Chapter = 08

Wave Motion and Sound

THEORY NOTES

SIMPLE HARMONIC MOTION:

DEFINITION:

If a body moves in a straight line such that its acceleration is always directed towards a fixed point on that line and its magnitude is proportional to the displacement from that point, then the body is said to execute simple harmonic motion.

EXAMPLES: Motion of a mass attached to one end of a spring.
Vibration of a string of sitar.
Motion of the bob of a simple pendulum
Motion of the pendulum of a clock.

CONDITIONS OF A BODY TO EXECUTE SIMPLE HARMONIC MOTION

(i) The motion of the body must be under elastic restoring force.

(ii) The acceleration of the body must be s proportional to the displacement and directed onwards the mean position.

(iii) The body must have inertia i.e. mass.

If these conditions are satisfied the body will execute simple harmonic motion.

MOTION UNDER ELASTIC RESORING FORCE

CASE I:-

As shown in the figure a mass 'm' is attached to one end of a spring placed on horizontal smooth surface, the other end of which is rigidly fixed. If the mass 'm' is pulled to the right through a distance x and then released, the mass 'm' will vibrate about its mean position.

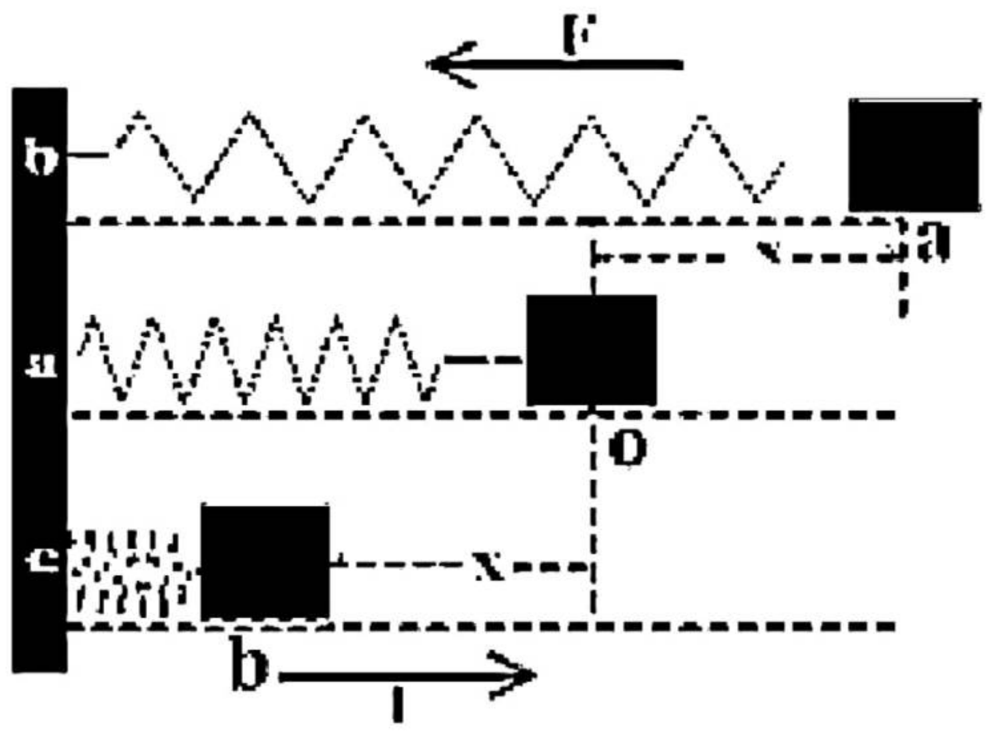
The force exerted on the spring will be proportional to the displacement.

$F \propto x$

or

$F = kx$

This is known as Hook's law.



Where k is a constant of proportionality, called force constant of spring. The spring also exerts an equal and opposite force to restore its shape. This force is called restoring force.

Restoring force = - Kx

EXPLANATION:

Consider the motion of mass 'm' in fig (a) there is no force on the mass 'm' because spring suffers no extension. In fig (b), the spring is pulled to the right through a distance x_0 the restoring force of the spring is F = - Kx

The work done in pulling the spring through a distance x_0 is stored in the form of potential energy of the spring. In return the spring applies a force to restore its position and the mass 'm' moves to the left. Thus potential energy changes into kinetic energy. At its mean position, the mass 'm' has a maximum speed and because of inertia the mass moves to the left. The spring then compresses and the motion of the mass 'm' retards. At its extreme left position all the energy is potential. This process repeats and the energy of the spring oscillates between potential and kinetic energy.

Let x be the displacement of the mass 'm' at any instant then.

$$F = -Kx$$

From Newton's second law of motion

$$F = ma$$

where a = acceleration

Hence

Or $a = -(K/m)\,x$

Or $\boxed{a \propto -x}$

Where k/m is a constant

The equation shows that the motion 'm' is vibratory, and its acceleration is directly proportional to the displacement and directed towards the mean position. This type of motion is called simple harmonic motion.

CONNECTION BETWEEN S.H.M. AND UNIFORM CIRCULAR MOTION

CASE II:-

Let us consider a particle of mass 'm' moving around a vertical circle of radius 'x_0' with constant angular velocity "ω". If 'θ' is the angular displacement swept during time 't' then $\theta = \omega t$

The projection 'Q' of particle 'P' on the diameter AB of the circle vibrates to and fro about the centre of the circle as 'P' moves along circular path. It is also observed that the projection 'Q' speeds up when it moves towards the centre 'O' and slows down when it moves away from the centre.Thus the instantaneous acceleration of projection 'Q' is directed towards the centre and it is in vibratory motion.

The motion of 'Q' is associated with the motion of 'P' hence the acceleration of 'Q' must be a component of the acceleration of the motion of particle P.The acceleration of the particle P is Centripetal acceleration i.e. directed towards centre of the circle along the line PO and it given by,

$$a_c = \frac{v^2}{r}$$

or $$a_c = -\frac{{v_p}^2}{x_o}$$ (x_0 and a_c are in opposite direction)

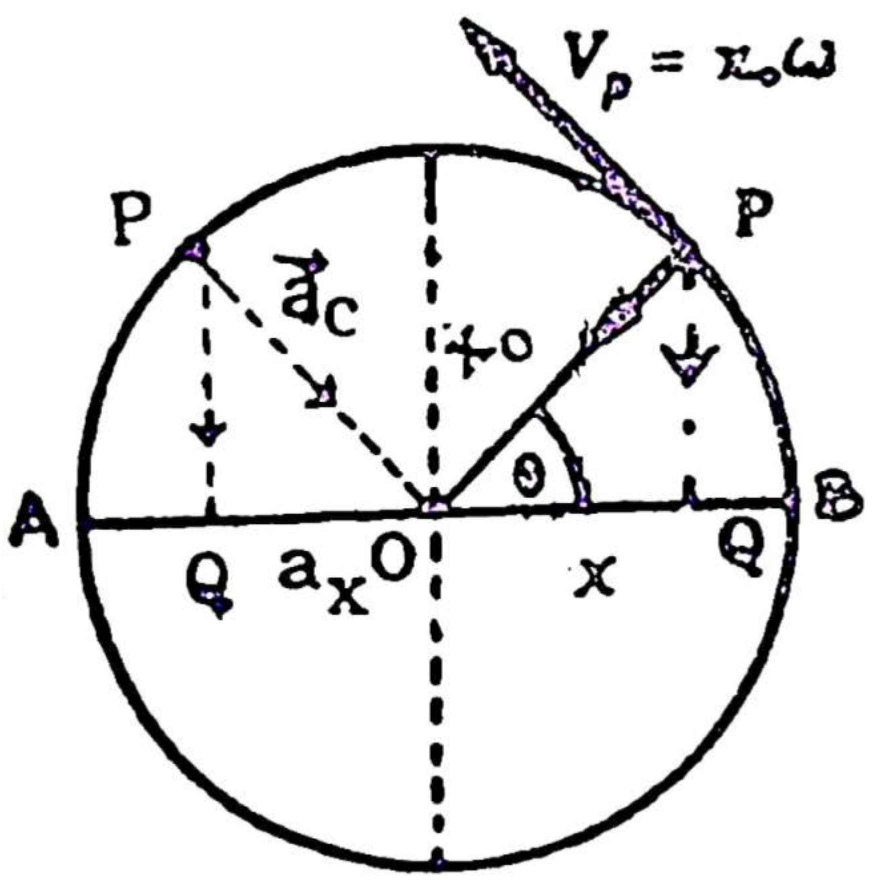
$$a_c = -\frac{(x_o\omega)^2}{x_o} \qquad \because v = r\omega$$

or $$a_c = -x_0\omega^2 \text{-------(i)} \quad \therefore v_p = x_0\omega$$

The acceleration of projection Q is equal to the component of acceleration of particle P along x axis .i.e.

$$a_x = a_c \cos\theta \text{ --------(ii)}$$

In triangle POQ



$$\cos\theta = \frac{\overline{OQ}}{\overline{OP}}$$

$$\cos\theta = \frac{x}{x_o} \text{ -------(iii)}$$

putting values from eq(i) and (iii) in eq(ii) we get ,

$$a_x = -x_0\omega^2 \frac{x}{x_o}$$

or $$a_x = -\omega^2 x$$

As we know that ω is constant, Therefore,

$$\boxed{a \propto -x}$$

Hence it is proved that the motion of projection Q of particle p executing uniform circular motion is S.H.M.

EQUATION OF DISPLACEMENT:

At some instant of time t, the angle between OP and x axis is ω t+ φ , where φ is the angle which

OP makes with the x axis at time t=0.This angle is known as initial phase angle.

In right angled triangle OPQ

$\cos\theta = \frac{\overline{OQ}}{\overline{OP}}$

$\cos(\omega\,t + \varphi) = \frac{x}{x_o}$ $\because\ \theta = \varphi + \omega t$

or

$x = x_o \cos(\omega\,t + \varphi)$

Where x_0=amplitude of S.H.M of Q and x =instantaneous displacement.

EQUATION FOR ACCELERATION:

As shown in the figure, a particle p is moving along the circumference of a circle of radius 'r; with its linear velocity V_p, its angular velocity ω is given by

$\omega = \frac{V_p}{r}$

or $v_p = r\omega$ ---------------- (1)

let the particle starts from 'A' and in time 't' it sweeps and angle θ then

$\theta = \omega\,t$

When the particle 'P' is at 'B' its projection 'Q' is at O. As the particle moves and reaches 'C', the projection of P (i.e.Q) along reaches C. When the particle is at D,Q again at O. And when P is at A, Q is also at A. Thus when the particle P moves along circular path. Q moves along AOC and back to COA. Hence the motion of O is along a straight line.

Since the particle P is moving along a circular path it centripetal acceleration a_c is given by

$a_c = -r\omega^2$

The acceleration of Q is along AOC, hence the component a_c along AOC will give the acceleration of O.The competent of a_c along AOC is given by

$a_o = a_c \cos\theta = \omega^2 r \cos\theta$

From figure r cosθ = x

$a_o = -\omega^2 x$

The negative sing shows that the acceleration of Q is directed towards the center and is proportional to x.

EQUATION OF VELOCITY:

The velocity of Q is equal to the X component of the velocity of P directed along AOC. Let Vo be the velocity of Q along AOC and Vp that of P along the tangent at any point on the circumference of the circle then

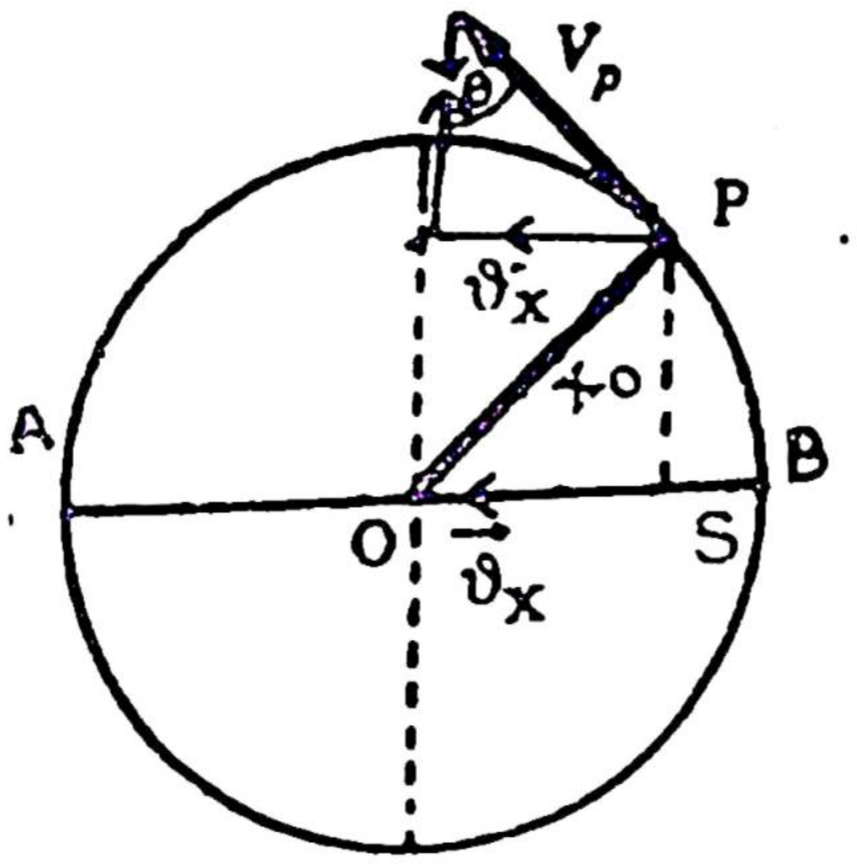
$V_Q = V_p \sin\theta = x_o\, \omega \sin\theta$ --------- (1)

From the relation $\sin^2\theta + \cos^2\theta = 1$, we get

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1 - \cos^2\theta}$

putting in equation (i)



$V_Q = x_0\omega\sqrt{1 - \left(\frac{x}{x_0}\right)^2}$

$V_Q = x_0\omega\sqrt{\frac{x_0^2 - x^2}{x_0^2}}$

$V_Q = \frac{x_0\omega}{x_0}\sqrt{x_0^2 - x^2}$

so,

$\boxed{V_Q = \omega\sqrt{x_0^2 - x^2}}$ ---------(ii)

i) At Extreme Position:

$x = x_0$, putting in eq (ii)

$V_Q = \omega\sqrt{x_0^2 - x_0^2}$

$V_Q = \omega\sqrt{0}$

$\boxed{V_Q = 0}$

The velocity of projection at the extreme position is equal to zero.

ii) At Mean Position:

$x = 0$, putting in eq (ii)

$V_Q = \omega\sqrt{x_0^2 - 0}$

$V_Q = \omega\sqrt{x_0^2}$

$$V_Q = \omega x_0$$

The velocity of projection at the extreme position is Maximum.

TIME PERIOD:

The time required to complete one cycle of motion is called time period. Denoted by "T".

A/c to the definition of angular velocity ω

$$\omega = \frac{\Delta\theta}{\Delta t}$$

For one complete cycle, $\Delta\theta = 360^0 = 2\pi, \Delta t = T$

$$\omega = \frac{2\pi}{T}$$

or

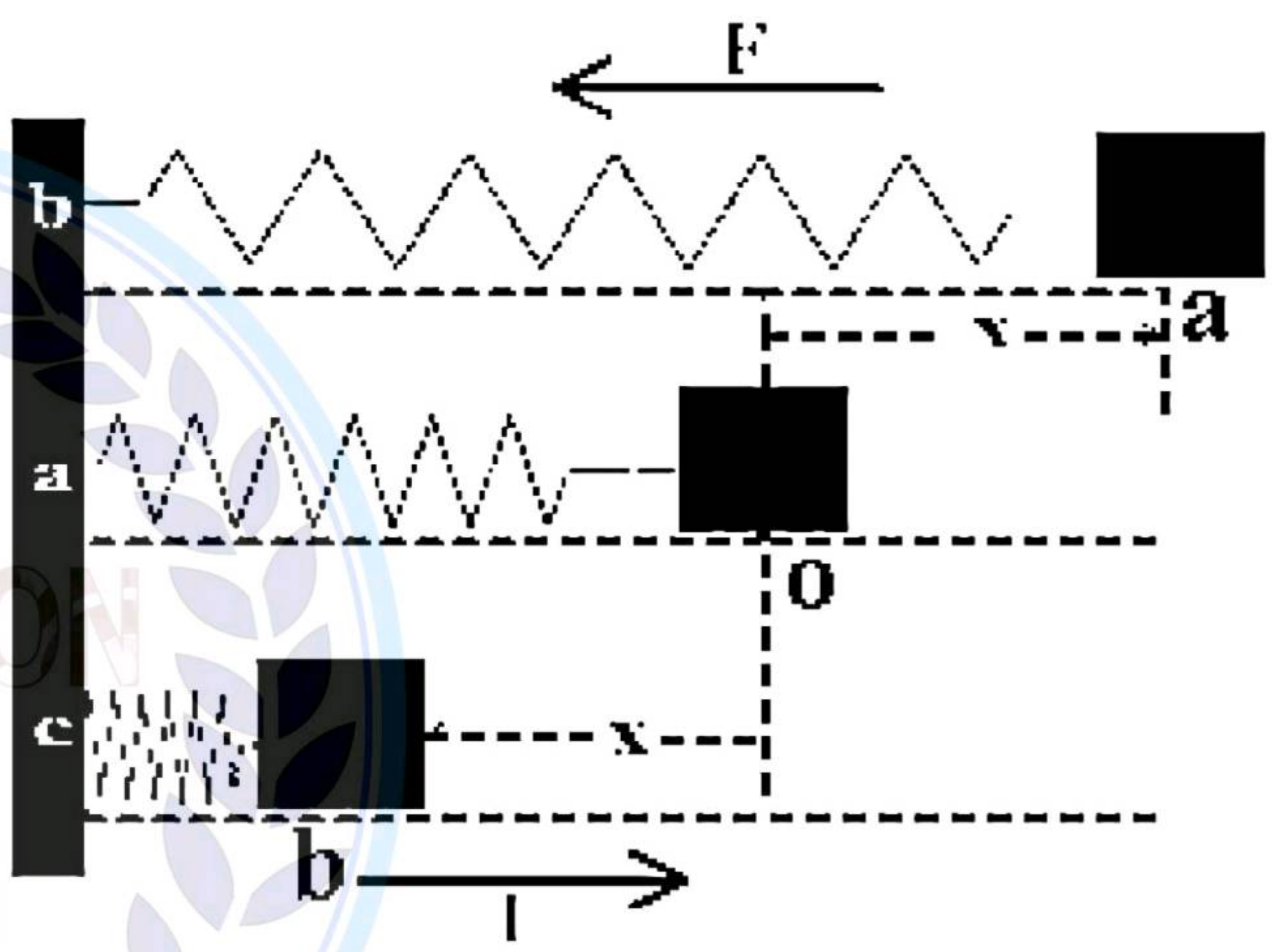
$$\boxed{T = \frac{2\pi}{\omega}}$$

ENERGY OF A BODY EXECUTION SIMPLE HARMONIC MOTION

Let us consider a mass "m" connected with one end of a string whose other end is connected with a rigid wall and it can execute SHM on friction less surface as shown in fig.

(I) KINETIC ENERGY:

The instantaneous velocity of the body when its displacement is x is given by,



$$V = \omega\sqrt{x_0^2 - x^2}$$

In case of Spring mass system $\omega = \sqrt{\frac{K}{m}}$

$$=> V = \sqrt{\frac{K}{m}}\sqrt{x_0^2 - x^2}$$

As we know that ,

K.E =1/2 mv²

K.E =1/2 m$\left(\sqrt{\frac{K}{m}}\sqrt{x_0^2 - x^2}\right)^2$

K.E =1/2 m$\frac{K}{m}(x_0^2 - x^2)$

$\boxed{\text{K.E} = 1/2\ K\,(x_0^2 - x^2)}$ --------(i)

(I) POTENTIAL ENERGY:

A/c to Hooke's law

F=Kx

At mean Position=> F=0

At extreme position => F=kx

Therefore the average force on mass "m" during the displacement x is

$$F = \frac{0+Kx}{2}$$

$$F = \frac{1}{2}kx$$

Now, A/c definition of P.E

P.E=avg. force x displacement

$$P.E = \frac{1}{2}kx \times x$$

$$\boxed{P.E = \frac{1}{2}kx^2} \text{------------(ii)}$$

(III) TOTAL ENERGY:

The energy of a body executing simple harmonic motion is the sum of potential energy and kinetic energy at that instant at a displacement x from the mean position.

E=K.E + P.E

putting values from eq(i) and eq(ii)

$$E = 1/2\ K\ (x_0^2 - x^2) + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}kx_0{}^2 - \frac{1}{2}kx^2$$

$$\boxed{E = \frac{1}{2}kx_0{}^2}$$

The above equation shows that the total energy of a particle executing simple harmonic motion is proportional to the square of its amplitude of vibration.

SIMPLE PENDULUM

DEFINITION:

An ideal simple pendulum consists of a point mass suspended from a light inextensible string tied to a grid and friction less support when the bob of the pendulum is vertical, the gravitational force W acts vertically downward and the tension T acts vertically upwards in this case the force W is balanced by the tension 'T' in the string.

CASE III:

EXPLANATION:

When the bob is slightly displaced from its mean position, it beings to perform oscillatory motion. Let us see the motion of the bob.

The bob of the pendulum is under the action of two force.

(i) The gravitational force W = mg acting vertically downwards

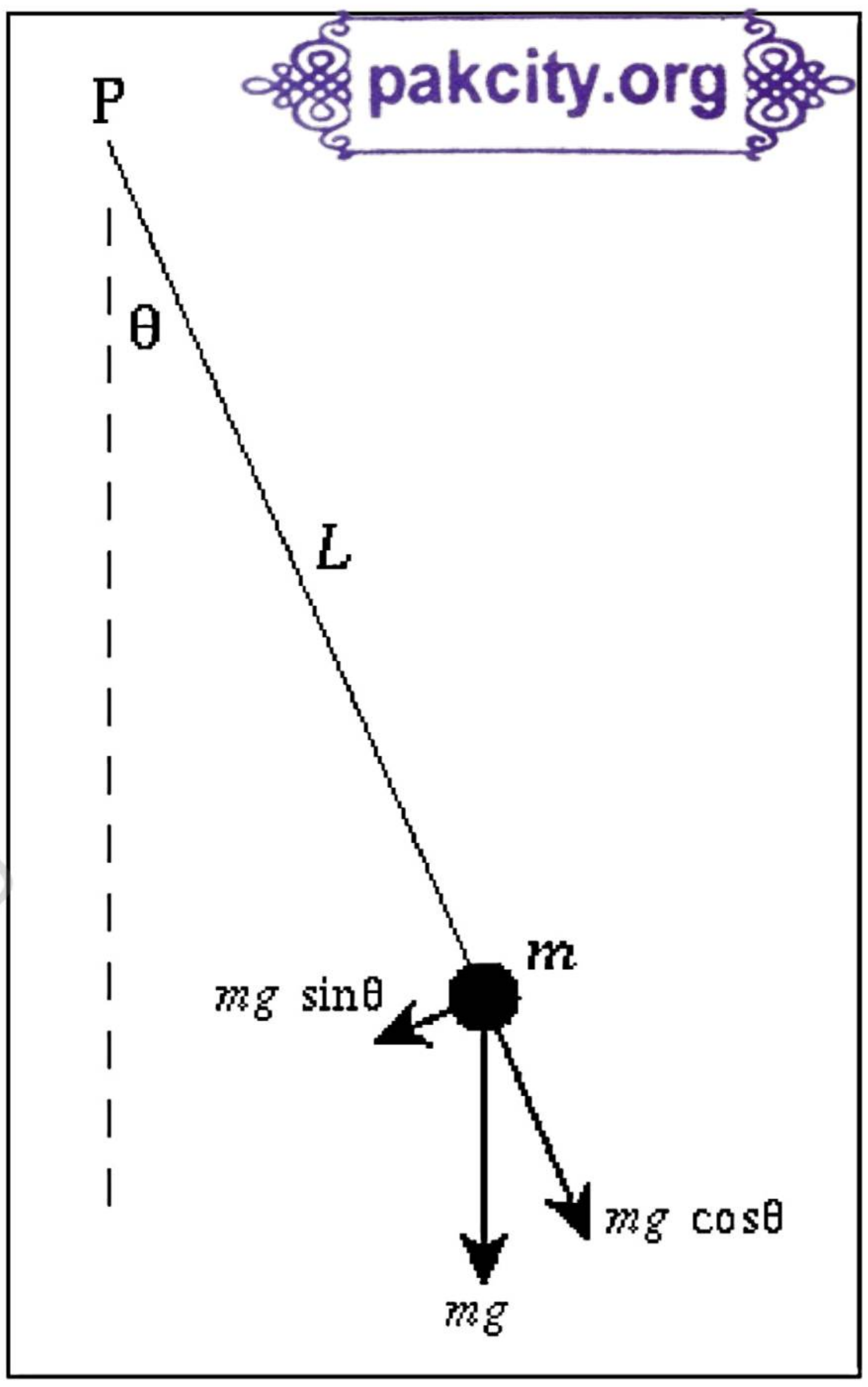
(ii) The tension T acts along the string.

The net force acting on the bob is $F_{Net} = W - T$

Resolving W into two components

(i) along the length of string (parallel) and

(ii) perpendicular to the string.



$$W_{\parallel} = mg\cos\theta \text{ and } W_{\perp} = mg\sin\theta$$

Since there is no motion of the bob along the string, a net force in the direction of string is zero. in this case.

$$mg\cos\theta = T$$

Hence the magnitude of the net force is $F_{Net} = mg\sin\theta$ brings the bob to its mean position.

From Newton's second law of motion $F_{Net} = ma$

$\cancel{m}a = \cancel{m}g \sin\theta$ (since the force is directed towards mean position, hence,)

$$a = -g\sin\theta$$

If θ is small $\sin\theta \approx \theta$

then $a = -g\theta$ -------------(i)

As we know that

$S = r\theta \quad \Rightarrow \theta = S/r$

In this case S=x and r=l (length of string)

so,

$$\theta = x/l$$

Putting in eq(i)

$$a = -g(x/l)$$

or $a = -(g/l)\,x$

where g and l are constant,therefore,

$$\boxed{a \propto -x}$$

Hence it is proved that the motion of pendulum is S.H.M.The acceleration of the simple pendulum is directly proportional to the displacement and directed towards the mean position. Thus the motion of simple pendulum is simple harmonic.

TIME PERIOD OF SIMPLE PENDULUM:

A/C to the definition of time period

$$T = \frac{2\pi}{\omega} \text{---------(ii)}$$

In case of circular motion

$$a = -\omega^2 x \text{-----------(iii)}$$

and

In case of Pendulum

$$a = -\frac{g}{l}x \text{-----------(iv)}$$

comparing eq(iii) and eq(iv), we get

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

putting in eq(ii),we get

$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}}$$

or

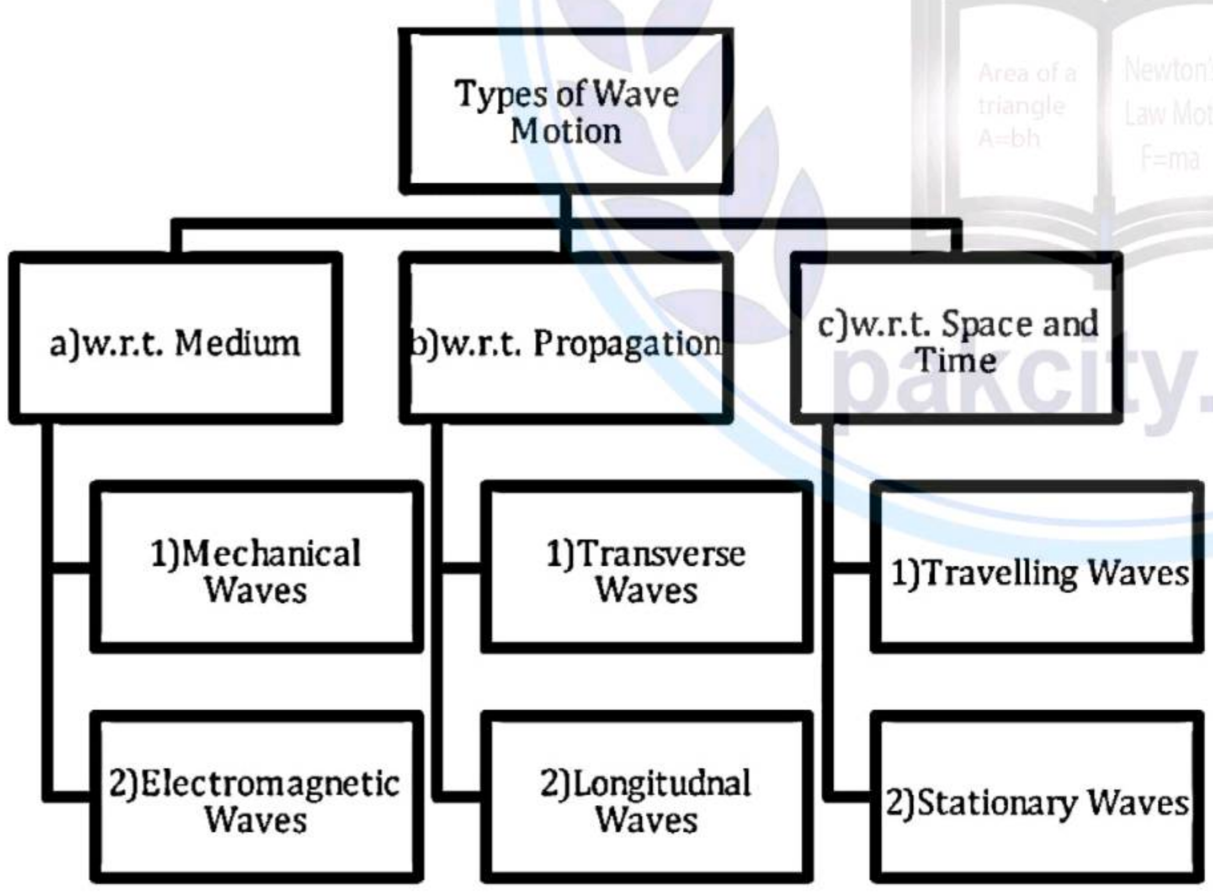
$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

WAVE MOTION

DEFINITION:

The mechanism by which energy transfers from one point to another is known as wave motion. Wave motion is a form of disturbance, which travels through a medium due to periodic motion of particles of the medium about their mean position.

TYPES OF WAVES:



A) W.R.T. MEDIUM:

1.MECHANICAL WAVES:

Those waves which require a particular medium for their propagation are known as mechanical waves.

For Example: Waves in water, Waves in String, Sound Waves e.t.c.

2. ELECTROMAGNETIC WAVES:

Those waves which do not require a particular medium for their propagation are known as electromagnetic waves.

For Example: Light waves, X rays, Microwaves e.t.c.

B) W.R.T. PROPAGATION:

1.TRANSVERSE WAVES:

Those waves in which particles of medium oscillate perpendicular to the direction of propagation of waves are called transverse waves. These waves consists of "Crests" and "Troughs". These transverse waves can be mechanical or electromagnetic in nature, water waves, waves in string are the examples of mechanical and transverse waves, while the x rays, radio waves, light waves are the example of electromagnetic transverse waves.

2.LONGITUDNAL WAVES:

Those waves in which particles of medium oscillate parallel to the direction of propagation of waves are called transverse waves. These waves consists of "Compressions" and "Rarefactions". These waves are produced in elastic materials like gases and springs. These waves are produced due to the high and low pressure zones in the medium. For Example: sound waves are longitudinal.

C) W.R.T. SPACE AND TIME:

1.TRAVELLING WAVES:

Travelling waves are those waves in which the displacement depends on both space and time. In case of travelling wave all particles of the medium vibrate simple harmonically with the frequency equal to the frequency of vibration of source that drives the wave into medium.

A travelling wave can be produced in a medium by disturbing its one end.

Mathematically the wave function is given by,

$$y=f(x)$$

Here y is the vertical displacement of particle from its mean position and x is the horizontal displacement. As we know that when the wave travels though a medium then its position changes w.r.t.

time also. So we can give the location of wave by the following equation.

y=f(x,t)

This is called wave function.

For travelling wave, moving along + x-axis the function will be, y=f(x - vt)

and

For travelling wave, moving along - x-axis the function will be, y=f(x + vt)

2.STATIONARY OR STANDING WAVES:

The waves which are formed due to the overlapping of two travelling waves of same amplitude and frequency moving in opposite direction in the same medium are called stationary waves. These waves are only the function of time not space.

STANDING OR STATIONARY WAVES IN A STRING:

When a stretched is fixed between two supports and is then plucked from the middle, the crest extends the whole length between the supports. At each end, the wave reflects from the denser medium and hence it suffers a phase change. The crest returns as a crest. Thus a wave is set up between two fixed points which lasts for a long time.At the end P and Q, the incident and reflected waves are always equal in amplitude and opposite in phase and hence the ends are stationery. Such waves are called stationery waves.

In a stationery waves the points of minimum displacement are called NODES (N) and the points of maximum displacement are called ANTI NODES (A).

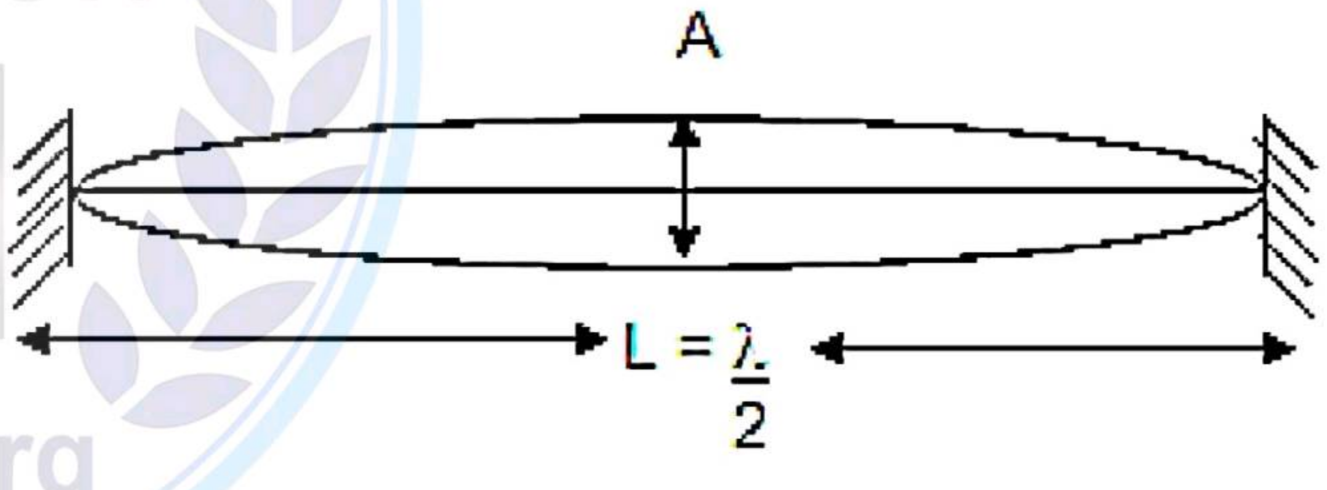
FUNDAMENTAL FREQUENCY AND HARMONICS:

When a stretched string is fixed between two ends and then plucked, stationery waves are produced due to superposition of wave's incident and reflected from the ends. The string can vibrate into several segments. There vibrations are called normal modes. Each mode has its characteristic frequency.

(i) For ONE loop:

When the string is plucked from the center it vibrates in one loop. The frequency of such a vibration is called fundamental frequency or first fundamental frequency or first harmonics and it is the lowest frequency with which the string can vibrate. In this case.

$$L = \frac{\lambda}{2} \quad or\ \lambda = 2L$$



Let ν_1, the frequency of first harmonics is given by

$$\because V = \nu\lambda \quad \text{or}\ \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_1 = \frac{V}{2L}}$$

(ii) For TWO loops:

When the string vibrates in two loops, the frequency is called second harmonics, In this case.

$$\lambda = L$$

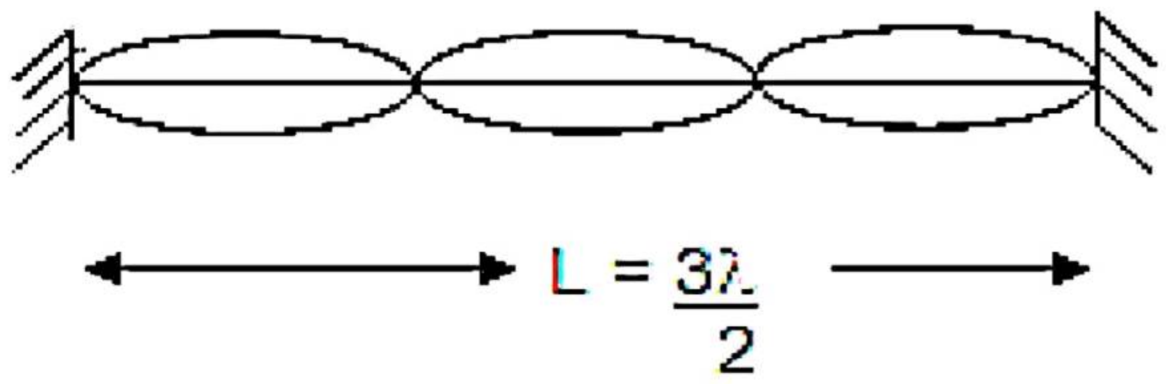
Let ν_2, the frequency of second harmonics is given by

$$\because V = \nu\,\lambda \quad \text{or } \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_2 = \frac{V}{L}}$$

(iii) For THREE loops: When the string vibrates in two loops, the frequency is called second harmonics, In this case.



$$L = 3\frac{\lambda}{2} \quad or\ \lambda = \frac{2L}{3}$$

Let ν_3, the frequency of third harmonics is given by

$$\because V = \nu\,\lambda \quad \text{or } \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_3 = \frac{3V}{2L}}$$

(iv) For "n" loops: When the string vibrates in two loops, the frequency is called second harmonics, In this case.

$$L = n\frac{\lambda}{2} \quad or\ \lambda = \frac{2L}{n}$$

Let ν_3, the frequency of third harmonics is given by

$$\because V = \nu\,\lambda \quad \text{or } \nu = \frac{V}{\lambda}$$

or

$$\boxed{\nu_n = \frac{nV}{2L}}$$

$$\because \nu_1 = \frac{V}{2L}$$

so,

$$\boxed{\nu_n = n\nu_1}$$

Thus we see that in case of a string fixed at both ends the harmonics are integral multiple of the fundamental frequency.

SUPERPOSITION PRINCIPLE

PRINCIPLE:

When two or more than two waves overlap each other, then a resultant wave is formed. The net wave displacement caused by the resultant wave is found equal to the algebraic sum of the individual wave displacements of all given waves. Mathematically we can write as,

$$Y = y_1 + y_2 + y_3 + \cdots + y_n$$

EXPLANATION:

Let us consider two sinusoidal waves with the same amplitude, frequency and wavelength and travelling in opposite direction. i.e.

$y_1 = A_0\ Sin(kx - \omega t)$ and
$y_2 = A_0\ Sin(kx + \omega t)$

where,

A_0=Amplitude of wave
$k = \frac{2\pi}{\lambda}$ = Angular wave number
$\omega = 2\ \pi f$
x= Position
t= Time

According to the superposition principle the resultant wave is given by,

$$Y = y_1 + y_2$$

By putting values ,

$$Y = A_0\ Sin(kx - \omega t)\ + A_0\ Sin(kx + \omega t)$$
$$Y = A_0[\ Sin(kx - \omega t)\ +\ Sin(kx + \omega t)\ ]$$
$$Y = A_0\ [2Sin(kx)Cos(\omega t)\ ] \qquad \because Sin(\alpha - \beta)\ +\ Sin(\alpha + \beta) = Sin \propto Cos\ \beta$$

This equation represents the wave function of the stationary wave in which the amplitude is equal to $2\ A_0\ Sin(kx)$.

POSITIONS OF NODES:

As we know that the nodes are the point of minimum amplitude or intensity ,therefore

$$Sin(kx) = 0$$
$$kx = \sin^{-1} 0$$
$$\frac{2\pi}{\lambda} x = 0^0, 180^0, 360^0, 540^0, \ldots$$

or $$\frac{2\pi}{\lambda} x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$$

or $$x = \frac{\lambda}{2\pi}(0, \pi, 2\pi, 3\pi, 4\pi, \ldots)$$

or

$$\boxed{x = 0, \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}, 2\lambda, \ldots}$$

These are the positions of Nodes in standing waves, which has the minimum amplitudes.

POSITIONS OF ANTINODES:

As we know that the antinodes are the point of maximum amplitude or intensity ,therefore

$$Sin(kx) = \pm 1$$

$$kx = \sin^{-1} \pm 1$$

$$\frac{2\pi}{\lambda}x = 90^0, 270^0, 450^0, 630^0, ...$$

or $$\frac{2\pi}{\lambda}x = \frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}, 7\frac{\pi}{2}, ...$$

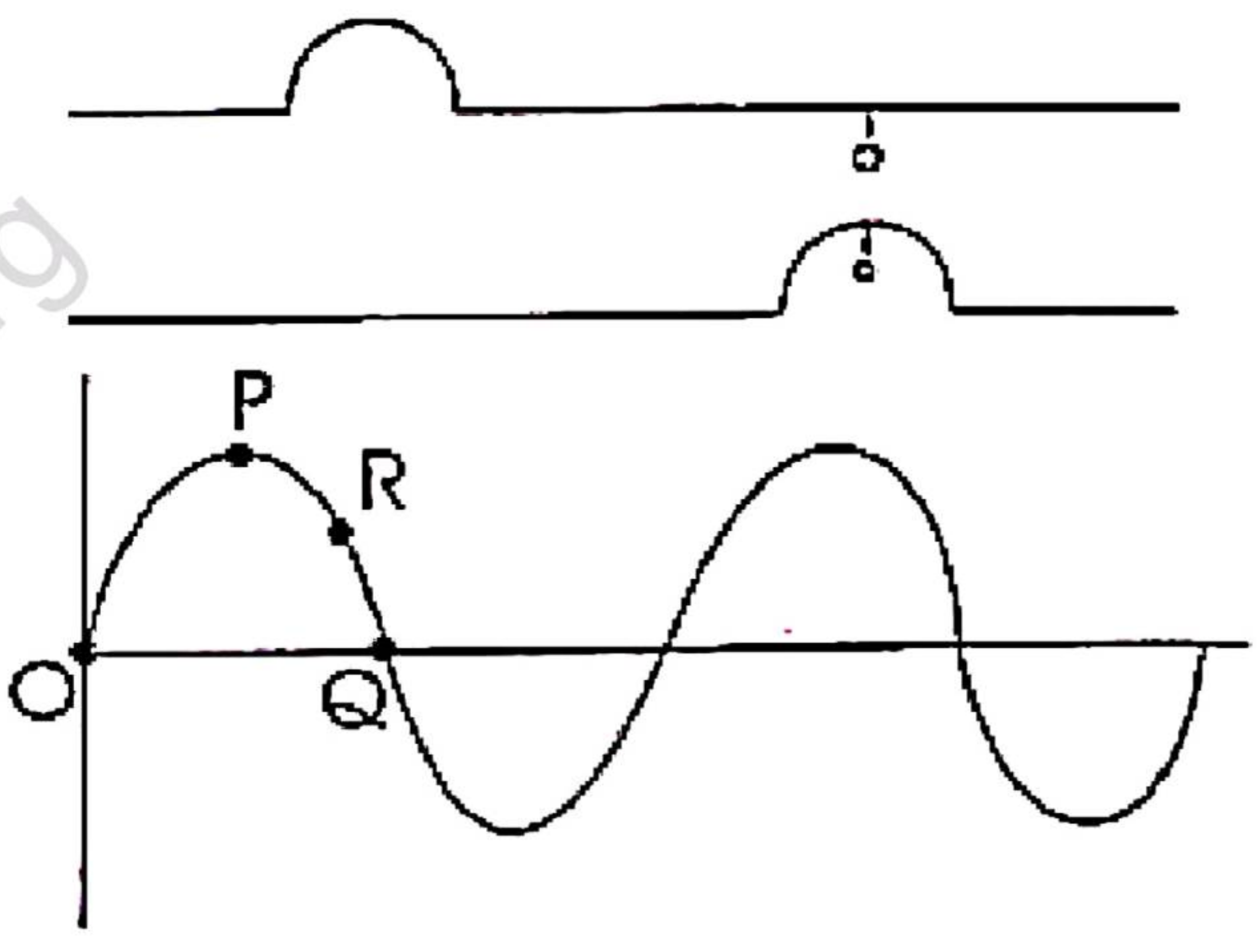
or $$x = \frac{\lambda}{2\pi}\left(\frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}, 7\frac{\pi}{2}, ...\right)$$

or

$$\boxed{x = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}, 7\frac{\lambda}{4}, ...}$$

These are the positions of Antinodes in standing waves, which has the maximum amplitudes.

ENERGY IN WAVES



Let a harmonic wave travelling along a string. The points P,Q and R represent various segments of the string which move vertically. The wave moves a distance equal to one wavelength 'λ' in time period 'T'. We know that every point of the string moves vertically up or down.Thus every segment of equal mass has the same total energy. The energy of the segment P is entirely potential energy since the segment is momentarily stationary. The energy of the segment Q is entirely kinetic energy and segment R has both kinetic & potential energies. Suppose at point Q, the mass of the segment of the string is Δm and it has maximum transverse velocity $V_{y\,MAX}$. Then the total energy of the segment is.

$$\Delta E = K.E$$

$$\Delta E = \tfrac{1}{2}\,\Delta m\,(V_{y\,MAX})^2$$

$$\Delta E = \tfrac{1}{2}\,\Delta m\,(y_0\omega)^2$$

$$\Delta E = \tfrac{1}{2}\,\Delta m\,y_0^2\omega^2$$

Now , power is defined as

$$P = \frac{\Delta E}{T}$$

$$P = \frac{\tfrac{1}{2}\,\Delta m\,y_0^2\omega^2}{T}$$

$$P = \frac{1}{T} \times \frac{1}{2}\,\Delta m\,y_0^2\omega^2$$

$$P = \frac{\lambda}{T} \times \frac{1}{2}\frac{\Delta m}{\lambda}\,y_0^2\omega^2$$

$$P = \lambda\nu \times \frac{1}{2}\frac{\Delta m}{\lambda}\,y_0^2\omega^2 \qquad \because \nu = \frac{1}{T}$$

$$P = v \times \frac{1}{2}\frac{\Delta m}{\lambda}\,y_0^2\omega^2 \qquad \because v = \lambda\nu$$

$$P = v \times \frac{1}{2}\mu\,y_0^2\omega^2 \qquad \because \mu = \frac{\Delta m}{\lambda}$$

or

$$P = \frac{1}{2} v \mu y_0^2 \omega^2$$

This result shows that the power transmitted by harmonic waves produced 0" a string is proportional to,

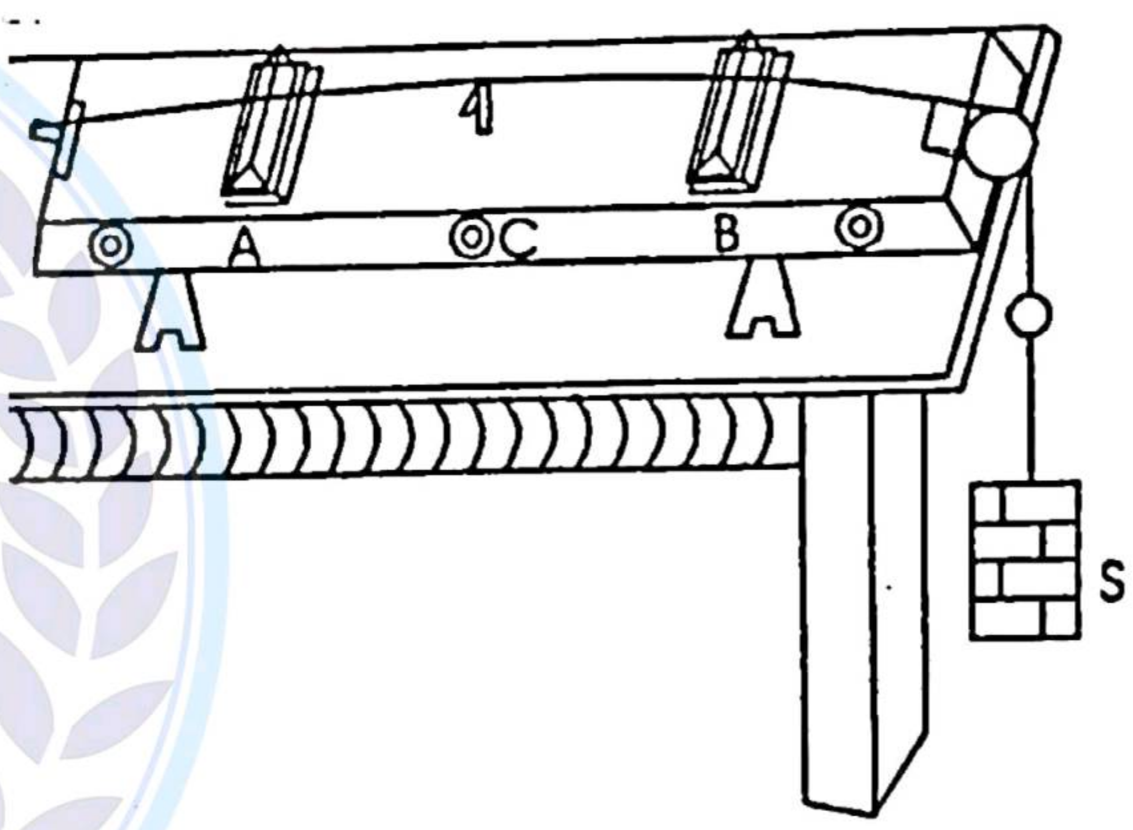
(i) The Velocity of the waves 'V'
(ii) The square of the frequency ω
(iii) The square of the amplitude y_o
(iv) The linear density of the medium (String) μ

SONOMETER

A Sonometer consists of a hollow wooden box over which steel wire a stretched. One end of the wire is tied to a peg and the other end passes over a friction – less pulley. A hanger is tied at the other end. The hanger carries slotted weights to change the tension 'T' in the wire. Two bridges C and D are placed below the wire. The length of the vibrating wire can be changed by changing the distance between the bridges C and D. The Sonometer is used for determining the frequency of a turning fork.

In order to find the frequency of a given tuning fork, a tension T = Mg is produced in the wire by keeping a mass M in the hanger. A thin piece of paper called rider is placed on the wire between C and D. The stem of the vibrating tuning fork is placed against the board of the Sonometer and the distance between the bridges is adjusted, when the frequency of the vibrating wire is in unison with the frequency of the tuning fork, the rider will jump off the wire. Let L be the distance between the bridges at this, position, and μ be mass per unit length (Linear density) of the wire, then the fundamental frequency of the tuning for will be,

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \text{--------------(i)}$$



Where $\sqrt{\frac{T}{\mu}}$ speed of a transverse wave in the wire and L is half the wavelength

Equation (1) also gives the three laws of transverse vibration of string.

(i) The frequency produced in a stretched string is inversely proportion to its length,

$$f \propto \frac{1}{L}$$

(ii) For a given mass per unit length and length of string the frequency is directly proportional to the square root of its tension.

$$f \propto \sqrt{T}$$

(iii) For a given tension and length of string the frequency is inversely proportional to the square root of its mass per unit length.

$$f \propto \frac{1}{\sqrt{\mu}}$$

These three laws can be verified by a Sonometer.

SOUND

DEFINITION:

"A vibration transmitted by air or other medium in the form of alternate compressions and rarefactions of the medium is known as Sound."

PRODUCTION OF SOUND:

Sound is produced by a vibrating body like a drum, bell, etc, when a body vibrates. due to the to and fro motion of the drum, compressions and rarefactions are produced and transmitted or pro pagated in air. When a body vibrates in air, it produces longitudinal waves by compressions and rarefactions. These compressions and rarefactions are traveled by the particles of the medium and transferred into the next particles. Due to this transference, sound propagates in a medium.

1. INFRA SONIC SOUND: The term "infrasonic" applied to sound refers to sound waves below the frequencies of audible sound, and nominally includes anything under 20 Hz.

2. AUDIBLE FREQUENCY RANGE: Usually "sound" is used to mean sound which can be perceived by the human ear, i.e., "sound" refers to audible sound unless otherwise classified. A reasonably standard definition of audible sound is that it is a pressure wave with frequency between 20 Hz and 20,000 Hz

3. ULTRA SONIC SOUND: The term "ultrasonic" applied to sound refers to anything above the frequencies of audible sound, and nominally includes anything over 20,000 Hz. Frequencies used for medical diagnostic ultrasound scans extend to 10 MHz and beyond.

SPEED OF SOUND WAVES

NEWTON'S FORMULA FOR THE SPEED OF SOUND WAVE:

In case of mechanical waves, the velocity of propagation depends upon the ratio between the elastic property of the medium (bulk modulus), and the inertial property of the medium (density).

Sound waves are compression waves which propagate through compressible medium such as air. For compression waves, the elastic property describes how the medium responds to changes in pressure with a change in volume. This is known as bulk modulus B .

$$B = \frac{-\Delta P}{\Delta V/V}$$

Where ΔP is the change in pressure and Δv is the change in volume V. The negative sign ensures that an increase in pressure ($\Delta P > o$) causes a decreases in volume ($\Delta v < o$).

The inertial property of a medium is gives by its density "ρ" .Hence the speed of sound wave in a medium is given by

$$v = \sqrt{\frac{B}{\rho}}$$

Newton's formula was based on the assumption that when compressions and refractions travel through medium (air or gas), the temperature of the air remains constant and Boyle's law is obtained under this isothermal process. So the Bulk modulus B is equal to the pressure P.

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

The above formula is known as Newton's formula for the speed of sound.The speed of sound in air as obtained by Newton's formula was not in good agreement with the experimental results. Theoretical value was less than the experimental value.

LAPLACE CORRECTION:-

Laplace suggested that when compressions and rarefaction travel through air, the temperature falls. Therefore, the compression and rarefactions occur adiabatically.

In such a case the bulk modulus of the gas is not equal to the pressure of the gas but, 'γ' times the pressure of the gas. Where, 'γ' is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume, For air, $\gamma = 1.4$.

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

This is known as Laplace correction.

For an ideal gas PV = nRT

$$v = \sqrt{\frac{\gamma nRT}{V\rho}}$$

As we know that, $V\rho$=m (mass of gas) so,

$$v = \sqrt{\frac{\gamma nRT}{m}}$$

or,

$$\boxed{v = \sqrt{\frac{\gamma RT}{M}}} \qquad \because M = n/m$$

The speed of sound wave is directly proportional to the square root of the temperature in Kelvin Scale.

BEATS

The periodic alternation of sound between a maximum and minimum loudness caused by the super position of two waves of nearly the same frequency are called beats.

PRODUCTION OF BEATS:

Take two tuning forks A and B of nearly the same frequencies say 32 and 30 hz. Respectively place them at equal distance from the ear. Let at time t = 0, the two forks are in phase and the right hand prongs of both the forks are sending compressions towards right. There two compressions will arrive at the ear together and thus a loud sound in heard.

As time goes on the fork B, vibrating at slightly lower frequency than A will begin to fall behind. After ¼ second the fork A will complete 8 vibrations and will just be sending out compressions. On the other hand the fork B will complete 71/2 vibrations and will sending out a rarefaction from B will reach the ear at the same time. They will cancel each other. Hence no sound will be heard.

As time passes the fork B will still fall behind A. After half a second the fork A will complete 16 vibrations while the fork B will complete 15 vibrations. Both the forks will be sending out compressions together and thus again a loud sound will be heard.

After ¾ second – fork A will complete 24 vibrations and fork B will complete 22½ vibrations. At this compression and fork B will be sending a rarefaction. B will be sending a rarefaction. Thus no sound will be heard.

After 1 second the fork A will complete 32 vibrations. Both the forks will be sending out compressions together and thus again a loud sound will be heard.

We have seen that in one second two beat are produced. The difference between the frequencies is also two. Thus we conclude that the number of beats per second is equal to the difference between the frequencies of the forks. The maximum beat frequency that the human ear can detect is 7 beats per second. When the beat frequency is number of beats produced per second) is greater than 7 we cannot hear them clearly.

ANALYTICAL TREATMENT OF BEATS:

Consider two waves with equal amplitude traveling through a medium in the same direction having slightly different frequencies f_1 and f_2. The displacement that each wave produces can be represented by the equation.

$Y_1 = A_o \cos 2\pi f_1 t$ ------------ (1)

$Y_2 = A_o \cos 2\pi f_2 t$ ------------ (2)

Where y1 and y2 are the instantaneous displacement of the waves (1)and (2). Let Y be the instantaneous displacement of the resultant wave, then by the principle of superposition of waves, the displacement Y of the new wave can be found out by adding the two displacements.

$Y = Y_1 + Y_2$

$Y = A_o (\cos 2\pi f_2 t + \cos 2\pi f_2 t)$ $\because \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\ \cos\frac{\alpha-\beta}{2}$

$Y = 2A_o \cos 2\pi \frac{(f_1 - f_2)}{2} t \times \cos 2\pi \frac{(f_1 + f_2)}{2} t$ --------- (3)

The resultant displacement y as expressed by equation (3) has an affective frequency equal to the average $(f_1 + f_2)$ and amplitude is given by

$A = 2A_o \cos 2\pi \frac{(f_1 - f_2)}{2} t$ ------------ (4)

Equation (4) shows that the amplitude varies in time with frequency given by $(f_1 - f_2)$ when f_1 close to f_2 the amplitude variation is shown by the dotted line of the resultant wave. A beat of maximum amplitude is detected whenever.

$\cos 2\pi \frac{(f_1 - f_2)}{2} t = \pm 1$

There will be two maxima in each cycle. Since the amplitude varies with the frequency as, the number of beats per second. Hence the beat frequency f_b is twice this value.

$f_b = f_1 - f_2$ --------------- (5)

CHARACTERISTICS OF MUSICAL SOUND

Musical sounds of tones can be distinguished from one other by the following characteristics.

(i) Intensity or loudness (ii) Pitch or frequency (iii) Quality.

INTENSITY AND LOUDNESS:

The characteristics of sound by which we can distinguish between Loud and Faint sound is called Loudness. The intensity of sound is defined as the amount of energy transmitted per sound through unit area held perpendicular to the direction of propagation of sound wave. It is denoted by I.

$$Intensity = \frac{Energy\ \ transmitted}{Time\ \ \times area}$$

or

$$I = \frac{E}{t \times A}$$

Unit: It's S.I unit is $\frac{Joule}{sec \times m^2}$ or $\frac{watt}{m^2}$

WEBER – FETCHNER LAW:

Weber and Fetchner found experimentally that the loudness depends upon the intensity as well as on ear. The ear operates on logarithmic scale rather than responding linearly to the intensity of sound.

The loudness (L) of the sound as received by the ear, is proportional to the logarithm of its intensity 'I'.

$$L \propto Log\ I$$

or

$$\boxed{L = K\ Log\ I}$$

Where k is a constant of proportionally.

INTENSITY LEVEL:

If Io and I be intensities of two sounds waves, then the difference in the loudness of sound ($L - L_0$) is known as the intensity level between them.

Intensity level $= L - L_o = K \log I - K \log I_o$

$$\boxed{S_L = K \log [I/I_o]}$$

Unit of intensity level (loudness) and is called as bel after the name of Alexandar Graham Bell. Since bel is a big unit of loudness and its submultiples are used for practical purposes.

1 deci bel = 0.1 bel.

1 centi bel = 0.01 bel.

PITCH OF SOUND:

The characteristics of sound by which a shrill sound can be distinguished from a grave sound is called pitch of sound.It depends upon the frequency of sound. More the frequency higher the pitch lowers the frequency lowers the pitch of sound. For example the pitch of sound produced by rats, bats, cats is higher than that of frog, dogs beating drums.

QUALITY OF SOUND:

The quality of sound is that characteristic of sound which enables the ear to recognize a sound also assigns its source.

The note played at piano to be different from the note played at the violin through both has same frequency and loudness it is because of the quality of two notes is different. A wave form which is a combination of fundamental frequency and second harmonic and a wave from which is a combination of the fundamental frequency and third harmonic will produce sound of different qualities though their pitch and loudness may be same.

DOPPLER'S EFFECT

The change in the pitch (frequency) of sound caused by the relative motion between the source and observer is called Doppler's Effect.

CASE 1(A): When listener moves towards the stationary source of sound.

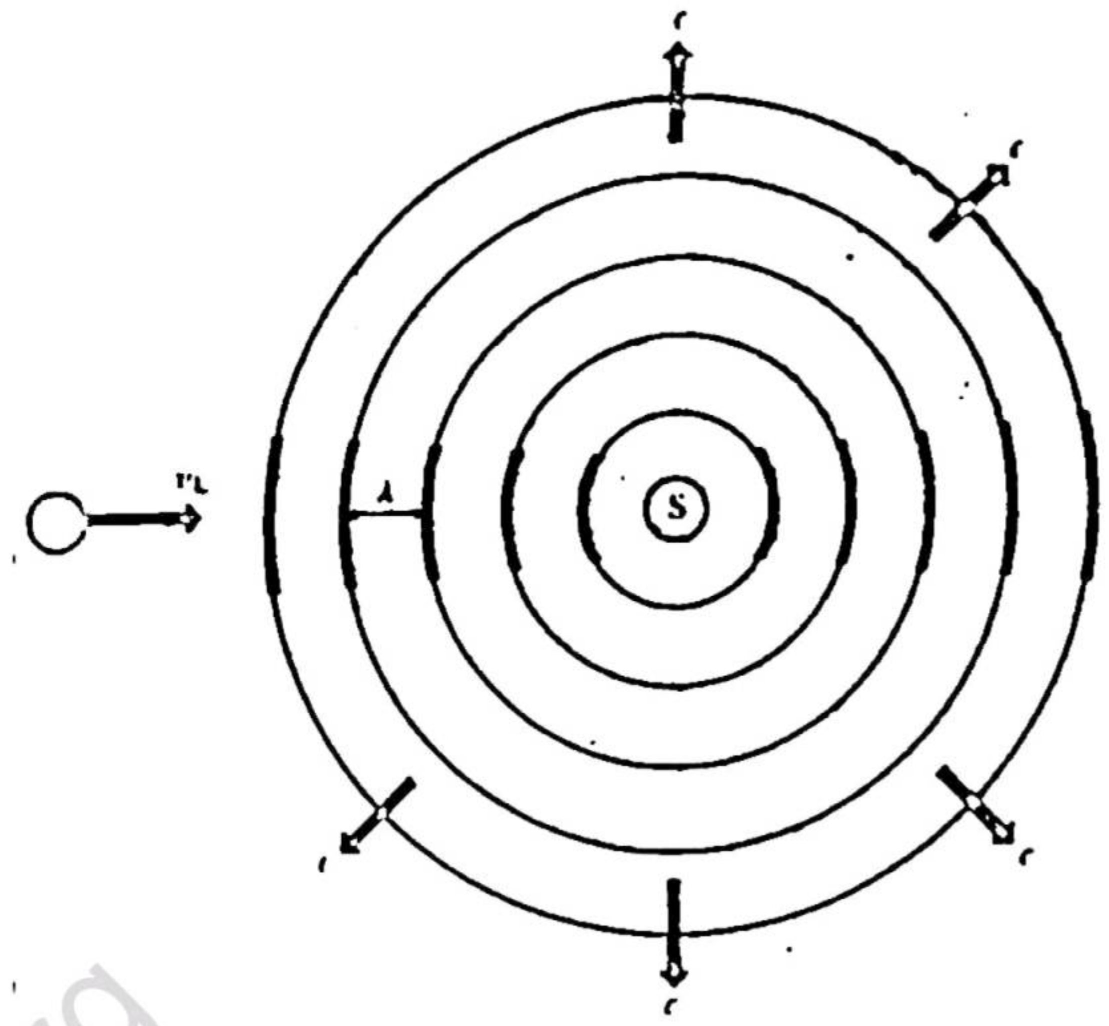
Let us consider the listener is moving towards the stationary source of sound emitting sound of frequency ν ,with velocity V_L.The speed of sound is V.In this case the apparent frequency heard by listener is ν'.

As we know that

$$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu} \text{----(i)}$$

and

$$\nu = \frac{V}{\lambda} \text{ (Real Frequency)}$$



But the relative velocity of sound for the listener will be $V+V_L$, then apparent frequency will be

$$\nu' = \frac{V+V_L}{\lambda}$$

putting the value of λ from eq(i),we get

$$\nu' = \frac{V+V_L}{\frac{V}{\nu}}$$

or

$$\boxed{\nu' = \left(\frac{V+V_L}{V}\right)\nu}$$

This expression shows that the apparent frequency will be greater than real frequency.

CASE 1(B): When listener moves away from the stationary source of sound.

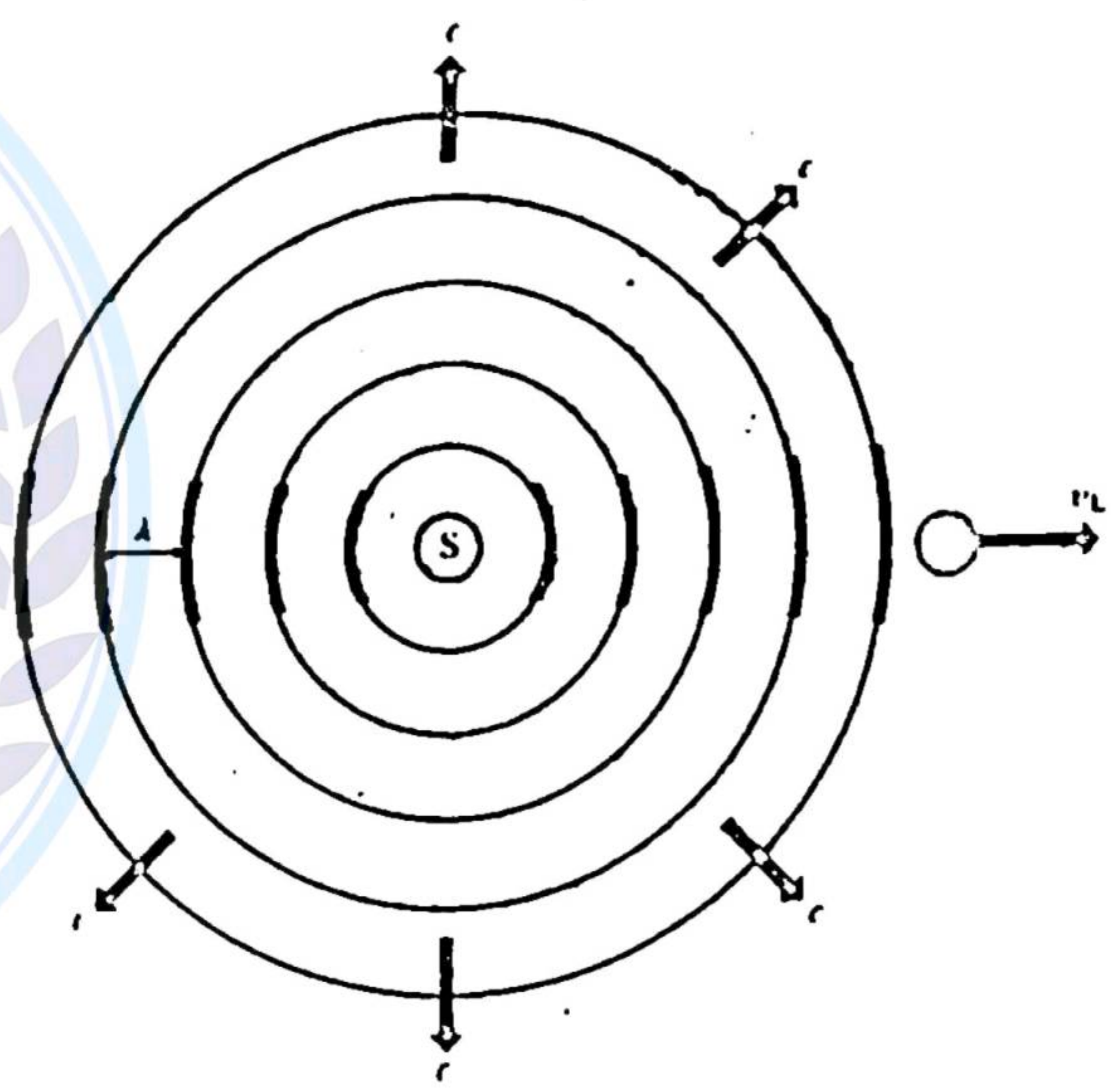
Let us consider the listener is moving towards the stationary source of sound emitting sound of frequency ν ,with velocity V_L.The speed of sound is V.In this case the apparent frequency heard by listener is ν'.

As we know that

$$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu} \text{----(ii)}$$

and

$$\nu = \frac{V}{\lambda} \text{ (Real Frequency)}$$



But the relative velocity of sound for the listener will be $V-V_L$, then apparent frequency will be

$$\nu' = \frac{V-V_L}{\lambda}$$

putting the value of λ from eq(ii),we get

$$\nu' = \frac{V-V_L}{\frac{V}{\nu}}$$

or

$$\boxed{\nu' = \left(\frac{V-V_L}{V}\right)\nu}$$

This expression shows that the apparent frequency will be less than real frequency.

CASE 2(A): When source of sound moves towards the stationary listener.

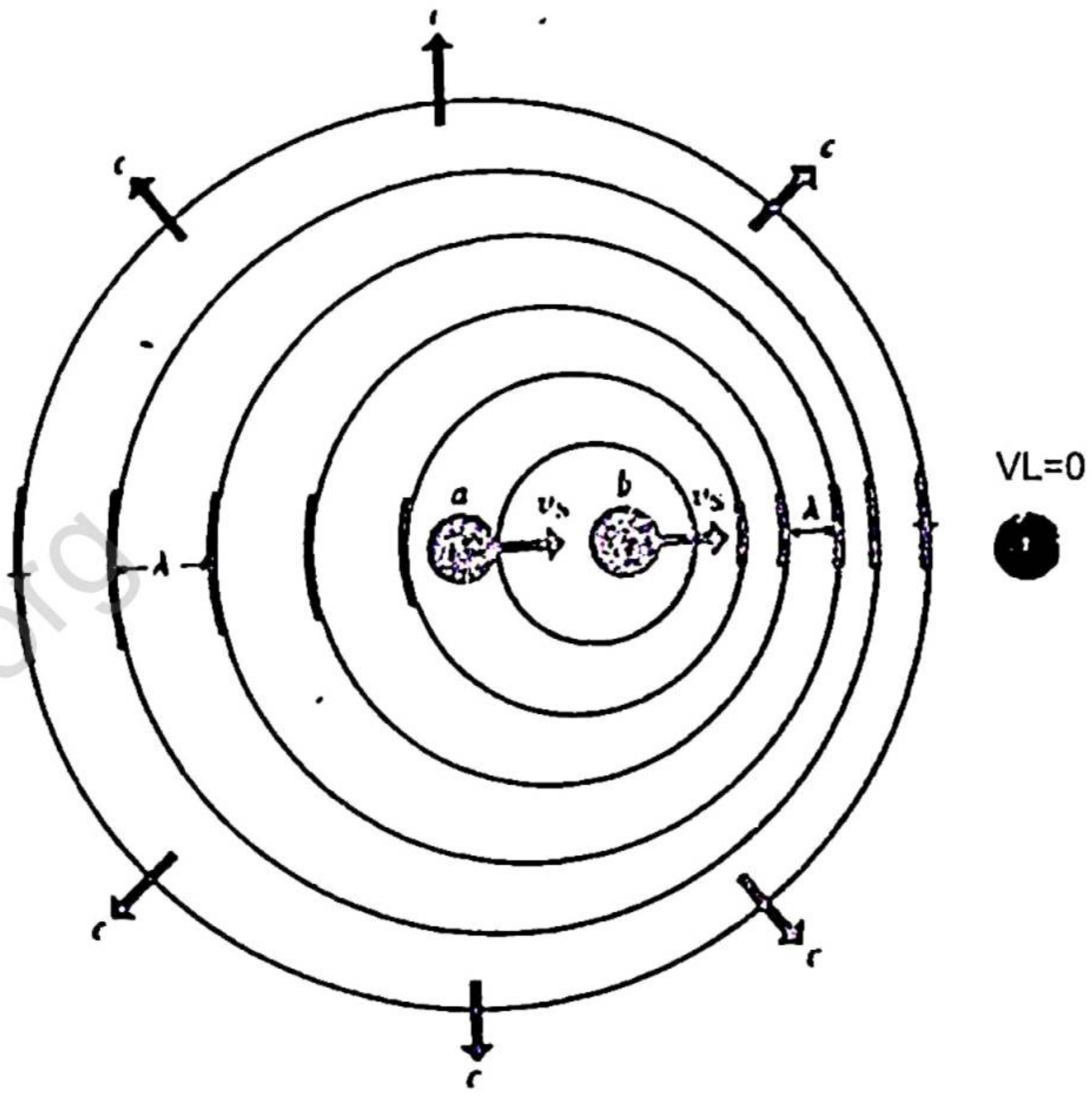
Let us consider a source of sound moving with velocity Vs towards stationary listener. The wave crests detected by the listener are closer together because the source is moving in the direction of outgoing wave resulting the shortening of wavelength.

As we know that

$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu}$ (Distance occupied by one wave)

and

$\nu = \frac{V}{\lambda}$ (Real Frequency) ------(i)



During each vibration source travels a distance equal to $\frac{V_S}{\nu}$ towards the listener then apparent wavelength is shortened.

$\lambda' = \frac{V}{\nu} - \frac{V_S}{\nu}$

or

$\lambda' = \frac{V - V_S}{\nu}$ ----------(ii)

The apparent frequency from eq(i) is given as,

$\nu' = \frac{V}{\lambda'}$

Putting value of λ' from eq(ii),we get

$\nu' = \frac{V}{\frac{V - V_S}{\nu}}$

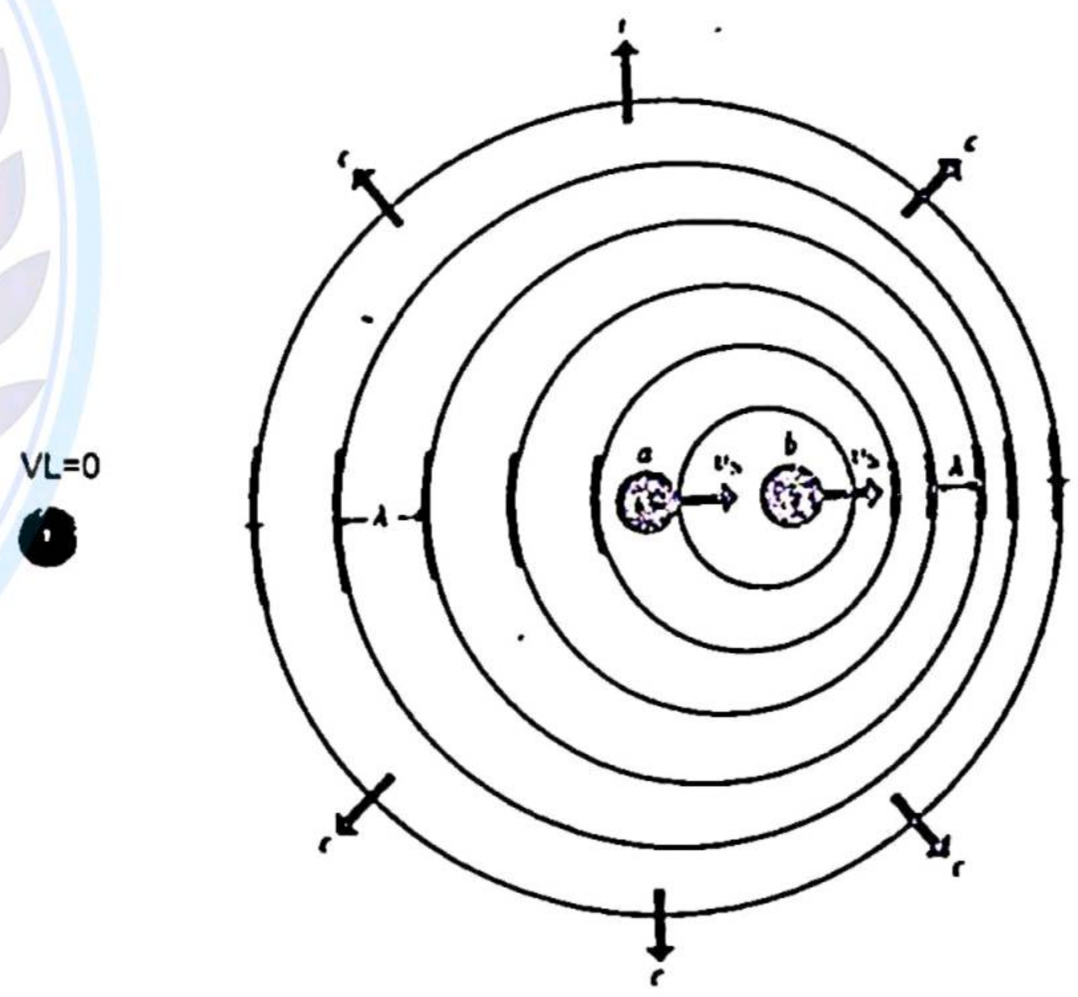
or

$$\boxed{\nu' = \left(\frac{V}{V - V_S}\right)\nu}$$

This expression shows that the apparent frequency will be greater than real frequency.

CASE 2(B): When source of sound moves away from the stationary listener.

Let us consider a source of sound moving with velocity Vs away from stationary listener. The wave crests detected by the listener are farther together because the source is moving in the opposite direction of outgoing wave resulting the increasing of wavelength.



As we know that

$V = \nu\,\lambda \qquad => \lambda = \frac{V}{\nu}$ (Distance occupied by one wave)

and

$\nu = \frac{V}{\lambda}$ (Real Frequency) ------(i)

During each vibration source travels a distance equal to $\frac{V_S}{\nu}$ away from the listener then apparent wavelength is increased.

$$\lambda' = \frac{V}{\nu} + \frac{V_S}{\nu}$$

or

$$\lambda' = \frac{V+V_S}{\nu} \text{----------(ii)}$$

The apparent frequency from eq(i) is given as,

$$\nu' = \frac{V}{\lambda'}$$

Putting value of λ' from eq(ii),we get

$$\nu' = \frac{V}{\frac{V+V_S}{\nu}}$$

or

$$\boxed{\nu' = \left(\frac{V}{V+V_S}\right)\nu}$$

This expression shows that the apparent frequency will be less than real frequency.

CASE 3(A): When both source of sound and listener move towards each other.

In this case,

$$\boxed{\nu' = \left(\frac{V+V_L}{V-V_S}\right)\nu}$$

This expression shows that the apparent frequency will rapidly increase.

CASE 3(B): When both source of sound and listener move away from each other.

In this case,

$$\boxed{\nu' = \left(\frac{V-V_L}{V+V_S}\right)\nu}$$

This expression shows that the apparent frequency will rapidly decrease.

ACOUSTICS

In the recording and reproduction of sound, one must avoid all possible sources of distortion. The size, shape of room, a studio or an auditorium the material used in the construction of the floor, ceiling walls, doors and windows etc. Also the number of chairs produced or reproduced in it.

A careful study of all these factors and their effect in the quality of sound produced in a room a studio or an auditorium is an important branch of radio engineering and is called ACOUSTICS.

Sound starting from a source S will reach a listener only along one path SL where as all other going spherical along all other path are absorbed all other path are absorbed or reflected by the walls, floor, ceiling etc. At the room ,the material of walls etc. absorbs sound waves of different frequencies by different amounts and remaining components of frequencies are reflected and approach to listener by several paths are shown.

REVERBERATION:

As some of sound wave energy is absorbed and some is reflected. The reflected portion travels back to hall and re-unite to form "echoes' which interfere to produce desirable or undesirable hearing. Even when the direct plus from source has ceased, some of the energy due to multi reflections from the walls, floor, ceiling etc is still on it way to ear. This cause prolongation or persistence of sound for some time.

The persistence of audible sound after the sources has ceased to operate is called reverberation and the time during which sound persists is called reverberation times.All the above sources which affect the quality of sound can be minimized by careful choice.

M.C.Qs.

1. The oscillatory motion in which the instantaneous acceleration is proportional to the displacement of the displacement of the oscillating bodies is called:

(a) Elastic motion (b) Translatory motion
(c) Transverse motion (d) Harmonic motion

2. Total energy of a particle performing SHM is directly proportional to:

(a) The amplitude
(b) The square root of amplitude
(c) Square of amplitude
(d) The reciprocal of amplitude

3. When a particle is executing SHM it is found that:

(a) The frequency depends upon the amplitude
(b) The periods depend on the amplitude.
(c) The period and frequency depend upon the amplitude
(d) The period and frequency are independent of the amplitude.

4. Beats are produced due to:

(a) diffraction of waves in time
(b) reflection of waves in time
(c) interference of waves in time
(d) polarization of waves in time

5. Which one of the following not undergoing a simple harmonic motion?

(a) Motion of a pendulum
(b) vibration of a violin string
(c) Motion of body in a rectilinear Path
(d) Oscillation of mass on a string

6.The product of frequency and time period is:

(a) 1 (b) 2 (c) 3 (d) 4

7. If a second pendulum is taken up on the moon, in order to have its time period same:

(a) The length of the pendulum must be increased (b) The length of the pendulum must be decreases
(c) The length of the pendulum must be kept the same
(d) None of the above

8. An ordinary clock loses time in summer this is because:

(a) The length of the pendulum increases
(b) The length of the pendulum decreases
(c) The length of the pendulum decreases and time period increases.
(d) The length the pendulum decreases and time period increases.

9.Weber Fechner law is:

(a) I α Log L (b) L α Log I
(c) I α 1/ Log L (d) I α Log L

10. Which one of the following contains a pair of transverse and longitudinal wave?

(a) Radio & X - rays
(b) Infra - red & ultra- violet
(c) Sound & radio wave
(d) Wave in a ripple tank & light

11. The velocity of a particle moving with a frequency 'f' and wave length 'λ' is:

(a) f λ (b) f/λ (c) λ/f
(d) fλ/2

12. Intensity of sound is measured in :

(a) watt/ m^2 (b) joule /m
(c) watt/ sec (d) watt/ m

13. Then temperature of air rises, the speed of sound waves increase because:

(a) wavelength of wave increases
(b) the frequency of wave increase
(c) both frequency and wavelength increases
(d) neither frequency nor wavelength increase

14. If the frequency of fifth harmonic of a vibrating string is 200 Hz ,its fundamental frequency is:

(a) 5 Hz (b) 25 Hz (c) 40 Hz (d) 100 Hz

15. It is common characteristics of all types of wave motion that without the transport of particles what transfers?

(a) Gravity (b) X rays
(c) Energy (d) Mass

16. The wave length of a radio wave when transmitted as a frequency of 150 MHz, will be:

(a) 20 m (b) 2 m (c) 10 m (d) 0.75 m

17. A simple pendulum completes one vibration in one second. If g = 981 cm/s² its length will be:

(a) 24.8 m (b) 24.8 cm
(c) 2.48 cm (d) 2.48 m

18. The range of audible sound is:

(a) 1 Hz – 10 Hz
(b) 20Hz – 20000Hz
(c) 21000Hz – 24000Hz
(d) 25000Hz – 50000Hz

19. When a string which is tied at both the ends is plucked from the centre the wave produced is:

(a) Transverse wave (b) Longitudinal wave
(c) Standing wave (d) Electromagnetic wave

20. Pitch depends upon:

(a) frequency (b) loudness
(c) time period (d) distance

21. Which of the following is not a transverse waves?

(a) x-rays (b) sound
(c) γ-rays (d) infrared

22. The distance between adjacent nodes or antinodes is:

(a) λ (b) λ/2 (c) λ/4 (d) 2λ

23. The velocity of sound in space is:
(a) zero m/s (b) 332 m/s
(c) 33200 cm/s (d) 3 x 10^8 m/s

24. The traveling wave in which particle of the disturbed medium move perpendicular to the direction of propagation of the wave is called:
(a) Longitudinal wave (b) Transverse wave

(c) Standing wave (d) Stationary wave

25. Earthquake waves are the example of:
(a) audio waves (b) infrasonic waves
(c) Ultrasonic waves (d) shock waves

26. In a stretched string, if the speed of the wave is four times, the tension will be ____ times the original:
(a) 2 (b) 4 (c) 8 (d) 16

27. Frequency of a stretched string is proportional to the:
(a) Tension (b) linear density
(c) reciprocal of the length d) Square of the tension

28. For a stationary wave in a string the points at which the particle is at maximum displacement from the mean position are called:
(a) Nodes (b) Anti nodes
(c) Compression (d) Rarefaction

29. A string fixed at two ends vibrates in two whole segment. The standing wave pattern set up is called:
(a) Fundamental (b) First harmonic
(c) Second harmonic (d) fourth harmonic

30. Sin θ= θ if θ is specially less than:
(a) 15^0 (b) 10^0 (c) 5^0 (d) 1 radian

PAST PAPER M.C.Qs.

2022

3.The range of audible sound is:
* 1 to 10 Hz   * **20Hz to 20,000 Hz**   * 21000 to 24000 Hz   * 25000 Hz onwards

19.The oscillatory of simple pendulum, the restoring force is
* **mg sinθ**   * mg cosθ   * mg tanθ   * mg

29.Pitch of sound depends upon:
*Amplitude   * Intensity   * frequency   * **loudness**

32.The distance between two consecutive nodes of a stationary wave is:
* $\frac{\lambda}{4}$ * λ * $\frac{\lambda}{2}$ * $\frac{\lambda}{3}$

2021

(x) The speed of sound in space (vaccum) is:
*332m/s   * 344m/s   *330m/s   ***Zero m/s**

(xi) A simple pendulum is performing S.H.M with time period T. If its length is doubled. The new time period will be

*2T *0.5T *2.5T **<u>* 1.414T</u>**

(xv) A body is executing S.H.M with amplitude A. Its potential energy is maximum when its displacement from mean position is:

*Zero *A/2 **<u>*A</u>** * A/ 4

(xxvii) If the mass of the bob of a simple pendulum is doubled, its time period will:

*be doubled *becomes triple **<u>* remain same</u>** *behalved

(xxviii) The unit of Intensity of sound is:

*watt/m^2 * watt-s * watt/s **<u>* watt/m</u>**

(xxxiii) It does not exhibit simple harmonic motion:

*A hanging spring supporting a weight *The motion of the prongs of tunning fork

<u>*The wheel of an automobile</u> *Motion of a string of a violin

<u>2019</u>

2.Beats are produced due to:

* diffraction of waves in time *reflection of waves in time

<u>*interference of waves in time</u> *polarization of waves in time

10.The product of frequency and time period is:

<u>*1</u> *2 *3 *4

17. Weber Fechner law is:

*I α Log L **<u>* L α Log I</u>** * I α 1/ Log L * I α Log L

<u>2018</u>

5. Intensity of sound is measured in :

<u>* watt/ m^2</u> *joule /m *watt/ sec *watt/ m

10. Then temperature of air rises, the speed of sound waves increase because:

<u>*wavelength of wave increases</u> *the frequency of wave increase

* both frequency and wavelength increases * neither frequency nor wavelength increase

<u>2017</u>

9. If the frequency of fifth harmonic of a vibrating string is 200 Hz ,its fundamental frequency is:

*5 Hz *25 Hz **<u>*40 Hz</u>** *100 Hz

10. The speed of sound in vacuum is:

<u>*zero m/s</u> *332 m/s *33200 cm/s` *3 x 10^8 m/s

11. The distance between two consecutive nodes of a transverse stationary wave is equal to:

*$\frac{\lambda}{4}$ *$\frac{\lambda}{2}$ *λ * 2λ

2016

5.The range of audible sound is:

*1 Hz – 10 Hz ***20Hz – 20000Hz**

*21000Hz – 24000Hz *25000Hz – 50000Hz

15. Two vibrating bodies, having slightly different frequencies, produce:

*Echo ***Beats** *Resonance *Polarization

2015

14. The velocity of a wave of wavelength ' λ ' and frequency 'ν' is given by:

* $\frac{\nu}{\lambda}$ * $\frac{\lambda}{\nu}$ ***$\nu\lambda$** * $\frac{1}{\lambda\nu}$

2014

3. The earth quake waves are the example of:

*Audible Waves *** Infrasonic waves** *Shock waves *Ultrasonic Waves

5. The distance between two consecutive nodes of a stationary wave will be:

* λ *** $\lambda/2$** * $\lambda/4$ * $\lambda/6$

15. If the mass of the bob of a simple pendulum is doubled, its time period will be:

*be doubled *become triple ***remain the same** *halved

2013

1. Power Law determines

*power *work *intensity ***loudness of sound**

10. The wave enters from one medium to another medium, no change occurs in its:

***frequency** *wavelength *amplitude *speed

15. The time period of simple pendulum depends upon:

*mass *length

*acceleration due to gravity ***both length and acceleration due to gravity**

3. The maximum number of beats per second which can be detected by the human ear is:

*2 *3 *5 ***7**

2012

2. The S.I unit of intensity level of sound is:

*watt * diopter * sone *** decibel**

6. The frequency of wave produced in a stretched string depends upon:

*length * tension * linear density *** all of these**

13. Sin θ= θ if θ is specially less than:

*15^0 * 10^0 * 5^0 *** 1 radian**

2011

1. Earthquake waves are the example of:

*audio waves *** infrasonic waves**

* Ultrasonic waves * shock waves

9. This is compression waves:

*light waves *x rays ***sound waves** *radio waves

10. If two tuning forks of frequencies 256 Hz and 260 Hz are sounded together, the number of beats per second will be:

*3 ***4** *5 *6

2010

4. Which of the following does not exhibit simple harmonic motion?

*A hanging spring supporting a weight

* The balance wheel of a watch

*** The wheel of an automobile**

*the string of violin

5. Pitch depends upon:

***frequency** * loudness

*time period * distance

6. The velocity of sound in space is:

***zero m/s** *332 m/s *33200 cm/s` *3 x 10^8 m/s

TEXTBOOK NUMERICALS

Q.1: An object is connected to one end of a horizontal spring whose other end is fixed. The object is pulled to the right (in the positive x-direction) by an externally applied force of magnitude 20 N causing the spring to stretch through a displacement of 1 cm (a) Determine the value of force constant if, the mass of the object is 4 kg (b) Determine the period of oscillation when the applied force is suddenly removed.

Data:

Applied Force $= F = 20\ N$

Displacement $= x = 1\ cm = 0.01\ m$

(a) Force Constant $= k = ?$

(b) Mass of Object $= m = 4\ kg$

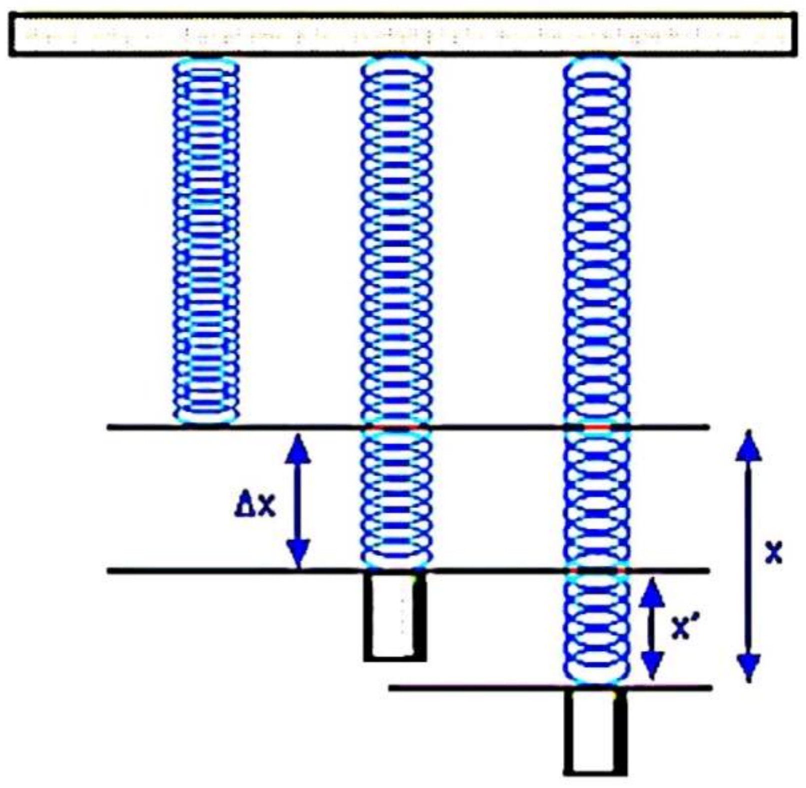
Time period of Oscillation $= T = ?$

Q.2: A body hanging from a spring is set into motion and the period of oscillation is found to be 0.50 s. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest?

Data:

Time period of Vibration $= T = 0.5$ sec

Displacement = x =?



Solution:

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ ------ (i)}$$

Also, $W = kx$

$$mg = kx$$

$$\frac{m}{k} = \frac{x}{g}$$

Putting in equation (i)

$$T = 2\pi\sqrt{\frac{x}{g}}$$

S.O.B.S

$$T^2 = 4\pi^2\left(\frac{x}{g}\right)$$

$$(0.5)^2 = 4(3.14)^2\left(\frac{x}{g}\right)$$

$$\boxed{x = \frac{(0.5)^2 \times 9.8}{4 \times 3.14^2} = 0.06\ m}$$

Result: The spring will be 0.06 m shorter when the body is removed.

Q.3: A pipe has a length of 2.46 m. (a) Determine the frequencies of the fundamental mode and the first two overtones if the pipe is open at both ends. Take υ = 344 m/s as the speed of sound in air. (b) What are the frequencies determined in (a) if the pipe is closed at one end? (c) For the case of open pipe, how many harmonics are present in the normal human being hearing range (20 to 20000 Hz)?

Data:

Length of Pipe $= L = 2.46\ m$

Speed of Sound $= v = 344\ m/s$

(a) For Open Pipe:

Frequecy of Fundamental Mode $= f_1 =?$

Frequecy of 2nd Harmonic $= f_2 =?$

Frequecy of 3rd Harmonic $= f_3 =?$

(b) For Closed Pipe:

Frequecy of Fundamental Mode $= f_1 =?$

Frequecy of 2nd Harmonic $= f_2 =?$

Frequecy of 3rd Harmonic $= f_3 =?$

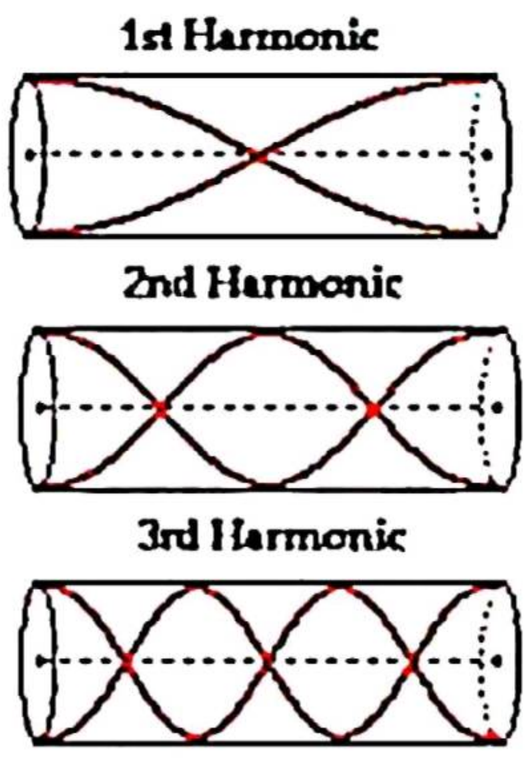
(c) Frequency of nth Harmonic $= f_n = 20000\ Hz$

No. of Harmonics $= n =?$

Solution:

(a) For Open Pipe:

Open at Both Ends   Frequency f



f_1

$2f_1$

$3f_1$

The fundamental Frequency is given by

$$f_1 = \frac{v}{2L} = \frac{344}{2 \times 2.46}$$

$$\boxed{f_1 = 70Hz}$$

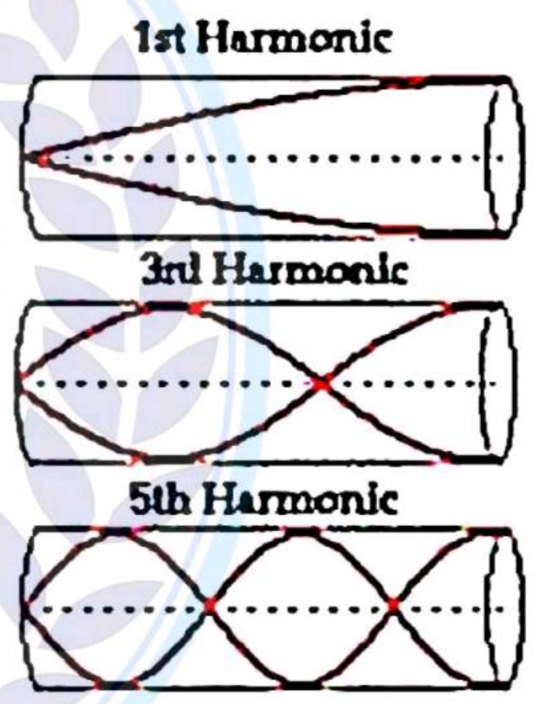
and then $\quad f_n = nf_1$

$$\boxed{f_2 = 2f_1 = 2 \times 70 = 140\ Hz}$$

and $\quad \boxed{f_3 = 3f_1 = 3 \times 70 = 210\ Hz}$

(b) For Closed Pipe:

Closed at One End   Frequency f



f_1

$3f_1$

$5f_1$

The fundamental Frequency is given by

$$f_1 = \frac{v}{4L} = \frac{344}{4 \times 2.46}$$

$$\boxed{f_1 = 35Hz}$$

and then $\quad f_n = nf_1$

$$\boxed{f_2 = 3f_1 = 3 \times 35 = 105\ Hz}$$

and $$\boxed{f_3 = 5f_1 = 5 \times 35 = 175\ Hz}$$

(c) For Open Pipe:

$$f_n = nf_1$$

then $$n = \frac{f_n}{f_1} = \frac{20000}{70}$$

$$\boxed{n = 285\ harmonics}$$

Result: (a)For Open pipe the frequencies are 70 Hz,140 Hz and 210 Hz.(b) For Closed pipe the frequencies are 35 Hz,105 Hz and 175 Hz (c) 285 harmonics are present in the normal human being hearing range.

Q.4: A standing wave is established in a 120 cm long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz (a) Determine the wavelength (b) What is the fundamental frequency?

Data:

Length of String = l=120 cm = 1.2 m

No. of loops = n = 4

Frequency of fourth harmonic = f_4 =120 Hz

Fundamental Frequency = f_1 = ?

Wavelength = λ = ?

Solution:

The fundamental Frequency is given by

$$f_n = n\frac{v}{2l}$$

Or $$f_n = nf_1$$

$$f_4 = 4f_1$$

$$\boxed{f_1 = \frac{f_4}{4} = \frac{120}{4} = 30\ Hz}$$

The wavelength of stationary wave is given by

$$L = \frac{n\lambda}{2}$$

$$L = \frac{(4)\lambda}{2}$$

$$L = 2\lambda$$

Or $$\boxed{\lambda = \frac{L}{2} = \frac{1.2}{2} = 0.6\ m}$$

Result: The fundamental frequency is 30 Hz and its wavelength is 0.6 m.

Q.5: Calculate the speed of sound in air at atmospheric pressure p = 1.01x10^5 N/m^2 , taking γ = 1.40 and ρ = 1.2kg/m^3 .

Data:

Speed of sound in air = v =?

Atmospheric pressure = $P = 1.01 \times 10^5\ Pa$

Ratio of molar specific heats = $\gamma = 1.40$

Density of air = $\rho = 1.2\ kg/m^3$

Solution:

The speed of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$v = \sqrt{\frac{1.40 \times 1.01 \times 10^5}{1.2}}$$

$$\boxed{v = 343.26\ m/s}$$

Result: The speed of sound in air at S.T.P. is 343 m/s.

Q.6: A sound wave propagating in air has a frequency of 4000 Hz. Calculate the percent change in wave length when the wave front, initially in a region where T = 27° C, enters a region where the air temperature decreases to 10° C.

Data:

Frequency of Sound = f = 500Hz

Initial temperature = T_1 = 27 ^{0}C + 273 = 300 K

Final temperature = T_2 = 10 ^{0}C +273= 283 K

Percent Fractional Change in wavelength = $\frac{\Delta\lambda}{\lambda_1}\%$ =?

Solution:

As we know that

$$v \propto \sqrt{T}$$

$$v = k\sqrt{T}$$

So, $$v_1 = k\sqrt{T_1} \text{ ---- (i)}$$

And $$v_2 = k\sqrt{T_2} \text{ ---- (ii)}$$

Dividing eq (ii) by eq (i)

$$\frac{v_2}{v_1} = \frac{k\sqrt{T_2}}{k\sqrt{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{283}{300}}$$

$$\frac{v_2}{v_1} = 0.971$$

$$\because v = f\lambda$$

$$\therefore \frac{f\lambda_2}{f\lambda_1} = 0.971$$

$$\boxed{\lambda_2 = 0.971\lambda_1}$$

Now, $\Delta\lambda = \lambda_1 - \lambda_2$

$$\Delta\lambda = \lambda_1 - 0.971\lambda_1$$

$$\Delta\lambda = 0.029\lambda_1$$

$$\frac{\Delta\lambda}{\lambda_1} = 0.029$$

Or $\boxed{\frac{\Delta\lambda}{\lambda_1}\% = 2.9\ \%}$

Result: The percent fractional change in wavelength will be 2.9%.

Q.7: The frequency of the second harmonic of an open pipe (open at both ends) is equal to the frequency of the second harmonic of a closed pipe (open at one end). (a) Find the ratio of the length of the closed pipe to the length of the open pipe.(b) If the fundamental frequency of the open pipe is 256 Hz, what is the length of pipe? (Use υ = 340 m/s).

Data:

(a) Ratio of Lengths of Pipe $= \frac{L_{Closed}}{L_{Open}} = ?$

Speed of Sound $= v = 340\ m/s$

(b) Fundamental Frequency of open Pipe $=$ $f_{1(Open)} = 256\ Hz$

Length of Open Pipe $= L_{Open} = ?$

Length of Closed Pipe $= L_{Close} = ?$

Solution:

(a) According to the given condition

$$f_{2(Open)} = f_{2(Closed)}$$

$$\frac{v}{L_{Open}} = \frac{3v}{4L_{Closed}}$$

$$\frac{1}{L_{Open}} = \frac{3}{4L_{Closed}}$$

$$\boxed{\frac{L_{Closed}}{L_{Open}} = \frac{3}{4}}$$

(b) **For Open Pipe:**

$$f_1 = \frac{v}{2L}$$

then

$$f_{1(open)} = \frac{v}{2L_{Open}}$$

$$L_{Open} = \frac{v}{2\times f_{1(open)}}$$

$$L_{Open} = \frac{340}{2\times 256}$$

$$\boxed{L_{Open} = 0.66\ m}$$

For Closed Pipe:

As we know that

$$\frac{L_{Closed}}{L_{Open}} = \frac{3}{4}$$

then

$$\frac{L_{Closed}}{0.66} = \frac{3}{4}$$

$$L_{Closed} = \frac{3}{4}\times 0.66$$

$$\boxed{L_{Closed} = 0.49\ m}$$

Result: The ratio of the length of the closed pipe to the length of the open pipe. is 3/4 and the length of open pipe is 0.66 m and closed pipe is 0.49 m.

Q.8: A 256 Hz tuning fork produces four beats per second when sounded with another fork of unknown frequency. What are two possible values for the unknown frequency?

Data:

Frequency of 1st Tuning Fork $= f_1 = 256\ Hz$

Beats Frequency $= f_b = 4\ beats/\ sec$

Frequency of 2nd Tuning Fork $= f_2 = ?$

Frequency of 2nd Tuning Fork $= f_2' = ?$

Solution:

For 1st Possible value:

$$f_b = f_1 - f_2$$

$$4 = 256 - f_2$$

$$f_2 = 256 - 4$$

$$\boxed{f_2 = 252\ Hz}$$

For 2nd Possible value:

$$f_b = f_2' - f_1$$

$$4 = f_2' - 256$$

$$f_2' = 256 + 4$$

$$\boxed{f_2' = 260\ Hz}$$

Result: The two possible values for unkown frequency are 252 Hz and 260Hz.

Q.9: An ambulance travels down a highway at a speed of 75 mi/h. Its siren emits sound at a frequency of 400 Hz . What frequency will heard by a person in a car traveling at 55 mi/h in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance.

Data:

Speed of Ambulance $= v_s = 75\ mi/h$

Actual Frequency $= f = 400\ Hz$

Speed of Car $= v_L = 55\ mi/h$

(a) Apparent frequncy $= f' = ?$

(b) Apparent frequncy $= f' = ?$

Speed of Sound $= v = 750\ mi/h$

Solution:

(a)When Source and listener move towards each other

$$f' = \left(\frac{V+V_L}{V-V_S}\right) f$$

$$f' = \left(\frac{750+55}{750-75}\right) \times 400$$

$$f' = \left(\frac{805}{675}\right) \times 400$$

$$\boxed{f' = 477\ Hz}$$

(b)When Source and listener move away from each other

$$f' = \left(\frac{V-V_L}{V+V_S}\right) f$$

$$f' = \left(\frac{750-55}{750+75}\right) \times 400$$

$$f' = \left(\frac{695}{825}\right) \times 400$$

$$\boxed{f' = 337\ Hz}$$

Result: When car and ambulance move towards each other the apparent frequency will be 477 Hz and when car and ambulance move away from each other the apparent frequency will be 337 Hz

Q.10: A car has siren sounding a 2 kHz tone. What frequency will be detected as stationary observer as the car approaches him at 80 km/h? Speed of sound = 1200 km/h.

Data:

Frequency of sound $= f = 2\ KHz = 2000\ Hz$

Apparent of frequency $= f' = ?$

Speed of source $= V_s = 80\ km/h$

Speed of Sound $= V = 1200\ km/h$

Solution:

When Source moves towards stationary listener

$$f' = \left(\frac{V}{V-V_S}\right) f$$

$$f' = \left(\frac{1200}{1200-80}\right) \times 2000$$

$$f' = (1.07) \times 2000$$

$$\boxed{f' = 2143\ Hz}$$

Result: The apparent frequency heard by the listener is 2143 Hz

PAST PAPER NUMERICALS

2022

x) Two cars approaching each other from opposite directions with same speed. The horn of one is blowing with the frequency of 3000 Hz and is heard by the people in the other car with the frequency of 3400 Hz. Find the speed of both cars, if speed of sound in air is 340 m/s .

Data:

Speed of Car $= v_s = v_L = v = ?$

Actual Frequency $= f = 3000\ Hz$

Apparent Frequency $= f = 3400\ Hz$

Speed of Sound $= v' = 340\ m/s$

Solution:

(a)When Source and listener move towards each other

$$f' = \left(\frac{v'+V_L}{v'-V_S}\right)f$$

$$3400 = \left(\frac{v'+v}{v'-v}\right)3000$$

$$\frac{3400}{3000} = \left(\frac{340+v}{340-v}\right)$$

$$1.13(340-v) = 340+v$$

$$384.2 - 1.13v = 340+v$$

$$384.2 - 340 = v + 1.13v$$

$$44.2 = 2.13v$$

$$\boxed{v = \frac{44.2}{2.13} = 20.7\ m/s}$$

Result: The speed of both cars is 20.7 m/s

2019

Q.2 (vii) A mass at the end of spring oscillates with a period of 0.4 sec. Find the acceleration when the displacement is 6 cm.

Data:

Time Period= T= 0.4 sec

Displacement = x = 6 cm =0.06 m

Acceleration= a =?

Solution:

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ ------ (i)}$$

Also, $F = kx$

$$ma = kx$$

$$\frac{m}{k} = \frac{x}{a}$$

Putting in equation (i)

$$T = 2\pi\sqrt{\frac{x}{a}}$$

S.O.B.S

$$T^2 = 4\pi^2\left(\frac{x}{a}\right)$$

$$(0.5)^2 = 4(3.14)^2\left(\frac{0.06}{a}\right)$$

$$\boxed{a = 14.8\ m/s^2}$$

Result: The acceleration of body is 14.8 m/s².

Q.2 (xiv) A string 2m long and mass 0.004 kg, is stretched horizontally by passing one end over a pulley and attaching a 1 kg mass to it. Find the speed of the transverse waves on the string and frequency of the second harmonic.

Data:

Length=l =2 m

Mass of string = m= 0.004 kg

Mass attached to string =M = 1 kg

Velocity of transverse wave = v=?

Frequency of Second harmonic = f_2 =?

Solution:

$$v = \sqrt{\frac{Mg}{(m/l)}}$$

$$v = \sqrt{\frac{1\times 9.8}{(0.004/2)}}$$

$$\boxed{v = 70\ m/s}$$

$$f_n = n\frac{v}{2l}$$

$$f_2 = 2\frac{70}{2(2)}$$

$$\boxed{f_2 = 35\ Hz}$$

Result: Velocity of transverse wave is 70 m/s and Frequency of Second harmonic 35 Hz.

2018

Q.2(vi) A 15 kg block is suspended by a spring of spring constant 5 x 10³ N/m. Calculate the frequency of vibration of the block displaced from its equilibrium position when it is released.

Data:

Mass of block= m = 15 kg

Spring constant = K = 5 x 10^3 N/m

Frequency of Vibration = f = ?

Solution:

The time period of spring mass system is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2(3.14)\sqrt{\frac{15}{5 \times 10^3}}$$

$$\boxed{T = 0.344\ s}$$

Now,

$$f = \frac{1}{T} = \frac{1}{0.344}$$

$$\boxed{f = 2.9\ Hz}$$

Result: The frequency of vibration of the block is 2.9 Hz.

Q.2(x) A standing wave is established in a 135 cm long string fixed at both ends. The string vibrates in four loops when driven at 130 Hz. Determine the fundamental frequency.

Data:

Length of String = l=135 cm = 1.35 m

No. of loops = n = 4

Frequency of fourth harmonic = f_4 =130 Hz

Fundamental Frequency = f_1 = ?

Solution:

The fundamental Frequency is given by

$$f_n = n\frac{v}{2l}$$

Or

$$f_n = nf_1$$

$$f_4 = 4f_1$$

$$\boxed{f_1 = \frac{f_4}{4} = \frac{130}{4} = 32.5\ Hz}$$

Result: The fundamental frequency is 32.5 Hz.

2017

Q.2(xv) A 100 cm long string vibrates into four loops at 50 Hz. The linear density of the string is 4 x 10^{-4} gm/cm. Calculate the tension in the string.

Data:

Length of string = l= 100 cm = 1m

No. of Loops = n = 4

Frequency of fourth harmonic = f_4 = 50 Hz

Mass per unit length of string = μ = 4 x 10^{-4} gm/cm

μ = 4 x 10^{-5} kg/m

Tension in the string= T = ?

Solution:

As we know that

$$f_n = n\frac{v}{2l}$$

Since

$$v = \sqrt{\frac{T}{\mu}}$$

Therefore

$$f_n = n\frac{\sqrt{\frac{T}{\mu}}}{2l}$$

$$50 = 4\frac{\sqrt{\frac{T}{4\times10^{-5}}}}{2(1)}$$

Or

$$25 = \sqrt{\frac{T}{4\times10^{-5}}}$$

S.O.B.S.

$$625 = T/4\times10^{-5}$$

Or

$$T = 625 \times 4 \times 10^{-5}$$

$$\boxed{T = 0.025\ N}$$

Result: The tension in the string is 0.025 N.

2016

Q.2 (vii) **Textbook Numerical 10**

Q.2 (xiii) A string 2 m long of mass 0.004 kg, is stretched horizontally by passing one end over a frictionless pulley and a mass of 1 kg is suspended, Find the speed of transverse waves on the string.

Data:

Length=l =2 m

Mass of string = m= 0.004 kg

Mass attached to string =M = 1 kg

Velocity of transverse wave = v=?

Solution:

$$v = \sqrt{\frac{Mg}{(m/l)}}$$

$$v = \sqrt{\frac{1 \times 9.8}{(0.004/2)}}$$

$$\boxed{v = 70\ m/s}$$

Result: Velocity of transverse wave is 70 m/s.

2015

Q.2 iv) A car emitted a note of frequency 490 Hz, if the car approaching towards a stationary listener at speed of 55 km/h, what frequency will be detected by the listener. Take speed of sound as 334 m/s.

Data:

Actual frequency of sound = f =490 Hz

Speed of car = Vs = 55 km/h = 55x1000/3600

Vs = 15.27 m/s

Apparent frequency of sound = f' =?

Speed of Sound = V = 334 m/s

Solution:

When Source moves towards stationary source

$$f' = \left(\frac{V}{V - Vs}\right) f$$

$$f' = \left(\frac{334}{334 - 15.27}\right) \times 490$$

$$\boxed{f' = 513.4\ Hz}$$

Result: The frequency heard by the listener is 513.4 Hz

Q.2 xiv) Textbook Numerical 2

2014

Q.2 (vi) Same as 2019 Q.2 (xiv)

2013

Q.2 (iii) A sound wave of frequency 500 Hz in air enters from a region of temperature 25 degrees C to a region of temperature 5 degrees C. Calculate the percent fractional change in wavelength.

Data:

Frequency of Sound = f = 500Hz

Initial temperature = T_1 = 25 ^{0}C + 273 = 298 K

Final temperature = T_2 = 5 ^{0}C +273= 278 K

Percent Fractional Change in wavelength = $\frac{\Delta\lambda}{\lambda_1}\% = ?$

Solution:

As we know that

$$v \propto \sqrt{T}$$

$$v = k\sqrt{T}$$

So, $v_1 = k\sqrt{T_1}$ ---- (i)

And $v_2 = k\sqrt{T_2}$ ---- (ii)

Dividing eq (ii) by eq (i)

$$\frac{v_2}{v_1} = \frac{k\sqrt{T_2}}{k\sqrt{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{278}{298}}$$

$$\frac{v_2}{v_1} = 0.965$$

$$\because v = f\lambda$$

$$\therefore \frac{f\lambda_2}{f\lambda_1} = 0.965$$

$$\boxed{\lambda_2 = 0.965\lambda_1}$$

Now,

$$\Delta\lambda = \lambda_1 - \lambda_2$$

$$\Delta\lambda = \lambda_1 - 0.965\lambda_1$$

$$\Delta\lambda = 0.035\lambda_1$$

$$\frac{\Delta\lambda}{\lambda_1} = 0.035$$

Or

$$\boxed{\frac{\Delta\lambda}{\lambda_1}\% = 3.5\ \%}$$

Result: The percent fractional change in wavelength will be 3.5%.

Q.2 (x) Textbook Numerical 2

2012

Q.2 (vii) Find the velocity of sound in a gas when two waves, of wavelengths 0.8m and 0.81, respectively, produce 4 beats per seconds.

Data:

Velocity of sound in gas = v =?

Wavelength of 1st wave = λ_1=0.8 m

Wavelength of 2nd wave = λ_2=0.81 m

Beat Frequency = f_b = 4 beats/sec

Solution:

The beat frequency is given by

$$f_b = |f_1 - f_2|$$

$$4 = \left|\frac{v}{\lambda_1} - \frac{v}{\lambda_2}\right|$$

$$4 = v\left|\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right|$$

$$4 = v\left|\frac{1}{0.8} - \frac{1}{0.81}\right|$$

$$4 = v|1.25 - 1.234|$$

$$4 = v|0.016|$$

$$v = \frac{4}{0.016}$$

$$\boxed{v = 250\ m/s}$$

Result: The speed of sound in gas is 250 m/s.

Q.2 (xiii) A string, lm long and of mass 0.004 kg, is stretched with a force. Calculate the force if the speed of the wave in the string is 140m/sec.

Data:

Length=l =1 m

Mass of string = m= 0.004 kg

Force Applies to string =T =?

Velocity of transverse wave = v= 140 m/s

Solution:

$$v = \sqrt{\frac{T}{(m/l)}}$$

$$140 = \sqrt{\frac{T}{(0.004/1)}}$$

S.O.B.S

$$(140)^2 = \left(\sqrt{\frac{T}{(0.004)}}\right)^2$$

$$19600 = \frac{T}{(0.004)}$$

$$\boxed{T = 19600 \times 0.004 = 78.4\ N}$$

Result: Force applied on the string is 78.4 N

2011

Q. 2(xiii) A note of frequency of 500 Hz is being emitted by an ambulance moving towards a listener at rest. If the listener detects a frequency of 526 Hz, calculate the speed of the ambulance. (speed of sound is 340 m/s at that moment)

Data:

$Frequency\ of\ sound = f = 500\ Hz$

$Apparent\ of\ frequency = f' = 526\ Hz$

$Speed\ of\ source = V_s = ?$

$Speed\ of\ Sound = V = 340\ m/s$

Solution:

When Source moves towards stationary listener

$$f' = \left(\frac{V}{V - V_s}\right)f$$

$$526 = \left(\frac{340}{340 - V_s}\right) \times 500$$

$$\frac{526}{500} = \left(\frac{340}{340 - V_s}\right)$$

$$1.052 = \left(\frac{340}{340 - V_s}\right)$$

$$340 - V_s = \frac{340}{1.052}$$

$$340 - V_s = 323.1$$

$$V_s = 340 - 323.1$$

$$\boxed{V_s = 16.8\ m/s}$$

Result: The speed of ambulance is 16.8 m/s.

2010

Q.2 (xi) **Same as 2019 Q.2 (xiv)**

Q.2 (xii) A simple pendulum completes 4 vibrations in 8 seconds on the surface of the earth. Find the time period on the surface of the moon where the acceleration due to gravity is one-sixth that of the earth.

Data:

No. of vibrations = N = 4

Time = t = 8 sec

Time period of Pendulum = T_m =?

Value of "g" on Moon = $g_m = \frac{1}{6}g$

Solution:

$$Time\ Period = \frac{Total\ \ time}{No\ .of\ Vibrations}$$

$$\boxed{T = \frac{t}{N} = \frac{8}{4} = 2\ sec}$$

The time period of pendulum on the surface of moon is given by

$$T_m = 2\pi\sqrt{\frac{L}{g_m}} \text{----(i)}$$

The time period of pendulum on the surface of Earth is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \text{----(ii)}$$

Dividing eq(ii) by eq(i)

$$\frac{T_m}{T} = 2\pi\sqrt{\frac{L}{g_m}} \div 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T_m}{T} = 2\pi\sqrt{\frac{L}{g_m}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\frac{T_m}{T} = \sqrt{\frac{g}{g_m}}$$

$$\frac{T_m}{2} = \sqrt{\frac{g}{\frac{1}{6}g}}$$

$$T_m = 2 \times \sqrt{6}$$

$$\boxed{T_m = 4.89\ sec}$$

Result: The time period of simple pendulum at the surface of moon is 4.89 sec.