# Chapter 14



## Exercise 14

Find value / solution of following equation $(0, 2\pi)$

### (i)

$\sin x = -\frac{\sqrt{3}}{2}$

$R.A = \frac{\pi}{3}$

As $\sin x$ is (-ve) in III & IV Quad

**In III Quad**

$x = \pi + \frac{\pi}{3}$

$x = \frac{3\pi+\pi}{3}$

$x = \frac{4\pi}{3}$

**IV Quad**

$x = 2\pi - \frac{\pi}{3}$

$x = \frac{6\pi-\pi}{3}$

$x = \frac{5\pi}{3}$

$S.S = \{\frac{4\pi}{3}, \frac{5\pi}{3}\}$

### (ii)

$\operatorname{cosec}\theta = 2$

$\frac{1}{\sin\theta} = 2$

$\sin\theta = \frac{1}{2}$

$R.A = \frac{\pi}{6}$

As $\operatorname{cosec}\theta$ is (+ve) in I & II

**I Quad**

$\theta = \frac{\pi}{6}$

**II Quad**

$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi-\pi}{6}$

$\theta = \frac{5\pi}{6}$

$S.S = \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

### (iii)

$\sec x = -2$

$\frac{1}{\cos x} = -2$

$\cos x = -\frac{1}{2}$

$R.A = \frac{\pi}{3}$

As $\sec\theta$ is -ve in II & III

**II Quad**

$x = \pi - \frac{\pi}{3}$

$x = \frac{3\pi-\pi}{3}$

$x = \frac{2\pi}{3}$

**III Quad**

$x = \pi + \frac{\pi}{3}$

$x = \frac{3\pi+\pi}{3}$

$x = \frac{4\pi}{3}$

$S.S = \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

### (iv)

$\cot\theta = \frac{1}{\sqrt{3}}$

$\frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$R.A = \frac{\pi}{3}$

As $\cot\theta$ is +ve in I & III

**I Quad**

$\theta = \frac{\pi}{3}$

**III Quad**

$\theta = \pi + \frac{\pi}{3}$

$\theta = \frac{3\pi+\pi}{3}$

$\theta = \frac{4\pi}{3}$

$S.S = \{\frac{\pi}{3}, \frac{4\pi}{3}\}$

# Question 2



## (i)

$\tan^2\theta = \frac{1}{3}$

take square root b/s

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$R.A = \frac{\pi}{6}$

**Firstly**

As $\tan\theta$ is (+ve) in I & III

**I Quad**

$\theta = \frac{\pi}{6}$

**III Quad**

$\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{6\pi + \pi}{6}$

$\theta = \frac{7\pi}{6}$

**Secondly**

As $\tan\theta$ is (-ve) in II & IV

**II Quad**

$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi - \pi}{6}$

$\theta = \frac{5\pi}{6}$

**IV Quad**

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{12\pi - \pi}{6}$

$\theta = \frac{11\pi}{6}$

$S.S = \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$



## (ii)

$\text{cosec}^2\theta = \frac{4}{3}$

Square root b.s

$\text{cosec}\theta = \pm\frac{2}{\sqrt{3}}$

$\frac{1}{\sin\theta} = \pm\frac{2}{\sqrt{3}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$R.A = \frac{\pi}{3}$

**Firstly**

As cosec $\theta$ is (+ve) in I & II

**I Quad**

$\theta = \frac{\pi}{3}$

**II Quad**

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{3\pi - \pi}{3}$

$\theta = \frac{2\pi}{3}$

**Secondly**

As cosec $\theta$ is (-ve) in III & IV

**III Quad**

$\theta = \pi + \frac{\pi}{3}$

$\theta = \frac{3\pi + \pi}{3}$

$\theta = \frac{4\pi}{3}$

**IV Quad**

$\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{6\pi - \pi}{3}$

$\theta = \frac{5\pi}{3}$

$S.S = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

(iii)

$\sec^2\theta = \frac{4}{3}$

Square root b/s

$\sec\theta = \pm\frac{2}{\sqrt{3}}$

$\frac{1}{\cos\theta} = \pm\frac{2}{\sqrt{3}}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$R.A = \frac{\pi}{6}$

**Firstly**

$\sec\theta$ (+ve) in I & IV

**I Quad**

$\theta = \frac{\pi}{6}$

**IV Quad**

$= 2\pi - \pi/6$

$= \frac{12\pi - \pi}{6}$

$= \frac{11\pi}{6}$

**Secondly**

As sec(-ve) in III & II

**II Quad**

$= \pi - \frac{\pi}{6}$

$= \frac{6\pi - \pi}{6}$

$\theta = \frac{5\pi}{6}$

**III Quad**

$\theta = 3\pi + \pi/6$

$\theta = \frac{6\pi + \pi}{6}$

$\theta = \frac{7\pi}{6}$

S.S = $\left\{\frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}\right\}$.



(iv)

$\cot^2\theta = \frac{1}{3}$

Square root b.s

$\cot\theta = \pm\frac{1}{\sqrt{3}}$

$\frac{1}{\tan\theta} = \pm\frac{1}{\sqrt{3}}$

$\tan\theta = \pm\sqrt{3}$

$R.A = \frac{\pi}{3}$

**Firstly**

As cotθ +ve in I & III

**I Quad**

$\theta = \frac{\pi}{3}$

**III Quad**

$\theta = \pi + \frac{\pi}{3}$

$\theta = \frac{3\pi + \pi}{3}$

$\theta = \frac{4\pi}{3}$

**Secondly**

As cotθ is (-ve) in II & IV.

**II Quad**

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{3\pi - \pi}{3}$

$\theta = \frac{2\pi}{3}$

**IV Quad**

$\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{6\pi - \pi}{3}$

$\theta = \frac{5\pi}{3}$

S.S = $\left\{\frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

(3)

find value of θ

$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

$(\sqrt{3}\tan\theta)^2 + 2(\sqrt{3}\tan\theta)(1) + (1)^2 = 0$

$(\sqrt{3}\tan\theta + 1)^2 = 0$

taking square root

$\sqrt{3}\tan\theta + 1 = 0$

$\sqrt{3}\tan\theta = -1$

$\tan\theta = \frac{-1}{\sqrt{3}}$

$R.A = \frac{\pi}{6}$

As $\tan\theta$ is (-ve) in II & IV Quad

**II Quad**

$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi - \pi}{6}$

$\theta = \frac{5\pi}{6}$

**IV Quad**

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{12\pi - \pi}{6}$

$\theta = \frac{11\pi}{6}$

$S.S = \left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

**Q(4)**

$\tan^2\theta - \sec\theta - 1 = 0$ $\because \tan^2\theta = \sec^2\theta - 1$

$\sec^2\theta - 1 - \sec\theta - 1 = 0$

$\sec^2\theta - \sec\theta - 2 = 0$

$\sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0$

$\sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$(\sec\theta + 1)(\sec\theta - 2)$

$\sec\theta + 1 = 0$

$\sec\theta = -1$

$\frac{1}{\cos\theta} = -1$

$\cos\theta = -1$

$R.A = \pi$

$\sec\theta - 2 = 0$

$\sec\theta = 2$

$\frac{1}{\cos\theta} = 2$

$\cos\theta = \frac{1}{2}$

$R.A = \frac{\pi}{3}$

$R.A = \frac{\pi}{3}$

As sec is (+ve) I & IV

**I Quad**

$\theta = \frac{\pi}{3}$

**IV Quad**

$\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{6\pi - \pi}{3}$

$\theta = \frac{5\pi}{3}$

$S.S = \left\{\pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

**Q(5)**

$2\sin\theta + \cos^2\theta - 1 = 0$ $\because 1 - \sin^2\theta = \cos^2\theta$

$2\sin\theta + (1 - \sin^2\theta) - 1 = 0$

$2\sin\theta + 1 - \sin^2\theta - 1 = 0$

$2\sin\theta - \sin^2\theta = 0$

$\sin\theta(2 - \sin\theta) = 0$

$\sin\theta = 0$

$\sin\theta = 0, \pi$

Quadratal angle

$2 - \sin\theta = 0$

$\sin\theta = 2$

Not Possible

$\because -1 \le y \le 1$

$[-1, 1]$

$S.S = \{0, \pi\}$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$

$\sin\theta = 0, \pi$

$2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\sin\theta = \frac{\pi}{6}$

As $\sin\theta$ is (+ve) in I & II

**I Quad**

$\theta = \frac{\pi}{6}$

**II Quad**

$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi - \pi}{6}$

$\theta = \frac{5\pi}{6}$

$S.S = \{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\}$



$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - \sqrt{3}\cdot\sqrt{3}\sin^2\theta = 0$

$3\cos^2\theta - 3\sqrt{3}\sin\theta\cos\theta + \sqrt{3}\sin\theta\cos\theta - \sqrt{3}\cdot\sqrt{3}\sin^2\theta = 0$

$3\cos\theta(\cos\theta - \sqrt{3}\sin\theta) + \sqrt{3}\sin\theta(\cos\theta - \sqrt{3}\sin\theta) = 0$

$(3\cos\theta + \sqrt{3}\sin\theta) = 0$

$3\cos\theta = -\sqrt{3}\sin\theta$

$\frac{-3}{\sqrt{3}} = \tan\theta$

$\frac{\sqrt{3}\sqrt{3}}{\sqrt{3}} = \tan\theta$

$-\sqrt{3} = \tan\theta$

$R.A = \pi/3$

As tan -ve in II & IV

**II Quad**

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

**IV Quad**

$\theta = 2\pi - \pi/3$

$\theta = \frac{5\pi}{3}$

$(\cos\theta - \sqrt{3}\sin\theta) = 0$

$\cos\theta = \sqrt{3}\sin\theta$

$\frac{1}{\sqrt{3}} = \tan\theta$

$R.A = \pi/6$

As tan (+ve) in I & III

**I Quad**

$\theta = \frac{\pi}{6}$

**III Quad**

$\theta = \pi + \frac{\pi}{6}$

$= \frac{6\pi + \pi}{6}$

$\theta = \frac{7\pi}{6}$

$S.S = \{2\pi/2, 5\pi/3, \pi/6, 7\pi/6\}$

$4\sin^2\theta - 8\cos\theta + 1 = 0$

$4(1-\cos^2\theta) - 8\cos\theta + 1 = 0$

$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

$5 - 4\cos^2\theta - 8\cos\theta = 0$

$-4\cos^2\theta - 8\cos\theta + 5 = 0$

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$4\cos^2\theta - 2\cos\theta + 8\cos\theta - 5 = 0$

$2\cos\theta(2\cos\theta - 1) + 5(2\cos\theta - 1) = 0$

$(2\cos\theta + 5)(2\cos\theta - 1) = 0$

$2\cos\theta + 5 = 0$

$\cos\theta = -5/2$

No possible

$2\cos\theta - 1 = 0$

$\cos\theta = 1/2$

$R.A = \pi/3$

As cos (+ve) in I & IV

**I Quad**

$\theta = \pi/3$

**IV Quad**

$\theta = 2\pi - \pi/3$

$\theta = \frac{5\pi}{3}$

$S.S = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$



$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan x = \sec x + 1$

Square b.s

$(\sqrt{3}\tan x)^2 = (\sec x + 1)^2$

$3\tan^2 x = \sec^2 x + 1 + 2\sec x$

$3(\sec^2 x - 1) = \sec^2 x + 1 + 2\sec x$

$3\sec^2 x - 3 - \sec^2 x - 1 - 2\sec x = 0$

$2\sec^2 x - 2\sec x - 4 = 0$

Divide by 2

$\sec^2 x - \sec x - 2 = 0$

$\sec^2 x - 2\sec x + \sec x - 2 = 0$

$\sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$(\sec x + 1) = 0$

$\frac{1}{\cos x} = -1$

$\cos x = -1$

$x = \pi$

(Quadratal angle)

$(\sec x - 2) = 0$

$\frac{1}{\cos x} = 2$

$\cos x = 1/2$

$R.A = \pi/3$

$R.A = \pi/3$

As cos is (+ve) in I & IV

**I** $x = \pi/3$

**IV** $x = 2\pi - \pi/3$

$x = \frac{5\pi}{3}$

**Checking:-**

At $x = \frac{\pi}{3}$

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan\frac{\pi}{3} - \sec\frac{\pi}{3} - 1 = 0$

$\sqrt{3}\cdot\sqrt{3} - 2 - 1 = 0$

$3 - 3 = 0$

$0 = 0$

At $x = \pi$

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan\pi - \sec\pi - 1 = 0$

$0 + 1 - 1 = 0$

$0 = 0$

At $x = 0$

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan 0 - \sec 0 - 1 = 0$

$-1 - 1 = 0$

$-2 \neq 0$

**For general value**

$x = \frac{\pi}{3} + 2n\pi, \quad x = \pi + 2n\pi$

$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \{\pi + 2n\pi\}$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$

$\sin\theta = 0, \pi$

$2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\sin\theta = \frac{\pi}{6}$

As $\sin\theta$ is (+ve) in I & II

**I Quad**

$\theta = \frac{\pi}{6}$

**II Quad**

$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi - \pi}{6}$

$\theta = \frac{5\pi}{6}$

$S.S = \{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\}$

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - \sqrt{3}\cdot\sqrt{3}\sin^2\theta = 0$

$3\cos^2\theta - 3\sqrt{3}\sin\theta\cos\theta + \sqrt{3}\sin\theta\cos\theta - \sqrt{3}\cdot\sqrt{3}\sin^2\theta = 0$

$3\cos\theta(\cos\theta - \sqrt{3}\sin\theta) + \sqrt{3}\sin\theta(\cos\theta - \sqrt{3}\sin\theta) = 0$

$(3\cos\theta + \sqrt{3}\sin\theta) = 0$

$3\cos\theta = -\sqrt{3}\sin\theta$

$\frac{-3}{\sqrt{3}} = \tan\theta$

$\frac{-\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}} = \tan\theta$

$-\sqrt{3} = \tan\theta$

R.A $= \pi/3$

As tan -ve in II & IV

**II Quad**

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

**IV Quad**

$\theta = 2\pi - \pi/3$

$\theta = \frac{5\pi}{3}$

$(\cos\theta - \sqrt{3}\sin\theta) = 0$

$\cos\theta = \sqrt{3}\sin\theta$

$\frac{1}{\sqrt{3}} = \tan\theta$

R.A $= \pi/6$

As tan (+ve) in I & III

**I Quad**

$\theta = \frac{\pi}{6}$

**III Quad**

$\theta = \pi + \frac{\pi}{6}$

$= \frac{6\pi + \pi}{6}$

$\theta = \frac{7\pi}{6}$

$S.S = \{2\pi/3, 5\pi/3, \pi/6, 7\pi/6\}$

(8)

## $4\sin^2\theta - 8\cos\theta + 1 = 0$

$4(1-\cos^2\theta) - 8\cos\theta + 1 = 0$

$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

$5 - 4\cos^2\theta - 8\cos\theta = 0$

$-4\cos^2\theta - 8\cos\theta + 5 = 0$

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$4\cos^2\theta - 2\cos\theta + 8\cos\theta - 5 = 0$

$2\cos\theta(2\cos\theta - 1) + 5(2\cos\theta - 1) = 0$

$(2\cos\theta + 5)(2\cos\theta - 1) = 0$

| $2\cos\theta + 5 = 0$ | $2\cos\theta - 1 = 0$ |
|---|---|
| $\cos\theta = -5/2$ | $\cos\theta = 1/2$ |
| No Possible | $R.A = \pi/3$ |

As cos (+ve) in I & IV

| I Quad | IV Quad |
|---|---|
| $\theta = \pi/3$ | $\theta = 2\pi - \pi/3$ |
| | $\theta = \frac{5\pi}{3}$ |

$S.S = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

(9)

## $\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan x = \sec x + 1$

Square b.s

$(\sqrt{3}\tan x)^2 = (\sec x + 1)^2$

$3\tan^2 x = \sec^2 x + 1 + 2\sec x$

$3(\sec^2 x - 1) = \sec^2 x + 1 + 2\sec x$

$3\sec^2 x - 3 - \sec^2 x - 1 - 2\sec x = 0$

$2\sec^2 x - 2\sec x - 4 = 0$

Divide by 2

$\sec^2 x - \sec x - 2 = 0$

$\sec^2 x - 2\sec x + \sec x - 2 = 0$

$\sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$(\sec x + 1) = 0$ | $(\sec x - 2) = 0$

$\frac{1}{\cos x} = -1$ | $\frac{1}{\cos x} = 2$

$\cos x = -1$ | $\cos x = 1/2$

$x = \pi$ | $R.A = \pi/3$

(Quadratal angle)

$R.A = \pi/3$

As $\cos x$ is (+ve) in I & IV

| I | IV |
|---|---|
| $x = \pi/3$ | $x = 2\pi - \pi/3$ |
| | $x = 5\pi/3$ |

### Checking :-

At $x = \frac{\pi}{3}$

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan\frac{\pi}{3} - \sec\frac{\pi}{3} - 1 = 0$

$\sqrt{3}\cdot\sqrt{3} - 2 - 1 = 0$

$3 - 3 = 0$

$0 = 0$

At $x = \pi$

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan\pi - \sec\pi - 1 = 0$

$0 + 1 - 1 = 0$

$0 = 0$

At $x = 0$

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\sqrt{3}\tan 0 - \sec 0 - 1 = 0$

$-1 - 1 = 0$

$-2 \neq 0$

**For general value**

$x = \frac{\pi}{3} + 2n\pi$, $x = \pi + 2n\pi$

$S.S = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \{\pi + 2n\pi\}$

$\cos 2x = \sin 3x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos^2 x - \sin^2 x = 3\sin x - 4\sin^3 x$

$1 - \sin^2 x - \sin^2 x = 3\sin x - 4\sin^3 x$

$1 - 2\sin^2 x = 3\sin x - 4\sin^3 x$

$1 - 2\sin^2 x + 4\sin^3 x - 3\sin x = 0$

$4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

| | | | | |
|---|---|---|---|---|
| 1 | 4 | −2 | −3 | 1 |
| | | 4 | 2 | −1 |
| | 4 | 2 | −1 | 0 |

$\sin x = 1$

$4\sin^2 x + 2\sin x - 1 = 0$

Quadratic formula

$\sin x = \frac{-2 \pm \sqrt{20}}{8}$

$\sin x = \frac{-2 + 2\sqrt{5}}{8}$

$\sin x = 2\left(\frac{-1 + \sqrt{5}}{8}\right)$

$\sin x = \frac{-1 + \sqrt{5}}{4}$

$x = \frac{\pi}{10}$

$\sin x = \frac{-2 - \sqrt{20}}{8}$

$\sin x = \frac{-2 - 2\sqrt{5}}{8}$

$\sin x = 2\left(\frac{-1 - \sqrt{5}}{8}\right)$

$\sin x = \frac{-1 - \sqrt{5}}{4}$

$x = \frac{3\pi}{10}$

$\sin x = 1$

$x = \frac{\pi}{2}$

$\sin$ (+ve) in I & II

| I | II |
|---|---|
| $= 0 + \frac{\pi}{2}$ | $= \pi - \frac{\pi}{2}$ |
| $= \frac{\pi}{2}$ | $= \frac{2\pi - \pi}{2}$ |
| | $= \frac{\pi}{2}$ |

As $\sin$ (+ve) in I & II

III & IV Quad

| I | II | III | IV |
|---|---|---|---|
| $= \frac{\pi}{10}$ | $= \pi - \frac{\pi}{10}$ | $= \pi + \frac{3\pi}{10}$ | $= 2\pi - \frac{3\pi}{10}$ |
| | $= \frac{10\pi - \pi}{10}$ | $= \frac{10\pi + 3\pi}{10}$ | $= \frac{20\pi - 3\pi}{10}$ |
| | $= \frac{9\pi}{10}$ | $= \frac{13\pi}{10}$ | $= \frac{17\pi}{10}$ |

Values of $x = \frac{\pi}{2}, \frac{\pi}{10}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$

$x = \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\}$

## (11)

$$\sec 3\theta = \sec\theta$$

$$\frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$$

$$\cos\theta = \cos 3\theta$$

$$\cos 3\theta - \cos\theta = 0$$

Using formula

$-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$$-2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)=0$$

$$-2\sin\frac{4\theta}{2}\sin\frac{2\theta}{2}=0$$

$$-2\sin 2\theta\sin\theta=0$$

$$\sin 2\theta\sin\theta=0$$

$\sin 2\theta = 0$

$2\theta = \sin^{-1}(0)$

$\theta = 0, \frac{\pi}{2}$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$= 0, \pi$

$\{\pi + 2n\pi\} \cup \{\frac{\pi}{2} + 2n\pi\}$

## (12)

$$\tan 2\theta + \cot\theta = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos\theta}{\sin\theta} = 0$$

$$\frac{\sin 2\theta\sin\theta + \cos\theta\cos 2\theta}{\cos 2\theta\sin\theta} = 0$$

$$\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = 0$$

$$\cos(2\theta - \theta) = 0$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

As (+ve) cos in I & IV

| I | IV |
|---|---|
| $= \frac{\pi}{2}$ | $= 2\pi - \frac{\pi}{2}$ |
| | $= \frac{4\pi-\pi}{2}$ |
| | $= \frac{3\pi}{2}$ |

Values of $\theta = \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

$\{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{3\pi}{2} + 2n\pi\}$

## (13)

$$\sin 2x + \sin x = 0$$

$2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$

$x = \sin^{-1} 0$

$x = n\pi$

$2\cos x + 1 = 0$

$\cos x = -1/2$

$x = \pi/3$

As cos -ve in II & III

| II | III |
|---|---|
| $= \pi - \frac{\pi}{3}$ | $= \pi + \frac{\pi}{3}$ |
| $= \frac{3\pi-\pi}{3}$ | $= \frac{3\pi+\pi}{3}$ |
| $= \frac{2\pi}{3}$ | $= \frac{4\pi}{3}$ |

$x = n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

$x = \{n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\} \cup \{\frac{4\pi}{3} + 2n\pi\}$

## (14)

$$\sin 4x - \sin 2x = \cos 3x$$

Using formula

$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

$$2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) - \cos 3x = 0$$

$$2\cos 3x\sin x - \cos 3x = 0$$

$$\cos 3x(2\sin x - 1) = 0$$

$\cos 3x = 0$

$3x = \pi/2$

cos +ve in I, IV

| I | IV |
|---|---|
| $= \pi/2$ | $= 2\pi - \pi/2$ |
| | $= 3\pi/2$ |

$2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \pi/6$

As sin +ve in I & II

| I | II |
|---|---|
| $= \pi/6$ | $= \pi - \pi/6$ |
| | $= 5\pi/6$ |

**14 Half)**

General value of $3x$

$3x = \frac{\pi}{2} + 2n\pi \;,\; \frac{3\pi}{2} + 2n\pi$

$x = \frac{\pi}{6} + \frac{2n\pi}{3} \;,\; \frac{\pi}{2} + \frac{2n\pi}{3}$

General value of $x$

$x = \frac{5\pi}{2} + 2n\pi \;,\; \frac{\pi}{6} + 2n\pi$

$S.S = \left\{\frac{\pi}{6} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{2} + \frac{2\pi n}{3}\right\} \cup \left\{\frac{5\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\}$



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**(15)**

**$\sin x + \cos 3x = \cos 5x$**

∵ Using formula

$-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = \cos\alpha - \cos\beta$.

$\cos 5x - \cos 3x - \sin x = 0$

$-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right) - \sin x = 0$

$-2\sin 4x \sin x - \sin x = 0$

$\sin x(-2\sin 4x - 1) = 0$

$\sin x = 0$

$x = \pi$

As sin +ve in I & II

| I | II |
|---|---|
| $= \pi$ | $= \pi - \pi$ |
| | $= 0$ |

$-2\sin 4x - 1 = 0$

$\sin 4x = -1/2$

$4x = \frac{\pi}{6}$

As sin (-ve) in III & IV

| III | IV |
|---|---|
| $= \pi + \frac{\pi}{6}$ | $= 2\pi - \pi/6$ |
| $= \frac{7\pi}{6}$ | $= \frac{12\pi - \pi}{6}$ |
| | $= \frac{11\pi}{6}$ |

Value for $4x$

$4x = \frac{7\pi}{6} \;;\; \frac{11\pi}{6}$

General value $4x = 2n\pi$,

$x = \frac{7\pi}{24} + \frac{n\pi}{2} \;,\; \frac{11\pi}{24} + \frac{n\pi}{2}$

General value $x$

$x = \pi + 2n\pi \;,\; 0 + 2n\pi = 2n\pi$

$S.S = \left\{2n\pi\right\} \cup \left\{\pi + 2n\pi\right\} \cup \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}$

---

**(16)**

**$\sin 3x + \sin 2x + \sin x = 0$**

∵ $2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \sin(\alpha+\beta)$

$\sin 3x + \sin x + \sin 2x = 0$

$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0$

$2\sin 2x \cos x + \sin 2x = 0$

$\sin 2x(2\cos x + 1) = 0$

$\sin 2x = 0$

$2x = \pi$

$2\cos x + 1 = 0$

$\cos x = -1/2$

$x = \pi/3$.

As sin +ve in I & II

| I | II |
|---|---|
| $= \pi$ | $= \pi - \pi$ |
| | $= 0$ |

As cos (-ve) in II & III

| II | III |
|---|---|
| $= \pi - \frac{\pi}{3}$ | $= \pi + \pi/3$ |
| $= \frac{3\pi - \pi}{3}$ | $= \frac{3\pi + \pi}{3}$ |
| $= \frac{2\pi}{3}$ | $= \frac{4\pi}{3}$ |

General value $2x = \pi + 2n\pi, 2n\pi, \frac{\pi}{3} + 2n\pi, n\pi$

$$S.S = \left\{n\pi\right\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$$

(17)

**$\sin 7x - \sin x = \sin 3x$**

$\because \sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\sin 7x - \sin x - \sin 3x = 0$

$2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) - \sin 3x = 0$

$2\cos 4x \sin 3x - \sin 3x = 0$

$\sin 3x(2\cos 4x - 1) = 0$

$\sin 3x = 0$

R.A $3x = \pi$

As sin (+ve) in I & II

| I | II |
|---|---|
| $= \pi$ | $= \pi - \pi$ |
| | $= 0$ |

Value for $3x = 0, \pi$

General value $= 2n\pi, \pi + 2n\pi$

$x = \frac{2n\pi}{3}, \frac{\pi}{3} + \frac{2n\pi}{3}$

$2\cos 4x - 1 = 0$

$\cos 4x = 1/2$

$4x = \pi/3$

As cos (+ve) in I & IV

| I | IV |
|---|---|
| $= \pi/3$ | $= 2\pi - \frac{\pi}{3}$ |
| | $= \frac{5\pi}{3}$ |

Value of $4x = \pi/3, 5\pi/3$.

General value $4x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

$x = \frac{\pi}{12} + \frac{2n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$

$$S.S = \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\}$$

**$\sin x+\sin 3x+\sin 5x=0$**

$\sin 5x+\sin x+\sin 3x=0$

$2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)+\sin 3x=0$

$2\sin 3x\cos 2x+\sin 3x=0$

$\sin 3x(2\cos 2x+1)=0$

$\sin 3x=0$ | $2\cos 2x+1=0$

$3x=\pi$ | $\cos 2x=-1/2$

$2x=\pi/3$

As sin +ve in I & II | As cos (-ve) in II & III

| I | II | II | III |
|---|---|---|---|
| $=\pi$ | $=\pi-\pi$ | $=\pi-\pi/3$ | $=\pi+\pi/3$ |
| | $=0$ | $=\frac{3\pi-\pi}{3}$ | $=\frac{3\pi+\pi}{3}$ |
| | | $=\frac{2\pi}{3}$ | $=\frac{4\pi}{3}$ |

General $x=\pi+2n\pi, 2n\pi$ | Value of $2x=\frac{2\pi}{3},\frac{4\pi}{3}$

Value $x=\frac{\pi}{3}+\frac{2n\pi}{3}, \frac{2n\pi}{3}$ | $x=\frac{2\pi}{3}+2n\pi, \frac{4\pi}{3}+2n\pi$

$x=\frac{\pi}{3}+n\pi, \frac{2\pi}{3}+n\pi$

$S.S=\left\{\frac{2}{3}n\pi\right\}\cup\left\{\frac{\pi}{3}+\frac{2}{3}n\pi\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+n\pi\right\}$





**$\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$**

$\sin 7\theta+\sin\theta+\sin 5\theta+\sin 3\theta=0$

$2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$

$2\sin 4\theta\cos 3\theta+2\sin 4\theta\cos\theta=0$

$2\sin 4\theta(\cos 3\theta+\cos\theta)=0$

Using formula $\cos(\alpha+\beta)$

$2\sin 4\theta\ 2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)=0$

$4\sin 4\theta\cos 2\theta\cos\theta=0$

$\sin 4\theta=0$ | $\cos 2\theta=0$ | $\cos\theta=0$

$4\theta=\pi$ | $2\theta=\pi/2$

As sin (+ve) in I & II | As cos +ve I & IV

| I | II | I | IV |
|---|---|---|---|
| $=\pi$ | $=\pi-\pi$ | $=\pi/2$ | $=2\pi-\pi/2$ |
| | $=0$ | | $=3\pi/2$ |

Value of $4\theta=\pi, 0$

General $\theta=\frac{\pi}{4}+\frac{n\pi}{2}, \frac{n\pi}{2}$

$\cos 2\theta$

Value of $2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

$\cos\theta = 0$

$\theta = \pi/2$

As cos (+ve) I & IV

$= \pi/2$

$= 2\pi - \pi/2$

$= 3\pi/2$

General value $\theta = \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi$

$S.S = \left[\frac{n\pi}{2}\right] \cup \left[\frac{\pi}{4} + \frac{n\pi}{2}\right] \cup \left[\frac{\pi}{4} + n\pi\right] \cup \left[\frac{3\pi}{4} + n\pi\right] \cup \left[\frac{\pi}{2} + 2n\pi\right] \cup \left[\frac{3\pi}{2} + 2n\pi\right]$

(20)

$\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

$\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

$\cos 7\theta + \cos\theta + \cos 3\theta + \cos 5\theta = 0$

$2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + 2\cos\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right) = 0$

$2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$

$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$

$2\cos 4\theta \, 2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) = 0$

$4\cos 4\theta \cos 2\theta \cos\theta = 0$

$4 \neq 0$

$\cos 4\theta = 0$

$4\theta = \pi/2$

As cos (+ve) I & IV

| I | IV |
|---|---|
| $= \pi/2$ | $= 2\pi - \pi/2$ |
| | $= 3\pi/2$ |

$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$G.V = \left[\frac{\pi}{8} + \frac{n\pi}{2}, \frac{3\pi}{8} + \frac{n\pi}{2}\right]$

$\cos 2\theta = 0$

$2\theta = \pi/2$

As cos (+ve) I & IV

| I | IV |
|---|---|
| $= \pi/2$ | $= 2\pi - \pi/2$ |
| | $= 3\pi/2$ |

Value of $2\theta = \frac{\pi}{2}, 3\pi/2$

$G.V\ \theta = \left[\frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi\right]$

$\cos\theta = 0$

$\theta = \pi/2$

As cos (+ve) in I & IV

| I | IV |
|---|---|
| $= \pi/2$ | $= 2\pi - \pi/2$ |
| | $= 3\pi/2$ |

$G.V\ \theta = \left[\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right]$

$S.S = \left[\frac{\pi}{8} + \frac{n\pi}{2}\right] \cup \left[\frac{3\pi}{8} + \frac{\pi n}{2}\right] \cup \left[\frac{\pi}{4} + n\pi\right] \cup \left[\frac{3\pi}{4} + \pi n\right] \cup \left[\frac{\pi}{2} + 2n\pi\right] \cup \left[\frac{3\pi}{2} + 2n\pi\right]$