## Unit 1 — Numer System

### History:-

In the very beginning, human life was simple. An early ancient herdsman compared Sheep (or cattle) of his herd with a pile of stones when the herd left for grazing and again on its return for missing animals. In the earliest systems probably the vertical strokes or bars such as; I, II, III etc, were used for the numbers 1, 2, 3, 4 etc. The Symbol "IIIII" was used by many people including the ancient Egyptians for the number of fingers of one hand.

Around 5000 B.C, the Egyptians had a number system based on 10. The Symbol ∩ for 10 and 9 for 100 were used by them. A Symbol was repeated as many times as it was needed. Different Peoples invented their own symbol for numbers. But these systems proved to be inadequate with advancement of Societies and were discarded. (ابزیب)

### Number System In Civilization:-

In Ancient Times, The humans used to live in caves. Some Twenty Thousand Years ago "Ishango Bones" were fined, it was observed that there was scratches on bones. This was the Idea that humans were trying to Inrent the Numbers.

# Civilizations:-

## (1) Egyptian Civilization (مصری تہذیب)

The Ancient Egyptians made up a Number System known as Egyptian Number System. They used Symbols


1 2 3 4 5 10 100 1000 10000 100,000 Milion

## (2) Mayan No. System:- (Mezo-Americans)

They used a Number system which was based on 20. For example the human hand and human foot have 20 fingers combined. This Number System was 1000 Times advanced then the Egyptian Number System. They used Nototains for Numbers

1 2 3 4 5 6 10 11 . . .

They also developed a Calender which was consisted on 18 Months and 20 Days, According to that which they said the earth will be demolished by 2012.

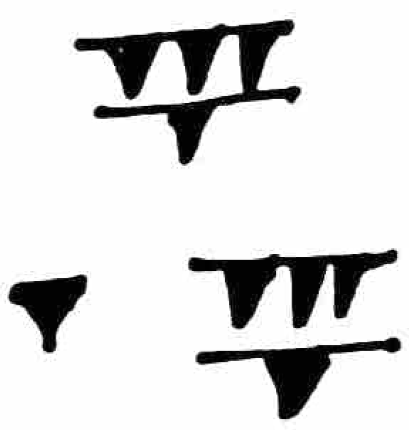
## (3) Babylon No System:- (Mesopotemians):-

There is evidence of Babylon No. System "The famous Pythagoras Theorem (1600–1900) which is kept in British Mueseum. The Number System was based on 60. They also fined that

1 Day = 24 hours
1 hr = 60 Minites
1 Minute = 60 Seconds

The Notations They used "

4 →
64 →



## (4) Chinease No. System:-

The Chinease Introduced the Number System that

| 1 | ll | lll | lllll | T | — | = | ≡ | ⊥ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 5 | 6 | 10 | 20 | 50 | 60 |

## (5) Greek No. Systems:-

The Greek Number System is consisted on 27 Greek letters.



| | | | |
|---|---|---|---|
| α → Alpha<br>β → Beta<br>γ → Gamma<br>δ → Delta<br>ρ → Rho<br>φ phi | ε → Epsilon<br>ζ → Zeta<br>η → eta<br>θ → Theta<br>σ Sigma<br>χ chi | ι → Iota<br>κ → Kappa<br>λ → lambda<br>μ → Mu<br>τ Tau<br>ψ psi | ν → Nu<br>ξ → Ksi<br>ο → omicron<br>π → Pi<br>ν upsilon<br>ω omega |



## (6) Roman No. System:-

The Romans attacked Greeks and Kill "Archimedas" (The Great Mathematician) and Then Romans Numbers were found.

| i | ii | iii | iv | v | vi | vii | viii | ix | x | xi ... |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

| XX | XXX | XL | L | LX | LXX | LXXX | XC | C |
|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

## (7) Indian Number System (Arabic Numbers) :-

It was observed that they have Instruments from which can measure the size or height of Object. 0.05, 0.1, 0.2, ... 500...

"Aryabatha" The great Indian Mathematician"

said (Hindi) Sthanam "Sthanam dasa Gunam"

English Meaning → Place to place Ten Time Time Value.

$$1 \overset{\times 10}{\curvearrowright} 2 \overset{\times 10}{\curvearrowright} 3 \rightarrow \begin{pmatrix} 100 \\ 20 \\ 3 \end{pmatrix}$$

"Aryabatha" Invented "0" Zero (French Word)

0 (zero) Means Nothing. Before that there was No Concept of zero in numbers.



## Number System

- **Decimal Numbers**
  which have base 10
  * 1, 2, 3, 4, 5, 6, 7, 8, 9
- **Binary Numbers**
  * which have base 2.
  0, 1
- **Octal Numbers**
  * which have base 8.
  0, 1, 2, 3, 4, 5, 6, 7
- **Hexa Decimal Numbers**
  * which have base 16
  (0 + 9), 10
  (A To F) → 6

* Decimal Number System:- It is frequently used in our every day Life.
* Binary Number System used for Programming.
* Octal Number System for heavy Machineries.
* Hexa Decimal Number System Used for high Level

# Numbers

N = The Set of Natural Numbers (قدرتی اعداد) = $\{1, 2, 3, \cdots\}$

W = The Set of Whole Numbers (مکمل اعداد) = $\{0, 1, 2, 3, \cdots\}$

Z = The Set of all Integers (صحیح اعداد) = $\{0, \pm 1, \pm 2, \cdots\}$

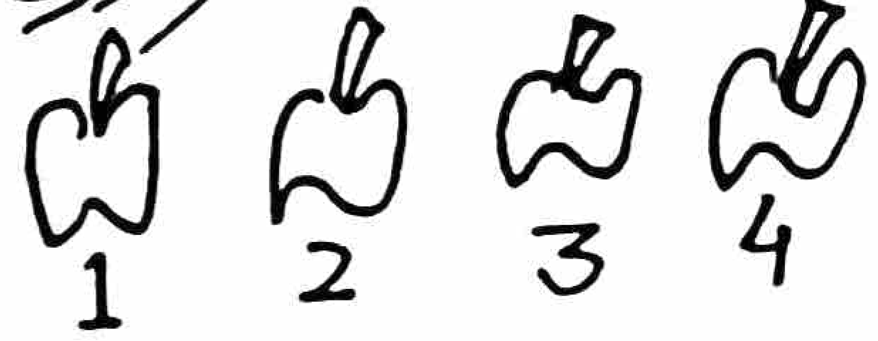
O = The Set of all odd Integers (طاق اعداد) = $\{\pm 1, \pm 3, \cdots\}$

E = The Set of all even Integers (جفت اعداد) = $\{0, \pm 2, \pm 4, \cdots\}$

Q = The Set of all Rational Numbers (ناطق اعداد) = $\{x \mid x = \frac{p}{q}, p, q \in Z\}$

Q′ = The Set of all Irrational Numbers (غیر ناطق اعداد) = $\{x \mid x \neq \frac{p}{q}, p, q \in Z, q \neq 0\}$

## The Natural Numbers:- (قدرتی اعداد)



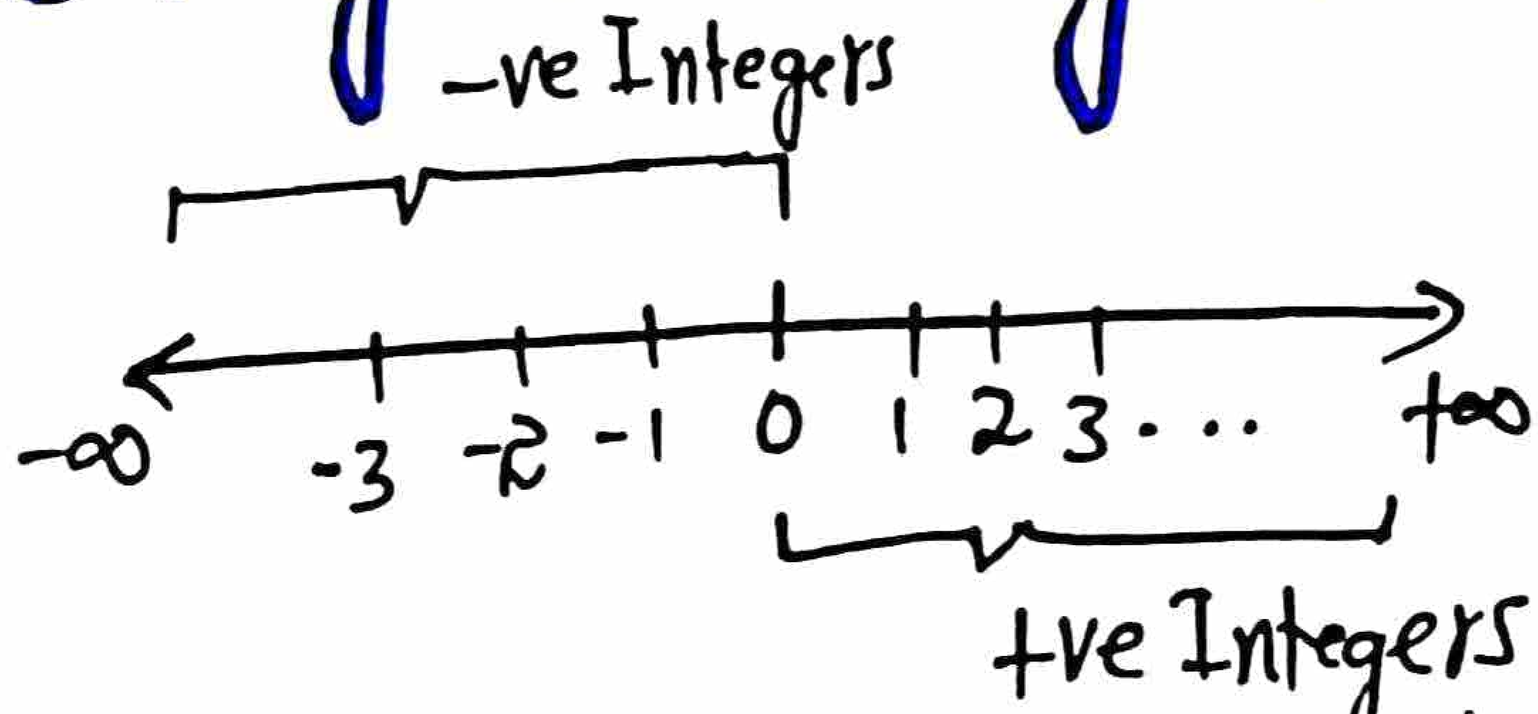

In every day life, if we have Apples we give them Number Notation 1, 2, 3, 4. In Mathematics, the Natural Number Starts with 1 and goes on.

## The Whole Numbers:- مکمل اعداد

In the Picture we see that there are three trees. We can count Them. But if there is no tree what will be Number. (The Number is 0 (zero). (Aryabatha Invented 0).

The Whole Numbers Starts with 0.

## (3) The Negative Integers:-




The Negative Integers are used to Measure Temperature. In Banking, when you withdraw Money from your Account, the Negative Numbers used.

Integer = Z = Zahlen is a word in German which Means Numbers.

## (4) The Rational Numbers:- (ناطق اعداد)

The Numbers that can be Put in the form of $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$, is called a Rational Number.

Examples:- All Integers are Rational Numbers.

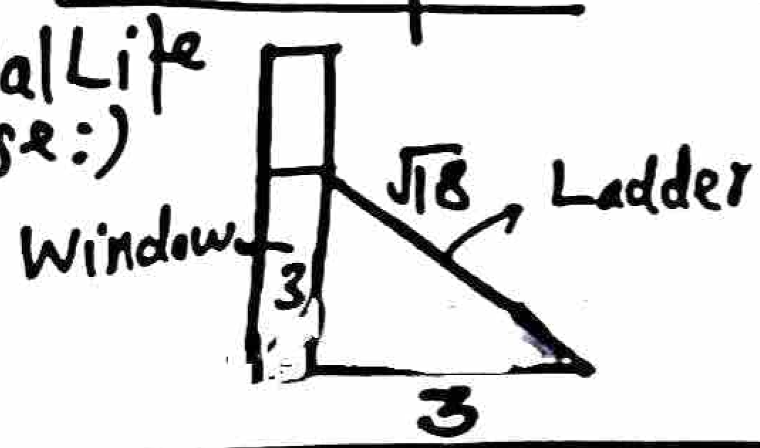
$\sqrt{16}$, 3, 7, 4, etc are Rational Number.
If the number under Square Root is a Perfect Square, then it will be Rational.

## (5) The Irrational Numbers:- (غیر ناطق اعداد)

The Numbers that cannot be Put in the form of $\frac{p}{q}$, where $p, q \in Z$, $q \neq 0$ are called Irrational Numbers.

Examples:- $\sqrt{2}$, $\sqrt{3}$, $\frac{7}{\sqrt{5}}$, $\sqrt{\frac{5}{16}}$ are Irrational Numbers

(Real Life Use:)




By Applying Pythagoras Theorem

## (1) Terminating Decimals:- (اعشاریہ والے حصے میں نمبرز Count ہو سکیں)

A Decimal which has only a finite Number of digits in its Decimal part, is called a Terminating Decimal. Thus 202.04, 0.0000415 are examples of Terminating Decimals.
Since a Terminating Decimal Can be Converted into a Common fraction, so every Terminating Decimal Representing a Rational Number.

## (2) Recurring Decimal:-

This is another type of Rational Numbers. In General, a recurring or Periodic decimal is a decimal in which one or more digits repeat Indefinitely. Every recurring decimal represents a Rational Number.
A Non-Terminating, non-recurring decimal is a decimal which Neither terminates nor it is recurring. It is not possible to convert Such a decimal into a Common fraction. Thus a Non-Terminating, Non-Recurring decimal Represents an Irrational Number.

**Example:-** (1) $.25 = \frac{25}{100}$ is a Rational Number.
(2) $.333 = \frac{1}{3}$ is a Recurring decimal, also Rational.
(3) $0.0100100010000 1\ldots$ is Non-Terminating. Non-Periodic decimal.

# Pi ($\pi$):-

In 1706 a little-known Mathematics teacher named "William Jones" first Used a symbol to represent the Platonic concept of Pi, an Ideal that in numerical terms can be approached but never reached. Later it was popularized by Swiss Mathematician "Leonhard Euler".

**Definition:-** It is an important irrational Number which denotes the Constant Ratio of the Circumference of any Circle to the length of its Diameter. i-e

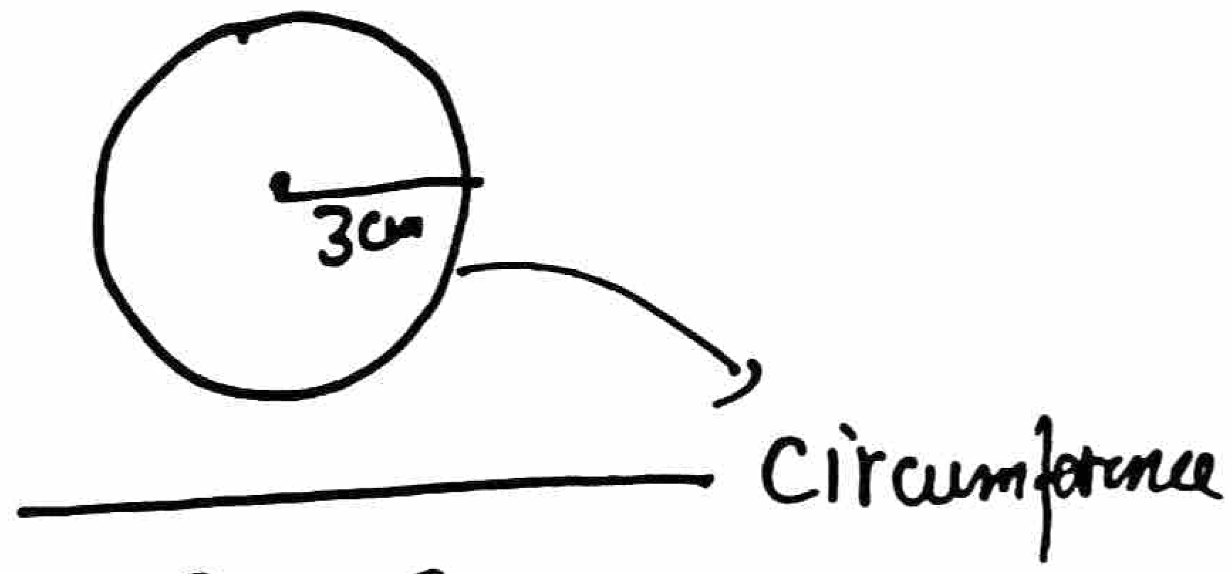
$$\pi = \frac{\text{Circumference of Any Circle}}{\text{Length of its Diameter}}.$$

An Approximate Value of $\pi$ is $\frac{22}{7}$, a better approximation is $\frac{355}{113}$ and still better approximation is 3.14159. The Value of $\pi$ correct to 5 lac places has been determined with help of Computer.

* $\pi$ is an Irrational Number because its decimal part is Non-Terminating Non Recurring.

**Example:-**

(1) We have Circle of Radius 3cm




$C = 18.85$

$\text{Diameter} = D = 2(3) = 6\text{cm}$

Then $\pi = \frac{18.85}{6}$

$\boxed{\pi = 3.14}$

(2) Circle of Radius 4cm


$C = 25.1$

$D = 2(4) = 8$

$\pi = \frac{25.1}{4}$

$\boxed{\pi = 3.14}$

F.Sc Part-I

Unit 1

# Complex Numbers

(Asad Ali Khalid
BS(H)Math
M.Phill Math, B.ed
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## History:-

In 16th Century "A Mathematician "Cardon" first started thinking Solutions of Quardratic equations with Negative Discimindants. In 18th Century, An English Mathematician "**Leonhard Euler**" give the Idea that $\sqrt{-1} = i$ called Imaginary Number.

## Complex Number:-

The Numbers of the form $x+iy$, where $x, y \in \mathbb{R}$ and $i = \sqrt{-1}$ are called Complex Numbers, here $x$ is called the real part and $y$ is called the Imaginary Part.



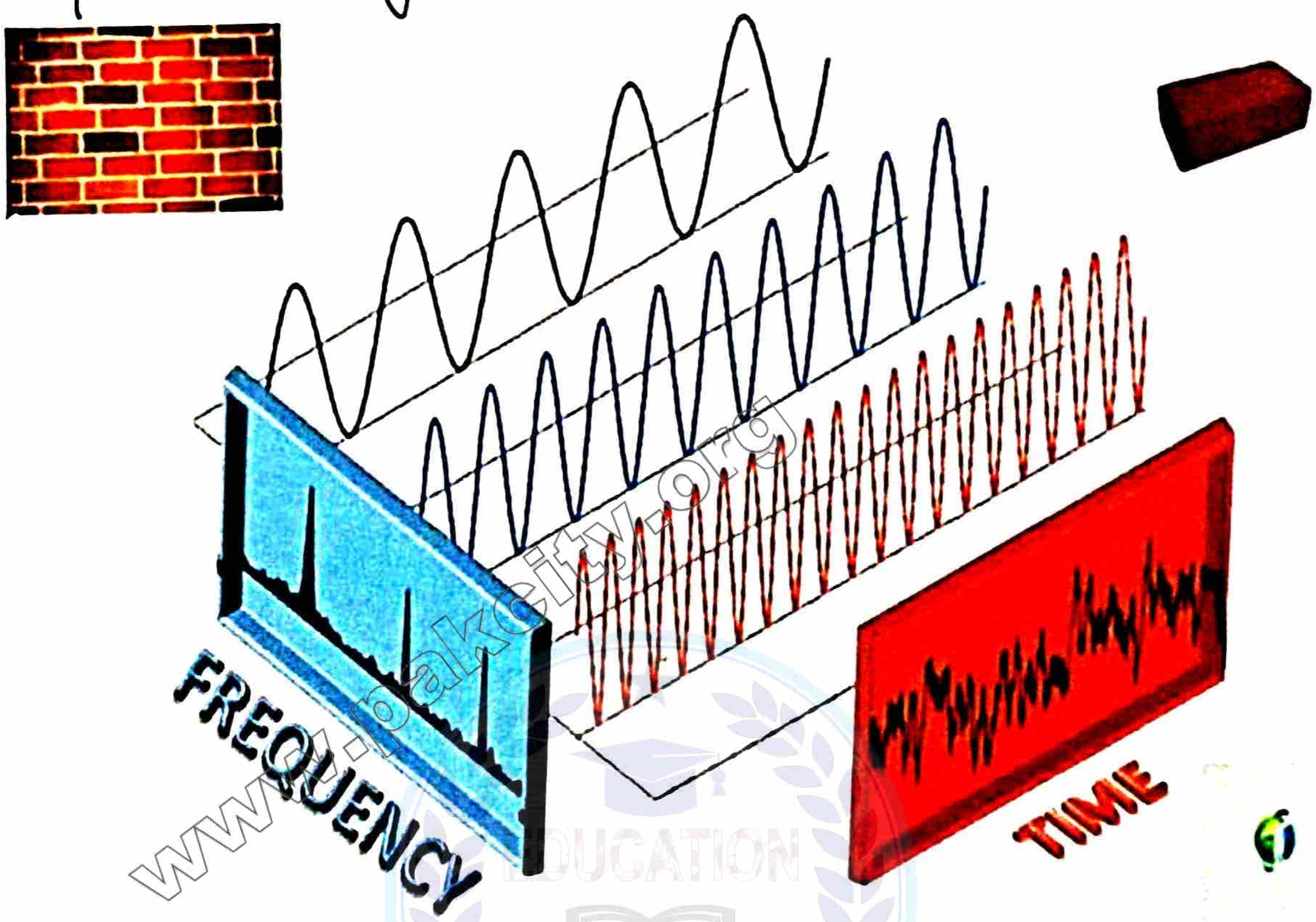
## Applications OF Complex Numbers

Complex Numbers are Used in designing and testing bridges strength. In 1940 "Washington USA" The Tacoma Bridge Collapsed. Beause in Modeling the equation, Mathematicians forget to Use the Imaginary Part (Vibration) and it was Collapsed. The Quardratic Equation was used and it Root was a Complex Number.

# Mobile Communication:-



If we want to process a signal, or Information we see it Domain that will be a Sinosidal wave, but in Time we cannot see the building blocks. But if we see in the frequency domain, we observe, which frequency waves are adding to form a Signal!



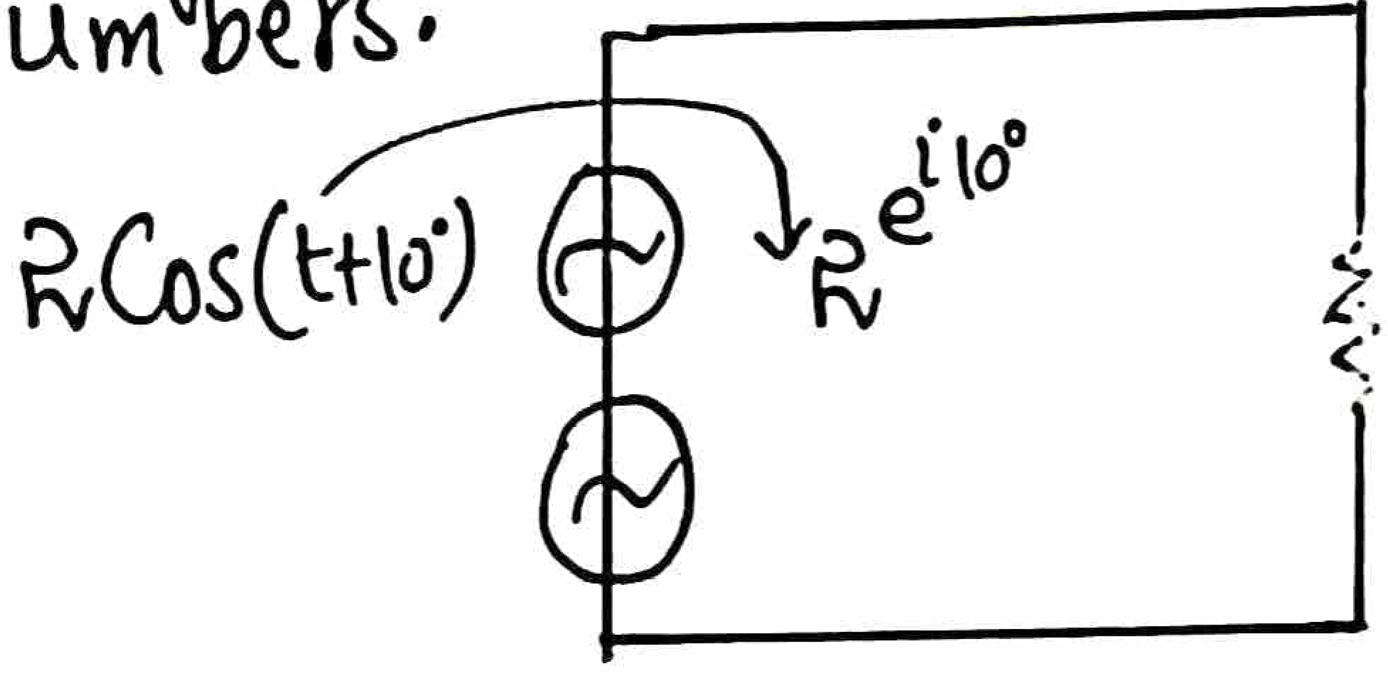

The above Process is called Fourier Transformation And it is helpfull in Mobile Application, Radar and Channel Modeling.

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{i2\pi ft}\, df$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t}\, d\omega$$

# Complex Numbers In AC Circuits:-

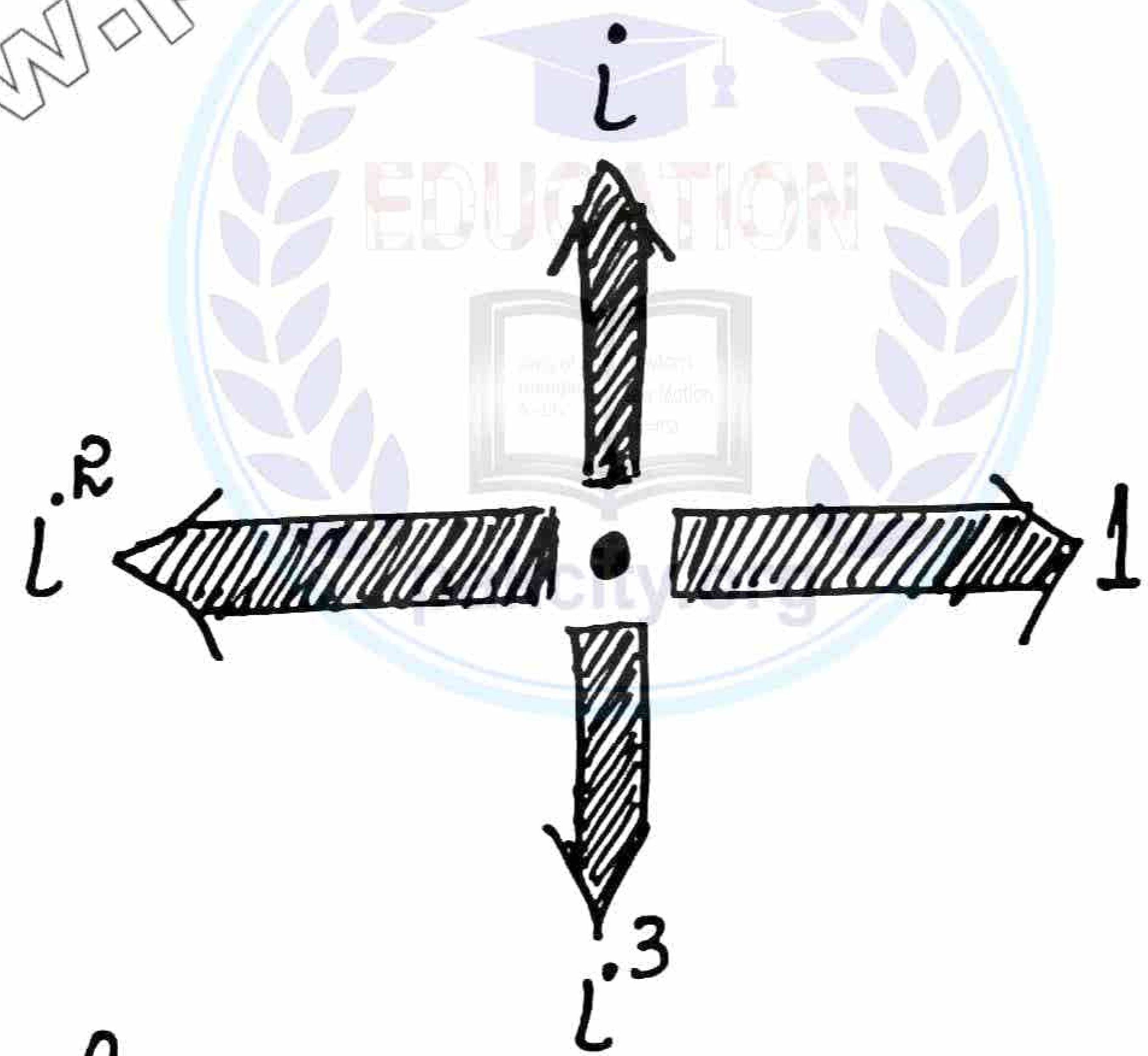
In the figure below if you have to combine two Voltages with different phases. We can use Complex Numbers.




When we put Capacitor —||— and inductor ~~~~ into a Circuit, that will shift Voltage and Current depending upon the values. Math represents that it is all due to Complex Numbers. We Use Imaginary Numbers to determine the shift in the Circuit.

## Complex Numbers Are Rotating Numbers:-

To Donate Rotation Complex Numbers are used.




$i^0 = 1 =$ No Rotation

$i^1 =$ Rotation by 90° Degrees

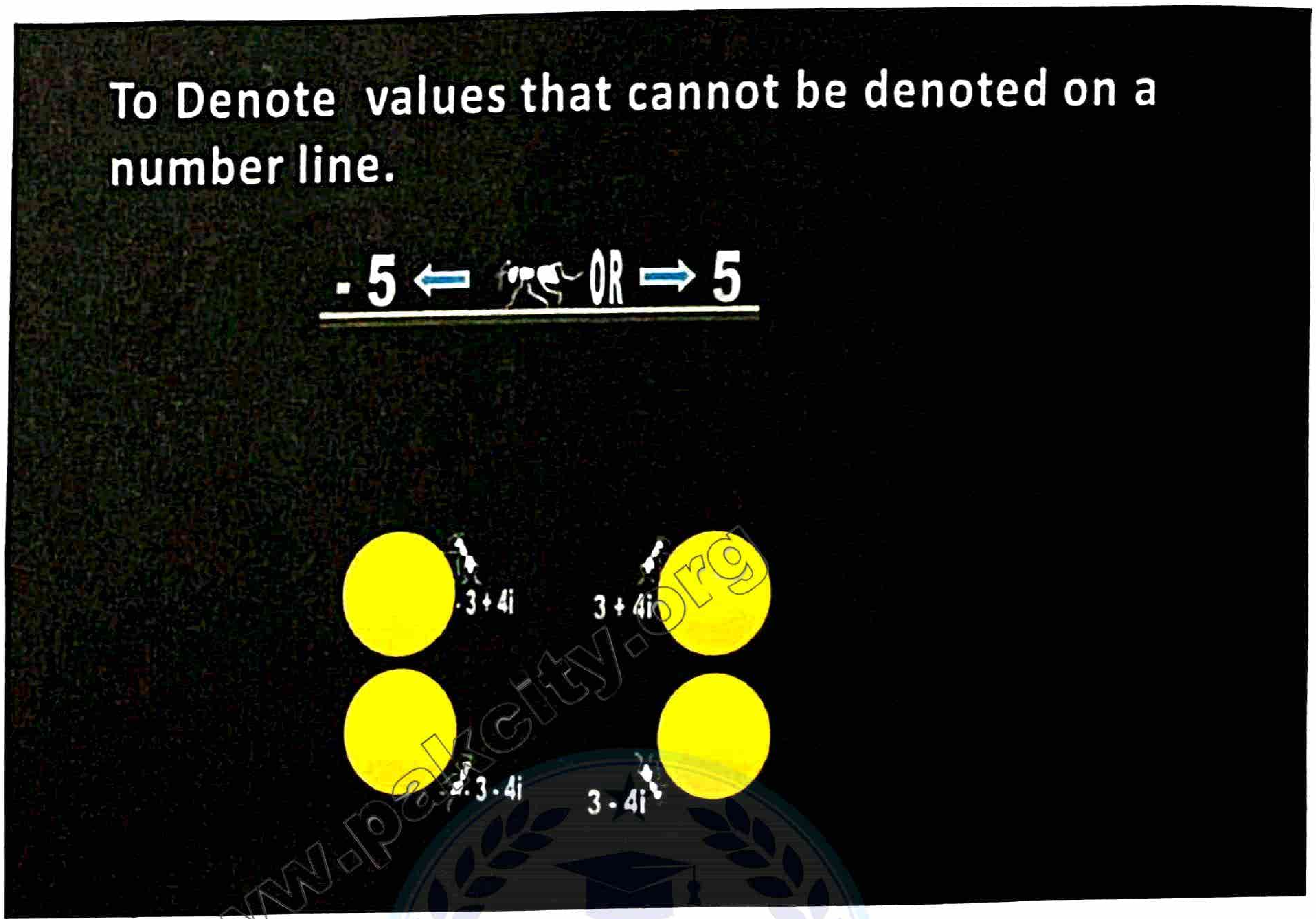
$i^2 =$ Rotation by 180° Degrees

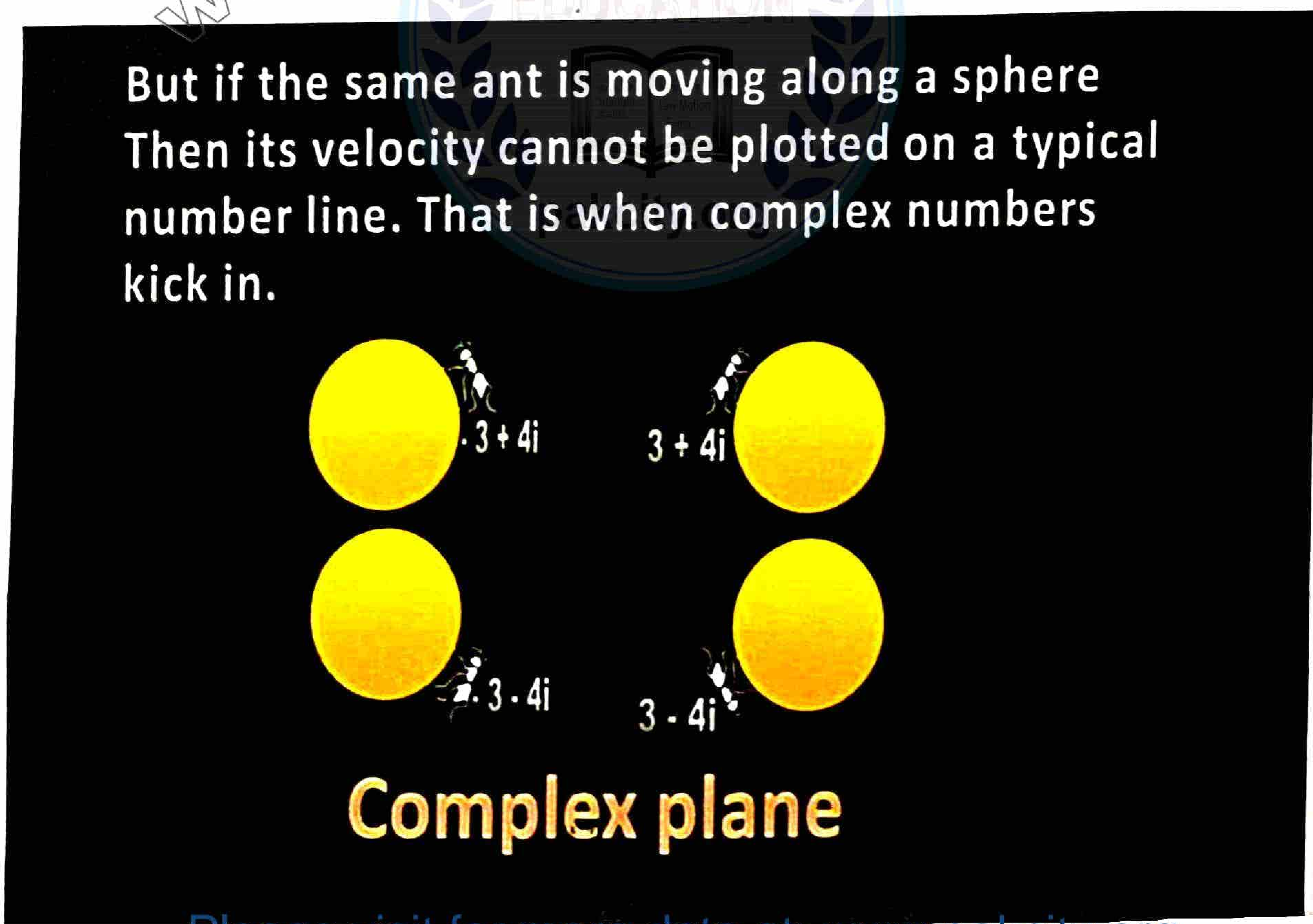
$i^3 =$ Rotation by 270° Degrees

$i^4 =$ Rotation by 360° Degrees

# Complex Numbers on Number Line:-

There are some values that can be not donated on Number line. Complex Numbers help in drawing values on Number Line



# LOGARITHMS

**Definition:-** A logarithim is the power to which a number must be raised in order to get some other number.

**History:-** Logarithims was Introduced by "John Napier" in 1610.

**Applications:-** Logarithims widely used to measure earthquake Intensity, Acid measurement Solutions (PH Value) Sound Intensity and Larger values.

**Types Of Logarithim:-**

(1) Basic logs: For Example $\log_3 9$, $\log 10000$

(2) Weirder logs: For Example $\log_2(\frac{1}{8})$, $\log 1$
$\log 0$, $\log(-1)$

(3) Natural logs: $\ln 1$, $\ln(e^3)$

(4) Even Weirder logs: $\log_x 64 = 6$, $\log_5 x = 3$, $\log_2 7$ (change of base formula)

**(1) Basic Logs:-**

(i) $\log_3 9$ (Read as log 9 to base 3)

$\log_3 9$ (Re-write in exponential Form)

$$\log_3 9 = x$$

equals

Raised to power of

$$9 = 3^x$$

$$3^x = 3^2$$

$$\Rightarrow \boxed{x = 2}$$

(ii) $\log_{10} 10000$

Rewrite in Exponential Form

$$\log_{10} 10000 = x$$

$$10000 = 10^x$$

$$10^x = 10\cdot10\cdot10\cdot10$$

$$10^x = 10^4$$

$$\Rightarrow \boxed{x = 4}$$

## (2) Weirder logs:-

(i) $\log_2\left(\frac{1}{8}\right)$

Let $x = \log_2\left(\frac{1}{8}\right)$

$\log_2\left(\frac{1}{8}\right) = x$

$\frac{1}{8} = 2^x$

$\frac{1}{2^3} = 2^x$

$2^x = 2^{-3}$

$\Rightarrow \boxed{x = -3}$

So $\log_2\left(\frac{1}{8}\right) = -3$

(ii)(a) $\log 1$

Let $\log_{10} 1 = x$

$1 = 10^x$

$\Rightarrow 10^x = 10^0$

$\Rightarrow \boxed{x = 0}$

$\boxed{\log_{10} 1 = 0}$

(b) $\log 0$

Let $\log_{10} 0 = x$

$0 = 10^x$

$10^x = 0 \rightarrow$ There is no Number that gives us zero!

Undefined

(iii) $\log(-1)$

Let $\log_{10}(-1) = x$

$-1 = 10^x$

$10^x = -1$

Undefined.

## (3) Natural logs:- (Special kind of Logarithim)

(i) $\ln_e 1 = \log_e 1$

Let $\log_e 1 = x$

$1 = e^x$

$e^0 = e^x$

$\Rightarrow \boxed{x = 0}$

$\boxed{\ln 1 = 0}$

(ii) $\ln(e^3) = \log_e(e^3)$

Let $\log_e(e^3) = x$

$e^3 = e^x$

$\Rightarrow \boxed{x = 3}$

$\boxed{\ln(e^3) = 3}$

## (4) Even Weirder logs:-

(i) Solve $\log_x 32 = 5$

$32 = x^5$

$\Rightarrow 2^5 = x^5$

$\Rightarrow \boxed{x = 2}$

(ii) Solve $\log_5 x = 3$

$5^3 = x$

$\boxed{x = 125}$

(iii) $\log_2 7$

Let $\log_2 7 = x$

$2^x = 7$ (Not possible)

So, we use change of base formula.

$\frac{\log 7}{\log 2} = x$

$\boxed{x = ?}$

# Properties OF Logarithims:-

**(1) Equality Property:-** $\log_b m = \log_b n \Leftrightarrow \boxed{m=n}$

**(2) Product Property:-** $\log_b mn = \log_b m + \log_b n$

**(3) Power Property:-** $\log_b m^p = p\log_b m$

**(4) Quotient Property:-** $\log_b \frac{m}{n} = \log_b m - \log_b n$

Application

(1) $\log(12x-7) = \log(3x+11) \rightarrow (A)$

$\log_{10}(12x-7) = \log_{10}(3x+11) \quad \because \log_b m = \log_b n$

$\Rightarrow \quad 12x-7 = 3x+11$

$9x = 18$

$\boxed{x=2}$

Put $x=2$ in (A)

$\log(12(2)-7) = \log(3(2)+11)$

$\boxed{\log 17 = \log 17}$

(2) $\log_2 x + \log_2(x-2) = 3 \rightarrow (A)$

$\log_2 x\cdot(x-2) = 3 \quad \because \log_b mn = \log_b m + \log_b n$

$x(x-2) = 2^3$

$x^2-2x=8$

$x^2-2x-8=0$

$x^2-4x+2x-8=0$

$x(x-4)+2(x-4)=0$

$(x+2)(x-4)=0$

$\boxed{x=-2}, \boxed{x=4}$

Unit NO. 1:-

Number Systems



# Definitions

Real Numbers:-

The union of rational and irrational number is called **real number**.

Rational Numbers:-

A **rational number** which can be put in the form $\frac{p}{q}$, where $p, q \in Z \wedge q \neq 0$.

For example:-

The numbers $\sqrt{16}$, 3, 7, 4 etc., are rational numbers.

Irrational Numbers:-

**Irrational numbers** are those which cannot be put into the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$.

For example:-

The number $\sqrt{2}, \sqrt{3}, \frac{7}{\sqrt{5}}$ are irrational numbers.

Terminating Decimals:-

A decimal which

has only a finite number of digits in its decimal part, is called a **terminating decimal**.

For Examples:-

202.04, 0.0000415 are the examples of terminating decimals.

$\pi$ (Pi):-

The ratio between the circumference of a circle and the length of the diameter of that circle is called $\pi$ (Pi).

Complex Numbers:-

The numbers of the form $x+iy$, where $x, y \in \mathbb{R}$ and $i = \sqrt{-1}$, are called **complex numbers**, here $x$ is called **real part** and $y$ is called **imaginary part** of the complex number.

For example:-

$3+4i$, $2-\frac{5}{7}i$ etc are complex numbers.

Powers of $i$:-

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$i^4 = i^2 \times i^2 = (-1)(-1) = 1$

$i^{13} = (i^2)^6 \cdot i = (-1)^6 \cdot i = i$

$i^6 = (i^2)^3 = (-1)^3 = -1$

## *Exercise 1.1

Q.NO.1:-

Which of the following sets have closure property w.r.t addition and multiplication?

(i) $\{1\} = B$

| Addition | Multiplication |
| --- | --- |
| $1 + 1 = 2$ | $1 \times 1 = 1$ |
| $\notin B$ | $\in B$ |
| Set B is not closed w.r.t (+). | Set B is closed w.r.t (·). |

(ii) $\{0\} = A$

| Addition | Multiplication |
| --- | --- |
| $0 + 0 = 0$ | $0 \times 0 = 0$ |
| $\in A$ | $\in A$ |
| Set A is closed w.r.t (+). | Set A is closed w.r.t (·). |

(iii) $\{0, -1\} = C$

| Addition | Multiplication |
|---|---|

| + | 0 | -1 |
|---|---|---|
| 0 | 0 | -1 |
| -1 | -1 | -2 |

Set C is not closed w.r.t (+).

| $\cdot$ | 0 | -1 |
|---|---|---|
| 0 | 0 | 0 |
| -1 | 0 | 1 |

Set C is not closed w.r.t ($\cdot$).

(iv) $D = \{1, -1\}$

Addition

| + | 1 | -1 |
|---|---|---|
| 1 | 2 | 0 |
| -1 | 0 | -2 |

Set D is not closed w.r.t (+).

Multiplication

| $\cdot$ | -1 | 1 |
|---|---|---|
| -1 | 1 | -1 |
| 1 | -1 | 1 |

Set D is closed w.r.t ($\cdot$).

Q.NO.4:-

Prove the following rules of addition.

(i) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

$= a \cdot \frac{1}{c} + b \cdot \frac{1}{c}$

$= (a+b) \cdot \frac{1}{c}$ Distributive Property

$$= \frac{a+b}{c}$$

(ii) $$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$= \frac{a}{b}\cdot 1 + \frac{c}{d}\cdot 1$$ Multiplicative Property

$$= \frac{a}{b}\left(d\cdot\frac{1}{d}\right) + \frac{c}{d}\left(b\cdot\frac{1}{b}\right)$$ Multiplicative Inverse

$$= \frac{a}{b}\cdot\frac{d}{d} + \frac{c}{d}\cdot\frac{b}{b}$$

$$= \frac{ad}{bd} + \frac{cd}{bd}$$ Commutative Property w.r.t (·)

$$= ad\cdot\frac{1}{bd} + bc\cdot\frac{1}{bd}$$

$$= (ad+bc)\cdot\left(\frac{1}{bd}\right)$$ Distributive property

Q. NO. 5:-

Prove that.

$$\frac{-7}{12} - \frac{5}{18} = \frac{-21-10}{36}$$

$$= \frac{-7}{12}\cdot 1 - \frac{5}{18}\cdot 1$$ Multiplicative identity

$$= \frac{-7}{12}\left(3\cdot\frac{1}{3}\right) - \frac{5}{18}\left(2\cdot\frac{1}{2}\right)$$ Multiplicative inverse

$= \frac{-7}{21} \cdot \frac{3}{3} - \frac{5}{18} \cdot \frac{2}{2}$

$= \frac{-21}{36} - \frac{10}{36}$

$= -21 \cdot \frac{1}{30} - 10 \cdot \frac{1}{36}$

$= (-21-10) \cdot \frac{1}{36}$ Distributive Property

$= \frac{-21-10}{36}$

Q. NO. 6:-

Simplify by justifying each step:-

(i) $\frac{4+16x}{4}$

$= \frac{1}{4} \cdot (4+16x)$

Distributive Property

$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16x$

$= 1 + 1 \cdot 4x$ Multiplicative Inverse

$= 1+4x$ Multiplicative Identity

(ii) $\frac{\left(\frac{1}{4} + \frac{1}{5}\right)}{\left(\frac{1}{4} - \frac{1}{5}\right)}$

$$= \left(\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}\right) \cdot 1$$

$$= \left(\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}\right) \cdot 20 \cdot \frac{1}{20}$$ Multiplicative Inverse

$$= \frac{\left(\frac{1}{4} + \frac{1}{5}\right) \cdot 20}{\left(\frac{1}{4} - \frac{1}{5}\right) \cdot 20}$$

$$= \frac{\frac{1}{4} \cdot 20 + \frac{1}{5} \cdot 20}{\frac{1}{4} \cdot 20 - \frac{1}{5} \cdot 20}$$ Distributive Property

$$= \frac{\left(\frac{1}{4} \cdot 4\right) 5 + \left(\frac{1}{5} \cdot 5\right) \cdot 4}{\frac{1}{4} \cdot 4 \cdot 5 + \frac{1}{5} \cdot 5 \cdot 4}$$

$$= \frac{1 \cdot 5 + 1 \cdot 4}{1 \cdot 5 - 1 \cdot 4}$$ Multiplicative Inverse

$$= \frac{5+4}{5-4}$$ Multiplicative Identity

$$= \frac{9}{1}$$

$$= 9$$

(iii) $$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$$

$$=\left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right)\cdot 1$$ Multiplicative Identity

$$=\left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right)\cdot bd\cdot\frac{1}{bd}$$ Multiplicative Inverse

$$=\frac{\left(\frac{a}{b}+\frac{c}{d}\right)\cdot bd}{\left(\frac{a}{b}-\frac{c}{d}\right)\cdot bd}$$

$$=\frac{\frac{a}{b}\cdot bd+\frac{c}{d}\cdot bd}{\frac{a}{b}\cdot bd-\frac{c}{d}\cdot bd}$$ Distributive Property

$$=\frac{a\cdot\frac{1}{b}\cdot b\cdot d+c\cdot\frac{1}{d}\cdot b\cdot d}{a\cdot\frac{1}{b}\cdot b\cdot d-c\cdot\frac{1}{d}\cdot b\cdot d}$$

$$=\frac{a\left(\frac{1}{b}\cdot b\right)d+c\left(\frac{1}{d}\cdot d\right)b}{a\left(\frac{1}{b}\cdot b\right)d-c\left(\frac{1}{d}\cdot d\right)b}$$ Commutative Property

$$=\frac{a\cdot 1\cdot d+c\cdot 1\cdot b}{a\cdot 1\cdot d-c\cdot 1\cdot b}$$ Multiplicative Inverse

$$= \frac{ad + cb}{ad - cb}$$ Multiplicative Identity

(iv) $$\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}\right) \cdot 1$$ Multiplicative Identity

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} - \frac{1}{b}}\right) \cdot ab \cdot \frac{1}{ab}$$ Multiplicative inverse

$$= \frac{\left(\frac{1}{a} - \frac{1}{b}\right) \cdot ab}{\left(1 - \frac{1}{a} \cdot \frac{1}{b}\right) \cdot ab}$$

$$= \frac{\frac{1}{a} \cdot ab - \frac{1}{b} \cdot ab}{ab \cdot \frac{1}{a} \cdot \frac{1}{b} \cdot ab}$$ Distributive Property

$$= \frac{\left(\frac{1}{a} \cdot a\right) b - \left(\frac{1}{b} \cdot b\right) a}{ab - \left(\frac{1}{a} \cdot a\right)\left(\frac{1}{b} \cdot b\right)}$$ Commutative Property

$$= \frac{1 \cdot b - 1 \cdot a}{ab - 1 \cdot 1}$$ Multiplicative Inverse

$$= \frac{b - a}{ab - 1}$$ Multiplicative identity

# Exercise 1.2



## Q.NO.4:-

Simplify the following:

(i) $i^9$

$= i^9$

$= i$

$= \sqrt{-1}$

$$\begin{array}{r} 2 \\ 4\,\overline{)\,9} \\ 8 \\ \hline 1 \end{array}$$

(ii) $i^{14}$

$= i^{14}$

$= i^2$

$= -1$

$$\begin{array}{r} 3 \\ 4\,\overline{)\,14} \\ 12 \\ \hline 2 \end{array}$$

(iii) $(-i)^{19}$

$= (-i)^{19}$

$= -i^3$

$= -i \cdot i^2$

$= -i \cdot (-1)$

$= i$

$$\begin{array}{r} 4 \\ 4\,\overline{)\,19} \\ 16 \\ \hline 3 \end{array}$$

(iv) $(-1)^{-\frac{21}{2}}$

$= \left(i^2\right)^{-\frac{21}{2}}$

$= (i)^{-21}$

$= \dfrac{1}{i^{21}}$

$= \dfrac{1}{i} \times \dfrac{i}{i}$

$$\begin{array}{r} 5 \\ 4\,\overline{)\,21} \\ 20 \\ \hline 1 \end{array}$$

$= \frac{i}{i^2} \Rightarrow = \frac{i}{-1}$

$= -i$

Q.NO. 5:-

Write in terms of i.

(i) $\sqrt{-1}\,b$

$= ib$

(ii) $\sqrt{-5}$

$= \sqrt{-1 \times 5}$

$= i\sqrt{5}$

(iii) $\sqrt{\frac{-16}{25}}$

$= \sqrt{\frac{-1 \times 16}{25}}$

$= \frac{4i}{5}$

(iv) $\sqrt{\frac{1}{-4}}$

$= \sqrt{\frac{1}{-1 \times 4}}$

$= \frac{1}{2i}$

Simplify the following.

Q.NO.6:-

$$(7,9)+(3,-5)$$

$= (7+9i)+(3-5i)$

$= 7+9i+3-5i$

$= 10-4i$

$= (10,-4)$

Q.NO.7:-

$$(8,-5)-(-7,4)$$

$= (8,-5i)-(-7+4i)$

$= 8-5i+7-4i$

$= 15-9i$

$= (15,-9)$

Q.NO.8:-

$$(2,6)(3,7)$$

$= (2+6i)(3+7i)$

$= 6+14i+18i+42i^2$

$= 6+32i-42$

$= -36+32i$

$= (-36,32)$

Q.NO.9:-

$$(5,-4)(-3,-2)$$

$= (5-4i)(-3-2i)$

$= -15 - 6i + 12i + 8i^2$

$= -15 + 6i - 8$

$= -23 + 6i$

$= (-23, 6)$

Q.NO.10:-

$(0,3)(0,5)$

$= (0+3i)(0+5i)$

$= 0+0+0+15i^2$

$= 15i^2 + 0$

$= -15 + 0i$

$= (-15, 0)$

Q.NO.11:-

$(2,6) \div (3,7)$

$= \frac{2+6i}{3+7i} \times \frac{3-7i}{3-7i}$

$= \frac{(2+6i)(3-7i)}{(3)^2 - (7i)^2}$

$= \frac{6 - 14i + 18i - 42i^2}{9 - 49(-1)}$

$= \frac{6 + 4i + 42}{9 + 49}$

$= \frac{68 + 6i}{58} \Rightarrow = \frac{48}{58} + \frac{6}{58} i$

Q.NO.12:-



$$(5,-4)\div(-3,-8)$$

$$= \frac{5-4i}{-3-8i} \times \frac{-3+8i}{-3+8i}$$

$$= \frac{(5-4i)(-3+8i)}{(-3)^2-(8i)^2}$$

$$= \frac{-15+40i+12i-32i^2}{9-64(-1)}$$

$$= \frac{-15+52i+32}{9+64}$$

$$= \frac{17+52i}{73}$$

$$= \frac{17}{73} + \frac{52}{73}i$$

Q.NO.13:-

Prove that the sum as well as the product of any two conjugate complex.

Let

$$z = x+iy$$

$$\bar{z} = x-iy$$

Condition-1

$$z+\bar{z} = x+iy+x-iy$$

$z + \bar{z} = 2x$

$\therefore$ Sum is real

Condition -2:- Product

$z\bar{z} = (x+iy)(x-iy)$

$= x^2 - i^2y^2 \qquad \therefore i^2 = -1$

$= x^2 - y^2$

$\therefore$ Product is real

Q.NO.14:-

Find the multiplicative inverse of each of the following numbers.

(i) $(-4, 7)$

$a = -4$

$b = 7$

$= \left(\dfrac{a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$

$= \left[\dfrac{-4}{(-4)^2+(7)^2}, \dfrac{-7}{(-4)^2+(7)^2}\right]$

$= \left(\dfrac{-4}{16+49}, \dfrac{-7}{16+49}\right)$

$= \left(\dfrac{-4}{65}, \dfrac{-7}{65}\right)$

(ii) $(\sqrt{2}, -\sqrt{5})$

$$a = \sqrt{2}$$

$$b = -\sqrt{5}$$

$$= \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

$$= \left[\frac{\sqrt{2}}{(\sqrt{2})^2+(-\sqrt{5})^2}, \frac{-(-\sqrt{5})}{(\sqrt{2})^2+(-\sqrt{5})^2}\right]$$

$$= \left(\frac{\sqrt{2}}{2+5}, \frac{\sqrt{5}}{2+5}\right)$$

$$= \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$$

(iii) $(1,0)$

$$a = 1$$

$$b = 0$$

$$= \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

$$= \left(\frac{1}{(1)^2+(0)^2}, \frac{0}{(1)^2+(0)^2}\right)$$

$$= \left(\frac{1}{1}, 0\right)$$

$$= (1, 0)$$

Q.NO.15:-

Factorize the following.

(i) $a^2+4b^2$

$= a^2 - (-4b^2)$

$= a^2 - (-1 \times 4b^2)$

$= a^2 - (4b^2 i^2)$

$= (a)^2 - (2bi)^2$

$= (a+2bi)(a-2bi)$

(ii) $9a^2 + 16b^2$

$= 9a^2 - (-16b^2)$

$= 9a^2 - (16b^2 \times -1)$

$= 9a^2 - 16b^2 i^2$

$= (3a)^2 - (4bi)^2$

$= (3a+4bi)(3a-4bi)$

(iii) $3x^2 + 3y^2$

$= 3(x^2+y^2)$

$= 3[x^2 - (-y^2)]$

$= 3[x^2 - (y^2 \times -1)]$

$= 3[x^2 - y^2 i^2]$

$= 3[(x)^2 - (yi)^2]$

$= 3(x+yi)(x-yi)$

Q.NO.16:-

Separate into real and imaginary parts.

(i) $\dfrac{2-7i}{4+5i}$

$= \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$

$= \frac{(2-7i)(4-5i)}{(4)^2-(5i)^2}$

$= \frac{8-10i-14i+35i^2}{16-25i^2}$

$= \frac{8-24i-35}{16+25}$

$= \frac{-27-24i}{41}$

$= \frac{-27}{41}, \frac{-24}{41}i$

(iii) $\frac{(-2+3i)^2}{1+i}$

$= \frac{[(-2)^2+2(-2)(3i)+(3i)^2]}{1+i}$

$= \frac{4-12i+3i^2}{1+i}$

$= \frac{1-12i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{(1-12i)(1-i)}{(1)^2-(i)^2}$

$= \frac{1-i-12i+12i^2}{1-(-1)}$

$$= \frac{1 - 13i - 12}{1+1}$$

$$= \frac{-11 - 13i}{2}$$

$$= \frac{-11}{2} ; \frac{-13}{2} i$$

(iii) $\dfrac{i}{1+i}$

$$= \frac{i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{i(1-i)}{(1)^2 - (i)^2}$$

$$= \frac{i - i^2}{1-(-1)}$$

$$= \frac{i-(-1)}{1+1}$$

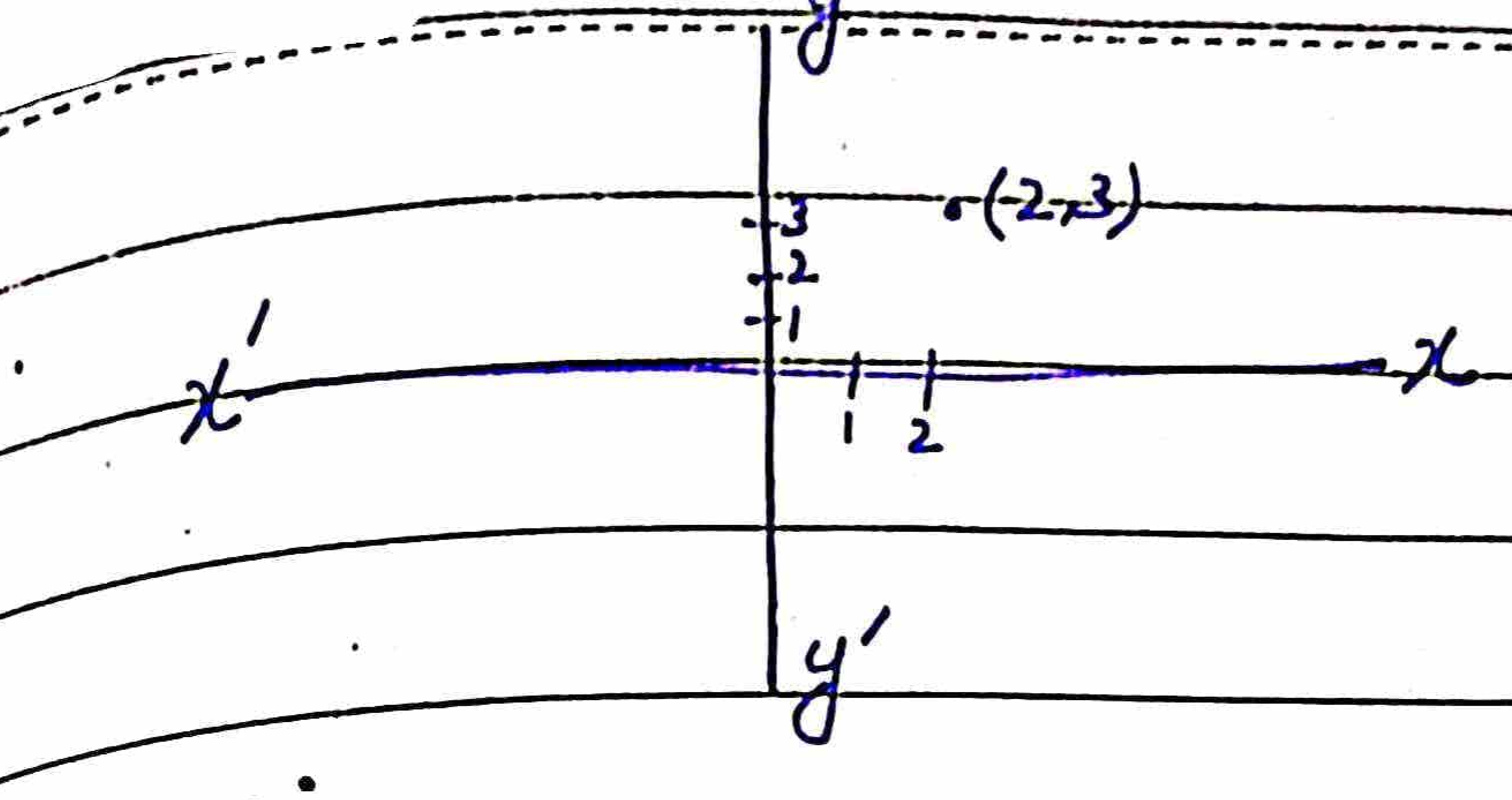
$$= \frac{i+1}{2}$$

$$= \frac{i}{2} + \frac{1}{2}$$

## *Exercise 1.3

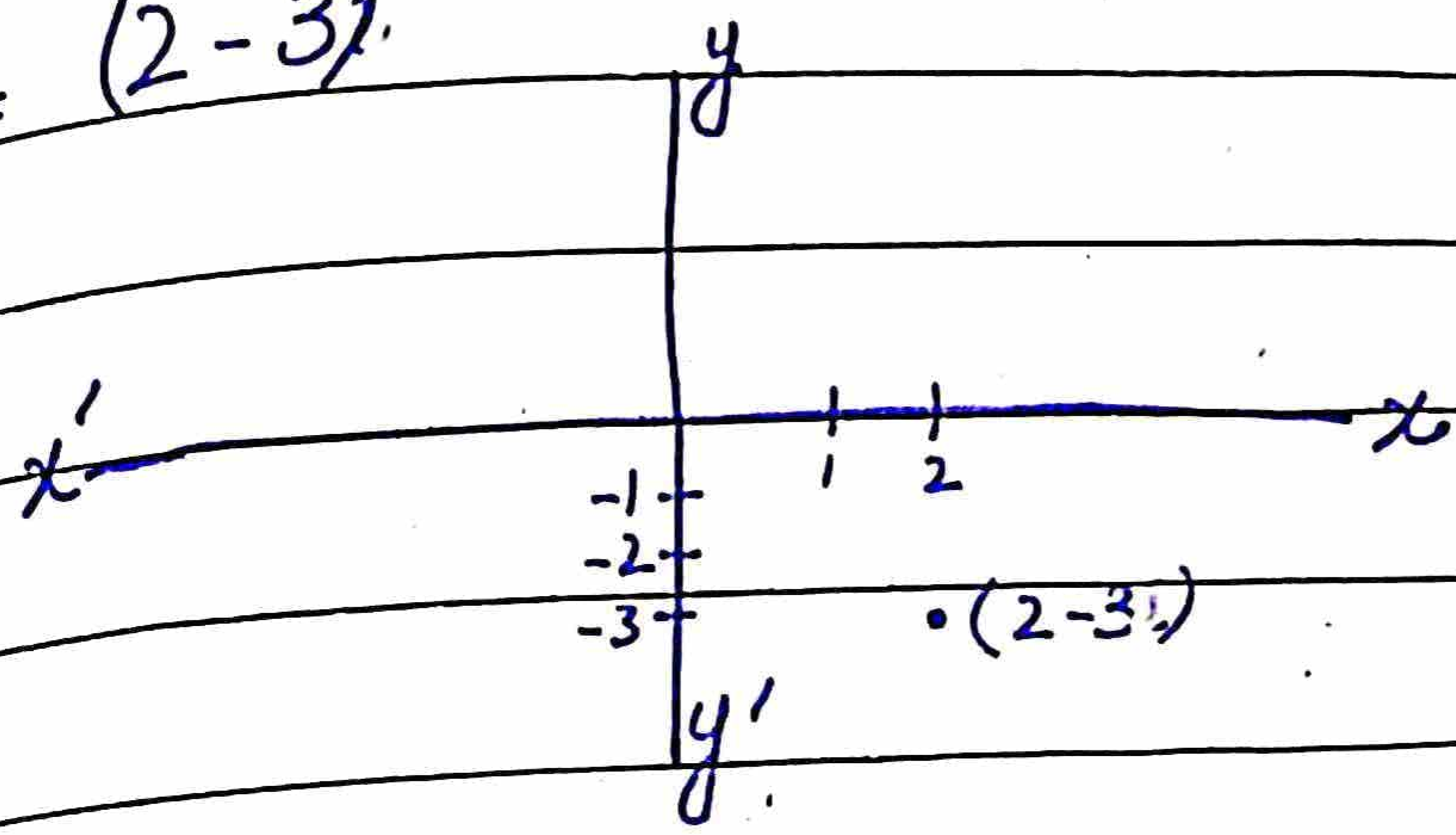
(i) $2+3i$

$= 2+3i$



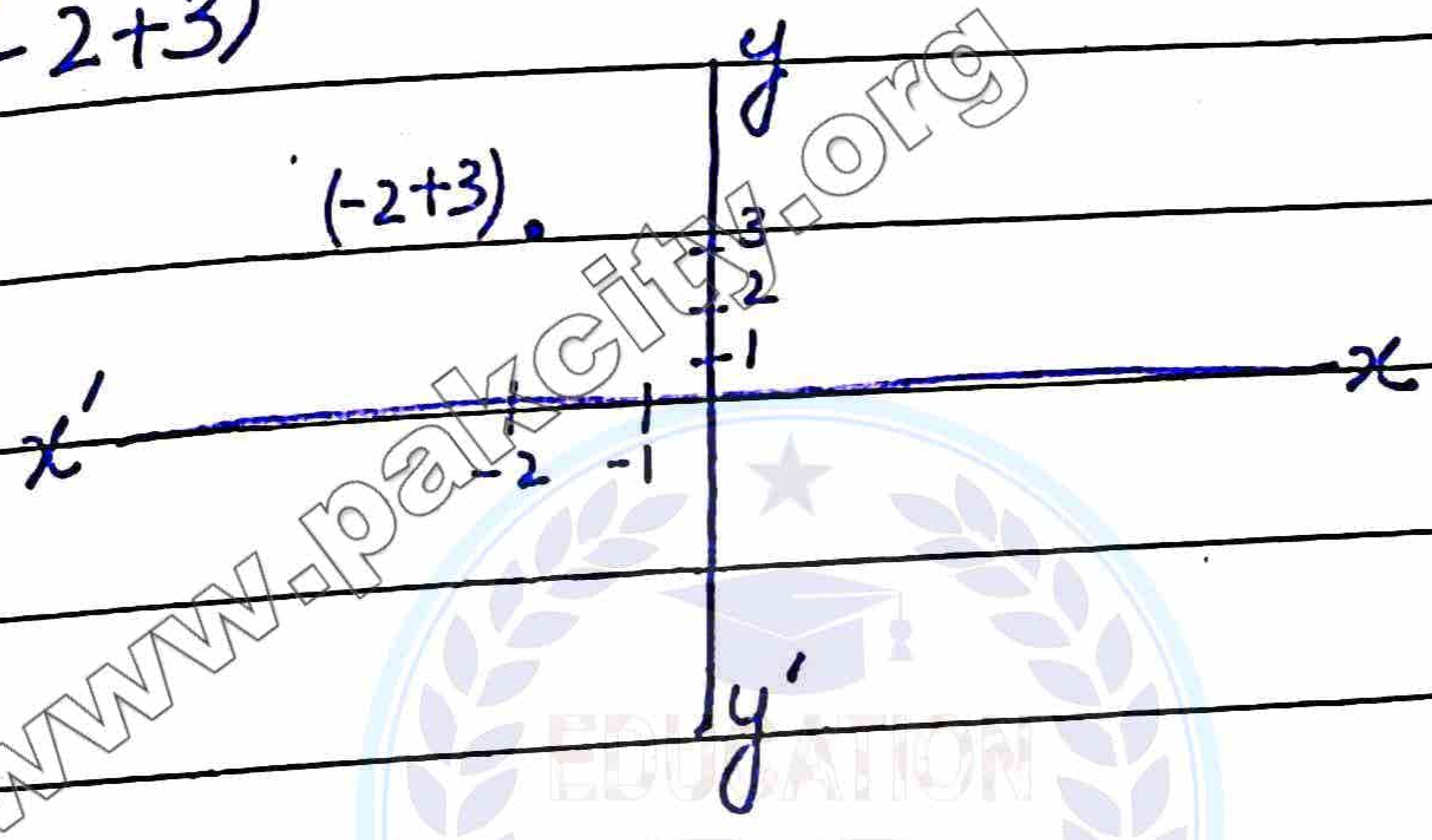

(ii) $2-3i$

$= (2-3)$



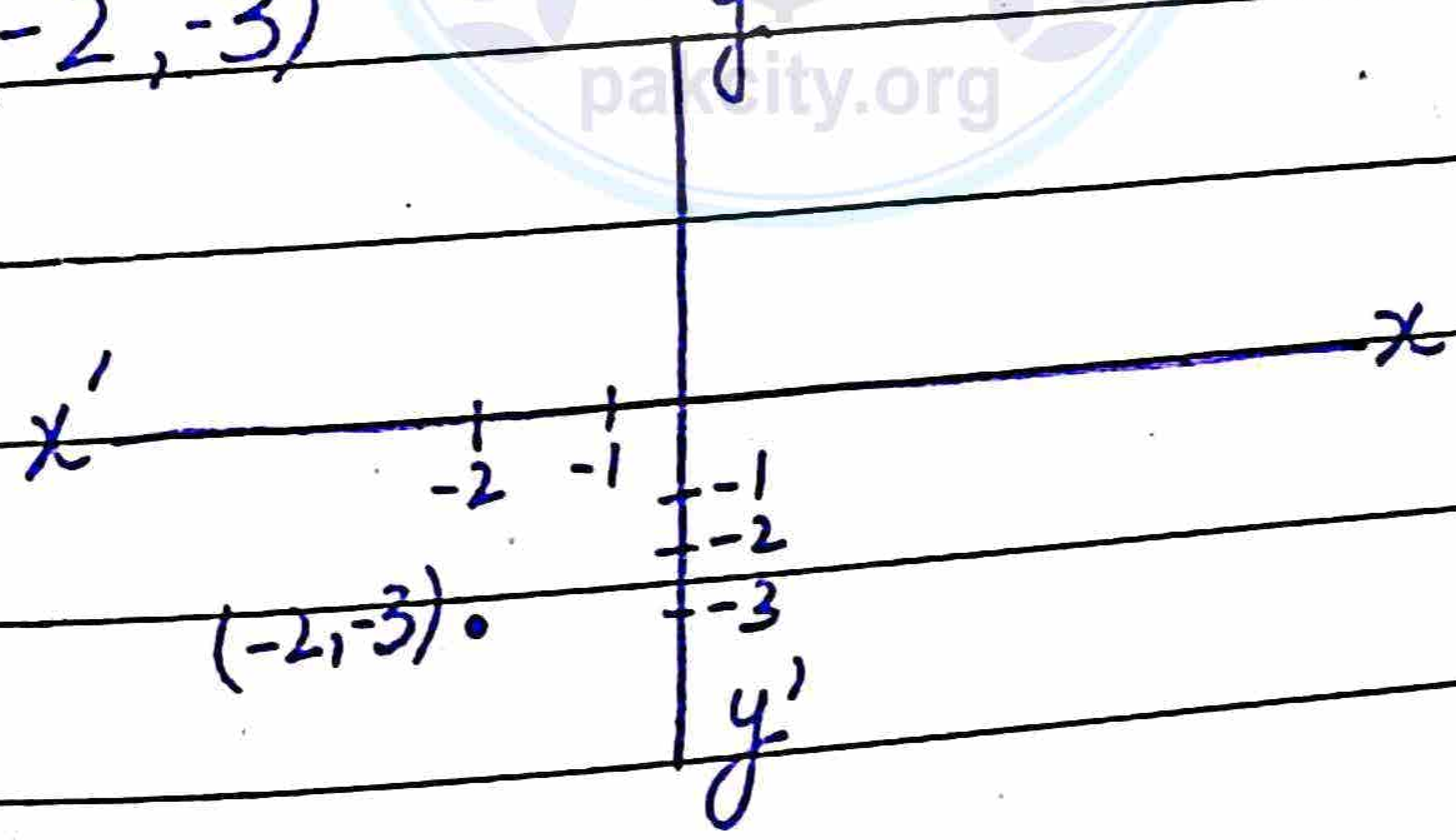

(iii) $-2+3i$

$= (-2+3)$



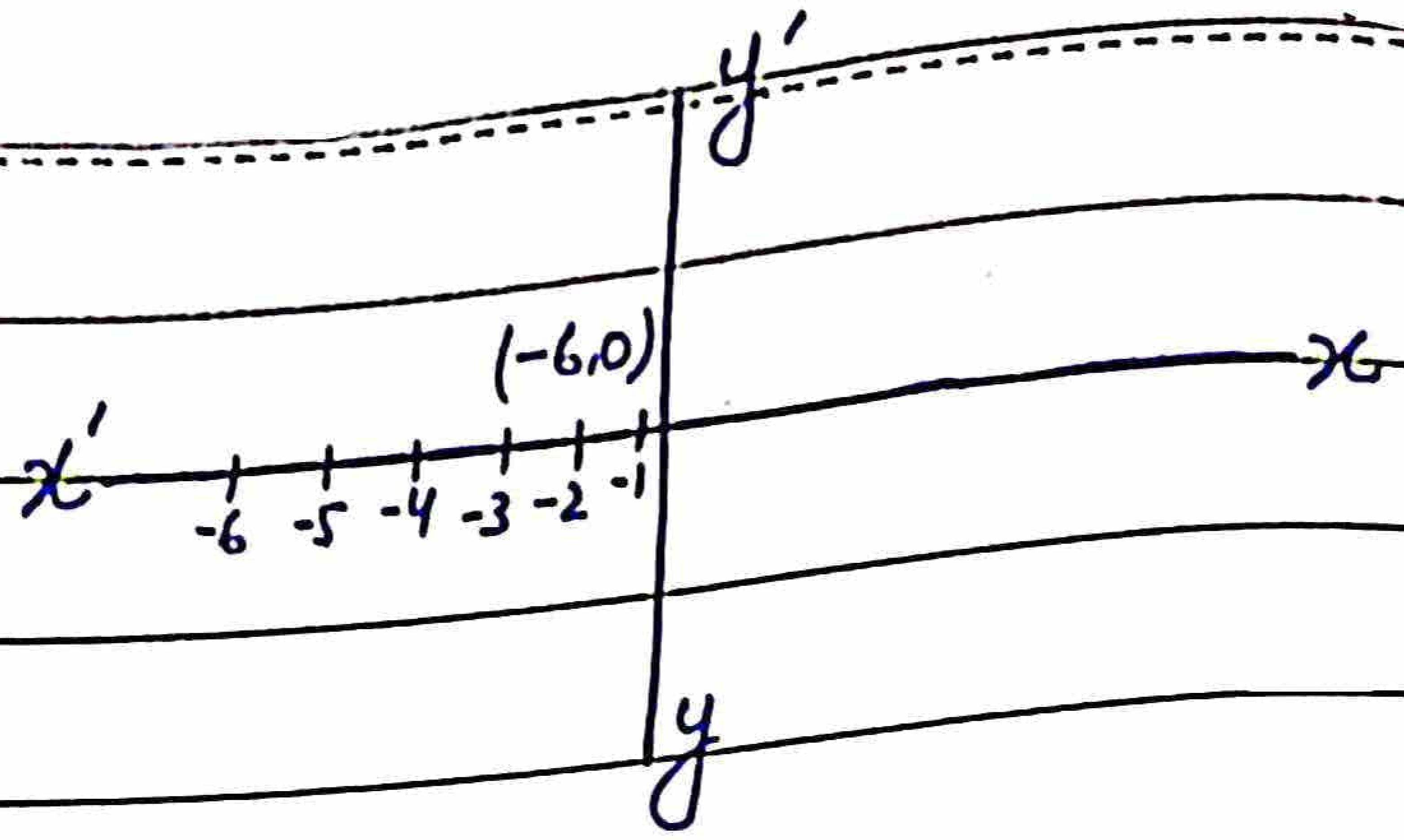

(iv) $-2-3i$

$= (-2,-3)$



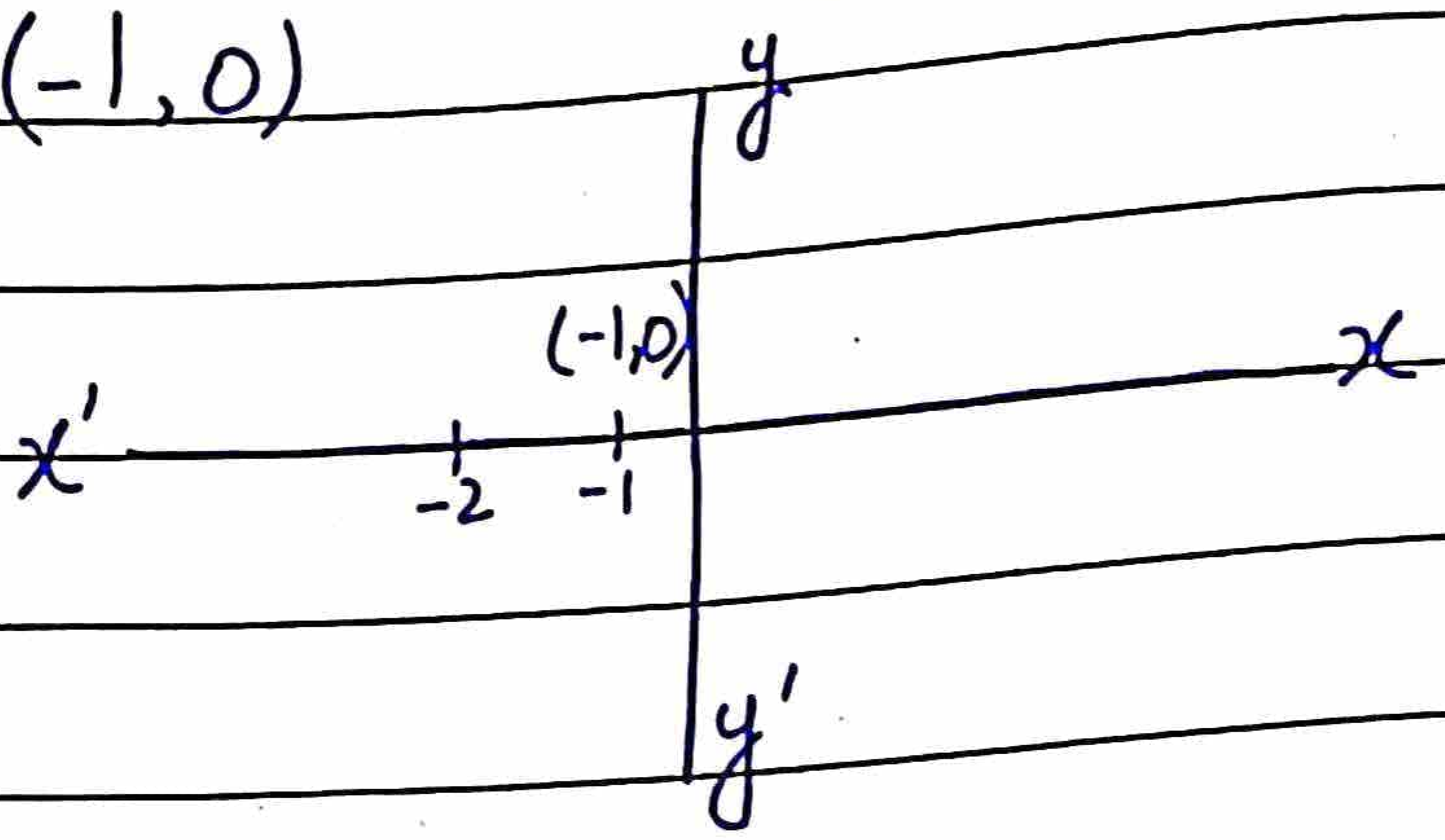

(v) $-6$

$= (-6,0)$




(vi) $i$

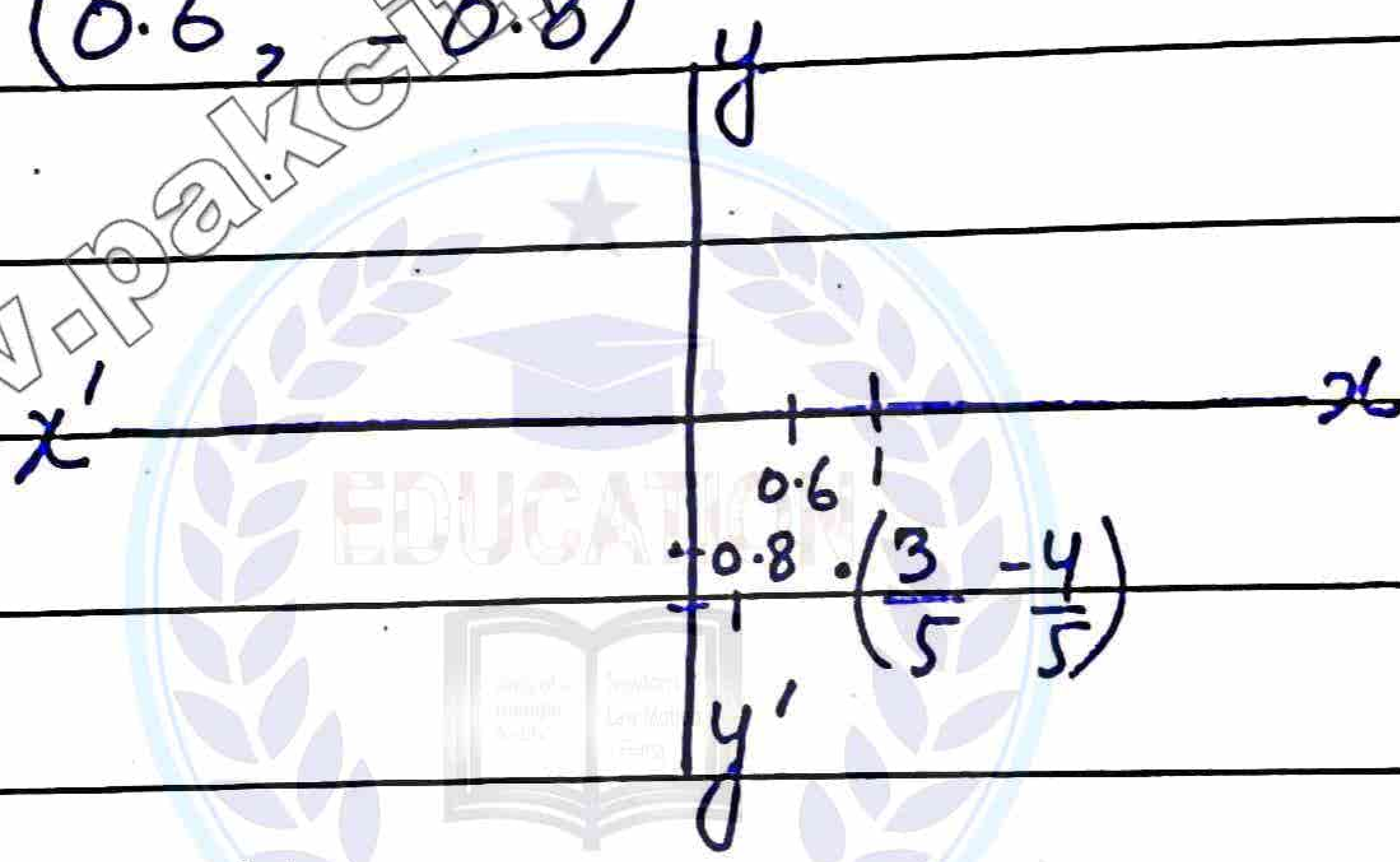
$= (-1, 0)$




(vii) $\frac{3}{5} - \frac{4}{5}i$

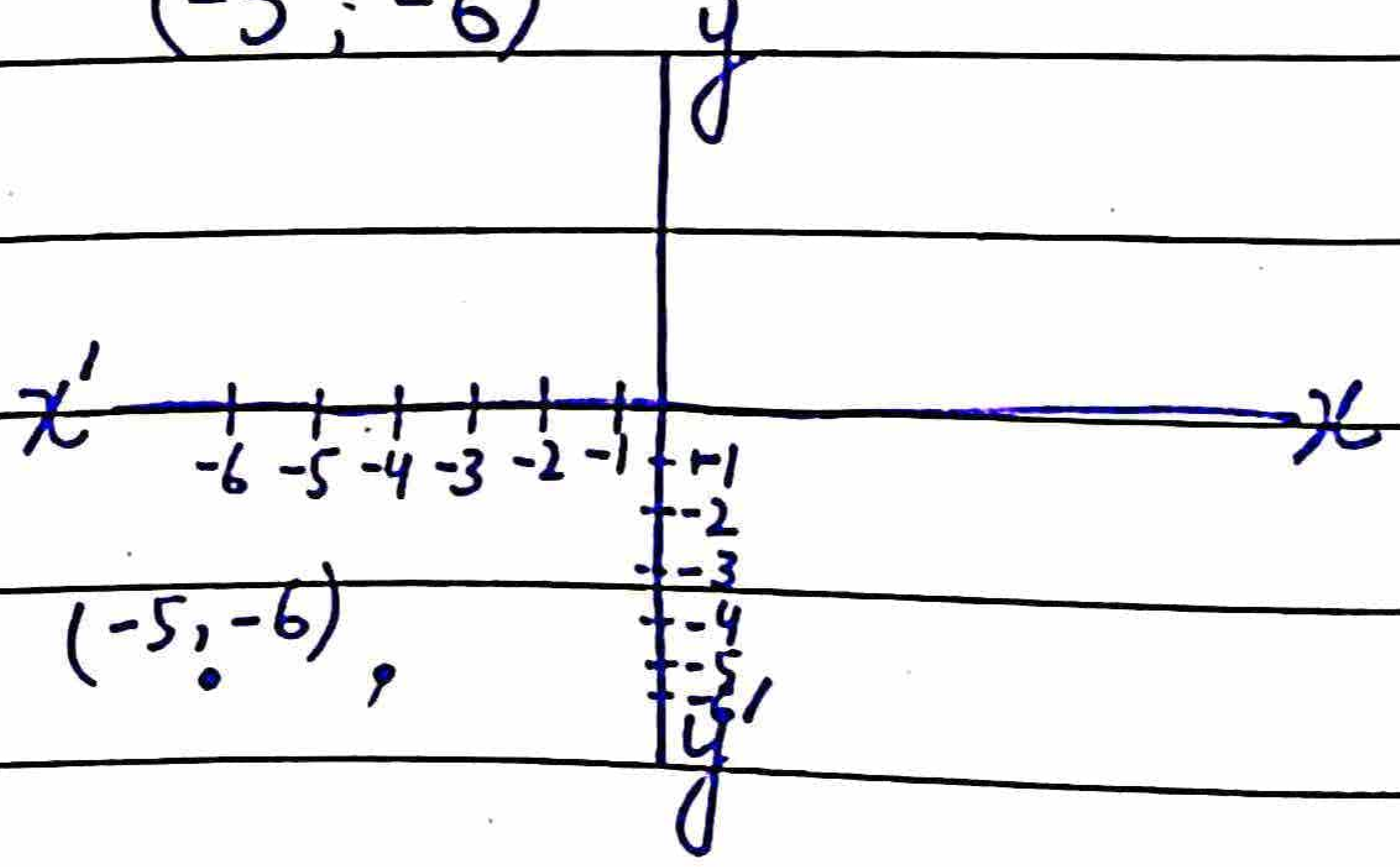
$= \left(\frac{3}{5}, -\frac{4}{5}\right)$

$= (0.6, -0.8)$




(viii) $-5, -6i$

$= (-5, -6)$



Q.NO.2:-



Find the multiplicative inverse.

(i) $-3i$

$= (0, -3) \quad a = 0 ; \ b = -3$

$= \left(\dfrac{a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$

$= \left(\dfrac{0}{(0)^2+(-3)^2}, \dfrac{-(-3)}{(0)^2+(-3)^2}\right)$

$= \left(0, \dfrac{3}{9}\right)$

(ii) $1-2i$

$= (1, -2) \quad a = 1 ; \ b = -2$

$= \left(\dfrac{a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$

$= \left(\dfrac{1}{(1)^2+(-2)^2}, \dfrac{-(-2)}{(1)^2+(-2)^2}\right)$

$= \left(\dfrac{1}{1+4}, \dfrac{2}{1+4}\right)$

$= \left(\dfrac{1}{5}, \dfrac{2}{5}\right)$

(iii) $-3-5i$

$= (-3, -5) \quad a = -3 ; \ b = -5$

$$= \left(\frac{a^2}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

$$= \left(\frac{-3}{(-3)^2+(-5)^2}, \frac{-(-5)}{(-3)^2+(-5)^2}\right)$$

$$= \left(\frac{-3}{9+25}, \frac{5}{9+25}\right)$$

$$= \left(\frac{-3}{34}, \frac{5}{34}\right)$$

(iv) $(1, 2)$

$a = 1 \;;\; b = 2$

$$= \left(\frac{a^2}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

$$= \left[\frac{1}{(1)^2+(2)^2}, \frac{-2}{(1)^2+(2)^2}\right]$$

$$= \left(\frac{1}{1+4}, \frac{-2}{1+4}\right)$$

$$= \frac{1}{5}, \frac{-2}{5}$$

Q.NO.3:-

Simplify.

(i) $i^{101}$

$= i^{100} \cdot i$

$= (i^2)^{50} \cdot i$

$= (-1)^{50} \cdot i$ $\quad \because i^2 = -1$

$= (1) \cdot i$

$= i$

(ii) $(-ai)^4$

$= a^4 \cdot i^4$

$= a^4 \cdot (i^2)^2$

$= a^4 \cdot (-1)^2$ $\quad \because i^2 = -1$

$= a^4$

(iii) $i^{-3}$

$= \frac{1}{i^3}$

$= \frac{1}{i^2 \cdot i}$

$= \frac{1}{(-1) \cdot i}$ $\quad \because i^2 = -1$

$= \frac{1}{-i} \Rightarrow = i$

(iv) $i^{-10}$

$= \frac{1}{i^{10}}$

$= \frac{1}{(i^2)^5}$

$= \frac{1}{(-1)^5}$ $\quad \because i^2 = -1$

$= \dfrac{1}{-1}$

$= -1$

Q.NO.4:-

Prove that $\bar{z} = z$ iff $z$ is real.

Condition -1

$\bar{z} = z$

let $z = a+bi$ , $a,b \in \mathbb{R}$

$\bar{z} = a-bi$

$a-bi = a+bi$

$0 = \cancel{a}+bi-\cancel{a}+bi$

$0 = 2bi$

$\boxed{0 = b}$

then $z = a+(0)i$

$\boxed{z = a}$

Hence $z$ is real.

Condition -2

$z = a+(0)i = a$

$\bar{z} = a-(0)i = a$

So, $\bar{z} = z$

Q.NO.5:-

Simplify by expressing in the form $a+bi$.

(i) $5+2\sqrt{-4}$

$= 5+2\sqrt{-1\times4}$

$= 5+2\sqrt{4}i$

$= 5+2(2i)$

$= 5+4i$

(ii) $(2+\sqrt{-3})(3+\sqrt{-3})$

$= (2+\sqrt{3}i)(3+\sqrt{3}i)$

$= 6+2\sqrt{3}i+3\sqrt{3}i+(\sqrt{3}i)^2$

$= 6+5\sqrt{3}i-3$

$= 3+5\sqrt{3}i$

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{-8}}$

$= \dfrac{2}{\sqrt{5}+\sqrt{-8}i} \times \dfrac{\sqrt{5}-\sqrt{8}i}{\sqrt{5}-\sqrt{8}i}$

$= \dfrac{2(\sqrt{5}-\sqrt{8}i)}{(\sqrt{5})^2-(\sqrt{8}i)^2}$

$= \dfrac{2(\sqrt{5}-\sqrt{8}i)}{5+8}$

$= \dfrac{2(\sqrt{5}-\sqrt{8}i)}{13} \Rightarrow = \dfrac{2(\sqrt{5}-2\sqrt{2}i)}{13}$

$= \dfrac{2\sqrt{5}-4\sqrt{2}i}{13}$

$= \dfrac{2\sqrt{5}}{13} - \dfrac{4\sqrt{2}}{13}i$

(iv) $\dfrac{3}{\sqrt{6}-\sqrt{-12}}$

$= \dfrac{3}{\sqrt{6}-\sqrt{12}i} \times \dfrac{\sqrt{6}+\sqrt{12}i}{\sqrt{6}+\sqrt{12}i}$

$= \dfrac{3(\sqrt{6}+\sqrt{12}i)}{(\sqrt{6})^2-(\sqrt{12}i)^2}$

$= \dfrac{3(\sqrt{6}+\sqrt{12}i)}{\cancel{(\sqrt{6})^2}\, 6+12}$

$= \dfrac{\cancel{3}(\sqrt{6}+\sqrt{12}i)}{\cancel{18}\,6}$

$= \dfrac{\sqrt{6}+\sqrt{4\times3}i}{6}$

$= \dfrac{\sqrt{6}+2\sqrt{3}i}{6}$

$= \dfrac{\sqrt{6}}{6} + \dfrac{\cancel{2}\sqrt{3}}{\cancel{6}\,3}i$

$= \dfrac{\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}\times\sqrt{6}} + \dfrac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\times\sqrt{3}}i$

$= \dfrac{1}{\sqrt{6}} + \dfrac{i}{\sqrt{3}}$

Q. NO. 6:-

Show that $\forall z \in C$

(i) $z^2+\bar{z}^2$ is a real number.

Let

$z = a+bi$, $a,b \in \mathbb{R}$

$\bar{z} = a-bi$

$z^2 + \bar{z}^2 = (a+bi)^2 + (a-bi)^2$

$= a^2+b^2i^2+\cancel{2abi}+a^2+b^2i^2-\cancel{2abi}$

$= 2a^2 + 2b^2i^2$

$= 2a^2 - 2b^2 \quad \because i^2 = -1$

$\therefore z^2 + \bar{z}$ is real numbers.

(ii) $(z-\bar{z})^2$ is a real number.

Let

$z = a+bi$, $a,b \in \mathbb{R}$

$\bar{z} = a-bi$

$z - \bar{z} = (a+bi)-(a-bi)$

$= \cancel{a}+bi-\cancel{a}+bi$

$= 2bi$

$(z-\bar{z})^2 = (2bi)^2$

$= 4b^2i^2 \quad \because i^2 = -1$

$= -4b^2$

Hence, $(z-\bar{z})^2$ is a real number.

Q.NO.78-

Simplify the following.

(i). $(a+bi)^2$

$= a^2 + b^2i^2 + 2abi$

$= (a^2-b^2) + 2abi$

(v) $(a+bi)^{-2}$

$= \dfrac{1}{(a+bi)^2}$

$= \dfrac{1}{a^2+b^2i^2+2abi}$

$= \dfrac{1}{(a^2-b^2)+2abi} \times \dfrac{(a^2-b^2)-2abi}{(a^2-b^2)-2abi}$

$= \dfrac{(a^2-b^2)-2abi}{(a^2-b^2)-(2abi)^2}$

$= \dfrac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2+4a^2b^2}$

$= \dfrac{(a^2-b^2)-2abi}{a^4+b^4+2a^2b^2}$

$= \dfrac{(a^2-b^2)-2abi}{(a^2+b^2)^2}$

$= \dfrac{a^2-b^2}{(a^2+b^2)^2} - \dfrac{2ab}{(a^2+b^2)^2}$

(viii) $(a-bi)^3$

$= a^3-b^3i^3-3a^2bi+3ab^2i^2$ $\quad \therefore i^2=-1$

$= a^3-b^3 \cdot i^2 \cdot i-3a^2bi-3ab^2$

$= a^3-3ab^2+b^3i-3a^2bi$

$= (a^3-3ab^2)+(b^3-3a^2b)i$

(ii) $\left(\frac{-1}{2}+\frac{\sqrt{3}i}{2}\right)^3$

$$= \left(\frac{-1}{2}\right)^3 + \left(\frac{\sqrt{3}i}{2}\right)^3 + 3\left(\frac{-1}{2}\right)^2\left(\frac{\sqrt{3}}{2}i\right) + 3\left(\frac{-1}{2}\right)\left(\frac{\sqrt{3}i}{2}\right)^2$$

$$= \frac{-1}{8} + \frac{3^{\frac{3}{2}}}{8}\cdot i^2\cdot i + \frac{3^{\frac{3}{2}}}{8}i - \frac{9}{8}i^2$$

$$= \frac{-1}{8} - \cancel{\frac{3^{\frac{3}{2}}}{8}i} + \cancel{\frac{3^{\frac{3}{2}}}{8}}i + \frac{9}{8}$$

$$= \frac{-1}{8} + \frac{9}{8}$$

$$= \frac{-1+9}{8}$$

$$= \frac{\cancel{8}^1}{\cancel{8}_1} \Rightarrow = 1$$



(iii) $\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

$$= \left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2+1}$$

$$= \left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1}$$

$$= \frac{1}{\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)} \times \frac{\frac{-1}{2}+\frac{\sqrt{3}}{2}i}{\frac{-1}{2}+\frac{\sqrt{3}}{2}i}$$

$$= \frac{\frac{-1}{2}+\frac{\sqrt{3}}{2}i}{\left(\frac{-1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2}$$

$$= \frac{\frac{-1}{2} + \frac{\sqrt{3}}{2} i}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{-1}{2} + \frac{\sqrt{3}}{2} i}{\frac{1+3}{4}}$$

$$= \frac{\frac{-1}{2} + \frac{\sqrt{3}}{2} i}{\cancel{\frac{4}{4}}}$$

$$= \frac{-1}{2} + \frac{\sqrt{3}}{2} i$$

(ii) $\left(\frac{-1}{2} - \frac{\sqrt{3}}{2} i\right)^3$

$$= \left(\frac{-1}{2}\right)^3 + \left(\frac{-\sqrt{3}}{2}\right)^3 + 3\left(\frac{-1}{2}\right)^2\left(\frac{-\sqrt{3}i}{2}\right) + 3\left(\frac{-1}{2}\right)\left(\frac{-\sqrt{3}i}{2}\right)^2$$

$$= \frac{-1}{8} - \frac{3^{\frac{3}{2}}}{8} \cdot i^2 \cdot i - \frac{3^{\frac{3}{2}}}{8} i - \frac{9}{8} i^2$$

$$= \frac{-1}{8} + \frac{3^{\frac{3}{2}} i}{8} - \frac{3^{\frac{3}{2}} i}{8} + \frac{9}{8}$$

$$= \frac{-1}{8} + \frac{9}{8}$$

$$= \frac{-1+9}{8}$$

$$= \frac{\cancel{8}}{\cancel{8}}$$

$$= 1$$

(viii) $(3-\sqrt{-4})^{-3}$



$= (3-2i)^{-3}$

$= \dfrac{1}{(3-2i)^3}$

$= \dfrac{1}{(3)^3+(-2i)^3+3(3)^2(-2i)+3(3)(-2i)^2}$

$= \dfrac{1}{27-8i^2\cdot i-54i+36i^2}$

$= \dfrac{1}{27+8i-54i-36}$

$= \dfrac{1}{-9-46i} \times \dfrac{-9+46i}{-9+46i}$

$= \dfrac{-9+46i}{(-9)^2-(46i)^2}$

$= \dfrac{-9+46i}{81+2116}$

$= \dfrac{-9+46i}{2197}$

$= \dfrac{-9}{2197} + \dfrac{46i}{2197}$

(vi) $(a+bi)^3$

$= a^3+b^3i^3+3a^2bi+3ab^2i^2 \quad \therefore i^2=-1$

$= a^3+b^3\cdot i^2\cdot i+3a^2bi-3ab^2$

$= a^3-b^3i+3a^2bi-3ab^2$

$= (a^3 - 3ab^2) - (b^3 - 3ab)i$

# IMPOTANT NOTES

→ It represents the complex number $x+yi$. The real number $\sqrt{x^2+y^2}$ is called the modulus of the complex number $a+ib$.

$$\therefore \overline{OM} = x, \quad \overline{MA} = y$$

$$|\overline{OA}|^2 = |\overline{OM}|^2 + |\overline{MA}|^2$$

$$\therefore |\overline{OA}| = \sqrt{x^2+y^2}$$

→ The modulus of a complex number is the distance from the origin of the point representing the number.

→ **De Moivre's Theorem:-**

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,$$

$\forall n \in Z$.

→ **Polar form of Complex Number**

Consider adjoining diagram representing the complex number $z = x+iy$. From the diagram, we see that $x = r\cos\theta$ and $y = r\sin\theta$, where $r = |z|$ and $\theta$ is called argument of $z$.

$$z = x + iy$$

$$x = r\cos\theta \rightarrow (i)$$

$$y = r\sin\theta \rightarrow (ii)$$

$$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$$

$$\sqrt{x^2+y^2} = \sqrt{r^2}$$

$$\sqrt{x^2+y^2} = r$$



Dividing (ii) from (i)

$$\frac{y}{x} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}$$

$$\frac{y}{x} = \tan\theta$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \theta$$

→ The graph of linear equation is always straight line.

→ If a point A of the coordinate plane corresponds to the ordered pair (a,b) then a, b are called the coordinates of A. a is called the x-coordinate or abscissa and b is called the y-coordinate or ordinate.

→ In this representation x-axis is called the real-axis and y-axis is called the imaginary axis. The coordinate plane itself is called the complex-plane or z-plane.

→ The members of a cartesian product are ordered pairs. The cartesian product $\mathbb{R} \times \mathbb{R}$ where $\mathbb{R}$ is the set of real numbers is called the cartesian plane.

→ Every real number is a complex number with 0 as its imaginary part.

## Theorems

(i) $|-z| = |z| = |\bar{z}| = |-\bar{z}|$

let $z = a+bi$

$\bar{z} = a-bi$

$-z = -a-bi$

$-\bar{z} = -a+bi$

$|-z| = \sqrt{(-a)^2+(-b)^2}$

$|-z| = \sqrt{a^2+b^2}$

$|z| = \sqrt{(a)^2+(b)^2}$

$|z| = \sqrt{a^2+b^2}$

$|\bar{z}| = \sqrt{(-a)^2+(-b)^2}$

$|\bar{z}| = \sqrt{a^2+b^2}$

$|-\bar{z}| = \sqrt{(-a)^2+(b)^2}$

$|-\bar{z}| = \sqrt{a^2+b^2}$

$\therefore$ Hence proved



(ii) $\bar{\bar{z}} = z$

let $z = a+bi$

$\bar{z} = a-bi$

Taking again conjugate on both sides

$\bar{\bar{z}} = a+bi$

$\therefore$ Hence proved

(iii) $z\bar{z} = |z|^2$

L.H.S $z = a+bi$

$\bar{z} = a-bi$

$z\bar{z} = (a+bi)(a-bi)$

$z\bar{z} = a^2 - b^2 i^2 \qquad \because i^2 = -1$

$z\bar{z} = a^2 + b^2$

R.H.S $|z|^2 = (\sqrt{(a)^2 + (b)^2})^2$

$|z|^2 = a^2 + b^2$

$\therefore$ Hence proved

(iv) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

L.H.S $z_1 = a+bi$

$\bar{z}_1 = a-bi$

$z_2 = c+di$

$\bar{z}_2 = c-di$

$z_1 + z_2 = a+bi + c+di$

$= (a+c) + (b+d)i$

$\overline{z_1 + z_2} = (a+c) - (b+d)i$

R.H.S $\bar{z}_1 + \bar{z}_2 = a - bi + c - di$

$= (a+c) - (b+d)i$

∴ Hence proved

(v) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ , $z_2 \neq 0$

L.H.S let $z_1 = a+bi$

$z_2 = c+di$

$z_1 = a-bi$

$z_2 = c-di$

$\frac{z_1}{z_2} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$

$= \frac{(a+bi)(c-di)}{(c)^2-(di)^2}$

$= \frac{ac-adi+cbi-bdi^2}{c^2-d^2i^2}$ ∴ $i^2=-1$

$= \frac{ac-adi+cbi+bd}{c^2+d^2}$

$= \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

$= \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$

$\overline{\left(\frac{z_1}{z_2}\right)} = \overline{\frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i}$

$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}i$$



vi) ~~$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$~~

R.H.S

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{a-bi}{c-di} \times \frac{c+di}{c+di}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{(a-bi)(c+di)}{(c)^2-(di)^2}$$

$$= \frac{ac+adi-bci-bdi^2}{c^2-d^2i^2} \quad \because i^2=-1$$

$$= \frac{ac+adi-bci+bd}{c^2+d^2}$$

$$= \frac{(ac+bd)-(bc-ad)i}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}i$$



(vi) $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

let $z_1 = a+bi$

$z_2 = c+di$

$$|z_1 \cdot z_2| = |(a+bi)(c+di)|$$

$$= |(ac+adi+bci+bdi^2)| \quad \because i^2=-1$$

$= |ac + adi + bci - bd|$

$= |(ac - bd) + (ad + bc)i|$

$= \sqrt{(ac-bd)^2 + (ad+bc)^2}$

$= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}$

$= \sqrt{(a^2+b^2)(c^2+d^2)}$

R.H.S

$|z_1| \cdot |z_2| = |a+bi| \cdot |c+di|$

$= \sqrt{(a)^2+(b)^2} \cdot \sqrt{(c)^2+(d)^2}$

$= \sqrt{a^2+b^2} \cdot \sqrt{c^2+d^2}$

$= \sqrt{(a^2+b^2)(c^2+d^2)}$

$\therefore$ Hence proved