

"Matrices and Determination"

Definitions

Matrix:-

A rectangular array (arrangement) of numbers enclosed by a pair of brackets such as $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 4 & 7 \end{bmatrix}$ is called a **matrix**.

Rows:-

The horizontal lines of numbers are called **rows**.

Columns:-

The vertical lines of numbers are called **columns**.

Entries or Elements:-

The numbers used in rows or columns are said to be the **entries or elements** of the matrix.

Row Matrix or Row vectors-

A matrix, which has only one row, i.e., a $1 \times n$ matrix of the form $[a_{i1} \ a_{i2} \ a_{i3} \ \dots \ a_{in}]$ is

said to be a **row matrix** or **row vector**.

Column Matrix or Column Vector:-

A matrix which has only one column i.e., an $m \times 1$ of the form $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$ is said to be a **column matrix** or **column vector**.

Rectangular Matrix:-

If $m \neq n$, then the matrix is called **rectangular matrix** of order $m \times n$, that is, the matrix in which the number of rows is not equal to the number of columns, is said to be a **rectangular matrix**.

For examples-

$$\begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -3 & 0 \\ 1 & 2 & 4 \\ 3 & -1 & 5 \\ 0 & 1 & 2 \end{bmatrix}$$

Square Matrix:-

If $m = n$, then the matrix

of order $m \times n$ is said to be a **square matrix** of m or n .

i.e., the matrix which has the same number of rows and columns is called a **square matrix**.

For example:-

$$[0], \begin{bmatrix} 2 & 5 \\ -1 & 6 \end{bmatrix} \text{ etc.}$$

Diagonal Matrix:-

Let $A = [a_{ij}]$ be a square matrix of order n . If $a_{ij} = 0$ for all $i \neq j$ and at least one $a_{ij} \neq 0$ for $i = j$, that is, some elements of the principal diagonal of A may be zero, but not all, then the matrix A is called **diagonal matrix**.

For examples:-

$$[7], \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Square Matrix-

Let $A = [a_{ij}]$ be a square matrix of order n .

If $a_{ij} = 0$ for all $i \neq j$ and $a_{ij} = k$ (some non-zero scalar) for all $i = j$, then the matrix A is called a **square matrix** of order n .

For examples:-

$$\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}, \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \text{ etc}$$

Unit Matrix or Identity Matrix-

Let $A = [a_{ij}]$ be a square matrix of order n . If $a_{ij} = 0$ for all $i \neq j$ and $a_{ij} = 1$ for all $i = j$, then the matrix A is called a **unit matrix** or

identity matrix of order n . We denote such a matrix by I_n and it is of the form.

For example:-

The identity matrix of order

3 is denoted by I_3 , $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Null Matrix or Zero Matrix:-

A square or rectangular matrix whose each element is zero, is called a **null or zero matrix**.

An $m \times n$ with all its elements equal to zero is denoted by $O_{m \times n}$. Null matrices may be of any order.

For example:-

$[0]$, $[0 \ 0 \ 0]$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ etc.

Equal Matrices:-

Two matrices of the same order are said to be equal if their corresponding entries are equal.

For example:-

$A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are equal, i.e., $A = B$ iff $a_{ij} = b_{ij}$ for $i = 1, 2, 3, \dots, m$, $j = 1, 2, 3, \dots, n$. In

other words, $A \times B$ represents the same matrix.

Transpose of a Matrix:-

If A is a matrix of order $m \times n$ then an $n \times m$ matrix obtained by interchanging the rows and columns of A , is called the transpose of A . It is denoted by A^T .

For example:-

$$\text{if } B = [b_{ij}]_{3 \times 4} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}$$

*Exercise 3.1

Q.NO. 1:-

$$\text{If } A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix},$$

then show that.

(i) $4A - 3A = A$

L.H.S:-

$$= 4 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} - 3 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

So L.H.S = R.H.S

$$3B - 3A = 3(B-A)$$

$$3 \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - 3 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = 3 \left(\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \right)$$

$$\begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} = 3 \begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix}$$

So L.H.S = R.H.S

Q.NO. 28-

If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$

$$A \cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i^2 + 0 & 0 - 0 \\ i - i & 0 + i^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1+0 & 0-0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = I_2$$

Hence proved.

Q. NO. 3:-

Find x and y , if

$$(i) \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$x+3 = 2 \quad ; \quad 3y-4 = 2$$

$$x = 2-3 \quad ; \quad 3y = 2+4$$

$$\boxed{x = -1} \quad ; \quad 3y = 6$$

$$y = \frac{6}{3} = 2$$

$$(ii) \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$$

$$x+3 = y$$

$$x-y = -3 \rightarrow (i)$$

$$3y-4 = 2x$$

$$-4 = 2x-3y$$

$$2x-3y = -4 \rightarrow (ii)$$

3(i) - (ii)

$$3x - 3y = -9$$

$$\pm 2x \mp 3y = \mp 4$$

$$\boxed{x = -5}$$

Put $x = -5$ in eq (i)

$$-5 - y = -3$$

$$-y = -3 + 5$$

$$-y = 2$$

$$\boxed{y = -2}$$

Q. NO. 4:-

$$\text{If } A = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix},$$

find the following matrices.

(i) $4A - 3B$

$$= 4 \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} - 3 \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4-0 & 8-9 & 12-6 \\ 4-3 & 0-(-3) & 8-6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

(ii) $A + 3(B - A)$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3 \left(\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 2+3 & 3+(-3) \\ 1+0 & 0+(-3) & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

Q. NO. 5:-

Find x and y if

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2x & 2y \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2x & x+2y \\ 1 & y+4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$2x = -2 \quad ; \quad y + 4 = 6$$

$$x = \frac{-2}{2} \quad ; \quad y = 6 - 4$$

$$\boxed{x = -1}$$

$$\boxed{y = 2}$$

Q.NO.8:-

$$\text{If } A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} \text{ and } A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

find the values of a and b .

$$A \cdot A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+2a & 4+2b \\ a+ab & 2a+b^2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1+2a & 4+2b \\ a+ab & 2a+b^2 \end{bmatrix}$$

$$0 = 1+2a$$

$$-1 = 2a$$

$$\boxed{\frac{-1}{2} = a}$$

$$0 = 4 + 2b$$

$$b = \frac{-4}{2}$$

$$\boxed{b = -2}$$

Q.NO.9:-

$$\text{If } A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \text{ and } A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

find the values of a and b .

$$A \cdot A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix}$$

$$1 = 1-a$$

$$1-1 = -a$$

$$\boxed{a = 0}$$

$$0 = -1-b$$

$$\boxed{b = -1}$$

Q.NO. 10:-

If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ and

$B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then show that

$$(A+B)^t = A^t + B^t$$

L.H.S:-

$$= \left(\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} \right)^t$$

$$= \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix}$$

$$R.H.S =$$

$$= A^t + B^t$$

$$A^t = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix}$$

$$\text{So } L.H.S = R.H.S$$

Q. NO. 11:-

Find A^3 if $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

$$A \cdot A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+5+6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0-0 & 0+0-0 & 0+0-0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q.NO. 12:-

Find the matrix X , if,

$$(ii) \quad X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$$

$$X A = B$$

$$X = B A^{-1}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix}$$

$$= (5)(1) - (2)(-2)$$

$$= 5 + 4$$

$$= 9$$

$$\text{Adj}^o A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$BA^{-1} = \frac{1}{9} \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$X = \frac{1}{9} \begin{bmatrix} -1+10 & 2+25 \\ 12+6 & -24+15 \end{bmatrix}$$

$$X = \frac{1}{9} \begin{bmatrix} 9 & 27 \\ 18 & -9 \end{bmatrix}$$

$$X = \begin{bmatrix} 9/9 & 27/9 \\ 18/9 & -9/9 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\text{ii) } \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$$A X = B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix}$$

$$|A| = 5 + 4$$

$$|A| = 9$$

$$\text{adj } A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$X = A^{-1} B$$

$$X = \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$$X = \frac{1}{9} \begin{bmatrix} 2 - 10 & 1 - 20 \\ 4 + 25 & 2 + 50 \end{bmatrix}$$

$$X = \begin{bmatrix} -8/9 & -19/9 \\ 29/9 & 52/9 \end{bmatrix}$$

Q. NO. 13:-

Find the matrix A, if,

$$(i) \begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$\text{let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5a-c & 5b-d \\ 0 & 0 \\ 3a+c & 3b+d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$5a - c = 3 \rightarrow \text{(i)}$$

$$3a + c = 7 \rightarrow \text{(ii)}$$

$$\text{(i)} + \text{(ii)}$$

$$5a - c = 3$$

$$3a + c = 7$$

$$8a = 10$$

$$a = \frac{10}{8}$$

$$\frac{5}{4}$$

$$\boxed{a = \frac{5}{4}}$$

Put a in (i)

$$5\left(\frac{5}{4}\right) - c = 3$$

$$\frac{25}{4} - 3 = c$$

$$4$$

$$25 - 12 = C$$

$$4$$

$$\boxed{\begin{array}{r} 13 \\ 4 \end{array}} = C$$

$$5b - d = -7 \rightarrow (iii)$$

$$3b + d = 2 \rightarrow (iv)$$

$$(iii) + (iv)$$

$$5b - d = -7$$

$$3b + d = 2$$

$$8b = -5$$

$$\boxed{\begin{array}{r} b = -5 \\ 8 \end{array}}$$

Put b in (iii)

$$5(-5) - d = -7$$

$$\frac{-25 + 7}{8} = d$$

$$\frac{-25 + 56}{8} = d$$

$$\boxed{\begin{array}{r} 31 \\ 8 \end{array}} = d$$

$$(ii) \quad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$\text{let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 2a-d & 2b-e & 2c-f \\ -a+2d & -b+2e & -c+2f \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$2a - d = 0 \rightarrow (i)$$

$$2b - e = f \rightarrow (ii)$$

$$2c - f = 8 \rightarrow (iii)$$

$$-a + 2d = 3 \rightarrow (iv)$$

$$-b + 2e = 3 \rightarrow (v)$$

$$-c + 2f = -7 \rightarrow (vi)$$

From eq (i)

$$2a - d = 0$$

$$2a = d$$

Put $2a = d$ in eq (iv)

$$-a + 2(2a) = 3$$

$$-a + 4a = 3$$

$$3a = 3$$

$$a = \frac{3}{3}$$

$$\boxed{a = 1}$$

As : $2a = d$

$$2(i) = d$$

$$\boxed{d = 2}$$

$$2(iii) + (vi)$$

$$4c - 2f = 16$$

$$-c + 2f = -7$$

$$3c = 9$$

$$c = 3$$

31

$$\boxed{c = 3}$$

Put $c = 3$ in eq (vi)

$$-c + 2f = -7$$

$$-3 + 2f = -7$$

$$2f = -7 + 3$$

$$2f = -4$$

$$f = -2$$

21

$$\boxed{f = -2}$$

$$2(v) + (ii)$$

$$-2b + 4e = 6$$

$$2b - e = -3$$

$$3e = 3$$

$$e = 1$$

31

$$\boxed{e = 1}$$

Put $e = 1$ in eq (v)

$$-b + 2e = 3$$

$$-b + 2(1) = 3$$

$$2 - 3 = b$$

$$\boxed{b = -1}$$

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$$

Q. NO. 14:-

Show that

$$\begin{bmatrix} r \cos \phi & 0 & -\sin \phi \\ 0 & r & 0 \\ r \sin \phi & 0 & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -r \sin \phi & 0 & r \cos \phi \end{bmatrix} = r I_3$$

$$= \begin{bmatrix} r \cos^2 \phi + 0 + r \sin^2 \phi & 0 + 0 + 0 & r \sin \phi \cos \phi + 0 - r \sin \phi \cos \phi \\ 0 + 0 + 0 & 0 + r + 0 & 0 + 0 + 0 \\ r \sin \phi \cos \phi + 0 - r \sin \phi \cos \phi & 0 + 0 + 0 & r \sin^2 \phi + 0 + r \cos^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} r(\cos^2 \phi + \sin^2 \phi) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(\sin^2 \phi + \cos^2 \phi) \end{bmatrix}$$

$$= \begin{bmatrix} r(1) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(1) \end{bmatrix}$$

$$= \gamma \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \gamma I_3$$

Q.NO. 68-

If $A = [a_{ij}]_{3 \times 3}$, show that

(i) $\lambda(A) = (\lambda A)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

L.H.S:-

$$\lambda A = \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\lambda(A) = \lambda \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix}$$

$$\lambda(A) = \begin{bmatrix} (\lambda a_{11})\lambda & (\lambda a_{12})\lambda & (\lambda a_{13})\lambda \\ (\lambda a_{21})\lambda & (\lambda a_{22})\lambda & (\lambda a_{23})\lambda \\ (\lambda a_{31})\lambda & (\lambda a_{32})\lambda & (\lambda a_{33})\lambda \end{bmatrix}$$

$$(\lambda A) = \begin{bmatrix} (\lambda A)a_{11} & (\lambda A)a_{12} & (\lambda A)a_{13} \\ (\lambda A)a_{21} & (\lambda A)a_{22} & (\lambda A)a_{23} \\ (\lambda A)a_{31} & (\lambda A)a_{32} & (\lambda A)a_{33} \end{bmatrix}$$

$$\lambda \mu A = \lambda \mu \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= (\lambda \mu) A$$

Hence proved

$$ii) (\lambda + \mu) A = \lambda A + \mu A$$

L.H.S:-

$$= (\lambda + \mu) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda + \mu)a_{11} & (\lambda + \mu)a_{12} & (\lambda + \mu)a_{13} \\ (\lambda + \mu)a_{21} & (\lambda + \mu)a_{22} & (\lambda + \mu)a_{23} \\ (\lambda + \mu)a_{31} & (\lambda + \mu)a_{32} & (\lambda + \mu)a_{33} \end{bmatrix}$$

R.H.S:-

$$\lambda A = \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix}$$

$$\mu A = \begin{bmatrix} \mu a_{11} & \mu a_{12} & \mu a_{13} \\ \mu a_{21} & \mu a_{22} & \mu a_{23} \\ \mu a_{31} & \mu a_{32} & \mu a_{33} \end{bmatrix}$$

$$\lambda A + \mu A = \begin{bmatrix} (\lambda + \mu)a_{11} & (\lambda + \mu)a_{12} & (\lambda + \mu)a_{13} \\ (\lambda + \mu)a_{21} & (\lambda + \mu)a_{22} & (\lambda + \mu)a_{23} \\ (\lambda + \mu)a_{31} & (\lambda + \mu)a_{32} & (\lambda + \mu)a_{33} \end{bmatrix}$$

$$\text{So } L.H.S = R.H.S$$

$$(iii) \lambda A - A = (\lambda - 1)A$$

L.H.S:-

$$\lambda A = \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix}$$

$$\lambda A - A = \begin{bmatrix} \lambda a_{11} - a_{11} & \lambda a_{12} - a_{12} & \lambda a_{13} - a_{13} \\ \lambda a_{21} - a_{21} & \lambda a_{22} - a_{22} & \lambda a_{23} - a_{23} \\ \lambda a_{31} - a_{31} & \lambda a_{32} - a_{32} & \lambda a_{33} - a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(\lambda - 1) & a_{12}(\lambda - 1) & a_{13}(\lambda - 1) \\ a_{21}(\lambda - 1) & a_{22}(\lambda - 1) & a_{23}(\lambda - 1) \\ a_{31}(\lambda - 1) & a_{32}(\lambda - 1) & a_{33}(\lambda - 1) \end{bmatrix}$$

$$= (\lambda - 1) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\lambda A - A = (\lambda - 1)A$$

Hence proved

Q. NO. 7:-

If $A = [a_{ij}]_{2 \times 3}$ and $B = [b_{ij}]_{2 \times 3}$,
show that $\lambda(A+B) = \lambda A + \lambda B$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

L.H.S:-

$$= \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$= \lambda \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} + \lambda b_{11} & \lambda a_{12} + \lambda b_{12} & \lambda a_{13} + \lambda b_{13} \\ \lambda a_{21} + \lambda b_{21} & \lambda a_{22} + \lambda b_{22} & \lambda a_{23} + \lambda b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \end{bmatrix} + \begin{bmatrix} \lambda b_{11} & \lambda b_{12} & \lambda b_{13} \\ \lambda b_{21} & \lambda b_{22} & \lambda b_{23} \end{bmatrix}$$

$$= \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \lambda \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$= \lambda A + \lambda B$$

Hence proved

* Exercise 3.2

Q.NO. 1:-

If $A = [a_{ij}]_{3 \times 4}$, then show that

i) $I_3 A = A$

L.H.S:-

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$I_3 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= A$$

Hence proved

(ii) $AT_4 = A$

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$AT_4 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= A$$

Hence proved

Q.NO. 28-

Find the inverse of the following matrices:

i) $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (3)(1) - (-1)(2)$$

$$= 3 + 2$$

$$= 5$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$$

ii) $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

Let $B = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

$$B^{-1} = \frac{\text{Adj } B}{|B|}$$

$$|B| = \begin{vmatrix} -2 & 3 \\ -4 & 5 \end{vmatrix}$$

$$= (-2)(5) - (3)(-4)$$

$$= -10 + 12$$

$$= 2$$

$$\text{Adj } B = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{2} \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$\text{Let } X = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$X^{-1} = \frac{\text{Adj } X}{|X|}$$

$$|X| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= (2i)(-i) - (i)(i)$$

$$= -2i^2 - i^2$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= 3$$

$$\text{Adj } X = \begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$$

$$X^{-1} = \frac{1}{3} \begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$$

$$= \begin{bmatrix} -i/3 & -i/3 \\ -i/3 & 2i/3 \end{bmatrix}$$

$$ii) \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$$

$$\text{Let } Z = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$$

$$Z^{-1} = \frac{\text{Adj } Z}{|Z|}$$

$$|Z| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix}$$

$$= (2)(3) - (1)(6)$$

$$= 6 - 6$$

$$= 0$$

Inverse does not exist

Q.NO:3:-

Solve the following system of linear equations:

$$(i) \begin{cases} 2x_1 - 3x_2 = 5 \\ 5x_1 + x_2 = 4 \end{cases}$$

$$\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$A X = B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix}$$

$$= (2)(1) - (-3)(5)$$

$$= 2 + 15$$

$$= 17$$

$$\text{Adj } A = \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$$X = \frac{1}{17} \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$X = \frac{1}{17} \begin{bmatrix} 5+12 \\ -25+8 \end{bmatrix}$$

$$X = \frac{1}{17} \begin{bmatrix} 17 \\ -17 \end{bmatrix}$$

$$X = \begin{bmatrix} 17/17 \\ -17/17 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{(iii)} \quad \begin{cases} 4x_1 + 3x_2 = 5 \\ 3x_1 - x_2 = 7 \end{cases}$$

$$\begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$A X = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (3)(3)$$

$$= -4 - 9$$

$$= -13$$

$$\text{Adj } A = \begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-13} \begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix}$$

$$X = \frac{1}{-13} \begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$X = \frac{1}{-13} \begin{bmatrix} -5 - 21 \\ -15 + 28 \end{bmatrix}$$

$$= \frac{1}{-13} \begin{bmatrix} -26 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\textcircled{iii} \quad \begin{cases} 3x - 5y = 1 \\ -2x + y = -3 \end{cases}$$

$$\begin{bmatrix} 3 & -5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$A X = B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 3 & -5 \\ -2 & 1 \end{vmatrix}$$

$$= (3)(1) - (-5)(-2)$$

$$= 3 - 10$$

$$= -7$$

$$\text{Adj } A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{-1}{7} \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$$

$$X = \frac{-1}{7} \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$X = \frac{-1}{7} \begin{bmatrix} 1 - 15 \\ 2 - 9 \end{bmatrix}$$

$$= \frac{-1}{7} \begin{bmatrix} -14 \\ -7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Q. NO. 4:-

If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$

and $C = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$, then find:

(i) $A - B$

$$A-B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-1 & 2+1 \\ 3-1 & 2-3 & 5-4 \\ -1-(-1) & 0-2 & 4-1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 2 & -2 & 3 \end{bmatrix}$$

(ii)

$$B-A$$

$$B-A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 1+1 & -1-2 \\ 1-3 & 3-2 & 4-5 \\ -1+1 & 2-0 & 1-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

(iii)

$$(A-B)-C$$

$$A - (B - C)$$

$$B - C = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 1-3 & -1+2 \\ 1+1 & 3-2 & 4-0 \\ -1-3 & 2-4 & 1+1 \end{bmatrix}$$

$$A - (B - C) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 4 \\ -4 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1+2 & 2-1 \\ 3-2 & 2-1 & 5-4 \\ -1+4 & 0+2 & 4-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}$$

Q.NO. 7:-

$$\text{If } A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix},$$

then find AA^t and A^tA .

$$A^t = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$$

(i) AA^t

$$= \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9+0 & 2-0+12-0 & -6-5+6+0 \\ 2-0+12-0 & 1+0+16+4 & -3+0+8+2 \\ -6-5+6-0 & -3+0+8+2 & 9+25+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 14 & -5 \\ 14 & 21 & 7 \\ -5 & 7 & 39 \end{bmatrix}$$

(ii) A^tA

$$= \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9 & -2+0-15 & 6+4-6 & 0-2+3 \\ -2+0-15 & 1+0+25 & -3+0+10 & 0+0-5 \\ 6+4-6 & -3+0+10 & 9+16+4 & 0-8-2 \\ 0-2+3 & 0+0-5 & 0-8-2 & 0+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -17 & 4 & 1 \\ -17 & 26 & 7 & -5 \\ 4 & 7 & 29 & -10 \\ 1 & -5 & -10 & 5 \end{bmatrix}$$

1. NO. 8:-

Solve the following matrix equations for X :

$$3X - 2A = B, \text{ if } A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$$

$$3X = B + 2A$$

$$X = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix} + 2 \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 6 & -4 \\ -2 & 2 & 10 \end{bmatrix}$$

$$3X = \begin{bmatrix} 2+4 & -3+6 & 1-4 \\ 5-2 & 4+2 & -1+10 \end{bmatrix}$$

$$3X = \begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix}$$

$$X = \begin{bmatrix} 2/3 & 1/3 & -1/3 \\ 1/3 & 2/3 & 3/3 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

(ii)

$$2X - 3A = B$$

$$\text{if } A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$2X = B + 3A$$

$$2X = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}$$

$$2X = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix}$$

$$2X = \begin{bmatrix} 3+3 & -1-3 & 0+6 \\ 4-6 & 2+12 & 1+15 \end{bmatrix}$$

$$2X = \begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$$

$$X = \begin{bmatrix} 3/2 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$$

Q.NO.9:-

Solve the following matrix equations for A:

$$i) \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A - \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} -1+2 & -4+3 \\ 3-1 & 6-2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$X A = B$$

$$A = X^{-1} B$$

$$X^{-1} = \frac{\text{Adj } X}{|X|}$$

$$|X| = \begin{vmatrix} 4 & 3 \\ 2 & 2 \end{vmatrix}$$

$$= (4)(2) - (3)(2)$$

$$= 8 - 6$$

$$= 2$$

$$\text{Adj } X = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$$

$$X^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$$

$$A = \frac{1}{2} \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2-6 & -2-12 \\ -2+8 & 2+16 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -4 & -14 \\ 6 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} -4/2 & -14/2 \\ 6/2 & 18/2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -7 \\ 3 & 9 \end{bmatrix}$$

$$\textcircled{ii} \quad A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$$

$$A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & 0+2 \\ -1+3 & 5+1 \end{bmatrix}$$

$$A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$AX = Y$$

$$A = YX^{-1}$$

$$X^{-1} = \frac{\text{Adj } X}{|X|}$$

$$|X| = \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix}$$

$$= (3)(2) - (1)(4)$$

$$= 6 - 4$$

$$= 2$$

$$\text{Adj } X = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$X^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$A = YX^{-1}$$

$$A = \frac{1}{2} \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2-8 & -1+6 \\ 4-24 & -2+18 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -6 & 5 \\ -20 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 5/2 \\ -10 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 5/2 \\ -10 & 8 \end{bmatrix}$$

Q. No. 5:-

$$\text{If } A = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}, B = \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix}$$

and $C = \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$, then show that

$$(i) \quad (AB)C = A(BC)$$

L.H.S

$$= \begin{pmatrix} i & 2i \\ 1 & -i \end{pmatrix} \begin{pmatrix} -i & 1 \\ 2i & i \end{pmatrix} \begin{pmatrix} 2i & -1 \\ -i & i \end{pmatrix}$$

$$= \begin{bmatrix} -i^2 + 4i^2 & 2i^2 + i \\ -i - 2i^2 & +1 - i^2 \end{bmatrix} \begin{pmatrix} 2i & -1 \\ -i & i \end{pmatrix}$$

$$= \begin{bmatrix} 3i^2 & -2 + i \\ -i - 2(-1) & 1 + 1 \end{bmatrix} \begin{pmatrix} 2i & -1 \\ -i & i \end{pmatrix}$$

$$= \begin{bmatrix} -3 & -2 + i \\ -i + 2 & 2 \end{bmatrix} \begin{pmatrix} 2i & -1 \\ -i & i \end{pmatrix}$$

$$= \begin{bmatrix} -6i - i(-2 + i) & 3 + i(i - 2) \\ 2i(-i + 2) - 2i & -1(-i + 2) + 2i \end{bmatrix}$$

$$= \begin{bmatrix} -6i - i^2 + 2i & 3 + i^2 - 2i \\ -2i^2 + 4i - 2i & i - 2 + 2i \end{bmatrix}$$

$$= \begin{bmatrix} -4i + 1 & 3 - 1 - 2i \\ 2 + 2i & 3i - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4i + 1 & 2 - 2i \\ 2 + 2i & 3i - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 4i & 2 - 2i \\ 2 + 2i & -2 + 3i \end{bmatrix}$$

R.H.S:-

$$A(BC) = A \begin{pmatrix} -i & 1 \\ 2i & i \end{pmatrix} \begin{pmatrix} 2i & -1 \\ -i & i \end{pmatrix}$$

$$= A \begin{bmatrix} -2i^2 - i & i + i \\ 4i^2 - i^2 & -2i + i^2 \end{bmatrix}$$

$$= A \begin{bmatrix} 2 - i & 2i \\ -3 & -2i - 1 \end{bmatrix}$$

$$= \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} \begin{bmatrix} 2 - i & 2i \\ -3 & -2i - 1 \end{bmatrix}$$

$$= \begin{bmatrix} i(2 - i) - 6i & 2i^2 + 2i(-2i - 1) \\ 2 - i + 3i & 2i - i(-2i - 1) \end{bmatrix}$$

$$= \begin{bmatrix} 2i - i^2 - 6i & 2i^2 - 4i^2 - 2i \\ 2 + 2i & 2i + 2i^2 + i \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 4i & -2i^2 - 2i \\ 2 + 2i & 3i + 2i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 4i & 2 - 2i \\ 2 + 2i & 3i - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 4i & 2 - 2i \\ 2 + 2i & -2 + 3i \end{bmatrix}$$

$$\text{So } L.H.S = R.H.S$$

$$\text{ii) } (A+B)C = AC + BC$$

$$L.H.S$$

$$(A+B)C = \left(\begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} + \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix} \right) C$$

$$= \begin{bmatrix} i - i & 2i + 1 \\ 1 + 2i & -i + i \end{bmatrix} C$$

$$= \begin{bmatrix} 0 & 2i+1 \\ 1+2i & 0 \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 0-i(2i+1) & 0+i(2i+1) \\ 2i(1+2i)+0 & -1(1+2i)+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2i^2-i & 0+2i^2+i \\ 2i+4i^2 & -1-2i \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & -2+i \\ 2i-4 & -1-2i \end{bmatrix}$$

R.H.S:-

$$AC = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 2i^2-2i^2 & -i+2i^2 \\ 2i+i^2 & -1-i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+2 & -i-2 \\ 2i-1 & -1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -i-2 \\ 2i-1 & 0 \end{bmatrix}$$

$$BC = \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} -2i^2-i & i+i \\ 4i^2-i^2 & -2i+i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & 2i \\ -4+1 & -2i-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & 2i \\ -3 & -2i-1 \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} 0 & -i-2 \\ 2i-1 & 0 \end{bmatrix} + \begin{bmatrix} 2-i & 2i \\ -3 & -2i-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2-i & -i-2+2i \\ 2i-1-3 & 0-2i-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & i-2 \\ 2i-4 & -2i-1 \end{bmatrix}$$

$$\text{So } L.H.S = R.H.S$$

Q. NO. 68-

If A and B are square matrices of the same order, then explain why in general;

i) $(A+B)^2 \neq A^2 + 2AB + B^2$

L.H.S

$$= (A+B)^2$$

$$= (A+B)(A+B)$$

$$= A^2 + AB + BA + B^2$$

$$\therefore AB \neq BA$$

$$\therefore AB + BA \neq 2AB$$

Hence,

$$(A+B)^2 \neq A^2 + 2AB + B^2$$

ii) $(A-B)^2 \neq A^2 - 2AB + B^2$

$$= (A-B)^2$$

$$= (A-B)(A-B)$$

$$= A^2 - AB - BA + B^2$$

$$\therefore -AB \neq -BA$$

$$\therefore -AB - BA \neq -2AB$$

Hence,

$$(A-B)^2 \neq A^2 - 2AB + B^2$$

$$(iii) \quad A^2 - B^2 \neq (A-B)(A+B)$$

R.H.S

$$= (A-B)(A+B)$$

$$= A^2 + AB - BA - B^2$$

$$\therefore AB \neq -BA$$

$$\therefore AB - BA \neq 0$$

Hence,

$$A^2 - B^2 \neq (A+B)(A-B)$$

*Exercise 3.3

Q.NO.18-

Evaluate the following determinants:-

$$(i) \quad \begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -1 & -3 & +2 & 3 \\ 1 & 2 & -2 & 2 & -2 & 3 & -1 \end{vmatrix}$$

$$= 5(-2+3) + 2(6-6) - 4(3-2)$$

$$= 5(1) + 2(0) - 4(1)$$

$$= 1$$

$$\text{ii) } \begin{vmatrix} 5 & 2 & -3 \\ 3 & -1 & 1 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -1 & 1 & -2 & 3 & 1 & -3 & 3 & -1 \\ 1 & -2 & -2 & -2 & -2 & -2 & -2 & 1 \end{vmatrix}$$

$$= 5(2-1) - 2(-6+2) - 3(3-2)$$

$$= 5(1) - 2(-4) - 3(1)$$

$$= 5 + 8 - 3$$

$$= 10$$

$$\text{iii) } \begin{vmatrix} 1 & 2 & -3 \\ -1 & 3 & 4 \\ -2 & 5 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 & 4 & -2 & -1 & 4 & -3 & -1 & 3 \\ 5 & 6 & -2 & 6 & -2 & 5 \end{vmatrix}$$

$$= 1(18-20) - 2(-6+8) - 3(-5+6)$$

$$= 1(-2) - 2(2) - 3(1)$$

$$= -2 - 4 - 3$$

$$= -9$$

(iv)

$$\begin{vmatrix} a+l & a-l & a \\ a & a+l & a-l \\ a-l & a & a+l \end{vmatrix}$$

$$= (a+l) \begin{vmatrix} a+l & a-l \\ a & a+l \end{vmatrix} - (a-l) \begin{vmatrix} a & a-l \\ a-l & a+l \end{vmatrix}$$

$$+ a \begin{vmatrix} a & a+l \\ a-l & a \end{vmatrix}$$

$$= (a+l)[(a+l)^2 - a(a-l)] - (a-l)[a(a+l) - (a-l)^2] + a[a^2 - (a+l)(a-l)]$$

$$= (a+l)[a^2 + 2al + l^2 - a^2 + al] - (a-l)[a^2 + al - (a^2 - 2al + l^2)] + a[a^2 - (a^2 - l^2)]$$

$$= (a+l)(l^2 + 3al) - (a-l)(a^2 + al - a^2 + 2al - l^2) + al^2$$

$$= al^2 + 3a^2l + l^3 + 3al^2 - (a-l)(3al - l^2) + al^2$$

$$= 5al^2 + 3a^2l + l^3 - (3a^2l - al^2 - 3al^2 + l^3)$$

$$= 5al^2 + 3a^2l + l^3 - 3a^2l + al^2 + 3al^2 - l^3$$

$$= 9al^2$$

(v)

$$\begin{vmatrix} 1 & 2 & -2 \\ -1 & 1 & -3 \\ 2 & 4 & -1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -3 & -2 \\ 4 & -1 \end{vmatrix} - 2 \begin{vmatrix} -1 & -3 \\ 2 & -1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix}$$

$$= 1[(-1)(-1) - (-3)(4)] - 2[(-1)(-1) - (3)(2)] - 2[(-1)(4) - (1)(2)]$$

$$= 1(-1+12) - 2(1+6) - 2(-4-2)$$

$$= 1(11) - 2(7) - 2(-6)$$

$$= 11 - 14 + 12$$

$$= 9$$

$$\begin{vmatrix} 2a & a & a \\ b & 2b & b \\ c & c & 2c \end{vmatrix}$$

$$= 2a \begin{vmatrix} 2b & b \\ c & 2c \end{vmatrix} - a \begin{vmatrix} b & b \\ c & 2c \end{vmatrix} + a \begin{vmatrix} b & 2b \\ c & c \end{vmatrix}$$

$$= 2a(4bc - bc) - a(2bc - bc) + a(bc - 2bc)$$

$$= 2a(3bc) - a(bc) + a(-bc)$$

$$= 6abc - abc - abc$$

$$= 4abc$$

Q.NO. 28-

Without expression show that

$$\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix}$$

$$C_2 - C_1, C_3 - C_2$$

$$= \begin{vmatrix} 6 & 7-6 & 8-7 \\ 3 & 4-3 & 5-4 \\ 2 & 3-2 & 4-3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 1 & 1 \\ 3 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\text{As, } C_2 = C_3$$

$$= 0$$

$$(ii) \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

$$C_2 + C_3$$

$$= \begin{vmatrix} 2 & 3-1 & -1-1 \\ 1 & 1+0 & 0 \\ 2 & -3+5 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 2 & -1 \\ 1 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix}$$

$$\text{As, } C_1 = C_2$$

$$= 0$$

$$(iii) \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

$$C_2 - C_1, C_3 - C_2$$

$$= \begin{vmatrix} 1 & 2-1 & 3-2 \\ 4 & 5-4 & 6-5 \\ 7 & 8-7 & 9-8 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & 1 \\ 7 & 1 & 1 \end{vmatrix}$$

$$As, C_2 = C_3$$

$$= 0$$

Q. NO. 38-



Show that.

$$\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = 9 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

L.H.S.:-

$$= \begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix}$$

Taking 3- common from R_2

$$= 3 \begin{vmatrix} 2 & 3 & 0 \\ 1 & 3 & 2 \\ 2 & 15 & 1 \end{vmatrix}$$

Taking 3- common from C_2

$$= 3 \times 3 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= 3 \times 3 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= 9 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= \text{R.H.S}$$

$$(iv) \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

L.H.S:-

$$= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix}$$

$$= \frac{xyz}{xyz} \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix}$$

$$= \frac{1}{xyz} \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ xyz & xyz & xyz \end{vmatrix}$$

Taking (xyz) -common from R_3

$$= \frac{xyz}{xyz} \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

Interchange R_2 and R_3

$$= - \begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ x^2 & y^2 & z^2 \end{vmatrix}$$

Interchanging R_1 and R_2

$$= -(-) \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = R.H.S$$

$$= \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

L.H.S

$$= \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= b+c \begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix} - a \begin{vmatrix} b & b \\ c & a+b \end{vmatrix} + a \begin{vmatrix} b & c+a \\ c & c \end{vmatrix}$$

$$= b+c [(c+a)(a+b) - bc] - a [ab + b^2 - bc] + a [bc - c^2 - ca]$$

$$= b+c (ca + cb + a^2 + ab - bc) - a (ab + b^2 - bc) + a (b - c^2 - ca)$$

$$= abc + a^2b + ab^2 + ac^2 + a^2c + abc - a^2b - ab^2$$

$$+ abc + abc - ac^2 - a^2c$$

$$= 4abc = R.H.S$$

$$(vi) \begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix} = a^3 + b^3$$

L.H.S:-

$$= \begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix}$$

$$= b \begin{vmatrix} b & 0 \\ a & b \end{vmatrix} + 1 \begin{vmatrix} a & 0 \\ 1 & b \end{vmatrix} + a \begin{vmatrix} a & b \\ 1 & a \end{vmatrix}$$

$$= b(b^2 - 0) + 1(ab - 0) + a(a^2 - b)$$

$$= b^3 + ab + a^3 - ab$$

$$= b^3 + a^3$$

= R.H.S

$$(vii) \begin{vmatrix} r \cos \phi & 1 & -\sin \phi \\ 0 & 1 & 0 \\ r \sin \phi & 0 & \cos \phi \end{vmatrix} = r$$

$$= r \cos \phi \begin{vmatrix} 1 & 0 \\ 0 & \cos \phi \end{vmatrix} - 1 \begin{vmatrix} -\sin \phi & 0 \\ r \sin \phi & \cos \phi \end{vmatrix} - \sin \phi \begin{vmatrix} 0 & 1 \\ r \sin \phi & 0 \end{vmatrix}$$

$$= r \cos \phi (\cos \phi - 0) - 1(0 - 0) - \sin \phi (0 - r \sin \phi)$$

$$= r \cos^2 \phi + r \sin^2 \phi$$

$$= r(\cos^2 \phi + \sin^2 \phi)$$

$$= r(1)$$

$$= r$$

$$= R.H.S$$

$$\text{iii) } \begin{vmatrix} a & b+c & a+b \\ b & c+a & b+c \\ c & a+b & c+a \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

$$= a \begin{vmatrix} c+a & b+c \\ a+b & c+a \end{vmatrix} - (b+c) \begin{vmatrix} b & b+c \\ c & c+a \end{vmatrix} + (a+b) \begin{vmatrix} b & c+a \\ c & a+b \end{vmatrix}$$

$$= a[(c+a)^2 - (a+b)(b+c)] - (b+c)[bc + ab - bc - c^2] + (a+b)[ab + b^2 - c^2 - ac]$$

$$= a[(c^2 + 2ac + a^2) - (ab + ac + b^2 + bc)] - (b+c)(ab - c^2) + a^2b + ab^2 - ac^2 - a^2c + ab^2 + b^3 - bc^2 - abc$$

$$= ac^2 + 2a^2c + a^3 - a^2b - ab^2 - abc - ab^2 + bc^2 - abc$$

$$+ c^3 + a^2b + ab^2 - ac^2 - a^2c + ab^2 + b^3 - bc^2 - abc$$

$$= a^3 + b^3 + c^3 - 3abc$$

$$= R.H.S$$

$$\text{iv) } \begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix} = \lambda^2(a+b+c+\lambda)$$

L.H.S:-

$$= \begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} a+\lambda & b+\lambda & c \\ b & c+\lambda & -b \\ a & c+\lambda & a \end{vmatrix} + \begin{vmatrix} a & b+\lambda \\ a & b \end{vmatrix}$$

$$= a+\lambda[(b+\lambda)(c+\lambda)-bc] - b[ac+a\lambda-ac] + c(ab-ab-a\lambda)$$

$$= a+\lambda(bc+b\lambda+c\lambda+\lambda^2-bc) - b(a\lambda) + c(-a\lambda)$$

$$= a+\lambda(b\lambda+c\lambda+\lambda^2) - ab\lambda - ac\lambda$$

$$= a\lambda^2 + b\lambda^2 + c\lambda^2 + \lambda^3 - ab\lambda - ac\lambda$$

$$= a\lambda^2 + b\lambda^2 + c\lambda^2 + \lambda^3$$

$$= \lambda^2(a+b+c+\lambda)$$

$$= R.H.S$$

$$(x) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$L.H.S:-$$

$$C_1 - C_2, C_2 - C_3$$

$$= \begin{vmatrix} 1-1 & 1-1 & 1-1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)[0-0+1(b+c-a-b)]$$

$$= (a-b)(b-c)[0-0+c-a]$$

$$= (a-b)(b-c)(c-a)$$

$$\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$$

L.H.S.-

$$C_1 + C_2$$

$$= \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} b+c+a & a & a^2 \\ a+b+c & b & b^2 \\ a+b+c & c & c^2 \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$R_1 - R_2, R_2 - R_3$$

$$= (a+b+c) \begin{vmatrix} 1-1 & a-b & a^2-b^2 \\ 1-1 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 0 & a-b & (a-b)(a+b) \\ 0 & b-c & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a+b+c)(a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a+b+c)(a-b)(b-c)[0-1(0-b-c)a+b(0-1)]$$

$$= (a+b+c)(a-b)(b-c)[b+c-a-b]$$

$$= (a+b+c)(a-b)(b-c)(c-a)$$

$$(iii) \begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix} = l^2(3a+l)$$

L.H.S:-

$$= \begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix}$$

$$= (a+l) \begin{vmatrix} a+l & a \\ a & a+l \end{vmatrix} - a \begin{vmatrix} a & a \\ a & a+l \end{vmatrix} + a \begin{vmatrix} a & a+l \\ a & a \end{vmatrix}$$

$$= (a+l)[(a+l)^2 - a^2] - a[a^2 + al - a^2] + a[a^2 - a^2 - al]$$

$$= (a+l)[a^2 + 2al + l^2 - a^2] - a(al) + a(-al)$$

$$= a+l(2al + l^2) - a^2l - a^2l$$

$$= 2a^2l + al^2 + 2al^2 + l^3 - 2a^2l$$

$$= 3al^2 + l^3$$

$$= l^2(3a+l)$$

$$(i) \begin{vmatrix} a_{11} & a_{12} & a_{13} + c_{13} \\ a_{21} & a_{22} & a_{23} + c_{23} \\ a_{31} & a_{32} & a_{33} + c_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & c_{13} \\ a_{21} & a_{22} & c_{23} \\ a_{31} & a_{32} & c_{33} \end{vmatrix}$$

R.H.S:-

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} + c_{13} \\ a_{21} & a_{22} & a_{23} + c_{23} \\ a_{31} & a_{32} & a_{33} + c_{33} \end{vmatrix}$$

Expand by C_3

$$= a_{13} + c_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} - (a_{23} + c_{23}) \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} + (a_{33} + c_{33}) \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$+ a_{33} + c_{33} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= \begin{vmatrix} a_{13} & a_{21} & a_{22} \\ a_{31} & a_{32} & \end{vmatrix} - a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} + a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$+ c_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} - c_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} + c_{33} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & c_{13} \\ a_{21} & a_{22} & c_{23} \\ a_{31} & a_{32} & c_{33} \end{vmatrix}$$

Q.NO. 56

Without expansion verify that

$$\begin{vmatrix} \alpha & \beta + \gamma & 1 \\ \beta & \gamma + \alpha & 1 \\ \gamma & \alpha + \beta & 1 \end{vmatrix} = 0$$

L.H.S:-

$$= \begin{vmatrix} \alpha + \beta + \gamma & \beta + \gamma & 1 \\ \alpha + \beta + \gamma & \gamma + \alpha & 1 \\ \alpha + \beta + \gamma & \alpha + \beta & 1 \end{vmatrix}$$

$$= \alpha + \beta + \gamma \begin{vmatrix} 1 & \beta + \gamma & 1 \\ 1 & \gamma + \alpha & 1 \\ 1 & \alpha + \beta & 1 \end{vmatrix}$$

Since, C_1 and C_2 are identical

$$= 0$$

$$\begin{matrix} \text{oo} \\ \text{(II)} \end{matrix} \begin{vmatrix} 1 & 2 & 3x \\ 2 & 3 & 6x \\ 3 & 5 & 9x \end{vmatrix} = 0$$

Common $(3x)$ from C_3

$$= 3x \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 5 & 3 \end{vmatrix}$$

As, C_1 and C_2 are identical, so

$$= 0$$

$$\begin{matrix} \text{ooo} \\ \text{(III)} \end{matrix} \begin{vmatrix} 1 & a^2 & a \\ & & bc \\ 1 & b^2 & b \\ & & ca \\ 1 & c^2 & c \\ & & ab \end{vmatrix} = 0$$

$$= \frac{1}{abc} \begin{vmatrix} 1 & 1 & a^2 \\ abc & 1 & b^2 \\ & 1 & c^2 \end{vmatrix} \begin{matrix} a \\ b \\ c \end{matrix} \times abc$$

$$= \frac{1}{abc} \begin{vmatrix} 1 & 1 & a^2 \\ abc & 1 & b^2 \\ & 1 & c^2 \end{vmatrix} \begin{matrix} a^2 \\ b^2 \\ c^2 \end{matrix}$$

$$= \frac{1}{abc} (0)$$

$$= 0 = \text{R.H.S}$$

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

$$C_1 + (C_2 + C_3)$$

$$= \begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$$

$$= \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$$

As, C_1 is zero

$$= 0$$

(v)

$$\begin{vmatrix} bc & ca & ab \\ 1 & 1 & 1 \\ a & b & c \\ a & b & c \end{vmatrix} = 0$$

$$= \frac{1}{abc} \begin{vmatrix} bc & ca & ab \\ abc & abc & abc \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} bc & ca & ab \\ bc & ac & ab \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc} (0)$$

$$= 0$$

(vi)

$$\begin{vmatrix} mn & l & l^2 \\ nl & m & m^2 \\ lm & n & n^2 \end{vmatrix} = \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$$

$$= \frac{1}{lmn} \begin{vmatrix} lmn & l^2 & l^3 \\ mnl & m^2 & m^3 \\ lmn & n^2 & n^3 \end{vmatrix}$$

Taking (lmn) common from C_1

$$= \frac{lmn}{lmn} \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$$

$$= \begin{vmatrix} 2a & 2b & 2c \\ a+b & 2b & b+c \\ a+c & b+c & 2c \end{vmatrix} = 0$$

Taking 2 - common from R_1

$$= 2 \begin{vmatrix} a & b & c \\ a+b & 2b & b+c \\ a+c & b+c & 2c \end{vmatrix}$$

$R_2 - R_1$, $R_3 - R_1$

$$= 2 \begin{vmatrix} a & b & c \\ a+b-a & 2b-b & b+c-c \\ a+c-a & b+c-b & 2c-b \end{vmatrix}$$

$$= 2 \begin{vmatrix} a & b & c \\ b & b & b \\ c & c & c \end{vmatrix}$$

Taking b and c common from

R_2 and R_3

$$= 2bc \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

As $R_2 = R_3$

$$= 2bc(0)$$

$$= 0$$

$$(ix) \begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix} = 0$$

L.H.S:-

$$R_1 \times b, R_2 \times c, R_3 \times a$$

$$= \begin{vmatrix} -ab & 0 & bc \\ 0 & ac & -bc \\ ab & -ac & 0 \end{vmatrix}$$

$$= R_1 + (R_2 + R_3)$$

$$= \begin{vmatrix} -ab+0+ab & 0+ac-ac & bc-bc+0 \\ 0 & ac & -bc \\ ab & -ac & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 0 & ac & -bc \\ ab & -ac & 0 \end{vmatrix}$$

$$\text{As } R_1 = 0$$

$$= 0$$

$$(viii) \begin{vmatrix} 7 & 2 & 6 \\ 6 & 3 & 2 \\ -3 & 5 & 1 \end{vmatrix} = \begin{vmatrix} 7 & 2 & 7 \\ 6 & 3 & 5 \\ -3 & 5 & -3 \end{vmatrix} + \begin{vmatrix} 7 & 2 & -1 \\ 6 & 3 & -3 \\ -3 & 5 & 4 \end{vmatrix}$$

R.H.S:-

$$= \begin{vmatrix} 7 & 2 & 7 \\ 6 & 3 & 5 \\ -3 & 5 & -3 \end{vmatrix} + \begin{vmatrix} 7 & 2 & -1 \\ 6 & 3 & -3 \\ -3 & 5 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 2 & 7-1 \\ 6 & 3 & 5-3 \\ -3 & 5 & -3+4 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 2 & 6 \\ 6 & 3 & 2 \\ -3 & 5 & 1 \end{vmatrix}$$

Q.NO. 14e-

Verify that $(AB)^{-1} = B^{-1}A^{-1}$ if

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$$

L.H.S:-

$$AB = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+8 & 1-2 \\ 3+0 & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 \\ 3 & -1 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{Adj} AB}{|AB|}$$

$$|AB| = \begin{vmatrix} 5 & -1 \\ 3 & -1 \end{vmatrix}$$

$$= (5)(-1) - (-1)(3)$$

$$= -5 + 3$$

$$= -2$$

$$\text{Adj } B = \begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$(AB)^{-1} = \frac{-1}{2} \begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -1/2 \\ 3/2 & 5/2 \end{bmatrix}$$

R.H.S:-

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$\text{Adj } A = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix}$$

$$= (1)(0) - (2)(-1)$$

$$= 0 + 2$$

$$= 2$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj } B}{|B|}$$

$$|B| = \begin{vmatrix} -3 & 1 \\ 4 & -1 \end{vmatrix}$$

$$= 3 - 4$$

$$= -1$$

$$\text{Adj } B = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 0-1 & 2-1 \\ 0-3 & 8-3 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -1/2 \\ 3/2 & -5/2 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 1 \\ 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

L.H.S. =

$$AB = \begin{bmatrix} 5 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20+2 & 15+1 \\ 8+4 & 6+2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 16 \\ 12 & 8 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{Adj}^{\circ} AB}{|AB|}$$

$$|AB| = \begin{vmatrix} 22 & 16 \\ 12 & 8 \end{vmatrix}$$

$$= (22)(8) - (16)(12)$$

$$= 176 - 192$$

$$= -16$$

$$\text{Adj}^{\circ} AB = \begin{bmatrix} 8 & -16 \\ -12 & 22 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{-16} \begin{bmatrix} 8 & -16 \\ -12 & 22 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} -1/2 & 1 \\ 3/4 & -11/8 \end{bmatrix}$$

∴ R.H.S:-

$$B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj}^{\circ} B}{|B|}$$

$$|B| = \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= (4)(1) - (3)(2)$$

$$= 4 - 6$$

$$= -2$$

$$\text{Adj}^{\circ} B = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-2} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & -3/2 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & -3/2 \\ 1 & -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}^{\circ} A}{|A|}$$

$$|A| = \begin{vmatrix} 5 & 1 \\ 2 & 2 \end{vmatrix}$$

$$= (5)(2) - (1)(2)$$

$$= 10 - 2$$

$$= 8$$

$$\text{Adj}^{\circ} A = \begin{bmatrix} 2 & -1 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} 2 & -1 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2/8 & -1/8 \\ -2/8 & 5/8 \end{bmatrix}$$

$$= \begin{bmatrix} 1/4 & -1/8 \\ -1/4 & 5/8 \end{bmatrix}$$

$$B^{-1}A^{-1} = -\frac{1}{2} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} \frac{1}{8} \begin{bmatrix} 2 & -1 \\ -2 & 5 \end{bmatrix}$$

$$= -\frac{1}{16} \begin{bmatrix} 2+6 & -1-15 \\ -4-8 & 2+20 \end{bmatrix}$$

$$= -\frac{1}{16} \begin{bmatrix} 8 & -16 \\ -12 & 22 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 1 \\ 3/4 & -11/8 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 1 \\ 3/4 & -11/8 \end{bmatrix}$$

Q. NO. 15B-

Verify that $(AB)^t = B^t A^t$ and if

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

L.H.S:-

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1-3+0 & 1-2-2 \\ 0+9+0 & 0+6-1 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} -2 & -3 \\ 9 & 5 \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & 9 \\ -3 & 5 \end{bmatrix}$$

R.H.S:-

$$B^t = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3+0 & 0+9+0 \\ 1-2-2 & 0+6-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 9 \\ -3 & 5 \end{bmatrix}$$

Q. NO. 16:-

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ verify that $(A^{-1})^t = (A^t)^{-1}$

L.H.S:-

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix}$$

$$= (1)(2) - (1)(-3)$$

$$= 2 + 3$$

$$= 5$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$$

$$(A^{-1})^t = \left(\begin{bmatrix} 1/5 & 1/5 \\ -3/5 & 2/5 \end{bmatrix} \right)^t$$

$$= \begin{bmatrix} 1/5 & -3/5 \\ 1/5 & 2/5 \end{bmatrix}$$

R.H.S-

$$A^t = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$$

$$(A^t)^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}$$

$$= (2)(1) - (3)(-1)$$

$$= 2 + 3$$

$$= 5$$

$$(A^t)^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & -3/5 \\ 1/5 & 2/5 \end{bmatrix}$$

So L.H.S = R.H.S

Q.NO.6:-

Find the values x if

$$\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$$

$$\begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & x \end{vmatrix} - \begin{vmatrix} -1 & 4 & +x \\ x & 0 & 1 \end{vmatrix} = -30$$

$$3[(3)(0) - (4)(1)] - 1[(-1)(0) - (4)(x)] + x[(-1)(1) - (3)(x)] = -30$$

$$3[0 - 4] - 1[0 - 4x] + x[-1 - 3x] = -30$$

$$-12 + 4x - x - 3x^2 = -30$$

$$-3x^2 + 3x - 12 = -30$$

$$-3x^2 + 3x - 12 + 30 = 0$$

$$-3x^2 + 3x + 18 = 0$$

$$-3(x^2 - x - 6) = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x+2)(x-3) = 0$$

$$x+2=0 \quad ; \quad x-3=0$$

$$\boxed{x-2} \quad ; \quad \boxed{x=3}$$

(ii)

$$\begin{vmatrix} 1 & x-1 & 3 \\ -1 & x+1 & 2 \\ 2 & -2 & x \end{vmatrix} = 0$$

$$1 \begin{vmatrix} x+1 & 2 \\ -2 & x \end{vmatrix} - (x-1) \begin{vmatrix} -1 & 2 \\ 2 & x \end{vmatrix} + 3 \begin{vmatrix} -1 & x+1 \\ 2 & -2 \end{vmatrix} = 0$$

$$1[x(x+1)+4] - (x-1)[-x-4] + 3[2-2(x+1)] = 0$$

$$1[x^2+x+4] + (x-1)(x+4) + 3[2-2x-2] = 0$$

$$x^2+x+4+x^2+4x-x-4-6x=0$$

$$2x^2-2x=0$$

$$2x(x-1)=0$$

$$2x=0 \quad ; \quad x-1=0$$

$$\boxed{x=0}$$

$$\boxed{x=1}$$

(iii)

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 3 & 6 & x \end{vmatrix} = 0$$

$$1 \begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} - 2 \begin{vmatrix} -2 & 2 \\ 3 & x \end{vmatrix} + 1 \begin{vmatrix} 2 & x \\ 3 & 6 \end{vmatrix} = 0$$

$$1[x^2-12] - 2[2x-6] + 1[12-3x] = 0$$

$$x^2-12-4x+12+12-3x=0$$

$$x^2 - 7x + 12 = 0$$

$$x^2 - 3x - 4x + 12 = 0$$

$$x(x-3) - 4(x-3) = 0$$

$$(x-4)(x-3) = 0$$

$$x-4=0 \quad ; \quad x-3=0$$

$$\boxed{x=4}$$

$$; \quad \boxed{x=3}$$

Q.NO.8:-

Show that

x	1	1	1
1	x	1	1
1	1	x	1
1	1	1	x

$$= (x+3)(x-1)^3$$

L.H.S:-

$$R_1 + (R_2 + R_3 + R_4)$$

$x+3$	$x+3$	$x+3$	$x+3$
1	x	1	1
1	1	x	1
1	1	1	x

Taking $(x+3)$ - common from R_1

$$= (x+3)$$

1	1	1	1
1	x	1	1
1	1	x	1
1	1	1	x

$$R_2 - R_1, R_3 - R_1, R_4 - R_1$$

$$= (x+3) \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & x-1 & 0 & 0 & 0 \\ 0 & 0 & x-1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-1 \end{array} \right]$$

Expand by C_1

$$= (x+3) \left[\begin{array}{ccc|c} x-1 & 0 & 0 & \\ 1 & 0 & x-1 & 0 \\ 0 & 0 & 0 & x-1 \end{array} \right] \begin{array}{c} \\ -0+0-0 \end{array}$$

$$= (x+3) \left[\begin{array}{ccc|c} x-1 & 0 & 0 & \\ 0 & x-1 & 0 & \\ 0 & 0 & x-1 & \end{array} \right]$$

Expand by R_1

$$= (x+3) \left[\begin{array}{cc|c} x-1 & 0 & -0+0 \\ 0 & x-1 & \end{array} \right]$$

$$= (x+3)(x-1)(x-1)^2 - 0$$

$$= (x+3)(x-1)^3 = R.H.S$$

Q.NO. 48-

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{bmatrix}$ and

$B = \begin{bmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & -2 \end{bmatrix}$, then find;

A_{12}, A_{22}, A_{32} and $|A|$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{vmatrix}$$

Expand by R_1

$$= 1 \begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 0 & 0 \\ -2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix}$$

$$= 1(-2+0) - 2(0+0) - 3(0-4)$$

$$= 1(-2) + 0 - 3(-4)$$

$$= -2 + 12$$

$$= 10$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ -2 & 1 \end{vmatrix}$$

$$= (-1)^3 [(0)(1) - (0)(-2)]$$

$$= -1(0)$$

$$= 0$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix}$$

$$= (-1)^4 [(1)(1) - (-3)(-2)]$$

$$= 1(1-6)$$

$$= 1(-5)$$

$$= -5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -3 \\ 0 & 0 \end{vmatrix}$$

$$= (-1)^5 [(1)(0) - (-3)(0)]$$

$$= -1(0 + 0)$$

$$= 0$$

ii) B_{21}, B_{22}, B_{23} and $|B|$

$$|B| = \begin{vmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & -2 \end{vmatrix}$$

Expand by R_1 :-

$$= 5 \begin{vmatrix} -1 & 4 \\ 1 & -2 \end{vmatrix} - (-2) \begin{vmatrix} 3 & 4 \\ -2 & -2 \end{vmatrix} + 5 \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix}$$

$$= 5[(-1)(-2) - (4)(1)] + 2[(3)(-2) - (4)(-2)] + 5[(3)(1) - (-1)(-2)]$$

$$= 5(2 - 4) + 2(-6 + 8) + 5(3 - 2)$$

$$= 5(-2) + 2(2) + 5(1)$$

$$= -10 + 4 + 5$$

$$= -1$$

$$B_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= (-1)^3 [(-2)(-2) - (5)(1)]$$

$$= (-1)(4 - 5)$$

$$= (-1)(-1)$$

$$= 1$$

$$B_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 5 \\ -2 & -2 \end{vmatrix}$$

$$= (-1)^4 (-10 + 10)$$

$$= (-1)(0)$$

$$= 0$$

$$B_{23} = (-1)^{2+3} \begin{vmatrix} 5 & -2 \\ -2 & 1 \end{vmatrix}$$

$$= (-1)^5 [(5)(1) - (-2)(-2)]$$

$$= (-1)(5 - 4)$$

$$= (-1)(1)$$

$$= -1$$

Q. NO. 17:-

If A and B are non-singular matrices, then show that:

$$(i) (AB)^{-1} = B^{-1}A^{-1}$$

$$= (AB)(B^{-1}A^{-1})$$

$$= A(BB^{-1})A^{-1} \therefore \text{Associative law}$$

$$\therefore BB^{-1} = I$$

$$= A(I)A^{-1}$$

$$= AA^{-1}$$

$$\therefore A(I) = A$$

$$= AA^{-1} = I$$

$$\text{Hence, } (AB)^{-1} = B^{-1}A^{-1}$$

$$(ii) (A^{-1})^{-1} = A$$

L.H.S:-

$$= A^{-1}A$$

$$= I$$

$$\text{Hence, } (A^{-1})^{-1} = A$$

Q.NO.9:-

$$(i) A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$|AA^t|$$

$$A^t = \begin{bmatrix} 3 & 2 \\ 2 & 1 \\ -1 & 3 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4+1 & 6+2-3 \\ 6+2-3 & 4+1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 5 \\ 5 & 14 \end{bmatrix}$$

$$|AA^t| = \begin{vmatrix} 14 & 5 \\ 5 & 14 \end{vmatrix}$$

$$= 196 - 25$$

$$= 171$$

$$|A^tA|$$

$$A^tA = \begin{bmatrix} 3 & 2 \\ 2 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4 & 6+2 & -3+6 \\ 6+2 & 4+1 & -2+3 \\ -3+6 & -2+3 & 1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 8 & 3 \\ 8 & 5 & 1 \\ 3 & 1 & 10 \end{bmatrix}$$

$$= 13 \begin{vmatrix} 5 & 1 \\ 1 & 10 \end{vmatrix} - 8 \begin{vmatrix} 8 & 1 \\ 3 & 10 \end{vmatrix} + 3 \begin{vmatrix} 8 & 5 \\ 3 & 1 \end{vmatrix}$$

$$= 13[5(10) - (1)(1)] - 8[(8)(10) - (1)(3)] + 3[(8)(1) - (5)(3)]$$

$$= 13(50 - 1) - 8(80 - 3) + 3(8 - 15)$$

$$= 13(49) - 8(77) + 3(-7)$$

$$= 637 - 616 - 21$$

$$=$$

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$$

, find $|AA^t|$

$$A^t = \begin{bmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+16 & 6+4 & 3+4 & 6+12 \\ 6+4 & 4+1 & 2+1 & 4+3 \\ 3+4 & 2+1 & 1+1 & 2+3 \\ 6+12 & 4+3 & 2+3 & 4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 10 & 7 & 18 \\ 10 & 5 & 3 & 7 \\ 7 & 3 & 2 & 5 \\ 18 & 7 & 5 & 13 \end{bmatrix}$$

$$|AA^t| = \begin{vmatrix} 25 & 10 & 7 & 18 \\ 10 & 5 & 3 & 7 \\ 7 & 3 & 2 & 5 \\ 18 & 7 & 5 & 13 \end{vmatrix}$$

$$C_3 + C_4$$

$$= \begin{vmatrix} 25 & 10 & 25 & 18 \\ 10 & 5 & 10 & 7 \\ 7 & 3 & 7 & 5 \\ 18 & 7 & 18 & 13 \end{vmatrix}$$

$$\text{As } C_1 = C_3$$

$$\text{So, } |AA^t| = 0$$

$$|A^t A|$$

$$|A^t A| = \begin{bmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4+1+4 & 12+2+1+6 \\ 12+2+1+6 & 16+1+1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 21 \\ 21 & 27 \end{bmatrix}$$

$$|A^t A| = \begin{vmatrix} 18 & 21 \\ 21 & 27 \end{vmatrix}$$

$$= [(18)(27) - (21)(21)]$$

$$= 486 - 441$$

$$= 45$$

Q. NO. 12:-

Which of the following matrices are singular and which of them are non-singular?

$$\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= 1 \begin{vmatrix} 1 & -1 & -0 \\ 2 & 4 & 0 \end{vmatrix} - 0 \begin{vmatrix} 3 & -1 \\ 0 & 4 \end{vmatrix} + 3 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1[(1)(4) - (-1)(2)] - 0 + 3[(3)(2) - (1)(0)]$$

$$= 1(4+2) - 0 + 3(6-0)$$

$$= 1(6) - 0 + 3(5)$$

$$= 6 - 0 + 18$$

$$= 24$$

Non-singular

$$(ii) \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$$

$$= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 0 & -3 \\ -3 & 5 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 0 & -1 \\ 2 & 5 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$$

$$= 2[(1)(5) - (0)(-3)] - 3[(1)(5) - (0)(2)] - 1[(1)(-3) - (1)(2)]$$

$$= 2(5 + 0) - 3(5 - 0) - 1(-3 - 2)$$

$$= 2(5) - 3(5) - 1(-5)$$

$$= 10 - 15 + 5$$

$$= 0$$

Singular

$$(iii) \begin{bmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & -1 & -3 \\ 2 & 3 & 1 & 2 \\ 3 & -1 & 3 & 4 \end{bmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & -1 & -3 \\ 2 & 3 & 1 & 2 \\ 3 & -1 & 3 & 4 \end{vmatrix}$$

$$R_2 - R_1, R_3 - 2R_1, R_4 - 3R_1$$

$$= \begin{vmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & -3 & -2 \\ 0 & 1 & -3 & 4 \\ 0 & -4 & -3 & 7 \end{vmatrix}$$

Expand by C_1

$$= 1 \begin{vmatrix} 1 & -3 & -2 \\ 1 & -3 & 4 \\ -4 & -3 & 7 \end{vmatrix} - 0 + 0 - 0$$

$$= 1 \begin{vmatrix} -3 & 4 \\ -3 & 7 \end{vmatrix} + 3 \begin{vmatrix} 1 & 4 \\ -4 & 7 \end{vmatrix} - 2 \begin{vmatrix} 1 & -3 \\ -4 & -3 \end{vmatrix}$$

$$= 1[(-3)(7) - (4)(-3)] + 3[(1)(7) - (4)(-4)] - 2[(1)(-3) - (-3)(-4)]$$

$$= 1(-21 + 12) + 3(7 + 16) - 2(-3 - 12)$$

$$= 1(-9) + 3(23) - 2(-15)$$

$$= -9 + 69 + 30$$

$$= 90$$

Non-singular

Q.No. 11:-

Find the values of λ if

A and B are singular

$$(i) \quad A = \begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$$

A is singular

$$0 = \begin{vmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{vmatrix}$$

$$0 = 4 \begin{vmatrix} 3 & 6 \\ 3 & 1 \end{vmatrix} - \lambda \begin{vmatrix} 7 & 6 \\ 2 & 1 \end{vmatrix} + 3 \begin{vmatrix} 7 & 3 \\ 2 & 3 \end{vmatrix}$$

$$0 = 4[(3)(1) - (6)(3)] - \lambda[(7)(1) - (6)(2)] + 3[(7)(3) - (3)(2)]$$

$$0 = 4(3 - 18) - \lambda(7 - 12) + 3(21 - 6)$$

$$0 = 4(-15) - \lambda(-5) + 3(15)$$

$$0 = -60 + 5\lambda + 45$$

$$0 = -15 + 5\lambda$$

$$\frac{15}{5} = \lambda$$

$$\lambda = 3$$

$$\boxed{3 = \lambda}$$

$$(ii) \quad B = \begin{bmatrix} 5 & 1 & 2 & 0 \\ 8 & 2 & 5 & 1 \\ 3 & 2 & 0 & 1 \\ 2 & \lambda & -1 & 3 \end{bmatrix}$$

B is singular

$$O = \begin{vmatrix} 5 & 1 & 2 & 0 \\ 8 & 2 & 5 & 1 \\ 3 & 2 & 0 & 1 \\ 2 & \lambda & -1 & 3 \end{vmatrix}$$

$$C_1 - 5C_2, \quad C_3 - 2C_2$$

$$O = \begin{vmatrix} 5-5(1) & 1 & 2-2(1) & 0 \\ 8-5(2) & 2 & 5-2(2) & 1 \\ 3-5(2) & 2 & 0-2(2) & 1 \\ 2-5\lambda & \lambda & -1-2(\lambda) & 3 \end{vmatrix}$$

$$O = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -2 & 2 & 1 & 1 \\ -7 & 2 & -4 & 1 \\ 2-5\lambda & \lambda & -1-2\lambda & 3 \end{vmatrix}$$

Interchanging C_1 and C_2

$$O = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & -2 & 1 & 1 \\ 2 & -7 & -4 & 1 \\ \lambda & 2-5\lambda & -1-2\lambda & 3 \end{vmatrix} \quad -0+0-0$$

or Expand by R_1

$$O = \begin{vmatrix} 1 & -2 & 1 & 1 \\ -7 & -4 & 1 & -0+0-0 \\ 2-5\lambda & -1-2\lambda & 3 & \end{vmatrix}$$

$$O = \begin{vmatrix} -2 & -4 & 1 & -1 & -7 & 1 & +1 & -7 & -4 \\ -1-2\lambda & 3 & 2-5\lambda & 3 & 2-5\lambda & -1-2\lambda \end{vmatrix}$$

$$0 = -2[(-4 \times 3) - (1)(-1-2\lambda)] - 1[(-7 \times 3) - (1)(2-5\lambda)] + 1[(-7 \times (-1-2\lambda)) - (-4)(2-5\lambda)]$$

$$0 = -2(-12+1+2\lambda) - 1(-21-2+5\lambda) + 1(7+14\lambda+8-20\lambda)$$

$$0 = -2(-11+2\lambda) - 1(-23+5\lambda) + 1(15-6\lambda)$$

$$0 = 22 - 4\lambda + 23 - 5\lambda + 15 - 6\lambda$$

$$0 = 60 - 15\lambda$$

$$\frac{60}{15} = \lambda$$

$$\boxed{4 = \lambda}$$

Q.NO. 13:-

Find the inverse of $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$

and show that $A^{-1}A = I_3$.

L.H.S:-

$$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

Expand by C_3

$$= 0 - 0 + 5 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 5[(2)(1) - (1)(1)]$$

$$= 5(2-1)$$

$$= 5(1)$$

$$= 5$$

$$\text{Adj } A = [\text{co-factors of } A]^t$$

$$= \begin{bmatrix} (5+0) & -(5-0) & (-3-2) \\ -(5+0) & (10-0) & -(-6-2) \\ (0-0) & -(0-0) & (2-1) \end{bmatrix}^t$$

$$\text{adj } A = \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{1}{5} \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix}$$

$$= A^{-1} A$$

$$= \frac{1}{5} \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 10-5+0 & 5-5-0 & 0-0+0 \\ -10+10+0 & -5+10-0 & 0+0+0 \\ -10+8+2 & -5+8-3 & 0+0+5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3 = R.H.S$$

Q.NO.10:-

If A is a square matrix of order 3, then show that $|KA| = k^3 |A|$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$KA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}$$

$$|KA| = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix}$$

Taking k -common from R_1, R_2 and R_3

$$= k \cdot k \cdot k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= k^3 |A|$$

$$= R.H.S$$

Q.No. 7:-

Evaluate the following determinants:

$$\begin{vmatrix} 3 & 4 & 2 & 7 \\ 2 & 5 & 0 & 3 \\ 1 & 2 & -3 & 5 \\ 4 & 1 & -2 & 6 \end{vmatrix}$$

$$R_1 - 3R_3, R_2 - 2R_3, R_4 - 4R_3$$

$$= \begin{vmatrix} 0 & -2 & 11 & -8 \\ 0 & 1 & 6 & -7 \\ 1 & 2 & -3 & 5 \\ 0 & -7 & 10 & -14 \end{vmatrix}$$

Expand by C_1

$$= 0 - 0 + 1 \begin{vmatrix} -2 & 11 & -8 \\ 6 & -7 & -0 \\ -7 & 10 & -14 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 11 & -8 \\ 1 & 6 & -7 \\ -7 & 10 & -14 \end{vmatrix}$$

Expand by R_1

$$= 2 \begin{vmatrix} 6 & -7 & -11 \\ 10 & -14 & -7 \end{vmatrix} - 11 \begin{vmatrix} 1 & -7 & -8 \\ -7 & -14 & -7 \end{vmatrix} + 8 \begin{vmatrix} 1 & 6 \\ -7 & 10 \end{vmatrix}$$

$$= 2[(6)(-14) - (-7)(10)] - 11[(1)(-14) - (-7)(-7)] - 8[(1)(10) - (6)(-7)]$$

$$= 2(-84 + 70) - 11(-14 - 49) - 8(10 + 42)$$

$$= -28 +$$

$$= -2(-14) - 11(-63) - 8(52)$$

$$= 28 + 693 - 416$$

$$= 305$$

(iii)

$$\begin{vmatrix} -3 & 9 & 1 & 1 \\ 0 & 3 & -1 & 2 \\ 9 & 7 & -1 & 1 \\ -2 & 0 & 1 & -1 \end{vmatrix}$$

$$R_2 + R_1, R_3 + R_1, R_4 - R_1$$

$$= \begin{vmatrix} -3 & 9 & 1 & 1 \\ -3 & 12 & 0 & 3 \\ 6 & 16 & 0 & 2 \\ 1 & -9 & 0 & -2 \end{vmatrix}$$

Expand by C_3

$$= \begin{vmatrix} -3 & 12 & 3 \\ 6 & 16 & 2 \\ 1 & -9 & -2 \end{vmatrix} \begin{vmatrix} -12 & 6 & 2 \\ 6 & 16 & 2 \\ 1 & -9 & -2 \end{vmatrix} + 3 \begin{vmatrix} 6 & 16 \\ 1 & -9 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 12 & 3 \\ 6 & 16 & 2 \\ 1 & -9 & -2 \end{vmatrix}$$

Expand by R_1

$$= -3 \begin{vmatrix} 16 & 2 \\ -9 & -2 \end{vmatrix} - 12 \begin{vmatrix} 6 & 2 \\ 1 & -2 \end{vmatrix} + 3 \begin{vmatrix} 6 & 16 \\ 1 & -9 \end{vmatrix}$$

$$\begin{aligned}
 &= -3[(16)(-2) - (2)(-9)] - 12[(6)(-2) - (2)(1)] + 3[(6)(-9) - (16)(1)] \\
 &= -3(-32 + 18) - 12(-12 - 2) + 3(-54 - 16) \\
 &= -3(-14) - 12(-14) + 3(-70) \\
 &= 42 + 168 - 210 \\
 &= 0
 \end{aligned}$$

$$\begin{vmatrix} 2 & 3 & 1 & -1 \\ 4 & 0 & 2 & 1 \\ 5 & 2 & -1 & 6 \\ 3 & -7 & 2 & -2 \end{vmatrix}$$

$$C_3 - 2C_4 + C_1 - 4C_2$$

$$\begin{vmatrix} 6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 1 \\ -19 & 2 & -13 & 6 \\ 11 & 7 & 6 & -2 \end{vmatrix}$$

Expand by R_2

$$\begin{vmatrix} 6 & 3 & 3 \\ -19 & 2 & -13 \\ 11 & -7 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 6 & 3 & 3 \\ -19 & 2 & -13 \\ 11 & -7 & 6 \end{vmatrix}$$

Expand by R_1

$$= 6 \begin{vmatrix} 2 & -13 & -3 \\ -7 & 6 & 11 \end{vmatrix} - 13 \begin{vmatrix} -19 & 6 \\ 11 & 6 \end{vmatrix} + 3 \begin{vmatrix} -19 & 2 \\ 11 & -7 \end{vmatrix}$$

$$= 6[(2)(6) - (-13)(-7)] - 13[(-19)(6) - (-3)(11)] + 3[(-19)(-7) - (2)(11)]$$

$$= 6(12 - 91) - 13(-114 + 143) + 3(133 - 22)$$

$$= 6(-79) - 13(29) + 3(11)$$

$$= -474 - 87 + 33$$

$$= -228$$

Definitions

Symmetric Matrix:-

A square matrix $A = [a_{ij}]_{n \times n}$ is called symmetric if $A^t = A$

For examples:-

$$\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

Skew-symmetric Matrix:-

A square matrix $A = [a_{ij}]_{n \times n}$ is called skew-symmetric or anti-symmetric if $A^t = -A$

For examples:-

$$B = \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}, \text{ then}$$

$$B^t = \begin{bmatrix} 0 & 4 & -1 \\ -4 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix} = (-1) \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix} = -B$$

Hermitian Matrixe-

A square matrix $A = [a_{ij}]_{n \times n}$ with complex entries, is called **hermitian** if $(\bar{A})^t = A$.

For examples-

$$A = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix}, \text{ then}$$

$$\bar{A} = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix} \Rightarrow (\bar{A})^t = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix} = A$$

Skew-hermitians-

A square matrix $A = [a_{ij}]_{n \times n}$ with complex entries is called **skew-hermitian** or

anti-hermitian if $(\bar{A})^t = -A$

For examples-

$$A = \begin{bmatrix} 0 & 2-3i \\ -2-3i & 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0 & 2+3i \\ -2+3i & 0 \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} 0 & -2+3i \\ 2+3i & 0 \end{bmatrix} = (-1) \begin{bmatrix} 0 & 2-3i \\ -2-3i & 0 \end{bmatrix} = -A$$

Rank of a Matrix-

Let A be a non-zero matrix. If r is the number of non-zero rows when it is reduced to the reduced echelon form, then r is called the rank of the matrix A .

* Exercise 3.4

Q.No.1:-

$$\text{If } A = \begin{bmatrix} 1 & -2 & 5 \\ -2 & 3 & -1 \\ 5 & -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & 2 \end{bmatrix}$$

then show that $A+B$ is symmetric.

$$A+B = \begin{bmatrix} 1 & -2 & 5 \\ -2 & 3 & -1 \\ 5 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & -2+1 & 5-2 \\ -2+1 & 3+0 & -1-1 \\ 5-2 & -1-1 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -1 & 3 \\ -1 & 3 & -2 \\ 3 & -2 & 2 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 3 & -2 \\ 3 & -2 & 2 \end{bmatrix}$$

$$(A+B)^t = A+B$$

Hence, $A+B$ is symmetric.

Q. NO. 28-

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$, show that

$A+A^t$ is symmetric

$$A^t = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A+A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+3 & 0-1 \\ 3+2 & 2+2 & -1+3 \\ -1+0 & 3-1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -1 \\ 5 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 5 & -1 \\ 5 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

$$(A + A^t)^t = A + A^t$$

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Hence, $A + A^t$ is symmetric

(ii) $A - A^t$ is skew-symmetric

$$A - A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-3 & 0+1 \\ 3-2 & 2-2 & -1-3 \\ -1-0 & 3+1 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$(A - A^t)^t = - \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$$(A - A^t)^t = A - A^t$$

Hence, $A - A^t$ is skew-symmetric

Q.NO. 48-

If the matrices A and B are symmetric and $AB = BA$, show that AB is symmetric.

$$A^t = A, \quad B^t = B \rightarrow (i)$$

$$AB = BA \rightarrow (ii)$$

AB is symmetric

$$(AB)^t = B^t A^t$$

Using eq (i)

$$(AB)^t = BA$$

Using eq (ii)

$$(AB)^t = AB$$

Hence, AB is symmetric

Q.NO. 70-

If A is symmetric or skew-symmetric, show that A^2 is symmetric.

A is symmetric

$$A^t = A$$

A is skew-symmetric

$$A^t = -A$$

Show:

$\therefore A^2$ is symmetric

$$= (A^2)^t$$

$$\begin{aligned}
 &= (A \cdot A)^t \\
 &= A^t \cdot A^t \\
 \text{As } , \quad A^t &= A \\
 &= A \cdot A \\
 &= A^2
 \end{aligned}$$

Hence, A^2 is symmetric

$\therefore A^2$ is skew-symmetric

$$\begin{aligned}
 &= (A^2)^t \\
 &= (A \cdot A)^t \\
 &= A^t \cdot A^t
 \end{aligned}$$

$$\begin{aligned}
 \text{As } , \quad A^t &= -A \\
 &= -A \cdot -A \\
 &= A^2
 \end{aligned}$$

Hence, A^2 is skew-symmetric

Q.NO.38-

If A is any square matrix of order 3, show that

(i) $A + A^t$ is symmetric

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^t = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 2a_{11} & a_{12} + a_{21} & a_{13} + a_{31} \\ a_{21} + a_{12} & 2a_{22} & a_{23} + a_{32} \\ a_{31} + a_{13} & a_{23} + a_{32} & 2a_{33} \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2a_{11} & a_{31} + a_{13} & a_{13} + a_{31} \\ a_{12} + a_{21} & 2a_{22} & a_{23} + a_{32} \\ a_{31} + a_{13} & a_{23} + a_{32} & 2a_{33} \end{bmatrix}$$

As, $A + A^t = (A + A^t)^t$

Hence, $A + A^t$ is symmetric

i) $A - A^t$ is skew-symmetric

$$A - A^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} - a_{11} & a_{12} - a_{21} & a_{13} - a_{31} \\ a_{21} - a_{12} & a_{22} - a_{22} & a_{23} - a_{32} \\ a_{31} - a_{13} & a_{32} - a_{23} & a_{33} - a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a_{12} - a_{21} & a_{13} - a_{31} \\ a_{21} - a_{12} & 0 & a_{23} - a_{32} \\ a_{31} - a_{13} & a_{32} - a_{23} & 0 \end{bmatrix}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & -a_{12} + a_{21} & -a_{31} + a_{13} \\ -a_{21} + a_{12} & 0 & -a_{32} + a_{23} \\ -a_{31} + a_{13} & -a_{23} + a_{32} & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & a_{12} - a_{21} & a_{13} - a_{31} \\ a_{21} - a_{12} & 0 & a_{23} - a_{32} \\ a_{31} - a_{13} & a_{32} - a_{23} & 0 \end{bmatrix}$$

As, $A - A^t = -(A - A^t)^t$

Hence, $A - A^t$ is skew-symmetric

Q.NO.68-

If $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$, show that

$$\bar{A} = \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

(i) $A + (\bar{A})^t$ is hermitian.

$$\begin{aligned} A + (\bar{A})^t &= \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} + \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2+i \\ 2-i & 0 \end{bmatrix} \end{aligned}$$

$$\overline{A + (\bar{A})^t} = \begin{bmatrix} 0 & 2-i \\ 2+i & 0 \end{bmatrix}$$

$$(\overline{A + (\bar{A})^t})^t = \begin{bmatrix} 0 & 2+i \\ 2-i & 0 \end{bmatrix}$$

So, $(\overline{A + (\bar{A})^t})^t = A + (\bar{A})^t$ is hermitian

$A - (\bar{A})^t$ is skew-hermitian

$$A - (\bar{A})^t = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} - \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

$$= \begin{bmatrix} i+i & 1+i-1 \\ 1-1+i & -i-i \end{bmatrix}$$

$$= \begin{bmatrix} 2i & i \\ i & -2i \end{bmatrix}$$

$$\overline{A - (\bar{A})^t} = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}$$

$$(\overline{A - (\bar{A})^t})^t = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}$$

$$= - \begin{bmatrix} 2i & i \\ i & -2i \end{bmatrix}$$

$$= - (A - (\bar{A})^t)$$

Hence, $A - (\bar{A})^t$ is skew-hermitian

Q. NO. 8:-

If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, find $A(\bar{A})^t$.

$$\bar{A} = \begin{bmatrix} 1 \\ 1-i \\ -i \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} 1 & 1-i & -i \end{bmatrix}$$

$$A(\bar{A})^t = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix} \begin{bmatrix} 1 & 1-i & -i \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 1-i^2 & -i-i^2 \\ 1 & i-i^2 & -i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 1+1 & -i+1 \\ 1 & i+1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 2 & 1-i \\ 1 & 1+i & 1 \end{bmatrix}$$

Q. NO. 5:-

Show that AA^t and A^tA are symmetric (and $AB = BA$, show that AB is sym^x) for any matrix of order 2×3 .

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A^tA = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

$$A^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11})^2 + (a_{12})^2 + (a_{13})^2 & a_{11} \times a_{21} + a_{12} \times a_{22} + a_{13} \times a_{23} \\ a_{21} \times a_{11} + a_{22} \times a_{12} + a_{23} \times a_{13} & (a_{21})^2 + (a_{22})^2 + (a_{23})^2 \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} (a_{11})^2 + (a_{12})^2 + (a_{13})^2 & a_{21} \times a_{11} + a_{22} \times a_{12} + a_{23} \times a_{13} \\ a_{11} \times a_{21} + a_{12} \times a_{22} + a_{13} \times a_{23} & (a_{21})^2 + (a_{22})^2 + (a_{23})^2 \end{bmatrix}$$

It is symmetric

$${}^tA = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11})^2 + (a_{21})^2 & a_{11} \times a_{12} + a_{21} \times a_{22} & a_{11} \times a_{13} + a_{21} \times a_{23} \\ a_{11} \times a_{12} + a_{21} \times a_{22} & (a_{12})^2 + (a_{22})^2 & a_{12} \times a_{13} + a_{22} \times a_{23} \\ a_{11} \times a_{13} + a_{21} \times a_{23} & a_{12} \times a_{13} + a_{22} \times a_{23} & (a_{13})^2 + (a_{23})^2 \end{bmatrix}$$

$$({}^tA)^t = \begin{bmatrix} (a_{11})^2 + (a_{21})^2 & a_{11} \times a_{12} + a_{21} \times a_{22} & a_{11} \times a_{13} + a_{21} \times a_{23} \\ a_{11} \times a_{12} + a_{21} \times a_{22} & (a_{12})^2 + (a_{22})^2 & a_{12} \times a_{13} + a_{22} \times a_{23} \\ a_{11} \times a_{13} + a_{21} \times a_{23} & a_{12} \times a_{13} + a_{22} \times a_{23} & (a_{13})^2 + (a_{23})^2 \end{bmatrix}$$

It is symmetric

Q.NO. 9:-

Find the inverse of the following matrices. Also find:

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj} A}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -2 & 0 \\ -2 & 2 \end{vmatrix} - 2 \begin{vmatrix} 0 & 0 \\ -2 & 2 \end{vmatrix} - 3 \begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix}$$

$$= 1[(-2)(2) - (0)(-2)] - 2[(0)(2) - (0)(-2)] - 3[(0)(-2) - (-2)(-2)]$$

$$= 1(-4 + 0) - 2(0 + 0) - 3(0 - 4)$$

$$= -4 - 0 + 12$$

$$= 8$$

$$\text{Adj} A = (\text{co-factors of } A)^t$$

$$= \begin{bmatrix} +(-4+0) & -(0+0) & +(0-4) \\ -(4-6) & +(2-6) & -(-2+4) \\ +(0-6) & -(0+0) & +(-2-0) \end{bmatrix}^t$$

$$= \begin{bmatrix} -4 & 0 & -4 \\ 2 & -4 & -2 \\ -6 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/8_2 & 1/8_4 & -3/8_4 \\ 0/8 & -1/8_2 & 0/8 \\ -1/8_2 & -2/8_4 & -7/8_4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/2 & 1/4 & -3/4 \\ 0 & -1/2 & 0 \\ -1/2 & -1/4 & -1/4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{\text{adj } B}{|B|}$$

$$|B| = \begin{vmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -1 & 3 & -2 & 0 & 3 & -1 & 0 & -1 \\ 0 & 2 & 1 & 2 & 1 & 1 & 0 \end{vmatrix}$$

$$1[(-1)(2) - (3)(0)] - 2[(0)(2) - (3)(1)] - 1[(0)(0) - (-1)(1)]$$

$$1(-2 - 0) - 2(0 - 3) - 1(0 + 1)$$

$$= 1(-2) - 2(-3) - 1(1)$$

$$= -2 + 6 - 1$$

$$= 3$$

$$\text{adj } B = [\text{co-factors of } B]^T$$

$$= \begin{bmatrix} +(-2-0) & -(0-3) & +(0+1) \\ -(4+0) & +(2+1) & -(-2+0) \\ +(6-1) & -(3+0) & +(-1-0) \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & 3 & 1 \\ -4 & 3 & 2 \\ 5 & -3 & -1 \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 1 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{3} \begin{bmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2/3 & -4/3 & 5/3 \\ 1/3 & 1/3 & -1/3 \\ 1/3 & 2/3 & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} -2/3 & -4/3 & 5/3 \\ 1 & 1 & -1 \\ 1/3 & 2/3 & -1/3 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$C^{-1} = \frac{\text{adj } C}{|C|}$$

$$|C| = \begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & -(-3) & 2 & 0 & +2 & 2 & 1 \\ -1 & 1 & & 0 & 1 & & 0 & -1 \end{vmatrix}$$

$$= 1[(1)(1) - (0)(-1)] + 3[(2)(1) - (0)(0)] + 2[(2)(-1) - (0)(1)]$$

$$= 1(1+0) + 3(2-0) + 2(-2-0)$$

$$= 1(1) + 3(2) + 2(-2)$$

$$= 1 + 6 - 4$$

$$= 3$$

$$\text{adj } C = [\text{co-factors of } C]^t$$

$$= \begin{bmatrix} +(1+0) & -(2-0) & +(-2-0) \\ -(-3+2) & +(1-0) & -(-1+0) \\ +(-0-2) & -(0-4) & +(1+6) \end{bmatrix}^t$$

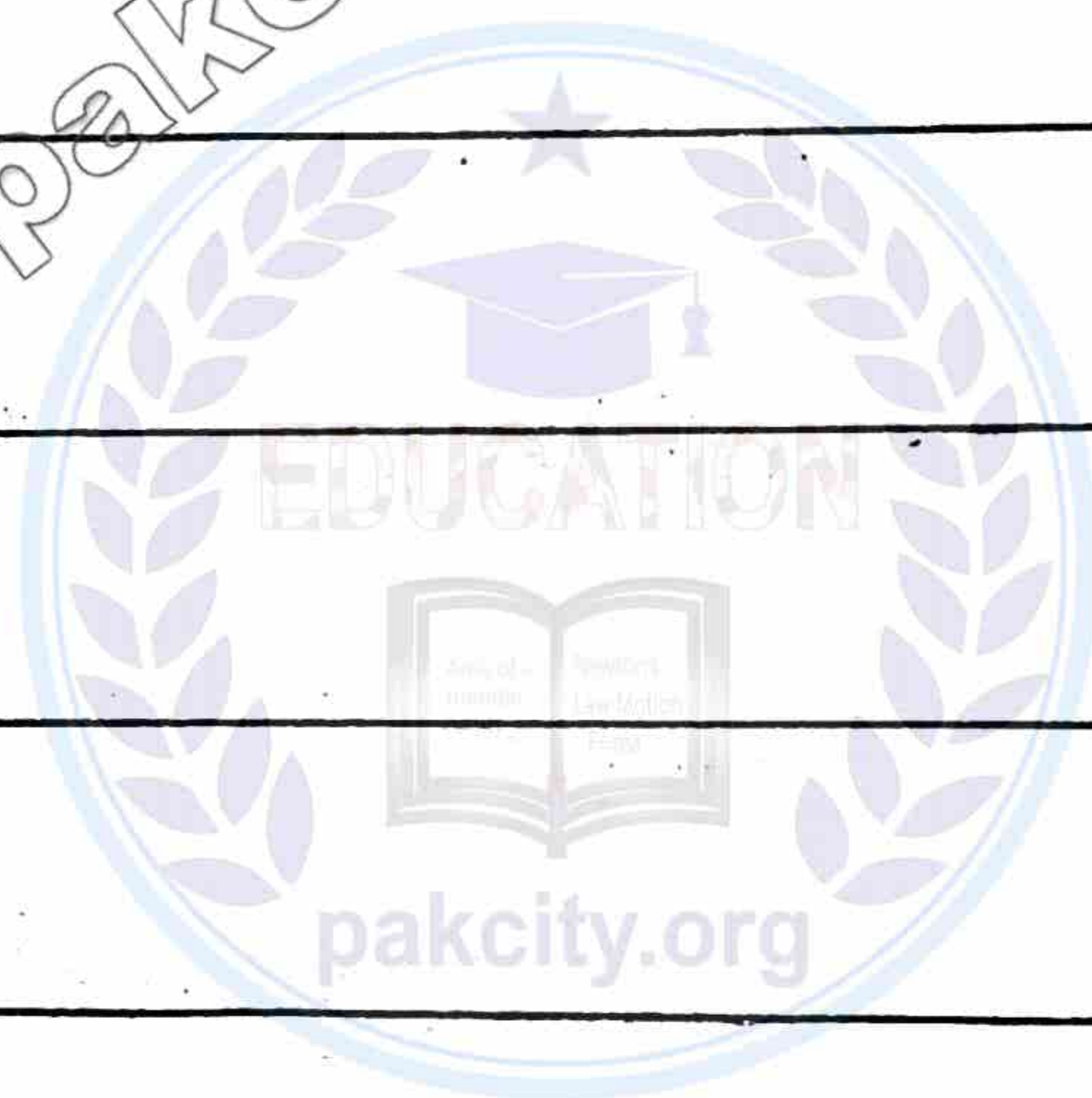
$$= \begin{bmatrix} 1 & -2 & -2 \\ 1 & 1 & 1 \\ -2 & 4 & 7 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 & -2 \\ -2 & 1 & 4 \\ -2 & 1 & 7 \end{bmatrix}$$

$$C^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & -2 \\ -2 & 1 & 4 \\ -2 & 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 & 1/3 & -2/3 \\ -2/3 & 1/3 & 4/3 \\ -2/3 & 1/3 & 7/3 \end{bmatrix}$$

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Exercise 3.5

Q. NO. 1:-

Solve the following systems of linear equations by Cramer's rule.

$$(i) \begin{cases} 2x + 2y + z = 3 \\ 3x - 2y - 2z = 1 \\ 5x + y - 3z = 2 \end{cases}$$

$$\begin{bmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$A \cdot X = B$$

$$x = \frac{|A_x|}{|A|}$$

$$y = \frac{|A_y|}{|A|}$$

$$z = \frac{|A_z|}{|A|}$$

$$|A| = \begin{vmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{vmatrix}$$

$$= 2 \begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2 \begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} + 1 \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= 2[(-2)(-3) - (-2)(1)] - 2[(3)(-3) - (-2)(5)] + 1[(3)(1) - (-2)(5)] \\
 &= 2(6+2) - 2(-9+10) + 1(3+10) \\
 &= 2(8) - 2(1) + 1(13) \\
 &= 16 - 2 + 13 \\
 &= 27
 \end{aligned}$$

$$|A_x| = \begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= 3 \begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2 \begin{vmatrix} -2 & 1 \\ 2 & -3 \end{vmatrix} + 1 \begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= 3[(-2)(-3) - (-2)(1)] - 2[(1)(-3) - (-2)(2)] + 1[(1)(1) - (-2)(2)] \\
 &= 3(6+2) - 2(-3+4) + 1(1+4) \\
 &= 3(8) - 2(1) + 1(5) \\
 &= 24 - 2 + 5 \\
 &= 27
 \end{aligned}$$

$$|A_y| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} - 3 \begin{vmatrix} -2 & -3 \\ 5 & -3 \end{vmatrix} + 1 \begin{vmatrix} -2 & 3 \\ 5 & 2 \end{vmatrix}$$

$$\begin{aligned}
 &= 2[(1)(-3) - (-2)(2)] - 3[(-2)(-3) - (-2)(5)] + 1[(3)(2) - (1)(5)] \\
 &= 2(-3+4) - 3(-9+10) + 1(6-5) \\
 &= 2(1) - 3(1) + 1(1)
 \end{aligned}$$

$$= 2 - 3 + 1$$

$$= 0$$

$$|Az| = \begin{vmatrix} 2 & 2 & 3 \\ 3 & -2 & 1 \\ 5 & 1 & 2 \end{vmatrix}$$

$$= 2 \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} + 3 \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$$

$$= 2[(-2)(2) - (1)(1)] - 2[(3)(2) - (1)(5)] + 3[(3)(1) - (-2)(5)]$$

$$= 2(-4 - 1) - 2(6 - 5) + 3(3 + 10)$$

$$= 2(-5) - 2(1) + 3(13)$$

$$= -10 - 2 + 39$$

$$= 27$$

$$x = \frac{|Ax|}{|A|} = \frac{27}{27} = 1$$

$$|A| = 27$$

$$y = \frac{|Ay|}{|A|} = \frac{0}{27} = 0$$

$$z = \frac{|Az|}{|A|} = \frac{27}{27} = 1$$

(ii)

$$2x_1 - x_2 + x_3 = 5$$

$$4x_1 + 2x_2 + 3x_3 = 8$$

$$3x_1 - 4x_2 - x_3 = 3$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 4 & 2 & 3 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix}$$

$$A X = B$$

$$x_1 = \frac{|Ax_1|}{|A|}$$

$$|A|$$

$$x_2 = \frac{|Ax_2|}{|A|}$$

$$|A|$$

$$x_3 = \frac{|Ax_3|}{|A|}$$

$$|A|$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 4 & 2 & 3 \\ 3 & -4 & -1 \end{vmatrix}$$

Expand by R_1

$$= 2 \begin{vmatrix} 2 & 3 \\ -4 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 2 \\ 3 & -4 \end{vmatrix}$$

$$= 2[(2)(-1) - (3)(-4)] + 1[(4)(-1) - (3)(3)] + 1[(4)(-4) - (2)(3)]$$

$$= 2(-2 + 12) + 1(-4 - 9) + 1(-16 - 6)$$

$$= 2(10) + 1(-13) + 1(-22)$$

$$= 20 - 13 - 22$$

$$|A| = -15$$

$$|Ax_1| = \begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & 3 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 5 \begin{vmatrix} 2 & 3 \\ -4 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 8 & 3 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 8 & 2 \\ 3 & -4 \end{vmatrix}$$

$$\begin{aligned}
 &= 5[(2)(-1) - (3)(-4)] + 1[(8)(1) - (3)(3)] + 1[(8)(-4) - (2)(3)] \\
 &= 5(-2 + 12) + 1(8 - 9) + 1(-32 - 6) \\
 &= 5(10) + 1(-1) + 1(-38) \\
 &= 50 - 17 - 38 \\
 &= -5
 \end{aligned}$$

$$|Ax_2| = \begin{vmatrix} 2 & 5 & 1 \\ 4 & 8 & 3 \\ 3 & 3 & -1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 8 & 3 \\ 3 & -1 \end{vmatrix} - 5 \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 8 \\ 3 & 3 \end{vmatrix}$$

$$= 2[(8)(-1) - (3)(3)] - 5[(4)(-1) - (3)(3)] + 1[(4)(3) - (8)(3)]$$

$$= 2(-8 - 9) - 5(-4 - 9) + 1(12 - 24)$$

$$= 2(-17) - 5(-13) + 1(-12)$$

$$= -34 + 65 - 12$$

$$= 19$$

$$|Ax_3| = \begin{vmatrix} 2 & -1 & 5 \\ 4 & 2 & 8 \\ 3 & -4 & 3 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 2 & 8 \\ -4 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 4 & 8 \\ 3 & 3 \end{vmatrix} + 5 \begin{vmatrix} 4 & 2 \\ 3 & -4 \end{vmatrix}$$

$$= 2[(2)(3) - (8)(-4)] + 1[(4)(3) - (8)(3)] + 5[(4)(-4) - (2)(3)]$$

$$= 2(6 + 32) + 1(12 - 24) + 5(-16 - 6)$$

$$= 2(38) + 1(-12) + 5(-22)$$

$$= 76 - 12 - 110$$

$$= 76 - 122$$

$$= 46$$

$$x_1 = \frac{|Ax_1|}{|A|} = \frac{+81}{+183} = \frac{1}{3}$$

$$x_2 = \frac{|Ax_2|}{|A|} = \frac{19}{-15}$$

$$x_3 = \frac{|Ax_3|}{|A|} = \frac{+46}{+15} = \frac{46}{15}$$

$$\begin{cases} 2x_1 - x_2 + x_3 = 8 \\ x_1 + 2x_2 + 2x_3 = 6 \\ x_1 - 2x_2 - x_3 = 1 \end{cases}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 1 \end{bmatrix}$$

$$A X = B$$

$$x_1 = \frac{|Ax_1|}{|A|}$$

$$x_2 = \frac{|Ax_2|}{|A|}$$

$$x_3 = \frac{|Ax_3|}{|A|}$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 2 & 2 \\ -2 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix}$$

$$= 2[(2)(-1) - (2)(-2)] + 1[(1)(-1) - (2)(1)] + 1[(1)(-2) - (2)(1)]$$

$$= 2(-2 + 4) + 1(-1 - 2) + 1(-2 - 2)$$

$$= 2(2) + 1(-3) + 1(-4)$$

$$= 4 - 3 - 4$$

$$= -3$$

$$|Ax_1| = \begin{vmatrix} 8 & -1 & 1 \\ 6 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 8 \begin{vmatrix} 2 & 2 \\ -2 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 6 & 2 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 6 & 2 \\ 1 & -2 \end{vmatrix}$$

$$= 8[(2)(-1) - (2)(-2)] + 1[(6)(-1) - (2)(1)] + 1[(6)(-2) - (2)(1)]$$

$$= 8(-2 + 4) + 1(-6 - 2) + 1(-12 - 2)$$

$$= 8(2) + 1(-8) + 1(-14)$$

$$= 16 - 8 - 14$$

$$= -6$$

$$|Ax_2| = \begin{vmatrix} 2 & 8 & 1 \\ 1 & 6 & 2 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 6 & 2 & -8 \\ 1 & -1 & 1 & -1 \end{vmatrix} + \begin{vmatrix} 2 & +1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 6 \\ 1 \end{vmatrix}$$

$$= 2[(6)(-1) - (2)(1)] - 8[(1)(-1) - (2)(1)] + 1[(1)(1) - (6)(1)]$$

$$= 2(-6 - 2) - 8(-1 - 2) + 1(1 - 6)$$

$$= 2(-8) - 8(-3) + 1(-5)$$

$$= -16 + 24 - 5$$

$$= 3$$

$$|Ax_3| = \begin{vmatrix} 2 & -1 & 8 \\ 1 & 2 & 6 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 6 & -(-1) \\ -2 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 6 & +8 \\ 1 & -2 \end{vmatrix} + \begin{vmatrix} 2 \\ 1 \end{vmatrix}$$

$$= 2[(2)(1) - (6)(-2)] + 1[(1)(1) - (6)(1)] + 8[(1)(-2) - (2)(1)]$$

$$= 2(2 + 12) + 1(1 - 6) + 8(-2 - 2)$$

$$= 2(14) + 1(-5) + 8(-4)$$

$$= 28 - 5 - 32$$

$$= -9$$

$$x_1 = \frac{|Ax_1|}{|A|} = \frac{+6}{+3} = 2$$

$$x_2 = \frac{|Ax_2|}{|A|} = \frac{-8}{-3} = -1$$

$$x_3 = \frac{|Ax_3|}{|A|} = \frac{+9}{+3} = 3$$

Q. NO. 2:-

Use matrices to solve the following system:

$$(i) \quad \left. \begin{aligned} x - 2y + z &= 1 \\ 3x + y - 2z &= 4 \\ y - z &= 1 \end{aligned} \right\}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$$A X = B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A|$$

$$|A| = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} - (-2) \begin{vmatrix} 3 & -2 \\ 0 & -1 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 1[(1)(-1) - (-2)(1)] + 2[(3)(-1) - (0)(-2)] + 1[(3)(1) - (1)(0)]$$

$$= 1(-1 + 2) + 2(-3 + 0) + 1(3 - 0)$$

$$= 1(1) + 2(-3) + 1(3)$$

$$= 1 - 6 + 3$$

$$= -2$$

$$\text{Adj } A = [\text{co-factors of } A]^t$$

$$= \begin{bmatrix} (-1+2) & -(-3+0) & (3-0) \\ -(2-1) & (-1-0) & -(1+0) \\ (4-1) & -(-2-3) & (1+6) \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ -1 & -1 & -1 \\ 3 & 5 & 7 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 3 & -1 & 5 \\ 3 & -1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 1 & -1 & 3 \\ 3 & -1 & 5 \\ 3 & -1 & 7 \end{bmatrix}$$

$$A^{-1}B = \frac{1}{-2} \begin{bmatrix} 1 & -1 & 3 \\ 3 & -1 & 5 \\ 3 & -1 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$$= \frac{-1}{2} \begin{bmatrix} -1-4+3 \\ -3-4+5 \\ -3-4+7 \end{bmatrix}$$

$$= \frac{-1}{2} \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/-2 \\ -2/-1 \\ 0/-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$x = 1, y = 1, z = 0$$

(ii)

$$\left. \begin{aligned} 2x_1 + x_2 + 3x_3 &= -3 \\ x_1 + x_2 - 2x_3 &= 0 \\ -3x_1 - x_2 + 2x_3 &= -4 \end{aligned} \right\}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & -2 \\ -3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ -4 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & -2 \\ -3 & -1 & 2 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix} - 1 \begin{vmatrix} -2 & 1 \\ -3 & 2 \end{vmatrix} + 3 \begin{vmatrix} 1 & 1 \\ -3 & -1 \end{vmatrix}$$

$$= 2[(1)(2) - (-2)(-1)] - 1[(-2)(2) - (-3)(-1)] + 3[(1)(-1) - (-1)(-3)]$$

$$= 2(2-2) - 1(2-6) + 3(-1+3)$$

$$= 2(0) - 1(-4) + 3(2)$$

$$= 0 + 4 + 6$$

$$= 10$$

$$\text{Adj } A = [\text{co-factors of } A]^t$$

$$= \begin{bmatrix} (2-2) & -(2-6) & (-1+3) \\ -(2+3) & (4+9) & -(-2+3) \\ (-2-3) & -(-4-3) & (2-1) \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 4 & 2 \\ -5 & 13 & -1 \\ -5 & 7 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & -5 & -5 \\ 4 & 13 & 7 \\ 2 & -1 & 1 \end{bmatrix}$$

$$A^{-1}B = \frac{1}{10} \begin{bmatrix} 0 & -5 & -5 \\ 4 & 13 & 7 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ -4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 0 - 0 + 20 \\ -12 + 0 - 28 \\ -6 - 0 - 4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 20 \\ -40 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} 20/10 \\ -46/10 \\ -10/10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4 \\ -1 \end{bmatrix}$$

$$x = 2, y = -4, z = -1$$

(iii)

$$x + y + 0z = 2$$

$$2x + 0y - z = 1$$

$$0x + 2y - 3z = -1$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$A X = B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 2 & -3 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & -1 \\ 2 & -3 \end{vmatrix} - 1 \begin{vmatrix} 2 & -1 \\ 0 & -3 \end{vmatrix} + 0$$

$$= 1[(0)(-3) - (-1)(2)] - 1[(2)(-3) - (-1)(0)] + 0$$

$$= 1(0+2) - 1(-6+0) + 0$$

$$= 1(2) - 1(-6) + 0$$

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$$= 2 + 6 + 0$$

$$= 8$$

$$\text{Adj}^o A = \begin{bmatrix} (0+2) & -(-6+0) & (4-0) \\ -(-3-0) & (-3-0) & -(2-0) \\ (-1-0) & -(-1-0) & (0-2) \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 6 & 4 \\ 3 & -3 & -2 \\ -1 & 1 & -2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 & -1 \\ 6 & -3 & 1 \\ 4 & -2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} 2 & 3 & -1 \\ 6 & -3 & 1 \\ 4 & -2 & -2 \end{bmatrix}$$

$$A^{-1}B = \frac{1}{8} \begin{bmatrix} 2 & 3 & -1 \\ 6 & -3 & 1 \\ 4 & -2 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 4+3+1 \\ 12-3-1 \\ 8-2+2 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 8 \\ 8 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8/8 \\ 8/8 \\ 8/8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x = 1, y = 1, z = 1$$