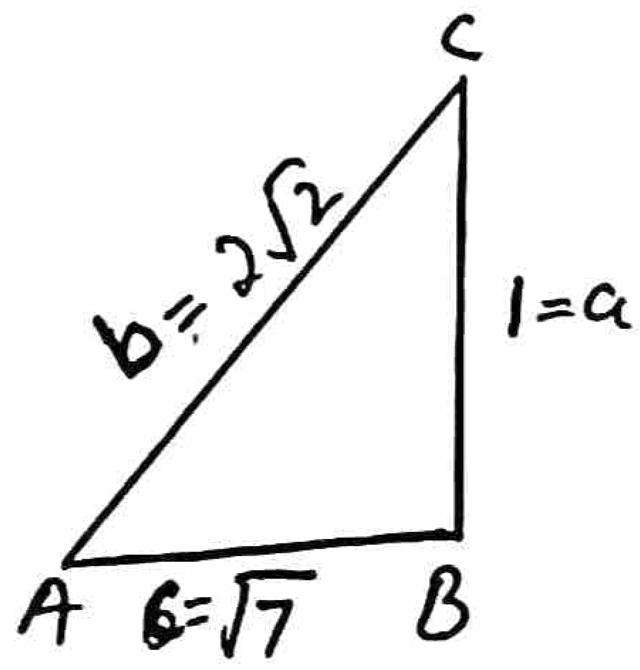
$$l = r\theta$$

(iv)

If $\tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in III quadrant, find value of $\frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta}$.

Answer: find value of $= \frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} \rightarrow (i)$

Given: $\tan\theta = \frac{1}{\sqrt{7}}$



By using pythagoras theorm:

$b^2 = a^2 + c^2$

$b^2 = (1)^2 + (\sqrt{7})^2$

$b^2 = 1 + 7$

$b^2 = 8$

$b = 2\sqrt{2}$

$\sin\theta = 1/2\sqrt{2}$

$\csc\theta = 2\sqrt{2}$

$\sec\theta = 2\sqrt{2}/\sqrt{7}$

$\because$ It is lies in not quadrant III

So, it is lies in quadrant I

Put values in eq (i)

$$= \frac{(2\sqrt{2})^2 - \left(2\sqrt{2}/\sqrt{7}\right)^2}{(2\sqrt{2})^2 + \left(2\sqrt{2}/\sqrt{7}\right)^2} \Rightarrow \frac{8 - 8/7}{8 + 8/7} \Rightarrow \frac{\frac{56-8}{\cancel{7}}}{\frac{56+8}{\cancel{7}}}$$

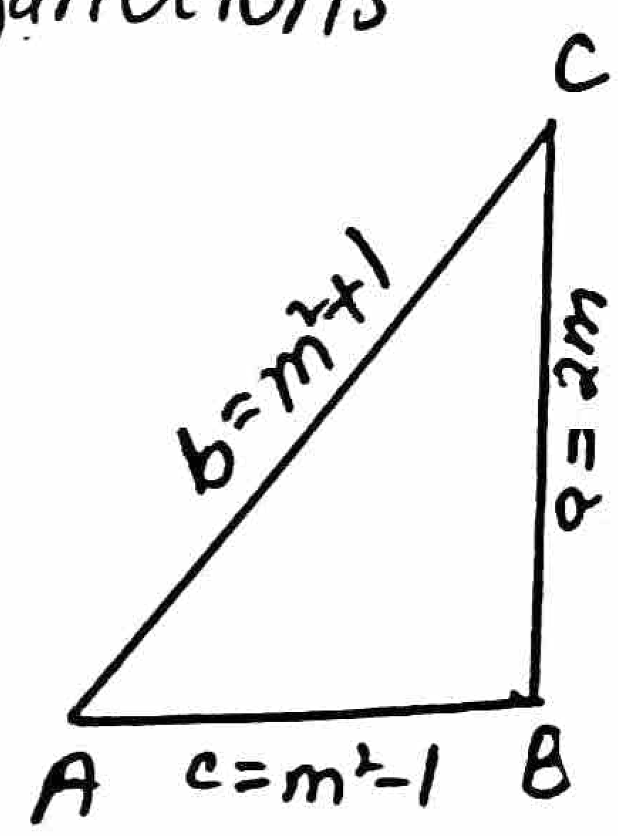
$$= \frac{48}{64} \Rightarrow 3/4.$$ (LHR- 2012+2014)

(v)

If $\csc\theta = \frac{m^2+1}{2m}$ and $m > 0 \left[0 < \theta < \frac{\pi}{2}\right]$, find the values of the remaining trigonometric functions.

Answer: (LHR-2013+18+22-G.I)

Given: $\csc\theta = \frac{m^2+1}{2m}$



By using pythagoras theorms

$$b^2 = a^2 + c^2$$
$$(m^2+1)^2 = (2m)^2 + c^2$$
$$m^4 + 1 + 2m^2 - 4m^2 = c^2$$
$$m^4 - 2m^2 + 1 = c^2$$
$$(m^2-1)^2 = c^2$$
$$m^2 - 1 = c.$$

$m > 0 \left[0 < \theta < \frac{\pi}{2}\right]$ is it lies in qudrant I

$\sin\theta = 2m/m^2+1$, $\operatorname{cosec}\theta = m^2+1/2m$

$\cos\theta = \frac{m^2-1}{m^2+1}$, $\sec\theta = m^2+1/m^2-1$

$\tan\theta = 2m/m^2-1$, $\cot\theta = m^2-1/2m$

(vi)

Find the values of the trignometric functions of angles $\frac{13\pi}{3}$

Answer: (LHR-2013-14-16.)

$$\frac{13\pi}{3} \Rightarrow \left(\frac{13\pi}{3} \times \frac{180}{\pi}\right)^\circ \Rightarrow 780^\circ$$

$$\frac{13\pi}{3} = 60^\circ + 2(360^\circ)(1) \quad k=1.$$

As we know:

$\theta = \theta + 2\pi k$ where $k \in z$.

$\sin\frac{13\pi}{3} = \sin 60^\circ = \sqrt{3}/2$, $\operatorname{cosec} 13\pi/3 = \operatorname{cosec} 60^\circ = 2/\sqrt{3}$

$\cos\frac{13\pi}{3} = \cos 60^\circ = 1/2$, $\sec 13\pi/3 = \sec 60^\circ = 2$

$\tan\frac{13\pi}{3} = \tan 60^\circ = \sqrt{3}$, $\cot 13\pi/3 = \cot 60^\circ = 1/\sqrt{3}$

(vii)

Prove that $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

Answer: $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

L.H.S $= \sin^6\theta - \cos^6\theta.$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3.$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta)$.

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta)$.

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$. **(LHR-2014+15)**

So, L.H.S = R.H.S.

(viii)

Prove tath $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

Answer: (LHR-2015) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

L.H.S $= \sin^6\theta + \cos^6\theta$.

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta)$

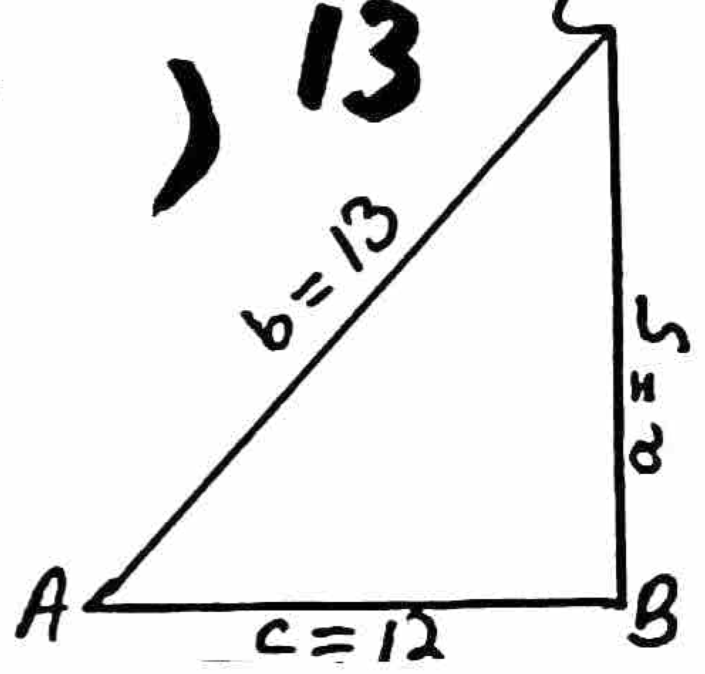
$= 1((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$.

$= ((\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta)$.

$= 1 - 3\sin^2\theta\cos^2\theta$ **So, L.H.S = R.H.S.**

(ix)

find values of other trigonometric functions $\cos\theta = \frac{12}{13}$ is not lies in I quadrant. **(LHR-2016)**



Using Pythagoras theorm:

$b^2 = a^2 + c^2$.

$$(13)^2 = a^2 + (12)^2$$
$$169 - 144 = a^2$$
$$25 = a^2$$
$$5 = a$$

θ lies in quadrant IV.

$\sin\theta = 5/13 \qquad \operatorname{cosec}\theta = 13/5$

$\cos\theta = 12/13 \qquad \sec\theta = 13/12$

$\tan\theta = 5/12 \qquad \cot\theta = 12/5.$

(x)

Verify: $$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

Answer:

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$\frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$\frac{1}{\frac{1-\cos\theta}{\sin\theta}} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\frac{1+\cos\theta}{\sin\theta}}$$

$$\frac{1}{(\sin\theta)(1-\cos\theta)} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{(1+\cos\theta)(\sin\theta)}$$

$$\frac{1}{\sin\theta - \sin\theta\cos\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\sin\theta + \sin\theta\cos\theta}$$

$$\cancel{\frac{1}{\sin\theta}} - \frac{1}{\sin\theta\cos\theta} - \cancel{\frac{1}{\sin\theta}} = \cancel{\frac{1}{\sin\theta}} - \cancel{\frac{1}{\sin\theta}} - \frac{1}{\sin\theta\cos\theta}$$

$$-\frac{1}{\sin\theta\cos\theta} = -\frac{1}{\sin\theta\cos\theta}$$

So, L.H.S = R.H.S. (LHR-2017)

(xi)

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$ Prove that (LHR-2017)

Answer: $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$

L.H.S $= \frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$ $\quad\because \sin^2\theta+\cos^2\theta=1.$

$$=\frac{1-\cancel{\sin\theta}+1+\cancel{\sin\theta}}{(1+\sin\theta)(1-\sin\theta)} \Rightarrow \frac{2}{1-\sin^2\theta}$$

$$=\frac{2}{\cos^2\theta} \Rightarrow 2\sec^2\theta.$$

So, L.H.S = R.H.S.

(xii)

Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$ (LHR-2019-G.I)

Answer: $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta.$ $\quad\because \frac{\sin\theta}{\cos\theta}=\tan\theta.$

L.H.S $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\times\sqrt{\frac{1-\sin\theta}{1-\sin\theta}} \Rightarrow \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$$=\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} \Rightarrow \frac{1-\sin\theta}{\cos\theta} \Rightarrow \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta} \Rightarrow \sec\theta-\tan\theta$$

So, L.H.S = R.H.S.

(xiii)

A railway train is running on a circular track of radius **500 m** at the rate of **30 km per hour**. Through what angle will it turn in **10 sec**? (LHR-2019)

Answer:- Given:-

$r = 500m$, $\vec{V} = 30 km/h \Rightarrow \frac{30 \times 1000}{60 \times 60} \Rightarrow 8.33 ms^{-1}$

find = θ.

$S = Vt$

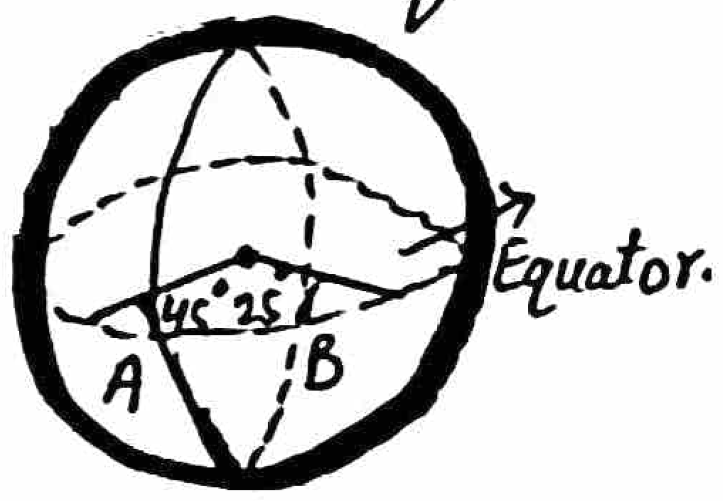
$S = (8.33)(10) \Rightarrow 83.33$.

formula: $l = r\theta$

$\theta = l/r \Rightarrow \frac{83.33}{500} \Rightarrow 0.16 rad$.

(xiv)

Two cities **A** and **B** lie on the equator such that their longitudes are **45°** E and **25°** W respectively. Find the distance between the two cities, taking radius of the Earth as **6400 kms**. (LHR-2021-G-I)



Answer:- Given:-

$r = 6400$, $\theta = 45° + 25° \Rightarrow 70° \Rightarrow \left(70 \times \frac{\pi}{180}\right) rad$

$\theta = 1.2217 rad$.

find = l

Using formula:

$l = r\theta$

$l = (6400)(1.2217)$

$l = 7819$ (Approximatly).

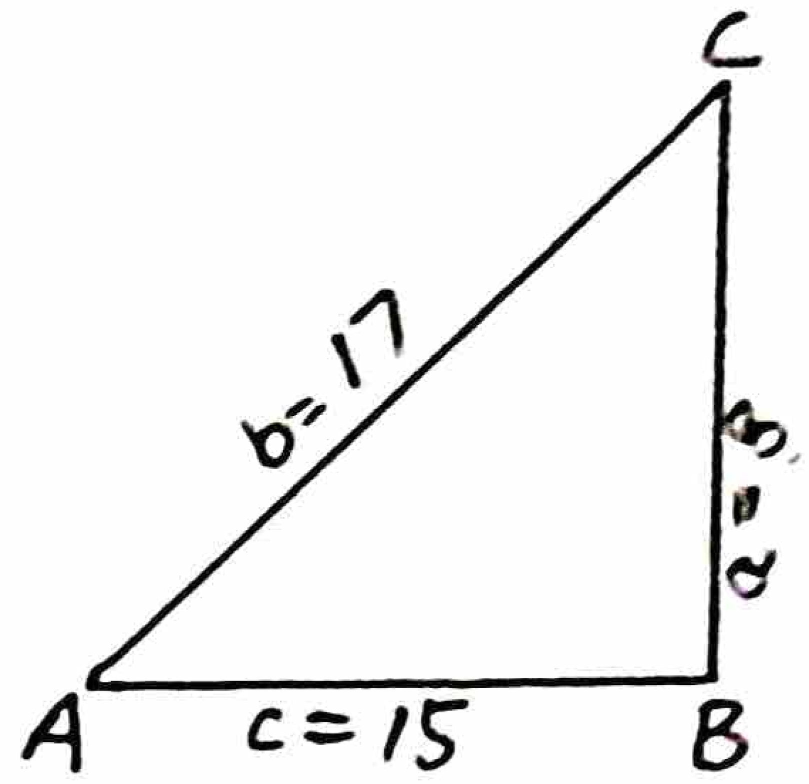
(XV)

If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quad-I find values of **$\cos\theta$ and $\operatorname{cosec}\theta$.**

Long-Q (LHR-2022.) + S.Q (2016 + 2021-LHR.)

Answer:

$$\cot\theta = \frac{15}{8}$$



Applying Pythagoras theorm:

$$b^2 = a^2 + c^2$$

$$b^2 = (8)^2 + (15)^2$$

$$b^2 = 64 + 225$$

$$b^2 = 289$$

$$b = 17.$$

Angle is not in first I-Q so it lies in III Q. so,

$$\cos\theta = -15/17$$

$$\operatorname{cosec}\theta = -17/8$$