Multiple Choice Questions
Chapter 1

1. If n is a prime, then $\sqrt{n}$ is :
   a) Rational number b) whole number c) natural number d) irrational number

2. The additive identity of real number is :
   a) 0 b) 1 c) 2 d) 3

3. The property $\forall\ a \in R; a = a$ is called :
   a) Reflexive b) symmetric c) transitive d) commutative

4. For $a, b \in R, a > b\ or\ a = b\ or\ a < b$ is the :
   a) Trichotomy property of real numbers
   b) Left distributive property of real numbers
   c) Right distributive property of real numbers
   d) Cancellation property of real numbers

5. Transitive property of order of the real numbers is that $\forall\ a, b, c\ \in R$
   a) $a < b\ \wedge b < c \Rightarrow a < c$ b) $a < b\ \wedge b < c \Rightarrow a = c$
   $c)$ $a < b\ \wedge b < c \Rightarrow a \geq c$ d) $a < b\ \wedge b < c \Rightarrow a > c$

6. Trichotomy is property of :
   a) division b) inequality c) equality d) subtraction

7. $\forall\ a, b\ \in R, a = b \Longrightarrow b = a,$ this property is called :
   a) Transitive b) symmetric c) reflexive d) additive

8. Which of the following sets has closure property with respect to multiplication?
   a) {-1,1} b) {-1} c) {-1,0} d) {0,2}

9. $(-\mathrm{i})^{19}$ is equal to :
   a) $-\mathrm{i}$ b) i c) 1 d) -1

10. Any real number a= :
   $a)$ ia b) $(0, a)$ c) $(a, 0)$ d) $(a, 1)$

11. Multiplicative inverse of (a, b) is:

a) $(\frac{1}{a}, \frac{1}{b})$ b) $(\frac{a}{a^2+b^2}, \frac{b}{a^2+b^2})$ c) $(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2})$ d) $(\frac{1}{a^2+b^2}, \frac{b}{a^2+b^2})$

12. Conjugate of $-2+3i$ is:

a) $-2-3i$ $b)$ $2-3i$ $c)$ $2+3i$ $d)$ $-2+3i$

13. If $z = a + ib, then$ $|\bar{z}| =$:

a) $\sqrt{a^2-b^2}$ b) $\sqrt{a^2-(ib^2)}$ c) a^2+b^2 d) $\sqrt{a^2+b^2}$

14. $|z^2|$=

a) z^2 $b)$ $z\bar{z}$ $c)$ $\bar{z}^2$ $d)$ z

15. $(-1)^{\frac{-21}{2}}$ is equal to :

a) $-i$ $b)$ i $c)$ 1 $d)$ -1

16. The polar form of a complex number is:

a) $r(tan\theta + icot\theta)$ $b)$ $r(sec\theta + icsc\theta)$

c) $r(cos\theta + i\,sin\theta)$ $d)$ $r(sin\theta + i\,cos\theta)$

17. $\forall\, n \in z$, $(cos\theta + i\,sin\theta)^n =$

a) $\csc n\,\theta +\ i\sec n\theta$ $b)$ $\tan n\,\theta +\ i\cot n\theta$

b) $\csc n\,\theta -\ isinn\theta$ d) $cosn\theta +\ isinn\theta$

18. The set of negative integers is closed with respect to :

a) Addition b) Multiplication c) Subtraction d) None of these

b) For all $x \in R, x = x$, this property is called:

a) Reflexive property b) Symmetric property

a) Transitive property d) Trichotomy property

c) $\sqrt{-1}$ belongs to the set of :

a) Real numbers b) Complex numbers

c) Prime numbers d) odd numbers

21. $z = (a, b)$, then $z^{-1} =$

a) $(\frac{1}{a}, \frac{1}{b})$ b) (-a,-b) c) $(\frac{a}{a^2+b^2}, \frac{b}{a^2+b^2})$ d) $(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2})$

22. Let x, y $\in R$ then $x + iy$ is purely imaginary if :
    a) $x \neq 0, y = 0$ b) $x = 0, y = 0$ c) $x = 0, y \neq 0$ d) $x \neq 0,\ y \neq 0$

23. Product of a complex number and its conjugate is:
    a) a real number b) irrational number c) a complex number d) none of these

24. Conjugate of a complex number $(-a, -b)$ is:
    a) $(-a, b)$ b) $(-a, -b)$ c) $(a, -b)$ d) none of these

25. The additive inverse of a real number a is :
    a) 0 b) -a c) a d) $\frac{1}{a}$

26. The set of all rational numbers between 2,3 is :
    a) an empty set b) an infinite set c) a finite set d) a power set

27. The multiplicative identity of real numbers is
    a) 0 b) 1 c) 2 d) -1

28. $z\bar{z} =$:
    a) 0 b) 1 c) $|z|^2$ d) none of these

29. Modulus of $15i + 20$ is :
    a) 20 b) 15 c) 25 d) none of these

30. $\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$ in Cartesian form is :
    a) 0 b) 1 c) i d) $-i$

1. a
2. a
3. a
4. a
5. a
6. b
7. b
8. a
9. b
10. c
11. c
12. a
13. d
14. b
15. a
16. c
17. d
18. a
19. a
20. b
21. d
22. c
23. a
24. a
25. b
26. b
27. b
28. c
29. c
30. c

Ch#1

(Number Systems)

Short Questions

1. Define rational and irrational numbers
2. Define the following properties of the real numbers
   a) Closure property w.r.t addition
   b) Associative property w.r.t addition
3. Define complex number and conjugate of a complex number.
4. Prove that the sum as well as the product of any complex number and its conjugate is a real number.
5. Define modulus of a complex number.
6. What is the polar form of a complex number.
7. Simplify the following:
   a) $-i^{19}$ b) $-1^{\frac{-21}{2}}$
8. Simplify the following
   a) $(7,9)+(3,5)$
   b) $(8,-5)-(-7,4)$
   c) $(2,6).(3,7)$
   d) $(2,6)\div(3,7)$
9. Find the multiplicative inverse of each of the following numbers
   a) $(-4,7)$
   b) $(\sqrt{2},-\sqrt{5})$
   c) $(1,0)$
10. Factorize the following:
    a) a^2+4b^2
    b) $9a^2+16b^2$
    c) $3x^2+3y^2$
11. Separate into real and imaginary parts
    a) $\frac{2-7i}{4+5i}$
    b) $\frac{(-2+3i)^2}{(1+i)}$
12. Show that $\forall z_{1,}z_2\in C, \overline{z_1z_2}=\overline{z}_1\overline{z}_2$
13. Express the complex number $1+i\sqrt{3}$ in polar form.
14. Find real and imaginary part of $(\sqrt{3+i})^3$
15. Show that $\forall\, Z\in C, Z^2+(\overline{z})^2$
16. Show that $(z-\overline{z})^2$ is a real number for all $z\in C$.

Important Definitions

Chapter #1

Q1. Define Rational and Irrational Numbers.

Ans : A rational number is a number which can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$

An Irrational number is a number which cannot be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$

Q2. State the commutative property of addition of real numbers.

Ans: $a + b = b + a, \forall a, b \in R$

Q3. State the closure property of multiplication of real numbers.

Ans: $\forall\, a, b \in R, a.b \in R$ (a.b is usually written as ab)

Q4. What is the Trichotomy property of the real numbers?

Ans: If a and b are two real numbers, then exactly one of the following holds:

$$a > b \text{ or } a = b \text{ or } a < b \qquad \forall\, a, b \in R$$

Q5. Write any two properties of inequalities.

Ans: Transitive Property: $a > b \wedge b > c \Rightarrow a > c \qquad \forall a, b, c \in R$

Additive Property: $a > b \Longrightarrow a + c > b + c \qquad \forall\, a, b, c \in R$

Q6. Define a Complex Number.

Ans: We can define complex numbers also by using ordered pairs.

Let C be the set of ordered pairs belonging to $R \times R$ which are subject to the following properties:

i. $(a, b) = (c, d) \Leftrightarrow a = c \wedge b = d$.
ii. $(a, b) + (c, d) = (a + c, b + d)$
iii. If k is any real number, then k(a,b)=(ka,kb)
iv. (a,b)(c,d)=(ac-bd,ad+bc) .Then C is called the set of Complex numbers

Important MCQs, SHORT & LONG QUESTIONS

CHAPTER # 9

Q.1 **Multiple Choice Questions**

i) π radians

a) $360°$ b) $360'$ c) $180°$ d) $45°$

ii) $\frac{\pi}{2}$ radian is an angle

a) Acute b) Obtuse c) Straight d) Quadrantal

iii) The vertex of an angle in standard form is at

a) (1,0) b) (0,1) c) (1,1) d) (0,0)

iv) If $cot\theta > 0$ and $sin\theta < 0$, then terminal arc of angle lies in quadrant.

a) I b) II c) III d) IV

v) $\theta°$ is measured in

a) Circular System b) Sexagesimal System c) MKS System d) CGS System

vi) $1 + cot^2\theta =$

a) $sec^2\theta$ b) $\frac{1}{sin^2\theta}$ c) $tan^2\theta$ d) $\frac{1}{sec^2\theta}$

vii) If $Sin\theta = \frac{\sqrt{3}}{2}$ then θ is

(a) $60°$ b) $30°$ c) $90°$ d) none of these

viii) $105° =$

(a) $\frac{7\pi}{12}$ (b) $\frac{2\pi}{3}$ (c) $\frac{5\pi}{12}$ (d) $\frac{5\pi}{6}$

ix) $\sec\theta \csc\theta \sin\theta \cos\theta =$

(a) 1 (b) 0 (c) $\sin\theta$ (d) $\cos\theta$

x) $\cot 45° =$

(a) 1 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 1

xi) $\cot^2\theta - \csc^2\theta =$

(a) 0 (b) 1 (c) -1 (d)2

xii) Value of $sin60°\ is$

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c)$2\sqrt{3}$ (d)None of these

xiii) Which one is true?

a) 1radian$< 1°$ b) 1radian$> 1°$ c) 1radian$= 1°$ d) 5radian$= 2°$

Answer key: **i) a** **ii)d** **iii)d** **iv)c** **v)b** **vi)b** **vii)a** **viii)a** **ix)a** **x)a** **xi)c** **xii)b** **xiii)b**

SHORT QUESTIONS

Q.1 Define Radian

Q.2 Prove that $\sin(180+\theta) = -\sin\theta$

Q.3 Convert $\frac{2\pi}{3}$ radian into degree

Q.4 Prove that $\sec\theta cosec\theta sin\theta cos\theta = 1$

Q.5 Show that $2\sin 45° + \frac{1}{2}\csc 45° = \frac{3}{\sqrt{2}}$

Q.6 Find r when l=56cm and $\theta = 45°$

Q.7 If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in first quadrant,

find the value $\cos\theta$ *and* $cosec\theta$.

Q.8 Verify that $sin^2\frac{\pi}{6} + sin^2\frac{\pi}{3} + tan^2\frac{\pi}{4} = 2$

Q.9 If $\tan\theta = -\frac{1}{3}$ and the terminal arm of the angle is in quad II find remaining

trigonometric functions.

Q.10 Find the radius of the circle in which the arms of central angle of measure 1

radian cut off an arc of length 35cm.

Q.11 Verify when $\theta = 30°, 45°$

a) $\cos 2\theta = 2\cos^2\theta - 1$ b) $\sin 2\theta = 2\sin\theta\cos\theta$

Q.12 $\cot^2\theta - \cos^2\theta = \cos^2\theta\cot^2\theta$

Q.13 Verify $cos2\theta = cos^2\theta - sin^2\theta$ when $\theta = 30°, 45°$

Q.14 Prove that $1 + \tan^2\theta = sec^2\theta$.

Q.15 Prove that $1 + cot^2\theta = cosec^2\theta$

Q.16 Show that $\frac{sin\theta}{1+cos\theta} + cot\theta = cosec\theta$

Q.17 Derive the fundamental identity $sin^2\theta + cos^2\theta = 1$

Q.18 Prove that $\frac{1+cos\theta}{1-cos\theta} = (cosec\theta + cot\theta)^2$

Q.19 Find $cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quad I, find the values of remaining trigonometric function

Q.20 Prove that $sin^3\theta + cos^3\theta = (sin\theta + cos\theta)(1 - sin\theta cos\theta)$

Q.21 Find the values of the remaining trigonometric functions:

If $tan\theta = \frac{-1}{\sqrt{2}}$ and the terminal arm of the angle is not in quad III.

Q.22 Prove that $cos^2\theta - sin^2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}$

LONG QUESTIONS

Q.1: If $cosec\theta = \frac{m^2+1}{2m}$ $m > 0,\ 0 < \theta < \frac{\pi}{2}$ find the value of the remaining trigonometric function.

Q.2: If $\cot\theta = \frac{5}{2}$ and terminal arm of the angle is in I quad find the value $\frac{3\sin\theta+4\cos\theta}{\cos\theta-\sin\theta}$

Q.3: Prove that $\sin^2\theta + \cos^2\theta = 1 - 3\sin^2\theta\cos^2\theta$

Q.4: Prove that $\frac{tan\theta+sec\theta-1}{tan\theta-\sec\theta+1} = tan\theta + sec\theta$

Q.5: Prove the identity $\frac{1}{cosec\theta-cot\theta} - \frac{1}{sin\theta} = \frac{1}{\sin\theta} - \frac{1}{cosec\theta+cot\theta}$

Q.6 Prove that $sin^6\theta - cos^6\theta = (sin^2\theta - cos^2\theta)(1 - sin^2\theta cos^2\theta)$

Q.7 If $tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in the III quadrant, find the value of $\frac{csc^2\theta-sec^2\theta}{csc^2\theta+sec^2\theta} = \frac{3}{4}$.

Q.8 Prove that $(tan\theta + cot\theta)^2 = sec^2\theta csc^2\theta$

Q.9 Prove that $\frac{1-sin\theta}{cos\theta} = \frac{cos\theta}{1+sin\theta}$

Q.10 Find the value of the other five trigonometric functions of θ, if $cos\theta = \frac{12}{13}$ and the terminal side of the angle is not in the I quadrant.

IMPORTANT (SHORT QUESTIONS) FROM Chap #09 – 14 (Sec C)

---

1. Verify $\sin^2 30^0 : \sin^2 45^0 : \sin^2 60^0 : \sin^2 90^0 = 1:2:3:4$
2. Prove that $\sin 2\alpha = 2\sin\alpha\cos\alpha$
3. Prove that $\cos 20^0 + \cos 100^0 + \cos 140^0 = 0$
4. Solve the triangle ABC if $a = 32$, $b = 40$ & $c = 66$
5. Prove that: $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$
6. Convert $16^0\,30'$ to circular measure
7. Solve equation $\sin 2x = \cos x$
8. Show that $r_1 = s\tan\alpha$
9. Prove that $r = (s-b)\tan\frac{\beta}{2}$
10. Prove that $\frac{Sin\ 2\alpha}{1+\cos\alpha} = \tan\alpha$
11. Convert 21.256^0 to D^0 M' S"
12. What is the Circular Measure of the angle between hands of a watch at 5 O´ clock?
13. Express $\sin(x + 45^0)\sin(x - 45^0)$ as sum or difference
14. Prove that: $\tan(\alpha + \beta) + \tan\gamma = 0$
15. If α, β, γ are the angles of ΔABC, prove that $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\ \tan\beta\ \tan\gamma$
16. Express $2\sin 7\theta\cos 3\theta$ as sum & difference.
17. Show that $\dfrac{\sin(360^0 - \theta)\cos(180^0 - \theta)\tan(180^0 + \theta)}{\sin(90^0 - \theta)\ \cos(90^0 - \theta)\ \tan(360^0 + \theta)} = 1$
18. Express $120'\,40''$ in radians
19. Define Radian.
20. Find θ, when $\ell = 10$cm and $r = 2$cm
21. State Fundamental Law of Trigonometry
22. What is the period of $3\cos\frac{x}{5}$?
23. Give the Cosine of Half the Angle in terms of the sides.
24. At the top of the cliff 80 m high, the angle of depression of a boat is 12^0. How far is the boat from the cliff?
25. Find the area of the triangle ABC, given the sides $a = 18$, $b = 24$, $c = 30$
26. Define Circum-Radius.
27. Show that $\cos^{-1}\frac{12}{13} = \sin^{-1}\frac{5}{13}$
28. Show that $\cos^{-1}(-x) = \pi - \cos^{-1}x$
29. Give or state hero's formula.
30. Prove that $\sec\theta\ \csc\theta\ \sin\theta\ \cos\theta = 1$
31. If α, β, γ are the angles of a triangle ABC then prove that $\sin(\alpha+\beta) = \sin\gamma$
32. Find the value of $\cos 15^0$
33. State Fundamental Law of Trigonometry.
34. Prove that $R = \frac{abc}{4\Delta}$

35. Convert $\frac{25\pi}{36}$ into the measure of sexagesimal system.
36. Solve $sinx\ cosx = \frac{\sqrt{3}}{4}$
37. Express sin 5x + sin x as a product .
38. Convert $75^0\ 6'\ 30''$ to radians
39. Write domain & range of cos x
40. Write domain & range of tanx
41. Solve the equation $cot^2\theta = \frac{1}{3}$
42. Prove that $\tan(45^0+A)\tan(45^0 - A) = 1$
43. Show that $(tan\theta + cot\theta)^2 = sec^2\theta\ cosec^2\theta$
44. Define In-circle
45. Find the Period of $\sin\frac{x}{5}$
46. Solve sin x + cos x = 0
47. Prove the identity $1 + \tan\alpha\tan 2\alpha = \sec 2\alpha$
48. Express $\cos 7\theta - \cos\theta$ as a product
49. Define Circum-circle
50. The area of a triangle is 2437. If a = 79, c = 97 then find the angle β
51. Prove that $\sin(\theta + \frac{\pi}{6}) + \cos(\theta + \frac{\pi}{3}) = \cos\theta$
52. Show that $\cos(2\sin^{-1}x) = 1 - 2x^2$
53. If $\cot\theta = \frac{15}{8}$ & the terminal arm of the angle is not in I quad, find the values of cos θ & cosec θ
54. Convert $54°45'$ into radians
55. Prove that $rr_1r_2r_3 = \Delta^2$
56. Express $2\sin 7\theta\sin 2\theta$ as a sum or difference
57. Define the Angle of Elevation
58. Convert $\frac{2\pi}{3}$ into radians
59. Find the solution of the equation $tan^2\theta - \sec\theta - 1 = 0$ which lie in [0, 2π]
60. Show that $\frac{1-sin\theta}{\cos\theta} = \frac{\cos\theta}{1+sin\theta}$
61. A ladder leaning against a vertical wall makes an angle of 24^0 with the wall. If its foot is 5m from the wall, find its length.
62. Express $\cos 12^0 + \cos 48^0$ as a product
63. Find the period of $3\tan\frac{x}{7}$
64. Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$
65. Prove that $\frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°} = \tan 37°$
66. Prove that $\cos 2\theta = \frac{1 - tan^2\theta}{1 + tan^2\theta}$
67. Show that $\frac{\sin\alpha - sin\beta}{\sin\alpha + \sin\beta} = \tan\left(\frac{\alpha-\beta}{2}\right)\cot\left(\frac{\alpha+\beta}{2}\right)$
68. Show that $\frac{\cos(90^0 + \theta)\ \text{Sec}(-\theta)\ \tan(180^0 - \theta)}{\text{Sec}(360^0 - \theta)\ \sin(180^0 + \theta)\ \cot(90^0 - \theta)} = -1$
69. Find the measure of the greatest angle, if sides of triangle are 16, 20, 33
70. The measures of side of a triangular plot are 413, 214 & 375 meters. Find the measure of the corner angles of the plot.
71. Prove that $(r_1 + r_2)\tan\frac{\gamma}{2} = a$
72. Prove that $(r_3 + r)\cot\frac{\gamma}{2} = c$

73. Prove that $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$
74. Solve $4cos^2x - 3 = 0$
75. Solve the trigonometric equation $sec^2\theta = \frac{4}{3}$
76. Verify $sin^2\frac{\pi}{6} : sin^2\frac{\pi}{4} : sin^2\frac{\pi}{3} : sin^2\frac{\pi}{2} = 1:2:3:4$
77. Find x, if $tan^2 45° - cos^2 60° = x \sin 45° \cos 45° \tan 60°$
78. Verify $sin^2\frac{\pi}{6} + sin^2 + Tan^2\frac{\pi}{4} = 2$
79. Prove that $sec^2 A + cosec^2 A = sec^2 A cosec^2 A$
80. Show that $\tan^{-1} A + \cot^{-1} A = \frac{\pi}{2}$
81. Prove that $\frac{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \sec\theta$
82. Prove that $\frac{cosec\ \theta + 2\ cosec\ 2\theta}{\sec\theta} = \cot\frac{\theta}{2}$
83. Prove that $\frac{2\tan\theta}{1 - tan^2\theta} = 2\ sin\theta cos\theta$
84. Prove that $2\tan^{-1} A = \tan^{-1}\frac{2A}{1-A^2}$
85. Prove that $cos^2\theta - sin^2\theta = \frac{1 - tan^2\theta}{1 + tan^2\theta}$
86. Prove that $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\frac{A-B}{1-AB}$
87. Give the range and domain of $\cos^{-1} x$
88. Prove that $\tan^{-1}\frac{120}{119} = 2\ \cos^{-1}\frac{12}{13}$
89. Show that $\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$
90. Prove that $\cos 2\alpha = 2\ cos^2\alpha - 1$
91. Write the Triple Angle Identity of tan 3α
92. Prove that $\sec\theta\ cosec\ \theta \sin\theta \cos\theta = 1$
93. What is the relation between a radian and a degree?
94. Is the relation $l = r\theta°$ valid?
95. Convert $\frac{19\pi}{32}$ into sexagesimal system.
96. What is the Circular Measure of the angle between hands of a watch at 8 O´ clock?
97. A horse is tethered to a peg by a rope of 9 meters length & it can move in a circle with the peg as center. If the horse moves along the circumference of the circle, keeping the rope tight, how far will it have gone when the rope has turned an angle of 55^0?
98. Define Co-terminal Angles
99. Define Allied Angles
100. $\frac{1 + cos\theta}{1 - cos\theta} = (cosec\ \theta + \cot\theta)^2$

LONG QUESTIONS

Question Bank

Chapter 2:

Q.1 Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Q.2 Convert $(A \cup B) \cup C = A \cup (B \cup C)$ into logical form and prove it by constructing the truth table.

Q.3 Give the logical proof of De Morgan's Law.

Q.4 Convert the theorem $(A \cup B)' = A' \cup B'$ to logical statement and prove them by constructing truth tables.

Q.5 Show that the set $\{1, \omega, \omega^2\}, \omega^3 = 1$, is an Abelian group w.r.t ordinary multiplication.

Q.6 Prove that 2x2 non singular matrices over the real field form a non-abelian group under multiplication.

Q.7 Consider the set S={1,-1,i-i}.Set up its multiplication table and show that the set S is an abelian group under multiplication.

Q.8 Give logical proofs of the following theorems

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $(A \cup B)' = A' \cap B'$

Q.9 If a,b are elements of a group G, solve the following equations:

i) $ax = b$ $\quad$ $ii)\ xa = b$

Chapter 3:

Q.1 Find x and y if $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$

Q.2 Solve the following system of linear equations: $3x - 5y = 1$; $-2x + y = -3$

Q.3 Solve the following matrix equation for A: $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A - \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$

Q.4 Show that $\begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix} = 0$

Q.5 Show that $\begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix} = \lambda^2(a+b+c+\lambda)$

Q.6 If A= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ then find A^{-1} by using adjoint of the matrix.

Q.7 Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$

Q.8 Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

Q.9 Without expansion ,verify that $\begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix} = 0$

Q.10 Solve the following systems of linear equations by Cramer's rule.

2x+2y+z=3

3x-2y-2z=1

5x+y-3z=2

Q.11 Use matrices to solve the following systems:

x - 2y + z = -1

3x + y - 2y = 4

y - z = 1

Q.12 Solve the system of linear equations by Cramer's rule.

$$3x_1 + x_2 - x_3 = -4$$
$$x_1 + x_2 - 2x_3 = -4$$
$$-x_1 + 2x_2 - x_3 = 1$$

Q.13 Use matrices to solve the system

$$x_1 - 2x_2 + x_3 = -4$$
$$2x_1 - 3x_2 + 2x_3 = -6$$
$$2x_1 + 2x_2 + x_3 = 5$$

Chapter 4:

Q.1 Solve by factorization $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$; $x \neq \frac{1}{a}, \frac{1}{b}$

Q.2 Solve by quadratic formula $(a+b)x^2 + (a + 2b + c)x + b + c = 0$

Q.3 Solve by quadratic formula

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

Q.4 Solve $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

Q.5 Solve $4.2^{2x+1} - 9.2^x + 1 = 0$

Q.6 Solve $3^{2x-1} - 12.3^x + 81 = 0$

Q.7 Show that $(1+\omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots 2n$ factors=1

Q.8 Find the condition that one root of $ax^2 + bx + c = 0$,$a \neq 0$ is square of the other.

Q.9 $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

Q.10 If the roots of $px^2 + qx + q = 0$ are α and β, prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

Q.11 If α, β are the roots of $5x^2 - x - 2 = 0$ form the equation whose roots are $\frac{3}{\alpha}$ and $\frac{3}{\beta}$.

Q.12 Show that the roots of $x^2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2(1 + m^2)$.

Q.13 Show that the roots of $(mx + c)^2 = 4ax$ will be equal if $c=\frac{a}{m}; m \neq 0$

Q.14 Prove that will have equal roots if $\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}$=1 will have equal roots if $c^2 = a^2m^2 + b^2$; ; $a \neq 0, b \neq 0$

Q.15 Solve $3x + 4y = 25$; $\frac{3}{x}+\frac{4}{y} = 2$

Q.16 Solve the system of equation : $x + y = a + b$ and $\frac{a}{x}+\frac{b}{y} = 2$

Q.17 Prove that sum of three cube roots of unity is zero.

Q.18 Prove that $(-1+\sqrt{-3})^4 + (-1-\sqrt{-3})^4 = -16$

Q.19 If α, β are the roots of $5x^2 - x - 2 = 0$, form the equation whose roots are

$\frac{3}{\alpha}$ *and* $\frac{3}{\beta}$.

Q.20 Solve $x^2 + (y+1)^2 = 18$; $(x+2)^2 + y^2 = 21$

Chapter 6:

Q.1 Find n so that $\frac{a^n+b^n}{a^{n+1}+b^{n+1}}$ may be the A.M. between a and b.

Q.2 The sum of 9 terms of an A.P. is 171 and its eight term is 31. Find the series.

Q.3The sum of three numbers in an A.P. is 24 and their product is 440. Find the numbers.

Q.4 Find the four numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

Q.5 Find three consecutive numbers in G.P. whose sum is 26 and their product is 216.

Q.6 Show that the reciprocals of the terms of the terms of the geometric sequence

$a_1, a_1 r^2, a_1 r^4, ...$ from another geometric sequence.

Q.7 If the sum of the four consecutive terms in G.P. is 80 and A.M. of the second and the fourth of them is 30.Find the terms.

Q.8 If a,b,c,d are in G.P. prove that $a^2 - b^2, b^2 - c^2, c^2 - a^2$ are in G.P.

Q.9 For what value of n, is $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ the positive geometric mean between two distinct numbers a and b?

Q.10 If $y=\frac{x}{2}+\frac{1}{4}x^2+\frac{1}{8}x^3+\cdots$ and if $0<x<2$, then prove that $x=\frac{2y}{1+y}$

Q.11 If $y=\frac{2}{3}x+\frac{4}{9}x^2+\frac{8}{27}x^3+\cdots$ and if $0<x<\frac{3}{2}$, then prove that $x=\frac{3y}{2(1+y)}$

Q.12 Find the five numbers in A.P. whose sum is 25 and sum of whose Squares is 135.

Q.13 If S_2, S_3, S_5 are the sums of 2n,3n,5n terms of an A.P.,show that $S_5=5(S_3-S_2)$

Q.14 Show that the sum of n A.Ms. between a and b is equal to n times their A.M.

Q.15 If $y=1+\frac{x}{2}+\frac{x^2}{4}+\cdots$ then show that $x=2(\frac{y-1}{y})$

Q.16 The sum of an infinite geometric series is 9 and the sum of the squares of its terms is $\frac{81}{5}$. Find the series.

Chapter 7:

Q.1Find the numbers greater than 23000 that can be formed from the digits 1,2,3,5,6 without repeating any digits.

Q.2 How many 6-digit numbers can be formed,without repeating any digit from the digits 0,1,2,3,4,5? In how many of them will 0 be at the tens place?

Q.3 Prove that ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$

Q.4 How many 6-digit numbers can be formed from the digits 2,2,3,3,4,4? How many Of them will lie between 400,000 and 430,000?

Q.5 In how many ways can be letters of the word MISSISSIPPI be arranged when all the letters are to be used?

Q.6 Prove from the first principle that

i) ${}^nP_r=n.{}^{n-1}P_{r-1}$ ii) ${}^nP_r={}^{n-1}P_r+r.{}^{n-1}P_{r-1}$

Q.7 Find the value of n when

i) ${}^{n}P_{2}=30$ ii) ${}^{11}P_{n}=11.10.9$

Q.8 How many numbers greater than 1000,000 can be formed from the digits 0,2,2,2,3,4,4?

Q.9 Find the value of n and r , when ${}^{n}C_{r}=35$ and ${}^{n}P_{r}=210$

Chapter 8:

Q.1 Use mathematical induction to prove that

i) $1+3+5+\cdots+(2n-1)=n^2$

ii) $1+4+7+\cdots+(3n-2)=\frac{n(3n-1)}{2}$

iii) $1+5+9+\ldots+(4n-3)=n(2n-1)$

Q.2 Find the term independent of x in the expansion

i) $(\sqrt{x}+\frac{1}{2x^2})^{10}$ ii) $(x-\frac{2}{x})^{10}$

Q.3 Find the term involving x^4 in the expansion of $(3-2x)^7$

Q.4 Use binomial theorem to show that $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{2.4.6}+\cdots=\sqrt{2}$

Q.5 If x is so small that its square and higher powers can be neglected, then show that

$\frac{1-x}{\sqrt{1+x}}=1-\frac{3}{2}x$

Q.6 If $y=\frac{1}{3}+\frac{1.3}{2!}(\frac{1}{3})^2+\cdots$,then prove that $y^2+2y-2=0$

Q.7 Find the coefficient of x^5 in the expansion of $(x^2-\frac{3}{2x})^{10}$

Q.8 If x is very nearly equal to 1, then prove that $px^p-qx^q=(p-q)x^{p+q}$

Q.9 Find the term involving x^{-2} in the expression of $\left(x-\frac{2}{x^2}\right)^{13}$

Q.10 Determine the middle term or terms in the following expansions $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$

Q.11 If x is so small that its square and higher powers can be neglected, then show that

$$\frac{1+x}{\sqrt{1-x}} = 1 + \frac{3}{2}x$$

Q.12 If $2y = \frac{1}{2^2} + \frac{1.3}{2!}\frac{1}{2^4} + \frac{1.3.5}{3!}\frac{1}{2^6} + \cdots$, then prove that $4y^2 + 4y - 1 = 0$

Chapter 9

Q.1 If $cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in first quadrant, find the value $cos\theta$ *and* $cosec\theta$.

Q.2 If $cosec\theta = \frac{m^2+1}{2m}$ and $0< \theta < \frac{\pi}{2}$ find the value of the remaining trigonometric ratio.

Q.3 Prove the identity $\frac{1}{cosec\theta - cot\theta} - \frac{1}{sin\theta} = \frac{1}{\sin\theta} - \frac{1}{cosec\theta + cot\theta}$

Q.4 Prove that $\frac{tan\theta + sec\theta - 1}{tan\theta + sec\theta + 1} = tan\theta + sec\theta$

Q.5 Prove that $\frac{1-sin\theta}{cos\theta} = \frac{cos\theta}{1+sin\theta}$

Q.6 $sin^6\theta - cos^6\theta = (sin^2\theta - cos^2\theta)(1 - sin^2\theta cos^2\theta)$

Q.7 $sin^6\theta + cos^6\theta = 1 - 3\,sin^2\theta cos^2\theta$

Q.8 If $tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in the III quadrant, find the value of $\frac{csc^2\theta - sec^2\theta}{csc^2\theta + sec^2\theta} = \frac{3}{4}$.

Q.9 Find the value of the other five trigonometric functions of θ, if $cos\theta = \frac{12}{13}$ and the terminal side of the angle is not in the I quadrant.

Q.10 Prove that $(tan\theta + cot\theta)^2 = sec^2\theta csc^2\theta$

Q.11 If $cot\theta = \frac{5}{2}$ and terminal arm of the angle is in the first quadrant, find the value of $\frac{3sin\theta + 4cos\theta}{cos\theta - sin\theta}$.

Chapter 10

Q.1 Prove the identity $\frac{sin3\theta}{sin\theta} - \frac{cos3\theta}{cos\theta} = 2$

Q.2 If $sin\alpha = \frac{4}{5}$ and $= \frac{40}{41}$, where $0< \alpha < \frac{\pi}{2}$ and $0< \beta < \frac{\pi}{2}$ Show that $\sin(\alpha - \beta)=\frac{113}{205}$

Q.3 Reduce $cos^4\theta$ to an expression involving only function of multiple of θ, raised to the first power.

Q.4 $\sqrt{\frac{1+sin\alpha}{1-sin\alpha}} = \frac{sin\frac{\alpha}{2}+cos\frac{\alpha}{2}}{sin\frac{\alpha}{2}-cos\frac{\alpha}{2}}$

Q.5 Show that $\cot(\alpha + \beta) = \frac{cot\alpha cot\beta - 1}{cot\alpha + cot\beta}$

Q.6 Reduce $sin^4\theta$ to an expression involving only function of multiple of θ, raised to the first power.

Q.7 Prove that $\frac{sin\theta+sin3\theta+sin5\theta+sin7\theta}{cos\theta+cos3\theta+cos5\theta+cos7\theta} = tan4\theta$

Q.8 Prove that $cos20°cos40°cos60°cos80° = \frac{1}{16}$

Q.9 Prove that $sin\frac{\pi}{9} sin\frac{2\pi}{9} sin\frac{\pi}{3} sin\frac{4\pi}{9} = \frac{3}{16}$

Q.10 Prove that $sin10°sin30°sin50°sin70° = \frac{1}{16}$

Q.11 Prove that $\frac{cos8°-sin8°}{cos8°+sin8°} = tan37°$

Q.12 If α, β, γ are the angles of triangle ABC ,show that

$$cot\frac{\alpha}{2} + cot\frac{\beta}{2} + cot\frac{\gamma}{2} = cot\frac{\alpha}{2} cot\frac{\beta}{2} cot\frac{\gamma}{2}$$

Q.13 Prove that $\frac{sinA+sin2A}{1+cosA+cos2A} = tanA$

Q.14 Show that $\cos(\alpha + \beta)\cos(\alpha - \beta) = cos^2\alpha - sin^2\beta = cos^2\beta - sin^2\alpha$

Chapter 12

Q.1 Solve the triangle ABC, in which $a = 3, c = 6, \beta = 36°20'$

Q.2 Solve the triangle ABC, in which $a = 7, b = 3, \gamma = 38°13'$

Q.3 Solve the triangle ABC, in which $a = 32, b = 40, c = 66$

Q.4 The sides of triangle are $x^2 + x + 1, 2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120°.

Q.5 Show that $r = asin\frac{\beta}{2}sin\frac{\gamma}{2}\ sec\frac{\alpha}{2}$

Q.6 Show that $r_1 = 4Rsin\frac{\alpha}{2}cos\frac{\beta}{2}cos\frac{\gamma}{2}$

Q.7 Show that $r_1 = stan\frac{\alpha}{2}$

Q.8 Prove that in equilateral triangle $r : R : r_1 = 1:2:3$

Q.9 Prove that $r = stan\frac{\alpha}{2}\ tan\frac{\beta}{2}tan\frac{\gamma}{2}$

Q.10 Prove that $(r_1+r_2)tan\frac{\gamma}{2} = c$

Q.11 With usual notations, prove that $R = \frac{abc}{4\Delta}$

Q.12 With usual notations, prove that $r = \frac{\Delta}{s}$

Q.13 Prove that Law of Cosine.

Q.14 Prove that Law of Sine.

Q.15 Show that $r = (s-a)tan\frac{\alpha}{2} = (s-b)tan\frac{\beta}{2} = (s-c)tan\frac{\gamma}{2}$

Q.16 Prove that $abc(sin\alpha + sin\beta + sin\gamma) = 4\Delta s$.

Q.17 Prove that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

Q.18 Prove that $r_1 + r_2 + r_3 - r = 4R$

Chapter 13

Q.1 Prove that $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=\cos^{-1}\frac{253}{325}$

Q.2 prove that $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}$

Q.3 Prove that $\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\frac{\pi}{4}$

Q.4 Prove that $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=\frac{\pi}{4}$

Q.5 Prove that $\sin^{-1}A+\sin^{-1}B=\sin^{-1}(A\sqrt{1-B^2}+B\sqrt{1-A^2}$

Q.6 Prove that $\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{15}{17}$

Q.7 Prove that $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

Q.8 Prove that $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$