

Chapter = 04

MOTION IN TWO DIMENSIONS

THEORY NOTES

**PROJECTILE****DEFINITION:**

An object falling freely in a gravitational field, having been projected with a velocity 'v' and at an angle of elevation 'θ' with the horizontal is called projectile.

PROJECTILE MOTION:

When an object is projected with a velocity 'v' it will move in a semi-parabolic path called 'trajectory'. Three assumptions are made, when considering a projectile motion.

ASSUMPTION:

- i) The acceleration due to gravity, 'g', is constant over the range of motion and is directed downward.
- ii) The effect of air resistance is negligible.
- iii) The rotation of earth does not affect the motion.

At any instant project motion can be described in two parts.

- a) Horizontal Motion: b) Vertical Motion:

When an object is projected with an initial velocity v_0 and with an angle 'θ' with the horizontal it travels in horizontal directions as well as in vertical direction.

The velocity can be resolved, at any instant, in two components. $V_x = V_{ox} = V_0 \cos \theta$

- a) **Horizontal Velocity:**

it remains unchanged throughout the motion because there is no acceleration in the motion.

- b) **Vertical Velocity:**

It continuously changes because of the force of attraction of the earth.

$$V_y = V_{oy} = V_0 \sin \theta$$

Thus,

$$\text{The net velocity} = V_0 = \sqrt{V_{ox}^2 + V_{oy}^2}$$

FOR HORIZONTAL MOTION:

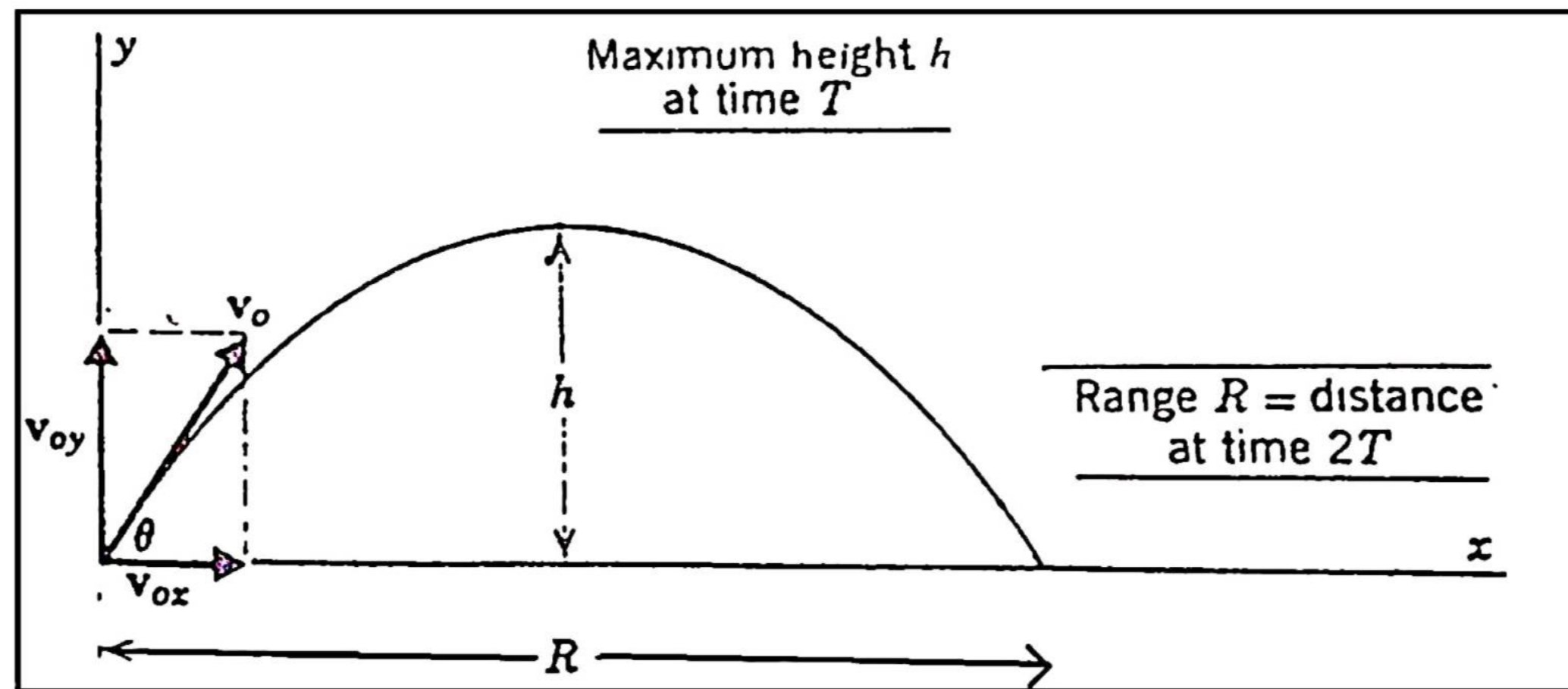
$$\text{Acceleration} = a_x = 0$$

$$\text{Velocity} = V_x = V_{ox}$$

$$\text{Displacement} = X = V_{ox} \cdot t$$

FOR VERTICAL MOTION:

$$\begin{aligned}\text{Acceleration} &= a_y = -g \\ \text{Velocity} &= V_y = V_{oy} - gt \\ \text{Displacement} &= y = V_{oy}t - \frac{1}{2}gt^2\end{aligned}$$

**DERIVATIONS:****A) TIME TO REACH MAXIMUM HEIGHT:**

$$\begin{aligned}\text{Initial Velocity} &= V_{oy} = V_o \sin \theta \\ \text{Final Velocity} &= V_y = 0 \\ \text{Time} &= T \\ \text{Acceleration} &= -g\end{aligned}$$

We know that

$$\begin{aligned}V_f &= V_i + at \\ V_y &= V_{oy} + (-g)T \\ 0 &= V_o \sin \theta - gT \\ gT &= V_o \sin \theta\end{aligned}$$

$$T = \frac{V_o \sin \theta}{g}$$

B) TIME OF FLIGHT:

$$\begin{aligned}\text{Total time of flight} &= T' \\ T' &= 2T\end{aligned}$$

$$T' = \frac{2V_o \sin \theta}{g}$$

C) MAXIMUM HEIGHT:

$$\begin{aligned}\text{Distance} = \text{Height} &= y = h_{\max} \\ \text{Initial Velocity} &= V_{oy} = V_o \sin \theta \\ \text{Acceleration} &= a = -g\end{aligned}$$

$$\text{Time} = T = \frac{V_o \sin \theta}{g}$$

We know that,

$$S = V_i t + \frac{1}{2} a t^2$$

$$Y = V_{oy} T + \frac{1}{2} (-g) T^2$$

$$h_{\max} = V_o \sin \theta \times \frac{V_o \sin \theta}{g} - \frac{1}{2} g \left(\frac{V_o \sin \theta}{g} \right)^2$$

$$h_{\max} = \frac{V_o^2 \sin^2 \theta}{g} - \frac{V_o^2 \sin^2 \theta}{2g}$$

$$h_{\max} = \frac{2V_o^2 \sin^2 \theta - V_o^2 \sin^2 \theta}{2g}$$

$$h_{\max} = \frac{V_o^2 \sin^2 \theta}{2g}$$

D) **RANGE:**

The maximum horizontal distance traveled by a projectile is called 'rang'.

$$\text{Distance} = X = R$$

$$\text{Velocity} = V_{ox} = V_o \cos \theta$$

$$\text{Time} = T'$$

Since,

$$S = V \times t$$

$$X = V_o \times T$$

$$R = V_o \cos \theta \times \frac{2 V_o \sin \theta}{g}$$

$$R = \frac{V_o^2 2 \cos \theta \sin \theta}{g}$$

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

E) **MAXIMUM RANGE:**

Horizontal Range is given as,

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

Above expression shows that, for constant velocity of projection (V_0) and gravitational acceleration (g), horizontal range depends on the factor $\sin 2\theta$ and it will be maximum at the maximum value of $\sin 2\theta$. The

maximum value of sin is 1.

$$\sin 2\theta = 1$$

or

$$2\theta = \sin^{-1}(1)$$

or

$$2\theta = 90^\circ$$

or

$$\theta = 45^\circ$$

It shows that, "when a projectile is projected with 45° , its horizontal range will be maximum."

F) **PROJECTILE TRAJECTORY:**



The path followed by a projectile is called its trajectory.

Knowing the displacement along horizontal and vertical direction, position of projectile can be determined.

A) **Horizontal displacement – X**

Since,

$$S = vt$$

$$X = V_o t$$

$$X = V_o \cos\theta \cdot t$$

$$t = \frac{X}{V_o \cos\theta} \text{-----(1)}$$

B) **Vertical displacement – Y**

Since,

$$S = Vit + \frac{1}{2} at^2$$

$$y = V_o y \cdot t + \frac{1}{2} (-g)t^2$$

$$y = V_o \sin\theta \frac{x}{V_o \cos\theta} - \frac{1}{2} g \cdot \frac{x^2}{V_o^2 \cos^2\theta}$$

$$y = x \frac{\sin\theta}{\cos\theta} - \frac{gx^2}{2V_o^2 \cos^2\theta}$$

$$y = x \tan\theta - \frac{gX^2}{2V_o^2 \cos^2\theta}$$

This expression is known as Equation of Trajectory.

ANGULAR MOTION OR CIRCULAR MOTION:

When a body is moving along a circular path, it is called as circular Motion or angular motion, in this type of motion, the change in position, of a moving body is measured by the angle subtended by it at the center of its circular path. The universe is full of a large number of objects such as the planets revolve around the sun and the moon moves around earth in nearly circular orbits.

ANGULAR DISPLACEMENT:

The angle, through which a body moves while moving along a circular path, is called as angular displacement. It is the angle subtended at the center during angular motion.

Angular displacement is measured in degree and radian.

(I) **DEGREE:** When a rotating object completes one revolution it subtends an angle of 360 degrees at the center of its circular path and thus its angular displacement is 360°.

II) **RADIAN:** It is the angle subtended at the center of a circle by an arc equal in length to its radius.

RELATION BETWEEN RADIAN AND DEGREE:

Consider a circle of radius “r” with o as center. Arc AB is taken in length equal to r. The circumference of the circle is equal in length to $2\pi r$. As the arc of length r subtends an angle of 1 radian at the center, so the whole circumference will subtend an angle of 2π radians at the center. But the whole circumference subtends an angle of 360° degree at the center, therefore,

$$2\pi \text{ radian} = 360 \text{ degrees}$$

$$\pi \text{ radian} = 180 \text{ degrees}$$

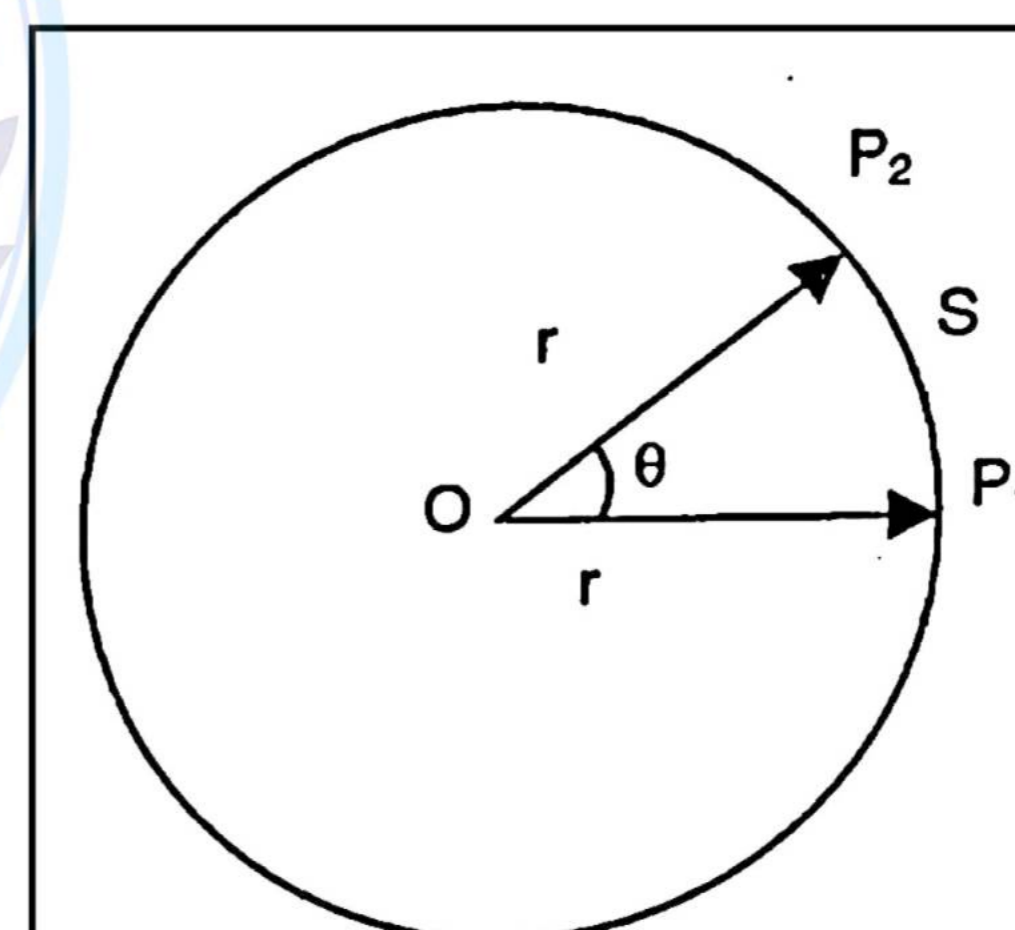
$$1 \text{ radian} = \frac{180}{\pi}$$

$$1 \text{ radian} = 57.3^\circ$$

RELATION BETWEEN LINEAR AND ANGULAR DISPLACEMENT:

It is clear from the figure that the arc length is directly proportional to the angle subtended at the center. Mathematically we can write as,

$$S \propto \theta$$



or

$$S = r\theta$$

Where, r is the radius of circle and angle is measured in Radian.

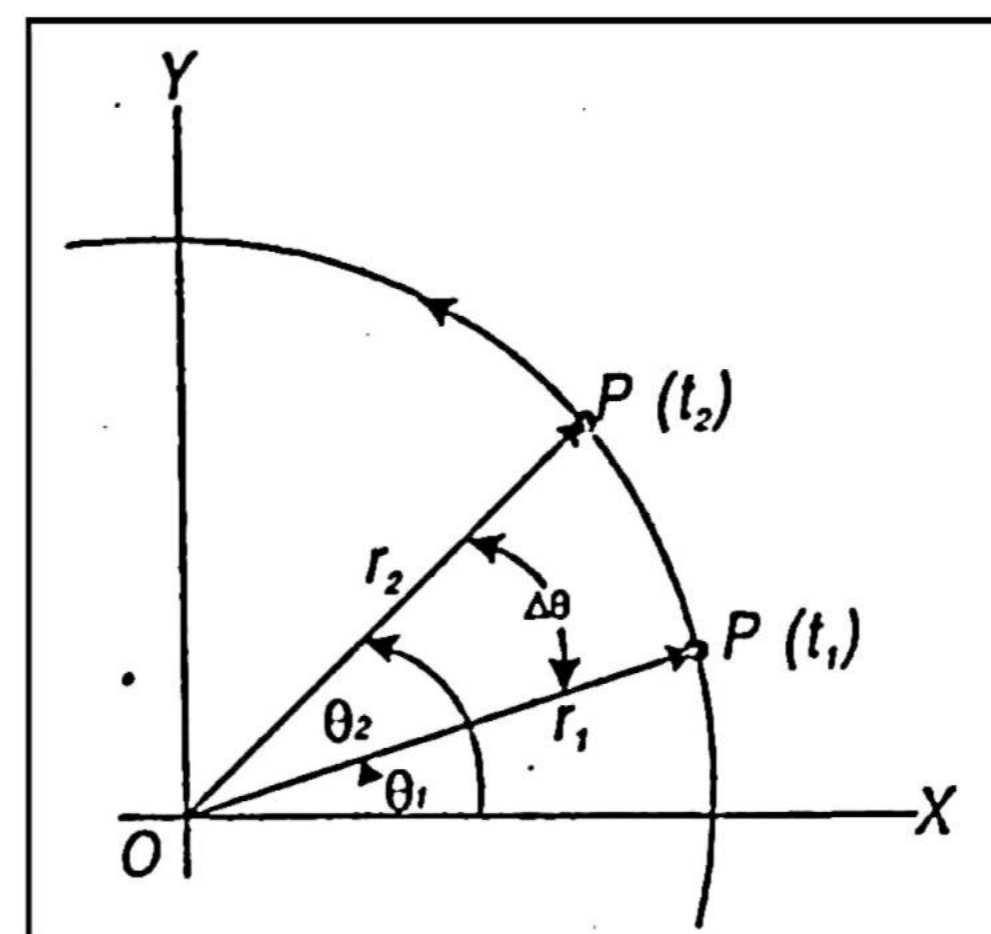
ANGULAR VELOCITY:



In circular motion of a particle P the rate of change of angular displacement is called angular velocity and it is denoted by " ω ". If ' $\Delta\theta$ ' is the change in angular displacement during time interval Δt due to the motion of particle P along a circular path then average angular velocity will be

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$



If the interval Δt is very small i.e., $\Delta t \rightarrow 0$ then angular velocity during such a short time interval will be called instantaneous angular velocity. i.e.

$$\omega_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t}$$

UNIT OF ANGULAR VELOCITY:

The angular velocity may be measured in deg / sec, rad / sec or rev / sec. But radian does not appear in the final answer so the unit of angular velocity is sec^{-1} , which is also the unit of frequency. So we may call ' ω ', as angular frequency.

The angular velocity is also measured in R.P.M.

$$\text{i.e } 1 \text{ revolution} = 2\pi \text{ rad}$$

$$1 \text{ rpm} = 2\pi / 60 \text{ rad/sec}$$

$$1 \text{ rpm} = 0.105 \text{ rad/sec}$$

DIRECTION OF ANGULAR VELOCITY:

The angular velocity is always directed along axis of rotation of the circle. If curl of fingers of right hand points the direction of rotation then thumb will point out the direction of omega " ω ".

In the case of counter clock wise rotation the ω will be directed out of the page. In case of Clock wise rotation ω will be directed into the page.

RELATION BETWEEN LINEAR AND ANGULAR VELOCITIES:

We know that

$$S = r\theta$$

But in case of small angular displacement due to circular motion of particle P along circle of radius r , we

may write this equation

$$\Delta S = r\Delta\theta$$

Divide both sides by Δt

$$\frac{\Delta S}{\Delta t} = r \frac{\Delta\theta}{\Delta t}$$

Taking limits on both sides

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \text{ -----(i)}$$

As we know that

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t}$$

And

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t}$$

Therefore eq(i) , becomes

$$V = r\omega$$

or

$$\vec{V} = \vec{r} \times \vec{\omega}$$

ANGULAR ACCELERATION:

The rate of change of angular velocity is called angular acceleration and it is denoted by ' α '. If $\Delta\omega$ is the change in angular velocity during time interval Δt due to the motion of a point along circular path then average angular acceleration will be,

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

Now if the time interval for this change is very small then angular acceleration on during such a short time interval is called instantaneous angular acceleration.i.e.

$$\alpha_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t}$$

UNIT OF ANGULAR ACCELERATION:

The angular acceleration may be measured in rad/sec^2 , deg / sec^2 or rev / sec^2 . But S. I Unit is rad/sec^2

DIRECTION OF ANGULAR ACCELERATION:

The direction of angular acceleration may either be parallel or anti parallel to the direction of $\vec{\omega}$ which is along the axis of rotation. If $\vec{\omega}$ increases then it will be parallel to the direction of $\vec{\omega}$. Similarly if it decreases then it will be in opposite direction to that of $\vec{\omega}$ along the axis of rotation.

RELATION BETWEEN LINEAR AND ANGULAR ACCELERATION:

We have the relation between linear and angular velocities as $V = r\omega$. Suppose an object rotating about a fixed axis, changes its angular velocity by $\Delta\omega$ in a time change Δt then the change in tangential velocity.

$$V_t = r\omega$$

Divide both sides by Δt

$$\frac{\Delta V_t}{\Delta t} = r \frac{\Delta \omega}{\Delta t}$$

Taking limits on both sides

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta V_t}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} \text{ -----(i)}$$

As we know that

$$a_t = \lim_{\Delta t \rightarrow 0} \frac{\Delta V_t}{\Delta t}$$

And

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$$

Therefore eq(i), becomes

$$a_t = r \alpha$$

or

$$\vec{a}_t = \vec{r} \times \vec{\alpha}$$

CENTRIPETAL ACCELERATION**DEFINITION:**

The acceleration in the motion of a body only due to the rate of change in direction of velocity is called centripetal acceleration as it is always directed towards the centre of the circle or centre of curvature of the track.

FORMULA:

$$a_c = \frac{v^2}{r}$$

DERIVATION:

Let us consider a particle of mass m moving with uniform speed ' v ' along a circular path of radius ' r '. Suppose its linear velocity vector at P_1 is \vec{v}_1 at time t_1 whereas velocity vector at P_2 is \vec{v}_2 at time t_2 , as shown in the figure.

In case of uniform motion

$$|v_1| = |v_2|$$

but from the figure

$$\vec{v}_1 + \Delta \vec{v} = \vec{v}_2$$

$$\Delta \vec{V} = \vec{V}_2 - \vec{V}_1$$

This change in velocity is only because of the change in its direction. So its rate of change will be called centripetal acceleration. As we know that the angle between perpendiculars to the two lines is same as the angle between these two lines, therefore, the angle between V_1 and V_2 is $\Delta\theta$ as the angle between radial lines is $\Delta\theta$.

It is clear from fig (i) and (ii) that the isosceles triangle ΔP_1OP_2 and ΔBAC are congruent. Then according to geometry,

$$\frac{\Delta S}{r} = \frac{\Delta V}{V}$$

$$\Delta V = \frac{V \Delta S}{r}$$

Divide both sides by Δt

$$\frac{\Delta V}{\Delta t} = \frac{V \Delta S}{r \Delta t}$$

Taking limit both sides as $\Delta t \rightarrow 0$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t} = \frac{V}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t} \text{-----(i)}$$

As we know that

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t}$$

and

$$a_c = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t}$$

Then eq (i) becomes

$$a_c = \frac{v^2}{r} \text{-----(ii)}$$

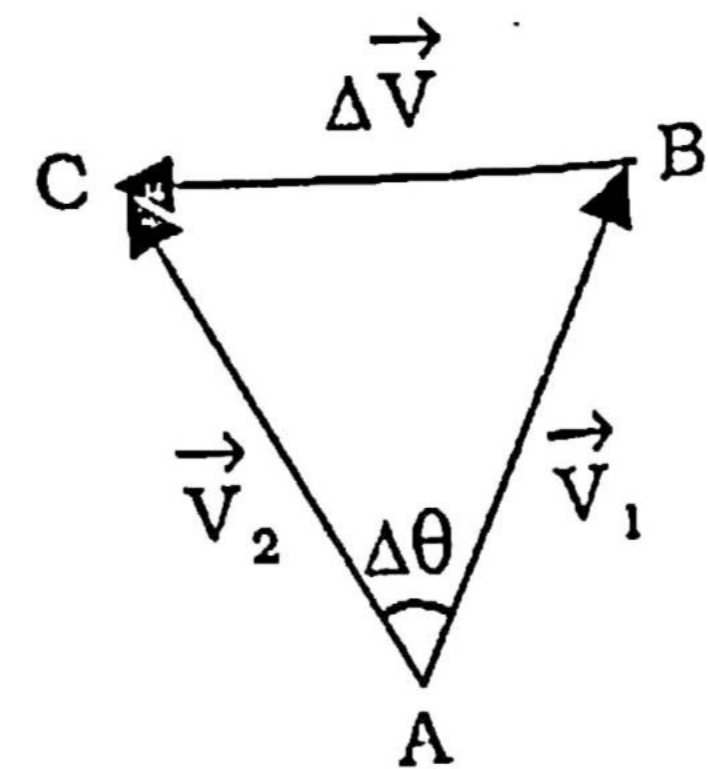
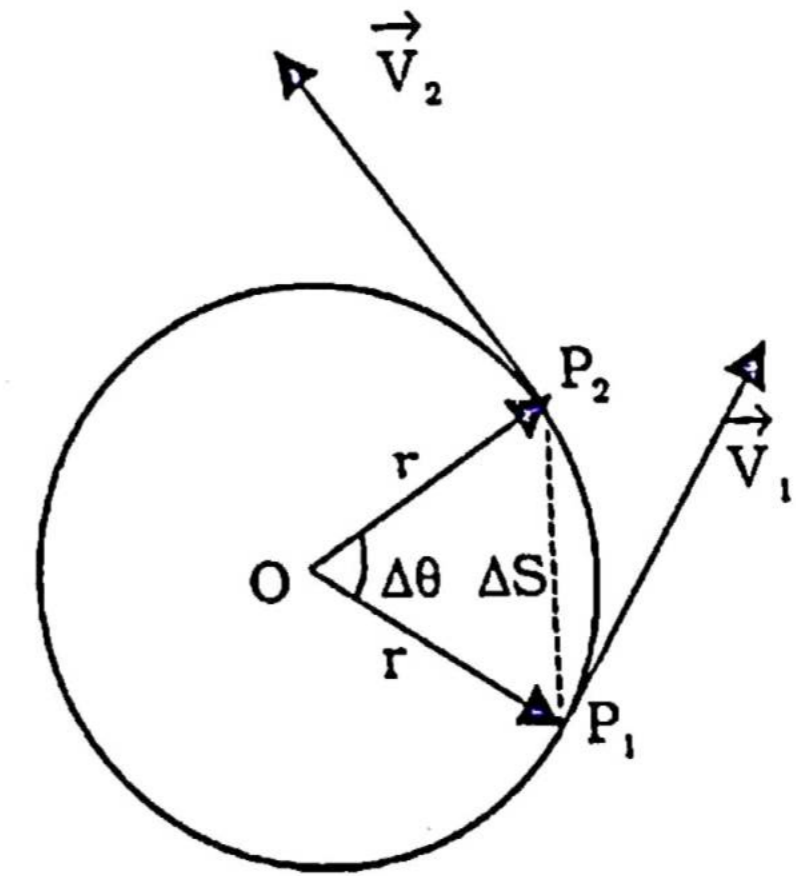
As we know that

$$V = r\omega$$

putting in eq(ii)

$$a_c = \frac{(r\omega)^2}{r}$$

$$a_c = r\omega^2$$



CENTRIPETAL FORCE:**DEFINITION:**

"The force that causes an object to move along a curve (or a curved path) is called centripetal force."

MATHEMATICAL EXPRESSION:

We know that the magnitude of centripetal acceleration of a body in a uniform circular motion is directly proportional to the square of velocity and inversely proportional to the radius of the path. Newton's Second Law of Motion:

$$F = ma$$

$$\Rightarrow F_c = mv^2/r$$

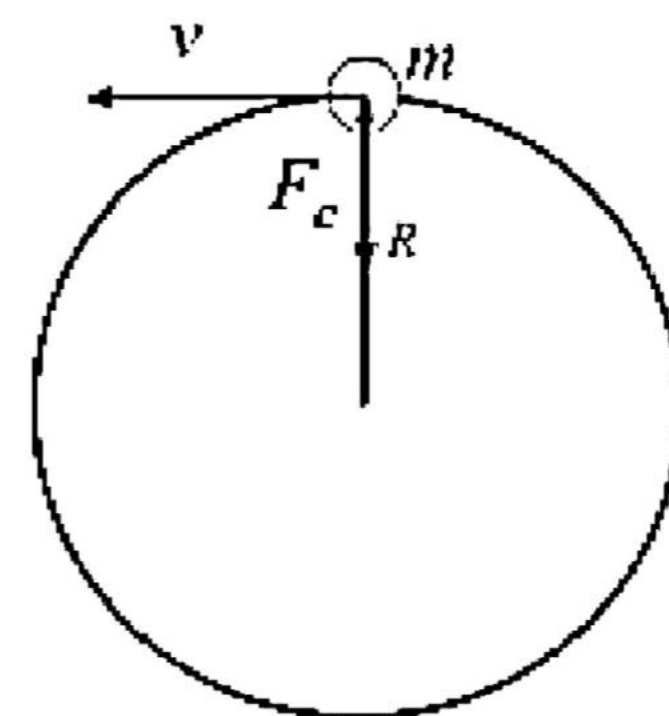
Where,

F_c = Centripetal Force

m = Mass of object

v = Velocity of object

r = Radius of the curved path



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1. In the absence of air friction projectile has maximum range when fired at an angle.

- (a) 30° with the horizontal
- (b) 45° with the vertical
- (c) 30° with the vertical
- (d) 60° with the horizontal

2. A horizontal range attained by a projectile can be found by the formula:

- (a) $\frac{v_o \sin \theta}{g}$
- (b) $\frac{2v_o \sin \theta}{g}$
- (c) $\frac{v_o^2 \sin \theta}{g}$
- (d) $\frac{v_o^2 \sin 2\theta}{g}$

3. During the projectile motion, the horizontal component of velocity:

- (a) Change with time
- (b) Becomes zero

- (c) Does not change but remains constant.
- (d) Increases with time

4. The maximum height of a projectile is directly proportional to.

- (a) The initial velocity
- (b) Launch angle
- (c) Square of the initial velocity
- (d) None of these

5. A body is moving in a circle at a constant speed which of the following statements about the body is true?

- (a) There is no acceleration.
- (b) There is no force acting on it
- (c) There is force acting at a tangent to the circle
- (d) There is force acting towards the centre of

the circle

6. The acceleration in uniform circular motion.

- (a) Varies inversely with the velocity of the particle.
- (b) Varies inversely with the radius of the orbit.
- (c) Varies directly with the square of the velocity.
- (d) both (b) and (c)



7. If a body is rotating in a circle with variable linear speed, it must have:

- (a) Only centripetal acceleration.
- (b) Only tangential acceleration
- (c) Both centripetal and tangent acceleration
- (d) None of these

8. The direction of angular velocity can be find out by

- (a) Left hand rule
- (b) Angular displacement
- (c) Direction of movement
- (d) Right hand rule

9. If a particle moves in a circle describing equal angles in equal intervals, then

- (a) Angular velocity change and linear velocity constant.
- (b) Angular velocity constant and linear velocity constant
- (c) Angular velocity constant and linear velocity changes.
- (d) None of these

10. The rate of change of angular displacement with time is called:

- (a) Angular acceleration.
- (b) Linear velocity
- (c) Angular velocity
- (d) None of these

11. The centripetal acceleration produced in a rotating body is commonly due to the change in _____ of the velocity:

- (a) Magnitude
- (b) Direction

- (c) Value
- (d) None of these

12. An object is launched in an arbitrary direction in space with a certain initial velocity and of moves freely under gravity. Its path will be a.

- (a) Straight line
- (b) circle
- (c) parabola
- (d) hyperbola

13. The velocity component with which a projectile covers certain vertical distance is minimum at the moment of:

- (a) Projection
- (b) Hitting the ground
- (c) Highest point
- (d) None of these

14. A body, moving along the circumference of a circle, completes two revolutions. If the radius of circle is R, the ratio of displacement to the covered path will be :

- (a) zero
- (b) πR
- (c) $2\pi R$
- (d) $4\pi R$

15. The angle between centripetal acceleration and tangential acceleration in circular motion is:

- (a) 180°
- (b) 0°
- (c) 90°
- (d) 45°

16. A projectile has its speed maximum at the moment of:

- (a) Projection
- (b) Hitting the ground
- (c) Both of these
- (d) None of these

17. The horizontal range of a projectile depend upon:

- (a) The angle of projection
- (b) The velocity of projection
- (c) Both of these
- (d) None of these

18. If a projectile is projected at an angle of 30° , it hits certain target. It will have the same range if it is projected at an angle of :

- (a) 45°
- (b) 55°
- (c) 90°
- (d) 60°

19. The linear and angular velocity of a particle, moving about the centre of a circle of radius r , are related by :

- (a) $v = \omega / r$ (b) $v = r \times \omega$
 (c) $\omega = v \times r$ (d) $\omega = r \times v$

20. A ball is thrown at 40 m/s with the angle of projection of 30° with the horizontal, the vertical velocity, of the projectile after 1 sec:

- (a) 20 m/s (b) 15 m/s
 (c) 10 m/s (d) Zero

21. A car moving at a constant speed of 20 ms⁻¹ on a circular path of radius 100m what is the acceleration?

- (a) 0.4 m/s² (b) 6 m/s²
 (c) 4.0 m/s² (d) 33 m/s²

22. The missile is fired at 20 m/s at 60° with respect to the horizontal, the horizontal and vertical component of the velocity at the maximum height is respectively:

- (a) 10 m/s, 10 m/s (b) 10 m/s, 5 m/s
 (c) 10 m/s, 0 (d) 0, 10 m/s

23. The cyclist cycling around a circular racing track skids because:

- (a) the centripetal force upon him is less than the limiting friction
 (b) the centripetal force upon him is greater than the limiting friction
 (c) the centripetal force upon him is equal to the limiting friction

(d) none of them

24. When the angular velocity of a disk increases, angular acceleration and angular velocity are:

- (a) parallel (b) non parallel
 (c) perpendicular (d) none of these

25. A 100 kg body is rotating in circular path of radius 200m, at 50 m/sec. find the centripetal force acting on the body:

- (a) 225 N (b) 1250 N
 (c) 525 N (d) 500 N

26. If a body covers 5 rotations in 2 seconds, around a path of radius 2m the linear velocity of body is:

- (a) π m/s (b) 10π m/s
 (c) 5π m/s (d) 20π m/s

27. The angular speed of an hour's hand of a watch in radian / minute is:

- (a) $\pi/6$ (b) $\pi/30$
 (c) $\pi/180$ (d) $\pi/360$

28. An angle subtended at its centre by an arc whose length is double to that of its radius is:

- (a) 2° (b) 57.3°
 (c) 80° (d) 114.6°

29. The unit of angular velocity is:

- (a) radian/cm (b) metre/sec
 (c) radian/sec (d) radian/sect

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7. The angle between centripetal and tangential acceleration in circular motion is:

*180° * zero * 90° *45°

23. The acceleration of projectile at the top of trajectory is:

* g *1/2 g * zero * 2g

2021

(vi) A car is travelling at a constant speed of 20 m/s round a curve of radius 100m. Its acceleration is:

*2m/s² * 3m/s² * 4m/s² *Zero m/s²

(xviii) During the projectile motion the acceleration of the projectile along the horizontal direction will:

*Decrease * Increase *be zero * remain constant

(xx) One radian is equal to:

*0.017° * 57.3° * 35.7° *0.117°

2019

7. An angle subtended at its centre by an arc whose length is double to that of its radius is:

*2° *57.3° *80° *114.6°

13. A projectile is thrown upward with a certain velocity. Its time of flight will be minimum, if it is launched at an angle of:

*30° *45° *60° *75°

2018

7. A body, moving along the circumference of a circle, completes two revolutions. If the radius of circle is R, the ratio of displacement to the covered path will be :

* zero * πR * 2 πR * 4πR

2017

2. The angular speed of the minute hand of a clock is:

* $\frac{\pi}{30}$ * $\frac{\pi}{60}$ * $\frac{\pi}{1800}$ * $\frac{\pi}{3600}$

3. A projectile is fired at an angle θ with the horizontal will be minimum at:

*the highest point *the point of projection
*all points of its path *the point of landing on ground

2016

11. The angle between centripetal acceleration and tangential acceleration in circular motion is:

*180° *0° *90° *45°

13. One radian is equal to:

*1° *75.3° *57.3° *0.017°

2015

7. The unit of angular velocity is:

*radian/cm * metre/sec * radian/sec * radian/sect

13. The angle between centripetal and tangential acceleration in circular motion is:

- *180° * zero * 90° *45°

2014

9. An angle subtended at its centre by an arc whose length is double to that of its radius is:

- *84.3° *57.3° *114.6° *168.6°

2012

3. If the axis of rotation of a rotating body passes through the body itself, then its motion is called:

- *linear motion * orbital motion
* spin motion * S.H.M

2011

6. When the angular velocity of a disk increases, angular acceleration and angular velocity are:

- *parallel *non parallel *perpendicular *none of these

2010

7. The cyclist cycling around a circular racing track skids because:

- *the centripetal force upon him is less than the limiting friction
* the centripetal force upon him is greater than the limiting friction
* the centripetal force upon him is equal to the limiting friction
*none of them

17. The horizontal range of a projectile depends upon:

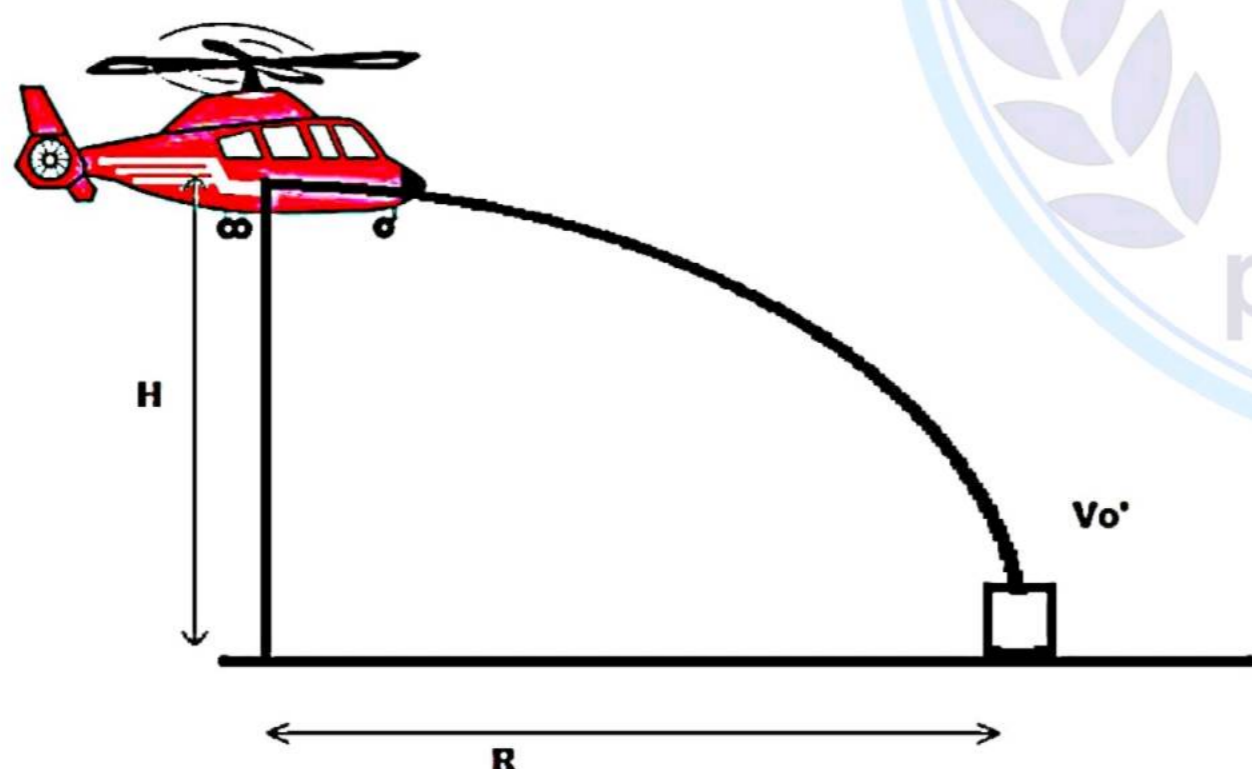
- * the angle of projection * 'g' at the place
* the velocity of projection * all of them



TEXTBOOK NUMERICALS

Q.1: A rescue helicopter drops a package of emergency ration to a stranded party on the ground. If the helicopter is traveling horizontally at 40 m/s at a height of 100 m above the ground, (a) where does the package strike the ground relative to the point at which it was released? (b) What are the horizontal and vertical component of the velocity of the package just before it hits the ground?

Data:



Horizontal velocity component = $v_{0x} = 40 \text{ m/s}$

Height of helicopter = $H = 100 \text{ m}$

(a) Range of projectile = $R = ?$

(b) X-component of Final Velocity = $v_{ox}' = ?$

Y-component of Final Velocity = $v_{oy}' = ?$

Solution:

(a) As we know that

$$s = v \times t$$

so,

$$R = v_{ox} \times T \text{ ---(i)}$$

For time (T):

Using Second equation of motion

$$s = v_i t + \frac{1}{2} a t^2$$

$$H = v_{oy} \times T + \frac{1}{2} (9.8) \times T^2$$

$$-100 = 0 \times T + \frac{1}{2} (-9.8) \times T^2$$

$$100 = 4.9 \times T^2$$

$$T^2 = 20.4$$

Or $T = 4.5 \text{ sec}$

Putting value in eq(i)

$$R = 40 \times 4.5$$

$$R = 180 \text{ m}$$

Now,

(b) Final Velocities:

Q.2: A long-jumper leaves the ground at an angle of 20° to the horizontal and at a speed of 11 m/s (a) How far does he jump? What is the maximum height reached? Assume the motion of the long jumper is that of projectile.

Data:Launch angle $= \theta = 20^\circ$ Initial Speed $= v_o = 11 \text{ m/s}$ (a) Range of long jumper $= R = ?$ (b) Maximum Height attained $= H_{max} = ?$ **Solution:**

(a) Range of projectile is given by

$$R = \frac{v_o^2 \sin 2\theta}{g}$$

$$R = \frac{(11)^2 \times \sin 2(20)}{9.8}$$

$$R = \frac{121 \times \sin (40)}{9.8}$$

$$R = 7.93 \text{ m}$$

(b) The Maximum height attained by projectile is given by

$$H_{max} = \frac{v_o^2 \sin^2 \theta}{2g}$$

$$H_{max} = \frac{(11)^2 \times (\sin 20)^2}{2 \times 9.8}$$

$$H_{max} = \frac{121 \times 0.116}{19.6}$$

$$H_{max} = 0.72 \text{ m}$$

Result: He jumped 7.93 m far and maximum height reached is 0.72 m .

Q.3: A stone is thrown upward from the top of a building at an angle of 30° to the horizontal and with an initial speed of 20 m/s . If the height of building is 45 m . (a) Calculate the total time the stone in flight (b) What is the speed of stone just before it strikes the ground? (c) Where does the stone strike the ground?

Data:Launch angle $= \theta = 30^\circ$

As we know that

$$v'_{ox} = v_{ox} = 40 \text{ m/s}$$

and

$$v_f = v_i + at$$

$$v'_{oy} = v_{oy} + gt$$

$$v'_{oy} = 0 - 9.8 \times 4.5$$

$$v'_{oy} = -44.1 \text{ m/s}$$

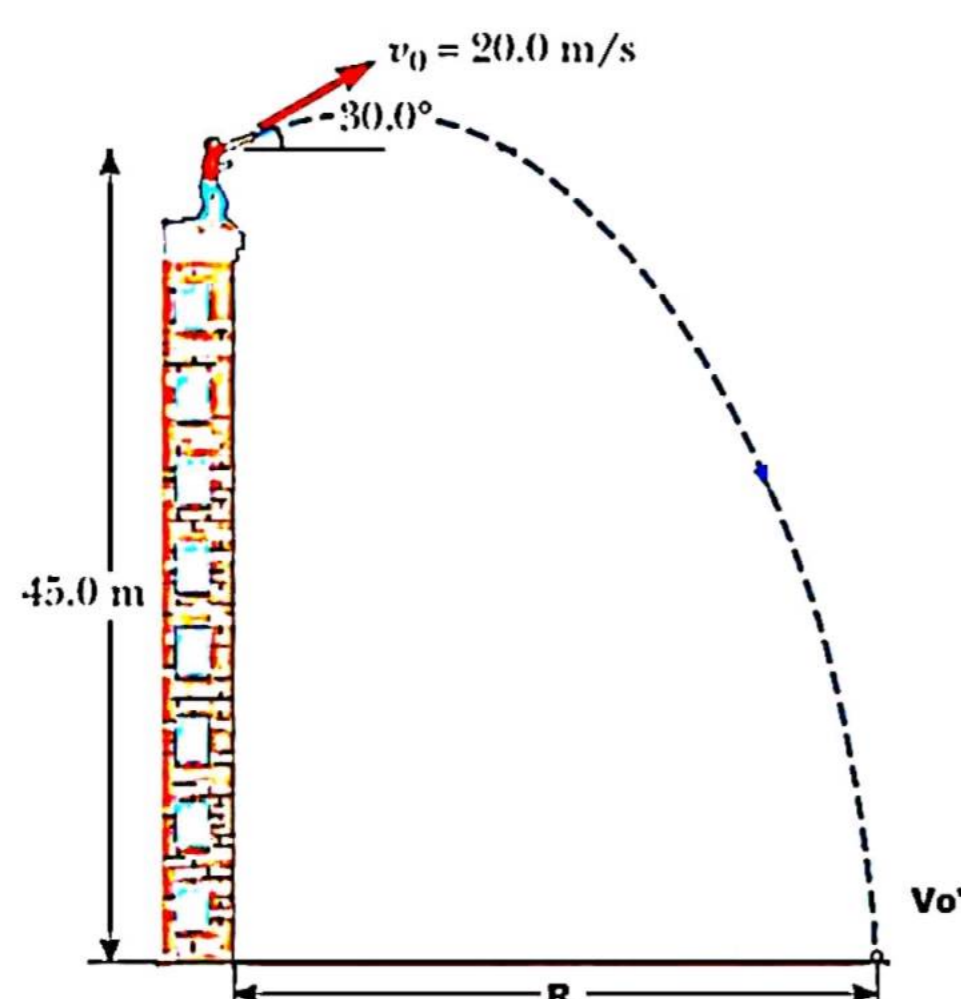
Result: The package strike the ground at 180 m relative to the point at which it was released and x component of final velocity is 40 m/s and y component is 44.1 m/s (downwards).



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- (a) Time of flight = $T' = ?$
 (b) Final speed = $v_o' = ?$
 (c) Range of stone = $R = ?$

**Solution:**

- (a) First we calculate time to reach maximum height

$$T_1 = \frac{v_o \sin \theta}{g}$$

$$T_1 = \frac{20 \times \sin 30}{9.8}$$

$$\boxed{T_1 = 1.02 \text{ s}}$$

Now, Height reached during this time

$$H_2 = \frac{v_o^2 \sin^2 \theta}{2g}$$

$$H_2 = \frac{(20)^2 \times (\sin 30)^2}{2 \times 9.8}$$

$$H_2 = \frac{400 \times 0.25}{19.6}$$

$$\boxed{H_2 = 5.1 \text{ m}}$$

Now, total height is given by

$$\boxed{H = H_1 + H_2 = 45 + 5.1 = 50.1 \text{ m}}$$

Now, using this height to find time for downward flight

$$s = v_i t + \frac{1}{2} a t^2$$

$$H = v_i T_2 + \frac{1}{2} a T_2^2$$

$$50.1 = 0 \times T_2 + \frac{1}{2} (9.8) (T_2^2)$$

$$50.1 = 4.9 (T_2^2)$$

$$T_2^2 = 10.22$$

Taking Square root O.B.S

$$\boxed{T_2 = 3.19 \text{ s}}$$

Now, Total Time is given by

$$\boxed{T = T_1 + T_2 = 1.02 + 3.19 = 4.21 \text{ s}}$$

(b) Final Velocities:

As we know that

$$v_{ox}' = v_{ox} = v_o \cos \theta$$

$$v_{ox}' = 20 \times \cos 30$$

$$\boxed{v_{ox}' = 17.3 \text{ m/s}}$$

and

$$v_f = v_i + at$$

$$v_{oy}' = v_{oy} + gT_2$$

$$v_{oy}' = 0 + 9.8 \times 3.19$$

$$\boxed{v_{oy}' = 31.26 \text{ m/s}}$$

Net Final velocity:

$$v_o' = \sqrt{(v_{ox}')^2 + (v_{oy}')^2}$$

$$v_o' = \sqrt{(17.3)^2 + (31.26)^2}$$

$$\boxed{v_o' = 35.7 \text{ m/s}}$$

(c) Range of stone:

$$R = v_{ox} \times T$$

$$R = 17.3 \times 4.21$$

$$\boxed{R = 73 \text{ m}}$$

Result: The total time the stone in flight is 4.21 s

(b) The speed of stone just before it strikes the ground is 35.7 m/s and the stone strikes the ground 73 m away.



Q.4: A ball is thrown in horizontal direction from a height of 10 m with a velocity of 21 m/s (a) How far will it hit the ground from its initial position on the ground? and with what velocity?

Data:

Initial Speed = $v_{ox} = 21 \text{ m/s}$

Height = $H = 10 \text{ m}$

(a) Range of stone = $R = ?$

(b) Final speed = $v_o' = ?$

Solution:

(a) The Range of ball is given by

$$R = v_{ox} \times T \text{ ---- (i)}$$

For time (T):

Using Second equation of motion

$$s = v_i t + \frac{1}{2} a t^2$$

$$H = v_{oy} \times T + \frac{1}{2}(9.8) \times T^2$$

$$-10 = 0 \times T + \frac{1}{2}(-9.8) \times T^2$$

$$10 = 4.9 \times T^2$$

$$T^2 = 2.04$$

Or $T = 1.42 \text{ sec}$

Putting value in eq(i)

$$R = 21 \times 1.42$$

$$R = 30 \text{ m}$$

(b) **Final Velocities:**

As we know that

$$v'_{ox} = v_{ox} = 21 \text{ m/s}$$

and

$$v_f = v_i + at$$

$$v'_{oy} = v_{oy} + gt$$

$$v'_{oy} = 0 - 9.8 \times 1.42$$

$$v'_{oy} = -13.9 \text{ m/s}$$

Net Final velocity:

$$v'_o = \sqrt{(v'_{ox})^2 + (v'_{oy})^2}$$

$$v'_o = \sqrt{(21)^2 + (13.9)^2}$$

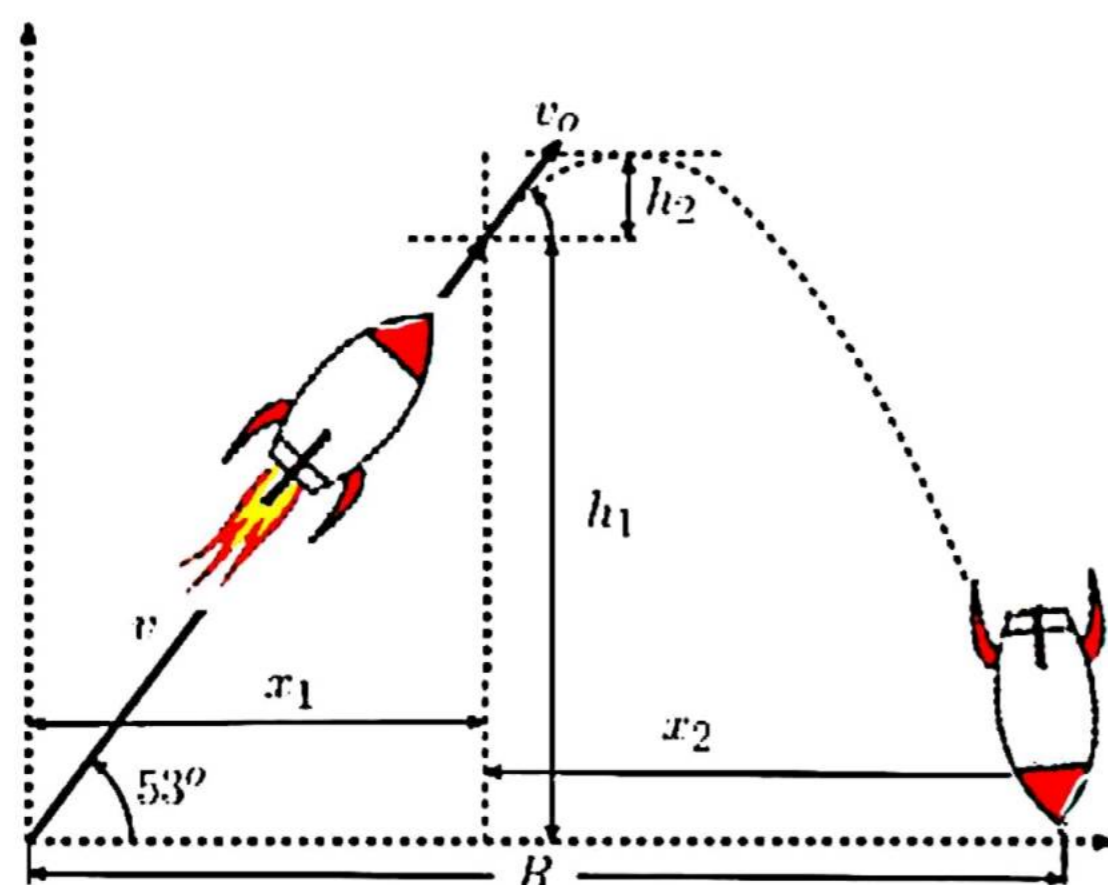
$$v'_o = 25.2 \text{ m/s}$$

Result: It will hit the ground 30m from its initial position on the ground and with velocity 25.2 m/s.



Q.5: A rocket is launched at an angle of 53° to the horizontal with an initial speed of 100 m/s. It moves along its initial line of motion with an acceleration of 30 m/s^2 for 3s. At this time the engine falls and the rocket proceeds to move as a free body. Find (a) the maximum altitude reached by the rocket (b) its total time of flight, and (c) its horizontal range.

Data:



Launch angle $= \theta = 53^\circ$

Initial Speed $= v_i = 100 \text{ m}$

Acceleration $= a = 30 \text{ m/s}^2$

Time $= t = 3 \text{ s}$

(a) Maximum altitude reached by the rocket $= H = ?$

(b) Time of flight $= T' = ?$

(c) Horizontal range $= R = ?$

Solution:

During Constant acceleration phase:

$$S = v_i t + \frac{1}{2} a t^2$$

$$S = 100 \times 3 + \frac{1}{2}(30) \times (3)^2$$

$$S = 300 + 135$$

$$S = 435 \text{ m}$$

i) Horizontal Distance(x_1):

$$x_1 = S \cos \theta$$

$$x_1 = 435 \times \cos 53^\circ$$

$$x_1 = 261.7 \text{ m}$$

ii) Vertical Distance(h_1):

$$h_1 = S \times \sin \theta$$

$$h_1 = 435 \times \sin 53^\circ$$

$$h_1 = 347.4 \text{ m}$$

Now at the time of failure of engine

$$v_f = v_i + at$$

$$v_f = 100 + 30 \times 3$$

$$v_f = 190 \text{ m/s}$$

Now, the rocket becomes a projectile and its initial velocity is 190 m/s

$$v_o = v_f = 190 \frac{\text{m}}{\text{s}}$$

During Projectile Motion phase:

Maximum height reached is given by

$$h_2 = \frac{v_o^2 \sin^2 \theta}{2g}$$

$$h_2 = \frac{(190)^2 \times (\sin 53)^2}{2 \times 9.8}$$

$$h_2 = \frac{36100 \times 0.63}{19.6}$$

$$h_2 = 1178.77 \text{ m}$$

Time to reach maximum height

$$t_1 = \frac{v_o \sin \theta}{g}$$

$$t_1 = \frac{190 \times \sin 53}{9.8}$$

$$t_1 = 15.4 \text{ s}$$

Now, Total height is given by

$$H = h_1 + h_2 = 347.4 + 1178.77 = 1526.17 \text{ m}$$

Now, Time for Downward Motion:

$$S = v_i t + \frac{1}{2} a t^2$$

$$H = 0 \times t_2 + \frac{1}{2} (9.8) \times (t_2)^2$$

$$1526.17 = 4.9 t_2^2$$

$$t_2^2 = 311.46$$

Taking Square root O.B.S

$$t_2 = 17.6 \text{ s}$$

Total Time of Flight is

$$T' = t + t_1 + t_2 = 3 + 15.4 + 17.6$$

$$T' = 36 \text{ s}$$

To calculate x_2 we have to find total time of projectile phase.

$$T = t_1 + t_2 = 15.4 + 17.6 = 33 \text{ s}$$

Now, range is given by

$$R = v_{ox} \times T$$

or

$$x_2 = v_o \cos \theta \times T$$

$$x_2 = 190 \times \cos 53 \times 33 = 3773.3 \text{ m}$$

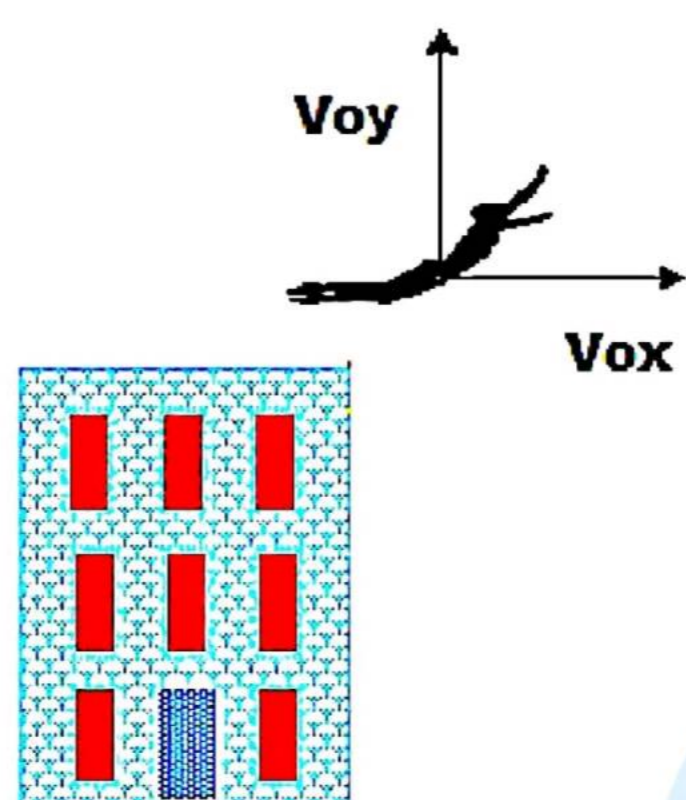
Now, Total Range is

$$R = x_1 + x_2 = 261.7 + 3773.3 = 4035 \text{ m}$$

Result: The maximum altitude reached by the rocket is 1526.17 m (b) its total time of flight is 36 s and (c) its horizontal range is 4035 m.

Q.6: A diver leaps from a tower with an initial horizontal velocity component of 7 m/s and upward velocity component of 3 m/s. find the component of her position and velocity after 1 second.

Data:



Initial Horizontal velocity = $v_{ox} = 7 \text{ m/s}$

Initial Vertical velocity = $v_{oy} = 3 \text{ m/s}$

Time = $t = 1 \text{ s}$

Final Horizontal velocity = $v'_{ox} = ?$

x component of position = $x = ?$

Final Vertical velocity = $v'_{oy} = ?$

y component of position = $y = ?$

Solution:

Since the x component of velocity remains constant

$$v'_{ox} = v_{ox} = 7 \text{ m/s}$$

Now her position

$$x = v_{ox} \times t$$

$$x = 7 \times 1$$

$$x = 7 \text{ m}$$

The y component of velocity is given by

$$v_f = v_i + at$$

$$\text{or } v'_{oy} = v_{oy} + (-g)t$$

$$v'_{oy} = 3 - (9.8 \times 1)$$

$$v'_{oy} = 3 - (9.8)$$

$$v'_{oy} = -6.8 \text{ m/s}$$

Now her position, using second equation of motion

$$s = v_i t + \frac{1}{2} a t^2$$

$$y = 3 \times 1 + \frac{1}{2} (-9.8)(1)^2$$

$$y = -1.9 \text{ m}$$

Result:

(i) The final x component of velocity will be 7 m/s and y component will be 6.8 m/s(downward)

(ii) Her displacement in x axis is 7 m and

in y axis is 1.9 m(downward)

Q.7: A boy standing 10m from a building can just barely reach the roof 12m above him when he throws a ball at the optimum angle with respect to the ground. Find the initial velocity component of the ball.

Data:

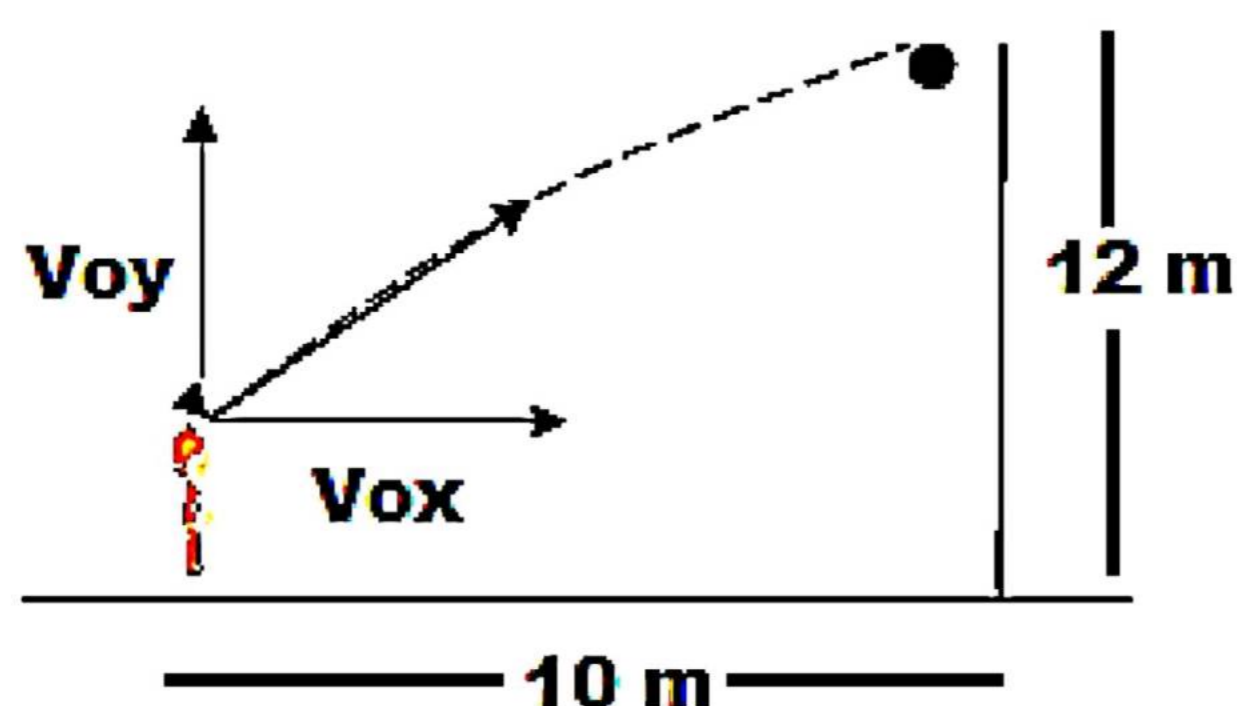
$$H_{max} = 12 \text{ m}$$

$$R = 10 \text{ m}$$

$$V_{0x} = ?$$

$$V_{0y} = ?$$

Solution:



As we know that

$$H_{max} = \frac{v_{0y}^2}{2g}$$

$$v_{0y}^2 = H_{max} \times 2g$$

$$v_{0y}^2 = 10 \times 2 \times 9.8$$

$$v_{0y} = \sqrt{196}$$

$$v_{0y} = 14 \text{ m/s}$$

Now,

$$s = v \times t$$

$$\text{Or } R = v_{0x} \times T \text{ ----- (i)}$$

The time to reach maximum height is given by

$$T = \frac{v_{0y}}{g} = \frac{14}{9.8} = 1.42 \text{ sec}$$

Putting values in eq (i)

$$12 = v_{0x} \times 1.42$$

$$\text{Or } v_{0x} = 8.4 \text{ m/s}$$

Result: The horizontal component of velocity is 8.4 m/s and the vertical component of velocity is 14 m/s



Q.8: A mortar shell is fired at a ground level target 500m distance with an initial velocity of 90 m/s. What is its launch angle?

Data:

$$\text{Range} = R = 500 \text{ m}$$

$$\text{Initial Velocity} = V_0 = 90 \text{ m/s}$$

$$\text{Launch Angle} = \theta = ?$$

Solution:

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$500 = \frac{(90)^2 \sin 2\theta}{9.8}$$

$$\sin 2\theta = \frac{500 \times 9.8}{8100}$$

$$\sin 2\theta = 0.604$$

$$2\theta = \sin^{-1}(0.604)$$

$$2\theta = 37.1$$

Or

$$\theta = 18.5^\circ$$

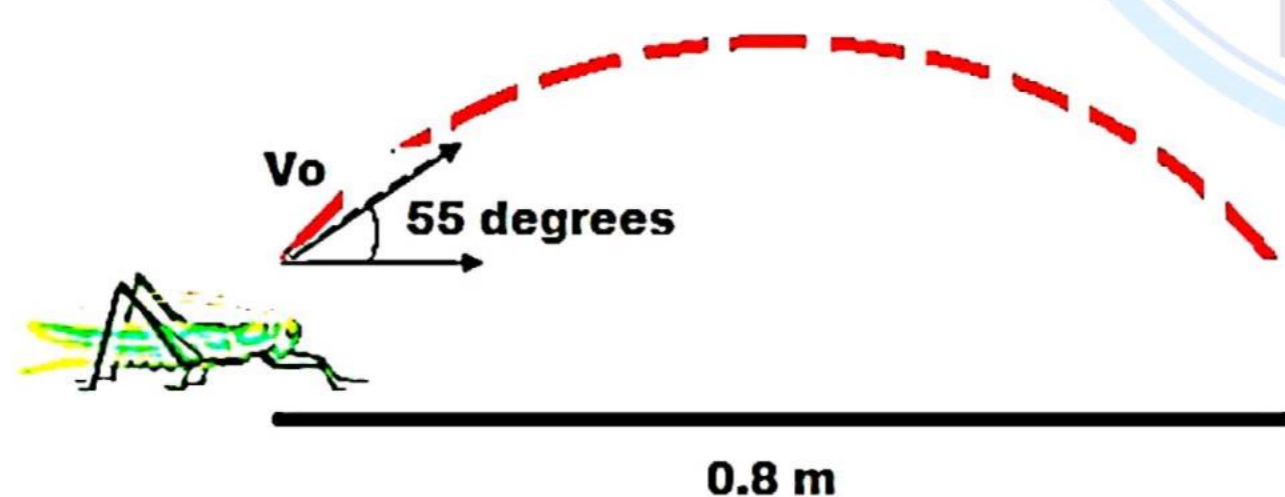
The larger angle is given by

$$\theta' = 90 - \theta = 90 - 18.5 = 71.4^\circ$$

Result: The two possible values of launch angles are 18.5° and 71.4° .

Q.9: What is the take off speed of a locust if its launch angle is 55° and its range is 0.8m?

Data:



$$\text{Range} = R = 0.8 \text{ m}$$

$$\text{Initial Velocity} = V_0 = ?$$

$$\text{Launch Angle} = \theta = 55^\circ$$

Solution:

The Range of Projectile is given by

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$0.8 = \frac{v_0^2 \sin 2(55)}{9.8}$$

$$7.84 = v_0^2 \times \sin 110^\circ$$

$$7.84 = v_0^2 \times 0.9396$$

$$v_0^2 = 8.34$$

Taking Square root on both sides

$$v_0 = 2.88 \text{ m/s}$$

Result: The take off speed of the locust is 2.88 m/s.



Q.10: A car is traveling on a flat circular track of radius 200m at 20 m/s and has a centripetal acceleration $a_c = 4.5 \text{ m/s}^2$ (a) If the mass of the car is 1000 kg, what frictional force is required to provide the acceleration? (b) if the coefficient of static frictions μ_s is 0.8, what is the maximum speed at which the car can circle the track?

Data:

Radius of Track = $R = 200 \text{ m}$

Centripetal Acceleration = $a_c = 4.5 \text{ m/s}^2$

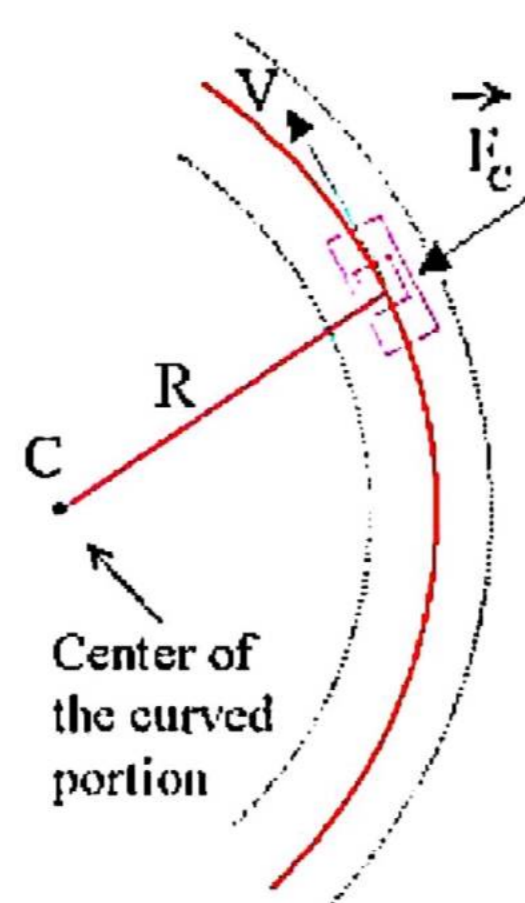
mass of Car = $m = 1000 \text{ kg}$

(a) Frictional Force = $f = ?$

(b) Co-efficient of friction = $\mu_s = 0.8$

Maximum speed of car = $v_{max} = ?$

Solution:

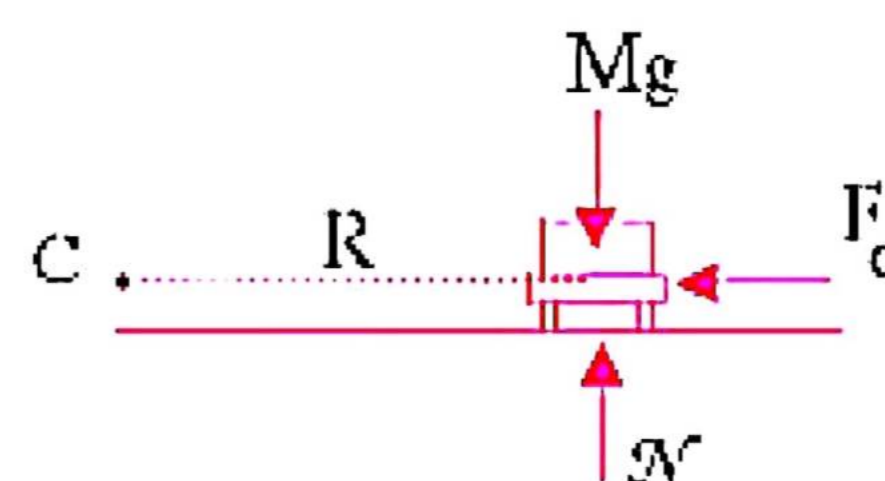


(a) As we know that Frictional force is equal to the centripetal force

$$f = F_c = ma_c$$

$$f = 1000 \times 4.5$$

$$f = 4500 \text{ N}$$



(b) According to the definition of co efficient of friction

$$\mu_s = \frac{F}{R} \text{ ---(i)}$$

In this Case

$$F = F_c = \frac{mv_{max}^2}{r}$$

and

$$R = W = mg$$

Putting values in eq(i)

$$\mu_s = \frac{\frac{mv_{max}^2}{r}}{mg}$$

$$\mu_s = \frac{v_{max}^2}{rg}$$

$$v_{max}^2 = \mu_s rg$$

$$v_{max}^2 = 0.8 \times 200 \times 9.8$$

$$v_{max}^2 = 1568$$

Taking Square root O.B.S.

$$v_{max} = 39.5 \text{ m/s}$$

Result: The frictional force required to provide the acceleration is 4500 N and the maximum speed at which the car can circle the track is 39.5 m/s.

Q.11: The turntable of a record player rotates initially at a rate of 33 rev/min and takes 20s to come to rest (a) What is the angular acceleration of the turntable, assuming the acceleration is constant? (b) How many rotation does the turntable make before coming to rest? (c) If the radius of the turntable is 0.14m, what is the initial linear speed of a bug riding on the rim?(d) What is the magnitude of the tangential acceleration

of the bug at time $t = 0$?

Data:

Initial angular velocity $= \omega_i = 33 \frac{\text{rev}}{\text{min}} = \frac{33 \times 2\pi}{60} = 3.45 \text{ rad/s}$

Time $= t = 20 \text{ s}$

Final angular velocity $= \omega_f = 0$

(a) Angular Acceleration $= \alpha = ?$

(b) No. of rotations $= \theta = ?$

(c) Radius of turntable $= r = 0.14 \text{ m}$

Initial Tangential velocity $= v_i = ?$

(d) Tangential Acceleration $= a = ?$

Solution:

(a) Using First Equation of Motion

$$\omega_f = \omega_i + \alpha t$$

$$0 = 3.45 + \alpha(20)$$

$$-3.45 = \alpha(20)$$

$$\alpha = -0.172 \text{ rad/s}^2$$

(b) Using Second Equation of Motion

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\theta = 3.45 \times 20 + \frac{1}{2} (-0.172) \times (20)^2$$

Q.12: Tarzan swings on a vine of length 4m in a vertical circle under the influence of gravity. When the vine makes an angle of $\theta = 20^\circ$ with the vertical, Tarzan has a speed of 5 m/s. Find (a) his centripetal acceleration at this instant, (b) his tangential acceleration, and (c) the resultant acceleration.

Data:

Length of vine $= r = 4 \text{ m}$

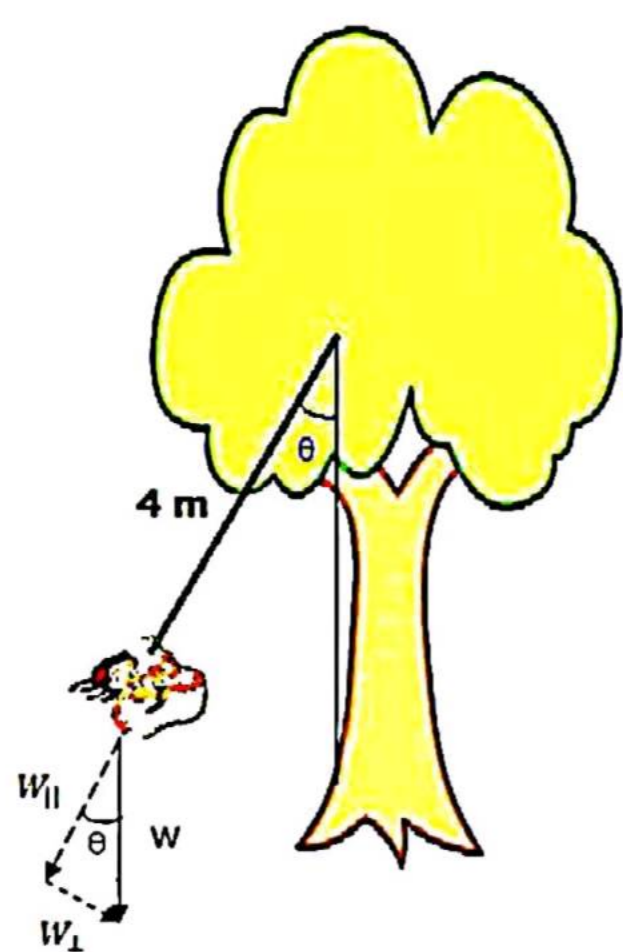
Angle with the vertical $= \theta = 20^\circ$

Speed of Tarzan $= v = 5 \text{ m/s}$

Centripetal Acceleration $= a_c = ?$

Tangential Acceleration $= a = ?$

Resultant Acceleration $= a_r = ?$



Solution:

$$\theta = 69 - 34.4$$

$$\theta = 34.6 \text{ radians}$$

In Rotations:

$$\theta = \frac{34.6}{2\pi} = \frac{34.6}{6.28}$$

$$\theta = 5.5 \text{ rotations}$$

(c) As we know that

$$v_i = r\omega_i$$

$$v_i = 0.14 \times 3.45$$

$$v_i = 0.483 \text{ m/s}$$

(d) As we know that

$$a = r\alpha$$

$$a = 0.14 \times (-0.172)$$

$$a = -0.024 \text{ m/s}^2$$

Result: (a) The angular acceleration of the turntable is -0.172 rad/s^2 (b) 5.5 rotations the turntable makes before coming to rest (c) The initial linear speed of a bug riding on the rim is 0.783 m/s (d) The tangential acceleration of the bug is -0.024 m/s^2

The centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(5)^2}{4}$$

$$a_c = 6.25 \text{ m/s}^2$$

The net force on Tarzan

$$F = W_{\perp}$$

$$F = W \sin \theta$$

$$ma = mg \sin \theta$$

$$a = g \times \sin \theta = 9.8 \times \sin 20^\circ$$

$$a = 3.35 \text{ m/s}^2$$

Now,

$$a_r = \sqrt{a^2 + a_c^2}$$

$$a_r = \sqrt{(3.35)^2 + (6.25)^2}$$

$$a_r = 7.09 \text{ m/s}^2$$

Result: The centripetal acceleration is 3.2 m/s^2 ,

tangential acceleration is 4.9 m/s^2 and resultant

acceleration is 7.09 m/s^2



PAST PAPER NUMERICALS

2022

vi) What is the ratio of maximum range and maximum height of a projectile for an angle at which range is maximum.

Solution:

The maximum range is given by

$$R_{max} = \frac{v_0^2}{g} \text{ -----(i)}$$

Maximum height is given by

$$H_{max} = \frac{v_0^2 \sin^2 \theta}{2g}$$

For maximum range $\theta = 45^\circ$

$$H_{max} = \frac{v_0^2 (\sin 45) ^2}{2g}$$

$$H_{max} = \frac{v_0^2}{4g} \text{ -----(ii)}$$

Dividing eq(i) by eq(ii)

$$\frac{R_{max}}{H_{max}} = \frac{v_0^2}{g} \div \frac{v_0^2}{4g}$$

$$\frac{R_{max}}{H_{max}} = \frac{v_0^2}{g} \times \frac{4g}{v_0^2}$$

$$\boxed{\frac{R_{max}}{H_{max}} = 4}$$

The ratio of maximum range and maximum height of a projectile for an angle at which range is maximum is 4:1

2019

No Numerical

2018

2(ii) Calculate the angle of projection for which the maximum height of projectile is equal to 1/3 of its horizontal range.

Data:

Angle of projection = $\theta = ?$

$$H_{max} = \frac{1}{3}R$$

Solution:

According to the given condition

$$H_{max} = \frac{1}{3}R$$

$$\frac{v_0^2 \sin^2 \theta}{2g} = \frac{1}{3} \left(\frac{v_0^2 \sin 2\theta}{g} \right)$$

$$\frac{\sin^2 \theta}{2} = \left(\frac{\sin 2\theta}{3} \right)$$

$$\because \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\therefore \frac{\sin^2 \theta}{2} = \left(\frac{2 \sin \theta \cos \theta}{3} \right)$$

$$\frac{\sin \theta}{\cos \theta} = \left(\frac{4}{3} \right)$$

$$\tan \theta = \left(\frac{4}{3} \right)$$

$$\theta = \tan^{-1} \left(\frac{4}{3} \right)$$

$$\boxed{\theta = 53.1^\circ}$$

Result: The angle of projection for which the maximum height of projectile is equal to 1/3 of its horizontal range is 53.1° .

2017

0.2 (viii)

Textbook Numerical 7

2016

Q.2 (iii) Tarzan swings on a vine, of length 5m, in a vertical circle, under the influence of gravity. When the vine makes an angle of 30° with the Vertical, Tarzan has a speed of 4 m/s. Find:

(a) Centripetal acceleration at this instant (b) His tangential acceleration

Data:

Length of vine = $r = 5\text{m}$

Angle with the vertical = $\theta = 30^\circ$

Speed of Tarzan = $v = 4\text{ m/s}$

Centripetal Acceleration = $a_c = ?$

Tangential Acceleration = $a = ?$

Solution:

The centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(4)^2}{5}$$

$$a_c = 3.2\text{ m/s}^2$$

The net force on Tarzan

$$F = W_\perp$$

$$F = W \sin \theta$$

$$ma = mg \sin \theta$$

$$a = g \times \sin \theta = 9.8 \times \sin 30$$

$$a = 4.9\text{ m/s}^2$$

Result: The centripetal acceleration is 3.2 m/s^2 and tangential acceleration is 4.9 m/s^2 .



2015

Q.2 vii) A boy whose mass is 100 kg when resting on the ground at the equator if the radius of earth 'Re' is $6.4 \times 10^6\text{ m}$. Calculate the centripetal acceleration and centripetal force.

Data:

Mass of boy = $m = 100\text{ kg}$

Radius of Earth = $R_e = 6.4 \times 10^6\text{ m}$

Centripetal Acceleration = $a_c = ?$

Centripetal Force = $F_c = ?$

Solution:

The centripetal Acceleration is given by

$$a_c = R\omega^2$$

And

$$\omega = \frac{2\pi}{T}$$

So,

$$a_c = R \left(\frac{2\pi}{T} \right)^2 = \frac{6.4 \times 10^6 \times 4 \times 3.14^2}{(86400)^2}$$

$$a_c = 0.034\text{ m/s}^2$$

Now, Centripetal force is given by

$$F_c = ma_c = 100 \times 0.034 = 3.4\text{ N}$$

Result: the centripetal acceleration is 0.034 m/s^2 and centripetal force is 3.4 N .

2014

Q.2 (ix) A mortar shell is fired at a ground level target of 400 m distance with an initial velocity 85m/sec. Calculate the maximum time to hit the target.

Data:

Range = $R = 400\text{ m}$

Initial Velocity = $V_0 = 85\text{ m/s}$

Max. Time to Hit = $T = ?$

Solution:

First we will find the angle of projection

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$400 = \frac{(85)^2 \sin 2\theta}{9.8}$$

$$\sin 2\theta = \frac{400 \times 9.8}{7225}$$

$$\sin 2\theta = 0.542$$

$$2\theta = \sin^{-1}(0.542)$$

$$2\theta = 32.8^\circ$$

Or

$$\theta = 16.4^\circ$$

The larger angle is given by

$$\theta' = 90 - \theta = 90 - 16.4 = 73.5^\circ$$

For maximum time we use greater angle and find time of flight

Q.2 (xii) Calculate the centripetal acceleration and centripetal force on a man whose mass is 80 kg when resting on the ground at the equator if the radius of earth is 6.4×10^6 metres.

Data:

Mass of man = $m = 80$ kg

Radius of Earth = $R_e = 6.4 \times 10^6$ m

Centripetal Acceleration = $a_c = ?$

Centripetal Force = $F_c = ?$

Solution:

The centripetal Acceleration is given by

$$a_c = R\omega^2$$

And

$$\omega = \frac{2\pi}{T}$$

$$T = 2 \frac{(85)\sin(73.5)}{9.8}$$

$$T = 16.6 \text{ sec}$$

Result: The maximum time to hit the target is 16.6 sec.

So,

$$a_c = R \left(\frac{2\pi}{T} \right)^2 = \frac{6.4 \times 10^6 \times 4 \times 3.14^2}{(86400)^2}$$

$$a_c = 0.034 \text{ m/s}^2$$

Now, Centripetal force is given by

$$F_c = ma_c = 80 \times 0.034 = 2.72 \text{ N}$$

Result: the centripetal acceleration is 0.034 m/s^2 and centripetal force is 2.72 N .

2013

Q.2 (viii) Textbook Numerical 12

2012

Q.2 (vi) A diver leaps from a tower with an initial horizontal velocity component of 7m/sec and upward velocity component of 5m/sec. Find the components of his velocity along x and y axis after 1.5 sec.

Data:

Initial Horizontal velocity = $v_{ox} = 7 \text{ m/s}$

Initial Vertical velocity = $v_{oy} = 5 \text{ m/s}$

Time = $t = 1.5 \text{ s}$

Final Horizontal velocity = $v'_{ox} = ?$

Final Vertical velocity = $v'_{oy} = ?$

Solution:

Since the x component of velocity remains constant

$$v'_{ox} = v_{ox} = 7 \text{ m/s}$$

The y component of velocity is given by

$$v_f = v_i + at$$

$$\text{or } v'_{oy} = v_{oy} + (-g)t$$

$$v'_{oy} = 5 - (9.8 \times 1.5)$$

$$v'_{oy} = 5 - (14.7)$$

$$v'_{oy} = -9.7 \text{ m/s}$$

Result: The final x component will be 7 m/s and y component will be 9.7 m/s(downward)

Q.2 (viii) Same as 2018 Q.2(ii)

2011

Q.2 (ix) Same as 2014 Q.2(xii)

2010

No Numerical