

1st Year Physics
Chapter # 03
Motion and Force



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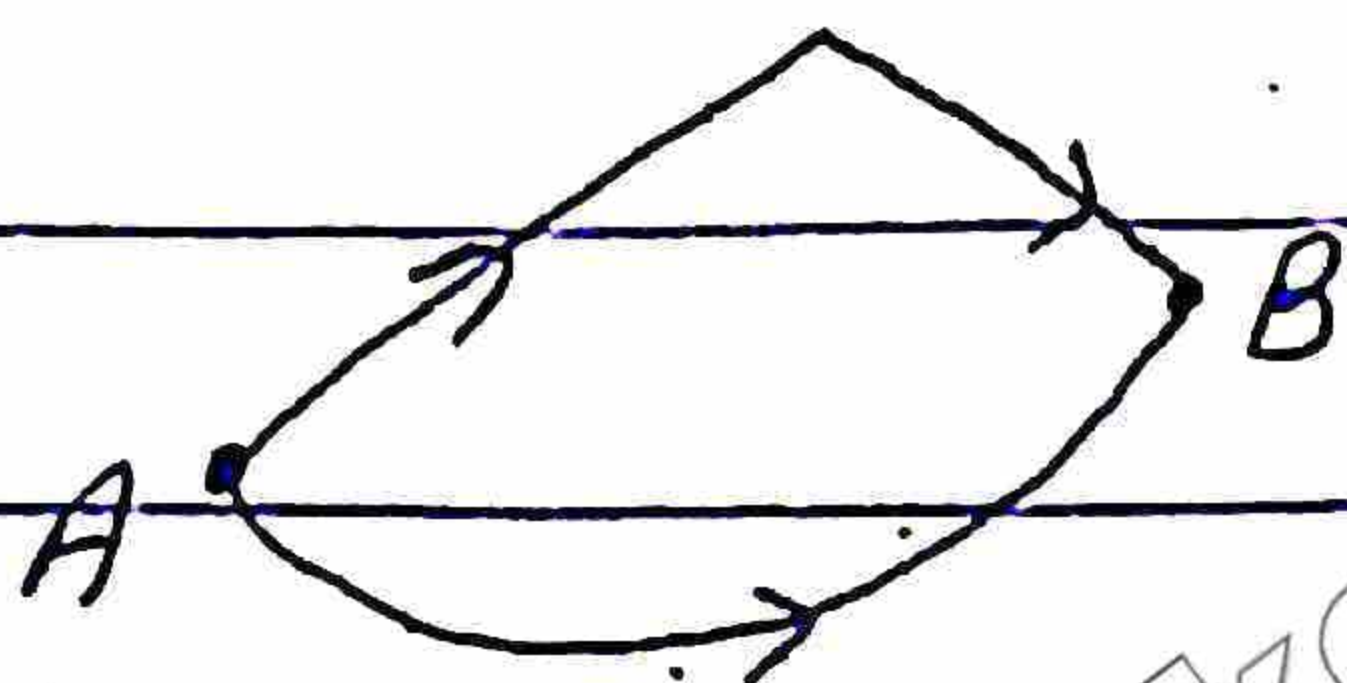
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Ch #03

Motion And Force

Distance

i- The length or path between two points is called distance

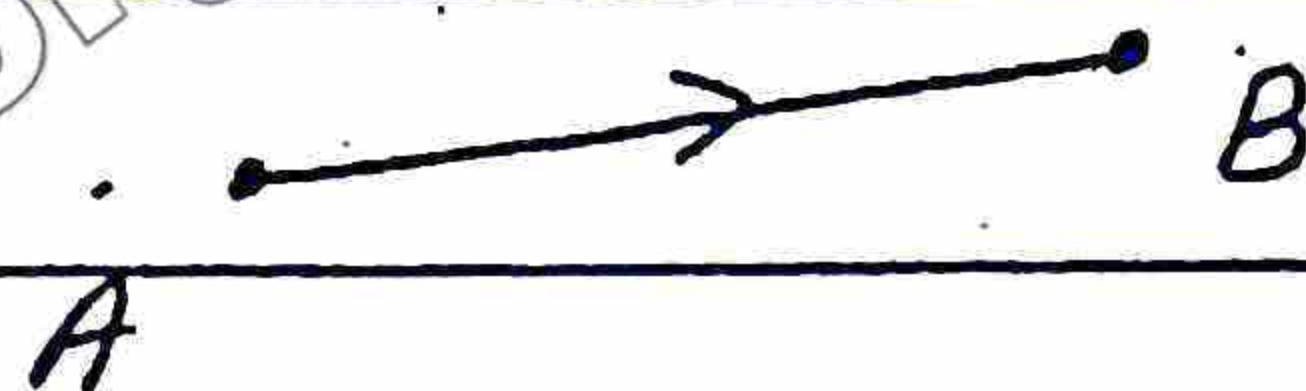


ii- It is a scalar quantity.

iii- Its SI unit is meter

Displacement

The shortest distance between two points is called displacement.



ii- It is a vector quantity.

iii- Its SI unit is meter.

Velocity: The rate of change of displacement is called velocity. It is represented by $\vec{v}$

$$\vec{v} = \frac{d\vec{s}}{t}$$

velocity is a vector quantity. Its SI unit is ms^{-1} .

Average velocity:

The total displacement covered by a body divided by total time is called average velocity.

$$\vec{v}_{av} = \frac{\vec{d}}{t}$$

Uniform velocity:

If a body covers equal displacement in equal intervals of time, then it has uniform velocity.

Variable velocity:

If a body does not cover equal displacement in equal intervals of time, then it has variable velocity.

Instantaneous velocity:

The velocity of a body at any instant of

time is called instantaneous velocity. Mathematically,

$$\vec{v}_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{d}}{\Delta t}$$

Here $\lim_{\Delta t \rightarrow 0}$ shows a very short interval of time.



Acceleration:

The rate of change of velocity of a body is called acceleration. It is represented by $\vec{a}$.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Acceleration is a vector quantity. Its SI unit is $m s^{-2}$.

Average Acceleration:

The total change in velocity divided by total time is called average acceleration.

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

Uniform acceleration:

If a body changes equal velocity in equal intervals of time, then it has uniform acceleration.

Variable acceleration:

If a body does not change equal velocity in equal intervals of time then it has variable acceleration.

Instantaneous acceleration:

The acceleration of a body at any instant of time is called instantaneous acceleration.

acceleration:

$$\vec{a}_{\text{ins}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$$

Here $\lim_{\Delta t \rightarrow 0}$ shows a very short interval of time.

Positive acceleration:

When the velocity of a body increases with time then it has positive acceleration. Its direction is same as the direction of motion of the body.

Negative acceleration:

When the velocity of a body decreases with time then it has negative acceleration. Its direction is opposite to the direction of motion of the body. Negative acceleration is also called

retardation or deceleration.

Velocity-Time Graph

Graph:

The pictorial way of representing information about various quantities is called graph.

Variable: The terms along which graph is plotted are called variables. One is called independent variable and other is named as dependent variable.

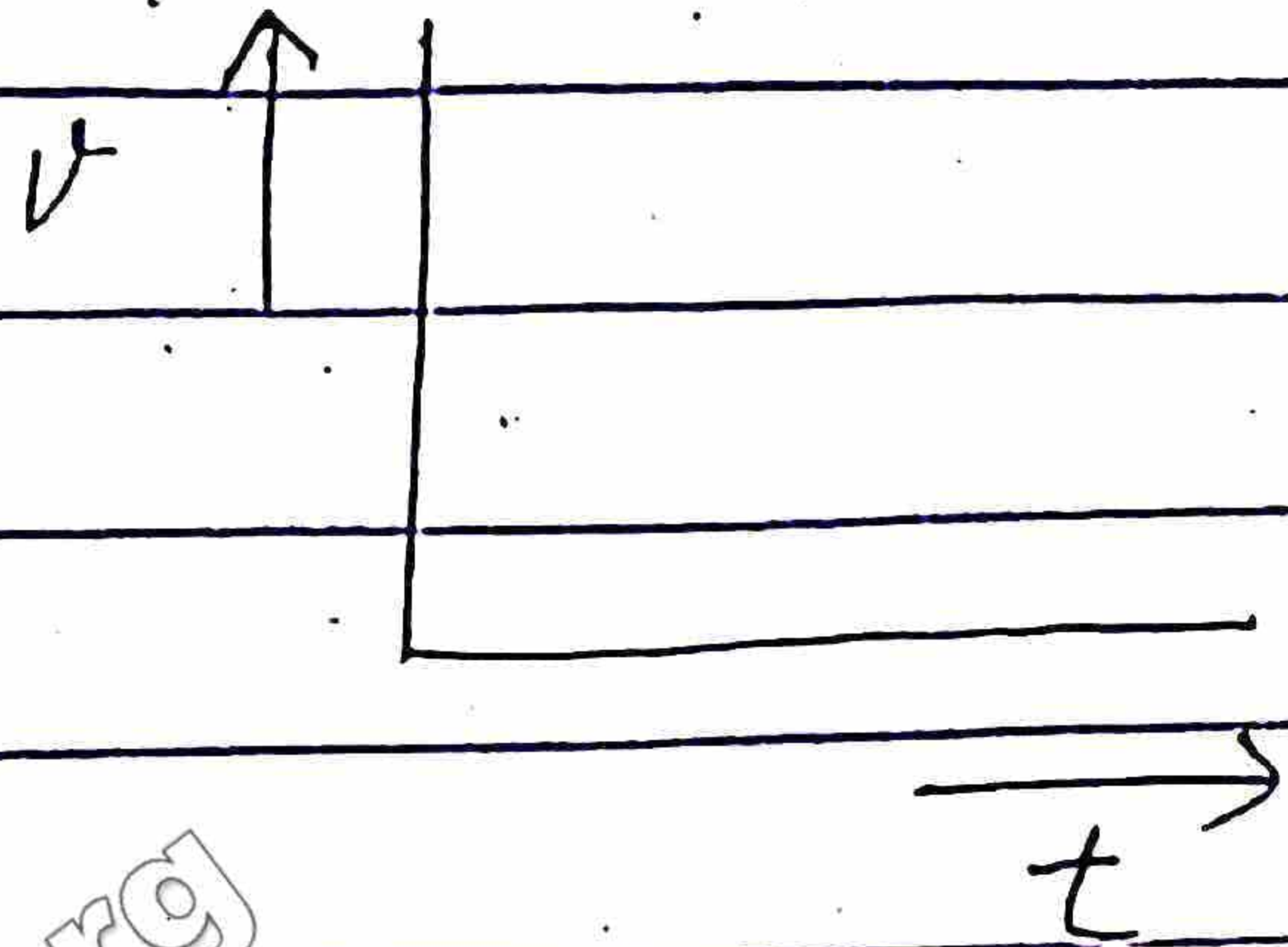


velocity-time graph:

The graph that shows the relation between velocity and time for a body is called velocity time graph. Time is independent quantity and taken along

X-axis (horizontal axis) while velocity is dependent quantity and taken along Y-axis (vertical axis).

Now we will discuss different conditions for velocity-time graph.

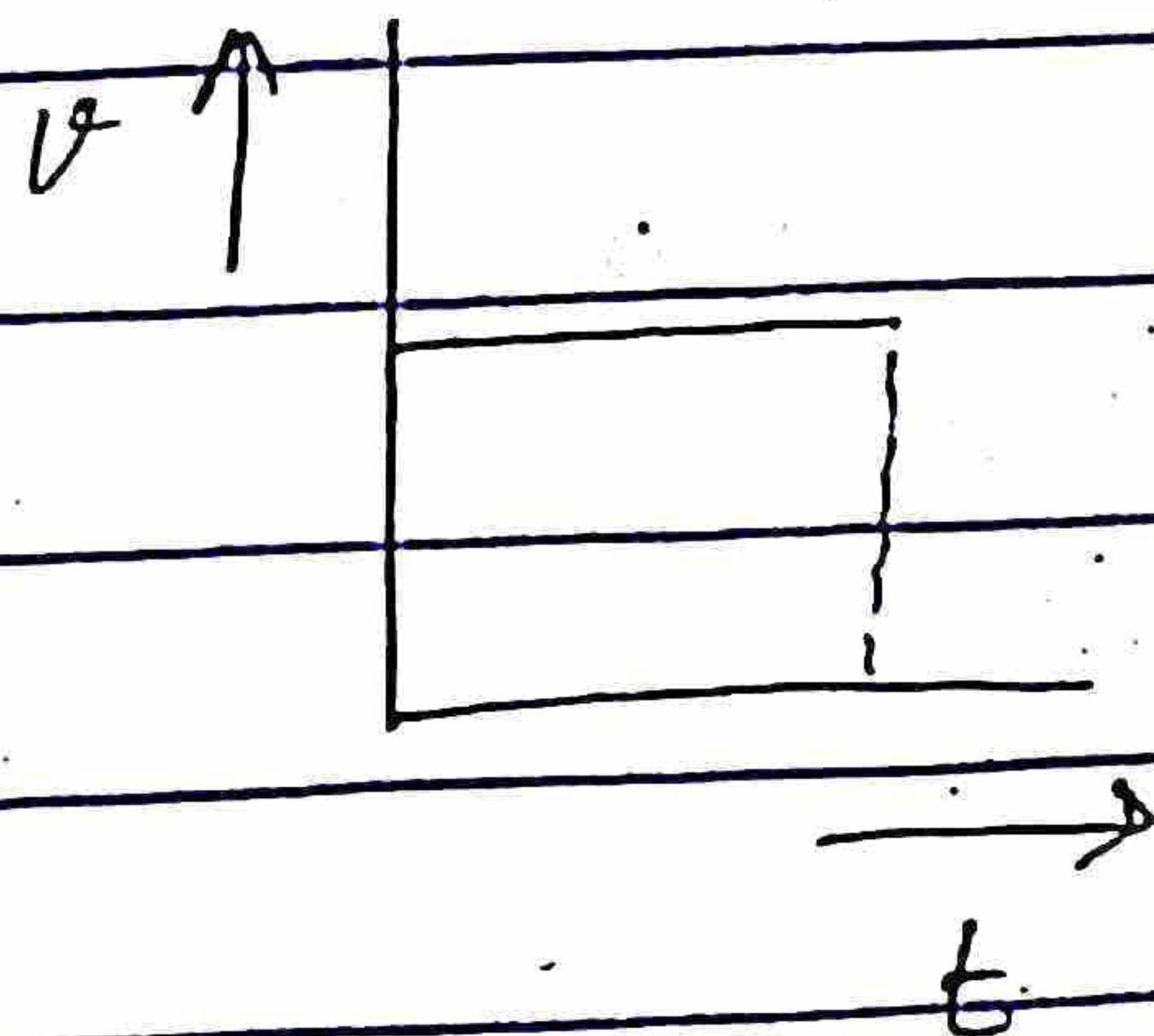


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Object moving with uniform velocity:

If the velocity of a body does not change with time then it has uniform velocity.

In the velocity-time graph, a straight line parallel to

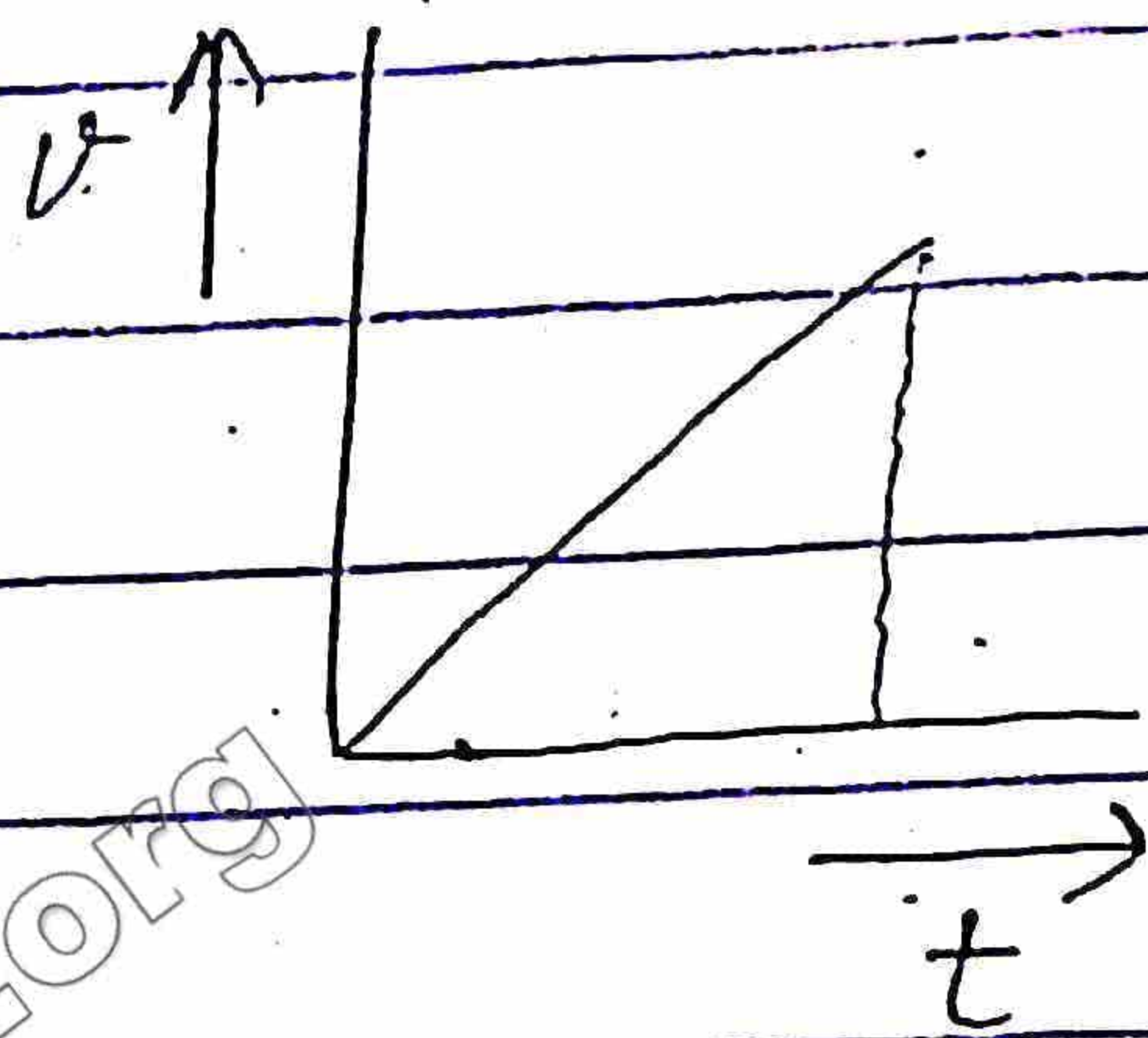


time axis shows that object is moving with uniform velocity.

Object moving with uniform acceleration:

If a body changes equal velocity in equal intervals of time then it has uniform acceleration.

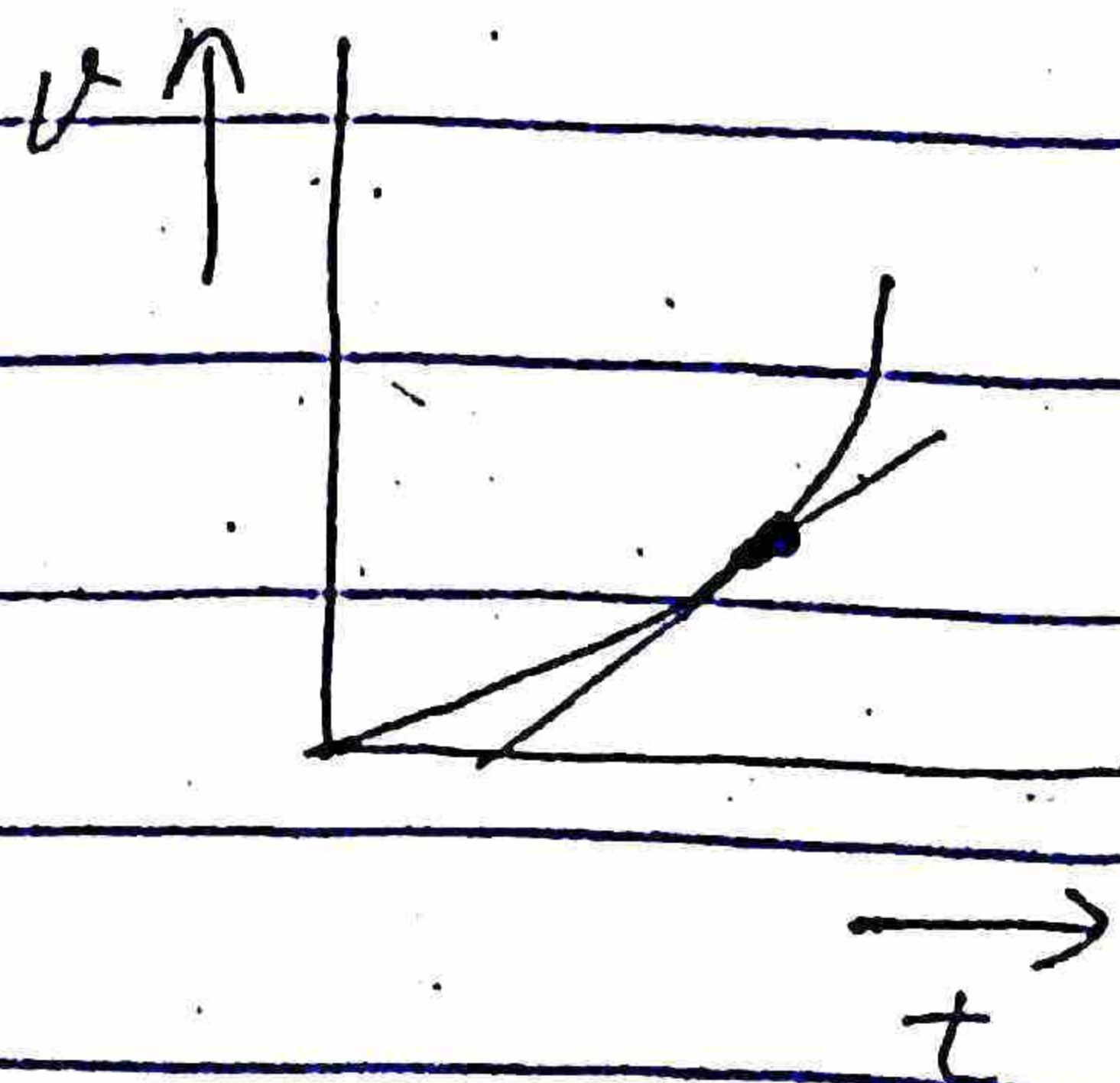
In the velocity-time graph an inclined shows that object is



moving with uniform acceleration. The value of acceleration can be found by calculating the slope of inclined.

Object moving with variable acceleration:

If a body does not change equal velocity in equal intervals



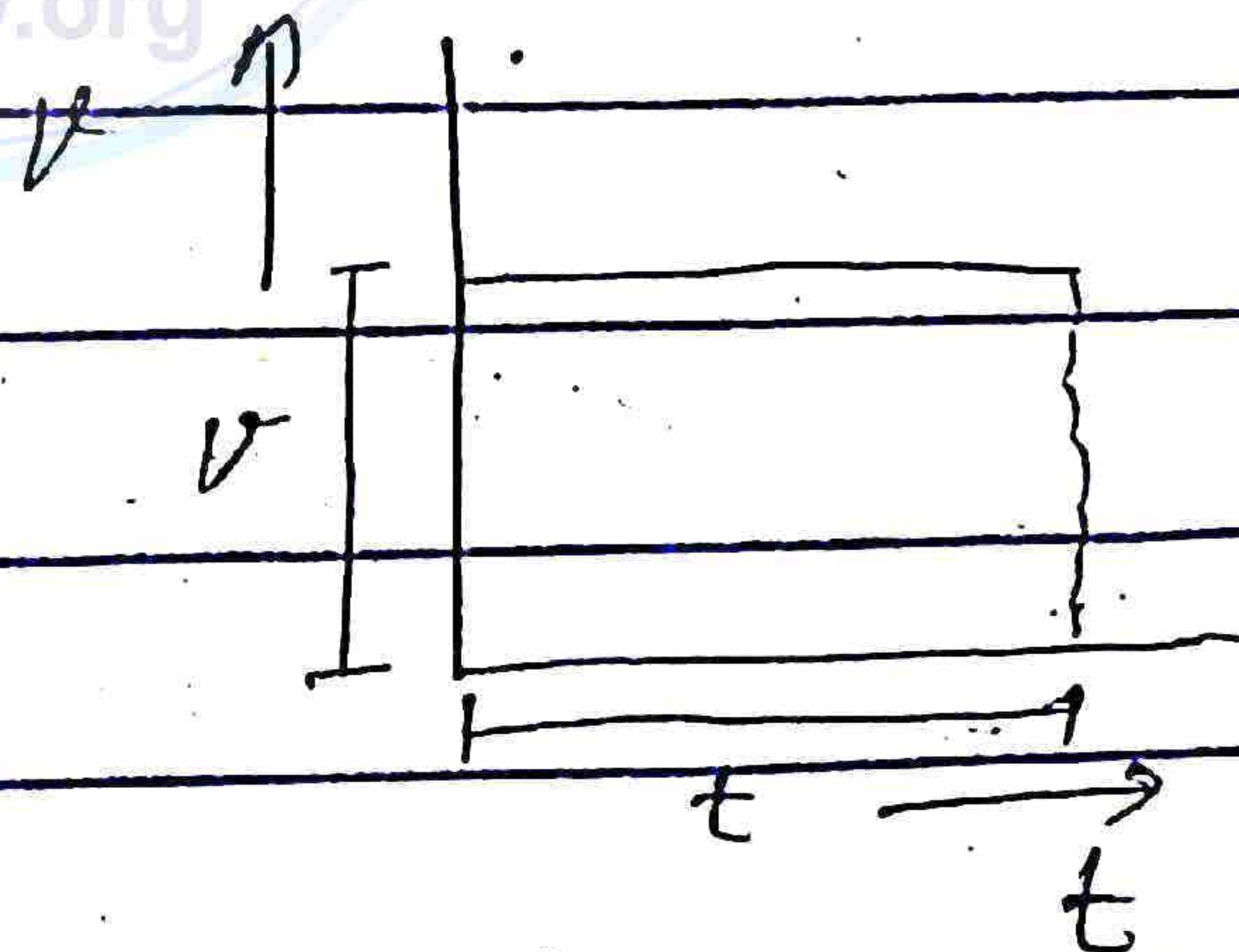
of time then it has variable acceleration.

In velocity-time graph, a non straight line shows variable acceleration. Its value can be found at any point by finding the slope at that point.

Distance covered by object:

We can find the distance covered by the object by calculating the area of velocity-time graph.

For example, for the shown graph.



Area of graph

= length $\times$ width.

$$A = (t)(v)$$

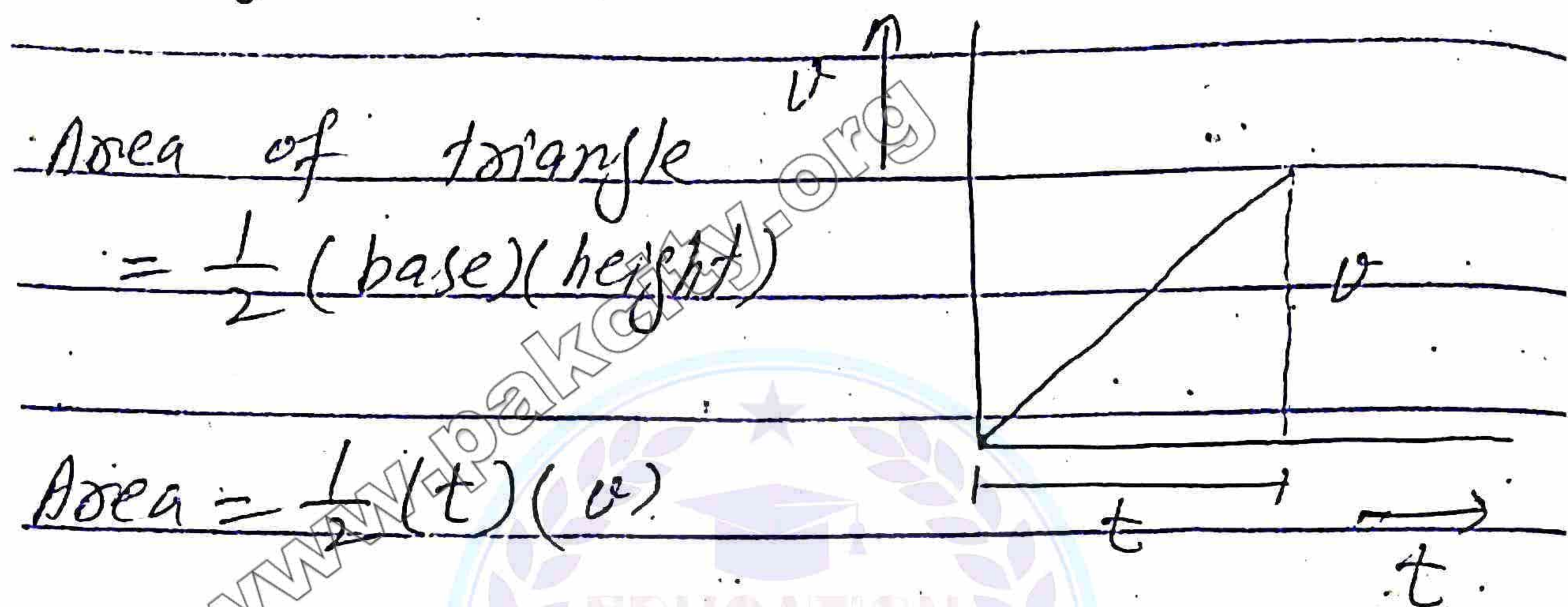
$$\text{Area} = vt$$

Also distance covered by the

object moving with uniform velocity ' v ' in time ' t ' is:

$$\text{distance} = S = vt$$

Take the graph for object moving with uniform acceleration.



$$\text{Area} = \frac{1}{2} vt$$

As the velocity of object increases from 0 to v .

$$\text{Average velocity} = v_{av} = \frac{0+v}{2}$$

$$v_{av} = \frac{v}{2}$$

So,

$$\text{distance} = S = v_{av} t$$

$$S = \frac{v}{2} (t) = \frac{1}{2} vt$$

Equations Of Motion:

Three equations of motion are:

1- $V_f = V_i + at$

2- $S = V_i t + \frac{1}{2} at^2$

3- $2aS = V_f^2 - V_i^2$

Here V_f is final velocity,
 V_i is initial velocity, a is
acceleration, t is time
and S is distance.

Gravitational Acceleration:

In the absence of air resistance,
all objects free fall near the surface
of the earth, moves towards the
Earth with a uniform acceleration.

This acceleration, known as
acceleration due to gravity,
is denoted by letter " g " and its
average value near the Earth surface
is taken as 9.8 m/s^2 in the
downward direction.

Newton's Laws Of Motion:

Newton's laws were given by Sir Isaac Newton who published them in 1687 in his famous book "Principia".

Newton's first law of motion:

Newton's first law states that: "A body at rest will remain in rest or a body moving with uniform velocity will continue its motion unless an unbalanced net force acts on it."

Newton's first law is also called law of inertia.

Inertia: Inertia is the property of a body due to which it resists any change in its state of rest or motion. Inertia

depends upon the mass of body.

Frame of reference:

The surrounding or conditions through which laws are applied is called frame of reference.

Inertial frame of reference:

The frame of reference through which law of inertia holds is called inertial frame of reference.

In this frame of reference acceleration will be zero.

Non inertial frame of reference:

The frame of reference through which law of inertia does not hold is called non inertial

frame of reference. It is
accelerated frame of
reference.

Newton's second law of motion:

"When a force is
applied on a body, it
produces acceleration in the
body. This acceleration will
be directly proportional to
force and inversely proportional
to the mass of the body."
Mathematically,

$$\vec{F} = m \vec{a}$$

Newton's third law of motion:

"To every action
there is an equal but
opposite reaction."

Momentum: The quantity of motion of a body is called its momentum.

OR

The product of mass of a body and its velocity is called momentum.

It is represented by " $\vec{p}$ "

$$\vec{p} = m \vec{v}$$

Momentum is a vector quantity. Its unit is kg m s^{-1} or N s .

Momentum and Newton's second law of motion:

According to Newton's second law:

$$\vec{F} = m \vec{a}$$

The rate of change of velocity is called acceleration.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

$$\text{So } \vec{F} = m \left(\frac{\vec{v}_f - \vec{v}_i}{t} \right)$$

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t}$$

So, the rate of change of momentum of a moving body is called force.

Impulse: when a large force acts on a body for a very short interval of time, then the product of force and time is called impulse.

$$\text{impulse} = \vec{F} \times t = m\vec{v}_f - m\vec{v}_i$$

Impulse is a vector quantity equal to change in momentum. Its unit is same as momentum i.e. kg m s^{-1} or Ns .

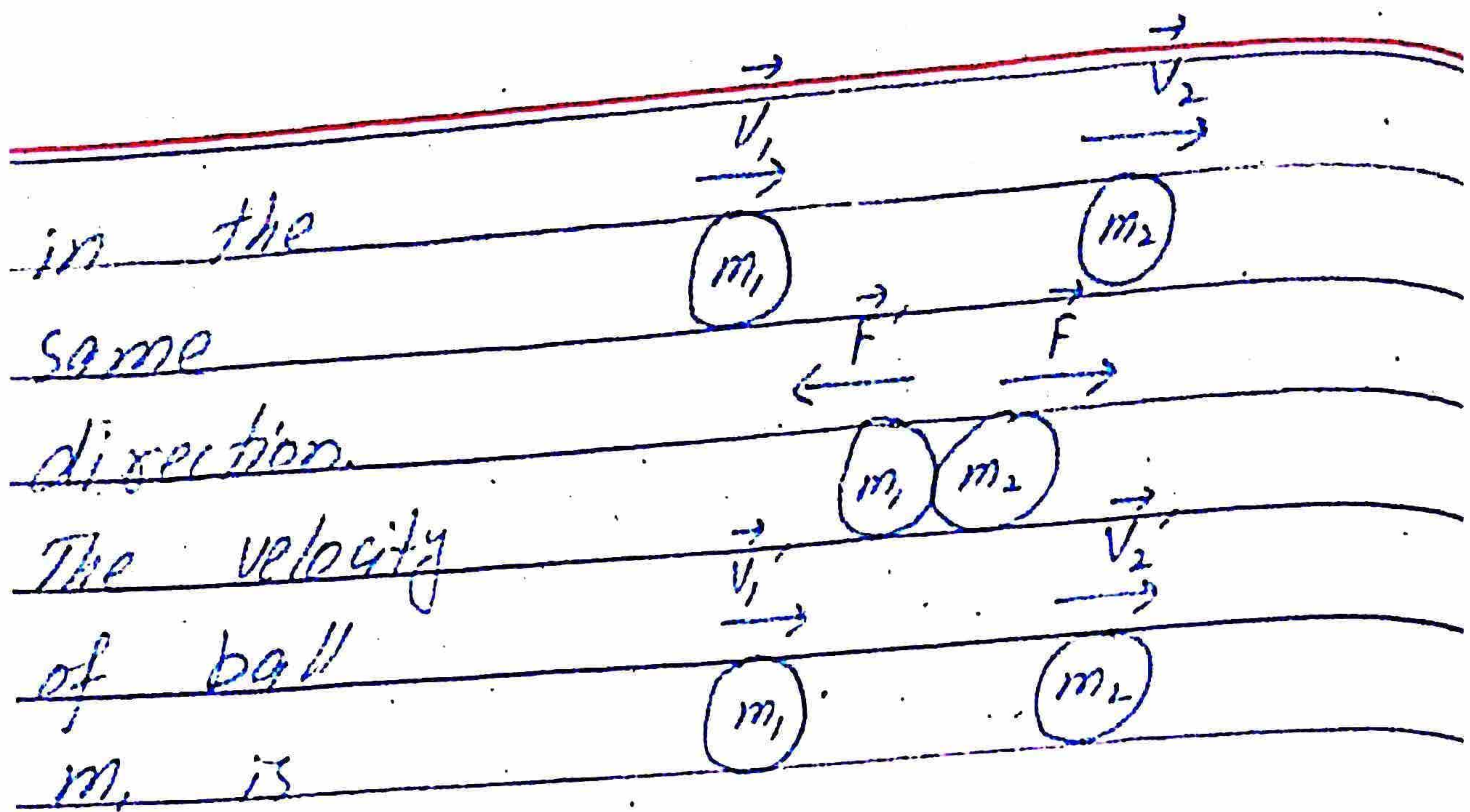
Law Of Conservation Of Momentum:

Isolated system: The system of two or more interacting bodies on which no external force acts is called isolated system.

For example, the gas molecules enclosed in a cylinder exert forces upon each other and on the walls of cylinder but no external force acts on them.

Statement: The total linear momentum of an isolated system remains constant.

Explanation: Consider two spherical balls of masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively.



in the
same
direction.

The velocity
of ball
 m_1 is

greater
than that of m_2 i.e. $\vec{v}_1 > \vec{v}_2$
So, the masses collide
and exert equal and
opposite forces upon each
other. After collision mass m_1
moves with velocity $\vec{v}_1'$ and
mass m_2 moves with
velocity $\vec{v}_2'$.

According to the
definition of impulse.

$$\vec{F} \times t = m \vec{v}_f - m \vec{v}_i$$

For mass m_1 ,

$$\vec{F}' \times t = m_1 \vec{v}_1' - m_1 \vec{v}_1$$

For mass m_2 ,

$$\vec{F} \times t = m_2 \vec{v}_2' - m_2 \vec{v}_2$$

By adding above equations

$$(\vec{F}' \times t) + (\vec{F} \times t) = m_1 \vec{v}_1' - m_1 \vec{v}_1 + m_2 \vec{v}_2' - m_2 \vec{v}_2$$

$$(\vec{F}' + \vec{F}) \times t = m_1 \vec{v}_1' - m_1 \vec{v}_1 + m_2 \vec{v}_2' - m_2 \vec{v}_2 \rightarrow (1)$$

As both the masses exert equal and opposite forces upon each other.

$$\vec{F}' = -\vec{F}$$

$$\vec{F}' + \vec{F} = 0$$

Now eq. (1) will become

$$0 = m_1 \vec{v}_1' - m_1 \vec{v}_1 + m_2 \vec{v}_2' - m_2 \vec{v}_2$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

Total momentum before collision = Total momentum after collision

Result: From the above discussion we can conclude

i- For an isolated system, the total initial momentum will be equal to total final momentum.

ii- For an isolated system, the change in momentum of the system is zero.

Elastic

Collision

i- The collision in which K.E and momentum of the system remains conserved is called elastic collision.

ii- For example, when a ball is dropped from a certain height it reaches to

In elastic

Collision

i- The collision in which K.E of the system does not remain conserved is called in elastic collision.

For example, when a ball is dropped from a certain height it does not reach

the same
height after
striking with
marble floor.

to the same height
after striking with
rough floor.



**Why K.E does not remain
conserved in most cases?**

When two tennis balls collide then, after collision, they will rebound with velocities less than the velocities before the impact. During this process a portion of K.E is lost, partly due to Δ and partly due to its change into heat and sound energies.

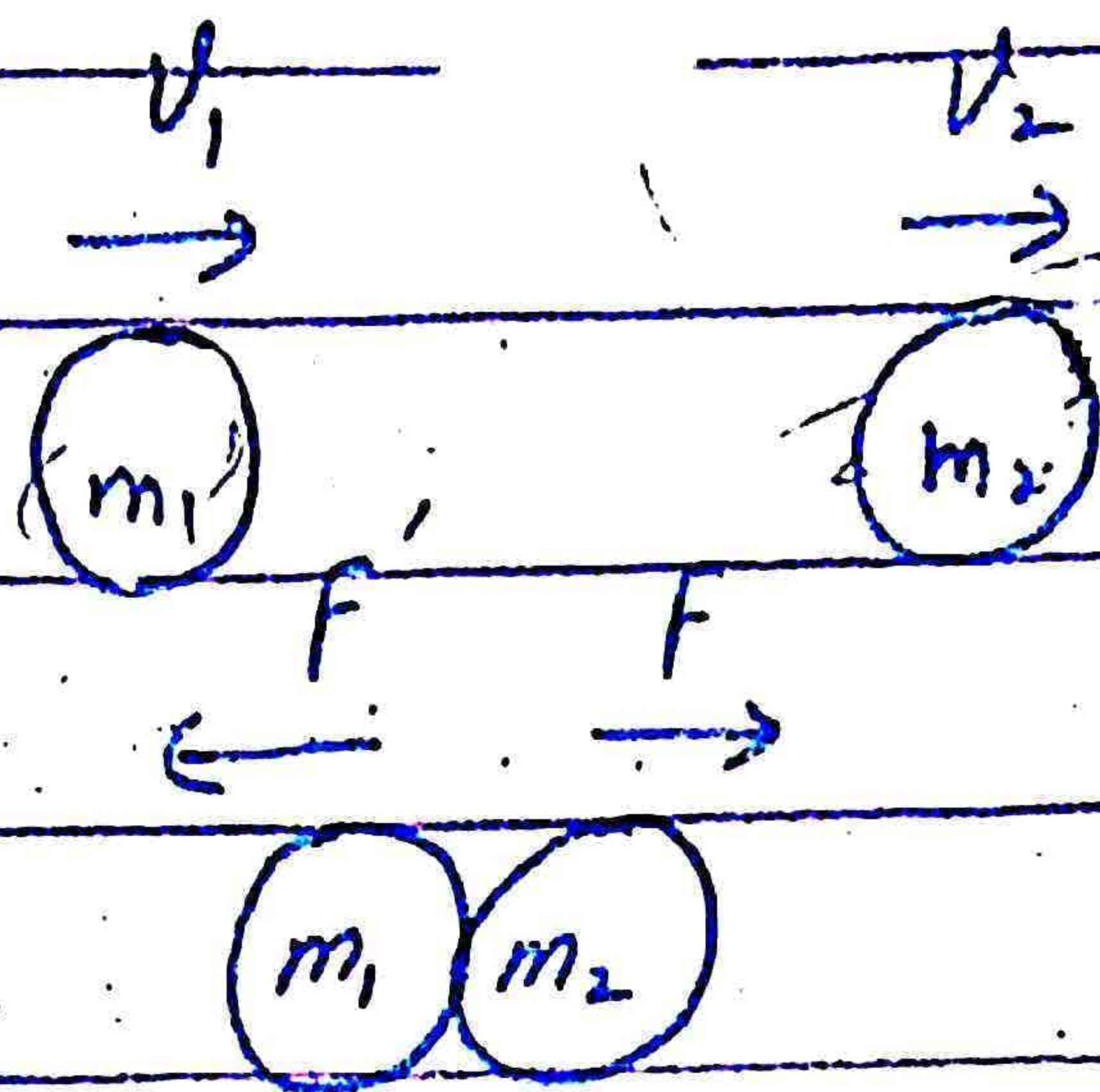
**Elastic Collision In One
Dimension:**

Elastic collision:

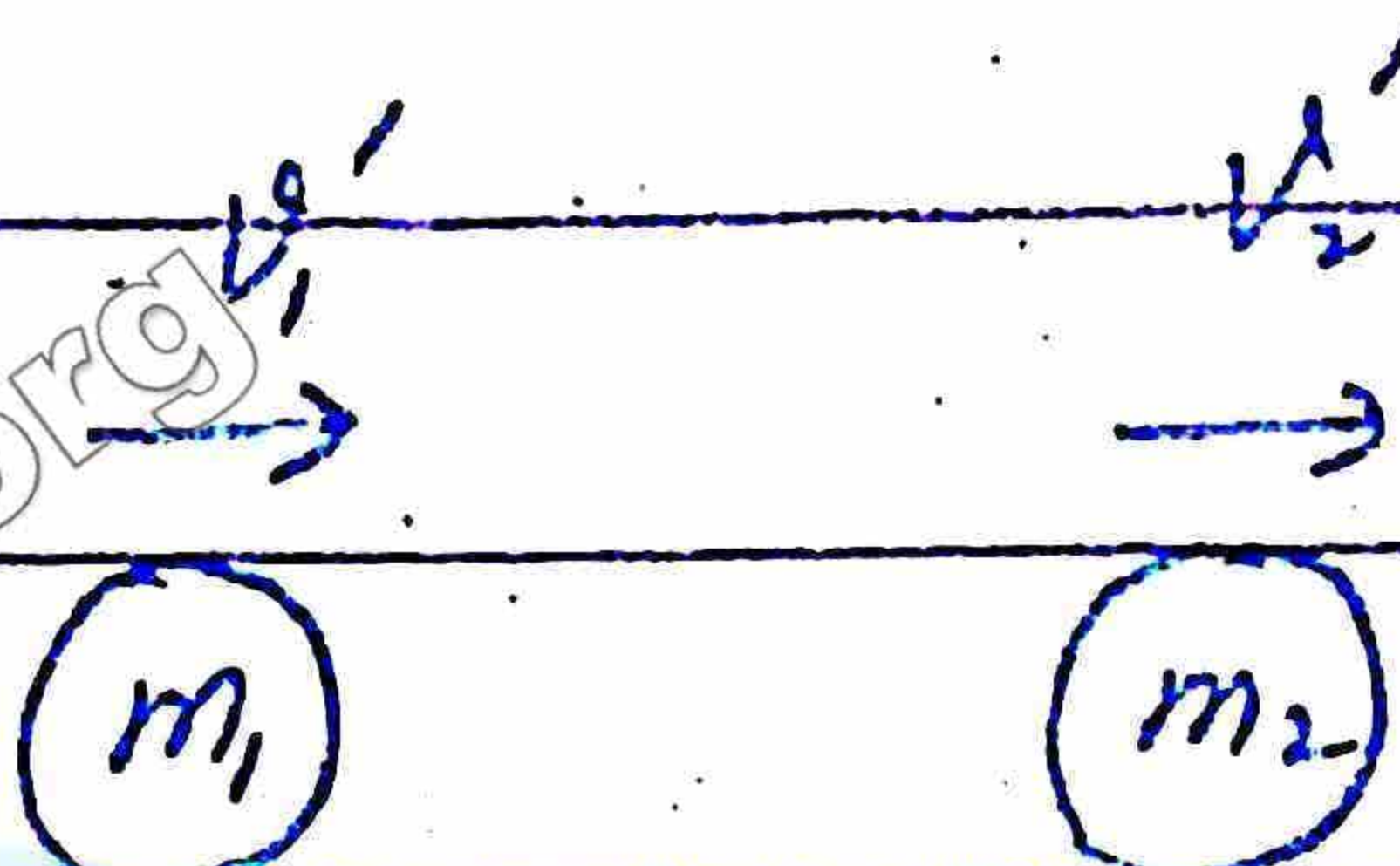
The collision in which the K.E and momentum of the system remains conserved is called Elastic collision.

Explanation:

Consider two spherical balls of masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$



respectively in the same direction.



The velocity of the ball m_1 is greater than the velocity of the ball m_2 , $v_1 > v_2$. So, the masses collide and exert equal and opposite forces upon each other. After collision, mass m_1 moves with velocity v_1' and mass m_2 moves with velocity v_2' .

According to law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 u_1' + m_2 u_2'$$

$$m_1 u_1 - m_1 u_1' = m_2 u_2' - m_2 u_2$$

$$m_1 (u_1 - u_1') = m_2 (u_2' - u_2) \rightarrow (1)$$

As the collision is elastic, so K.E of the system will also remain conserved.

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2$$

$$\frac{1}{2} (m_1 u_1^2 + m_2 u_2^2) = \frac{1}{2} (m_1 u_1'^2 + m_2 u_2'^2)$$

$$m_1 u_1^2 - m_1 u_1'^2 = m_2 u_2'^2 - m_2 u_2^2$$

$$m_1 (u_1^2 - u_1'^2) = m_2 (u_2'^2 - u_2^2)$$

$$m_1 (u_1 + u_1') (u_1 - u_1') = m_2 (u_2' + u_2) (u_2' - u_2) \rightarrow (2)$$

By dividing eq. (2) by eq. (1)

$$\frac{m_1 (u_1 + u_1') (u_1 - u_1')}{m_1 (u_1 - u_1')} = \frac{m_2 (v_2 + v_2') (v_2 - v_2')}{m_2 (v_2' - v_2)}$$

$$u_1 + u_1' = v_2' + v_2$$

$$u_1 - u_2 = v_2' - v_1'$$

$$u_1 - u_2 = -(v_1' - v_2')$$

Result:

- i- Relative velocities before and after collision have the same magnitude but are reversed after the collision.
- ii- The magnitude of relative velocity of approach is equal to magnitude of relative velocity of separation.

By solving the above equations we can get the mathematical expressions for final velocities.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Case.1: when both balls have same masses:

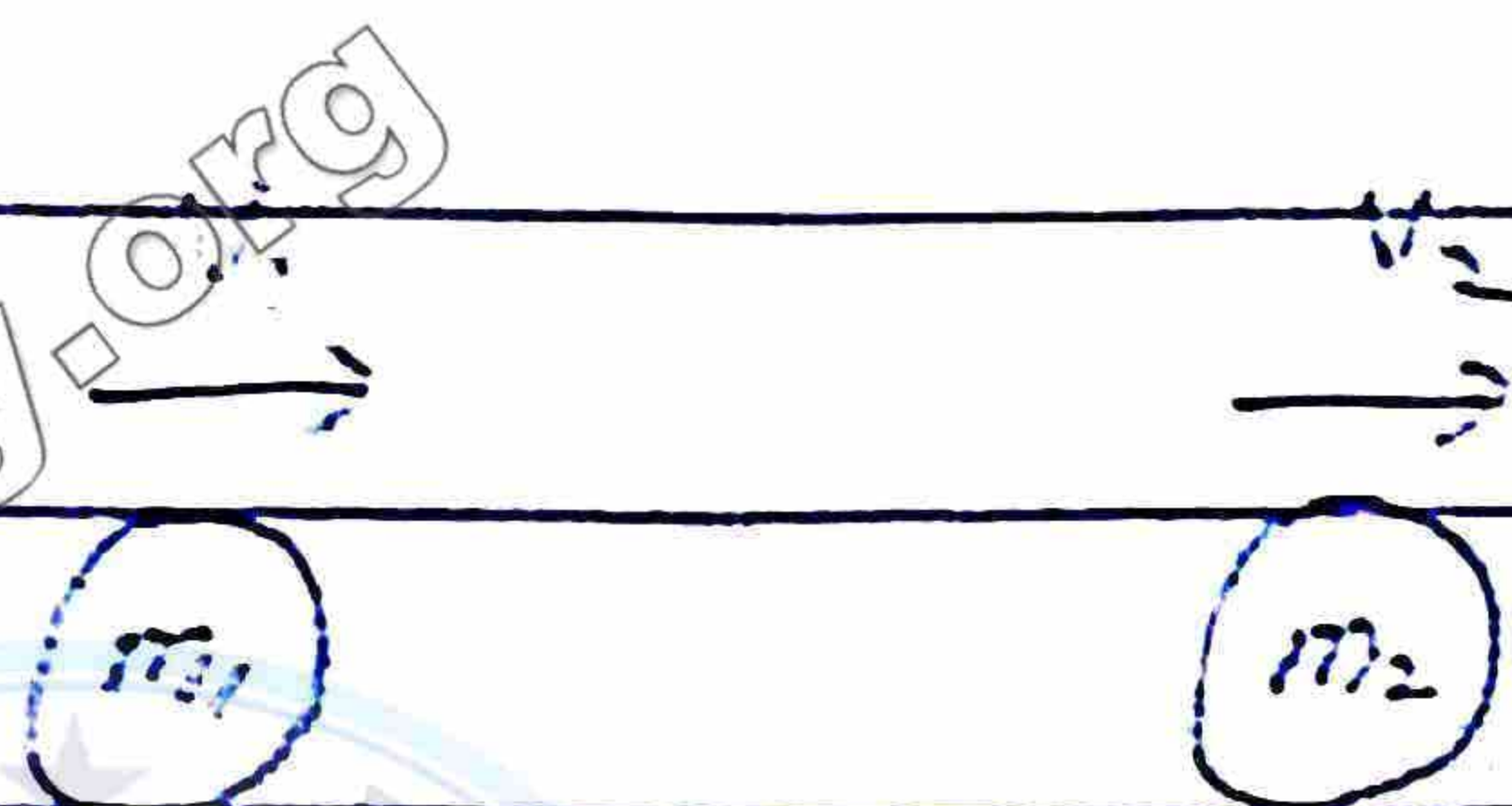
when both

the balls

have same

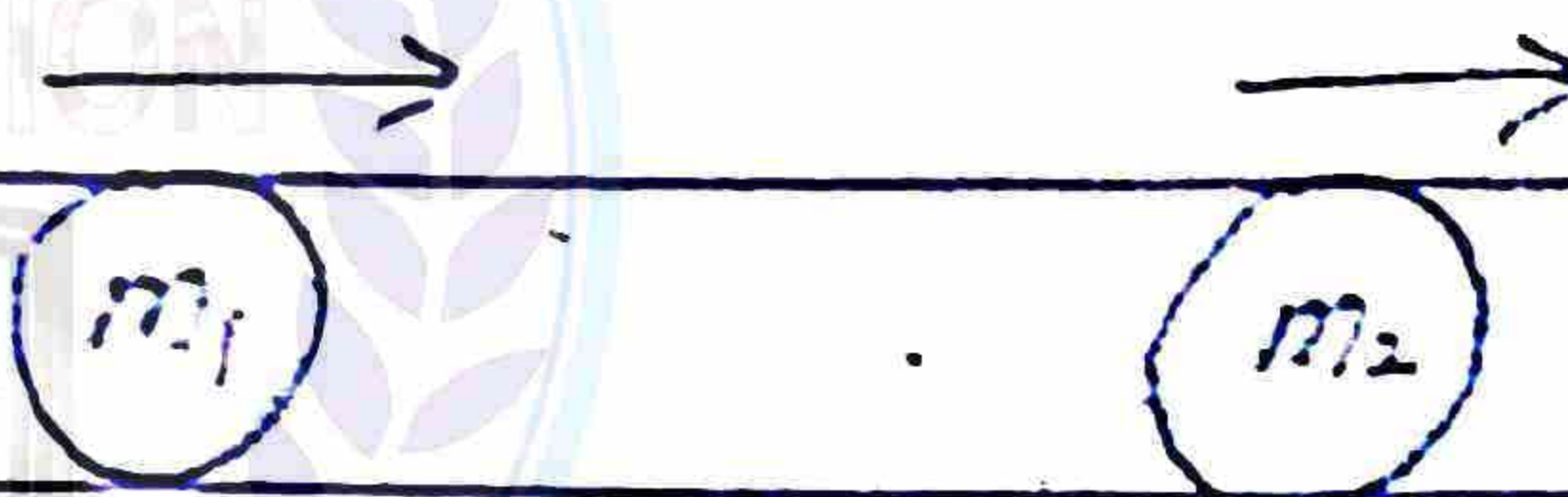
masses. Then

$$m_1 = m_2 = m$$



$$v_1' = v_2'$$

$$v_2' = v_1'$$



$$v_1' = \frac{m - m}{m + m} v_1 + \frac{2m}{m + m} v_2$$

$$= 0 + \frac{2m}{2m} v_2$$

$$v_1' = v_2$$

Now

$$v_2' = \frac{2m}{m + m} v_1 + \frac{m - m}{m + m} v_2$$

$$v_2' = \frac{2m}{2m} v_1 + 0$$

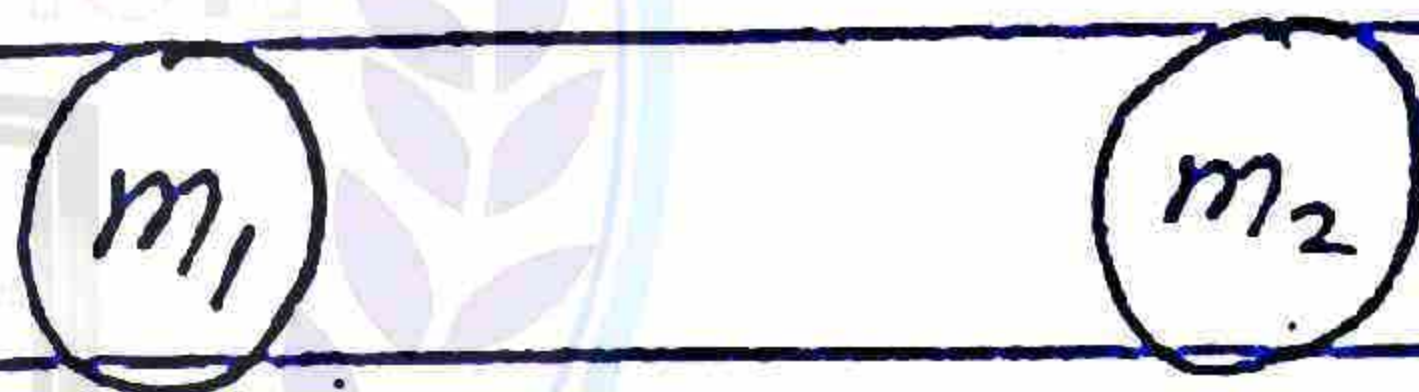
$$v_2' = v_1$$

Result: when both the balls have same masses then after collision they interchange their velocities.

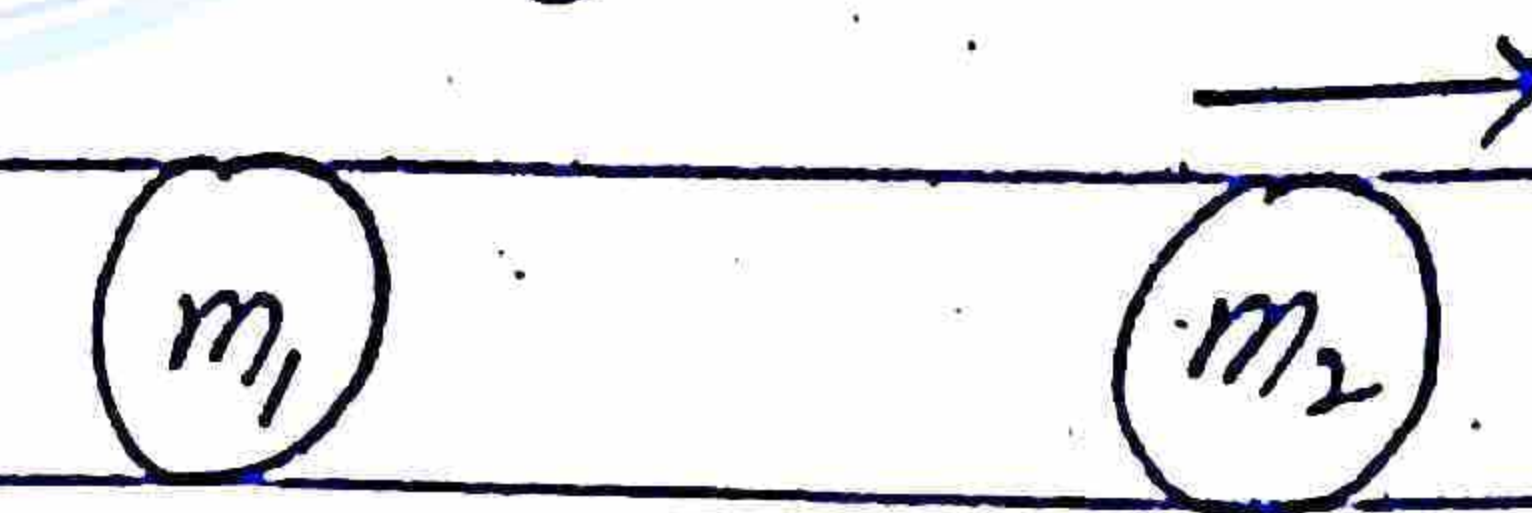
Case 2: when both balls have same masses and target is at rest:

When both the balls have same masses and target is at rest, we put

$$v_1 \rightarrow \quad v_2 = 0$$



$$v_1' = 0 \quad v_2' = v_1$$



$$m_1 = m_2 = m$$

$$v_2 = 0$$

$$v_1' = \frac{m-m}{m+m} v_1 + \frac{2m}{m+m} (0)$$

$$= 0 + 0 \Rightarrow v_1' = 0$$

$$v_2' = \frac{2m}{m+m} v_1 + \frac{m-m}{m+m} (0)$$

$$= \frac{2m}{2m} v_1 + 0$$

$$v_2' = v_1$$

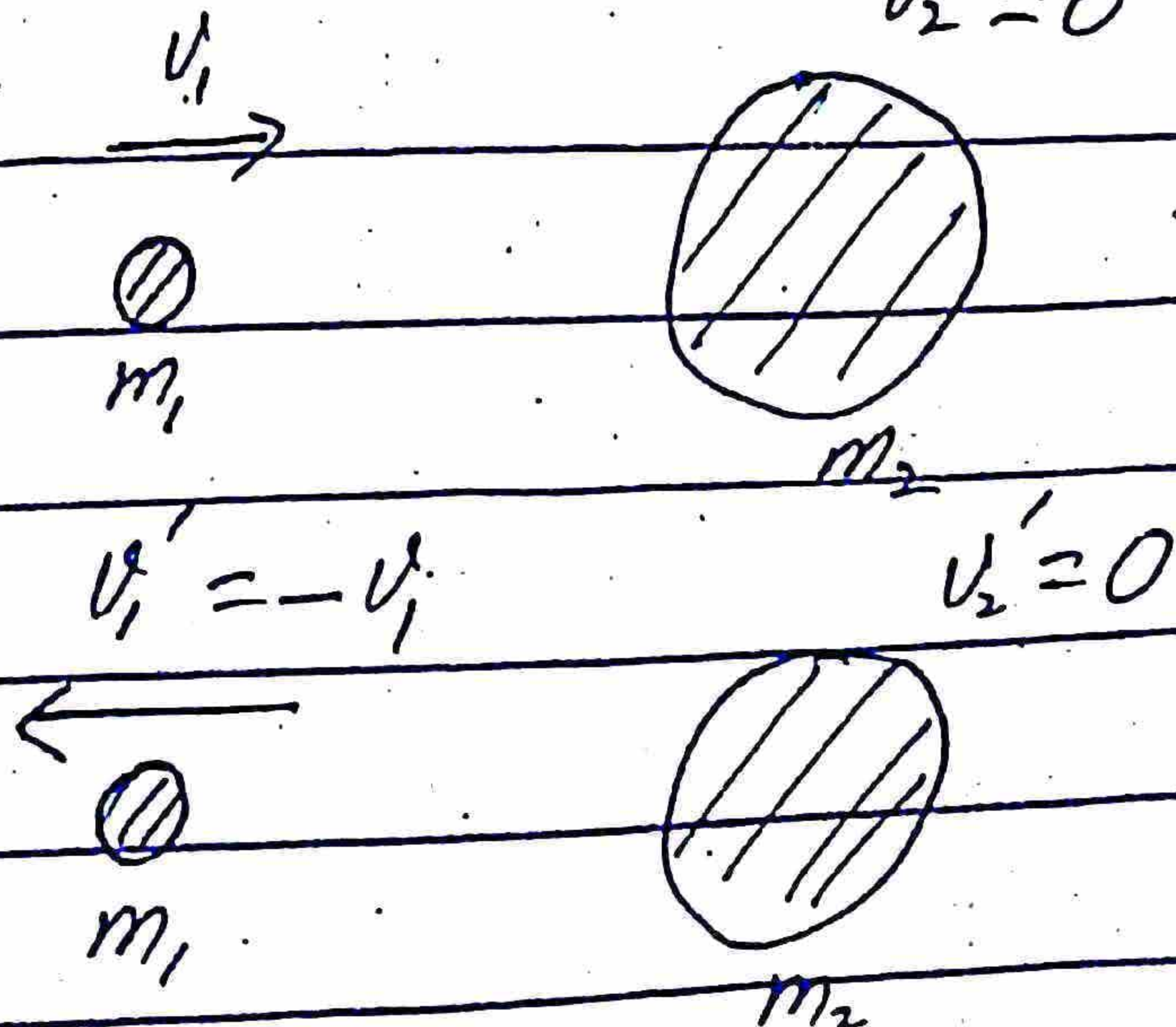


Result: When both the balls have same masses and target is at rest. Then after the collision first mass comes to rest and second mass starts moving with the velocity of first.

Case. 3 When a light body collides with a massive body at rest:

When a light body collides with

a massive body at rest. we



neglect light body.

$m_1 \ll m_2$ take $m_1 = 0$

and $u_2 = 0$

$$v_1' = \frac{0 - m_2}{0 + m_2} v_1 + \frac{2m_2}{0 + m_2} (0)$$

$$= -\frac{m_2}{m_2} v_1 + 0$$

$$v_1' = -v_1$$

now

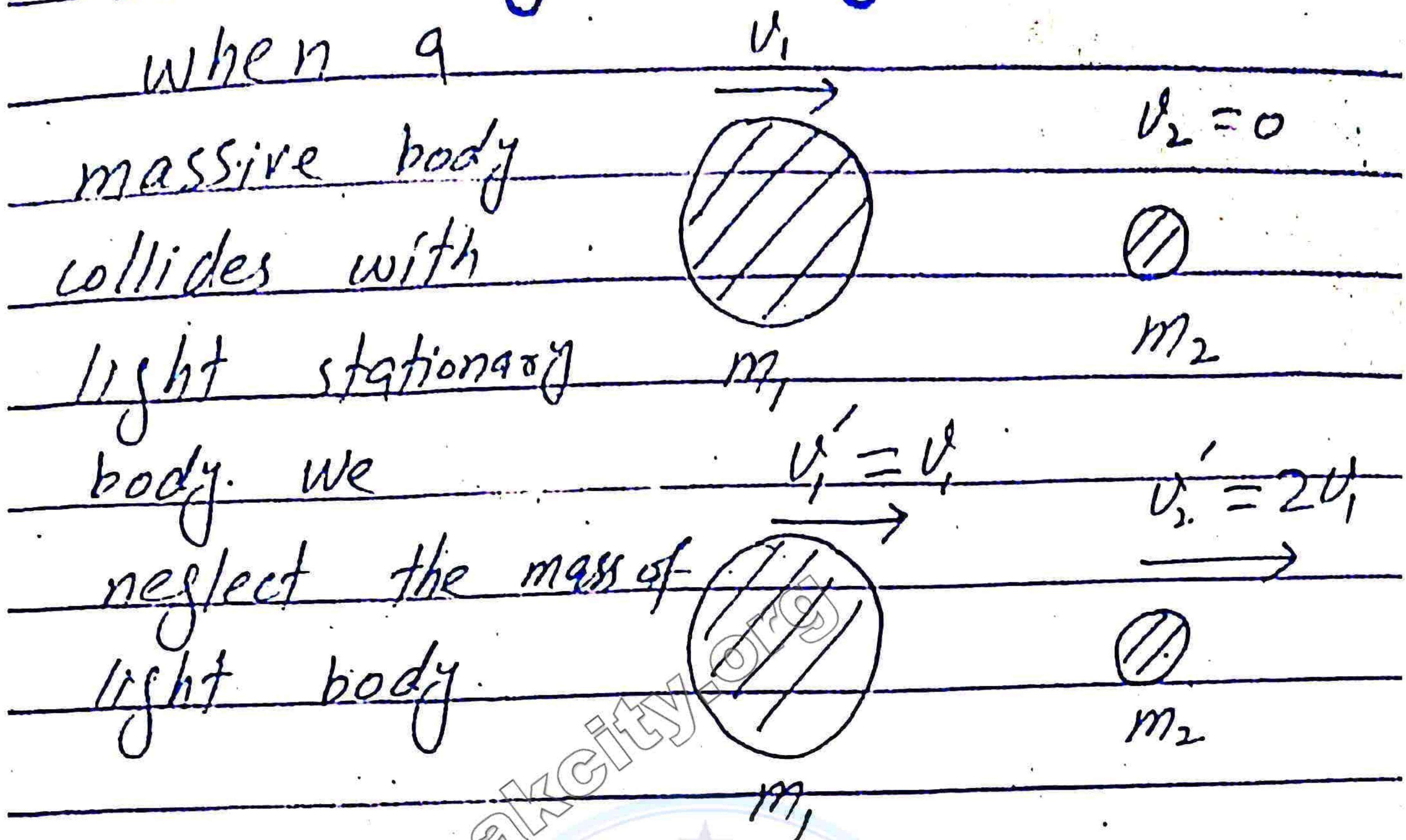
$$v_2' = \frac{2(0)}{0 + m_2} v_1 + \frac{m_2 - 0}{0 + m_2} (0)$$

$$= 0 + 0$$

$$v_2' = 0$$

Result: When a light body collides with a massive body at rest. Then after collision, light body moves back with same velocity but massive body will remain at rest.

Case 4 When a massive body collides with light stationary body:



$$m_2 \ll m_1$$

$$\text{take } m_2 = 0$$

$$\text{and } v_2 = 0$$

Now

$$v_1' = \frac{m_1 - 0}{m_1 + 0} v_1 + \frac{2(0)}{m_1 + 0} (0)$$

$$= \frac{m_1}{m_1} v_1 + 0$$

$$v_1' = v_1$$

Now

$$v_2' = \frac{2m_1}{m_1 + 0} v_1 + \frac{0 - m_1}{m_1 + 0} (0)$$

$$u_2' = \frac{2m_1}{m_1} u_1 + 0$$

$$u_2' = 2u_1$$

Result: When a massive body collides with light stationary body. Then after the collision massive body will keep moving with same velocity and light body will start moving with the velocity double as massive.

Discuss force due to water flow.

ANS. Consider water of mass " m " moving with velocity " v " strikes to a wall and comes to rest.

$$\begin{aligned} \text{change in velocity} &= v_f - v_i \\ &= 0 - v \end{aligned}$$

$$\Delta v = -v$$

we know that when water

strikes to wall, it exerts force on wall and wall will exert equal and opposite force on water.

$$\text{Force exerted on water} = \frac{mv_f - mv_i}{t}$$

$$= \frac{m(v_f - v_i)}{t}$$

$$= \frac{m(-v)}{t}$$

$$= -\left(\frac{m}{t}\right)(v)$$

$$\text{force exerted} = - \left(\text{mass per second} \right) \left(\text{change in velocity} \right)$$

on water

According to Newton's third

law:

$$\text{Force exerted on wall by water} = - \left(- \frac{mv}{t} \right)$$

$$F = \frac{mv}{t}$$

Q. Write an application of law of conservation of momentum.

Ans. When a shell explodes in mid air, its fragments fly off in different directions. The total momentum of all its fragments equals the initial momentum of the shell.

Q. Calculate the velocity of rifle when fired.

Ans. Consider a rifle of mass M and bullet of mass m . Before firing both are at rest and total initial momentum will be zero. When we fire the rifle bullet moves forward and rifle moves back to conserve its momentum. According to law of conservation of momentum:

final momentum = initial momentum
after fire before fire

$$mv + Mv' = 0$$

$$Mv' = -mv$$

$$v' = -\frac{m}{M}v$$

Here v' is velocity of rifle and v is velocity of bullet.

The above equation shows that:

i- The velocity of bullet will be greater than velocity of rifle.

ii- Both rifle and bullet will move in opposite direction.

Rocket Propulsion

Principle: The motion of rocket can be understood by two principles

i- Law of conservation of momentum; when the gases

are ejected through the rocket they gain momentum. To conserve this momentum rocket moves upward.

ii- Newton's third law;
When the gases are ejected through rocket it is action so, as a reaction rocket moves upward.

Speed:

Instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

Fuel of rocket:

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore, work at great heights where very little or no air is present. In order to

provide enough upward thrust to overcome gravity, a typical rocket consumes about 1000 kg s^{-1} of fuel and ejects the burnt gases at speeds of over 4000 m s^{-1} . In effect, more than 80% of the launch mass of a rocket consists of fuel only.



How can we overcome the problem of mass of fuel for rocket?

One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together. When one rocket has done its job, it is discarded leaving others to carry the spacecraft further up at ever greater speed.

Acceleration of rocket,

is the mass of the gases ejected $g \cdot m$

per second with velocity v relative to the rocket. The acceleration 'a' of the rocket is:

$$\vec{a} = \frac{m\vec{v}}{M}$$

Where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.



Projectile Motion

Defination:

The two dimensional motion of a body under constant acceleration due to gravity is called projectile motion.

Examples:

i- The motion of football hit by player.

ii- motion of cricket ball hit by player.

iii- motion of rocket.

Projectile:

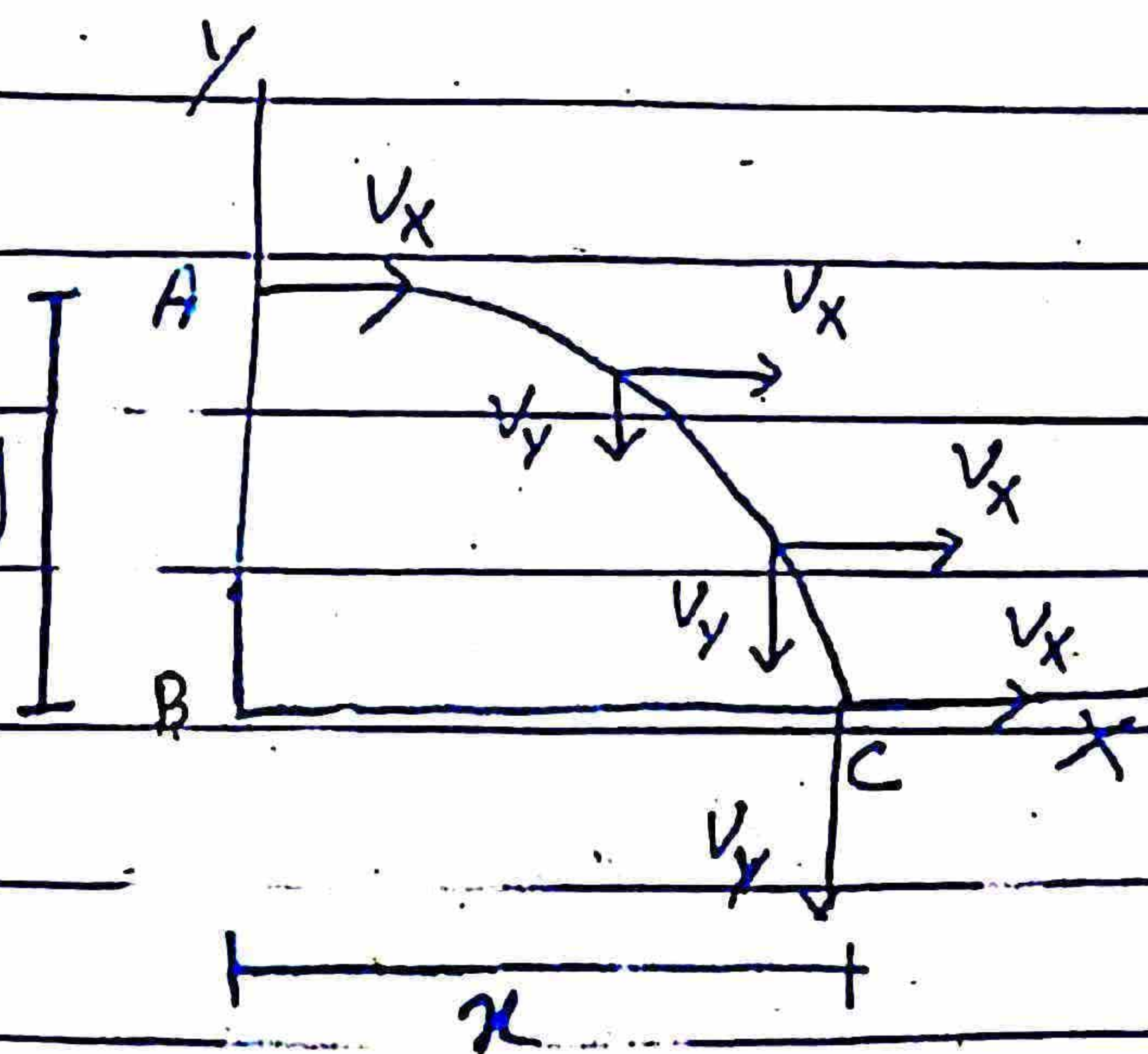
The object / body that performs projectile motion is called projectile.

Trajectory:

The path followed by projectile is called its trajectory. The trajectory of projectile is parabolic.

Explanation:

Consider a body of mass m moved horizontally from height y .



with velocity v_x , it moves forward as well as falls down due to gravity.

The horizontal component of velocity v_x remains constant but vertical component v_y increases with its motion. Here horizontal component of acceleration will be zero. i.e. $a_x = 0$

Now we will calculate mathematical relation for horizontal distance " x " and vertical distance " y ".

Horizontal distance (x):

we know that

$$S = vt$$

Here $S = x$, $v = v_x$

So,

$$x = v_x t$$

Vertical height (y):

According to second equation of motion:

$$S = v_i t + \frac{1}{2} a t^2$$

Here

$$S = y, \quad v_i = v_{iy} = 0, \quad a = g$$

So,

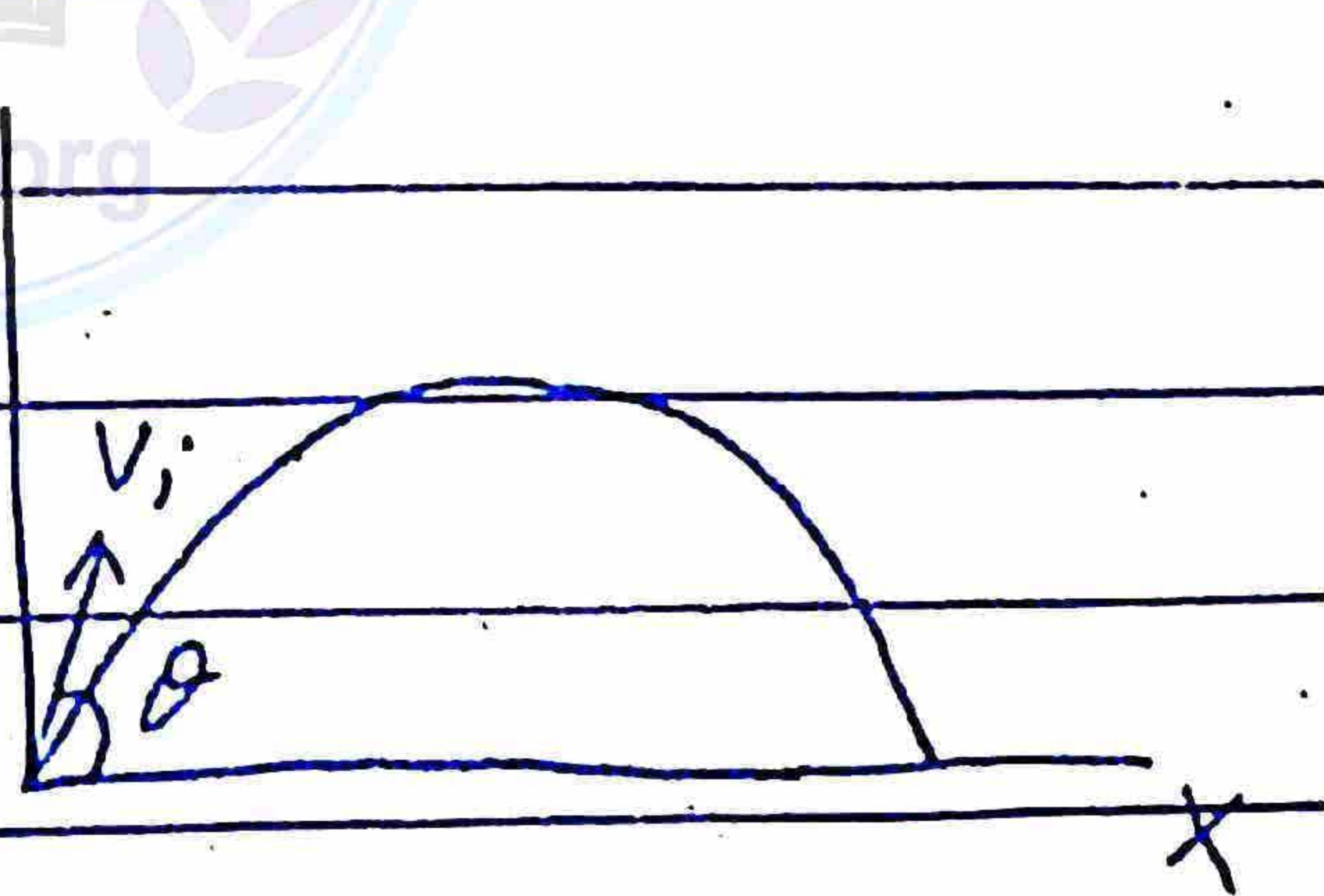
$$y = (0)t + \frac{1}{2} g t^2$$

$$y = \frac{1}{2} g t^2$$

Motion making angle " θ ":

Now consider

the projectile motion of body making angle " θ " with x-axis.



The object is thrown with initial velocity v_i , its horizontal component remains same throughout its motion.

$$V_{ix} = V_{fx} = V_i \cos \theta$$

But the vertical component of velocity changes.

$$V_{iy} = V_i \sin \theta$$

And according to first equation of motion.

$$V_f = V_i + at$$

Here

$$V_f = V_{fy}, \quad V_i = V_{iy} = V_i \sin \theta$$

$$a = -g$$

So,

$$V_{fy} = V_i \sin \theta - gt$$

The magnitude velocity at any instant is:

$$V = \sqrt{V_{fx}^2 + V_{fy}^2}$$

The angle ϕ made by velocity with x-axis is:

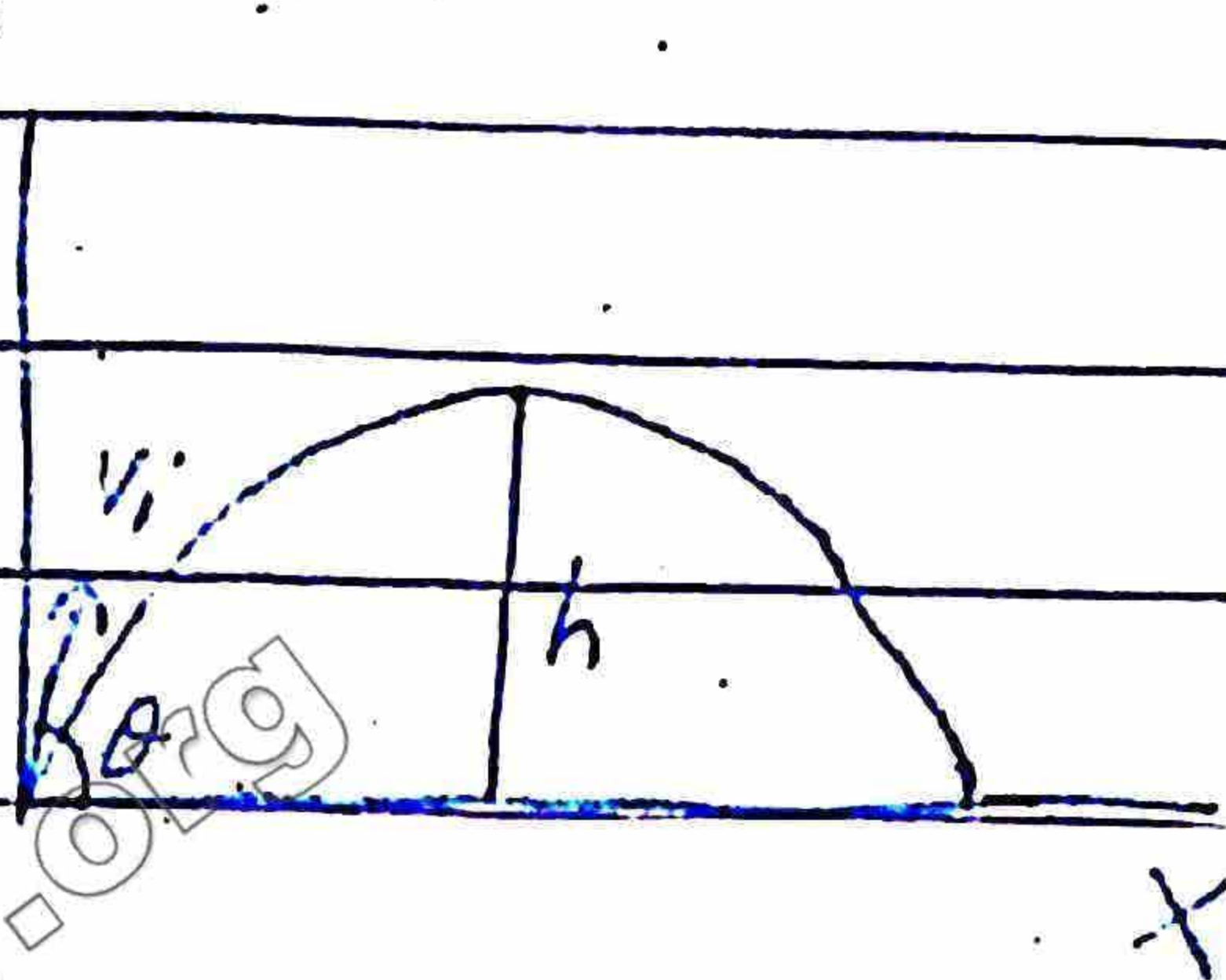
$$\phi = \tan^{-1} \frac{V_{fy}}{V_{fx}}$$

Height of projectile:

The maximum distance covered by projectile along y-axis is called height of projectile.

According to third

equation of motion:



$$2as = v_f^2 - v_i^2$$

Here $a = -g$, $s = h$

$$v_f = 0, \quad v_i = v_{iy} = v_i \sin \theta$$

so,

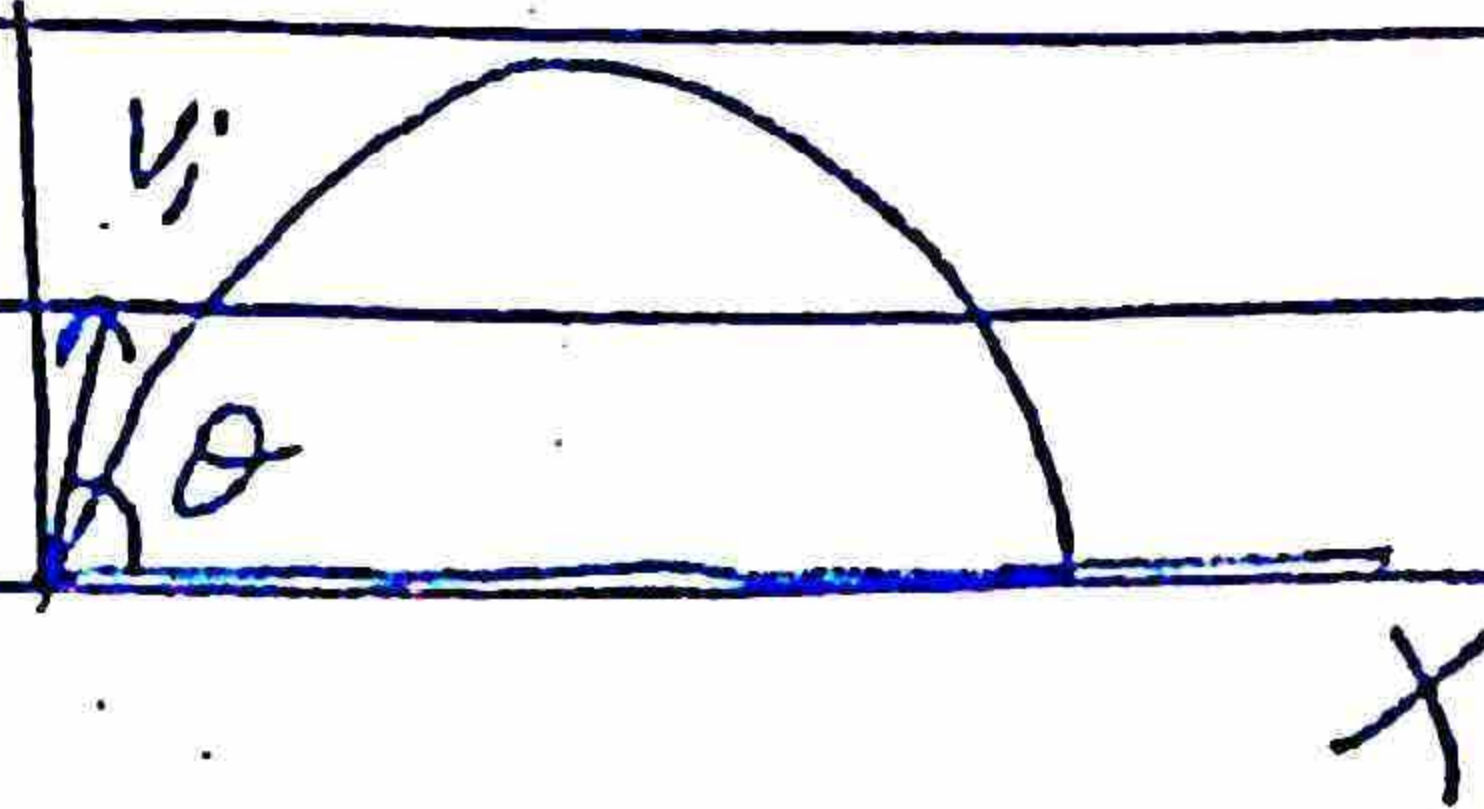
$$2(-g)(h) = (0)^2 - (v_i \sin \theta)^2$$

$$-2gh = -v_i^2 \sin^2 \theta$$

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

Time of flight:

The time taken by projectile to



covers the distance from the place of projection to the place where it hits the ground is called time of flight.

OR

The time taken by projectile to cover its trajectory is called time of flight.

According to second equation of motion:

$$S = v_i t + \frac{1}{2} a t^2$$

Here

$$S = h = 0, \quad v_i = v_{iy} = v_i \sin \theta$$

$$a = -g$$

So,

$$0 = v_i \sin \theta t - \frac{1}{2} g t^2$$

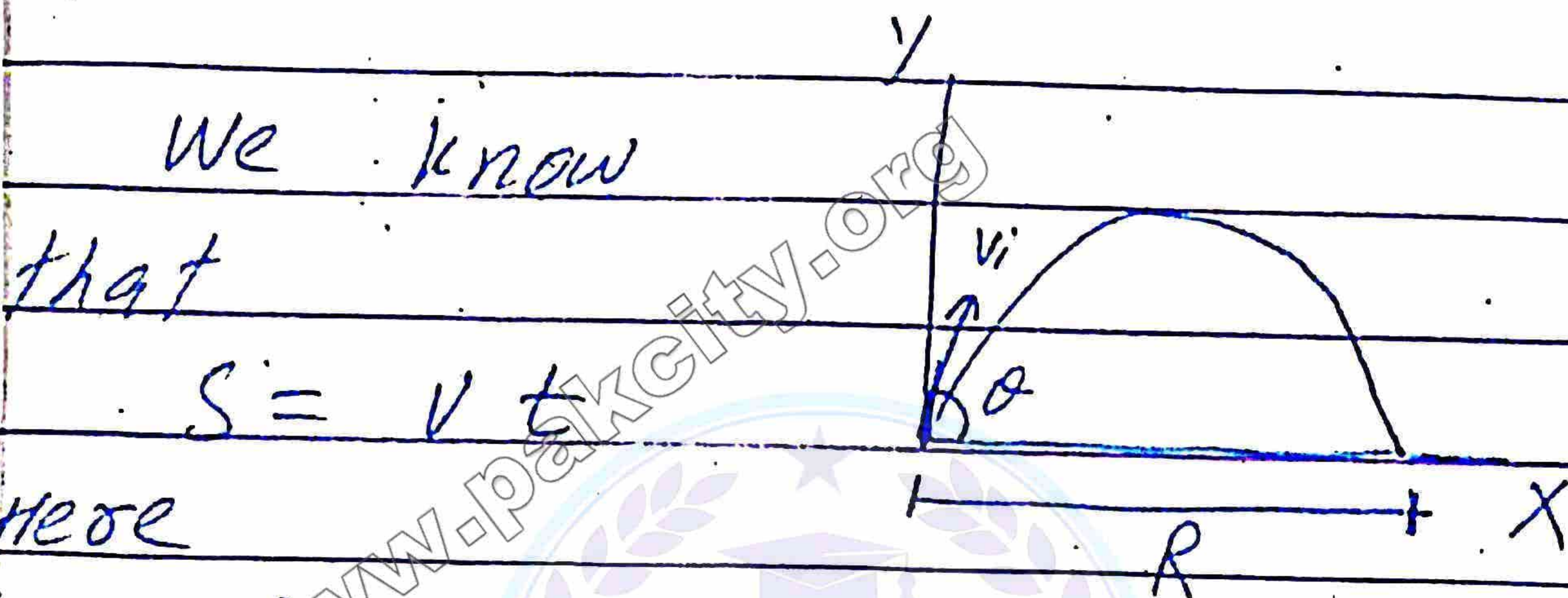
$$\frac{1}{2} g t^2 = v_i \sin \theta t$$

$$gt = 2v_i \sin \theta$$

$$t = \frac{2v_i \sin \theta}{g}$$

Range of the projectile:

The distance covered by the projectile along x-axis is called range of projectile.



$$S = R$$

$$v = v_{ix} = v_i \cos \theta, \quad t = \frac{2v_i \sin \theta}{g}$$

So,

$$R = (v_i \cos \theta) \left(\frac{2v_i \sin \theta}{g} \right)$$

$$R = \frac{v_i^2 (2 \sin \theta \cos \theta)}{g}$$

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Maximum range:

for the maximum
range of projectile:

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = \frac{90^\circ}{2}$$

$$\theta = 45^\circ$$

Therefore, the range
of projectile is maximum
for angle 45° mathematically.

$$R_{\max} = \frac{v_i^2}{g}$$



Ballistic flight / Ballistic missile / Ballistic trajectory:

A ballistic flight is that in which a projectile is given an initial push and then allowed to move freely due to inertia and under the action of gravity.

An un-powered and un-guided missile is called a ballistic missile and the path followed by its ballistic trajectory:

M.C.Qs

1- For short ranges and flat Earth approximation, the trajectory is parabolic but dragless ballistic trajectory for spherical Earth should actually be elliptical.

2- The ballistic missiles are useful only for short ranges. For long ranges and greater precision, powered and remote control guided missiles are used.

Short Questions

3.1:

Uniform velocity: If a body covers equal displacement in equal intervals of time, then it has uniform velocity.

Variable velocity: If a body does not cover equal displacement in equal intervals of time, then it has variable velocity.

The SI unit of velocity is: ms^{-1}

Acceleration: The rate of change of velocity of a body is called Acceleration. It is represented by $\vec{a}$.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

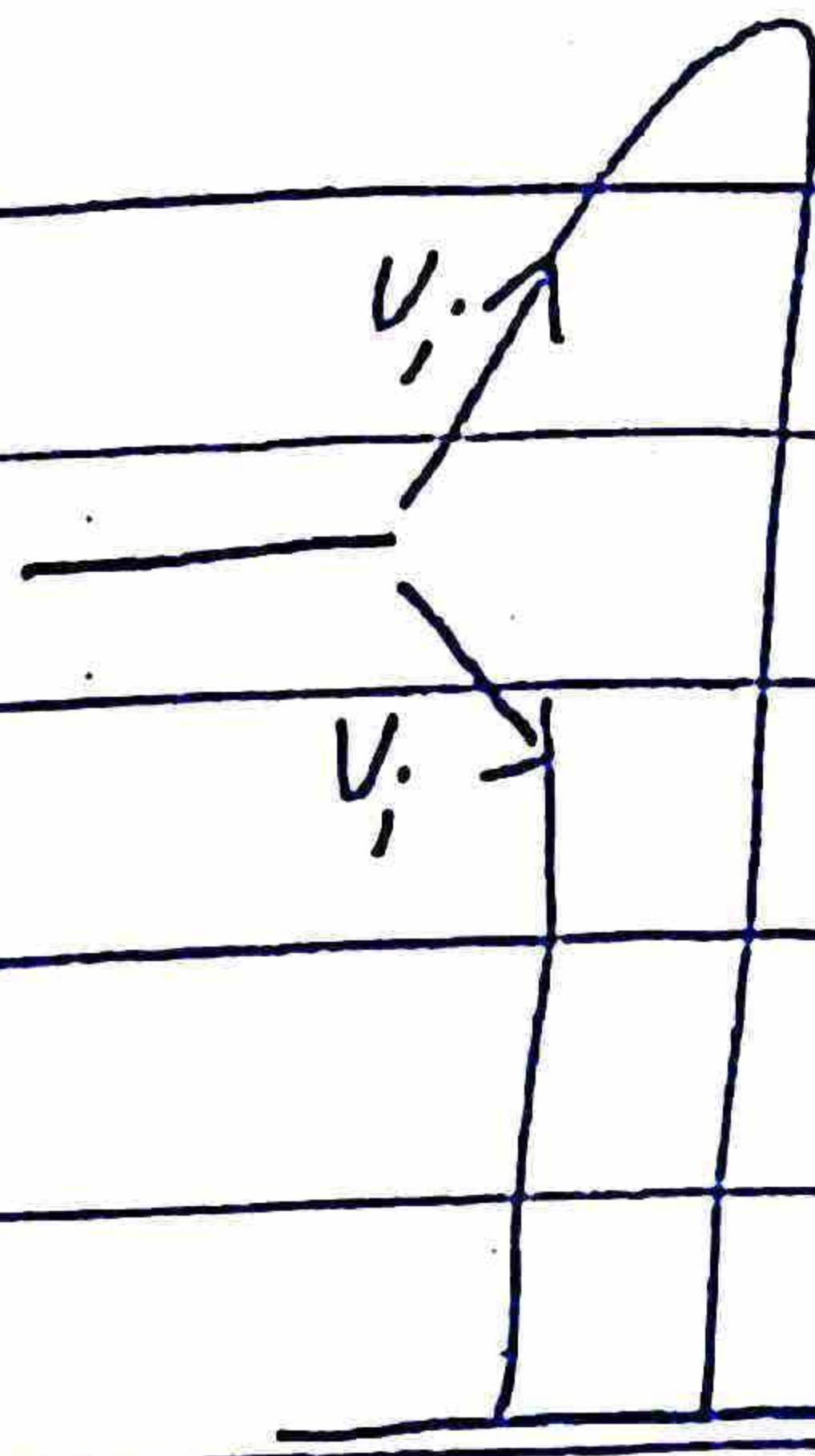
Acceleration is a vector quantity. Its SI unit is ms^{-2} .

3.2: When the object is thrown vertically upward the acceleration due to gravity will be negative. After reaching the highest point it moves down now acceleration due to gravity will be positive.

3.3: Yes, the velocity of an object can reverse its direction when acceleration is constant.

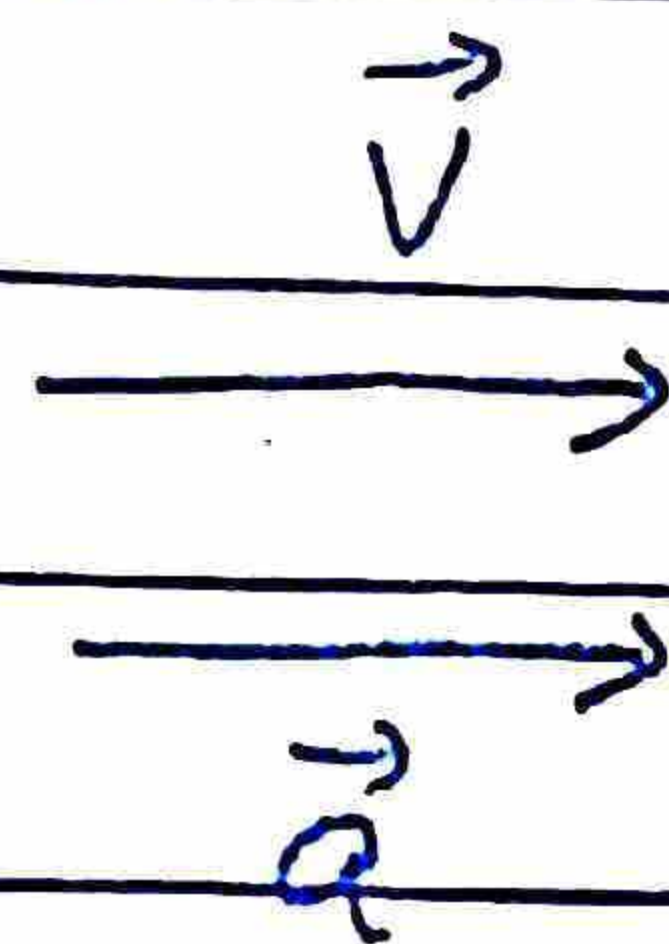
For example, when an object is thrown vertically upward its velocity decreases. After reaching the highest point it moves down, now its velocity increases gradually. But magnitude of acceleration due to gravity remains constant.

3.5: A man standing on the top of a tower throws a ball straight up with initial



velocity v_i and at the same time throws a second ball straight downward with the same speed. While striking the ground both the ball will have same speed. The only difference will be of time.

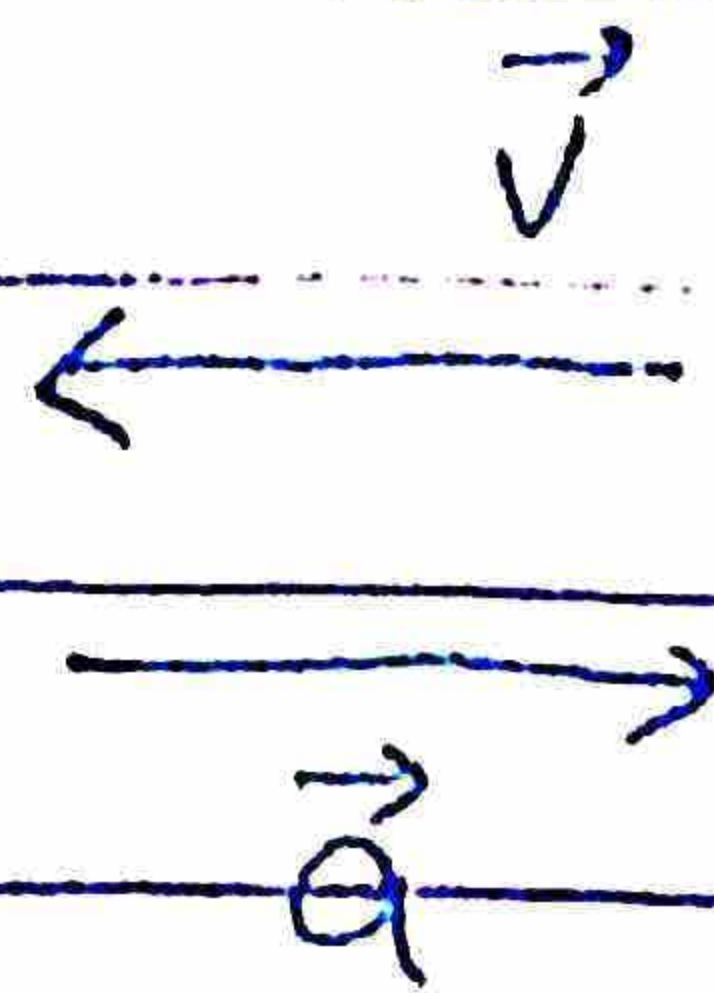
3.6: i- When the velocity of car is increasing



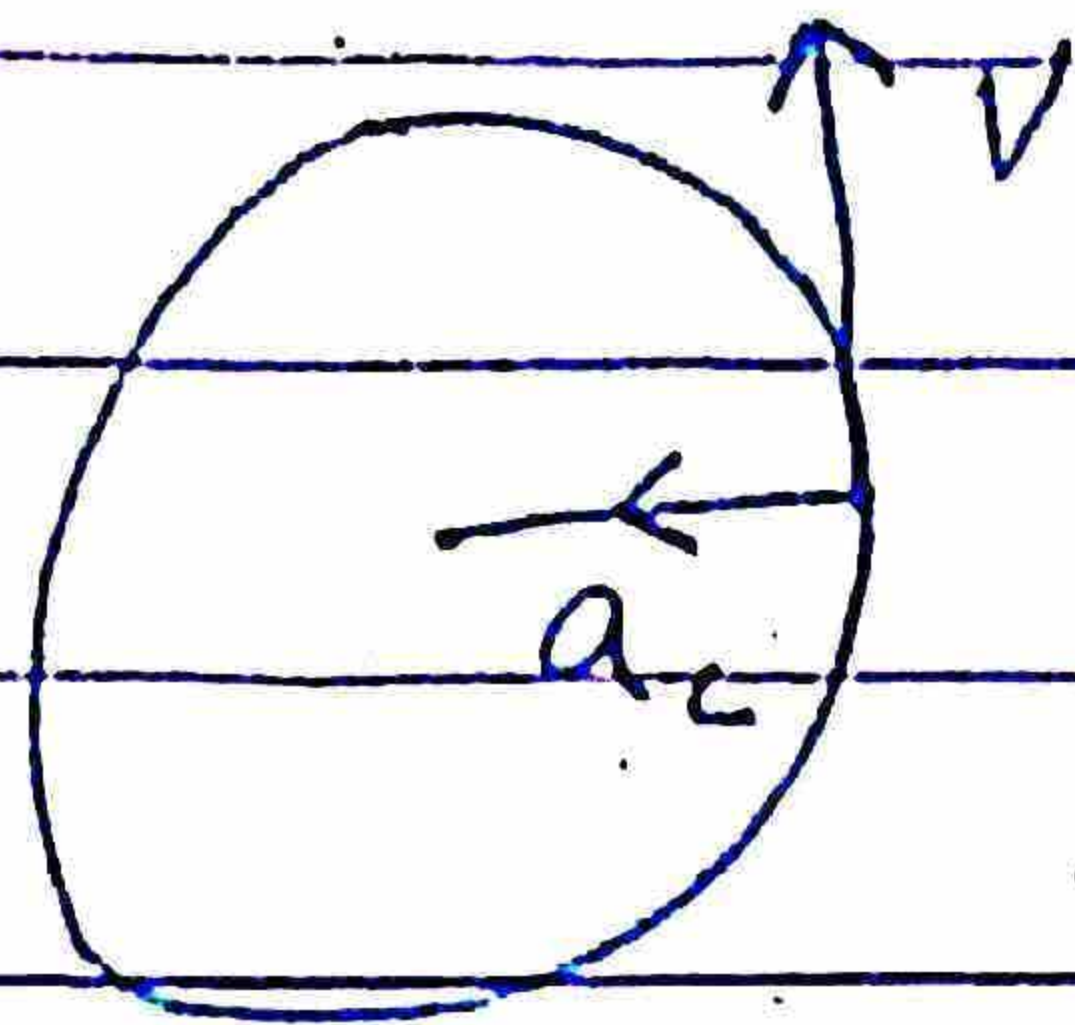
Then velocity $\vec{v}$ and acceleration $\vec{a}$ will be parallel.

ii- When brakes are applied

to car. Then
velocity $\vec{v}$ and
acceleration $\vec{a}$
will be
anti-parallel.



iii- When the
car is moving
in a circle.



Then velocity $\vec{v}$
and acceleration $\vec{a}$ will
be perpendicular.

iv- When the brakes are
applied strongly then velocity
of car will become zero
But it will cover some
distance before coming to
rest. So, acceleration $\vec{a}$
is not zero.

v- When the car is moving
with uniform velocity. Then

acceleration will be zero.

3.7: When the velocity of any object is constant its acceleration is zero.

Here acceleration remains zero and does not change.

So, we can say that acceleration is also constant.

3.8: According to Newton's Second law:

$$\vec{F} = m\vec{a}$$

The rate of change of velocity is called acceleration.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

$$\text{So, } \vec{F} = m \left(\frac{\vec{v}_f - \vec{v}_i}{t} \right)$$

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t}$$

So, the rate of

change of momentum of a moving body is called force.

3.9: When a large force acts on a body for a very short interval of time, then the product of force and time is called impulse.

$$\text{impulse} = \vec{F} \times t = m\vec{v}_f - m\vec{v}_i$$

Impulse is a vector quantity equal to change in momentum. Its SI unit is same as momentum i.e. kgms^{-1} or Ns .

3.10: Statement: The total linear momentum of an isolated system remains constant.
Isolated system: The system of two or more interacting

bodies on which no external force acts is called isolated. **For example:** the gas molecules enclosed in a cylinder exert forces upon each other and on the walls of cylinder but no external force acts on them.

In our practical life, any system can not be completely isolated because force gravity and air must act on it.

So, we neglect these forces to apply law of conservation of momentum.

3.11: Elastic collision

i- The collision in which k.E and momentum of the system remains

In-elastic collision

i- The collision in which the k.E of the system does not remain

conserved.

for example

ii - When a

ball is dropped
from a certain
height it reaches
to the same

height

striking with

marble floor.

conserved

ii - For example:

When a ball is

dropped from a

certain height it

does not reach to

the same height after

striking with

rough floor.

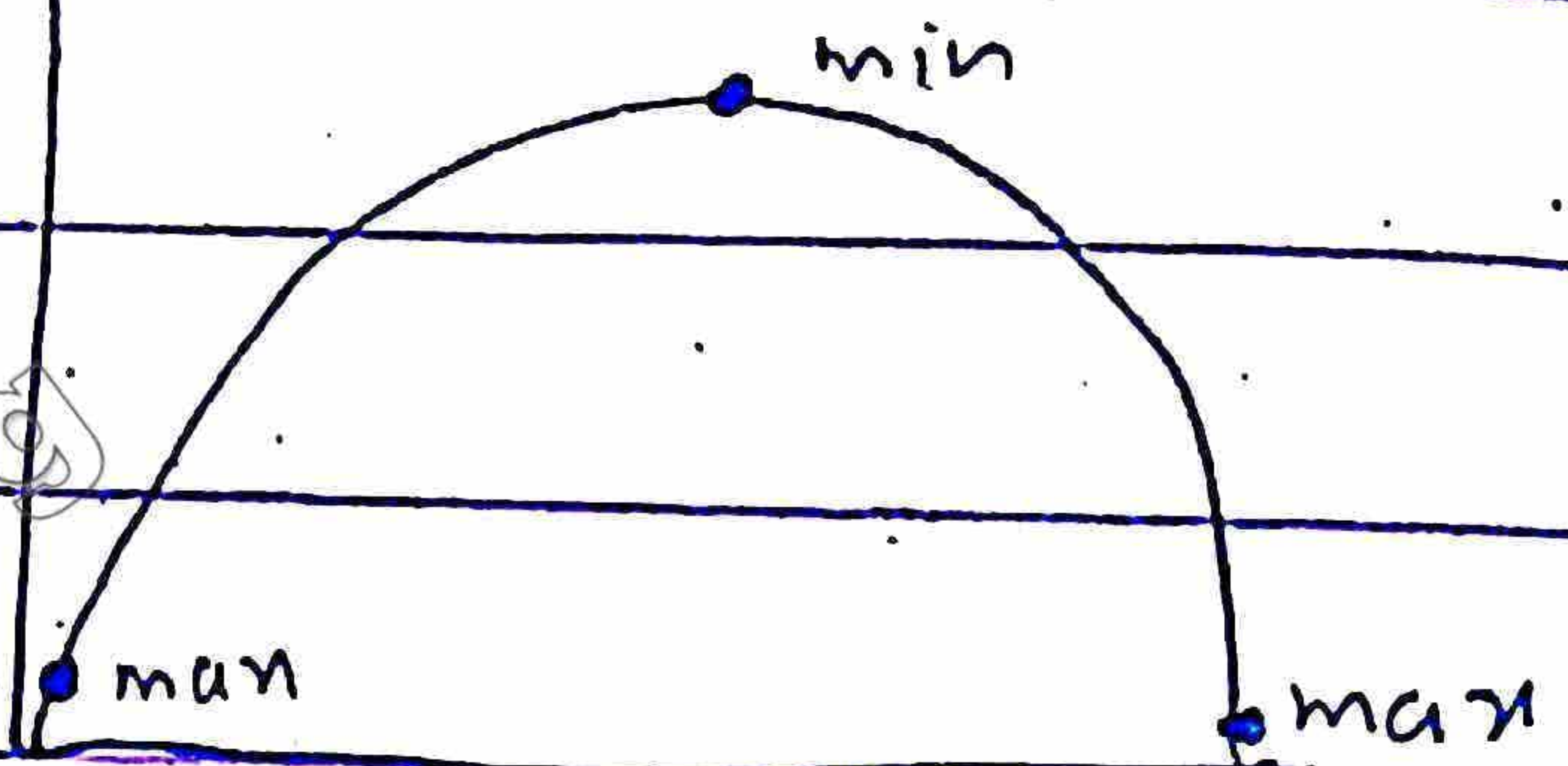
K.E does not conserved in most
cases:

When two tennis ball collide,
then after collision, they will
rebound with velocities less than
the velocities before the impact.

During this process a portion
of K.E is lost, partly due to
its friction and partly due to
change into heat and sound
energies.

3.13:

The projectile
have maximum
speed at the
point of projection
and at point
just before



striking the ground: The projectile
has minimum speed at its
maximum height.

Numerical Problems

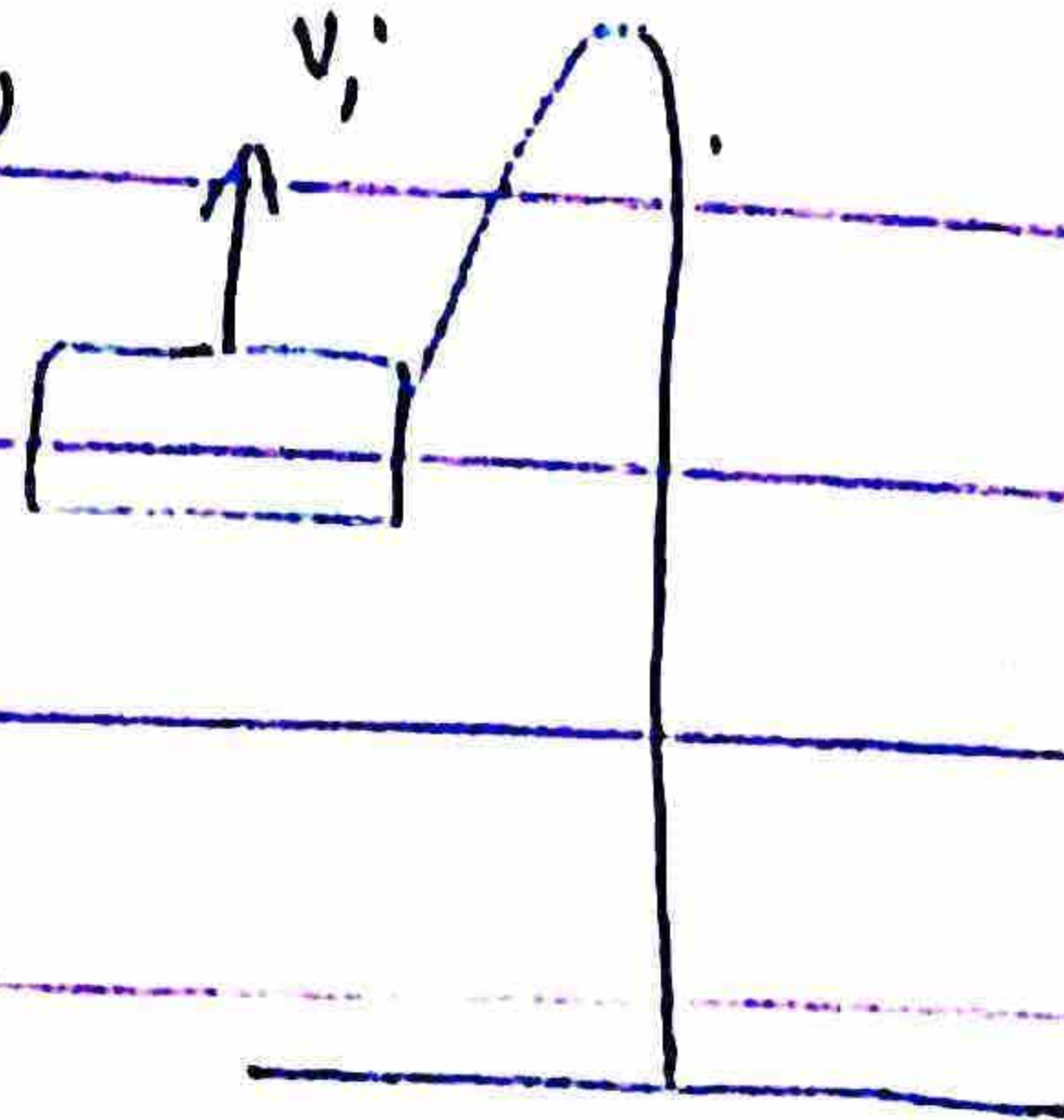
3.1 Data

$$\text{velocity} = v_i = 19.6 \text{ m s}^{-1}$$

$$\text{height} = h = -156.8 \text{ m}$$

$$\text{time} = t = ?$$

$$g = 9.8 \text{ m s}^{-2}$$



Solution:

$$s = v_i t + \frac{1}{2} a t^2$$

$$\text{here } s = h, \quad a = -g$$

$$h = v_i t - \frac{1}{2} g t^2$$

$$-156.8 = 19.6 t - \frac{1}{2} (9.8) t^2$$

$$-156.8 = 19.6 t - 4.9 t^2$$

$$4.9 t^2 - 19.6 t - 156.8 = 0$$

$$4.9 (t^2 - 4t - 32) = 0$$

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t+4)(t-8) = 0$$

$$t+4=0, \quad t-8=0$$

$$t = -4$$

so,

$$t = 8 \text{ s}$$

Time can not
be negative.

3.3 Data

$$\text{initial speed} = v_i = 1 \times 10^7 \text{ m s}^{-1}$$

$$\text{distance} = s = 0.02 \text{ cm} = \frac{0.02}{100} \text{ m}$$

$$s = 2 \times 10^{-4} \text{ m}$$

$$\text{final speed} = v_f = 2 \times 10^6 \text{ m s}^{-1}$$

$$\text{retardation} = a = ?$$

$$\text{time} = t = ?$$



Solution:

$$2as = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2s}$$

$$= \frac{(2 \times 10^6)^2 - (1 \times 10^7)^2}{2(2 \times 10^{-4})}$$

$$= \frac{-9.6 \times 10^{13}}{4 \times 10^{-4}}$$

$$a = -2.4 \times 10^{17} \text{ m s}^{-2}$$

Negative sign shows
retardation.

$$a = 2.4 \times 10^{17} \text{ m s}^{-2}$$

$$\text{Now } a = \frac{v_f - v_i}{t}$$

$$t = \frac{v_f - v_i}{a}$$

$$= \frac{(2 \times 10^6) - (1 \times 10^7)}{-2.4 \times 10^{17}}$$

$$t = 3.3 \times 10^{-11} \text{ s}$$



3.4 Data:

Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What will be the ratio of the magnitude of their velocities after the spring has been released?

Solution

Initially both masses are at rest. So,

$$\text{initial momentum} = 0$$

$$\text{final momentum} = m_1 v_1 + m_2 (-v_2)$$

-ve sign shows that

both masses will move in opposite direction.

$$\text{Final momentum} = m_1 v_1 - m_2 v_2$$

According to law of conservation of momentum.

$$\text{Final momentum} = \text{initial momentum}$$

$$m_1 v_1 - m_2 v_2 = 0$$

$$m_1 v_1 = m_2 v_2$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$



3.6 Data

$$\text{mass of can} = m_1 = 40\text{g} = \frac{40}{1000} \text{kg}$$

$$m_1 = 0.04 \text{kg}$$

$$\text{mass of wooden block} = m_2 = 200\text{g}$$

$$m_2 = \frac{200}{1000} \text{kg} = 0.2 \text{kg}$$

$$\text{speed of block} = v_2 = 3 \text{ms}^{-1}$$

$$\text{speed of can} = v_1 = ?$$

Solution: According to law of conservation of momentum

$$\text{Final momentum} = \text{initial momentum}$$

$$m_1 v_1 + m_2(-v_2) = 0$$

-ve sign shows that car and block will move in opposite direction

$$m_1 v_1 - m_2 v_2 = 0$$

$$m_1 v_1 = m_2 v_2$$

$$v_1 = \frac{m_2 v_2}{m_1}$$

$$= \frac{(0.2)(3)}{0.04}$$

$$v_1 = 15 \text{ m s}^{-1}$$

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3.7 Data

mass of electron = $m_1 = 9.1 \times 10^{-31} \text{ kg}$

velocity of electron = $v_1 = 2 \times 10^7 \text{ m s}^{-1}$

mass of hydrogen atom = $m_2 = 1.67 \times 10^{-27} \text{ kg}$

velocity of hydrogen atom = $v_2 = 0$

final velocity of hydrogen atom
= $v_2' = ?$

Solution

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

$$V_2' = \frac{2(9.1 \times 10^{-31})(2 \times 10^7)}{(9.1 \times 10^{-31}) + (1.67 \times 10^{-27})} + 0$$

$$V_2' = \frac{3.64 \times 10^{-23}}{1.67 \times 10^{-27}}$$

$$V_2' = 21796 \text{ ms}^{-1}$$

$$= 2.1796 \times 10^4 \text{ ms}^{-1}$$

$$V_2' = 2.2 \times 10^4 \text{ ms}^{-1}$$

3.8 Data

mass of truck = $m_1 = 2500 \text{ kg}$

velocity of truck = $V_1 = 21 \text{ ms}^{-1}$

velocity of car = $V_2 = 0$

mass of car = $m_2 = 1000 \text{ kg}$

common velocity = $V_1' = V_2' = V = ?$

after collision

Solution: According to law of conservation of momentum

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$m_1 V_1 + m_2 V_2 = m_1 V + m_2 V$$

$$(2500)(21) + (1000)(0) = 2500V + 1000V$$

$$52500 + 0 = 3500 V$$

$$\frac{52500}{3500} = V$$

$$V_1' = V_2' = V = 15 \text{ m s}^{-1}$$

3.9 Data



mass of first block = $m_1 = 2 \text{ kg}$

mass of second block = $m_2 = 0.5 \text{ kg}$

elastic. P.E = 10 J

velocity of first block = $V_1 = ?$

velocity of second block = $V_2 = ?$

Solution: According to law of conservation of momentum

Final momentum = initial momentum

$$m_1 V_1 + m_2 V_2 = 0$$

$$2 V_1 + 0.5 V_2 = 0$$

$$0.5 V_2 = -2 V_1$$

$$V_2 = -\frac{2}{0.5} V_1$$

$$V_2 = -4 V_1 \rightarrow (1)$$

According to law of conservation of energy

$$K.E = P.E$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 10$$

$$\frac{1}{2} (2) v_1^2 + \frac{1}{2} (0.5) (-4v_1)^2 = 10$$

$$v_1^2 + 0.25 (16 v_1^2) = 10$$

$$v_1^2 + 4 v_1^2 = 10$$

$$5 v_1^2 = 10$$

$$v_1^2 = \frac{10}{5}$$

$$v_1^2 = 2$$

$$\sqrt{v_1^2} = \sqrt{2}$$

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$$v_1 = 1.4 \text{ m s}^{-1}$$

now put in eq. (1)

$$v_2 = -4 v_1$$

$$= -4 (1.4)$$

$$v_2 = -5.6 \text{ m s}^{-1}$$

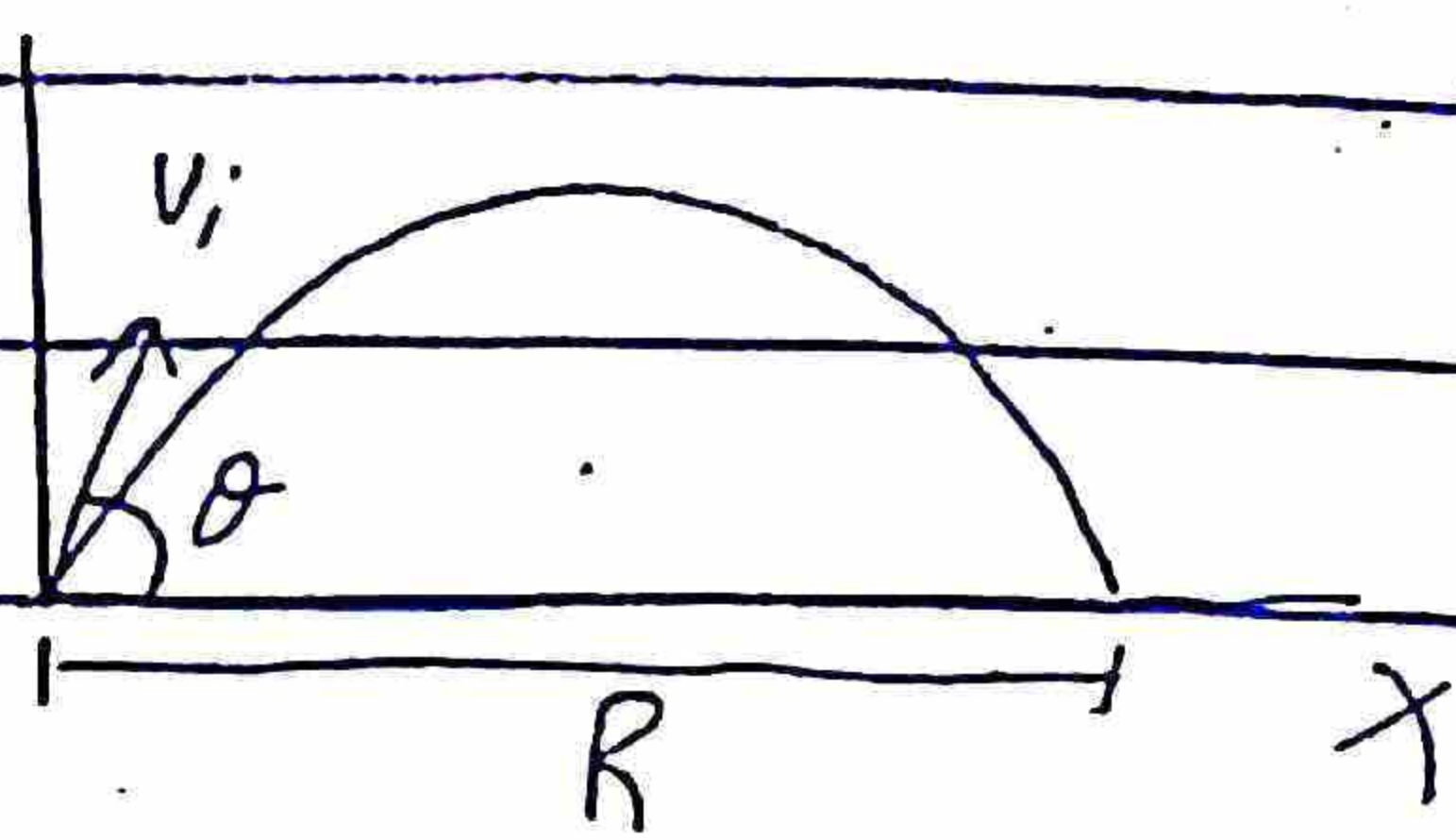
3.10 Data

$$\theta = 30^\circ$$

$$\text{range} = R = 40 \text{ m}$$

$$\text{initial speed} = v_i = ?$$

$$g = 9.8 \text{ m s}^{-2}$$



Solution

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R g = v_i^2 \sin 2\theta$$

$$(40)(9.8) = v_i^2 \sin 2(30^\circ)$$

$$392 = v_i^2 \sin 60^\circ$$

$$392 = v_i^2 (0.866)$$

$$\frac{392}{0.866} = v_i^2$$

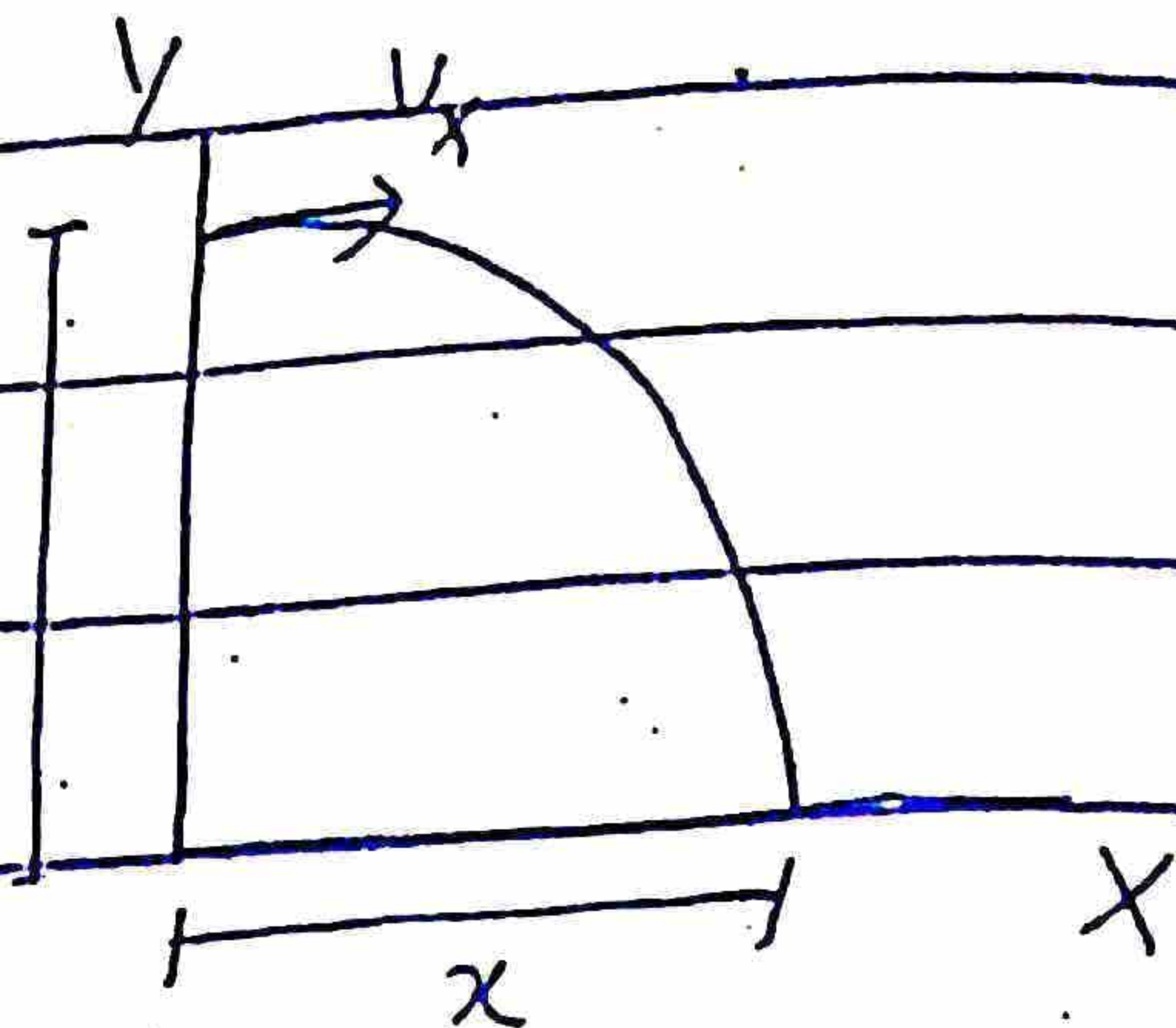
$$v_i^2 = 453$$

$$\sqrt{v_i^2} = \sqrt{453}$$

$$v_i = 21 \text{ m s}^{-1}$$

3.11 Data

height = $y = 10\text{ m}$



horizontal velocity

$$= v_x = 21 \text{ m s}^{-1}$$

horizontal distance = $x = ?$

$$\text{velocity} = v = ? , g = 9.8 \text{ m s}^{-2}$$

Solution:

$$y = \frac{1}{2} g t^2$$

$$2y = g t^2$$

$$\frac{2y}{g} = t^2$$

$$t^2 = \frac{2(10)}{9.8} = \frac{20}{9.8}$$

$$\sqrt{t^2} = \sqrt{2.04}$$

$$t = 1.43 \text{ s}$$

now

$$x = v_x t$$

$$= (21)(1.43)$$

$$x = 30 \text{ m}$$

As

$$V = \sqrt{V_{fx}^2 + V_{fy}^2}$$

here

$$V_{fx} = V_x = 21 \text{ m s}^{-1}$$

and

$$V_f = V_i + at$$

$$\text{So, } V_{fy} = V_{iy} + gt$$

$$= 0 + (9.8)(1.43)$$

$$V_{fy} = 14 \text{ m s}^{-1}$$

now

$$V = \sqrt{(21)^2 + (14)^2}$$

$$V = 25 \text{ m s}^{-1}$$

3.12 Data

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$$g = 9.8 \text{ m s}^{-2}$$

$$\text{height} = y = 490 \text{ m}$$

horizontal velocity

$$= V_x = 300 \text{ km h}^{-1}$$

$$= \frac{300 \text{ km}}{h}$$

$$V_x = \frac{300000 \text{ m}}{3600 \text{ s}} = 83.3 \text{ m s}^{-1}$$

time = $t = ?$

horizontal distance = $x = ?$

Solution:

$$y = \frac{1}{2} g t^2$$

$$\frac{2y}{g} = t^2$$

$$t^2 = \frac{2(490)}{9.8} = 100$$

$$\sqrt{t^2} = \sqrt{100}$$

$$t = 10 \text{ s}$$

now

$$x = v_x t$$

$$= (83.3)(10)$$

$$x = 833 \text{ m}$$

3.13 Data

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angle of projection = $\theta = ?$

if $h = R$

Solution Given that

$$h = R$$

$$\frac{v_i^2 \sin^2 \theta}{2g} = \frac{v_i^2 \sin 2\theta}{g}$$

$$\sin^2 \theta = 2 \sin 2\theta$$

$$\sin^2 \theta = 2 (2 \sin \theta \cos \theta)$$

$$\sin \theta = 4 \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = 4$$

$$\tan \theta = 4$$

$$\theta = \tan^{-1}(4)$$

$$\theta = 76^\circ$$

3.14 Data

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$$\theta = 45^\circ$$

$$\theta_1 = 45^\circ + 15^\circ = 60^\circ$$

$$\theta_2 = 45^\circ - 15^\circ = 30^\circ$$

we have to prove

$$R_1 = R_2$$

Solution

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R_1 = \frac{v_i^2 \sin 2\theta_1}{g} \quad ; \quad R_2 = \frac{v_i^2 \sin 2\theta_2}{g}$$

$$= \frac{v_i^2 \sin 2(60^\circ)}{g} \quad ; \quad = \frac{v_i^2 \sin 2(30^\circ)}{g}$$

$$R_1 = \frac{v_i^2 \sin 120^\circ}{g}, \quad R_2 = \frac{v_i^2 \sin 60^\circ}{g}$$

$$R_1 = \frac{v_i^2 (0.866)}{g}, \quad R_2 = \frac{v_i^2 (0.866)}{g}$$

Therefore, $R_1 = R_2$

3.15 Data

$$\text{range} = R = 3000 \text{ km}$$

$$R = 3000 \text{ m}$$

$$R = 3 \times 10^6 \text{ m}$$

$$\text{angle} = \theta = 45^\circ, \quad g = 9.8 \text{ m/s}^2$$

$$\text{initial velocity} = v_i = ?$$

$$\text{time} = t = ?$$

Solution

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$\frac{Rg}{\sin 2\theta} = v_i^2$$

$$v_i^2 = \frac{(3 \times 10^6)(9.8)}{\sin 2(45^\circ)}$$

$$= \frac{2.94 \times 10^7}{\sin 90^\circ}$$

$$\sqrt{V_i^2} = \sqrt{2.94 \times 10^7}$$

$$V_i = 5422 \text{ m s}^{-1}$$

$$= 5.422 \times 10^3 \text{ m s}^{-1}$$

$$V_i = 5.42 \text{ km s}^{-1}$$

$$t = \frac{2V_i \sin \theta}{g}$$

$$= \frac{2(5422) \sin 45^\circ}{9.8}$$

$$t = 782 \text{ s}$$

$$t = \frac{782}{60} \text{ min}$$

$$t = 13 \text{ min}$$