Chapter = 02

Vector and Equilibrium

What are Physical Quantities? Describe the Types on the basis of direction

Physical Quantities: All measurable quantities are called Physical Quantities. E.g length, temperature etc.

Types: There are two types of Physical quantities on the basis of direction

Scalar Quantities	Vector Quantities
The quantities which have only magnitude and no direction are called scalar quantities.	The quantities which have magnitude as well as direction are called vector quantities
For example mass, density, temperature etc.	For example force, velocity, acceleration etc

What are the Methods For Representation Of Vector?

There are two methods for representation of vector quantity

Symbolic Representation	Graphical Representation
It is represented by bold face letter. Like **A,B** It is also represented by a letter with arrow head above or below it like $\vec{A}$, and magnitude is represented by light face letter A or $\lvert\vec{A}\rvert$	It is represented by a straight line with an arrow head at its one end. The length of line show magnitude and arrow show direction of vector. Like → etc

Explain Rectangular co-ordinate system

Rectangular co-ordinate system: Two lines drawn perpendicular to each other are called co-ordinate axis and system of co-ordinate axis is called rectangular co-ordinate system.

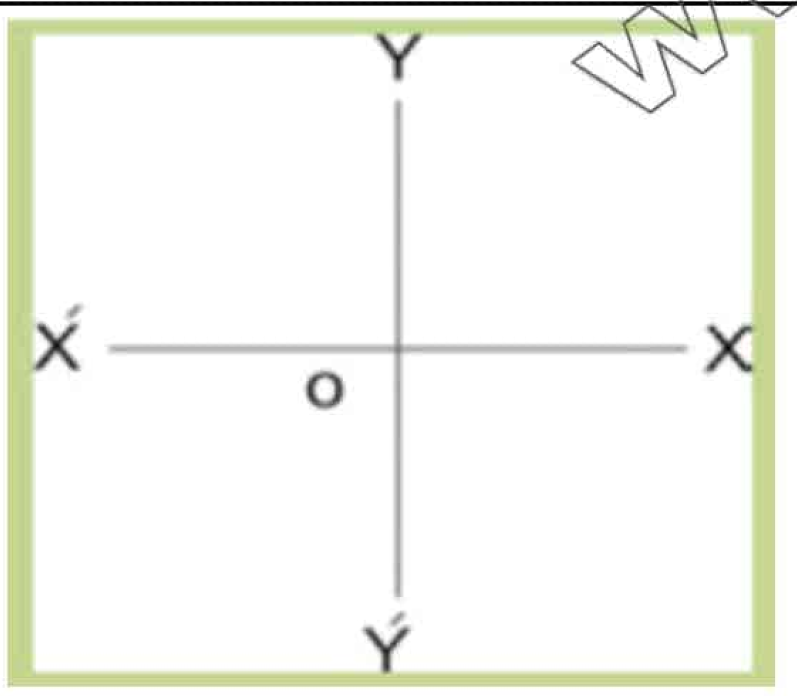
Horizontal line (axis) is called X-axis and vertical line (Axis) is called Y-axis.

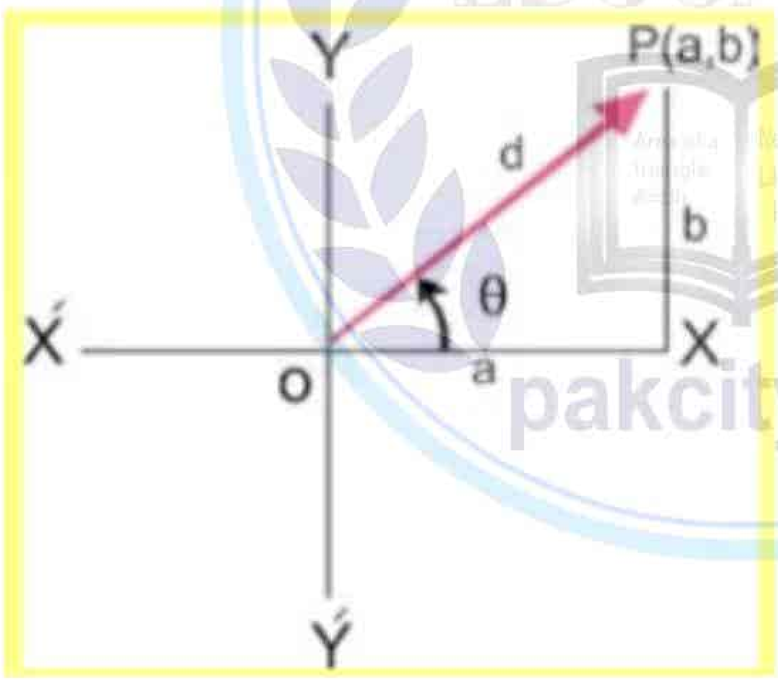
Origin: The point of intersection of two axis is called origin. And line right to and above origin is taken as positive and line left and below origin is taken as negative.

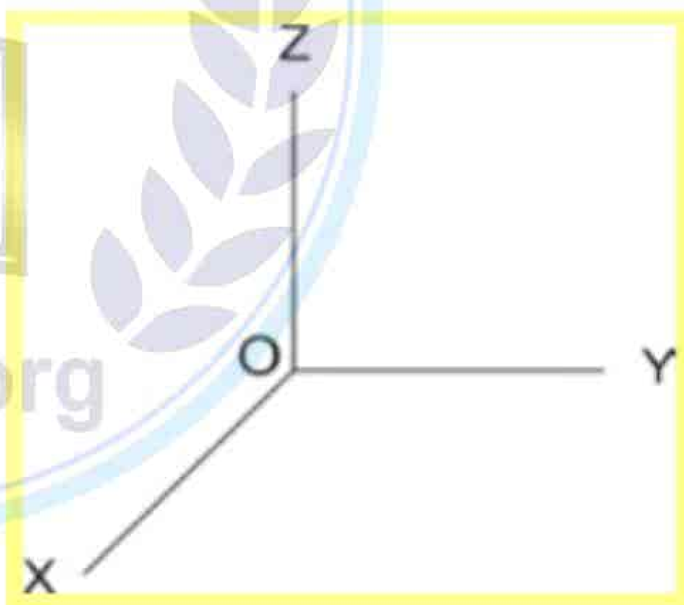
Two dimensional co-ordinate system: Such a system in there are two perpendicular lines is called two dimensional

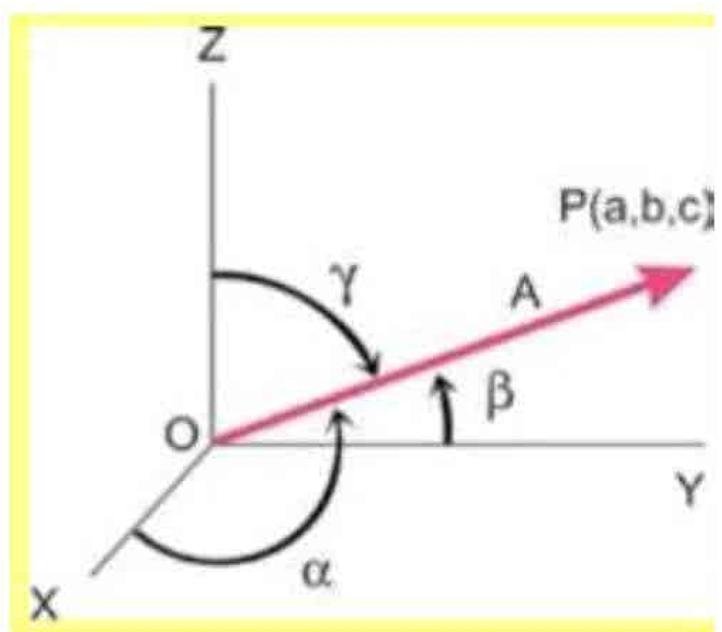
The direction of vector in plane is represented by angle which the vector makes with positive x-axis in anti-clock direction.

Three dimensional co-ordinate system: such a system in there are three perpendicular lines is called three dimensional co-ordinate system. Direction of vector in space is represented by three angle with the vector makes with x,y,z axis.







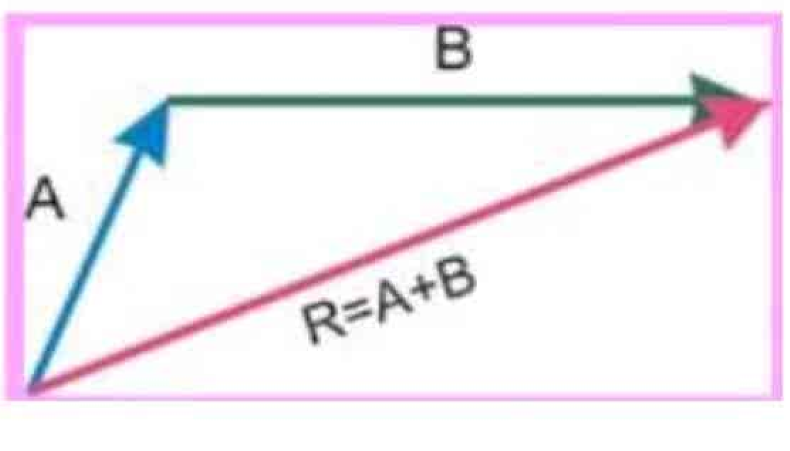


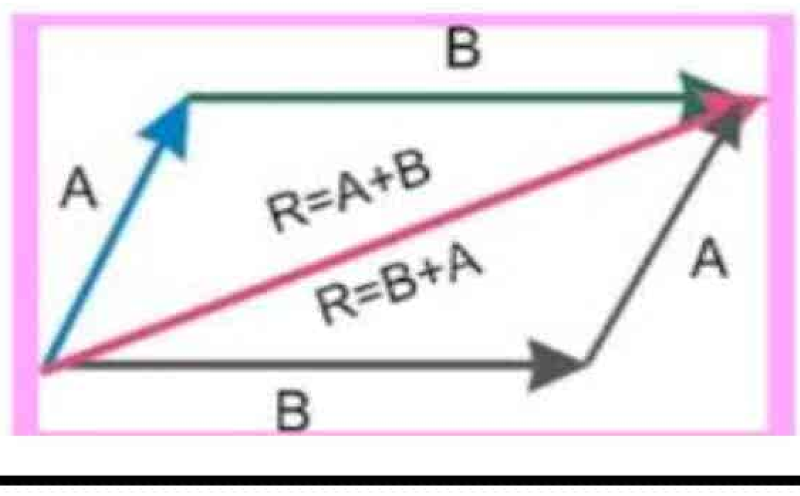
How two vectors are added (Explain head to tail rule of vector addition). OR Prove that A+B=B+A

Such a graphical method to add two vectors is called head to tail rule. There are following steps of vector addition by head to tail rule

i. Draw a representative lines vector **A&B**
ii. Join the tail of Vector **B** with head of vector **A**
iii. Now join the tail of vector **A** with head of **B** which gives resultant vector **R**.

As the vector sum **A+B** and **B+A** has the same results so **A+B**=**B+A**

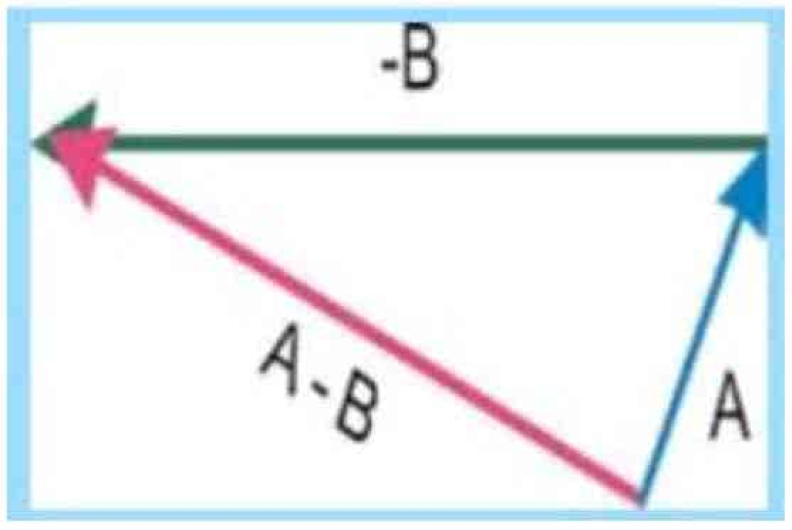


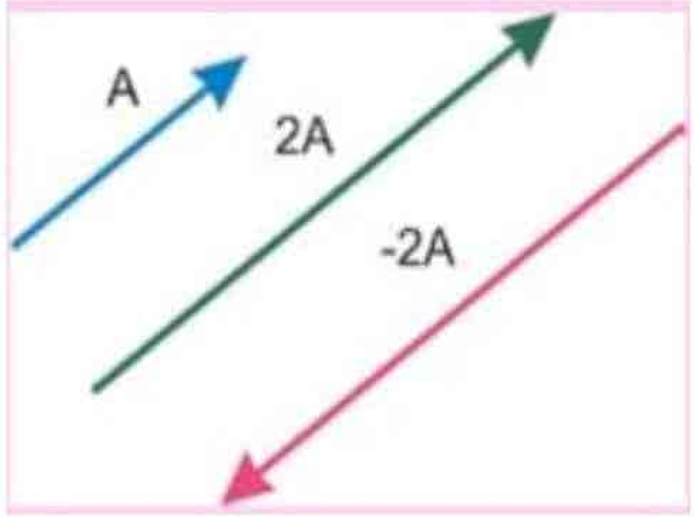


Resultant vector: Sum of two or more vector Result into a single vector is called resultant vector.

Vector Subtraction OR How Two Vectors Are Subtracted?

The subtraction of a vector is equivalent to the addition of same vector with its direction reversed.

Subtraction 



Multiplication

What is the Multiplication Of Vector

When a vector **A** is multiplied by a positive number n>0 then its magnitude is n$\vec{A}$ and in case of negative number direction is reversed.

What is Unit Vector? Write its formula.

A vector whose magnitude is one and used to show the direction of given vector is called unit vector. Its formula is $\hat{A}=\frac{\vec{A}}{|\vec{A}|}$, unit vector along X-axis is $\hat{\imath}$, along Y-axis is $\hat{\jmath}$ and along Z-axis is $\hat{k}$.

What is Null Vector Or Zero Vector?

A vector having zero magnitude and arbitrary direction is called null vector. $\vec{A}+(-\vec{A})=\vec{0}$ For example of position vector origin is null vector.

What are Equal Vectors?

Two vectors are said to be equal if they have same magnitude and same direction regardless of initial position.

What is Position Vector? Write its formula.

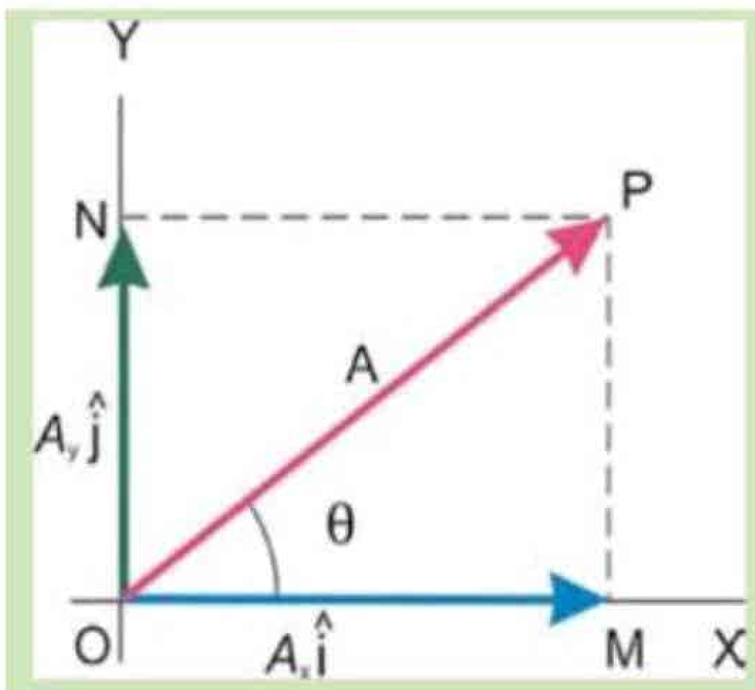
The vector which locates the position of particle with respect to origin is called position vector. $\vec{r}=a_{\hat{i}}+b_{\hat{j}}$ And magnitude $|\vec{r}|=\sqrt{a^2+b^2}$ in three dimensional $\vec{r}=a_{\hat{i}}+b_{\hat{j}}+c_k$ and magnitude $|\vec{r}|=\sqrt{a^2+b^2+c^2}$.

What are Rectangular Components Of A Vector? Explain.

Component of a vector: The effective values of a vector in given direction is component of a vector.

Rectangular components of a vector: The components of vector which are perpendicular to each other are called rectangular components of vector.

Explanation: Let us consider a vector $\vec{A}$ makes an angle Θ with x-axis. Draw a projection OM of vector OP on x-axis and projection ON(ON=MP)of vector OP on y-axis as shown in figure.



Using head to tail rule **OP=OM+MP** $\vec{A}=Ax\hat{i}+Ay\hat{j}$

X- Component of vector: In right angle triangle OPM $Cos\theta=\frac{OM}{OP}$

$Cos\theta=\frac{Ax}{A}$ $Ax=ACos\theta$ -------- (1)

Y- Component of vector: In same triangle $Sin\theta=\frac{MP}{OP}$

$Sin\theta=\frac{Ay}{A}$ $Ay=ASin\theta$ --------- (2)

Vector and its magnitude: Squaring and adding both equation or applying Pythagoras theorem

$OP^2=OM^2+MP^2$ $A^2=A^2x+A^2y$ $A=\sqrt{Ax^2+Ay^2}$

Direction of vector: The direction of vector can be found by dividing eq (2) by eq (1)

$\frac{A\sin\theta}{A\cos\theta}=\frac{Ay}{Ax} \Rightarrow \quad Tan\theta=\frac{Ay}{Ax} \quad \Rightarrow \theta=\tan^{-1}(\frac{Ay}{Ax})$

This method is also called composition of vector

Write a note on Vector addition by rectangular components

Let us consider two vectors $\vec{A}$ and $\vec{B}$ represented by lines OM and ON, using head to tail rule the resultant $\vec{R}=\vec{A}+\vec{B}$.

Step01: To find x and y components of all given vectors: To resolve the vector $\vec{R}$,$\vec{A}$ and $\vec{B}$ into rectangular components, draw perpendiculars MQ and PR from points "M" and "P" on x-axis.

Step 02: To find the resultant of X-components: As horizontal line X-axis

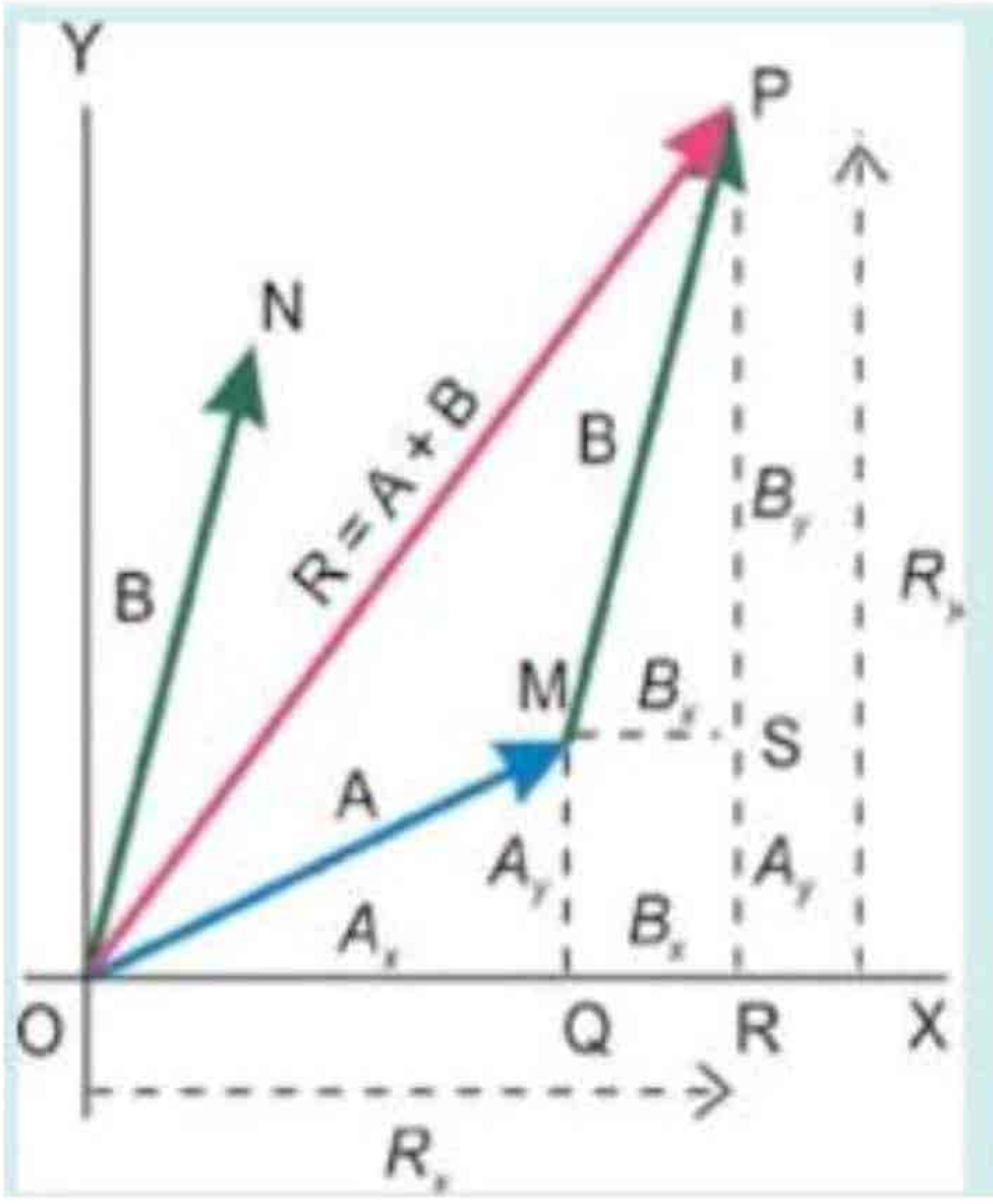
OR=OQ+QR OR=OQ+MS (As QR=MS)

Rx=Ax+Bx ------------------- (1)

Step 03: To find the resultant of Y-components: As Vertical components are

PR=RS+SP PR=MQ+SP (As RS=MQ)

Ry=Ay+By ------------------ (2)



Now we can find resultant of Resultant vector R by adding (1) and (2)

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

$$\vec{R} = (Ax+Bx)\hat{i} + (Ay+By)\hat{j}$$

Step 04: To find the magnitude of Resultant vector $\vec{R}$: Magnitude can be found By taking the magnitude of R or using Pythagoras theorem.

$$\left|\vec{R}\right|^2 = (Ax+Bx)^2 + (Ay+By)^2$$

$$\left|\vec{R}\right| = \sqrt{(Ax+Bx)^2 + (Ay+By)^2}$$

Step 05: To find the Direction of Resultant vector $\vec{R}$: The direction can be found by

$$\text{Tan}\theta = \left(\frac{Ry}{Rx}\right) \Rightarrow \quad \theta = \text{Tan}^{-1}\left(\frac{Ry}{Rx}\right) \Rightarrow \quad \theta = \text{Tan}^{-1}\left(\frac{Ay+By}{Ax+Bx}\right)$$

For any number of coplanar vectors Magnitude can be written as

$\left|\vec{R}\right| = \sqrt{(Ax+Bx+Cx+....)^2 + (Ay+By+Cy+....)^2}$ And Direction can be written as

$\theta = \text{Tan}^{-1}\left(\frac{Ay+By+Cy+....}{Ax+Bx+Cx+....}\right)$ This is also called reverse process of vector addition.

Determination of Angle by rectangular components

First Quadrant: Rx=+ and Ry=+ angle ϴ=ϕ

2nd Quadrant: Rx=-, Ry=+ angle ϴ=180°-ϕ

Third Quadrant: Rx=- and Ry=- angle ϴ=180°+ϕ

4th Quadrant: Rx=+, Ry=- angle ϴ=360°-ϕ

Q. What is Scalar/Dot product? Explain its characteristics.

Definition: If the product of two vectors result into a scalar quantity then this product is called scalar product.

Mathematically it can be written as $\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\cos\theta = AB\cos\Theta$,

Physically $\vec{A}.\vec{B}$=Magnitude of Vector A (Projection of B on A)=A(BcosΘ)= ABcosΘ shown in fig

Example: Work is an example which is scalar product of force and displacement $W=\vec{F}.\vec{d}=Fd\cos\Theta$

Characteristics:

(1) Scalar product is commutative $\vec{A}.\vec{B}=\vec{B}.\vec{A}$, as ABcosΘ=BAcosΘ

(2) Scalar product of two perpendicular vector is zero, i.e Θ=90°, $\vec{A}.\vec{B}=AB\cos 90°=0$, where in case of unit vectors

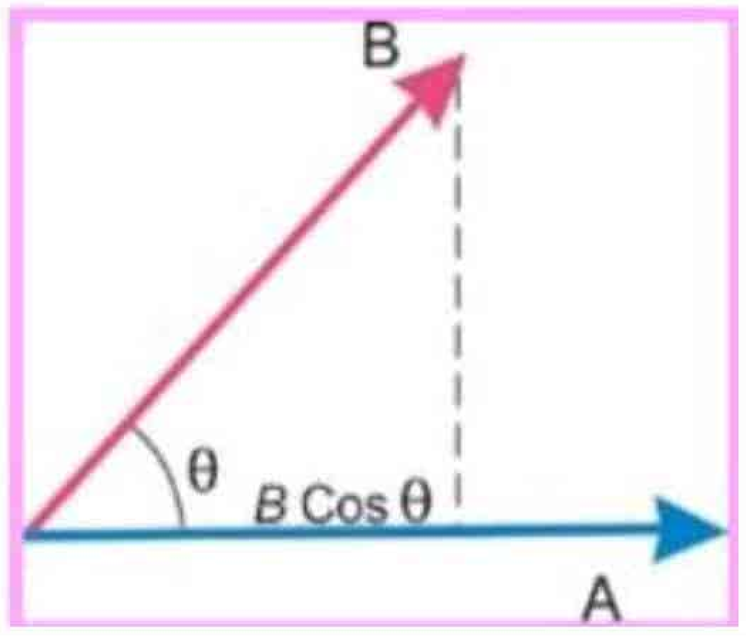
$\hat{i}.\hat{j}=(1)(1)\cos 90^o=0$ In same case $\hat{j}.\hat{k}=0$ and $\hat{k}.\hat{i}=0$

3) Scalar product of two parallel is equal to the product of their magnitudes i.e Θ=0°, $\vec{A}.\vec{B}=AB\cos 0°=AB$, in case of unit vector $\hat{i}.\hat{i}=(1)(1)\cos 0^o=(1)(1)(1)=1$ In same case $\hat{j}.\hat{j}=1$ and $\hat{k}.\hat{k}=1$

4) Scalar product for two anti-parallel vector Θ=180°, $\vec{A}.\vec{B}=AB\cos 180°=-AB$

5) Self product of a vector A is equal to square of its magnitude A. $\vec{A}.\vec{A}=AA\cos 0°=A^2(1)=A^2$

6) In case of rectangular components,



$$\vec{A}=Ax\hat{i}+Ay\hat{j}+Az\hat{k},\quad \vec{B}=Bx\hat{i}+By\hat{j}+Bz\hat{k}$$

$$\vec{A}.\vec{B}=(Ax\hat{i}+Ay\hat{j}+Az\hat{k})(Bx\hat{i}+By\hat{j}+Bz\hat{k})$$

$$AB\cos\theta = AxBx+AyBy+AzBz$$

$$Cos\theta=\frac{AxBx+AyBy+AzBz}{AB}\quad \Rightarrow \theta=Cos^{-1}\left(\frac{AxBx+AyBy+AzBz}{AB}\right)$$

What is Vector/Cross product? Explain its characteristics

Definition: If the product of two vectors results into a vector quantity then this product is called vector or cross product. $\vec{A}x\vec{B}=AB\sin\theta\hat{n}$.In this case AB sinθ give magnitude and $\hat{n}$ give direction, which is found by right hand rule

Right Hand Rule: Rotate the fingers of your right hand through some possible angle then erect thumb will show the direction of vector product.

Example: (1) Torque $\vec{\tau}=\vec{r}*\vec{F}=rF\sin\theta\hat{n}$. (2) Angular momentum $\vec{L}=\vec{r}*\vec{P}=rP\sin\theta\hat{n}$

Characteristics: Properties of Vector/ cross product are as follows.

(1) Vector product is not commutative as $\vec{A}x\vec{B}\neq\vec{B}x\vec{A}$ but $\vec{A}x\vec{B}=-\vec{B}x\vec{A}$

(2) Vector product of two mutually perpendicular vector has maximum value Θ=90°, $\vec{A}x\vec{B}=AB\sin 90°\hat{n}=AB\hat{n}$,

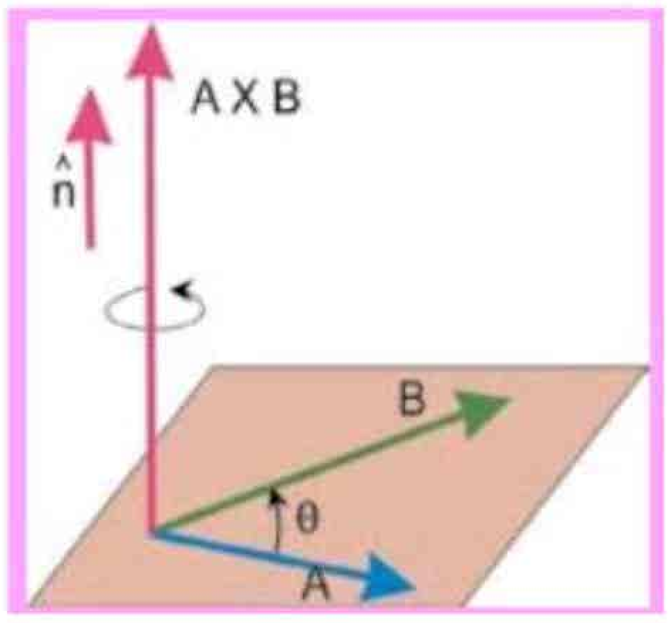
$\hat{i}x\hat{j}=\hat{k}$, $\hat{j}x\hat{k}=\hat{i}$, $\hat{k}x\hat{i}=\hat{j}$, where in reverse $\hat{j}x\hat{i}=-\hat{k}$, $\hat{k}x\hat{j}=-\hat{i}$, $\hat{i}x\hat{k}=-\hat{j}$ unit vector case

Proof : $\hat{i}x\hat{j}=(1)(1)\sin 90^o\hat{k}=(1)(1)(1)\hat{k}=\hat{k}$

(3) Vector/Cross product two parallel or anti-parallel vector is null vector i.e. Θ=0°,180°, $\vec{A}x\vec{B}=AB\sin 0°\hat{n}=\vec{0}$

$\hat{i}x\hat{i}=\hat{j}x\hat{j}=\hat{k}x\hat{k}=\vec{0}$ as $\hat{i}x\hat{i}=(1)(1)\sin 0^o=\vec{0}$

(4) Cross product in terms of rectangular components is expressed in determinant form



$$\vec{A}x\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ Ax & Ay & Az\\ Bx & By & Bz\end{vmatrix}=\hat{i}\begin{vmatrix}Ay & Az\\ By & Bz\end{vmatrix}-\hat{j}\begin{vmatrix}Ax & Az\\ Bx & Bz\end{vmatrix}+\hat{k}\begin{vmatrix}Ax & Ay\\ Bx & By\end{vmatrix}$$

$$\vec{A}x\vec{B}=(AyBz-AzBy)\hat{i}+(AzBx-AxBz)\hat{j}+(AxBy-AyBx)\hat{k}$$

(5) The magnitude of $\vec{A}x\vec{B}$ is equal to area of parallelogram with two A and B adjacent sides.

What Is Torque? Calculate The Torque Acting On Rigid Body.

Definition: The turning effect produced in a body about axis of rotation is called torque.

Equation: $\vec{\tau} = \vec{r} * \vec{F} = rF\sin\theta\,\hat{n}$ Its **SI unit** is Nm **Dimension** $[ML^2T^{-2}]$

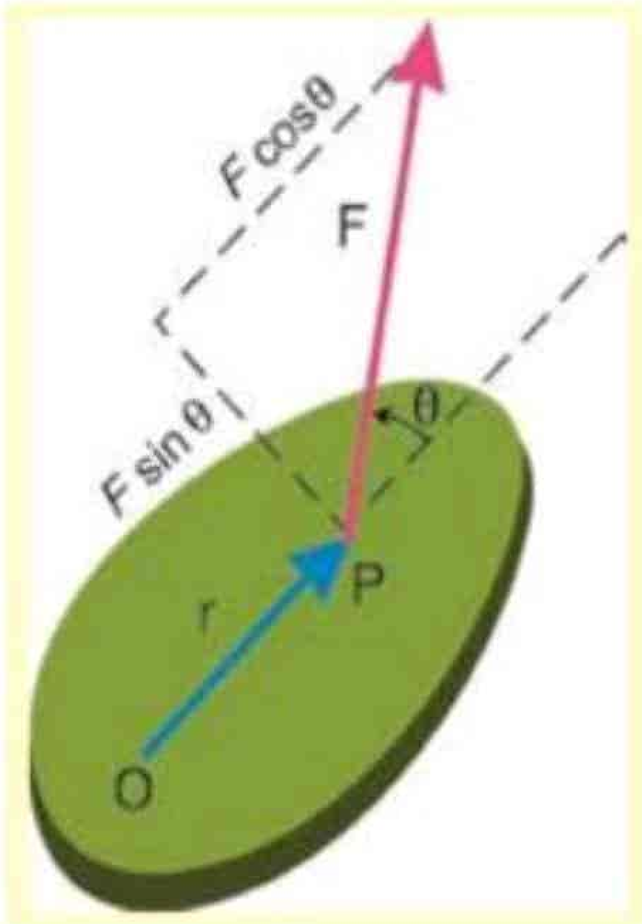
Moment Arm: The perpendicular distance from axis of rotation to line of action of force is called moment arm. The nut is easier to turn with moment arm of large value.

Example: Tightening and loosening of nut with a spanner.

Torque on rigid body: Consider force $\vec{F}$ is acting on rigid body at point P whose position vector relative to axis of rotation is $\vec{r}$. the Force can be resolved into two rectangular components.

(i) FsinΘ is perpendicular to $\vec{r}$

(ii) FcosΘ is along the direction of $\vec{r}$ (Torque due to this components is zero as it passes from axis of rotation)



The torque is produced due to FsinΘ only about O, which is given by

$\tau = r(F\sin\theta) = rF\sin\theta$ in vector form $\vec{\tau} = rF\sin\theta\hat{r}$ or $\vec{\tau} = \vec{r}x\vec{F}$ ....(a)

Similarly if we resolve the position vector r into its components,
Then only component which produce torque is rsinΘ

$\tau = F(r\sin\theta) = rF\sin\theta$ in vector form $\vec{\tau} = rF\sin\theta\hat{r}$ or $\vec{\tau} = \vec{r}x\vec{F}$ ....(b)

Important points about torque:

- Torque is count part of force for rotational motion
- Torque is also called moment of force
- Torque determine angular acceleration in body
- Clock wise torque is taken negative and anti-clock wise torque is taken positive.

What is Equilibrium of forces? Define its types and conditions.

Equilibrium: A body is said to be in equilibrium if it is at rest or moving with uniform velocity under the action of number of forces.

Types of Equilibrium: There are two types of equilibrium

Static Equilibrium: If a body is at rest, it is said to be in static equilibrium for example book lying on a table.

Dynamics Equilibrium: If a body is moving with uniform velocity, it is said to be in dynamic equilibrium. For example A car moving with uniform velocity.

Conditions of Equilibrium: There are two conditions of equilibrium

First condition: Sum of all the forces acting on a body is equal to zero $\Sigma\vec{F} = 0$

2nd condition: Sum of torques acting on a body is equal to zero $\Sigma\vec{\tau} = 0$

Translational Equilibrium: When first condition of equilibrium is satisfied and body has zero linear acceleration then is in translational equilibrium.

Rotational Equilibrium: When 2nd condition of equilibrium is satisfied and body has zero angular acceleration then it is in rotational equilibrium.

Complete Equilibrium: When both conditions of equilibrium are satisfied then it is said to be in complete equilibrium.

Why do you keep your legs far apart when you have to stand in the aisle of a bumpy riding bus?

When you stand in the aisle of a bumpy riding bus, you are in unstable position and you may fall. To make you stable you keep your legs far apart.

Exercise short questions

1: Define the terms (i) unit vector (ii) Position vector and (iii) Components of a vector?

Unit vector: A vector whose magnitude is one and used to show the direction of given vector is called unit vector. Its formula is $\hat{A} = \frac{\vec{A}}{A}$.

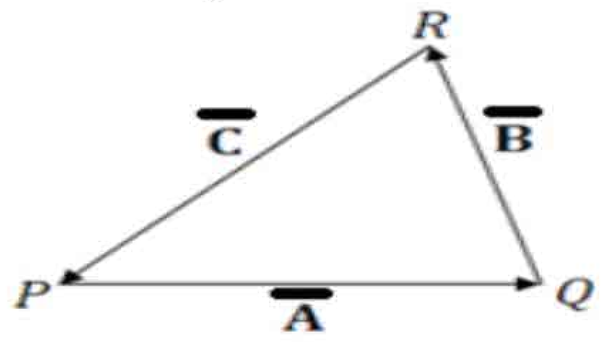
Position vector: The vector which locates the position of particle with respect to origin is called position vector. $\vec{r} = a\hat{i} + b\hat{j}$ And magnitude $r = \sqrt{a^2 + b^2}$.

Components of vector: The effective values of a vector in a given direction are components of a vector.

2. The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

If the three vectors are represented by the sides of triangle joined by head to tail rule at angle of 60°, there sum will be zero as shown in figure.

$\vec{A}+\vec{B}+\vec{C}=\vec{0}$



3) Vector A lies in the xy plane. For what orientation will both of its rectangular components be negative? For what orientation will its components have opposite signs?

When the vector lies in 3rd quadrant, then both of its rectangular components of vector will negative.

ii) The components of a vector have opposite sign when the vector lies in 2nd or 4th quadrant.

4) If one of the components of a vector is not zero, can its magnitude be zero? Explain.

No, its magnitude cannot be zero. As we know that magnitude of A is $\sqrt{Ax^2 + Ay^2}$ which shows that magnitude of vector will be zero only when all of its rectangular components are zero.

5) Can a vector have a component greater than the vector's magnitude?

No, the component of a vector can never be greater than the vector's magnitude because the component of a vector is its effective value in a specific direction and it is the part of vector and part is always less than full. So A≥Ax&A≥Ay.

6) Can the magnitude of a vector have a negative value?

No, its magnitude can never be zero. As we know that magnitude of A is $\sqrt{Ax^2 + Ay^2}$ which shows that square of real values is always positive.

7) If A + B = 0, what can you say about the components of the two vectors?

$$\vec{A} + \vec{B} = \vec{o} \Rightarrow \vec{A} = -\vec{B}$$

In terms of rectangular components

$$Ax\hat{i} + Ay\hat{j} = -(Bx\hat{i} + By\hat{j})$$

$Ax = -Bx, \quad Ay = -By$ Hence the components of both vectors are equal in magnitude but opposite in direction.

8) Under what circumstances would a vector have components that are equal in magnitude?

It is possible only when the vector makes angle of 45° with x-axis.

$$Ax = Ay \Rightarrow A\cos\theta = A\sin\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1) = 45^\circ$$

9) Is it possible to add a vector quantity to a scalar quantity? Explain.

No it is not possible to add a vector to a scalar quantity because both are different quantities as scalars have only magnitude while vector quantities have both magnitude as well direction so cannot added be add to each other.

10) Can you add zero to a null vector?

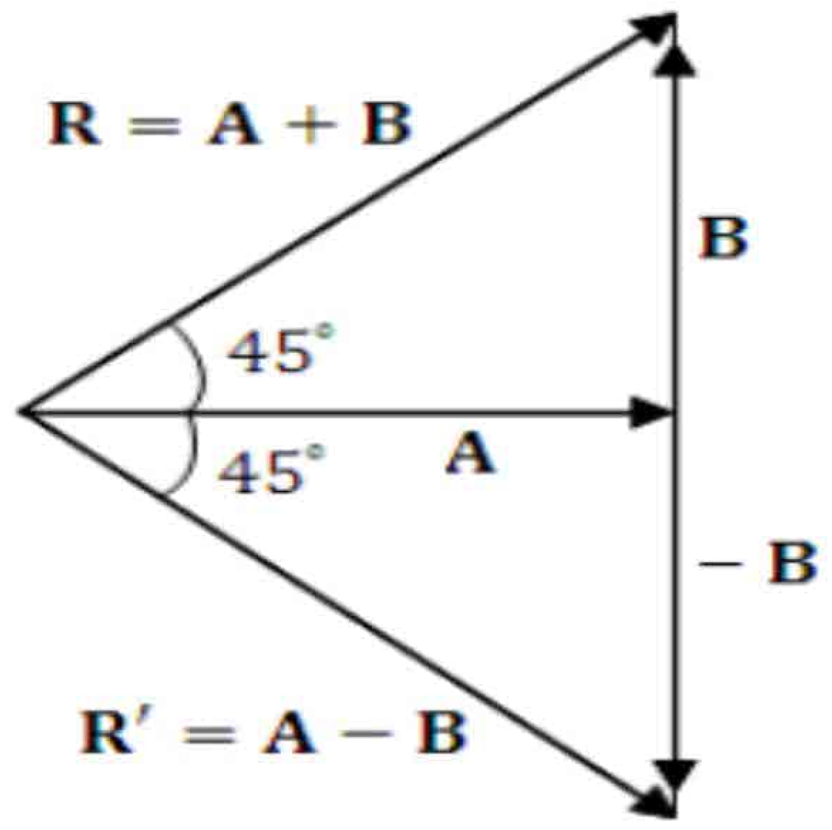
No it is not possible to add zero to a null vector because zero is scalar and null vector is vector and scalar is not added to vector quantity due to different quantities.

11) Two vectors have unequal magnitudes. Can their sum be zero? Explain.

No, the sum of two vectors having unequal magnitudes can't be zero. The sum of two vectors will be zero only when their magnitudes are equal and they act in opposite direction.

12) Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length?

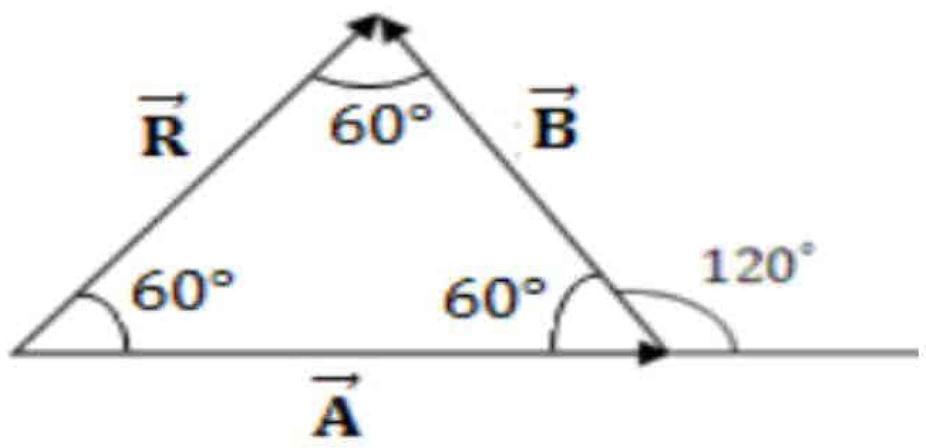
Consider two vectors **A** and **B** of equal **A=B** magnitude which are perpendicular to each other



(**A+B**).(**A-B**)=A^2-B^2=A^2-A^2=0, when dot product of two vectors is zero then they are perpendicular.

13.How would the two vectors of the same magnitude have to be oriented, were to be combined to give a resultant equal to a vector of the same magnitude?

It is possible only when the angle b/w two vectors is 120°. If the two vectors are shown by two sides of equilateral triangle then third side shows their resultant A=B=R.



14)The two vectors to be combined have magnitudes 60N and 35N. Pick the correct answer from those given below and tell why it is the only one of the three that is correct. (i)100N (ii)70N (iii)20N.

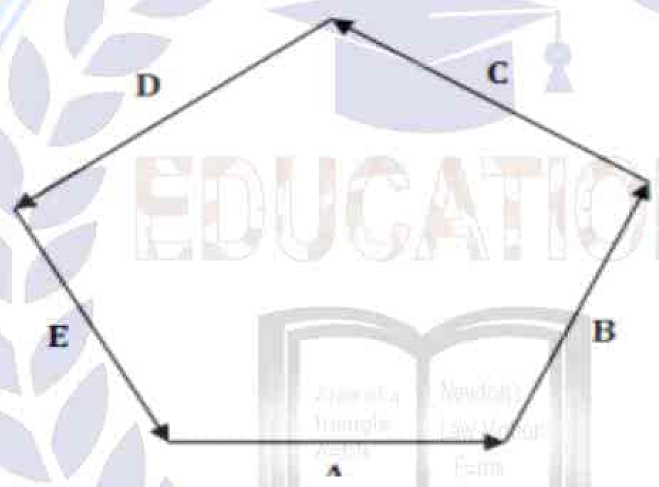
The correct answer is 70 N.

Sum of two vector is maximum when they are parallel to each other as 60+35=95 N, sum of two vector is minimum when opposite as 60+(-35)=25N, this shows that range of resultant is from 25 N to 95 N so correct answer is 70 N

15) **Suppose the sides of a closed polygon represent vector-arranged head to tail. What is the sum of these vectors?**

Sum of these vectors will be zero, in this case the head of last vector coincides with tail of first vector as

A+B+C+D+E+F=O



16)Identify the correct answer:

i)The actual direction of motion will be due to west

ii) Fcosɵ-mgsinɵ is correct answer by converting into rectangular components along the inclined plane

17)If all the components of the vectors A_1 *and* A_2 were reversed, how would this alter $A_1 \times A_2$?

It would not be changed when all the components of a vector were reversed.

-$\mathbf{A}_1$x-$\mathbf{A}_2$= $\mathbf{A}_1$x$\mathbf{A}_2$

18) **Name the three different conditions that could make $A_1 \times A_2 = 0$.**

This is zero when

i. A_1 or A_2 is a null vector

ii. A_1 and A_2 are parallel vector(ɵ=0°) As A_1*A_2sin0°=**0**

iii. A_1 and A_2 are anti-parallel (ɵ=180°) As A_1*A_2sin180°=**0**

19) Identify true of false statements and explain the reason. (a) A body in equilibrium implies that is not moving nor rotating. (b)If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

a) This statement is false because in dynamic equilibrium body may move or rotate with uniform velocity.

b) This statement is true only as first condition if satisfied body is said to be in translational equilibrium.

20) A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the strings will be minimum.

If picture is suspended from wall by two strings and tension is resolved into its rectangular components then Tsinɵ + Tsinɵ = W, 2Tsinɵ = W T = W/2sinɵ, tension will be minimum if Sinɵ is maximum so at 90° tension will be minimum.

21) Can a body rotate about its center of gravity under the action of its weight?

No, A body cannot rotate about its center of gravity under the action of its weight because in this case line of action of force passes through axis of rotation so moment arm is zero and : force = 0s o torque acting on it is zero.

Numerical problems

2.1: Suppose, in a rectangular coordinate system, a vector A has its at the point P (-2, -3) and its tip at Q (3,9). Determine the distance between these two points.

$$\text{Sol : Points P((-2,-3) and Q(3,9)},\ \vec{r}_1 = -2\hat{i} - 3\hat{j}\ ,\vec{r}_2 = 3\hat{i} + 9\hat{j},\ d = ?$$

$$\vec{d} = \vec{r}_2 - \vec{r}_1 = (3\hat{i} + 9\hat{j}) - (-2\hat{i} - 3\hat{j}) = (3\hat{i} + 2\hat{i}) + (9\hat{j} + 3\hat{j}) = 5\hat{i} + 12\hat{j}$$

$$d = \sqrt{x^2 + y^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13 \text{ units}$$

2.2: A certain corner of a room is selected as the origin of a rectangular coordinate system, If an insect is sitting on an adjacent wall at a point having coordinates (2,1), where the units are in meters, what is the distance of the insect from this corner the room?

$$\text{Sol : Points P((2,1) and O(0,0)},\ \vec{r} = 2\hat{i} + \hat{j}\quad d = ?$$

$$d = \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5} = 2.24 \text{ units}$$

2.3: What is the unit vector in the direction of the vector $A = 4\hat{i} + 3\hat{j}$.

$$\text{sol} : \vec{A} = 4\hat{i} + 3\hat{j}\ \ \hat{A} = ?$$

$$\hat{A} = \frac{\vec{A}}{A} = \frac{\vec{A}}{\sqrt{A_x^{\ 2} + A_y^{\ 2}}} = \frac{4\hat{i} + 3\hat{j}}{\sqrt{4^2 + 3^2}} = \frac{4\hat{i} + 3\hat{j}}{\sqrt{16+9}} = \frac{4\hat{i} + 3\hat{j}}{\sqrt{25}} = \frac{4\hat{i} + 3\hat{j}}{5}$$

2.4: Two particles are located at $r_1 = 3\hat{i} + 7\hat{j}$ and $r_2 = -2\hat{i} + 3\hat{j}$ respectively. Find both the magnitude of the vector $(r_{2\_}r_1)$ and its orientation with respect to the x-axis.

$$\text{Sol :},\ \vec{r}_1 = 3\hat{i} + 7\hat{j}\ \ ,\vec{r}_2 = -2\hat{i} + 3\hat{j}, \vec{r}_2 - \vec{r}_1 = ?$$

$$\vec{r}_2 - \vec{r}_1 = (-2\hat{i} + 3\hat{j}) - (3\hat{i} + 7\hat{j}) = (-2\hat{i} - 3\hat{i}) + (3\hat{j} - 7\hat{j}) = -5\hat{i} + -4\hat{j}$$

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41} = 6.4 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{-4}{-5}\right) = 38.6^o, \text{As in 3rd quad so angle} = 180^o + 38.6^o = 218.6^o \approx 219^o$$

2.5: If a vector 'B' is added to vector A, the result is $6\hat{i} + \hat{j}$ .If 'B' is subtracted from A, the result is $-4\hat{i} + 7\hat{j}$. What is the magnitude of vector 'A'?

$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j},\quad \vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}\quad A = ?$$

$$(\vec{A} + \vec{B}) + (\vec{A} - \vec{B}) = (6\hat{i} + \hat{j}) + (-4\hat{i} + 7\hat{j}) = 2\hat{i} + 8\hat{j}$$

$$2\vec{A} = 2\hat{i} + 8\hat{j} \Rightarrow \vec{A} = \hat{i} + 4\hat{j}$$

$$A = \sqrt{A_x^{\ 2} + A_y^{\ 2}} = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17} = 4.1$$

2.6: Given that $A = 2\hat{i} + 3\hat{j}$ and $B = 3\hat{i} - 4\hat{j}$, find the magnitude and angle of (a) C=A+B, and (b) D=3A-2B.

Sol (a) : $\vec{C} = \vec{A} + \vec{B} \quad \Rightarrow \vec{C} = (2\hat{i} + 3\hat{j}) + (3\hat{i} - 4\hat{j}) = 5\hat{i} - \hat{j}$

$|\vec{C}| = \sqrt{(5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26} = 5.1$

Direction $= \theta = \tan^{-1}(\frac{-1}{5}) = 11^o$ As φ lies in fourth quad so orintation $360^o - 11^o = 349^o$

(b) $\vec{D} = 3\vec{A} - 2\vec{B} \quad \Rightarrow \vec{D} = 3(2\hat{i} + 3\hat{j}) - 2(3\hat{i} - 4\hat{j}) = (6\hat{i} + 9\hat{j}) - (6\hat{i} + 8\hat{j}) = 0\hat{i} + 17\hat{j}$

$|\vec{C}| = \sqrt{(0)^2 + (17)^2} = \sqrt{0 + 289} = 17$

Direction $= \theta = \tan^{-1}(\frac{0}{17}) = 90^o$ As φ lies in *F*irst quad

2.7: Find the angle between the two vectors, $A = 5\hat{i} + \hat{j}$ and $B = 2\hat{i} + 4\hat{j}$.

Given data : $\vec{A} = 5\hat{i} + \hat{j}, \vec{B} = 2\hat{i} + 4\hat{j}$ angle $= \theta = ?$

Using equation of scalar product for two vectors $AB\cos\theta = AxBx + AyBy$

$$\cos\theta = \frac{AxBx + AyBy}{AB} = \frac{(5)(2) + (1)(4)}{(\sqrt{5^2 + 1^2})(\sqrt{2^2 + 4^2})} = \frac{10+4}{\sqrt{26}\sqrt{20}} = \frac{14}{5.1 * 4.5}$$

$$\theta = \cos^{-1}(\frac{14}{5.1*4.5}) \quad \Rightarrow \theta = 52^o$$

2.8: Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line from the point (2,-1) to the point (6, 4).

Given data : $\vec{F} = 3\hat{i} + 2\hat{j}$, point(2,-1) $\vec{r}_1 = 2\hat{i} - \hat{j}$, point(6,4), $\vec{r}_2 = 6\hat{i} + 4\hat{j}$ W = ?

$\vec{d} = \vec{r}_2 - \vec{r}_1 = (6\hat{i} + 4\hat{j}) - (2\hat{i} - \hat{j}) = 4\hat{i} + 5\hat{j}$

$W = \vec{F}.\vec{d} = (3\hat{i} + 2\hat{j}).(4\hat{i} + 5\hat{j}) = 12 + 10 = 22\,J$

2.9: Show that the three vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Given Data : $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$

We know that two vectors are perpendicular if if $\vec{A}.\vec{B} = ABCos90^o = 0 \Rightarrow \vec{A}.\vec{B} = 0$

$\vec{A}.\vec{B} = (\hat{i} + \hat{j} + \hat{k}).(2\hat{i} - 3\hat{j} + \hat{k}) = 2 - 3 + 1 = 3 - 3 = 0$

$\vec{A}.\vec{C} = (\hat{i} + \hat{j} + \hat{k}).(4\hat{i} + \hat{j} - 5\hat{k}) = 4 + 1 - 5 = 5 - 5 = 0$

$\vec{B}.\vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}).(4\hat{i} + \hat{j} - 5\hat{k}) = 8 - 3 - 5 = 8 - 8 = 0$

Hence prove that given three vectors are mutually perpendicular

2.10: Given that $A = \hat{i} - 2\hat{j} + 3\hat{k}$ and $B = 3\hat{i} - 4\hat{k}$, find the projection of A on B.

Given Data : $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{B} = 3\hat{i} - 4\hat{k}$ Projection of A on B = $A\cos\theta = ?$

$$\text{As } \vec{A}.\vec{B} = AB\cos\theta \Rightarrow A\cos\theta = \frac{\vec{A}.\vec{B}}{B} = \frac{AxBx + AyBy + AzBz}{B} = \frac{(1)(3) + (-2)(0) + ((3)(-4)}{\sqrt{(3)^2 + 0^2 + (-4)^2}} = \frac{-9}{5}$$

2.11: Vectors A, B and C are 4 units north, 3 units west and 8 units east, respectively. Describe carefully (a) $A\times B$ **(b)** $A\times C$ **(c)** $B\times C$

Given Data : $\vec{A} = 4$ unit North, $\vec{B} = 3$ units west, $\vec{C} = 8$ unit east, $\vec{A}x\vec{B} = ?$ $\vec{A}x\vec{C} = ?$ $\vec{B}x\vec{C} = ?$

$\vec{A}x\vec{B} = AB\sin\theta\,\hat{n} = (4)(3)\sin 90^o = 12$ units vertically upward (Using Right hand rule)

$\vec{A}x\vec{C} = AC\sin\theta\,\hat{n} = (4)(8)\sin 90^o = 32$ units vertically downward (using right hand rule)

$\vec{B}x\vec{C} = BC\sin\theta\,\hat{n} = (3)(8)\sin 0^o = 0$

2.12: The torque or turning effect of force about a given point is given by $r\times F$ **where 'r' is the vector from the given point to the point of application of F. Consider a force** $F=-3\hat{i}+\hat{j}+5\hat{k}$ **(Newton) acting on the point** $7\hat{i}+3\hat{j}+\hat{k}(m)$**. What is the torque in Nm about the origin?**

Given Data : $\vec{F} = -3\hat{i}+\hat{j}+5\hat{k}$, $\vec{r} = 7\hat{i}+3\hat{j}+\hat{k}$ torque $= \vec{\tau} = ?$

$$\vec{\tau} = \vec{r}\ x\ \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = \hat{i}\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} 7 & 1 \\ -3 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} 7 & 3 \\ -3 & 1 \end{vmatrix} = \hat{i}(15-1)-\hat{j}(35-(-3))+\hat{k}(7-(-9))$$

$\vec{\tau} = 14\hat{i}-38\hat{j}+16\hat{k}$

2.13: The line of action of force, $F=\hat{i}-2\hat{j}$**, passes through a point whose position vector is** $\left(-\hat{j}+\hat{k}\right)$ **.Find (a)the moment of F about the origin, (b) the moment of F about the point of which the position vector is** $\hat{i}+\hat{k}$ **.**

Given Data : $\vec{F} = \hat{i}-2\hat{j}$, $\vec{r} = -\hat{j}+\hat{k}$ torque $= \vec{\tau} = ?$

$$(a)\ \vec{\tau} = \vec{r}\ x\ \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 1 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = \hat{i}(0-(-2)) - \hat{j}(0-1) + \hat{k}(0-(-1)) = 2\hat{i}+\hat{j}+\hat{k}$$

(b) first of all to find r, $\vec{r} = \vec{r}_2 - \vec{r}_1 = (-\hat{j}+\hat{k})-(\hat{i}+\hat{k}) = -\hat{j}+\hat{k}-\hat{i}-\hat{k} = -\hat{i}-\hat{j}$ so $\vec{r} = -\hat{i}-\hat{j}$ and $\vec{F} = \hat{i}-2\hat{j}$

$$\vec{\tau} = \vec{r}\ x\ \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 0 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(2-(-1)) = 3\hat{k}$$

2.14: The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ **and 6 respectively. Find the angle between the vectors.**

Given Data : $AB\cos\theta = 6\sqrt{3}$, $AB\sin\theta = 6$ angle $= \theta = ?$

dividing both equations, $\dfrac{AB\sin\theta}{AB\cos\theta} = \dfrac{6}{6\sqrt{3}} \Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\theta = \tan^{-1}(\frac{1}{\sqrt{3}}) = 30^o$

2.15: A load of 10.0N is suspended from a clothes line. This distorts the line so makes an angle of 15º with the horizontal at each end. Find the tension in the clothes line.

Given Data : Weight $= W = 10$ N, Angle $= \theta = 15^o$, T = ?

As Tension due to X - components is zero as $\sum Fx = 0$

Along Y – axis $T\sin\theta + T\sin\theta = W \Rightarrow 2T\sin\theta = W$

$T = \dfrac{W}{2\sin\theta} = \dfrac{10}{2\sin 15^o} = 19.3N$

Previous board exam multiple choice question

	Questions	Option A	Option B	Option C	Option D
1)	The magnitude of cross product and dot product are equal at angle of	**45˚**	90˚	180˚	Zero ˚
A.B=**AxB**.. ABcosɵ=ABsinɵ, sinɵ/cos=1, tanɵ=1 ɵ=45°					
2)	Magnitude of rectangular components are equal at angle of	**45˚**	90˚	180˚	Zero ˚
3)	i^.(j^xk^)=?	**1**	0	J	I
(j^xk^)=i^ so i^.i^=1					
4)	Projection of B along A is written as	A.B	A	**A.B**/B	**A.B**/A
5)	A force of 10N acting on 30˚ with y axis then magnitude of X-component will be	**5N**	8.66N	10N	Zero
F= 10 N, angle with y axis is 30° then with x-axis will be 60° so Fx=Fcosɵ=10cos60°=5N					
6)	The resultant of two force 5N and 10N cannot be?	**4N**	6N	9N	13N
Max ans is 5+10=15 N and min ans=10-5=5 N, ans range is 5-15					
7)	Resultant of two forces 30N and 40N acting at angle of 90° is	**50N** Apply Pythagoras theorem to get result	30N	40N	70N
8)	The unit vector along y axis is	i^	**j^**	k^	y^
9)	If the angle between two vectors of magnitude 12 and 4 is 60˚, then dot product	6	12	**24**	48
A.B=ABcosɵ= (12)(4)cos60°=48(0.5)=24					
10)	Resultant magnitude of 6N force acting on right angle with force of 8N	6N	8N	**10N** Apply Pythagoras theorem to get result	14N
11)	A body is in a static equilibrium when it is at	**Rest**	Moving with uniform velocity	Moving with variable velocity	All of these
12)	If body is at rest or rotating with uniform angular velocity then torque will	Maximum	**Zero**	Negative	Positive
13)	The magnitude of vector can never be	Positive	**Negative**	Both A&B	None of these
14)	The vector in space has components	Two	**Three**	Four	One
15)	Dot product of vector A with itself is	A	2A	**A^2**	0
16)	A body will be in translational equilibrium if	**ΣF=0**	Σt=0	Both A&B	None of these
17)	Two forces of 10 N and 20 N act on a body in direction making angle 30°, Resultant of X-component is	**25.98 N**	12.5 N	30.98 N	36.36 N
18)	If second condition of equilibrium is satisfied then body will be in	Translational equilibrium	**Rotational equilibrium**	Dynamic equilibrium	Complete equilibrium
19)	The magnitude of resultant of two perpendicular vector of magnitude A will be?	A	**$\sqrt{2}$A** Apply Pythagoras theorem to get result	A	A^2
20)	Name the quantity which is vector?	Speed	**Force**	Temperature	Density
21)	A force 2i + j has moved its point of application from (2,3) to (6,5). What is work done?	-10	-18	+18	**+10**
W=**F.d**, **d=r2-r1**=(6i+5j)-(2i+3j)=4i+2j, W=(2i+j).(4i+2j)=8+2=10J					
22)	If a force of 10N acting on y axis then its x component will be	8.66 N	5 N	**0 N**	10N
For Fx=Fcosɵ=10cos90°=0 as w.r.t X component angle is 90° w.r.t y axis					
23)	The direction of torque is along	Position vector	Force	Parallel to plane contain r and F	**Perpendicular to plane contain r&F**
24)	The magnitude of cross and dot product are6 and 6$\sqrt{3}$then what is angle b/w them	0˚	**30˚**	45˚	60˚

See solution of numerical 2.14					
47+	Two vector of 60N and 35N combined then correct answer will be	15N	20N	**<u>70N</u>**	100N
Apply Pythagoras theorem to get result , also its max ans=60+35=95N and min ans=60-35=25N, its ans range 95-25 so					
48+	A single vector having the same effect as all the original vectors taken together called	**<u>Resultant vector</u>**	Equal vector	Unit vector	Position vector
49+	Unit vector in the direction of vector 2i − 4j will be:	$\frac{2i - 4j}{\sqrt{6}}$	$\frac{4i - 2j}{\sqrt{10}}$	<u>$\frac{i-2j}{\sqrt{5}}$</u>	$\frac{i - 2j}{\sqrt{7}}$
$A=\sqrt{2^2+(-4)^2}=\sqrt{20}=\sqrt{4*5}=2\sqrt{5}, \hat{A}=\frac{2i-4j}{2\sqrt{5}}=\frac{2(i-2j)}{2\sqrt{5}}=\frac{i-2j}{\sqrt{5}}$					
4:+	The angle of A=Axi-Ayj with x-axis in b/w	0° and 90°	90° and 180°	180° and 270°	**<u>270° and 360°</u>**
As resultant lies in 4th quadrant so angle is b/w 270° and 360°					
4;+	If the resultant of two vectors each of magnitude F is also of magnitude F, the angle between them will be ?	60°	30°	90°	**<u>120°</u>**
See solution of exp 2.3 for explanation					
52+	If |A+B|=|A-B| then angle between A&B is	**<u>90°</u>**	0°	180°	45°
Sum and difference of equal vectors are perpendicular to each other					
53+	If the force of magnitude 8 N acts on a body in direction making an angle 30°, its X and Y components will be:	$F_x = 3\sqrt{3}$ $F_y = 4$	$F_x = 4\sqrt{3}$ $F_y = 8$	**<u>$F_x = 4\sqrt{3}$ $F_y = 4$</u>**	$F_x = 8$ $F_y = 4\sqrt{3}$
F=8N, Fx=Fcosθ=8cos30°=8√3/2=4√3 Fy=Fsinθ=8sin30°=8(1/2)=4					
54+	If A=2i and B=3i+4j then A.B	1	0	14	**<u>6</u>**
A.B=(2i).(3i+4j)=6(i,i)=6					
55D	Angle between Ax and Az is	**<u>90°</u>**	180°	270°	360°
56+	If Fx=2N and Fy=2N then F along X-axis	0°	90°	**<u>45°</u>**	60°
57+	The scalar product of i and k is:	**<u>Zero</u>**	1	90°	−1
58+	A force of 15 N makes an angle of 90° with x-axis, its y component will be	15 N	**<u>0 N</u>**	100 N	15 N
59+	If vector A lies along x-axis then its component along y-axis will be?	A sinΘ	A cosΘ	A tanΘ	**<u>Zero</u>**
5:+	The result of 120 N and 20 N forces cannot	**<u>141 N</u>**	100 N	101 N	**<u>130 N</u>**
5;+	When a vector is multiplied by -1 then its direction is changed by?	90°	120°	360°	**<u>180°</u>**
40)	If F=2i+3j and d=4i+4j then work will be?	12J	**<u>20J</u>**	32J	40J
41)	If the two unit vectors perpendicular to each other are added, magnitude of resultant	1	**<u>√2</u>**	4	3
By Pythagoras theorem magnitude $\sqrt{1^2+1^2}=\sqrt{2}$					
42)	If the magnitude of then angle between $A.B=\frac{1}{2}AB$ A and B is	30°	45°	**<u>60°</u>**	90°
A.B=ABcosθ=ABcos60°=1/2 AB as Cos 60°=1/2 or 0.5					
43)	Torque of force t=rxF then r and F are at angle of	0°	**<u>90°</u>**	45°	60°
44)	When a vector A is added to negative vector-A then resultant will be	2A	A	0	**<u>Null vector</u>**
45)	A body will be in complete equilibrium when it satisfies	First condition	2nd condition	**<u>Both A&B</u>**	None of these

46)	If we double the moment arm the value of torque becomes	**Two times**	Three times	Four times	Half
47)	The position vector r in xz plane	$\underline{x\hat{i}+z\hat{k}}$	$y\hat{i}+z\hat{k}$	$y\hat{i}+x\hat{k}$	$y\hat{j}+x\hat{i}$
48)	The resultant of two forces 3N and 4N acting at right angle to each other	**5N**	6N	1N	7N
Apply Pythagoras theorem					
49)	What is angle between two vectors A=5i+j and B=2i+4j	66^o	**52^o**	25^o	33^o
See solution of numerical no 2.7 to get the result					
50)	The vector product rxdp/dt is	F	I	**torque**	Momentum
51)	|i-j-3k|=?	$\sqrt{5}$	$\sqrt{15}$	$\underline{\sqrt{11}}$	$\sqrt{7}$
Apply formula of magnitude $\sqrt{(a^2+b^2+c^2)}$ put a=1 b=-1 c=-3 to get the result					
52)	If position vector r and F are in same direction then torque will be	Maximum	Minimum	**Zero**	Same
53)	Torque has zero value if angle between r and F is	**0°**	90°	45°	60°
54)	The cross product k^ x j^	i^	j^	K^	**-i^**
55)	The cross product i^ x k^	i^	j^	K^	**-j^**
56)	For maximum torque, the angle between **r&F** is	0°	**90°**	45°	60°
57)	If the scalar product of two vectors is $2\sqrt{3}$ and magnitude of their vector product is 2, the angle b/w them is	120°	**30°**	60°	180°
$AB\cos\theta = 2\sqrt{3},\ \ ABSin\theta = 2,\ \frac{ABsin\theta}{ABcos\theta} = \frac{2}{2\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}(\frac{1}{\sqrt{3}}) = 30^o$					
58)	The resultant of two forces 30 N and 40 N acting parallel to each other is:	30 N	40 N	**70 N**	10 N
For parallel forces, forces are sum up so 30+40=70 N					
59)	Which is correct formula?	$\vec{\tau} = rF$	$\vec{\tau} = rF\sin\theta$	$\underline{\vec{\tau} = \vec{r}x\vec{F}}$	$\vec{\tau} = rF\cos\theta$
60)	A force of 100 N is acting on y axis 60° with y axis then its horizontal component will be	50 N	60N	70N	**86.6 N**
F= 100 N, angle with y axis is 60° then with x-axis will be 30° so Fx=Fcoso=100cos30°=86.6N					