

Chapter = 10

## SIMPLE HARMONIC MOTION AND WAVES



### OSCILLATORY MOTION

**Definition:** - The “to and fro” motion of a body about its mean position is known as oscillatory motion. OR

The repeated **back and forth** motion of a body about a certain **equilibrium** (mean) **position** is known as oscillatory motion.

**Other Name:** - It is also called vibratory motion.

**Oscillator:** - The body which perform oscillatory motion is known as oscillator.

**Cause:** - A vibratory motion is a general phenomenon and is closely related to the

(1) Elastic property of matter.

(2) Inertial property of matter.

**Examples:**

(i) Motion of simple pendulum.

(ii) Motion of mass attached to a spring.

(iii) Motion of metallic strip.

(iv) Motion of molecules in solids etc.

### PERIODIC MOTION

**Definition:-** Any motion which repeats itself in equal intervals of time is known as periodic motion.

**Explanation:-**

(i) Every vibratory motion is periodic motion. But

(ii) Every periodic motion is not vibratory motion.

(iii) The simplest periodic vibratory is known as simple harmonic motion.

**Examples:-**

(i) Motion of simple pendulum .

- (ii) Motion of mass attached to a spring.
- (iii) Motion of earth around the sun.
- (iv) Motion of hands of a watch etc.

## DEFINITION OF SOME IMPORTANT TERMS OR TERMINOLOGY OF OSCILLATORY MOTION

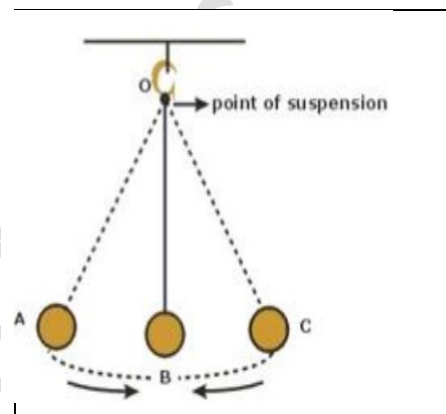
### VIBRATION



**Definition:** - One complete round trip of a vibrating body about its mean position is known as vibration. OR  
A vibration means one complete round trip of the vibrating body.

**Other Name:-** It is also called "Oscillation" OR "Cycle".

**Example:** - Motion of bob of pendulum from "A" to "C" and back from "C" to "A" is one vibration as show in figure.



### TIME PERIOD

**Definition:** - The time required to complete one vibration or one oscillation is known as time period. OR  
The time taken by a vibrating body to complete one vibration is known as time period.

**Symbol:-**It is denoted by "T".

**Mathematical form:** -

$$T = \frac{1}{f}$$

**Unit :-** Its S I unit is **second** (sec) .

**Note:-** The reciprocal of frequency is known as time period.

## FREQUENCY



**Definition:-** The number of vibrations completed by a body in one second is known as frequency. OR

The number of vibrations or cycles of a vibrating body in one second is known as frequency.

**Symbol:-** It is denoted by “f”.

**Mathematical form:-**  $f = \frac{1}{T}$

**Unit:-** Its units are

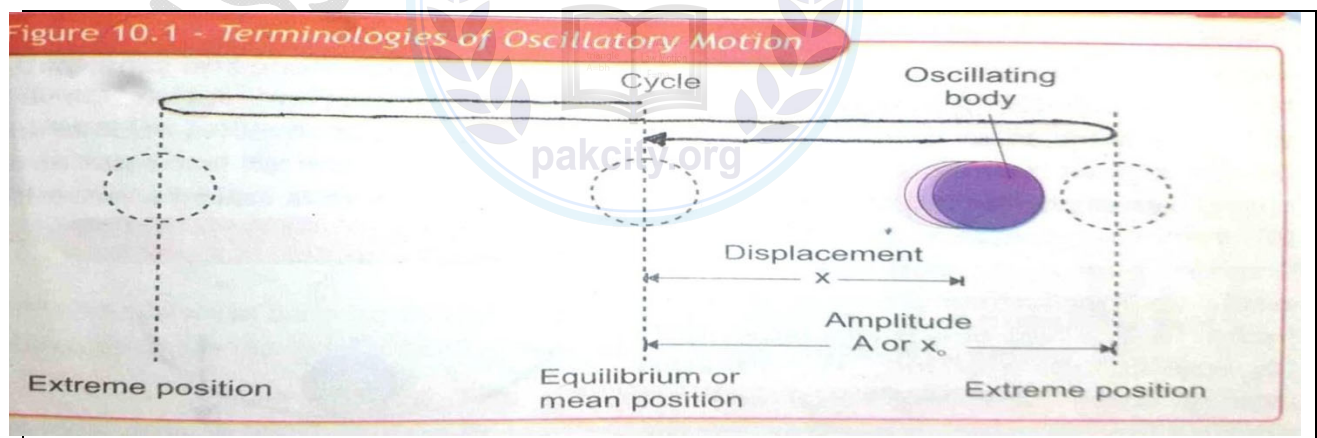
- (i) Vib /sec (vib . sec<sup>-1</sup>)
- (ii) Cycle / sec (cycle. Sec<sup>-1</sup>) OR
- (iii) Hertz (HZ) .

**Note:-** The reciprocal of time period is known as frequency.

### RELATION BETWEEN TIME PERIOD (T) AND FREQUENCY (f)

The time period and frequency are reciprocal of each other.

$$T = \frac{1}{f}$$



## DISPLACEMENT



**Definition:** - The distance of oscillating body from the mean position at any instant of time is known as displacement. OR

The minimum distance covered by the vibrating body from its mean position is known as displacement.

**Symbol:** - It is denoted by "X".

**Unit:** - Its unit is meter (m).

## AMPLITUDE

**Definition:** - The maximum displacement of a vibrating body on either side of its mean position is known as amplitude. OR

The maximum distance covered by the vibrating body from its mean position is known as amplitude. OR

The maximum displacement of a body from its mean position is known as amplitude.

**Symbol:** - It is represented by " $X_0$ " OR "A".

**Unit:** - Its unit is meter (m).

## SIMPLE HARMONIC MOTION

**Definition:** - The type of vibratory motion in which the acceleration " $a$ " is directly proportional to the displacement and always directed towards the mean position.

**Abbreviation:** It is abbreviated by S.H.M.

**Mathematical form:** -  $a \propto -X$  ----- (1)

In equation (1):-

(i) " $a$ " shows acceleration of body (SHO).

(ii) " $X$ " shows displacement of body (SHO).



(iii) The negative sign shows that the acceleration is always directed towards the mean position.



**Conditions:-** The basic condition for a system to execute S.H.M are

- (a) The system must have “**inertia**”.
- (b) The system must obey “**Hook’s Law**”.
- (c) The system should have **elastic restoring force acting**.
- (d) The system should be **frictionless**.

**Simple Harmonic Oscillator (S.H.O):-** It is an object or body which performs S.H.M.

**Examples:-**

- (i) Motion of simple pendulum
- (ii) Motion of a swing.
- (iii) Motion of mass attached to a spring etc.

**NOTE: - The vibrating bodies produce waves.**

## CHARACTERISTIC FEATURES OF SHM

- (i) In SHM of a body vibrates about a mean position.
- (ii) A body always moves on a straight line.
- (iii) Displacement “ $x$ ” is directly proportional to the applied force  $F_{app}$ .
- (iv) Acceleration “ $a$ ” is directly proportional to the displacement “ $x$ ” i.e.  $a \propto x$ .
- (v) Acceleration “ $a$ ” is always directed towards the mean position.
- (vi) K.E is maximum at mean position.
- (vii) K.E is minimum at extreme position.
- (viii) P.E is **maximum** at extreme position.
- (ix) P.E is **minimum** at mean position.
- (x) Acceleration “ $a$ ” is **zero** at mean position.
- (xi) Acceleration “ $a$ ” is **maximum** at extreme position.
- (xii) Velocity “ $v$ ” is **zero** at extreme position.
- (xiii) Velocity “ $v$ ” is **maximum** at mean position.

(xiv) The total energy **remains constant**.

(Xv) The time period of simple pendulum is  $T = 2\pi\sqrt{l/g}$  .

(xvi) The time period of mass attached to a spring is  $T = 2\pi\sqrt{m/k}$  .



## RESTORING FORCE

**Definition:** - The force which tends to move the body to its original position when the applied force is removed is known as restoring force. OR  
The force which brings the body to its mean position is known as restoring force.

**Symbol:-** It is represented by " $F_{res}$ " .

**Mathematical form:** -  $F_{res} = -kX$

**Explanation:**

(i) It is always directed towards mean position.

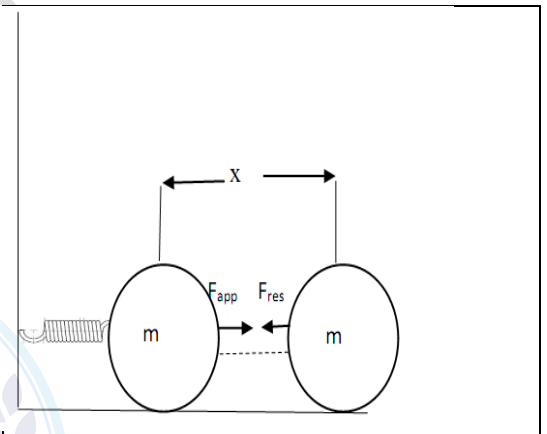
(ii) It is always equal in magnitude to the external force but opposite in direction.

(iii) It is assigned negative.

(iv) It is a vector quantity.

(v) It is a derived quantity.

**Unit:** - Its unit is Newton (N).



## MOTION OF MASS ATTACHED TO A SPRING

OR

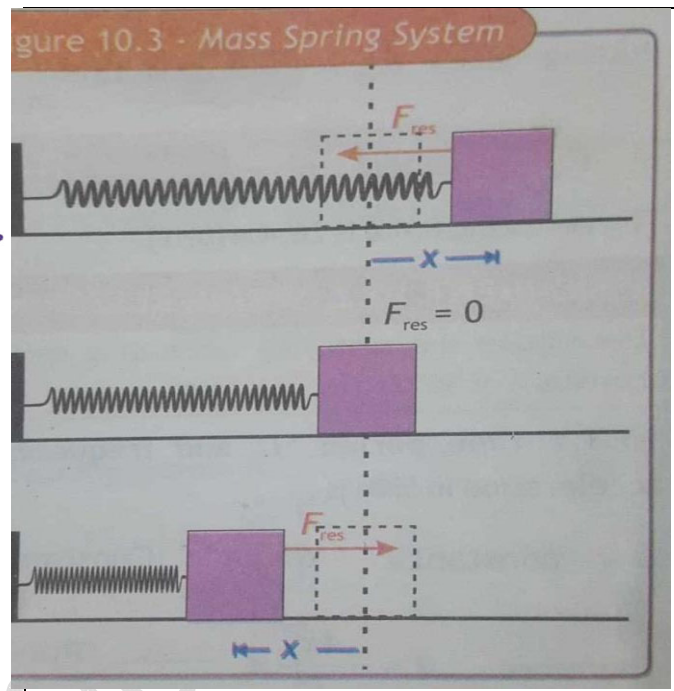
**SHOW THAT MASS ATTACHED TO A SPRING PERFORM S.H.M**

Consider a body of mass “m” attached to one end of an elastic spring, which can move freely on a frictionless horizontal surface as shown in the figure 10.3.



When the block is displaced from its mean position then released it will start vibrations under the action of restoring force of the spring. This force produces acceleration in the body.

Suppose the body is at a distance “X” from its mean position at any instant then the restoring force acting on it to bring it back to its initial position is given by.



$$F_{res} = -kx \dots\dots\dots (1)$$

According to Newton's 2<sup>nd</sup> law of motion

$$F_{res} = ma \dots\dots\dots (2)$$

By comparing equation (1) and (2) we get

$$ma = -kx$$

$$a = \frac{-kx}{m}$$

$$a = \frac{k}{m} (-x)$$

$$a = \text{Constant} (-x)$$

OR

$$a \propto -x \dots\dots\dots (3)$$

$$\text{Constant} = \frac{K}{m}$$

Equation (3) show that mass attached to a spring perform S.H.M.

**NOTE:-** If the restoring force obeys Hook's law precisely , the oscillatory motion of mass attached to spring is simple harmonic.



## ANGULAR FREQUENCY

**Definition:-** The angular displacement per unit time is known as angular frequency.

**Symbol:-** It is denoted by “ $\omega$ ” (Omega).

**Mathematical Form:-**

$$\text{Angular Frequency} = \frac{\text{Angular Displacement}}{\text{Time}}$$

$$\omega = \frac{2\pi}{T}$$

## Time period “ T ” and frequency “ f ” of mass spring system

**For time period (T) :-** As we know that incase of SHM

$$a \propto -X \quad \text{OR} \quad a = \text{Constant} (-X)$$

$$a = \omega^2 (-X)$$

$$\text{OR} \quad a = \left(\frac{2\pi}{T}\right)^2 (-X)$$

$$a = -\frac{4\pi^2}{T^2} (x) \dots\dots\dots (1)$$

$$\text{Constant} = \omega^2$$

$$\omega = \frac{2\pi}{T}$$

**In case of mass-spring system:-**  $a = -\frac{k}{m} x \dots\dots\dots (2)$

By comparing equation (1) and (2) we get

$$\text{pakcity.org} - \frac{4\pi^2}{T^2} x = -\frac{k}{m} x$$

$$-\frac{4\pi^2}{T^2} \cancel{x} = -\frac{k}{m} \cancel{x} \text{ OR } \frac{4\pi^2}{T^2} = \frac{k}{m} \dots\dots\dots (4)$$

On re-arranging the equation (4) becomes  $T^2 = 4\pi^2 \frac{m}{k} \dots\dots\dots (5)$

Taking square root on both sides of equation (5) we have

$$\sqrt{T^2} = \sqrt{4\pi^2} \sqrt{\frac{m}{k}} \text{ OR } T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2\pi \sqrt{\frac{m}{k}} \dots\dots\dots (6)$$

**Results:-** From eq (6) we conclude that the time period of mass-spring system depends upon the:-

(i) Mass of body (m) :-  $T \propto \sqrt{m}$ .

**For frequency of mass-spring system:-** As we know that

$$f = \frac{1}{T} \dots\dots\dots (7)$$

Now by putting equation (6) in equation (7) we get

$$f = \frac{1}{2\pi \sqrt{\frac{m}{k}}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \dots\dots\dots (8)$$

**Result:-** From equation (8) we conclude that the frequency of mass-spring system depends upon the:-

Mass of the body (m) i-e  $f \propto \frac{1}{\sqrt{m}}$ .

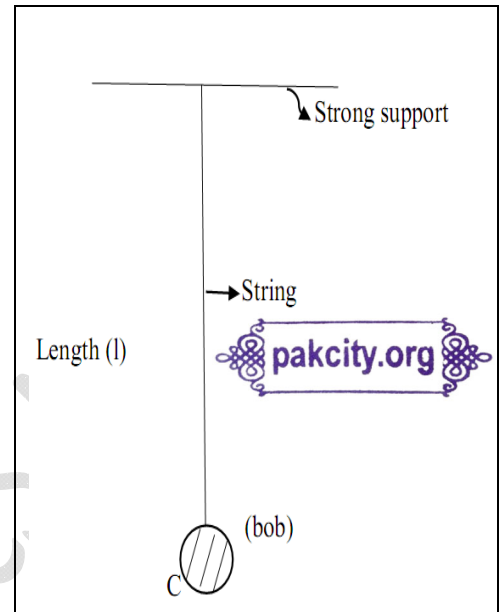
## SIMPLE PENDULUM

**History:** - Simple pendulum was first of all introduced by an **ITALIAN** Scientist **Galileo** in **1583**.

**Definition:** - A point mass suspended by a weightless, inextensible string supported from a fixed frictionless support is known as simple pendulum.

**Construction:** - It consists of

- (1) A small mass (bob).
- (2) Weightless and inextensible string
- (3) Fixed (Strong) support. As shown in figure.



## SHOW THAT THE MOTION OF SIMPLE PENDULUM IS S.H.M ( $a \propto -x$ )

**Proof:** - Consider a pendulum whose length is " $l$ " as shown in figure.

**At mean position:** - There are two forces acting on the bob which are

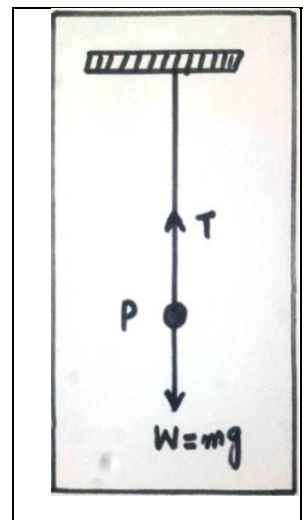
- (i) Weight of bob =  $w = mg$  (downward)
- (ii) Tension in string =  $T = mg$  (upward)

**Net force F:** -  $F = W - T$

or  $F = mg - mg$

$$F = \cancel{mg} - \cancel{mg} = 0$$

So the bob is at rest at point C because  $F = 0$



**At the extreme position:** - If we displace the bob from its mean position to any extreme position and then release and start oscillations as shown in figure (10.5).



For  $\Delta QRS$  we resolve the weight ( $W = mg$ ) in to two components. So there are three forces acting on the bob at the extreme position which are.



(i) Tension =  $T = mg \cos \theta$

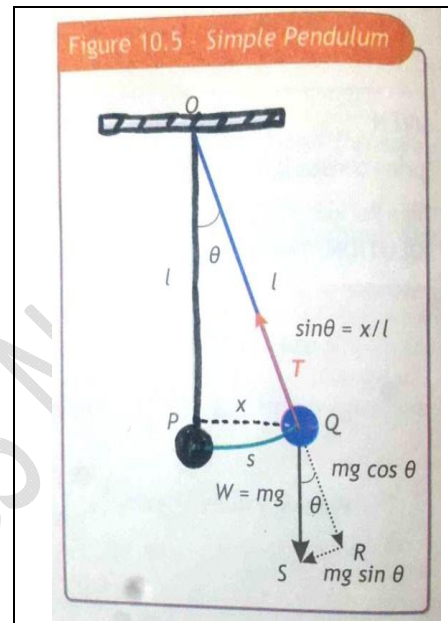
(ii)  $mg \cos \theta = W_x$

(iii)  $mg \sin \theta = W_y$

Both  $mg \cos \theta$  and  $T$  are equal in magnitude but opposite in direction so they cancel the effect of each other as shown in figure Z (b)-

**Net force (F):-** The restoring force is only provided by components " $mg \sin \theta$ ". Therefore

$$F = - mg \sin \theta \dots\dots\dots (i)$$



From newton 2nd law of motion  $F = ma$  then equation (i) become

$$ma = - mg \sin \theta$$

$$\cancel{m}a = -\cancel{m}g \sin \theta$$

$$a = - g \sin \theta \dots\dots\dots (ii)$$

**Now from  $\Delta OPQ$ :-**  $\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{PQ}{OQ} = \frac{x}{l}$

Then equation (ii) becomes

$$a = - g \left( \frac{x}{l} \right) = \frac{g}{l} (-x)$$

$$\frac{g}{l} = \text{Constant}$$

$$a = \text{Constant} (- x)$$

$$a \propto - x \dots\dots\dots (R)$$

So equation (R) represents the mathematical form of SHM.

## TIME PERIOD "T" AND FREQUENCY "f" OF SIMPLE PENDULUM



**For time period (T) :-** As we know that incase of SHM

$$a \propto -X \quad \text{OR} \quad a = \text{Constant} (-X)$$

$$a = \omega^2 (-X)$$

$$\text{OR} \quad a = \left(\frac{2\pi}{T}\right)^2 (-X)$$

$$a = -\frac{4\pi^2}{T^2} (X) \dots\dots\dots (1)$$

$$\text{Constant} = \omega^2$$

$$\omega = \frac{2\pi}{T}$$

**In case of Simple pendulum: -**  $a = -\frac{g}{l} X \dots\dots\dots (2)$

By comparing equation (1) and (2) we get

$$-\frac{4\pi^2}{T^2} X = -\frac{g}{l} X$$

$$-\frac{4\pi^2}{T^2} \cancel{X} = -\frac{g}{l} \cancel{X} \quad \text{OR} \quad \frac{4\pi^2}{T^2} = \frac{g}{l} \dots\dots\dots (4)$$

On re-arranging the equation (4) becomes  $T^2 = 4\pi^2 \frac{l}{g} \dots\dots\dots (5)$

Taking square root on both sides of equation (5) we have

$$\sqrt{T^2} = \sqrt{4\pi^2} \sqrt{\frac{l}{g}} \quad \text{OR} \quad T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2\pi \sqrt{\frac{l}{g}} \dots\dots\dots (6)$$

**Results:-** From equation (6) we conclude that the time period of simple pendulum depends upon the:-

- (i) Length of bob ( $l$ ) :-  $T \propto \sqrt{l}$ .
- (ii) Gravitational Constant :-  $T \propto \frac{1}{\sqrt{g}}$

**For frequency of Simple Pendulum:** - As we know that



$$f = \frac{1}{T} \dots\dots\dots (7)$$

Now by putting equation (6) in equation (7) we get

$$f = \frac{1}{2\pi \sqrt{\frac{l}{g}}} = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \dots\dots\dots (8)$$

**Result:** - From equation (8) we conclude that the frequency of simple pendulum depends upon the:-

- (i) Length of the bob ( $l$ ) i-e  $f \propto \frac{1}{\sqrt{l}}$ .
- (ii) Gravitational acceleration ( $g$ ) i-e  $f \propto \sqrt{g}$


**NOTE:** - (i) The time period of the pendulum does not depends on the mass of the pendulum bob.

(ii) The time period of a pendulum also does not depend on its amplitude.

## DAMPING

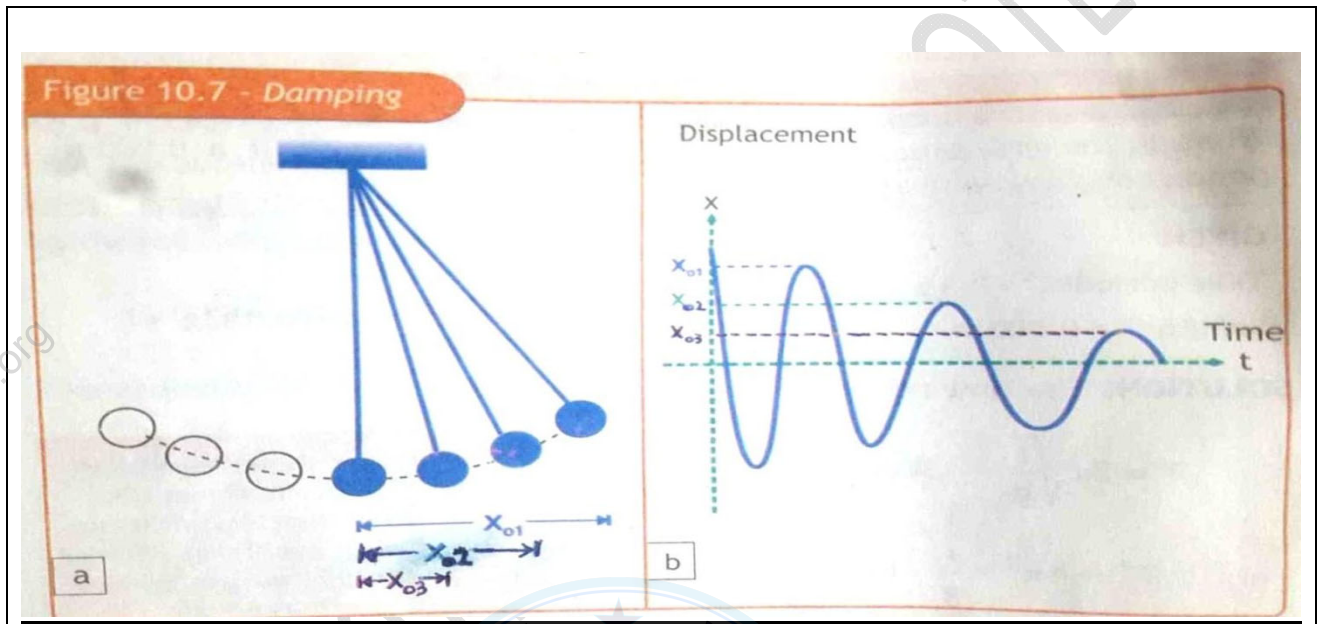
**Definition:-** Any effect that tends to reduce the amplitude of vibrations is known as damping. OR

The process by which energy of the oscillating system is dissipated is known as damping.

**Damped Oscillation**: - Those oscillations in which the amplitude decreases steadily with time are known as damped oscillations. 

**Cause**:- Damping occurs due to work done against the friction.

**Example**: - Oscillation of real simple pendulum.

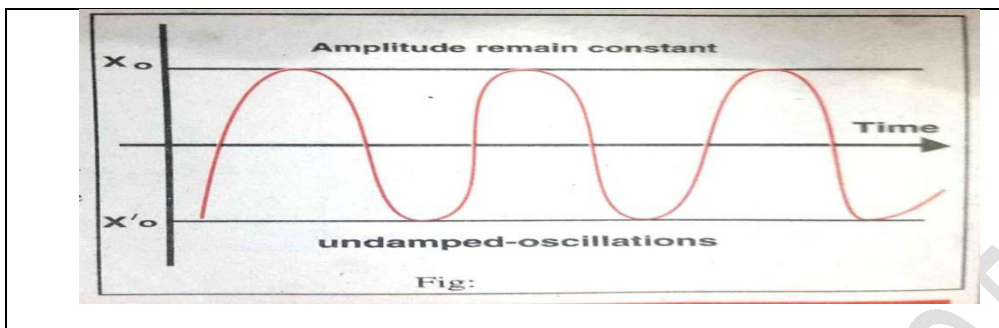


**Applications of Damping**:- The concept of damping is used in the suspension system of a car, motor cycle etc. Damping system is required to ensure a comfortable ride for the passengers when the car, bus, motor cycle etc moving on a bumpy, rough road by producing excessive oscillations by damping using shock absorbers in such vehicles.

**Un-Damped Oscillations**:-

**Definition**:- Those oscillations in which the amplitude oscillation remains constant are known as un-damped oscillations.

**Example:-** When we set a simple pendulum into oscillation in a vacuum. It will continue to execute SHM (oscillation) of constant amplitude.



## DIFFERENCE BETWEEN DAMPED OSCILLATION AND UN- DAMPED OSCILLATION

DAMPED OSCILLATIONS	UN- DAMPED OSCILLATIONS
Those oscillations in which the amplitude decreases steadily with time.	Those oscillations in which the amplitude remains same with time.
They occur in the presence of frictional forces.	They occur in the absence of frictional forces.
In this case energy is dissipated from the oscillating system.	In this case energy is not dissipated from the oscillating system.
Oscillation of real simple pendulum.	Oscillation of an ideal simple pendulum.



## NATURE OF WAVES AND THEIR TYPES

### WAVES MOTION

**Definition:** - The transmission of energy in a medium due to the oscillatory motion of the particles of the medium about their mean position is known as wave motion. OR The mechanisms by which energy can be transferred from one point to another point is known as wave motion. OR

Wave is a phenomenon of transferring energy from one place to another without the transfer of matter.

**Generation and propagation of waves:** -

When a pebble is dropped into quite pool of water, the circular ripples are produced at a point where the pebble touches the water. These ripples spread towards the edges in all directions on the surface of water with some speed as shown in figure.



## WAVES

**Definition:** - A disturbance of some kind by means of which energy is transmitted from one place to another place is known as Wave.

**Examples:** -


- (i) Sound Waves
- (ii) Light Waves
- (iii) Radio Waves etc .

## COMMON FEATURS OF WAVES

There are two common features which are given below.

- (1) A wave is a traveling disturbance.
- (2) A wave carries energy from place to place.



**Explanation:-** A wave is a disturbance that transfer energy trough a medium. While the disturbance and energy that it carries, moves through the medium, the matter does not experience net moment. Instead, each particle in the medium vibrates about some mean (or set) position as the waves passes. 

## ACTIVITY: WATER WAVES AS MEANS OF ENERGY TRANSFER

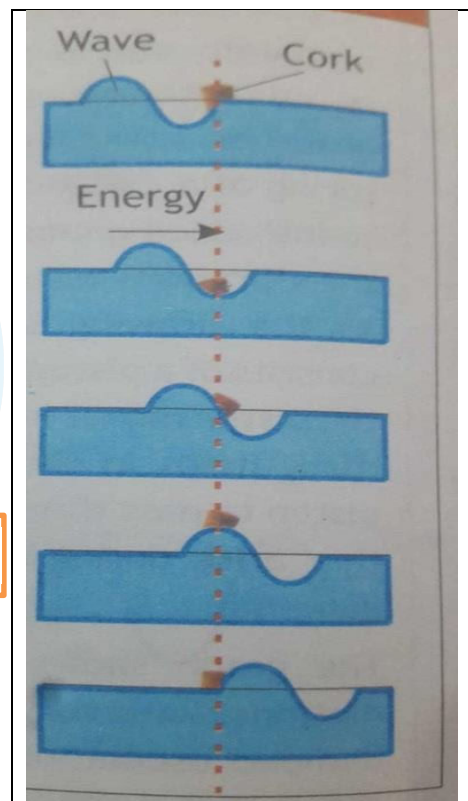
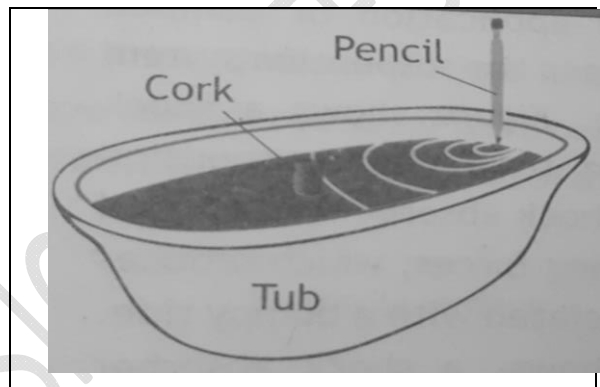
**APPRATUS:-** (i) Tub.(ii)Pencil. (iii)Cork.(iv)Water.

**WORKING:-** Take a tub full of water, move a pencil up and down at one edge of the tub. Waves are produced on the water surface which move away from point of impact of the pencil.

Place a cork in the middle of the tub. You can see that as the waves passes through the cork it will move up and down about its place.

The energy which is spent in moving the pencil up and down reaches the cork by means of water waves due to which it is also moves up and down.

**Result:-** Notice that that during this process that the cork does not move with waves, it only moves up and down which shows that the particles of matter (water) does not move forward with waves instead they oscillate about their mean position.



## TYPES OF WAVES OR CATEGORIES OF WAVES

There are two types of waves which are given below.

- (1) Mechanical Wave
- (2) Electromagnetic waves

## MECHANICAL WAVES

**Definition:** The waves which require any material medium for their propagation are known as mechanical wave. OR



The waves produced by oscillation of material particles are known as mechanical waves.

**Examples: -**

- (i) Sound Waves
- (ii) Waves on the surface of water.
- (iii) Waves produce on the string and spring.
- (iv) Seismic waves etc.

**Mechanical waves can exist only within a material medium.**

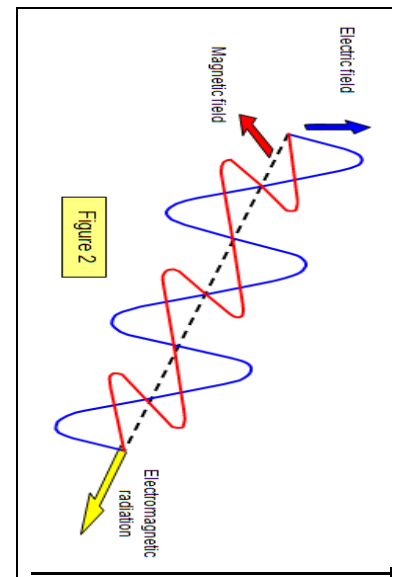
## ELECTROMAGNETIC WAVES

**Definition:** -The waves which do not require any material medium for their propagation are known as electromagnetic waves. OR

The waves that propagate by oscillation of electric and magnetic fields are known as electromagnetic waves.

**Examples:**

- (i) Light Waves
- (ii) Radio Waves .
- (iii) Heat Waves
- (iv) X-Rays waves etc.



**Electromagnetic waves is a combination traveling electric and magnetic fields.**

**NOTE:-**

**MATTER WAVES:-**

**Definition:** - Those waves which are associated with microscopic particles moving with very high velocity are known as matter waves.

**Other Name:** - They are also called particles waves.

**Examples:** - 

Electrons moving with very high velocity behave like matter waves etc.

## TYPES OF MECHANICAL WAVES

There are two type of mechanical waves which are given below.

- (1) Transverse Waves.
- (2) Longitudinal Waves.

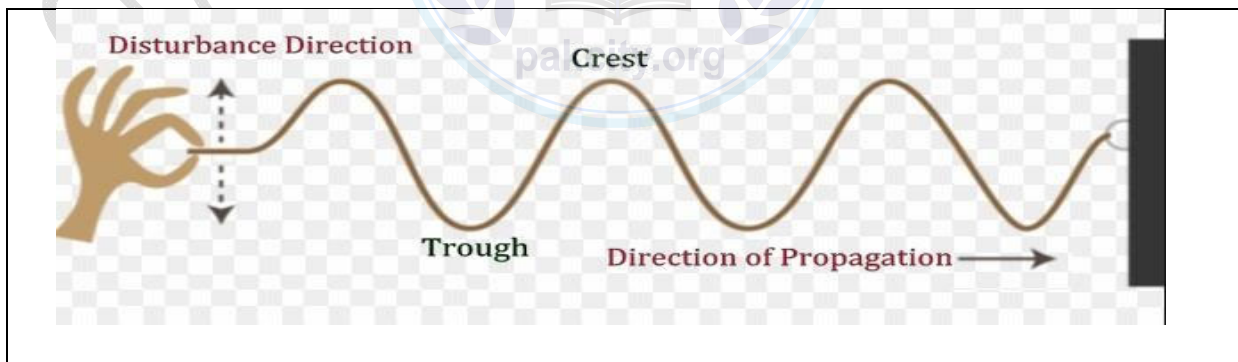
### (1) TRANSVERSE WAVES

**Definition:-** The waves in which the vibration of the individual particles of the medium is perpendicular to the direction of propagation of wave are known as transverse wave. OR

A transverse wave is one which the disturbance occurs perpendicular to the direction of the waves.

**Explanation:**

- (i) These waves consists of crests and troughs.
- (ii) The distance between two consecutive crests or trough is known as wave length.



**Examples: -**

(i) Waves on the surface of water.

(ii) Waves in a stretched string.

(iii) Radio waves . 

(iv) Light waves .

(v) Microwaves etc.

**Crest:** The part of transverse waves where the medium of propagation is above the mean position. OR

The portion of a wave above the mean level is known as crest.

**Trough:** The part of transverse waves where the medium of propagation is below the mean position. OR

The portion of a wave below the mean level is known as trough.

**Transverse waves also travel on the strings of instruments such as guitars and banjos.**

## LONGITUDINAL WAVES

**Definition:-** The wave in which the particles of the medium vibrate about their mean position parallel to the direction of propagation of waves are known longitudinal waves. OR

A longitudinal wave is one which the disturbance occurs parallel to the line of travel of the wave.

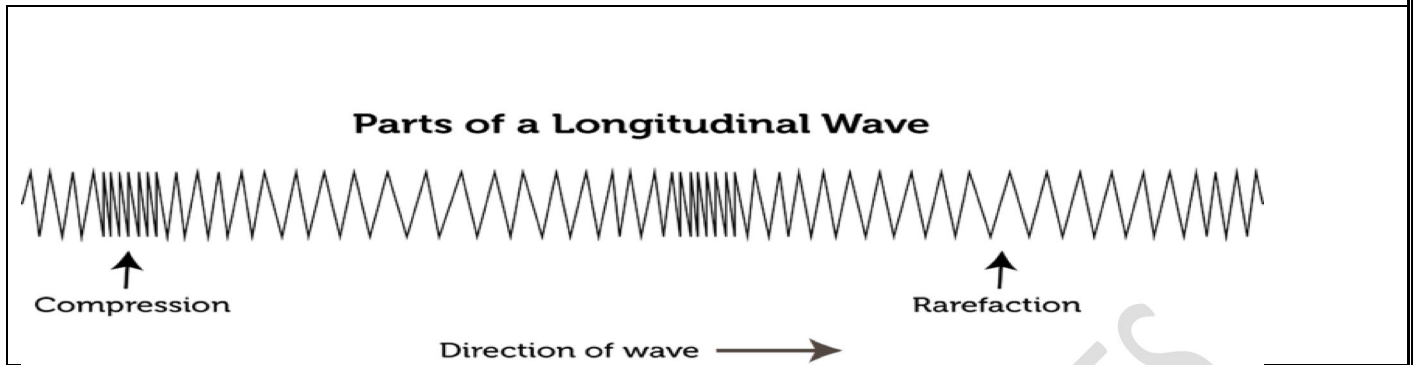
**Other Name:** -They are also called compressional waves.

**Explanation:**

(i) These waves consist of compressions and rarefactions.

(ii) The distance between two consecutive compressions or rarefactions is known as wave length.





**Examples: -**

(i) Sound Waves

(ii) Waves along a spring etc.

**Compression:-** The portion of the medium where the particles are over crowded is known as compression.

**Rarefaction:-** The portion of the medium where its particle are least over crowded.

## CHARACTERISTICS WAVE PARAMETERS

The characteristics feature of wave are given below.

### (1) WAVE LENGTH

**Definition:** In case of

a. **Transverse Waves:** The distance between two consecutive crest or trough is known as wave length.

b. **Longitudinal Wave:** The distance between two consecutive compression or rarefaction is known as wave length. OR

The shortest distance between points where the wave pattern repeats itself is known as wavelength.

**Symbol:-** It is denoted by Greek letter lambda " $\lambda$ ".

**Mathematical Form:-**  $\lambda = \frac{v}{f}$

**Unit:-** Its unit is meter (m).

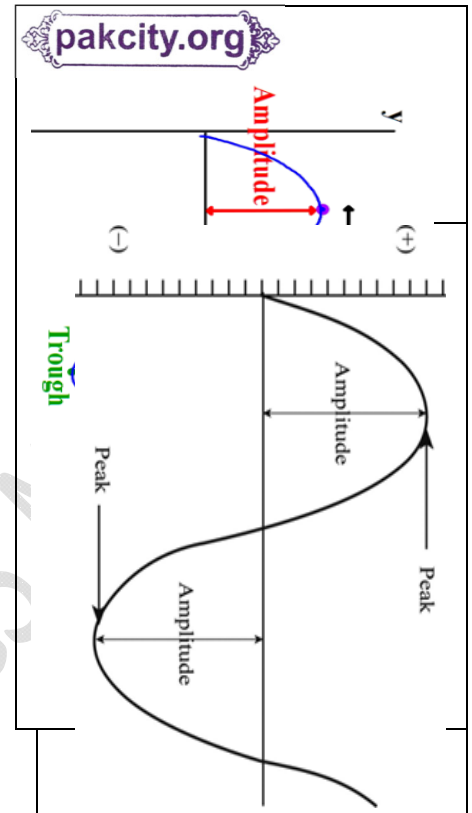
## (2) AMPLITUDE

**Definition:-** The maximum displacement of the particles of the medium from their original position is known as amplitude. OR

The maximum displacement covered by a vibrating particle from its mean position on either side is known as amplitude.

**Symbol:-** It is denoted by " $X_0$ " OR " $A$ ".

**Unit:-** Its SI unit is meter.

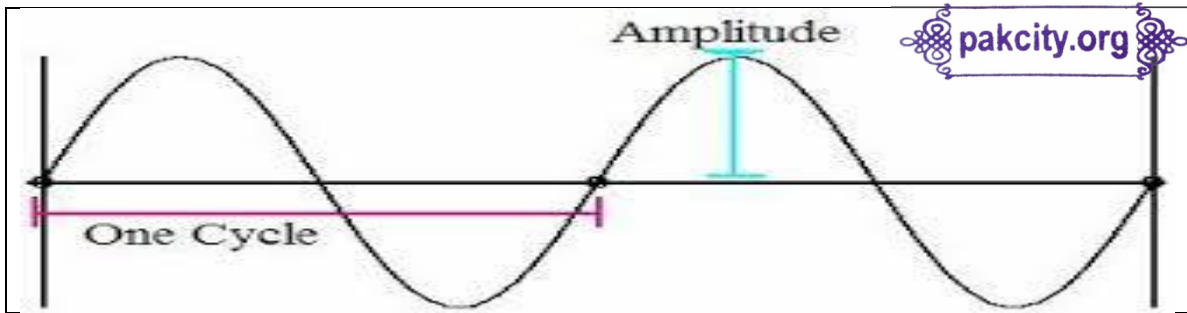


## (3) WAVE CYCLE

**Definition:-** Wave cycles are pairs of a maximum value and a corresponding minimum value.

**Explanation:-** As a wave passes given point along its path, that undergoes cyclic motion. The point is displaced first one direction and then in the other direction. Finally, the point returns to its original equilibrium position, thereby completing one cycle.





## (4) FREQUENCY

**Definition:-** The number of wave cycles (N) passing through a certain point (P) in unit time is known as frequency.

**Symbol:-** It is denoted by “f”.

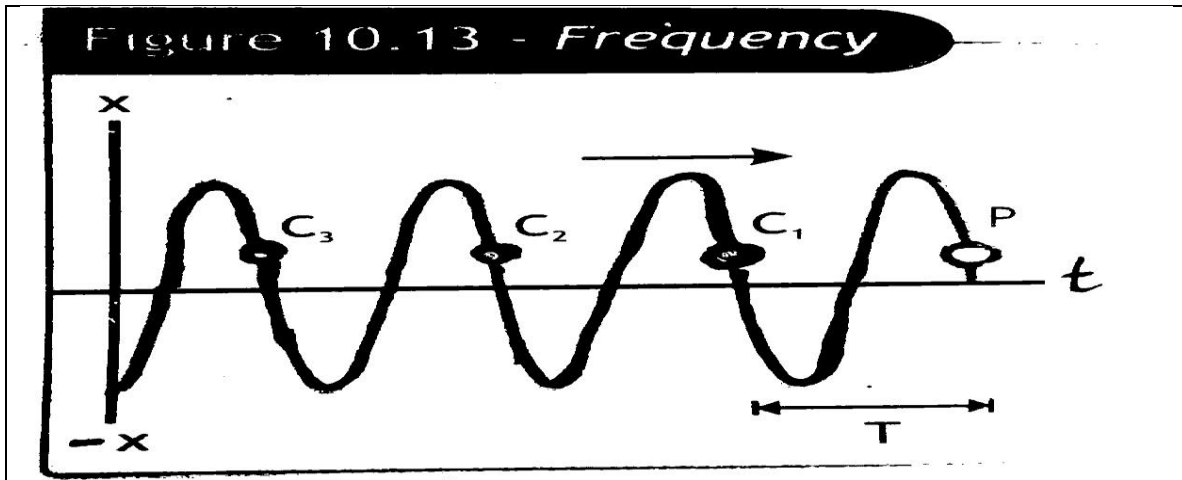
**Mathematical Form:-**

$$\text{Frequency} = \frac{\text{Numbers of waves cycles}}{\text{Time}}$$

$$f = \frac{N}{t}$$

**Unit:-** Its SI unit is hertz (Hz).

**Explanation:-** Figure 10.13 shows that three cycles  $C_1$ ,  $C_2$  and  $C_3$  approaching point “P” if these cycles cross point “P” in one second then frequency will be 3 hertz.



### (5) TIME PERIOD

**Definition:-** The time required for one wave cycle to pass through a certain point is known as time period.

**Symbol:-** It is denoted by "T".

**Mathematical Form:-**  $T = \frac{1}{f}$

**Unit:-** Its unit is second (sec).

### (6) WAVE SPEED

**Definition:-** The distance covered by a wave in unit time is known as velocity of wave

**Symbol:-** It is denoted by "V"



**Mathematical Form:-**

$$\text{Velocity} = \frac{\text{Distance travelled}}{\text{time taken}}$$

$$V = \frac{s}{t}$$

**Unit:** its SI unit is m/s ( m s<sup>-1</sup> ) .

**Factors:** velocity of a wave depends upon the

- (i) Nature of the waves.
- (ii) Elasticity of medium.
- (iii) Density of medium.
- (iv) Temperature of medium.

**NOTE:-**

- (i) Light and radio wave travel through air at about  $3 \times 10^8$  m/s (ms<sup>-1</sup>)
- (ii) Sound waves travel through air about 330 m/s (ms<sup>-1</sup>) at 0°C



### Relationship Between velocity, frequency and wavelength of a wave

OR Show that  $V = f\lambda$

**Proof:-** As we know that

$$V = S/t \text{ ----- (1)}$$

**In case of waves:-**

$$S = \lambda \quad \text{and} \quad t = T$$

Then equation (1) becomes

$$V = \frac{\lambda}{T}$$

OR

$$V = \lambda \left( \frac{1}{T} \right) \text{ ----- (2)}$$

We also know that:  $f = \frac{1}{T}$

Then equation (2) become



$$V = (f) (\lambda)$$

OR

$$V = (f) (\lambda)$$

OR

$$V = f\lambda \text{ ----- (4)}$$

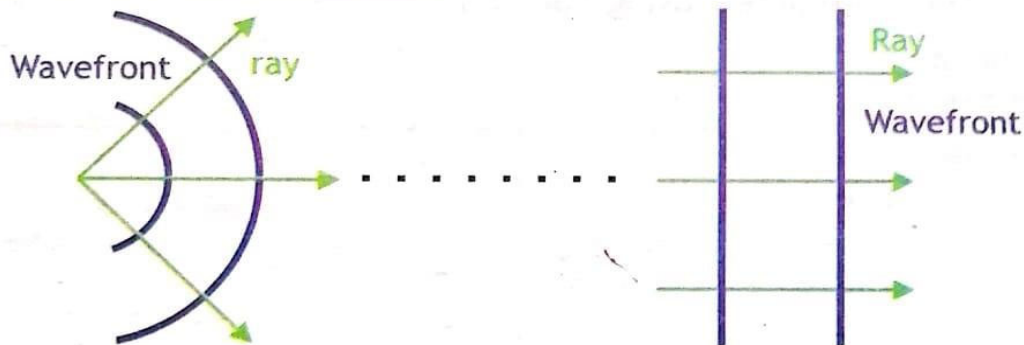
Equation (4) is the required proof. OR

Equation (4) represents the relationship between velocity frequency and wavelength of the wave.

**WAVE FRONT:-** The locus of all points in a medium which have the same phase of vibration.

**Ray:-** The arrows to indicate the direction of wave fronts.

Figure        - Rays and wavefronts



*Rays, signifying the direction of wave motion, are always perpendicular to the wave fronts (wave crests). (a) Circular or spherical waves near the source. (b) Far from the source, the wave fronts are nearly straight or flat, and are called plane waves.*

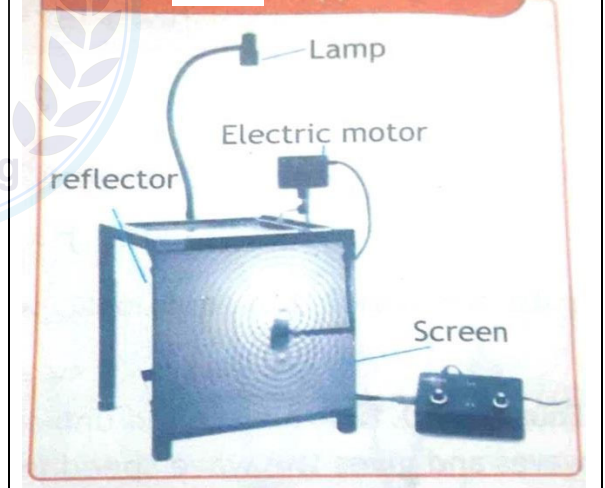
## RIPPLE TANK

**Definition:-** It is a device to produce water waves and to study their characteristics

**Construction:-** It consists of

- (i) Rectangular tray with transparent glass bottom.
- (ii) A bulb or A lamp
- (iii) White screen

Figure        - Ripple Tank



(iv) Vibrator.

**Working:** - Circular waves are produced by wave generator in a rectangular tray contains water. At the same time bright and dark circular fringes are seen on the screen.



**Reason:-**

(i) Water waves act as crest and trough and behave like convex and concave lens.

(ii) Crests converge light and produces bright circles.

(iii) Troughs divers light and produce circles.

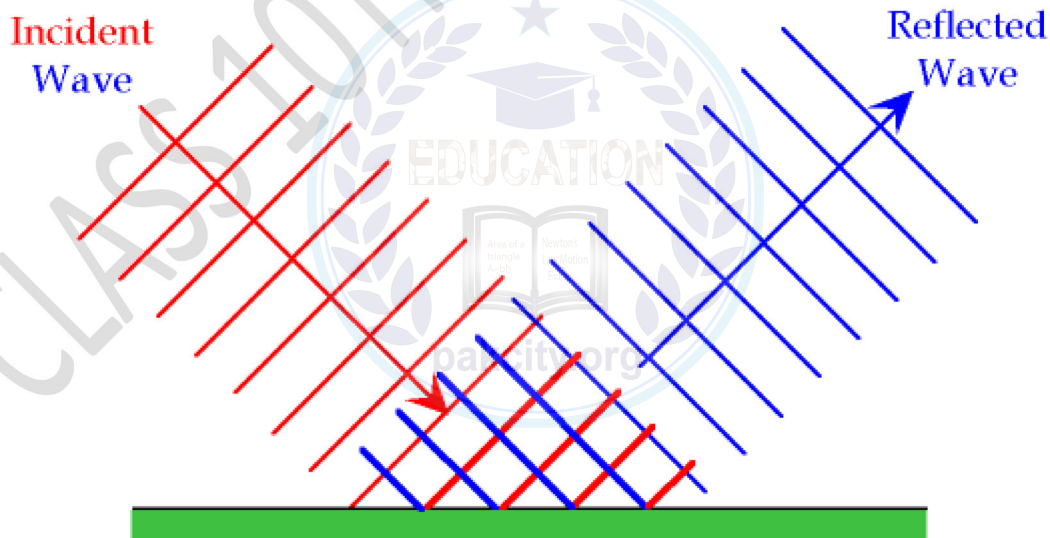
With the help of ripple tank, we can study all the basic properties of waves including reflection, refraction diffraction etc.

### **(1) Reflection of Wave: -**

**Definition:** - The bouncing back of waves after striking from some obstacles is known as reflection of waves.

**Explanation: -**

In a ripple tank, reflection can be demonstrated by placing an upright barrier in water as shown in figure (H).



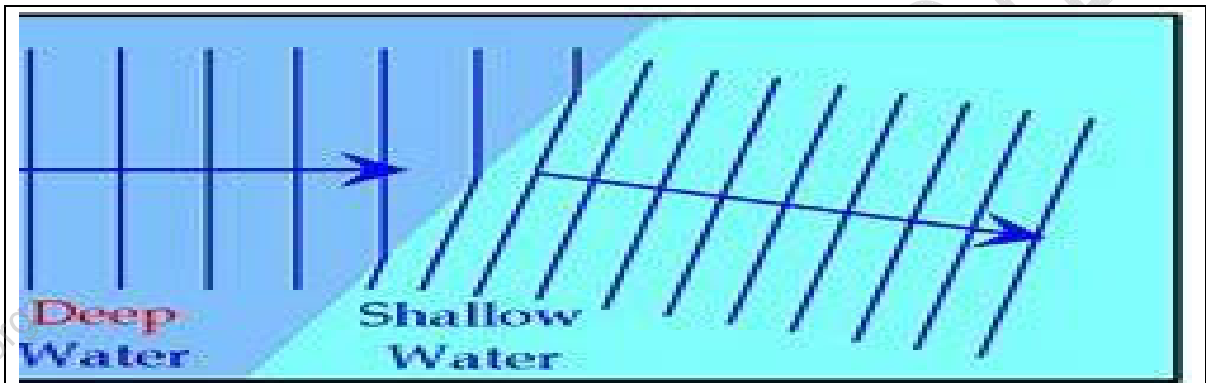
## **(2)Refraction of waves:**

**Definition:** - The slight bending of waves from its original path is known as refraction of waves.



### **Explanation: -**

This phenomenon can be demonstrated by the waves in ripple tank place a plastic sheet in the bottom portion of the tray observe that the incident waves refract at the edge of the plastic sheet as shown in figure (R).



### **NOTE:**

The speed of waves in the deep water is greater than their speed in the shallower water.

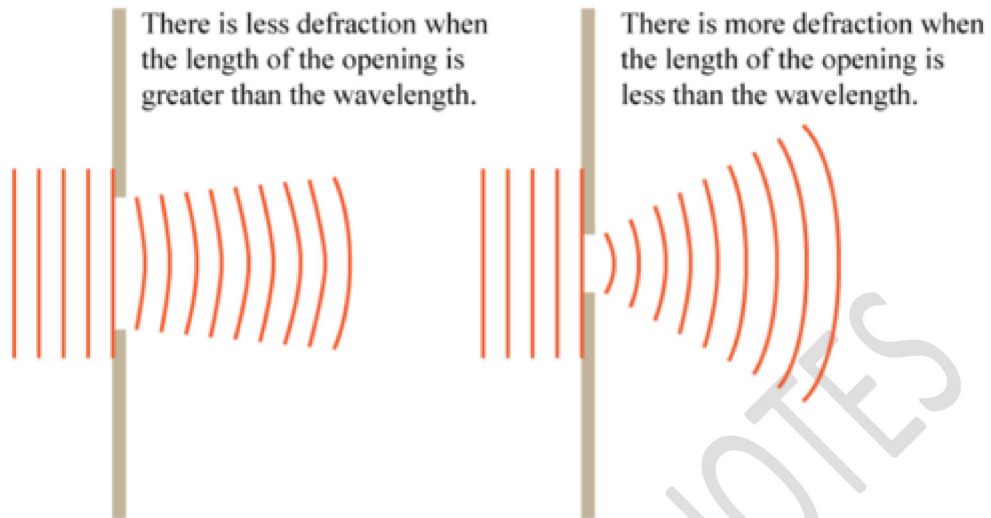
## **Diffraction of waves:**

**Definition:** - The bending of waves around the corners of an obstacle is known as diffraction of waves.

### **Explanation:**

In ripple tank, the diffraction of waves can be demonstrate by placing two obstacle in such a ways that a small opening is left between them as shown in figure (h) and (k) .





**Note:-** This effect will be

- (i) Greatest when the size of the opening is less than or equal to the size of the wavelength of generated wave.
- (ii) Weakest when the size of the opening is greater than the size of the wavelength of generated wave.
- (iii) It must be noted that wavelength ( or Speed) of the wave is not affected by diffraction.



## CONCEPTUAL QUESTIONS

**Q # 01: Is every oscillatory motion simple harmonic? Give examples.** 

**Ans:- Statement**:- No, every oscillatory motion is not simple harmonic.

**Reason**:- It is because in every oscillatory motion the necessary condition of SHM does not fulfill.

**Explanation**:- As we know that in every SHM

(1)  $F_{\text{res}} \propto -X$

(2)  $a \propto -X$

**Examples**:-

(i) A ball dropped from a certain height onto the floor and keeps bouncing does not execute SHM because it does not fulfill the necessary condition of SHM.

(ii) An electrocardiogram traces the periodic pattern of a beating heart, but the motion of the recording needle is not a SHM because it does not fulfill the necessary condition of SHM.

**Conclusion**:- As conclusion we find that every oscillatory motion is not simple harmonic.

**Q # 02: For a particle with simple harmonic motion, at what point of the motion does the velocity attain maximum magnitude? Minimum/ magnitude ?**

**Ans:- Statement**:- For a particle with simple harmonic motion, at mean position its velocity attains maximum magnitude and minimum magnitude at extreme position.

**Reason**:- It is because

(i) At mean position  $X = 0$ .

(ii) At extreme position  $X = X_0$

**Explanation**:- As we know that the instantaneous velocity of the oscillator is

$$V = \omega \sqrt{X_0^2 - X^2} \dots\dots\dots (1)$$

(a) **For Maximum velocity**:- At mean position  $X = 0$  then equation (1) becomes.

$$V = \sqrt{X_0^2 - X^2} = \omega \sqrt{X_0^2 - (0)^2} = \omega \sqrt{X_0^2 - 0} = \omega \sqrt{X_0^2}$$

OR  $V_{max} = \omega X_0$  ..... (2)

**(b) For Minimum velocity:-** At extreme position  $X = X_0$  then equation (1) becomes

$$V = \omega \sqrt{X_0^2 - X^2} = V = \omega \sqrt{X_0^2 - X_0^2} = \omega (0)$$

OR  $V_{min} = 0$  ..... (3)

**Conclusion:-** From equation (2) and (3) as conclusion we find that for a particle with simple harmonic motion, at mean position its velocity attains maximum magnitude and minimum magnitude at extreme position

**Q # 03: Is the restoring force on a mass attached to spring in simple harmonic motion ever zero? If so, where?**

**Ans:- Statement:-** The restoring force on a mass attached to spring in simple harmonic is zero only at mean position.

**Reason:-** It is because Restoring Force  $\propto$  Displacement

$$F_{res} \propto -x$$

**Explanation:-** As we know that

$$F_{res} = -kx$$
 ..... (1)

**Condition:-** In case of SHM at mean position displacement =  $x = 0$

Then equation (1) becomes.

$$F_{res} = -k(0) = 0$$

**Conclusion:-** As conclusion we find that the restoring force on a mass attached to spring in simple harmonic is not ever zero. It can be zero only at mean position.

**Q # 04: If we shorten the string of a pendulum to half its original length, what is the effect on its time period and frequency?**

**Ans:- Statement:-** If we shorten the string of a pendulum to half of its original length, then its time period will be decreased and frequency will be increased.


**Reason:-** It is because

(i)  $T \propto \sqrt{L}$

(ii)  $f \propto \frac{1}{T}$

**Explanation:-**

**For time period of pendulum:-** As we know that

  $T = 2\pi \sqrt{\frac{L}{g}} \dots\dots\dots (1)$

**Condition:-** If  $L = \frac{1}{2} L$  then equation (1) becomes.

$$T' = 2\pi \sqrt{\frac{\frac{1}{2}L}{g}} = 2\pi \sqrt{\frac{L}{2 \times g}} = \sqrt{\frac{1}{2}} (2\pi \sqrt{\frac{L}{g}})$$

OR  $T' = \sqrt{\frac{1}{2}} T \dots\dots\dots (2)$

**For frequency of pendulum:-** As we know that

$$f = \frac{1}{T} = \frac{1}{2\pi \sqrt{\frac{L}{g}}} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \dots\dots\dots (3)$$

**Condition:-**  $L = \frac{1}{2} L$  then equation (3) becomes.

$$f' = \frac{1}{2\pi} \sqrt{\frac{g}{\frac{1}{2}L}} = \frac{1}{2\pi} \sqrt{\frac{2 \times g}{L}} = \sqrt{2} \left( \frac{1}{2\pi} \sqrt{\frac{g}{L}} \right)$$

OR  $f' = \sqrt{2} f \dots\dots\dots (4)$

**Conclusion:-** From equation (2) and (4) we conclude that If we shorten the string of a pendulum to half its original length, then its (i)  $T' = \frac{1}{\sqrt{2}} T$  (ii)  $f' = \sqrt{2} f$

**Q # 05:-**A thin rope hangs from dark high tower so that its upper end is not visible. How can the length of rope be determined?

**Ans:- Statement:-** A wire hangs from a dark high tower so that its upper end is not visible we can find its length by using simple pendulum time period formula.

**Reason:**  $T \propto \sqrt{\frac{l}{g}}$

**Explanation:** As we know that



$$T = 2\pi \sqrt{\frac{l}{g}} \text{ ----- (1)}$$

Squaring both sides of equation (1) we get

$$(T)^2 = (2\pi \sqrt{l/g})^2$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

OR

$$l = \frac{T^2 g}{4\pi^2} \text{ ----- (2)}$$

Now putting the values of  $g$ ,  $T$ , and  $\pi$  in equation (1) we can easily find the value of ' $l$ '.

**Q # 06:** Suppose you stand on a swing instead of sitting on it. Will your frequency of oscillation increase or decrease?

**Ans:- Statement:-** If we stand on a swing instead of sitting on it. Our frequency of oscillation will increase.

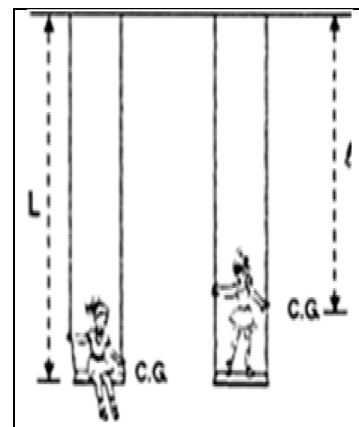
**Reason:-** It is because of  $f \propto \frac{1}{\sqrt{l}}$

**Explanation:-** As we know that

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \text{ ----- (1)}$$

From equation (1) it is an established fact that :-

(i) Greater the length less will be the frequency of



oscillation and vice versa.

We are swinging on a swing in sitting position. When we stand up, the distance of CG from the point of suspension decreases. Hence the frequency of oscillation increases.



**Conclusion:-** As conclusion we find that If we stand on a swing instead of sitting on it.

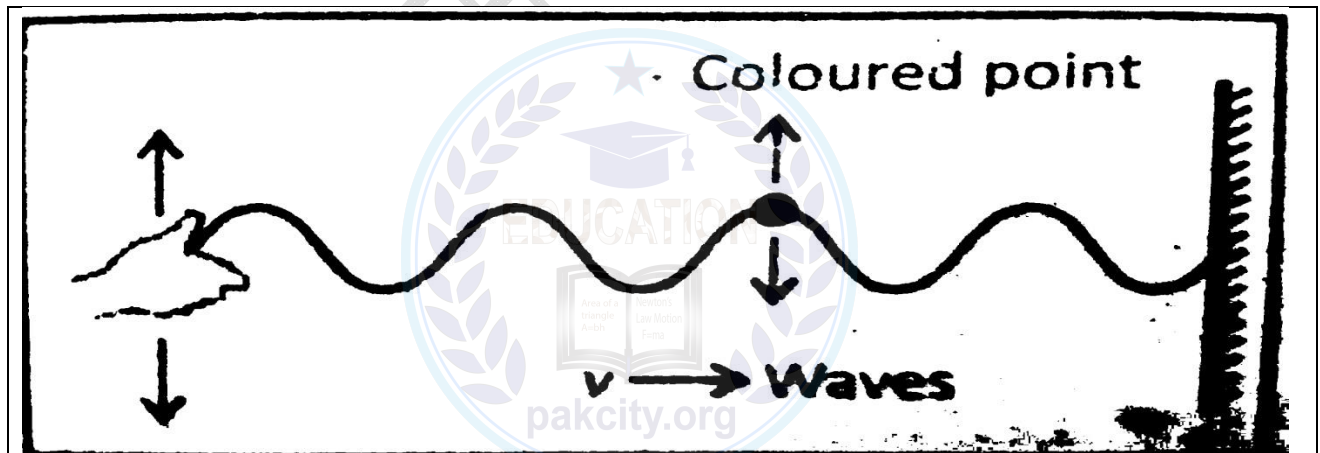
Our frequency of oscillation will increase.

**Q # 07:** Explain the difference between the speed of a transverse wave travelling along a cord and the speed of a tiny colored part of the cord.

**Ans:- Transverse Waves:-**

**Definition:-** The waves in which the particles of the medium oscillates perpendicular to the direction of motion of wave are known as transverse waves.

**Explanation:-** Take a cord and color a part of it. Attach one end of the cord to the wall and wiggle the other end regularly and continuously. The number of waves will be produced forming wave train. Observe the color marking, it will execute oscillations about certain mean position as shown in figure.



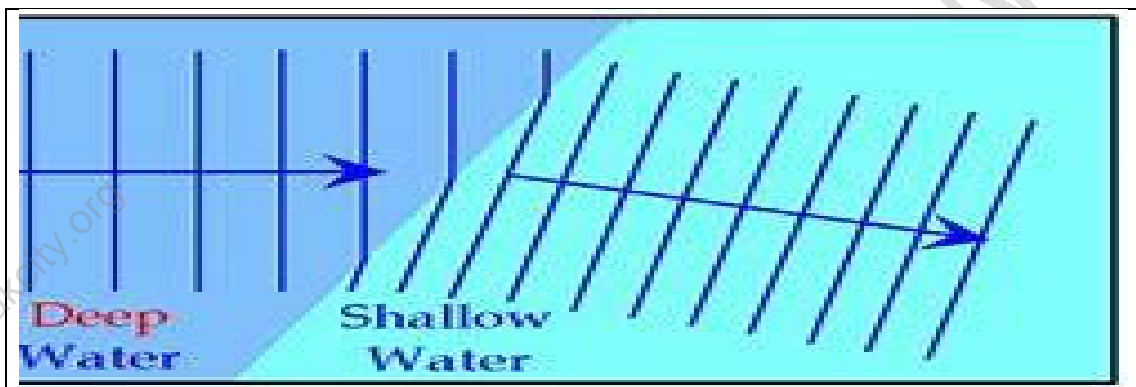
**Q # 08 :** Why waves refract at the boundary of shallow and deep water?



**Ans:- Statement:-** Waves refract at the boundary of shallow and deep water.

**Reason:-** It is because the speed of the wave is different in different media.

**Explanation:-** As we know that when waves travel from one medium into another their speed changes. This phenomenon is known as refraction. The speed of waves is different in different media. In case of water waves, the speed of waves depends upon the depth of water. Hence two portions of water having different depths can be considered to be different media for the wave propagation. When the waves travel from shallower to deeper water or from deeper to shallower water, change taking place in its wavelength and speed. The speed of waves in deep water is greater than the speed of waves in shallow water.



**Conclusion:** As conclusion we find that Waves refract at the boundary of shallow and deep water.

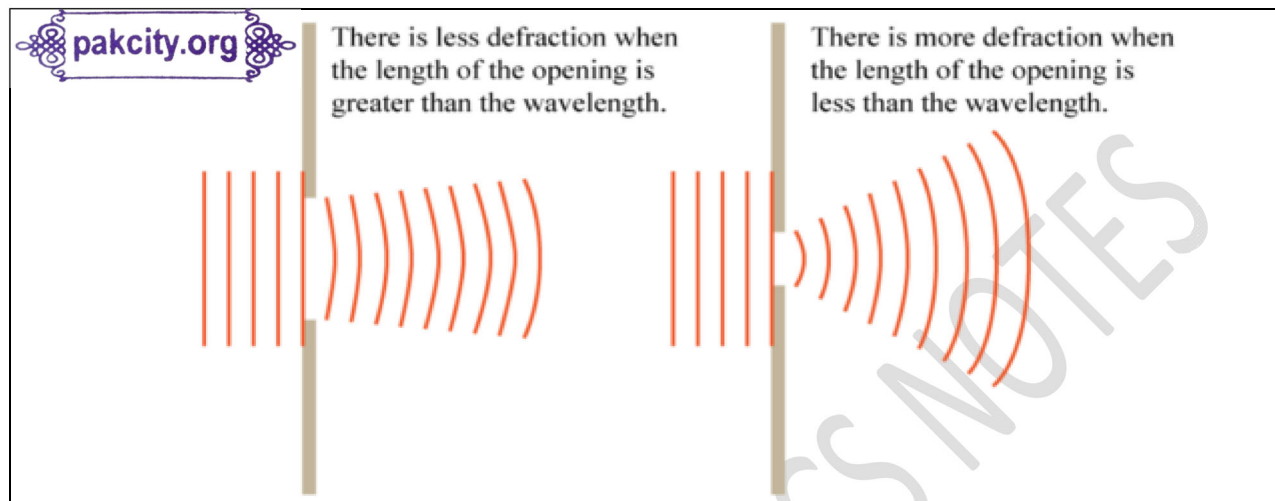
**Q # 09: What is the effect on diffraction if the opening is made small?**

**Ans:- Statement:-** The diffraction of waves increases if the opening is made small.

**Reason:-** Because  $\text{Diffraction of waves} \propto \frac{1}{\text{Size of obstacle (opening)}}$

**Explanation:-** As we know that diffraction is the process of bending of waves around the corners of the obstacle. When waves pass through an aperture or opening it spreads out due to diffraction. This effect will be greatest when the size of the opening is less (small) than or equal to the size of the wavelength of the generated waves. It must be noted that wavelength (or speed) of the waves is affected by diffraction.

**Conclusion:** - As a result we conclude that the diffraction of waves increases if the opening is made small.



## NUMERICAL QUESTIONS

**Pb#01:** A mass hung from a spring vibrates 15 times in 12 s. Calculate (a) the frequency and (b) the period of the vibration.

### **SOL:- GIVEN DATA:-**

No. of vibrations =  $n = 15$

Time taken =  $t = 12\text{sec}$

### **REQUIRED DATA:-**

a) Frequency =  $f = ?$

b) Time period =  $T = ?$

### **SOLUTION:-**

**(a) For Frequency =  $f$  :-**

**FORMULA:-** As we know that

$$f = \frac{\text{Number of vibrations}}{\text{Total time taken}} = \frac{n}{t} \dots\dots\dots (1)$$

### **CALCULATION:-**

By putting values in equation (1) we get.

$$f = \frac{n}{t} = \frac{15}{12} = 1.25 \text{ Hz}$$

**(b) For Time period =  $T$  :-**

**Formula:-** As we know that

$$T = \frac{1}{f} \dots\dots\dots (2)$$

**Calculation:-** By putting values in equation (2) we get

$$T = \frac{1}{1.25} = 0.8 \text{ sec}$$

### **RESULT:-**

(a) Frequency =  $f = 1.25 \text{ Hz}$

(b) Time period =  $T = 0.8 \text{ sec}$

**Pb# 02: A spring requires a force of 100.0 N to compress it to a displacement of 4 cm. What is its spring constant?**

**ANSWER:- GIVEN DATA:-**

Force =  $F = 100 \text{ N}$

Displacement =  $x = 4 \text{ cm} = \frac{4}{100} \text{ m} = 0.04 \text{ m}$

**REQUIRED DATA:-**

Spring constant =  $k = ?$

**SOLUTION:-**

**FORMULA:-** From Hooks law

$$F = K x \quad \text{OR} \quad K = \frac{F}{x} \dots\dots\dots (1)$$

**Calculation:-** By putting values in equation (1) we get

$$K = \frac{F}{x} = \frac{100}{0.04} = 2500 \text{ N/m} = 2.5 \times 10^3 \text{ N/m}$$

**RESULT:-**

So the spring constant =  $K = 2.5 \times 10^3 \text{ N/m}$

**P#03: A second pendulum is a pendulum with period of 2.0 s. How long must a second pendulum be on the Earth ( $g = 9.81 \text{ ms}^{-2}$ ) and Moon (where  $g = 1.62 \text{ ms}^{-2}$ )? What is the frequency of second pendulum at Earth and on Moon?**



**ANSWER:- GIVEN DATA:-**

Time period of 2<sup>nd</sup> pendulum =  $T = 2.0 \text{ sec}$

Gravitational acceleration on earth =  $g_E = 9.8 \text{ m/s}^2$ .

Gravitational acceleration on earth =  $g_m = 1.62 \text{ m/s}^2$ .

**REQUIRED DATA:-**

(i) Length of pendulum on earth =  $L_e = ?$

(ii) Length of pendulum on moon =  $L_m = ?$

(iii) Frequency of pendulum on earth =  $f_e = ?$

(iv) Frequency of pendulum on moon =  $f_m = ?$

**SOLUTION:-**

**Formula:-**  $T = 2\pi \sqrt{\frac{L}{g}} \dots\dots\dots (1)$

**(i) For length of pendulum on earth =  $L_E$  :-**

$$T = 2\pi \sqrt{\frac{L_e}{g_e}} \dots\dots\dots (2)$$

Squaring both sides of eq (2) we get

$$\text{OR } L_e = \frac{g_e T^2}{2\pi} \dots\dots\dots (3)$$

**Calculation:-** By putting values in equation (1) we get.

$$L_e = \frac{9.8 \times (2)^2}{4 \times (3.14)^2} = \frac{9.8 \times 4}{4 \times 9.85} = \frac{39.2}{39.4} = 0.99 \text{ m}$$

**(ii) For length of pendulum on moon =  $L_m$  :-**

$$L_m = \frac{g_m T^2}{2\pi} \dots\dots\dots (4)$$

**Calculation:-** By putting values in equation (4) we get.

$$L_m = \frac{1.6 \times (2)^2}{4 \times (3.14)^2} = \frac{1.6 \times 4}{4 \times 9.85} = \frac{6.4}{39.4} = 0.16 \text{ m}$$

**(iii) For frequency on the surface earth =  $f_e$  :-**

$$\text{Formula:- } f_e = \frac{1}{2\pi} \sqrt{\frac{g_e}{L_e}} \dots\dots\dots (5)$$

**Calculation:-** By Putting values in equation (5) we get

$$f_e = \frac{1}{2 \times 3.14} \sqrt{\frac{9.8}{0.99}} = \frac{1}{6.28} \sqrt{9.8}$$

$$f_e = 0.15 \times 3.13 = 0.49 \text{ Hz} \approx 0.5 \text{ Hz}$$

**(iv) For frequency on the surface earth =  $f_e$  :-**

$$\text{Formula:- } f_m = \frac{1}{2\pi} \sqrt{\frac{g_m}{L_m}} \dots\dots\dots (6)$$

**Calculation:-** By Putting values in equation (6) we get

$$f_e = \frac{1}{2 \times 3.14} \sqrt{\frac{1.6}{0.16}} = \frac{1}{6.28} \sqrt{10}$$

$$f_e = 0.15 \times 3.16 = 0.5 \text{ Hz}$$

**RESULT:-** 

(i)  $L_e = 0.99 \text{ m}$

(ii)  $L_m = 0.16 \text{ m}$

(iii)  $f_e = 0.49 \text{ Hz} \approx 0.5 \text{ Hz}$

(iii)  $f_e = 0.5 \text{ Hz}$

**Pb# 04:** Calculate the period and frequency of a propeller on a plane if it completes 250 cycles in 5.0 s.

**ANSWER:- GIVEN DATA:-**

Number of cycles (vibrations) =  $n = 250$

Total time taken =  $t = 5.0 \text{ sec}$

**REQUIRED DATA:-**

a) Frequency =  $f = ?$

b) Time period =  $T = ?$

**SOLUTION:-**

(a) For Frequency =  $f$  :-

**FORMULA:-**

$$f = \frac{\text{no. of vibration}}{\text{total time taken}} = \frac{n}{t} \dots\dots (1)$$

**Calculation:-** By Putting values in equation

(1) we get

$$f = \frac{n}{t} = \frac{250}{5} = 50 \text{ Hz}$$

**(b) Time period = T:-**

**Formula:-**  $T = \frac{1}{f} \dots\dots (2)$

**Calculation:-** By Putting values in equation in equation (2) we get

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

**RESULT:-** So as a result the

(a) Frequency =  $f = 50 \text{ Hz}$

(b) Time period =  $T = 0.02 \text{ sec}$

**Pb#05:** Water waves with wavelength 2.8 m, produced in a ripple tank, travel with a speed of 3.80 m/s. What is the frequency of the straight vibrator that produced them?

**ANSWER:- GIVEN DATA:-**

Wavelength of water waves =  $\lambda = 2.8\text{m}$

Speed of water waves =  $v = 3.80\text{ m/s}$

**REQUIRED DATA:-**

Frequency =  $f = ?$

**SOLUTION:-** As we know that

**FORMULA:-**  $V = f \lambda$  OR  $f = \frac{V}{\lambda}$  ..... (1)

**Calculation:-** By putting values in equation (1) we get

$$f = \frac{V}{\lambda} = \frac{3.80}{2.8} = 1.357\text{ Hz}$$

**RESULT:-**

So the frequency of the vibrator  $f = 1.357\text{Hz}$ .

**Pb#06:** The distance between successive crests in a series of water waves in 4.0 m, and the crests travel 9.0 m in 4.5 s. What is the frequency of the water waves?

**ANSWER:- GIVEN DATA:-**

Wavelength of water waves =  $\lambda = 4.0\text{ m}$

Distance =  $S = 9.0\text{ m}$

Time =  $t = 4.5\text{ sec}$

**REQUIRED DATA:-**

Frequency =  $f = ?$

**SOLUTION:-**

**FORMULA:-** As we know that

$$V = f \lambda \quad \text{OR} \quad f = \frac{V}{\lambda} \dots\dots\dots (1)$$

**For Velocity (V) :-** First we find the value of "V":-

$$V = \frac{S}{t} = \frac{9.0}{4.5} = 2\text{ m/s}$$

**Calculation:-** By putting values in equation (1) we get.

$$f = \frac{V}{\lambda} = \frac{2}{4.0} = 0.5\text{ Hz}$$

**RESULT:-**

So the frequency of the waves =  $f = 0.5\text{ Hz}$

**Pb# 07:-**A station broadcasts an AM radio wave whose frequency is  $1230 \times 10^3\text{ Hz}$  (1230 kHz on the dial) and an FM radio wave whose frequency is  $91.9 \times 10^6\text{ Hz}$  (91.9 MHz on the dial). Find the distance between adjacent crests in each wave.

**ANSWER:- GIVEN DATA:-**

Frequency of AM radio waves =  $f_1 = 1230 \times 10^3\text{ Hz}$

Frequency of FM radio waves =  $f_2 = 91.9 \times 10^6\text{ Hz}$

Velocity waves =  $V = 3 \times 10^8\text{ m/s}$

**REQUIRED DATA:-**

(1) Wavelength of AM radio waves =  $\lambda_1 = ?$

(2) Wavelength of FM radio waves =  $\lambda_2 = ?$



(1) For Wavelength of AM radio waves =  $\lambda_1$ :-

**Calculation:-** By putting values in equation (A) we get

$$\lambda_1 = \frac{v}{f_1} = \frac{3 \times 10^8}{1230 \times 10^3} = 0.00243 \times 10^{-3}$$

$$\lambda_1 = 0.00243 \times 10^5 = 243 \text{ m}$$

**(2) For Wavelength of FM radio waves =  $\lambda_2$ :-**

**Calculation:-** By putting values in equation (B) we get

$$\lambda_2 = \frac{v}{f_2} = \frac{3 \times 10^8}{91.9 \times 10^6} = 0.0326 \times 10^{-6}$$

$$\lambda_2 = 0.0326 \times 10^2 = 3.26 \text{ m}$$

**RESULT:-** (1)  $\lambda_1 = 243 \text{ m}$  (2)  $\lambda_2 = 3.26 \text{ m}$