Chapter 14

Magnetism and Electromagnetism

Magnet:

It is a natural substance which attracts the things made up of iron, cobalt and nickel. If it is suspended freely, it always points towards geographical north and south.

Magnetic Field:

The region around the magnet in which its effect can be experienced is called magnetic field.

Magnetic Force:

The force experienced by a magnetic substance due to a magnet is called magnetic field.

Magnetic Lines of Force:

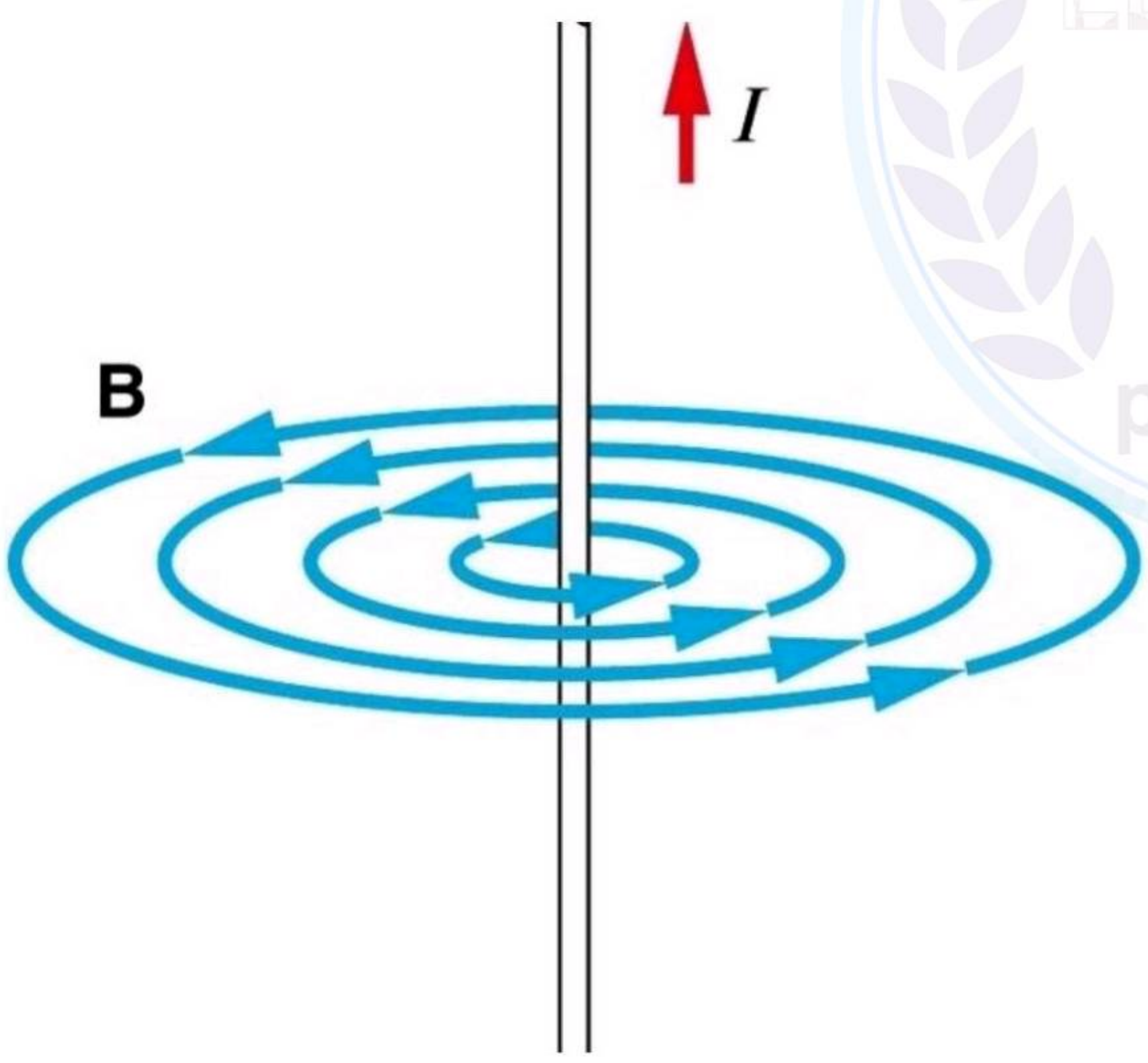
In the magnetic field the effect of magnet is caused by special lines of force which are called magnetic lines of force.

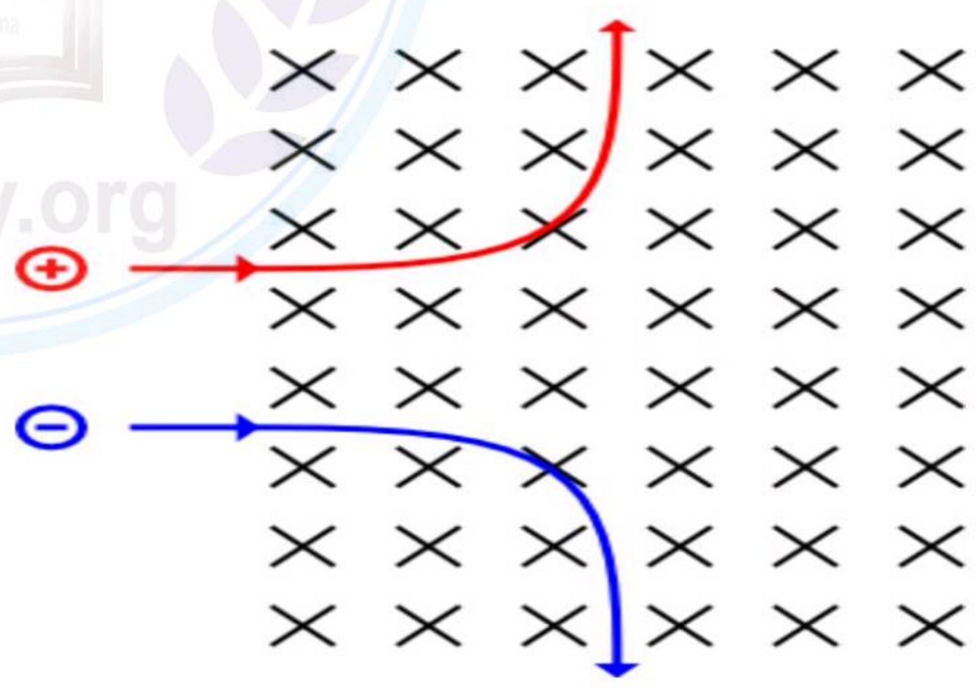
Properties of Magnetic Lines of Force:

i) The magnetic lines of force start from the North Pole and end at south pole.
ii) Inside the magnet, these lines continue from the South Pole to the North Pole.
iii) They do not intersect each other.
iv) They pass through iron more easily as compared to air.

Magnetic Field of a Straight Wire:

When a current is flowing in a straight wire, a magnetic field is produced around the wire. It consists of concentric circular magnetic lines.



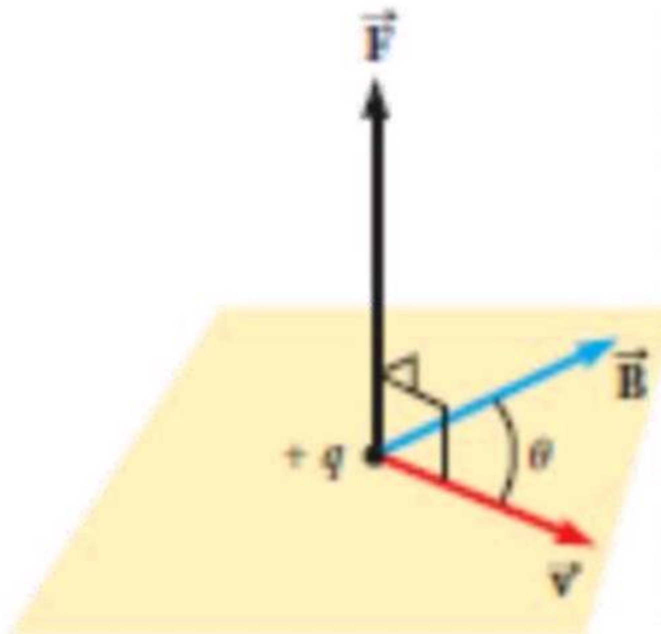


Force on Charge Particle in a Uniform Magnetic Field:

Experimentally it is found that when a charge particle moves in a magnetic field it experiences a force due to magnetic field called magnetic force.

Mathematical Expression:

Mathematically the magnetic force on a charged particle is directly proportional to the component of velocity of particle perpendicular to magnetic field, strength of magnetic field and magnitude of charge.



$$F \propto V_{\perp}$$

Or

$$F \propto v sin\theta$$

$$F \propto B$$

$$F \propto q$$

Combing above

$$F \propto qvBsin\theta$$

Or

$$F = kqvBsin\theta$$

Practically $k = 1$, Hence

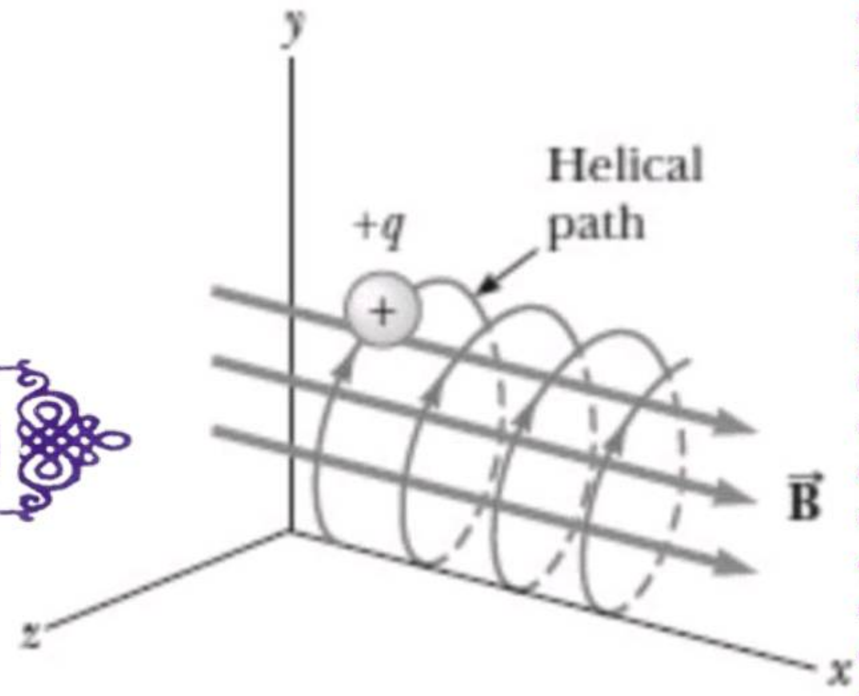
$$\boxed{\boldsymbol{F = qvBsin\theta}}$$

In vector form

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

Trajectory of a charged particle moving in a uniform magnetic field:

Consider a charged article moving in a uniform magnetic field of strength $\boldsymbol{B}$ such that it is making angle $\boldsymbol{\theta}$ with the magnetic field vector $\vec{\boldsymbol{B}}$. The components of velocity vector parallel and perpendicular to the field vectors can be written as



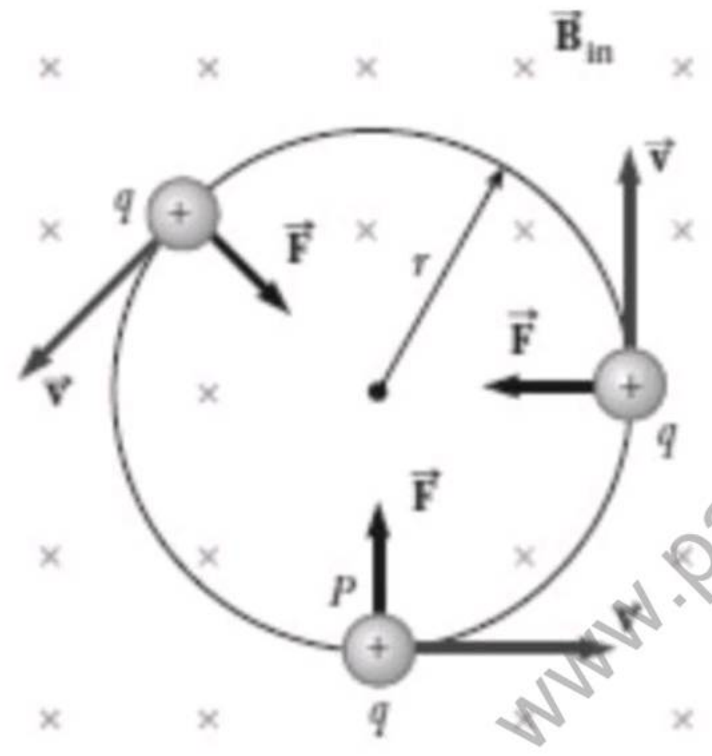
$$v_{\perp} = v\ sin\theta$$

$$v_{\parallel} = v\ cos\theta$$

The particle will experience magnetic force which will alter $\boldsymbol{v}_{\perp}$ and it will move in a helical path along magnetic field. The radius of helical path can be written as

Magnetic force on particle is
$F_m = qvBsin\theta$

Due to circular motion Centripetal force is



$$F_c = \frac{m\,v_{\perp}^2}{r} = \frac{m\,(v\ sin\theta)^2}{r}$$

Comparing above

$$F_m = F_c$$

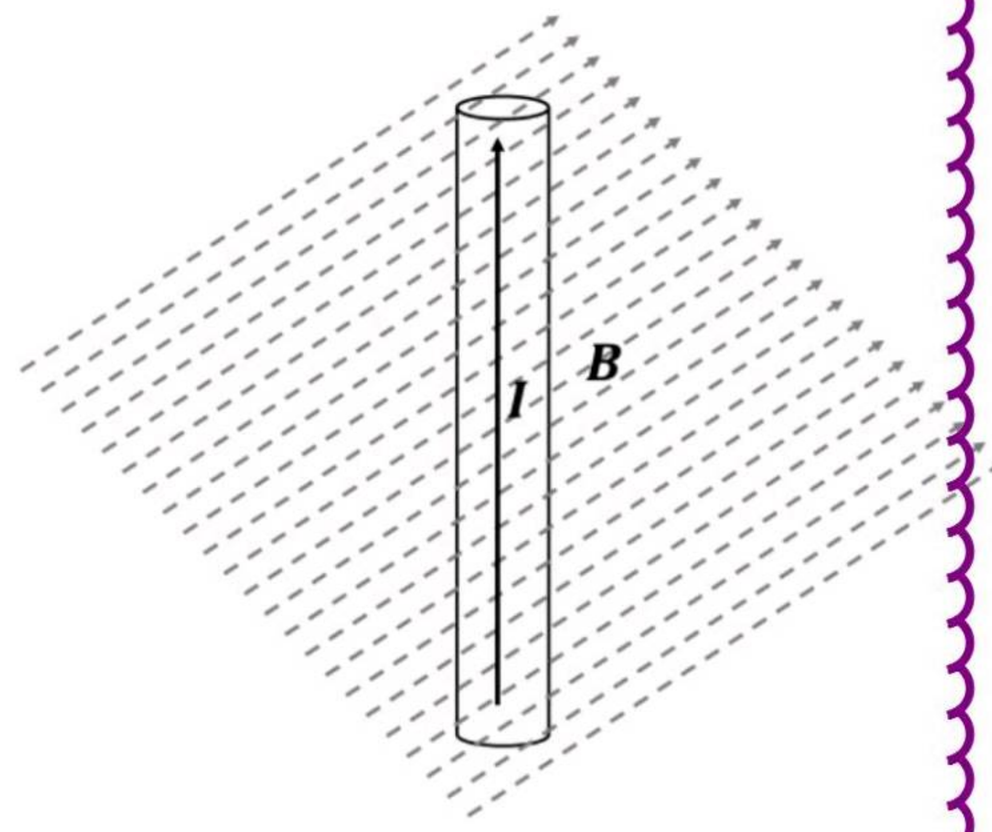
$$q\cancel{v}B\cancel{sin\theta} = \frac{m(vsin\theta)^{\cancel{2}}}{r}$$

$$qB = \frac{mvsin\theta}{r}$$

$$\boxed{\boldsymbol{r = \frac{mvsin\theta}{qB}}}$$

Force on a current carrying conductor in a uniform magnetic field:

Consider conductor of length $\boldsymbol{l}$ placed in a uniform magnetic field of strength $\boldsymbol{B}$. If the current flowing through the conductor is $\boldsymbol{I}$ and the angle between velocity vector $\vec{\boldsymbol{v}}$ and magnetic field vector $\vec{\boldsymbol{B}}$ is $\boldsymbol{\theta}$ then the force experience by the conductor can be written as

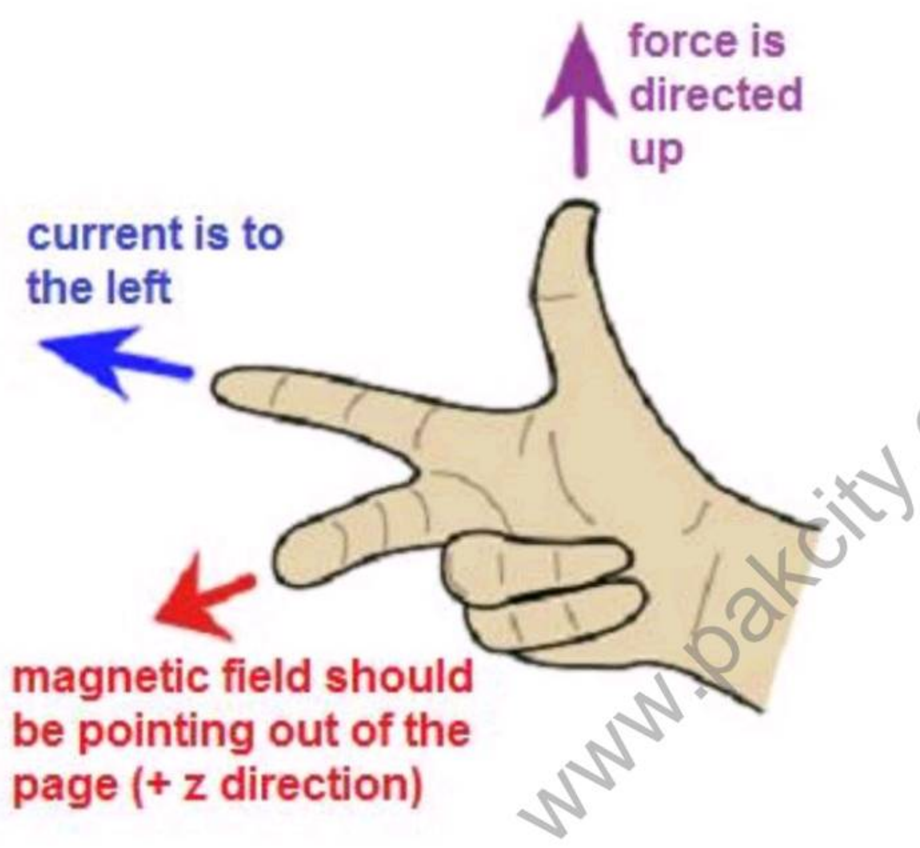


$$\vec{F} = q(\vec{v} \times \vec{B})$$

As the charges moves from one end of the conductor to other end of the conductor in time $\boldsymbol{t}$ the velocity of charges can be written as

$$\vec{v} = \frac{\vec{l}}{t}$$

Putting in above we get



$$\vec{F} = q\left(\frac{\vec{l}}{t} \times \vec{B}\right)$$

Using property of cross product $m\vec{A} \times \vec{C} = m\left(\vec{A} \times \vec{C}\right)$

$$\vec{F} = \frac{q}{t}(\vec{l} \times \vec{B})$$

Or

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

Now the magnitude of force can be written as

$$\boxed{F = IlBsin\theta}$$

As the magnitude of force changes with the inclination of the conductor with respect to the magnetic field, therefore, $\vec{L}$ is taken as a vector, its magnitude is equal to length of the conductor and its direction is same as the direction of conventional current.

The magnitude of force experienced by a straight current carrying conductor in a uniform magnetic field is given by

$$F = ILB \sin\theta$$

When conductor is perpendicular to the field $\theta=90^o$, force experienced by the conductor will be maximum and is given by:

$$F = ILB \qquad (Sin90^o = 1)$$

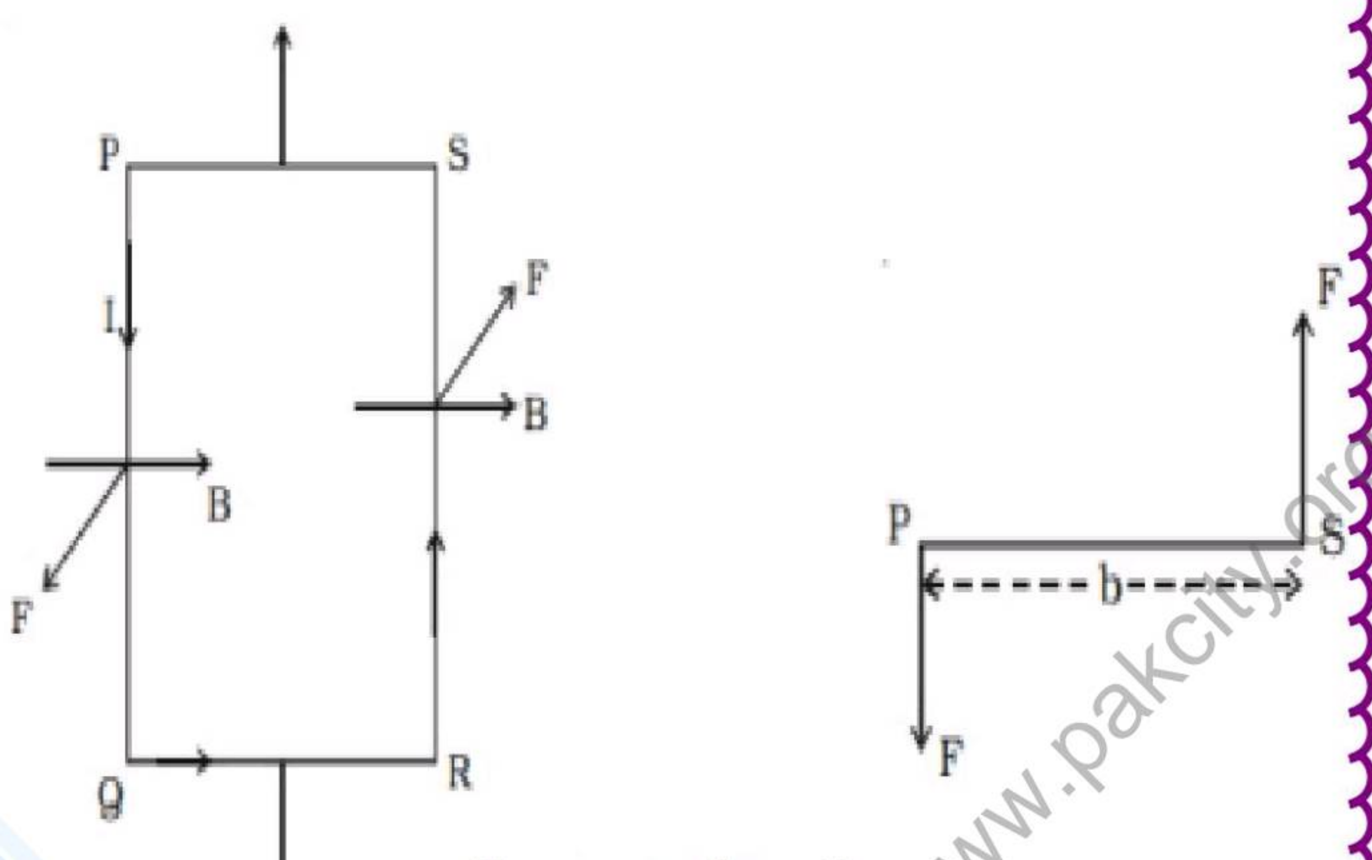
When the conductor is held parallel to the field $\theta=0°$ then;

$$F=0 \qquad (Sin0^o=0)$$

"Force experienced by a current carrying conductor in a magnetic field is always perpendicular to length of the conductor as well as the magnetic field and its direction can be determined by right hand rule".

Torque on a Current Carrying Coil in a Uniform Magnetic Field:

When current is passed through a coil placed in a magnetic field, a couple is developed which rotates the coil. A coil "PQRS" is placed in a uniform magnetic field $\vec{B}$ and capable of rotation about an axis xx'.



Torque on the coil

L=length of coil
b = breath of the coil.
As current **I** pass through the coil, force is produced on each length of the coil.

$$\because \vec{F} = I\left(\vec{L} \times \vec{B}\right)$$

$$F = BIL \sin 90^o$$

$$F = BIL\,(1)$$

$$F = BIL$$

Torque of couple is given by,

$$\tau = Force \times Couple\ arm$$

$$\tau = F \times b$$

$\tau = BIL \times b = BIA$
for coil of N turns;

$$\tau = BIA \times N$$

$$\tau = BINA.....................(i)$$

Eq(i) applies when the plane of the coil is parallel to the direction of magnetic field. When the plane of coil makes an angle "α" with the direction of magnetic field then;

$$\tau = \text{Force x Couple arm}$$
$$\tau = \text{BIL x b cos}\alpha$$
$$\tau = \text{BIA cos}\alpha \qquad (\because \text{A} = \text{Lb})$$

For N turns;

$$\tau = BINA \cos\alpha$$

• Torque is maximum when the plane of coil is parallel to the magnetic field i.e. when α=0.
• Torque is zero when the plane of the coil is perpendicular to the direction of magnetic field i.e. α= 90.

Magnetic Flux:

The number of magnetic lines of force passing normally through the surface is called "Magnetic flux".

Mathematical Definition:

Magnetic flux is equal to the dot product of magnetic field of induction "$\vec{B}$" and the vector area "$\Delta\vec{A}$" of the surface, provided the magnetic field of induction is uniform over the given area of the surface. It is denoted by $\boxed{\Phi_m}$.

$$\boxed{\Delta\Phi_m = \vec{B}.\Delta\vec{A}}$$

$$\Delta\Phi_m = B\Delta A\cos\theta$$

The unit of magnetic flux is "Weber".

Magnetic Flux Density:

"The magnetic flux per unit area of a surface which is held normal to the field is called Magnetic Flux Density"
It is denoted by "B".

$$B = \frac{\Delta\Phi_m}{\Delta A}$$

The Unit of magnetic flux density is "W/m^2" or "Tesla".

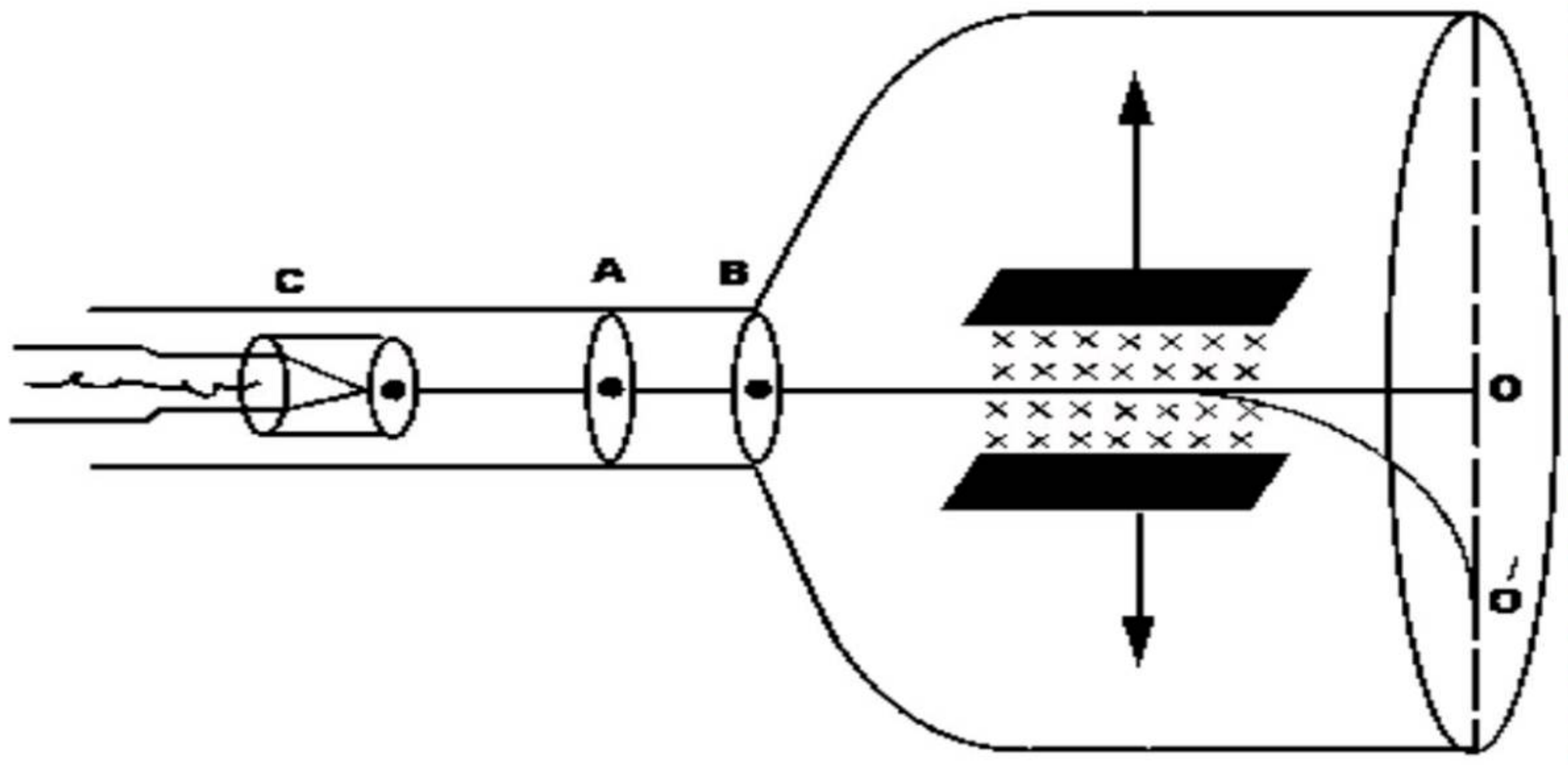
J.J Thomson Experiment for charge to mass ratio of electron:

Significance:

It was the first direct measurement of charge to mass ration of an electron which led to the measurement of mass of electron.

Experimental Setup and working:

It consists of a highly evacuated glass tube, fitted with electrodes. Electrons are produced by heating a tungsten filament electrically. Electrons are made to accelerate and form a beam by passing through discs A and B. They are passed through electric and magnetic field. Finally, they fall on zinc sulphide screen.



Calculations:

As the electron beam is moves in a circular arc when passing through magnetic field its centripetal force is equal to the magnetic force i-e

$$\boldsymbol{F_m = F_c}$$

$$\boldsymbol{evB = \frac{mv^2}{r}}$$

$$\boxed{\boldsymbol{\frac{e}{m} = \frac{v}{Br}}}$$

Calculation for velocity:

Method#1:

As the particle is moving under the influence of an electric field its kinetic energy can be written as

$$K.E = eV$$

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

$$v = \sqrt{\frac{2eV}{m}}$$

Putting in equation (1)

$$\frac{e}{m}=\sqrt{\frac{2eV}{m}}\times\frac{1}{Br}$$

Squaring on both sides

$$\left(\frac{e}{m}\right)^{\cancel{2}}=\frac{2\cancel{e}V}{\cancel{m}B^2r^2}$$

$$\boxed{\boldsymbol{\frac{e}{m}=\frac{2V}{B^2r^2}}}$$

Method#2:

If an electric field is applied perpendicular to the magnetic field such that it will cancel the effect of magnetic force and the electron beam passes by in a straight line then

$$F_E=F_m$$

$$eE=evB$$

$$v=\frac{E}{B}$$

Putting in equation (1) we get

$$\frac{e}{m}=\frac{E}{B}\times\frac{1}{Br}$$

$$\boxed{\boldsymbol{\frac{e}{m}=\frac{E}{B^2r}}}$$

Calculation for Radius:

If the electron beam strikes the screen at point E as shown in figure then using law of chords

$$\overline{DE}\times\overline{EB}=\overline{AE}\times\overline{EC}$$

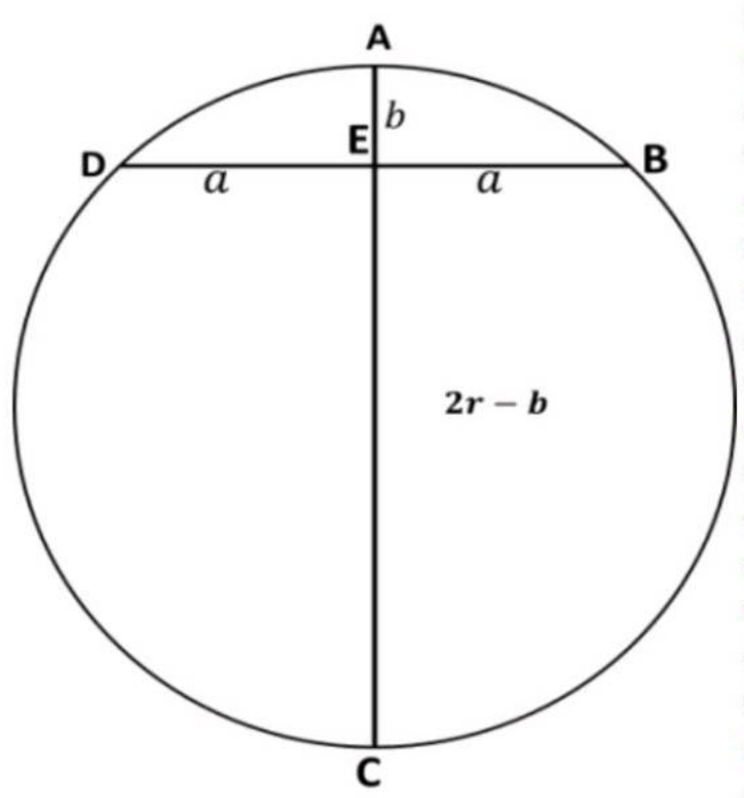
$$a\times a=b\times(2r-b)$$

$$a^2=2rb-b^2$$

Since b^2 is negligible

$$a^2=2rb$$

$$\boxed{\boldsymbol{r=\frac{a^2}{2b}}}$$



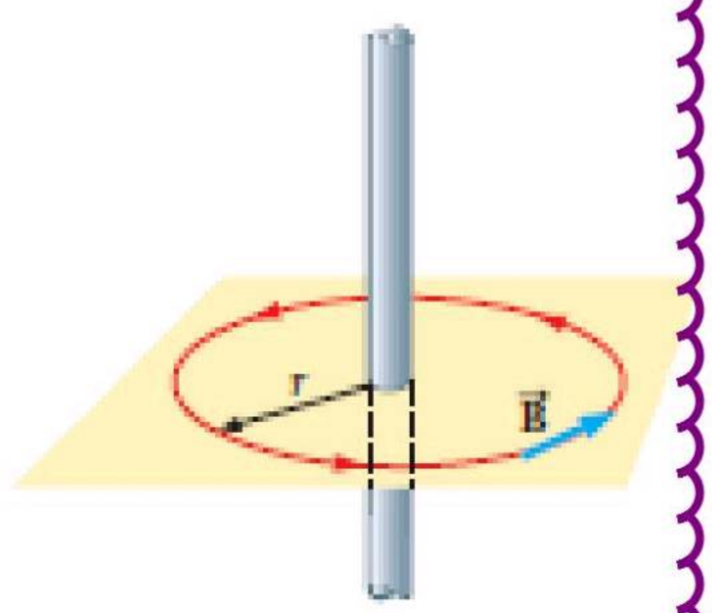
Biot-Savart Law:

Statement:

"Strength of magnetic field ($\boldsymbol{B}$) around a conductor is directly proportion to the current ($\boldsymbol{I}$) flowing through a conductor and inversely proportional to the distance ($\boldsymbol{r}$) from the conductor."

Mathematical Expression:

According to Biot-Savart law



$$B \propto I$$

$$B \propto r$$

Combining above we get

$$B \propto \frac{I}{r}$$

$$\boxed{\boldsymbol{B = \frac{\mu_o}{2\pi}\frac{I}{r}}}$$

Here $\boldsymbol{\frac{\mu_o}{2\pi}}$ is constant of proportionality and its Numerical value of $\boldsymbol{\mu_o}$ is

$$\boxed{\boldsymbol{\mu_o = 4\pi \times 10^{-7}\, TA^{-1}.m^{-1}}}$$

Ampere's Law:

Statement:

"The sum of the product of the tangential component of magnetic field of induction and the length of an element of a closed curve taken in a magnetic field is "i" times the current which passes through the area bounded by this curve".

Mathematical Representation:

$$\sum_{i=1}^{i=n}\left(\vec{B}.\Delta\vec{L}\right)_t = \mu_0 \times I_{enclosed}$$

Proof:

Consider a straight current carrying conductor through which current **"I"** is flowing. Experimentally, it has been observed that the strength of the magnetic field produced at any point near the conductor is directly proportional to twice the current **"I"** and inversely proportional to the distance ***"r"*** from the conductor.

$$B \propto I$$

$$B \propto \frac{1}{r^2}$$

by combining both observations,

$$B \propto \frac{I}{r}$$

$$\mathrm{B} = (\text{constant})\ \frac{\mathrm{I}}{\mathrm{r}}$$

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

$\frac{\mu_0}{2\pi}$ is the constant of proportionality and it is known as permeability of free-space.

The value of "μ_0" is $4\pi \times 10^{-7}$ **Web/Amp-m**

$$B = \frac{\mu_0 I}{2\pi r} \ldots\ldots\ldots\ldots(i)$$

The above relation shows that the value of **"B"** at all the points on the circle will be the same if a straight conductor is at the center of the circle. Hence the magnitude of magnetic field of induction "B" at any point on the surface of a circular closed path can be calculated with the help of equation (*i*) the above formula is valid only for a circular closed path surrounding the conductor.

To derive a general formula, we will divide the circle into many small elements each of length "Δl". The tangential component of magnetic field of induction for an element is "$B\cos\theta$" hence, the product of tangential component of "B" and length of an element "Δl" is given as;

$$(B\cos\theta)\Delta l = B\Delta l \cos\theta$$

But,

The sum of these products for all the elements is given by:

$$\sum \vec{B}.\Delta\vec{l} = \sum B\Delta l Cos\theta$$

In this case the angle *0* between $\vec{B}$. and $\Delta\vec{l}$ each and every point is zero, because the circular path coincides exactly with the magnetic field.

$$\sum \vec{B}.\Delta\vec{l} = \sum B\Delta l Cos\theta$$

$$\sum \vec{B}.\Delta\vec{l} = \sum B\Delta l$$

$$\sum \vec{B}.\Delta\vec{l} = B\sum \Delta l$$

$\sum \Delta l = 2\pi r$ (Total length of the circular closed path), But $B=\dfrac{\mu_0 I}{2\pi r}$ circular closed path, from equation (i).

$$\sum \vec{B}.\Delta \vec{l} = \frac{\mu_0 I}{2\pi r} \times 2\pi r$$

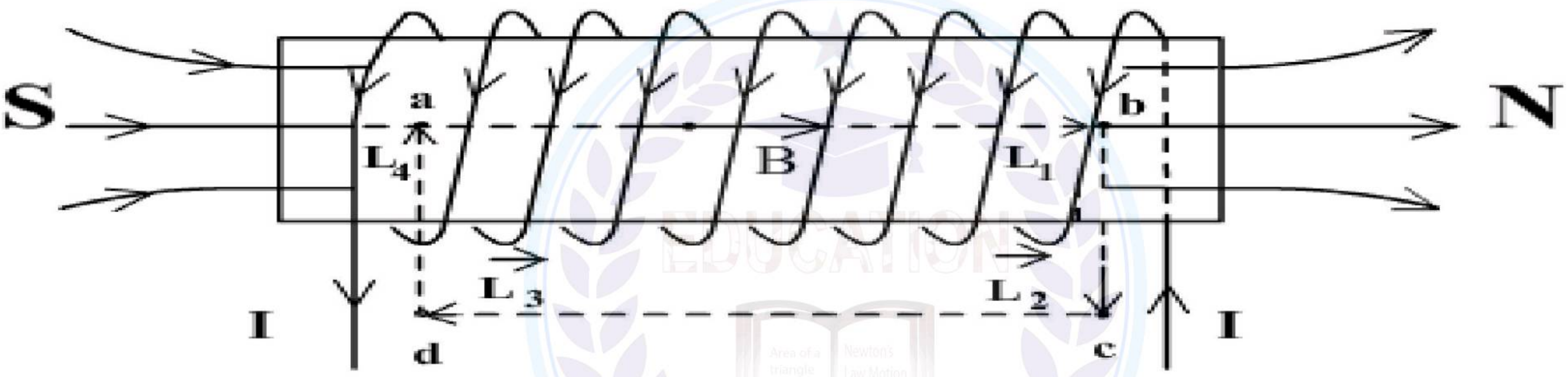
$$\sum_{i=1}^{i=n} (\vec{B}.\Delta\vec{L})_i = \mu_0 I \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Equation (ii) is the general form of Ampere's circular law. It is independent of the distance of elements form the conductor; therefore, it is applicable to closed curve of any shape taken in the magnetic field.

Application of Ampere's Law:

With the help of Ampere's law the magnetic field of induction B due to a current can be determined provided $\sum \vec{B}.\Delta\vec{l}$ for an imaginary closed curve around the conductor is known.

Determination of "B" Inside a Long Solenoid:



A coil of insulated wire wound on a cylindrical core is called Solenoid. When a strong current pass through the loops a magnetic field is formed inside the "Core" of Solenoid. Outside the core, the field is very weak so that the force of induction is negligible outside the core.
Consider rectangular amperian loop, the length of its sides is l_1, l_2, l_3 *and* l_4.

To Determine the Line Integral of Magnetic Induction

The line integral of magnetic induction on Amperian loop is given by;

$$\sum \vec{B}.\Delta\vec{l} = \vec{B}.\vec{l}_1 + \vec{B}.\vec{l}_2 + \vec{B}.\vec{l}_3 + \vec{B}.\vec{l}_4 \ldots\ldots\ldots\ldots(i)$$

Now, for $\vec{B}.\vec{l}_1$

since $\vec{B}$ and $\vec{l}_1$ are parallel, therefore (i.e. θ = 0°)

$$\vec{B}.\vec{l}_1 = B l_1 \cos 0^o$$
$$\vec{B}.\vec{l}_1 = B\, l_1 (1)$$
$$\vec{B}.\vec{l}_1 = B l_1$$

Now for $\vec{B}.\vec{l}_2$

since $\vec{B}$ and $\vec{l}_2$ are perpendicular, (i.e. θ = 90°)

$$\vec{B}.\vec{l}_2 = B l_2 \cos 90^o$$
$$\vec{B}.\vec{l}_2 = B\, l_2 (0)$$
$$\vec{B}.\vec{l}_2 = 0$$

Now for $\vec{B}.\vec{l}_3$

since $\vec{B}$ and $\vec{l}_3$ are anti- parallel, therefore (i.e., θ = 180°)

$$\vec{B}.\vec{l}_3 = B l_3 \cos 0^o$$

Since at l_3 the field strength is zero (i.e., B = 0)

$$\vec{B}.\vec{l}_3 = (0)\, l_3 (-1)$$
$$\vec{B}.\vec{l}_3 = 0$$

Now for Bl_4

Since, $\vec{B}$ and $\vec{l}_4$ are perpendicular, (i.e., θ = 90°)

$$\vec{B}.\vec{l}_4 = B l_4 \cos 90^o$$
$$\vec{B}.\vec{l}_4 = B\, l_4 (0)$$
$$\vec{B}.\vec{l}_4 = 0$$

Now;

$$\sum \vec{B}.\Delta \vec{l} = Bl_1 + 0 + 0 + 0$$

$$\sum \vec{B}.\Delta \vec{l} = Bl_1$$

To Determine the Total Current Passes Through Amperian Loop:

Suppose the no. of turns per unit length of solenoid is "n", therefore the no. of turns on "l_1" is "nl_1". If "I" be the current passes through "one" turn, hence the total current enclosed by the rectangular Amperian loop will be "nl_1I".

Now, applying Ampere's Law;

$$\sum \vec{B}.\Delta \vec{l} = \mu_0 (I_{enclosed})$$

$$Bl_1 = \mu_0 (nl_1 I)$$

$$B = \mu_0 nI$$

OR

$$\boxed{B = \frac{\mu_o N I}{L} \; ; \quad \left(\therefore n = \frac{N}{L} \right)}$$

Where, "N" and "L" are the number of turns and length of solenoid respectively.

If a medium other than vacuum is present at the core of the solenoid the value of "B" is given by;

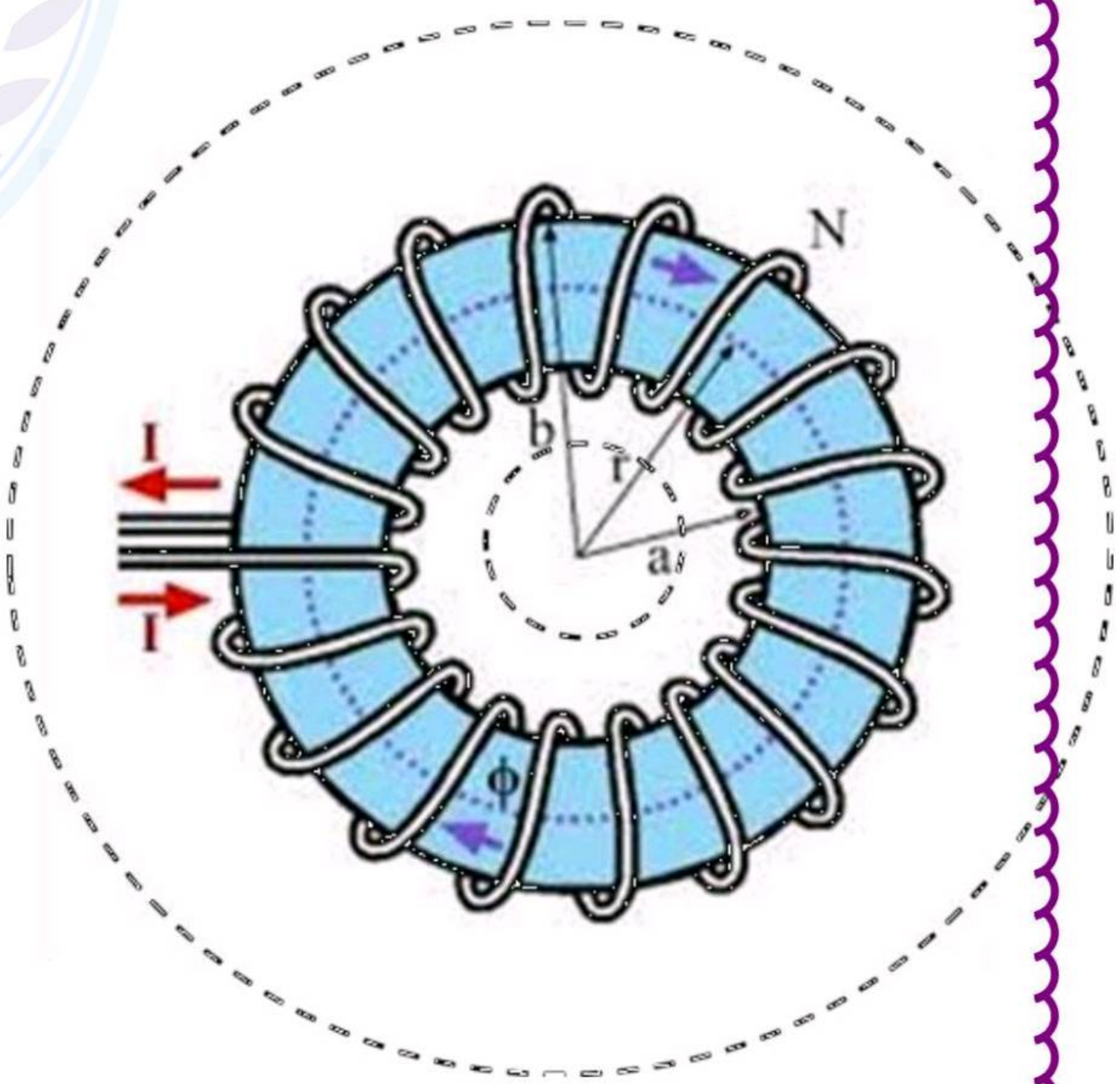
$$B = \mu_m n I$$

where "μ_m" is the permeability of the medium.

$$\boxed{B = \mu_m n I = \frac{\mu_r \mu_o N I}{L}}$$

Toroidal Field:

Consider a toroid of outer radius $\boldsymbol{b}$, inner radius $\boldsymbol{a}$ and mean radius $\boldsymbol{r}$ having $\boldsymbol{N}$ turns such that the current flowing through each turn is $\boldsymbol{I}$. If we divide toroid in n small equal segments such that each segment is parallel to the magnetic field vector tangent to it. Now, according to Ampere's law:



$$\Sigma \overrightarrow{B}.\overrightarrow{\Delta l} = \Sigma B \Delta l_n \, Cos\theta$$

Since $\theta = 0°$

$$\Sigma\vec{B}.\vec{\Delta l} = \Sigma B\Delta l_n = B\Sigma\Delta l$$

$$\Sigma\vec{B}.\vec{\Delta l} = B(2\pi r)$$

Using ampere's law

$$B(2\pi r) = \mu_o NI$$

$$\boxed{B = \frac{\mu_o NI}{2\pi r}}$$

Now Applying Amperes Law in toroid. Consider an ampere loop of radius $\boldsymbol{R}$.

If $R > b$:

If $R > b$ then the net current through the ampere loop is zero (current in is equal to current out) hence:

$$B = \frac{\mu_o NI}{2\pi R} = \frac{\mu_o N(0)}{2\pi r}$$

$$\boxed{B = 0}$$

If $R < a$:

If $R < a$ then the net current through the ampere loop is zero, hence:

$$B = \frac{\mu_o NI}{2\pi R} = \frac{\mu_o N(0)}{2\pi r}$$

$$\boxed{B = 0}$$

If $R = r$:

If $R > r$ then we get

$$B = \frac{\mu_o NI}{2\pi R}$$

$$\boxed{B = \frac{\mu_o NI}{2\pi r}}$$

Hence Magnetic field exists inside toroid only.

Electromagnetic Induction:

Changing magnetic flux in a coil or loop produces an emf in it. This emf is called induced emf and the phenomenon is known as electromagnetic induction. Induced emf causes current in the loop which is called induced current.

Faraday's Law:

According to faraday's law of electromagnetic induction,

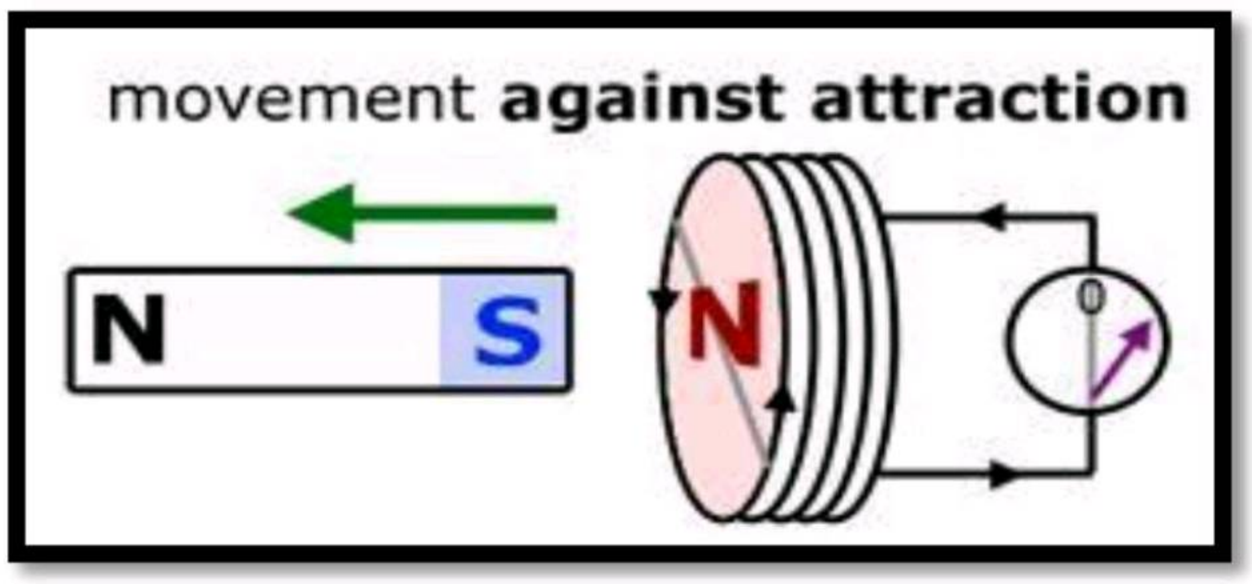
1. "An emf is induced in a coil through which the magnetic flux is changing".
2. The magnitude of induced emf depends only upon the number of turns and the time rate of change of flux linked with the circuit.
   Hence,
3. $\xi = -N\dfrac{\Delta\phi}{\Delta t}$

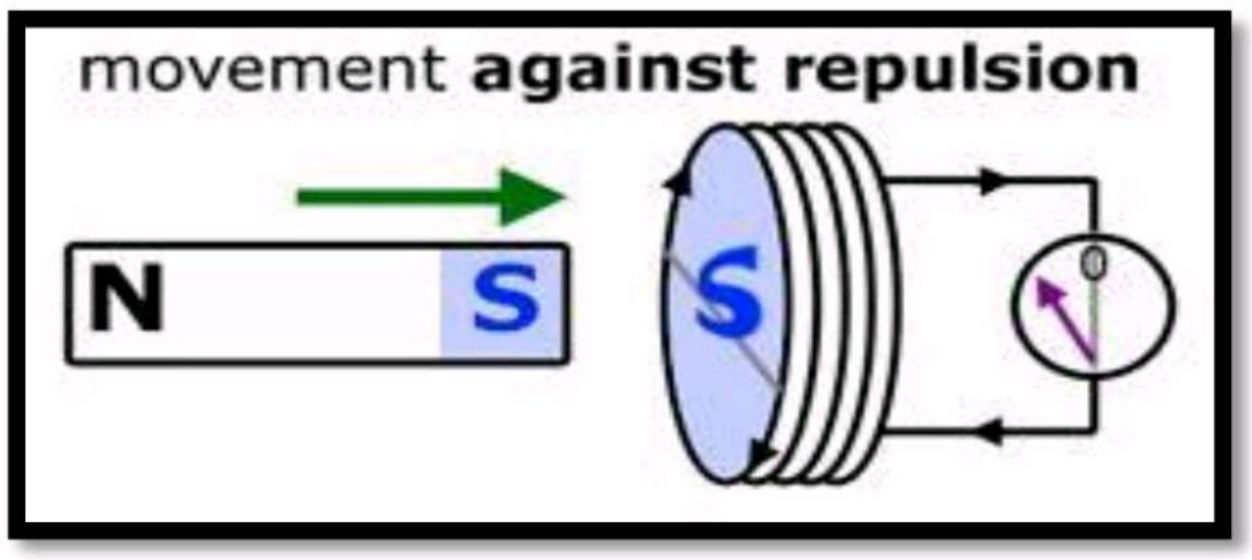
Where "N" represents the number of turns of coil, $\Delta\phi$ is the change of magnetic flux in time Δt. The negative sign is introduced according to Lenz's Law, to indicate the direction of induced emf. "$N\Delta\phi$" is called Flux Linkage.

The emf will be induced if there is a change of magnetic flux, no emf will be induced if flux either becomes zero or becomes constant.

Lenz's Law:

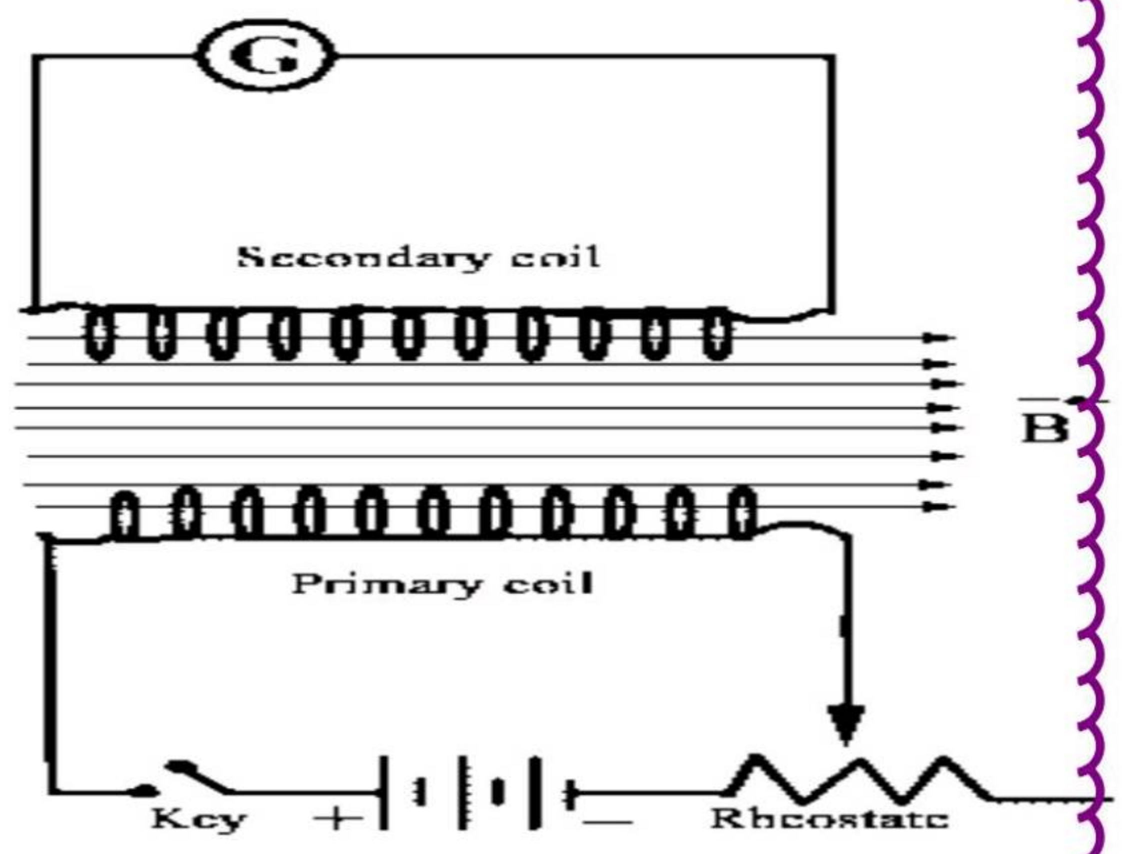
According to Lenz's Law, "The direction of induced emf and hence the direction of induced current is always such that it opposes the change which produces it".





Mutual-Induction:

The phenomenon in which an E.M.F is induced in to a coil due to change in current in a nearby coil is called Mutual-Induction.



Explanation:

If we place a coil (secondary coil) near another coil (primary coil) through which current is passing the magnetic flux of primary coil will pass through secondary coil. When current flowing through primary coil changes it changes magnetic flux through the secondary coil due to which an E.M.F is induced in it which is according to Faraday's law of EM induction.

Mathematical Expression:

Consider secondary coil having N_s turns. If the current flowing through the primary coil is change by amount ΔI_s in time Δt then the induced E.M.F can be written as

$$\boxed{\boldsymbol{\xi_S = -M\frac{\Delta I_P}{\Delta t}}}$$

Here $\boldsymbol{M}$ is called Mutual-inductance and it depends upon physical and geometrical properties of both coils.

According to Faraday's Law of electromagnetic induction

$$\xi_S = -N_S\frac{\Delta\phi_S}{\Delta t}$$

Comparing above we get

$$-N_S\frac{\Delta\phi_S}{\Delta t} = -M\frac{\Delta I_P}{\Delta t}$$

$$N\,\Delta\phi_S = M\Delta I_P$$

Or

$$\boxed{\boldsymbol{M = N\frac{\Delta\phi_S}{\Delta I_P}}}$$

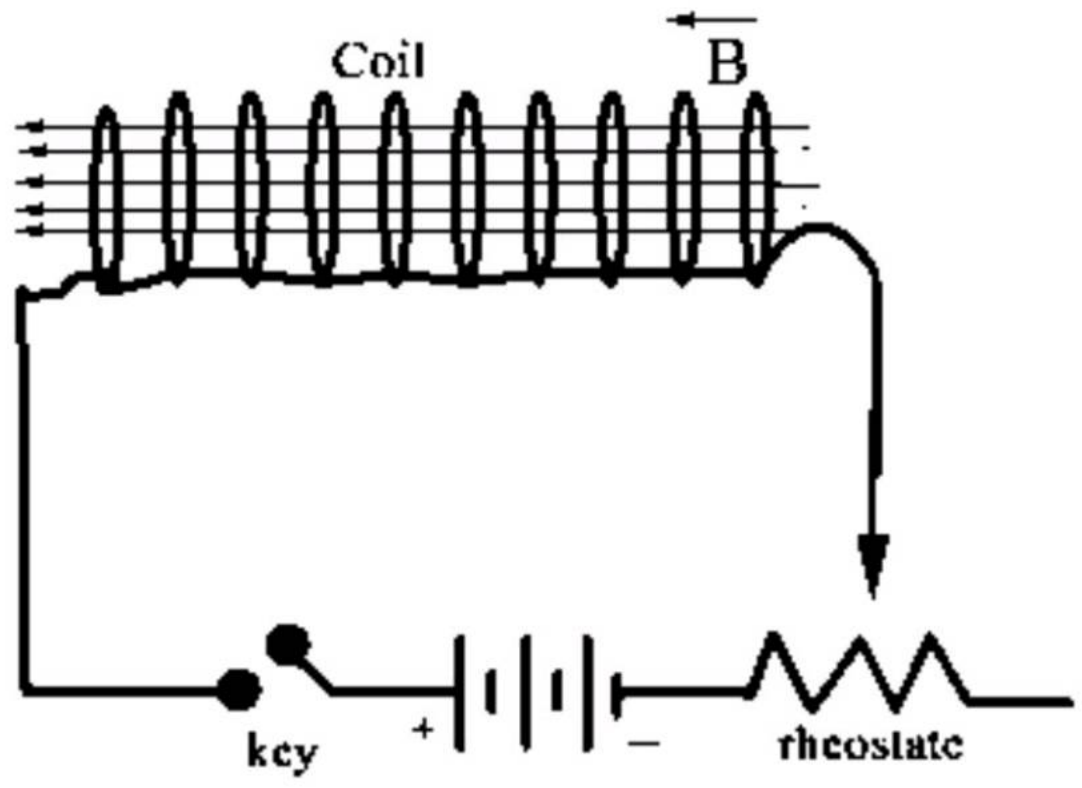
Unit of Mutual Inductance:

The unit of mutual inductance is **"Henry".**

Self-Induction:

The phenomenon in which an E.M.F is induced in to a coil due to change in current in that coil is called Self-Induction.

Explanation:



When current flowing through a coil changes it changes magnetic flux through the coil due to which an E.M.F is induced in to the coil which is according to Faraday's law of EM induction.

Mathematical Expression:

Consider a coil having N turns. If the current flowing through the coil is change by amount ΔI in time Δt then the induced E.M.F can be written as

$$\boxed{\xi = -L\,\frac{\Delta I}{\Delta t}}$$

Here L is called self-inductance and it depends upon physical and geometrical properties of coil.

According to Faraday's Law of electromagnetic induction

$$\xi = -N\,\frac{\Delta \phi}{\Delta t}$$

Comparing above we get

$$-N\,\frac{\Delta \phi}{\Delta t} = -L\,\frac{\Delta I}{\Delta t}$$

$$N\,\Delta \phi = L \Delta I$$

Or

$$\boxed{L = N\frac{\Delta \phi}{\Delta I}}$$

Unit of Self Inductance:

The unit of self-inductance is **"Henry".**

Motional E.M.F:

When a conductor is moved across a magnetic field, a potential difference is established between its ends. The potential difference is called motional emf.

Explanation:

When the conductor is moved with velocity v, the free electrons in the conductor also move along with it with same velocity. A force acts on each electron which is given by.

$$\vec{F} = e(\vec{B} \times \vec{v})$$

This force pushes the free electrons from end "b" to end "a" of the wire. As a result, upper end becomes more and more positive and lower end negative. Transfer of electrons stops when force F is balanced by the electrostatic attraction between ends "a" and "b". Hence, under given conditions, a certain value of emf is obtained.

Derivation of Formula:

Suppose;

q = Total charge transferred from end "b" to "a".

B =Flux density of uniform magnetic field.

θ =Angle between $\vec{v}$ and $\vec{B}$.

l=Length of the conductor.

Force on charge "q" is;

$$\vec{F} = q(\vec{B} \times \vec{v})$$
$$F = qBv \sin\theta$$

Work done on the charge from "b" to "a"

$$W = F\ l$$
$$W = qBv\sin\theta \times l$$

Motional emf is

$$\xi = \frac{W}{q} = \frac{qBv\sin\theta \times l}{q}$$
$$\xi = Bvl\sin\theta$$

Electromechanical Device:

An electromechanical device is that which converts electrical energy into mechanical energy or mechanical energy into electrical energy.

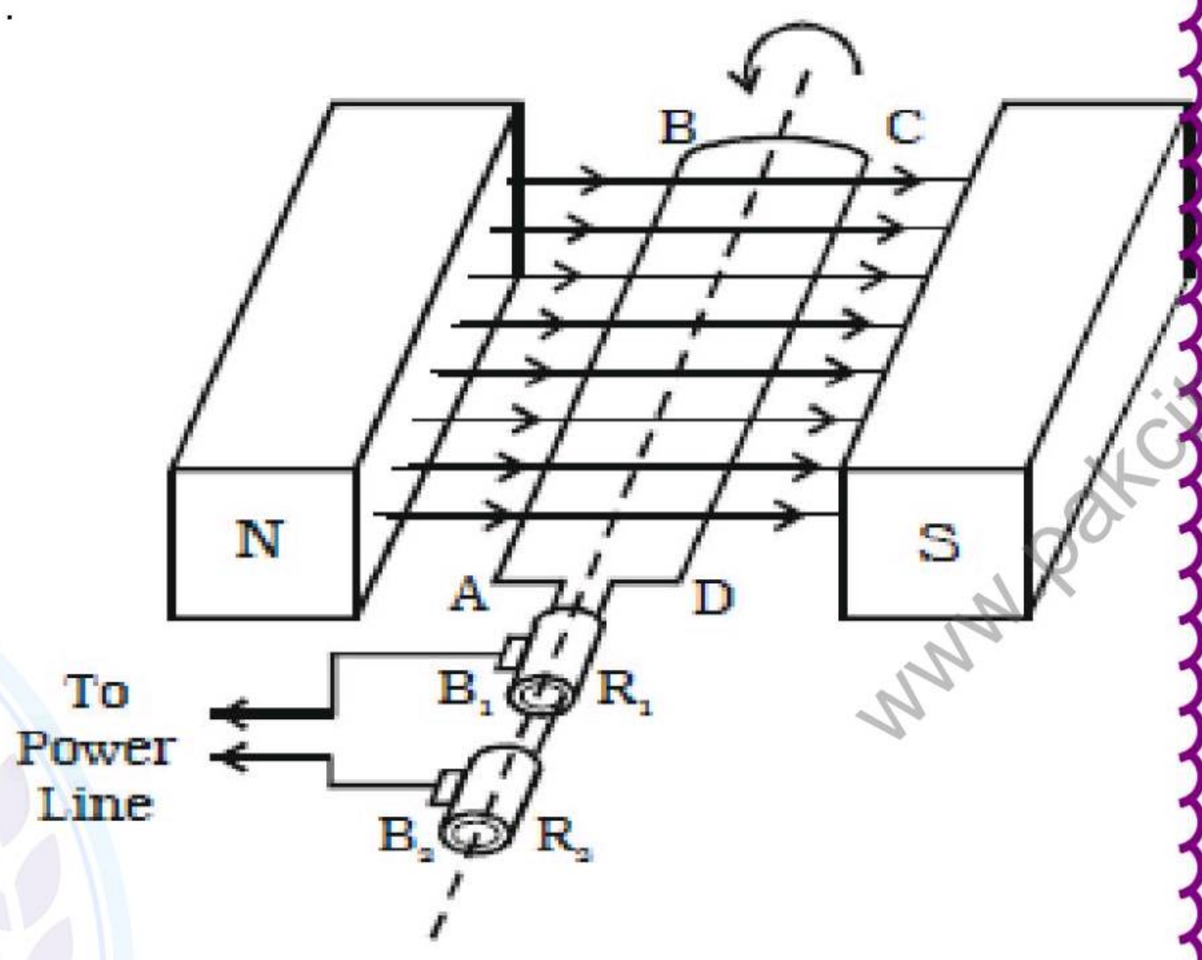
Electric Motor: A motor is an electromechanical device which converts electrical energy into mechanical energy:

Generator: A generator is a device which converts mechanical energy into electrical energy.

Alternating Current Generator (Dynamo):

It is an electrical machine that converts mechanical energy into electrical energy and gives output in the form of alternating current using the concept of Motional EMF.

Construction:



An AC generator consist of following parts

- A wire loop or Coil
- A magnet
- Slip rings
- Carbon brushes
- Armature

Working:

- As the coil rotates in the magnetic field an EMF is induced in along its lengths$(AB = CD = l)$.
- As these lengths are in series with each other the sum of the EMFs in AB and CD is received at output through the slip rings rubbing along carbon brushes connected to the output leads.
- The magnitude of EMF varies with time due to change in angle between velocity vector of coil and magnetic field vector.
- Due to the rotation of coil the direction of current changes in each half cycle therefore the output current is Alternating in nature.

Expression for EMF of an AC generator:

If the coil is rotating with linear speed v in the magnetic field of strength B then the instantaneous output EMF ξ can be written as

$$\xi = 2vBl \sin \omega t$$

If the breadth of the coil $AD = b$, then the linear speed v can be written as

$$v = \frac{b}{2}\omega$$

Hence

$$\xi = \cancel{2} \times \frac{b}{\cancel{2}} \omega Bl \sin \omega t$$

$$\xi = \omega Bbl \sin \omega t$$

Since area of coil is $A = bl$

$$\xi = AB\omega \sin \omega t$$

If coil has N turns then

$$\xi = ANB\omega \sin \omega t$$

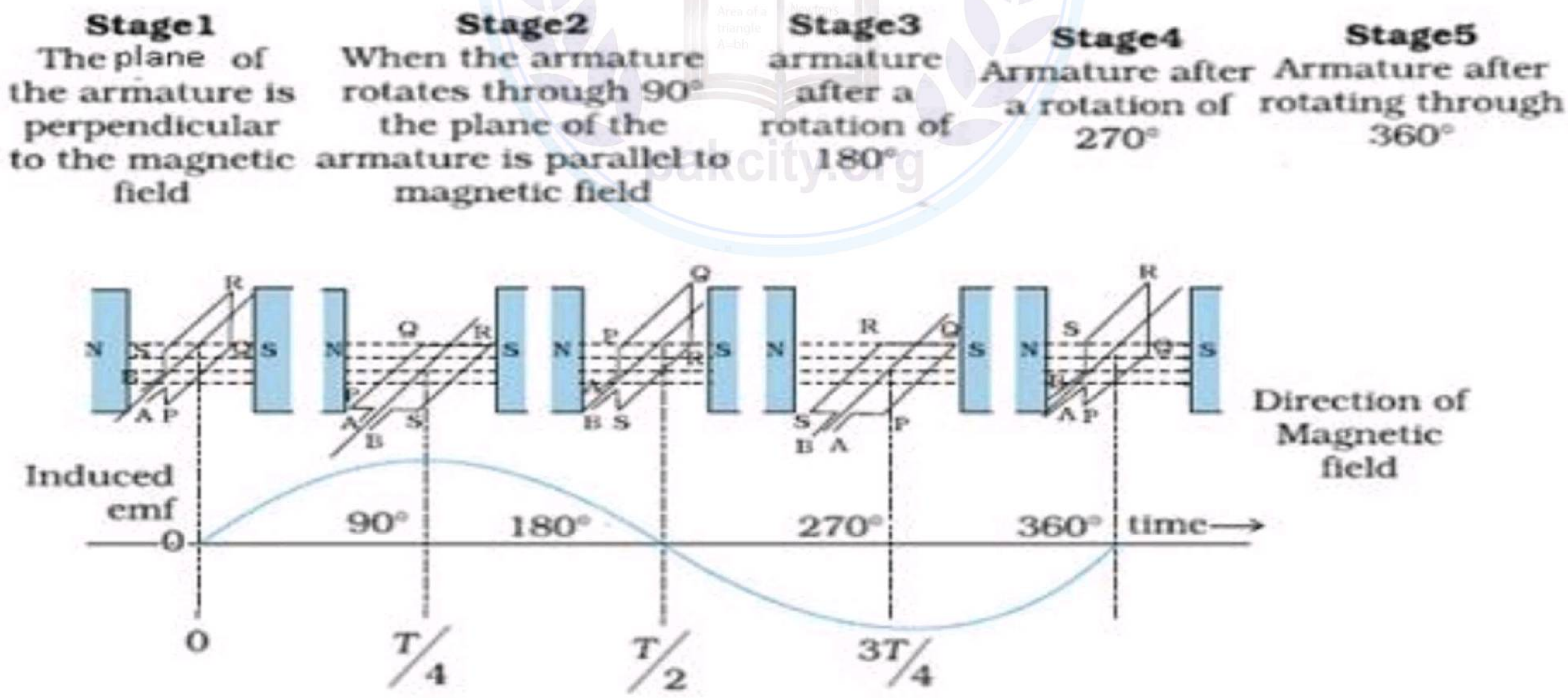
The maximum output EMF ξ_o will be obtain when coil is perpendicular to the field ($i.e: \theta = \omega t = 90°$) can be written as

$$\xi_o = ANB\omega \sin(90°)$$

$$\xi_o = ANB\omega$$

Now instantaneous output EMF can be written as

$$\boxed{\xi = \xi_o \sin \omega t}$$



Expression For Maximum emf:

The emf induced in the coil will be maximum when *sin ωt*=1, which is possible when the angle "ϑ" between $\vec{v}$ and $\vec{B}$ is 90°, at this particular moment plane of the coil will be exactly parallel to the magnetic field,

$$\xi_{max} = N\omega AB$$

Relation Between Instantaneous emf "ξ" and maximum emf "ξ_{max}"

The instantaneous and maximum emf induced in the coil are related by:

$$\xi = \xi_{max} \sin \omega t$$

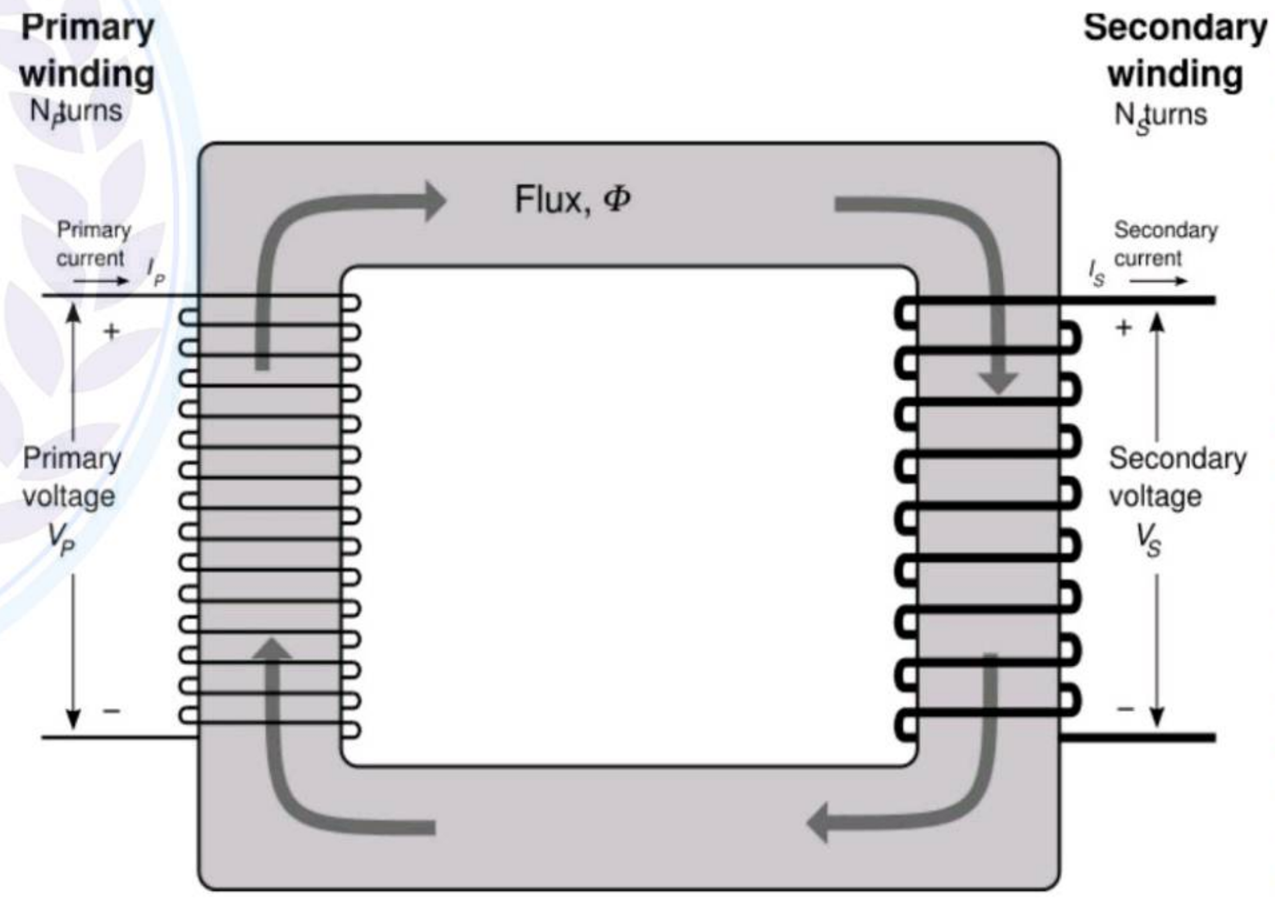
$$\omega = 2\pi f$$

In the rotating coil emf induced changes in magnitude and direction with time, such an emf is known as alternating emf. The current caused by alternating emf will also change continuously in magnitude and direction. Under the influence of alternating emf free electrons of the conductor will simply vibrate about their mean position.

Transformer:

Transformer is an electric device which is used either to step up or step down an alternating emf (Voltage). It works on the principle of "Mutual Induction".

Construction & Working:



A transformer consists of two coils known as "primary" and "secondary". These coils of insulated copper wires are wound one on top of the other on a laminated soft iron core. On one of the coils, known as the "primary coil", an alternating emf (Voltage) is applied. Due to self-induction an emf (ξ_P) Is induced in the primary coil,

given by; $\xi_P = -N_P \dfrac{\Delta\phi_m}{\Delta t}$ ....(i)

Where "Np" is the no. of turns in primary coil $\dfrac{\Delta\phi_m}{\Delta t}$ and is the rate of change of flux through primary coil. Since, the secondary coil is wound on top of the primary coil; therefore, the flux linked with the two coils will be practically equal. In other words, the rate of change of flux $\dfrac{\Delta\phi_m}{\Delta t}$ for both coils are same.

Due to mutual induction, emf Induced ξ_S in the secondary coil is given by:

$$\xi_S = -N_S \frac{\Delta\phi_m}{\Delta t} \text{.................}(ii)$$

Where, "Ns' is the no. of turns in secondary coil.
Dividing eq. (ii) by eq. (i)

$$\frac{\xi_S}{\xi_P} = \frac{-N_S \frac{\Delta\phi_m}{\Delta t}}{-N_P \frac{\Delta\phi_m}{\Delta t}}$$

$$\frac{\xi_S}{\xi_P} = \frac{N_S}{N_P}$$

Efficiency of Transformer:

Efficiency may be defined as;
"The ratio of power output to power input".

Power Output'= $\xi_S I_S$

Power Input = $\xi_P I_P$

If the power loses in a transformer are neglected then,

Power Output = Power Input

$$\xi_S I_S = \xi_P I_P$$

$$\frac{\xi_S}{\xi_P} = \frac{I_P}{I_S}$$

This relation shows that in a step-up transformer for which $(\xi_S > \xi_P)$, "I_P" will greater than ***"I_S".***

Step Up Transformer:

If $N_S > N_P$ then "ξ_S" also be greater than "ξ_P", A transformer in which $(\xi_S > \xi_P)$ is known as "Step up Transformer". It increases the applied voltage, whereas $(I_S < I_P)$.

Step Down Transformer:

A transformer in which $N_S < N_P$, gives lower emf through the secondary coil i.e. $(\xi_S < \xi_P)$ is known as "Step down Transformer". It decreases the applied voltage whereas $(I_P < I_S)$.

Uses of Transformer:

1. Step up transformer is used for sending electricity to long distance.
2. Step down transformer is used to decrease the large voltage up to 220 volts.
3. In electric bell the step-down transformer is used to set the voltage up to 4 volts.

Power Losses in A Transformer:

1. Power loses in transformer is mainly due to resistance of its wires.
2. To minimize this source of power loss, thick copper wire is used in the coil which carries larger current.
3. Eddy current induced on the surface of iron core due to the changing magnetic flux, produce heating effect.
4. To minimize this effect core is made up of thin sheets of soft iron separated by a thin layer of varnish.

CHAPTER-14 **NUMERICALS from Past Papers**

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1985

Q.4. (c) A toroidal coil has 300 turns and its mean radius 12cm. Calculate the magnetic field of induction 'B' inside the coil when a current of 5 amperes passes through it. (**2.5×10^{-3}Wb/m^2**)

Q.5. (c) Current of 2 amperes passes through an inductive circuit. What is the self-inductance of the circuit if the current falls to zero in 0.1 seconds? The average value of induced e.m.f. is 20 volts. (**1.0 Henry**)

1986

Q.5. (c) A train is moving directly towards south with a uniform speed of 10m/s, if the vertical component of the earth's magnetic field induction is 5.4×10^{-5} Tesla. Compute the e.m.f. induced in the axle 1.2m long.

(**6.48×10^{-4} volts**)

1987

Q.4. (c) An electron having a speed of 1.6×10^6 m/s is moving along a circle of radius 1.82×10^{-6}m entering perpendicularly in a uniform magnetic field. Find the value of magnetic field. **(5 Tesla)**

1988

Q.5. (c) A solenoid of diameter 5.0cm is 25cm long and has 250 turns. If the current flowing in it is 5 amperes, find B inside the solenoid. (**6.28×10^{-3} web/m^2**)

1990

Q.4. (c) Find the current required to produce a field of induction B = 2.51×10^{-3} web/m^2 in a 50cm long solenoid having 4000 turns of wire. (**0.25A**)

1992

Q.4. (c) A current of 2 amperes is passing through a solenoid. If the solenoid has 24 turns per cm of its length, find the value of B. (**6.03×10^{-3} web/m^2**)

Q.6. (c) A transformer has 1000 turns in its primary coil. If the input voltage of the transformer is 200 volts, what should be the number of turns of the secondary coil to obtain an output of 6.0 volts? (**30 turns**)

1993

Q.5. (c) A 10eV electron is moving in a circular orbit in a uniform magnetic field of strength 10^{-4}weber/m^2. Calculate the radius of the circular path. (**0.107m**)

Q.6. (c) A coil having an area of cross section 0.05m^2 and number of turns 100 is placed perpendicular to the magnetic field of induction 0.08 weber/m^2. How much e.m.f. will be induced in it if the field is reduced to 0.02 weber/m^2 in 0.01 seconds? (**30 volts**)

1994

Q.6. (c) A solenoid 25cm long has a cross-section of 5 square cm with 250 numbers of turns on it. If a current if 5 amperes is passed through it, find "B" in it. (**6.28×10^{-3} web/m^2**)

1995

Q.4. (c) An electron is moving along a circle of radius 1.8×10^{-7}m. Calculate the speed of the electron on entering perpendicularly in a uniform magnetic field of 5.0 Tesla. (**15.81×10^4 m/s**)

1996

Q.4. (c) A coil of 100 turns and area (4cm x 2cm) is placed in a uniform magnetic field of 0.45 T. The coil carries a current of 1.5 amperes. Calculate the torque on the coil when the plan is at 60° with B. **(0.0054 Nm)**

Q.5. (c) An airplane is flying in a region where the vertical component of earth's magnetic field is 3.2×10^{-4}T. If the wingspan of the airplane is 50m and its velocity is 360 km/hour, find the potential difference between the tips of the wing of airplane. **(1.6V)**

1997

Q.4. (c) An α particles are accelerated from rest at a P.D. of 1 KV. They then enter a magnetic field B=0.2T perpendicular to their direction. Calculate the radius.
Given m = 6.68×10^{-27} kg & q = 2e. **(0.032m)**

1998

Q.4. (c) A solenoid 20 cm long has three layers of windings of 300 turns each. If a current of 3 amperes is passed through it, find the value of the magnetic field of Induction. **(0.016 web/m^2)**

Q.5. (d) A 500 turn coil in A.C. Generator having an area of 1000 cm^2 rotates in a magnetic field of value 50 Tesla. In order to generate 220 volts maximum, how fast is the coil to be rotated? Express your answer in terms of the number of revolutions per second. **(0.088 rad/sec, 0.014rev/s)**

1999

Q.5. (d) A 10eV electron is moving in a circular orbit in a uniform magnetic field of strength 10^{-4} weber/m^2; calculate the radius of the circular path. **(0.107m)**

Q.6. (d) A transformer has 1000 turns in the primary coil. If the input voltage of the transformer is 200 volts, what should be the number of turns of the secondary coil to obtain an output of 6.0 volts? **(30 turns)**

2001

Q.5. (d) What will be the mutual inductance of two coils when the change of a current of a 3 amperes in one coil produces the change of flux of 6×10^{-4} Weber in the second coil having 2000 turns? **(400mH)**

Q.6. (d) An electron is accelerated by the potential difference of 1000 volts. It then enters into a uniform magnetic field of induction B = 2.5, weber/m^2 at an angle of 45° with the direction of the field, find the value of the path described by the electron. **(6.04 x 10^{-5} m)**

2002 (Pre–Med)

Q.5. (d) A long solenoid is wound with 10 turns per cm and carries a current of 10 amperes; find the magnetic flux density within it. **(0.0125 web/m^2)**

2002 (Pre–Engg)

Q.5. (d) An e.m.f. of 45 milli-volts is induced in a coil of 500 turns, when the current in a neighboring coil changes from 10 amperes to 14 amperes in 0.2 seconds.

a) What is the Mutual Inductance of the coils?
b) What is the rate of change of flux in the second coil? **(2.25mH, 9 x 10^{-5} web/sec)**

2003 (Pre–Med)

Q.5. (d) The current in a coil of 325 turns is changed from zero to 6.32 amperes thereby producing a flux of 8.46×10^{-4} Webers. What is the self-inductance of the coil? (**43.2mH**)

2003 (Pre–Engg)

Q.5. (d) Calculate the speed of an electron entering perpendicularly in a uniform magnetic field of 5.0 w/m^2 which moves along a circle of radius 1.8×10^{-6} m in the field. (**1.58×10^6 m/sec**)

Q.6. (d) A coil of 50 turns is wound on an ivory frame 3cm x 6 cm which rotates in a magnetic field of induction B = 2 web/m^2. What will be the torque acting on it if a current of 5 amp passes through it and the plane of the coil makes an angle of 45° with the field. (**0.6363 Nm**)

2004

Q.5. (d) How fast must a proton of mass 1.67×10^{-27} kg be moving if it is to follow a circular path of radius 2.0 cm in a magnetic field of 0.7 Tesla? (**1.34×10^6 m/sec**)

Q.6. (d) The current in a coil of 500 turns is changed from zero to 5.43 amps. Thereby producing a magnetic flux of 8.52×10^{-4} Webers. What is the Self-Inductance of the coil? (**78.45mH**)

2005

Q.5. (d) A proton accelerated through 1000 volts is projected normal to a 0.25 Tesla magnetic field. Calculate the following:

a) The Kinetic energy of the proton on entering the magnetic field.

b) The radius of the circular path of the proton (**1.6×10^{-16} J, 0.0182 m**)

Q.6. (d) A step-down transformer having 4000 turns in primary is used to convert 4400 volts to 220 volts. The efficiency of the transformer is 90% and 9KWatt output is required. Determine the Input power, the Number of turns in the secondary coil and the current in the primary and secondary coils? (**10000W, 200turns, 2.23A, 40.9A**)

2006

Q.5. (d) A long solenoid is wound with 35 turns in 10cm and carries a current of 10A. Find the magnetic field in it. (**4.3×10^{-3} wb/m^2**)

2007

Q.4. (d) An airplane is flying in a region where the vertical component of earth's magnetic field is 3.2×10^{-4}T. If the wingspan of the airplane is 50m and its velocity is 360 km/hour, find the potential difference between the tips of the wing of airplane. (**1.6V**)

20

Q.5. (d) A step-down transformer reduces 1100V to 220V. The power output is 12.5KW and overall efficiency of the transformer is 90%. The primary winding has 1000 turns. How many turns do the secondary have? What is the power input? What is the current in each coil?

(**200 turns, 13.8×10^3 Watt, 12.6A, 56.81A**)

2009

Q.4. (d) A pair of adjacent coil has a mutual inductance of 850mH. If the current in the primary coil changes from 0 to 20A in 0.1 sec; what is the change in the magnetic flux in the secondary coil of 800 turns? (**2.12×10^{-2}webers**)

2010

Q.2. (x) Find the current required to produce a field of induction B = 2.512 x 10^{-3} T in a 50 cm long solenoid having 4000 turns of wire. ($\mu_o = 4\pi \times 10^{-7} Wb/A\text{-}m$

(0.785A)

2011

Q.2. (viii) An alternating current Generator operating at 50 Hz has a coil of 200 turns, while the coil has an area of 120cm^2. Calculate the magnetic field intensity applied to rotate the coil to produce the maximum voltage of 240V. **(0.31831T)**

Q.2. (xiii) The inner and the outer diameters of the toroid are 22cm and 26cm. If a current of 5.0 amp is passed which produces 0.025 tesla flux density inside the core, find the approximate length of the wire wound on the toroid. ($\mu_o = 4\pi \times 10^{-7} Wb/A\text{-}m$).

(226.2m)

2012

Find the current required to produce a field of induction B = 2.512 x 10^{-3} T in a 50 cm long solenoid having 4000 turns of wire. ($\mu_o = 4\pi \times 10^{-7} Wb/A\text{-}m$).

2013

An alternating current generator operates in at 79 Hz the area of coil is 500 cm^2. Calculate the number of turns in the coil when a magnetic field of induction 0.06 web/m^2 produces a maximum potential difference of 149 V.

2014

Q.2. (iii) An iron core of solenoid 500 turns has a cross section of 5 cm^2. A current of 2.3 A passing through produces of flux of B = 0.53 T. How large an e.m.f is induced in it, if the current is turned off in 0.1 second? What is the self-inductance of the solenoid?

2015

Q.2(ix) An e.m.f. of 45 milli-volts is induced in a coil of 500 turns, when the current in a neighboring coil changes from 15 amperes to 4 amperes in 0.2 seconds.

a) What is the Mutual Inductance of the coils?
b) What is the rate of change of flux in the second coil?