(Short Questions)

(i)

Find the value: with out using table:
$\tan(1110^\circ)$ (LHR-2011+22)

Answer:- $\tan(1110^\circ)$

$$90\overline{)1110}\ \ 12, \quad 1080, \quad 30^\circ$$

$= \tan(12\times 90^\circ + 30^\circ)$

$= \tan 30$

$= 1/\sqrt{3}$

(ii)

Express $2\sin 7\theta \cos 3\theta$ as sum and difference (LHR-2011+16)

Answer:-

$2\sin 7\theta \cos 3\theta$

using formula:
$\because \sin(\alpha+\beta)+\sin(\alpha+\beta) = 2\sin\alpha\cos\beta$

$= \sin(7\theta+3\theta)+\sin(7\theta-3\theta)$

$= \sin 10\theta + \sin 4\theta$

(iii)

If α, β, γ are angle of triangle ABC prove that $\tan\alpha+\tan\beta+\tan\gamma = \tan\alpha\tan\beta\tan\gamma$ (LHR-2013+11)

Answer:-

$\tan\alpha+\tan\beta+\tan\gamma = \tan\alpha\tan\beta\tan\gamma$

let:- As α, β, γ are the angles of triangle ABC let: $\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta=180^\circ-\gamma$

Multiply tan on both side:

$\tan(\alpha+\beta)=\tan(180^\circ-\gamma)$

$\tan(\alpha+\beta)=-\tan\gamma$

$\because$ Using formula:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=-\tan\gamma \Rightarrow \tan\alpha+\tan\beta=-\tan\gamma+\tan\alpha\tan\beta\tan\gamma$$

$$\tan\alpha+\tan\beta+\tan\gamma=\tan\alpha\tan\beta\tan\gamma$$

Hence, prove.

(iv)

If α,β,γ are angles of triangle ABC prove $\tan(\alpha+\beta)+\tan\gamma=0$. (LHR-2012+13+14+17+18)

Answer:- let, $\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta=180^\circ-\gamma$

Multiply tan both side:-

$\tan(\alpha+\beta)=\tan(180^\circ-\gamma)$

$\tan(\alpha+\beta)=-\tan\gamma$

$\tan(\alpha+\beta)+\tan\gamma=0$ Hence, proved

(v)

Prove:- $\dfrac{\sin 3x-\sin x}{\cos x-\cos 3x}=\cot 2x$ (2011+12+13 ......)

Answer:-

$$\frac{\sin 3x-\sin x}{\cos x-\cos 3x}=\cot 2x$$

$$L.H.S = \frac{\sin 3x - \sin x}{\cos x - \cos 3x}$$

using formula:-

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$= \frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}$$

$$= \frac{2\cos 2x \sin x}{-2\sin 2x(-\sin x)} \Rightarrow \frac{\cos 2x}{\sin 2x} \Rightarrow \cot 2x.$$

So, L.H.S = R.H.S

(vi)

Prove: $1+\tan\alpha\tan 2\alpha = \sec 2\alpha$. (LHR-2012+14)

Answer:- $1+\tan\alpha\tan 2\alpha = \sec 2\alpha$

$$L.H.S = 1+\tan\alpha\tan 2\alpha$$

$$= 1 + \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin 2\alpha}{\cos 2\alpha} \Rightarrow \frac{(\cos\alpha)(\cos 2\alpha)+\sin\alpha\sin 2\alpha}{(\cos\alpha)(\cos 2\alpha)}$$

$$= \frac{\cos\alpha\cos 2\alpha + \sin\alpha\sin 2\alpha}{\cos\alpha\cos 2\alpha} \Rightarrow \frac{\cos(\alpha-2\alpha)}{\cos\alpha\cos 2\alpha} \Rightarrow \frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos 2\alpha}$$

$$= \frac{1}{\cos 2\alpha} = \sec 2\alpha$$

So, L.H.S = R.H.S.

(7)

Write expression: $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ (LHR-2012)

Answer:-

- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(8)

Prove: $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta$ (LHR-2012+16)

Answer:- $L.H.S = \sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)$

$$=\left(\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right)+\left(\cos\theta\cos\frac{\pi}{3}-\sin\theta\sin\frac{\pi}{3}\right)$$

using formula:

$\because \sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\because \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\left(\sin\theta\frac{\sqrt{3}}{2}+\cos\theta\frac{1}{2}\right)+\left(\cos\theta\frac{1}{2}-\sin\theta\frac{\sqrt{3}}{2}\right)$$

$$=\cancel{\sin\theta\frac{\sqrt{3}}{2}}+\cos\theta\frac{1}{2}+\cos\theta\frac{1}{2}-\cancel{\sin\theta\frac{\sqrt{3}}{2}} \Rightarrow \frac{\cancel{2}\cos\theta}{\cancel{2}}$$

$$=\cos\theta$$

So, L.H.S = R.H.S.

(9)

Express following (X) as (+) and (−).

$\sin 12^\circ \sin 46^\circ$ (LHR-2012)

Answer:- $\sin 12^\circ \sin 46^\circ$

÷ and X by −2

Using formula:-

$\because \cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$

$$=\frac{1}{-2}\left(-2\sin 12\sin 46^\circ\right)$$

$$=-\frac{1}{2}\left(\cos(12+46)-\cos(12-46)\right) \Rightarrow -\frac{1}{2}\left(\cos 58^\circ-\cos 34^\circ\right)$$

(10)

Express (+) and (−) into (X) $\sin 5x+\sin 3x$. (LHR-2013)

Answer:- $\sin 5x+\sin 3x$

Using formula:-

$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$.

$$=2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$$

$$=2\sin 4x\cos x$$

(11)

Prove $\cos 3\alpha$. (LHR-2013)

Answer: $\cos 3\alpha = \cos(2\alpha+\alpha)$

$= \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha$

$= (2\cos^2\alpha - 1)\cos\alpha - (2\sin\alpha\cos\alpha)\sin\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\sin^2\alpha\cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2(1-\cos^2\alpha)\cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$

$= 4\cos^3\alpha - 3\cos\alpha$

(12)

Prove: $\dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$ (LHR-2013+19)

Answer: R.H.S $= \tan 56^\circ$

Using formula:- $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \tan(45^\circ + 11^\circ)$

$= \dfrac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ} \Rightarrow \dfrac{1 + \tan 11^\circ}{1 - \tan 11^\circ} \Rightarrow \dfrac{1 + \dfrac{\sin 11^\circ}{\cos 11^\circ}}{1 - \dfrac{\sin 11^\circ}{\cos 11^\circ}}$

$= \dfrac{\dfrac{\cos 11^\circ + \sin 11^\circ}{\cancel{\cos 11^\circ}}}{\dfrac{\cos 11^\circ - \sin 11^\circ}{\cancel{\cos 11^\circ}}} \Rightarrow \dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$

So, L.H.S = R.H.S.

(13)

Prove $\tan(270^\circ - \theta) = \cot\theta$ (LHR-2013)

Answer:- $\tan(270^\circ-\theta)=\cot\theta$

Method - I

$$L.H.S = \tan(270^\circ-\theta)$$
$$= \tan\left(3\frac{\pi}{2}-\theta\right)$$
$$= \cot\theta$$

Method - II

$$L.H.S = \tan(270^\circ-\theta)$$

Using: $\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$.

$$= \frac{\tan 270^\circ - \tan\theta}{1+\tan 270^\circ \tan\theta} \Rightarrow \frac{\frac{\sin 270^\circ}{\cos 270^\circ} - \frac{\sin\theta}{\cos\theta}}{1+\frac{\sin 270^\circ}{\cos 270^\circ}\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin 270^\circ\cos\theta - \cos 270^\circ\sin\theta}{\cancel{\cos 270\cos\theta}}}{\frac{\cos 270\cos\theta + \sin\theta\sin 270}{\cancel{\cos 270\cos\theta}}} \Rightarrow \frac{-\cos\theta}{-\sin\theta} \Rightarrow \cot\theta.$$

(16)

$\tan(180^\circ+\theta)=\tan\theta$.

$$L.H.S = \tan\theta$$

(14)

$\cos(\theta-180^\circ) = -\cos\theta$

$L.H.S$
$$= +\cos\theta(\theta-\pi)$$
$$= -\cos\theta. \quad \text{(LHR-2013.)}$$

So, $L.H.S = R.H.S.$

16

(15)

$\sin(180^\circ+\theta) = -\sin\theta$

$L.H.S.$
$$= \sin(\pi+\theta)$$
$$= -\sin\theta. \quad \text{(LHR-2015.)}$$

So, $L.H.S = R.H.S.$

(17)

$\sin 105^\circ$

$$= \sin(60^\circ+45^\circ)$$
$$= \sin 60^\circ\cos 45^\circ + \cos 60^\circ \sin 45^\circ$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\,\frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)$$
$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right) \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(18)

$\cos 105^\circ$

$$= \cos(60^\circ+45^\circ)$$
$$= \cos 60^\circ\cos 45 - \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)$$
$$= \frac{1}{\sqrt{2}}\left(\frac{1-\sqrt{3}}{2}\right) \Rightarrow \frac{1-\sqrt{3}}{2\sqrt{2}}$$

(19)

Define allied angle and illustrated with ex? (LHR-2014)

Answer

|| The angles associated with basic angles ||

of measure θ to a right angle triangle or its multiple are called allied angle.

Example:— Angle of measure of $90°\pm\theta$, $180°\pm\theta$, $270°\pm\theta$ $360°\pm\theta$ are know as allied angles

$= \sin\left(\frac{\pi}{2} - \theta = \cos\theta\right)$ etc.

(20)

Prove $\sin(180°+\alpha)\sin(90°-\alpha) = -\sin\alpha\cos\alpha$.

(LHR– 2017)

Answer:

$\sin(180°+\alpha)\sin(90°-\alpha) = -\sin\alpha\cos\alpha$

$L.H.S = \sin(180°+\alpha)\sin(90°-\alpha)$

$= +\sin(2\times90°+\alpha)\sin(90-\alpha) \Rightarrow -\sin\alpha\cos\alpha$

Note:— Because 180 is 2×90 and 2 is even multiple and 90° has 1 multiple and it is odd multiple.

(21)

(22)

cos 315° (LHR-2016)

solve:— $\cos 315°$

$$90\overline{)315}^{\,3}, \quad 270, \quad 45°$$

$= \cos(3\times90°+45°)$

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$

sin 540° (LHR-2018)

solve:— $\sin 540°$

$$90\overline{)540}^{\,6}, \quad 540, \quad 0$$

$= \sin(6\times90°+0°)$

$= \sin 0°$

$= 0$

(23)

$\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$

(LHR-2017)

(24)

$\cos(\alpha+\beta) = -\cos\gamma$

(LHR - 2016)

solve:- let:

$\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta=180^\circ-\gamma$

Divide both side by 2

$\frac{\alpha}{2}+\frac{\beta}{2}=\frac{180^\circ}{2}-\frac{\gamma}{2}$

$\frac{\alpha+\beta}{2}=90^\circ-\frac{\gamma}{2}$

Apply cos on both side:-

$\cos\left(\frac{\alpha+\beta}{2}\right)=\cos\left(90^\circ-\frac{\gamma}{2}\right)$

$\cos\left(\frac{\alpha+\beta}{2}\right)=\sin\frac{\gamma}{2}$

solve:- let:

$\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta=180^\circ-\gamma.$

Apply cos on both side:-

$\cos(\alpha+\beta)=\cos(180^\circ-\gamma)$

$\cos(\alpha+\beta)=\cos(2\times90^\circ-\gamma)$

$\cos(\alpha+\beta)=\cos(2\times\frac{\pi}{2}-\gamma)$

$\cos(\alpha+\beta)=-\cos\gamma$

(25)

Without using table find values of all sin 75°. (LHR-2015)

Answer:-

$\sin 75^\circ$

$\because$ Using formula:-

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta.$

$=\sin(45^\circ+30^\circ)$

$=\sin45^\circ\cos30^\circ+\cos45^\circ\sin30^\circ$

$=\left(\frac{1}{\sqrt2}\right)\left(\frac{\sqrt3}{2}\right)+\left(\frac{1}{\sqrt2}\right)\left(\frac{1}{2}\right)\Rightarrow\frac{\sqrt3+1}{2\sqrt2}$

(26)

$\sin(45^\circ+\alpha)=\frac{1}{\sqrt2}(\sin\alpha+\cos\alpha)$ (LHR-2017)

Answer:

L.H.S $=\sin(45^\circ+\alpha)$

$\because$ Using formula:

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\sin45^\circ\cos\alpha+\cos45^\circ\sin\alpha$

$=\frac{1}{\sqrt2}\cos\alpha+\frac{1}{\sqrt2}\sin\alpha$

$=\frac{1}{\sqrt2}(\sin\alpha+\cos\alpha)$

(27)

$\cos(\alpha+45^\circ)=\frac{1}{\sqrt2}(\cos\alpha-\sin\alpha).$ (LHR-2015+17.)

Answer:

L.H.S $=\cos(\alpha+45^\circ)$

$\because$ Using formula:

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\cos\alpha\cos45^\circ-\sin\alpha\sin45^\circ$

$=\cos\alpha\frac{1}{\sqrt2}-\sin\alpha\frac{1}{\sqrt2}$

$=\frac{1}{\sqrt2}(\cos\alpha-\sin\alpha)$

(28)

Prove: $\tan(45^\circ+A)\tan(45^\circ-A)=1$. (LHR-2014.)

Answer:- $\tan(45^\circ+A)\tan(45^\circ-A)=1$.

Using formulas:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$L.H.S=\tan(45^\circ+A)\tan(45^\circ-A)$$

$$=\left(\frac{\tan45^\circ+\tan A}{1-(\tan45^\circ)(\tan A)}\right)\left(\frac{\tan45^\circ-\tan A}{1+\tan45^\circ\tan A}\right)$$

$$=\left(\frac{\cancel{1+\tan A}}{\cancel{1-\tan A}}\right)\left(\frac{\cancel{1-\tan A}}{\cancel{1+\tan A}}\right)=1.$$

So, L.H.S = R.H.S.

(29)

Prove: $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)=0$. (LHR-2018+22)

Answer:- L.H.S $=\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)$

Using formulas:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$=\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)+\left(\frac{\tan\frac{3\pi}{4}+\tan\theta}{1-\tan\frac{3\pi}{4}\tan\theta}\right)$$

$$=\left(\frac{1-\tan\theta}{1+\tan\theta}\right)+\left(\frac{-1+\tan\theta}{1+\tan\theta}\right)$$

$$=\frac{\cancel{1}-\cancel{\tan\theta}-\cancel{1}+\cancel{\tan\theta}}{1+\tan\theta}\Rightarrow\frac{0}{1+\tan\theta}=0.$$

SO, L.H.S = R.H.S

(30)

Prove: $\dfrac{\cos8^\circ-\sin8^\circ}{\cos8^\circ+\sin8^\circ}=\tan37^\circ$ (LHR-2016)

Answer: R.H.S $=\tan37^\circ$

$$= \tan(45^\circ - 8^\circ)$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \tan 8^\circ}$$

∴ using formula: $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$$= \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ} \Rightarrow \frac{1 - \frac{\sin 8^\circ}{\cos 8^\circ}}{1 + \frac{\sin 8^\circ}{\cos 8^\circ}} \Rightarrow \frac{\frac{\cos 8^\circ - \sin 8^\circ}{\cancel{\cos 8^\circ}}}{\frac{\cos 8^\circ + \sin 8^\circ}{\cancel{\cos 8^\circ}}}$$

$$= \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

So, L.H.S = R.H.S.

(31)

$$\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A. \quad (\text{LHR} - 2015)$$

Answer:- $L.H.S = \frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$

∴ using formulas: $\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\cos 2\alpha = 2\cos^2\alpha - 1$

$$= \frac{\sin A + 2\sin A\cos A}{\cancel{1} + \cos A + 2\cos^2 A - \cancel{1}} \Rightarrow \frac{\sin A(\cancel{1 + 2\cos A})}{\cos A(\cancel{1 + 2\cos A})} \Rightarrow \tan A$$

So, L.H.S = R.H.S.

(32)

$$\cot\alpha - \tan\alpha = 2\cot 2\alpha. \quad (\text{LHR} - 2018)$$

Answer:- $R.H.S = 2\cot 2\alpha$

Using formula: $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

$$= 2\left(\frac{1}{\tan 2\alpha}\right) = \cancel{2}\left(\frac{1 - \tan^2\alpha}{\cancel{2}\tan\alpha}\right) \Rightarrow \frac{1}{\tan\alpha} - \frac{\tan^{\cancel{2}}\alpha}{\cancel{\tan\alpha}} \Rightarrow \cot\alpha - \tan\alpha$$

So, L.H.S = R.H.S

(33)

$$\frac{1 - \cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}. \quad (\text{LHR} - 2019)$$

Answer:- $L.H.S = \frac{1 - \cos\alpha}{\sin\alpha} \Rightarrow \frac{2\sin^2\alpha/2}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

∴ $\sin\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}}$

$\sin^2\frac{\alpha}{2} = \frac{1 - \cos\alpha}{2}$

$2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha.$

$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$$= \frac{\sin\alpha/2}{\cos\alpha/2} \Rightarrow \tan\alpha/2$$

(34)

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}.$$ (LHR-2022)

$\because$ Using formulae $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$.

Answer:- $L.H.S = \cot(\alpha+\beta)$

$$= \frac{1}{\tan(\alpha+\beta)} = \frac{1}{\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}} = \frac{1-\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$$

$$= \frac{1-\dfrac{1}{\cot\alpha}\dfrac{1}{\cot\beta}}{\dfrac{1}{\cot\alpha}+\dfrac{1}{\cot\beta}} \Rightarrow \frac{\dfrac{\cot\alpha\cot\beta-1}{\cancel{\cot\alpha\cot\beta}}}{\dfrac{\cot\beta+\cot\alpha}{\cancel{\cot\alpha\cot\beta}}} \Rightarrow \frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}$$

So, $L.H.S = R.H.S$.

(35)

$$\frac{\sin3\theta}{\cos\theta} + \frac{\cos3\theta}{\sin\theta} = 2\cot2\theta$$ (LHR-2015)

Using formula:-
$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

Answer:- $L.H.S = \dfrac{\sin3\theta}{\cos\theta} + \dfrac{\cos3\theta}{\sin\theta}$

$$= \frac{\sin3\theta\sin\theta + \cos3\theta\cos\theta}{\sin\theta\cos\theta} \Rightarrow \frac{\cos3\theta\cos\theta+\sin3\theta\sin\theta}{\sin\theta\cos\theta} \Rightarrow \frac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}$$

Multiply and divide 2

$$= \frac{2\cos2\theta}{2\sin\theta\cos\theta} = \frac{2\cos2\theta}{\sin2\theta} = 2\cot2\theta$$

So, L.H.S = R.H.S

(36)

$$\frac{\sin3x - \sin x}{\cos x - \cos3x} = \cot2x$$ (LHR-2011+12+13.) Repeat.

$$\cos20^\circ + \cos100^\circ + \cos140^\circ = 0.$$ (LHR-2019+22)

Answer:- $L.H.S = \cos20^\circ + \cos100^\circ + \cos140^\circ$

$$= \cos20^\circ + 2\cos\left(\frac{100+140}{2}\right)\cos\left(\frac{100-140}{2}\right)$$

$= \cos 20° + 2\cos 120° \cos 20°$

$= \cos 20° + \cancel{2}(-\frac{1}{\cancel{2}})\cos 20°$

$= \cos \cancel{20}° - \cos \cancel{20}° = 0.$ So, L.H.S = R.H.S.

(37)

Express following (X) as (+) and (−)

$2\sin 3\theta \cos\theta$

$\because$ Using formula: (LHR-2015)

$\sin(\alpha+\beta)+\sin(\alpha-\beta) = 2\sin\alpha\cos\beta$

$2\sin 3\theta\cos\theta = \sin(3\theta+\theta)+\sin(3\theta-\theta)$

$= \sin 4\theta + \sin 2\theta.$

(39)

$\sin(x+45°)\sin(x-45°)$

X and ÷ by −2 (LHR-2018)

$= -\frac{1}{2}(-2\sin(x+45°)\sin(x-45°))$

$= -\frac{1}{2}(\cos(x+45°+x-45°)-\cos(x+45°-x+45°))$

$= -\frac{1}{2}(\cos 2x - \cos 90°)$

(41)

$2\sin 7\theta\cos 2\theta$

$\because \sin(\alpha+\beta)+\sin(\alpha-\beta) = 2\sin\alpha\cos\beta.$ (LHR-2017)

$= \sin(7\theta+2\theta)+\sin(7\theta-2\theta)$

$= \sin 9\theta + \sin 5\theta.$

(38)

Express (+) and (−) to (X).

$2\sin 7\theta\sin 2\theta$

$\because$ Using formulas:- (LHR-2017+21)

$= -(-2\sin 7\theta\sin 2\theta)$

$\cos(\alpha+\beta)-\cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

$= -(\cos(7\theta+2\theta)-\cos(7\theta-2\theta))$

$= -(\cos 9\theta - \cos 5\theta) \Rightarrow \cos 5\theta - \cos 9\theta$

(40)

Express (+),(−) as (X)

$\cos 7\theta - \cos\theta$

Using formula:- (LHR-2016+19)

$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$= -2\sin\left(\frac{7\theta+\theta}{2}\right)\sin\left(\frac{7\theta-\theta}{2}\right)$

$= -2\sin\left(\frac{8\theta}{2}\right)\sin\left(\frac{6\theta}{2}\right)$

$= -2\sin 4\theta\sin 3\theta$

(42)

$\cos 12 + \cos 48°$

By formula: (LHR-2014)

$= 2\cos\left(\frac{12+48}{2}\right)\cos\left(\frac{12-48}{2}\right)$

$= 2\cos\left(\frac{60}{2}\right)\cos\left(\frac{36}{2}\right)$

$= 2\cos 30°\cos 18°$

find the value of $\sin\theta$ and $\cos\theta$ when $\theta=18°$. (LHR-2015.)

Answer:-

$\theta=18°$

Multiply both side by 5

$5\theta = 90°$

$2\theta+3\theta = 90°$

$2\theta = 90°-3\theta°$

Applying $\sin\theta$

$\sin 2\theta = \sin(1\times 90°-3\theta°)$

$\sin 2\theta = \cos 3\theta$

$2\cancel{\cos\theta}\sin\theta = \cancel{\cos\theta}(4\cos^2\theta-3)$

$2\sin\theta = 4\cos^2\theta-3$

$2\sin\theta = 4(1-\sin^2\theta)-3$

$2\sin\theta = 4-4\sin^2\theta-3$

$4\sin^2\theta+2\sin\theta-1=0$

$a=4,\ b=2,\ c=-1$

$$\sin\theta = \frac{-2\pm\sqrt{4-4(4)(1)}}{2(4)}$$

$$\sin\theta = \frac{-2\pm\sqrt{20}}{8}$$

$$\sin\theta = \frac{-2\pm\sqrt{4\times5}}{8}$$

$$\sin\theta = \frac{-2\pm2\sqrt{5}}{8}$$

$$\sin\theta = \frac{2(-1\pm\sqrt{5})}{8}$$

$$\sin\theta = \frac{-1\pm\sqrt{5}}{4}$$

Now θ lies in I Quadrant

$$\sin\theta \neq \frac{-1-\sqrt{5}}{4}$$

$$\sin\theta = \frac{\sqrt{5}-1}{4}$$

$$\boxed{\sin 18° = \frac{\sqrt{5}-1}{4}}$$

Now:

$$\cos 18 = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{1-\left(\frac{5+1-2\sqrt{5}}{6}\right)}$$

$$\cos 18° = \sqrt{\frac{16-6-2\sqrt{5}}{16}}$$

$$\boxed{\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4}}$$

(44)

Express in form of $r\sin(\theta+\phi)$ or $\sin(\theta+\phi)$.

$\sin\theta+\cos\theta$ (LHR-2013.)

Answer:- $\sin\theta+\cos\theta = r\sin(\theta+\phi) \rightarrow (i)$

$\sin\theta+\cos\theta = r(\sin\theta\cos\phi+\cos\theta\sin\phi)$ $\quad \because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin\theta+\cos\theta = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

By comparing:

$1 = r\cos\phi \longrightarrow$ (ii)

$1 = r\sin\phi \longrightarrow$ (iii)

Add and squar of eq (ii)(iii)

$1+1 = r^2(\sin^2\phi + \cos^2\phi)$

$\sqrt{2} = r.$

Divide eq (iii) by (ii)

$\frac{\cancel{r}\sin\phi}{\cancel{r}\cos\phi} = 1$

$\tan\phi = 1$

$\tan^{-1}1 = \phi$

So,

$= \sqrt{2}\sin(\theta+\phi)$

where $\phi = \tan^{-1}1$

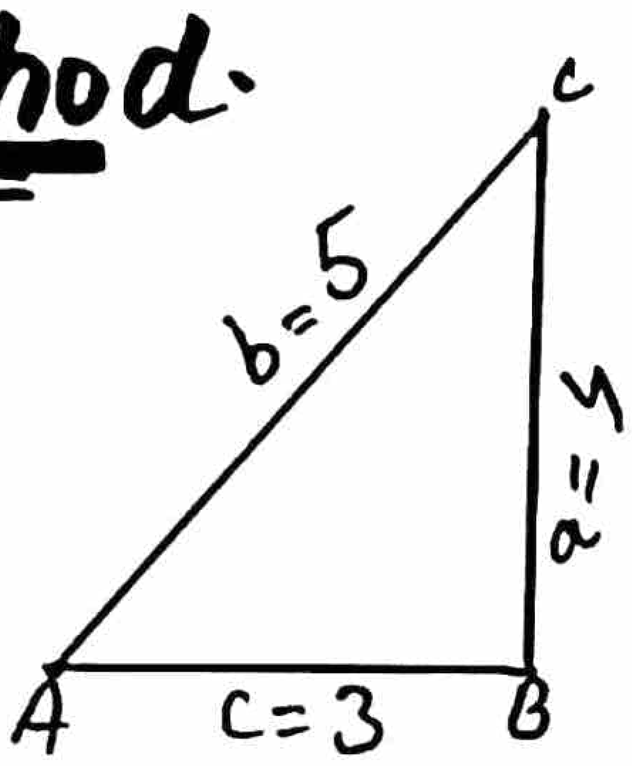
44

find: $\cos 2\alpha = ?$

$\cos\alpha = \frac{3}{5} \qquad 0 < \alpha < \frac{\pi}{2}. \quad (\qquad)$

Answer: $\cos\alpha = \frac{3}{5}$

I-method.



$b^2 = a^2 + c^2$

$5^2 = a^2 + 3^2$

$25 - 9 = a^2$

$16 = a^2 \Rightarrow 4 = a \qquad \sin\alpha = 4/5.$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha.$

$\cos 2\alpha = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 \Rightarrow \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

II-method: $\cos\alpha = 3/5$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos 2\alpha = 2\left(\frac{3}{5}\right)^2 - 1 \Rightarrow \frac{18}{25} - 1 \Rightarrow -\frac{7}{25}$

$$\boxed{\cos 2\alpha = -\frac{7}{25}}$$

Long Questions

(1)

$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$. (LHR-2011+13)

Answer:- $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

$L.H.S = \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

$= \sin 10^\circ \frac{1}{2} \sin 50^\circ \sin 70^\circ$

$= \frac{1}{2} \sin 10^\circ \sin 50^\circ \sin 70^\circ$

$= (x)(\div) -2 \Rightarrow \frac{1}{2}\sin 10^\circ\left(-\frac{1}{2}\right)(-2\sin 50^\circ \sin 70^\circ)$

$= -\frac{1}{4}(\sin 10^\circ \cos(50^\circ+70^\circ) - \cos(50^\circ-70^\circ))$

$= -\frac{1}{4}\sin 10^\circ \cos 120^\circ - \cos 20^\circ$

$=$

$= -\frac{1}{4}\sin 10^\circ\left(-\frac{1}{2} - \cos 20^\circ\right)$

$= \frac{1}{8}\sin 10^\circ + \frac{1}{4}\cos 20^\circ \sin 10^\circ$

$= \frac{1}{8}\sin 10^\circ + \frac{1}{8}(\sin(20+10) - \sin(20-10))$

$= \frac{1}{8}\sin 10^\circ + \frac{1}{8}(\sin 30^\circ - \sin 10^\circ)$

$= \frac{1}{8}\cancel{\sin 10^\circ} + \frac{1}{8}\sin 30^\circ - \frac{1}{8}\cancel{\sin 10^\circ}$

$= \frac{1}{8}\left(\frac{1}{2}\right) \Rightarrow \frac{1}{16}$

(2)

$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$ (LHR-2011+12+15+18)

Answer:- $L.H.S = \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$= \cos 20^\circ \cos 40^\circ \frac{1}{2} \cos 80^\circ$

$= \frac{1}{2}\cos 20^\circ \cos 40^\circ \cos 80^\circ$

$= \frac{1}{2}\cos 20 \frac{1}{2}(2\cos 40^\circ \cos 80^\circ)$

$= \frac{1}{4}\cos 20^\circ(\cos(40+80^\circ) + \cos(40-80^\circ))$

$= \frac{1}{4}\cos 20^\circ(\cos 120^\circ + \cos 40^\circ)$

$= \frac{1}{4}\cos 20^\circ\left(-\frac{1}{2} + \cos 40^\circ\right)$

$= -\frac{1}{8}\cos 20^\circ + \frac{1}{4}\cos 20 \cos 40^\circ$

$= -\frac{1}{8}\cos 20^\circ + \frac{1}{4\times 2}(2\cos 20 \cos 40^\circ)$

$= -\frac{1}{8}\cos 20^\circ + \frac{1}{8}(\cos(20+40^\circ) + \cos(20-40^\circ))$

$= -\frac{1}{8}\cos 20^\circ + \frac{1}{8}(\cos 60^\circ + \cos 20^\circ)$

$= -\frac{1}{8}\cancel{\cos 20^\circ} + \frac{1}{8}\cos 60^\circ + \frac{1}{8}\cancel{\cos 20^\circ}$

$= \frac{1}{8}\left(\frac{1}{2}\right) \Rightarrow \frac{1}{16}$

(3)

$$\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{2}\cos 2\theta \cdot \text{(LHR-2014)}$$

Answer:- L.H.S $=\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)$

Method - I

$$\left(\sin\frac{\pi}{4}\cos\theta-\cos\frac{\pi}{4}\sin\theta\right)\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right)$$

$$\left(\frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{2}}\sin\theta\right)\left(\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)$$

$$=\left(\frac{1}{\sqrt{2}}\right)^2\cos^2\theta-\left(\frac{1}{\sqrt{2}}\right)^2\sin^2\theta$$

$$=\frac{1}{2}\cos^2\theta-\frac{1}{2}\sin^2\theta$$

$$=\frac{1}{2}(\cos^2\theta-\sin^2\theta)$$

$$=\frac{1}{2}\cos 2\theta$$

Method - II:

= Devide or X by -2

$$=-\frac{1}{2}\left(-2\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)\right)$$

$$=-\frac{1}{2}\left(\cos\left(\frac{\pi}{4}-\cancel{\theta}+\frac{\pi}{4}+\cancel{\theta}\right)-\cos\left(\cancel{\frac{\pi}{4}}-\theta-\cancel{\frac{\pi}{4}}-\theta\right)\right)$$

$$=-\frac{1}{2}\left(\cos\frac{\pi}{2}-\cos 2\theta\right)$$

$$=-\frac{1}{2}(0-\cos 2\theta)$$

$$=\frac{1}{2}\cos 2\theta$$

(4)

$$\frac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\sin 7\theta}=\tan 4\theta\cdot\text{(LHR-2017+21.)}$$

Answer:- L.H.S $=\dfrac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cancel{\sin}\cos 7\theta}$

$$=\frac{(\sin\theta+\sin 7\theta)+(\sin 3\theta+\sin 5\theta)}{(\cos\theta+\cos 7\theta)+(\cos 3\theta+\cos 5\theta)}$$

$\because \sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\because \cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$$=\frac{\left(+2\sin\left(\frac{\theta+7\theta}{2}\right)\cos\left(\frac{\theta-7\theta}{2}\right)\right)+\left(2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta+5\theta}{2}\right)\right)}{\left(2\cos\left(\frac{\theta+7\theta}{2}\right)\cos\left(\frac{\theta-7\theta}{2}\right)\right)+\left(2\cos\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)\right)}$$

$$=\frac{2\sin 4\theta\cos 3\theta+2\sin 4\theta\cos 4\theta}{2\cos 4\theta\cos 3\theta+2\cos 4\theta\cos\theta}\Rightarrow\frac{\cancel{2}\sin 4\theta\,\cancel{(\cos 3\theta+\cos\theta)}}{\cancel{2}\cos 4\theta\,\cancel{(\cos 3\theta+\cos\theta)}}$$

$$=\tan 4\theta$$

So, L.H.S = R.H.S

(5)

$\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9} = 3/16$. (LHR-2018)

$L.H.S = \sin\frac{\pi}{9}\sin 2\frac{\pi}{9}\sin\frac{\pi}{3}\sin 4\frac{\pi}{9}$

$= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}\sin 20^\circ \sin 40^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}\sin 20^\circ - \frac{1}{2}(-2\sin 40^\circ \sin 80^\circ)$

formula: $(-2\sin\alpha\sin\beta) = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= -\frac{\sqrt{3}}{4}\sin 20^\circ(\cos 120^\circ - \cos 40^\circ)$

$= -\frac{\sqrt{3}}{4}\sin 20^\circ\left(-\frac{1}{2} - \cos 40^\circ\right)$

$= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{4}\sin 20^\circ \cos 40^\circ$

$= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{8}(2\sin 20^\circ \cos 40^\circ)$

$= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{8}\sin 60^\circ - \frac{\sqrt{3}}{8}\sin 20^\circ$

$= \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2}$

$= 3/16$

(6)

Reduce $\cos^4\theta$ in an expression only θ.

Answer:- (LHR-2012+16)

$\cos^4\theta = (\cos^2\theta)^2 = \left[\frac{1+\cos 2\theta}{2}\right]^2$ $\qquad \because \cos 2\theta = 2\cos^2\theta - 1$, $\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$= \frac{1 + 2\cos 2\theta + \cos^2 2\theta}{4}$

$= \frac{1}{4}[1 + 2\cos 2\theta + \cos^2 2\theta] \Rightarrow \frac{1}{4}\left[1 + 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right]$

$= \frac{1}{4}\times 2[2 + 4\cos 2\theta + 1 + \cos 4\theta] \Rightarrow \frac{1}{8}[3 + 4\cos 2\theta + \cos 4\theta]$

(7)

$\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\alpha/2 + \cos\alpha/2}{\sin\alpha/2 - \cos\alpha/2}$ (LHR-2015)

Answer:- $L.H.S = \sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}$ $\quad\because \sin 2\alpha = 2\sin\alpha\cos\alpha$, $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$$= \sqrt{\frac{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}+\sin\alpha}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}-\sin\alpha}} \Rightarrow \sqrt{\frac{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$$

$$= \sqrt{\frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)^2}} \Rightarrow \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$$ So, L.H.S = R.H.S

(8)

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2.$$ (LHR-2017)

Answer:- $L.H.S = \dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta}$ $\quad\because \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$

$$= \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta} \Rightarrow \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta} \Rightarrow \frac{\sin 2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cancel{2\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} = 2$$ **So, L.H.S = R.H.S.**

(9)

$$\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta.$$ (LHR-2021)

Answer:- $L.H.S = \dfrac{\cos 3\theta}{\cos\theta} + \dfrac{\sin 3\theta}{\sin\theta}$

$$= \frac{\sin\theta\cos 3\theta + \cos\theta\sin 3\theta}{\sin\theta\cos\theta} \Rightarrow \frac{\sin(\theta+3\theta)}{\sin\theta\cos\theta} \Rightarrow \frac{\sin 4\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin 2(2\theta)}{\sin\theta\cos\theta} \Rightarrow \frac{2\sin 2(2\theta)}{2\sin\theta\cos\theta} \Rightarrow \frac{2\; 2\sin 2\theta\cos 2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{2(\cancel{2\sin 2\theta}\cos 2\theta)}{\cancel{\sin 2\theta}} \Rightarrow 4\cos 2\theta$$ **So, L.H.S = R.H.S.**

(10)

Reduse $\sin^4\theta$ to an expression involving only functions of multiples of θ raised to first power (LHR-2019+22)

Answer:- $\sin^4\theta$

$\because \cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$= (\sin^2\theta)^2 \Rightarrow \left(\frac{1-\cos 2\theta}{2}\right)^2$$

$$= \frac{(1-\cos 2\theta)^2}{4}$$

$\because$ formula:- $(a-b)^2 = a^2 + b^2 - 2ab$

$$= \frac{1+\cos^2 2\theta + 2\cos 2\theta}{4} \Rightarrow \frac{1}{4}\left(1+\cos^2 2\theta - 2\cos 2\theta\right)$$

$\because \cos 2\theta = 2\cos^2\theta - 1$

$1 + \cos 2\theta = 2\cos^2\theta$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\cos^2 2\theta = \dfrac{1+\cos 4\theta}{2}$

$$= \frac{1}{4}\left(1 - 2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)\right)$$

$$= \frac{1}{4}\left(\frac{2 - 4\cos 2\theta + 1 + \cos 4\theta}{2}\right)$$

$$= \frac{1}{8}\left(2 - 4\cos 2\theta + 1 + \cos 4\theta\right)$$

$$= \frac{1}{8}\left[3 - 4\cos 2\theta + \cos 4\theta\right]$$

(11)

If α, β, γ are the angles of the Triangle ABC show that $\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$. (LHR-2013+14+19+22)

Answer:- If α, β and γ are the angles of triangle ABC so,

let: $\alpha + \beta + \gamma = 180°$

$\alpha + \beta = 180 - \gamma$

Divide both side by 2

$$\frac{\alpha+\beta}{2} = \frac{180-\gamma}{2}$$

$$\frac{\alpha}{2}+\frac{\beta}{2} = \frac{180}{2}-\frac{\gamma}{2}$$

$$\frac{\alpha}{2}+\frac{\beta}{2} = 90-\frac{\gamma}{2}$$

Apply tan on both side:-

$$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \tan\left(90-\frac{\gamma}{2}\right)$$

$$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \cot\frac{\gamma}{2}$$

Using formula :-

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\frac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = \frac{1}{\tan\frac{\gamma}{2}}$$

$$\tan\frac{\gamma}{2}\left(\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}\right) = 1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

$$\tan\frac{\alpha}{2}\tan\frac{\gamma}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2} = 1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

$$\tan\frac{\alpha}{2}\tan\frac{\gamma}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = 1$$

Divide both side by $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

$$\frac{1}{\tan\frac{\beta}{2}}+\frac{1}{\tan\frac{\alpha}{2}}+\frac{1}{\tan\frac{\gamma}{2}} = \frac{1}{\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}}$$

$$\cot\frac{\beta}{2}+\cot\frac{\alpha}{2}+\cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$$\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$