Chapter = 11

Heat

Heat:

Heat is a form of energy that always flows from high temperature body to low temperature body. OR

"In an isolated system, heat is defined as the sum of kinetic energies of molecules or atoms."

Unit:

The S.I unit of heat is Joule. In C.G.S system unit of heat is Caloric and in British Engineering System the unit of heat is Btu (British thermal unit).

1 Btu = 252 Cal

1 Btu = 1055 Joule

1 Cal = 4.1865 Joule

Different modes of transfer of heat:

1. Convection
2. Conduction
3. Radiation

Convection:

When heat is transferred from hot body to cold body by actual movements of molecules of atoms then it is called convection process. Gases and liquids are heated by this mode of transfer of heat.

Conduction:

In conduction heat is transferred from hot body to cold body by vibration of atoms of molecules when they are in contact with each other. In metals heat is transferred by conduction process.

Radiation:

In radiation heat is transferred from hot body to cold body in form of electromagnetic waves. Sunlight and Laser light are examples of radiation.

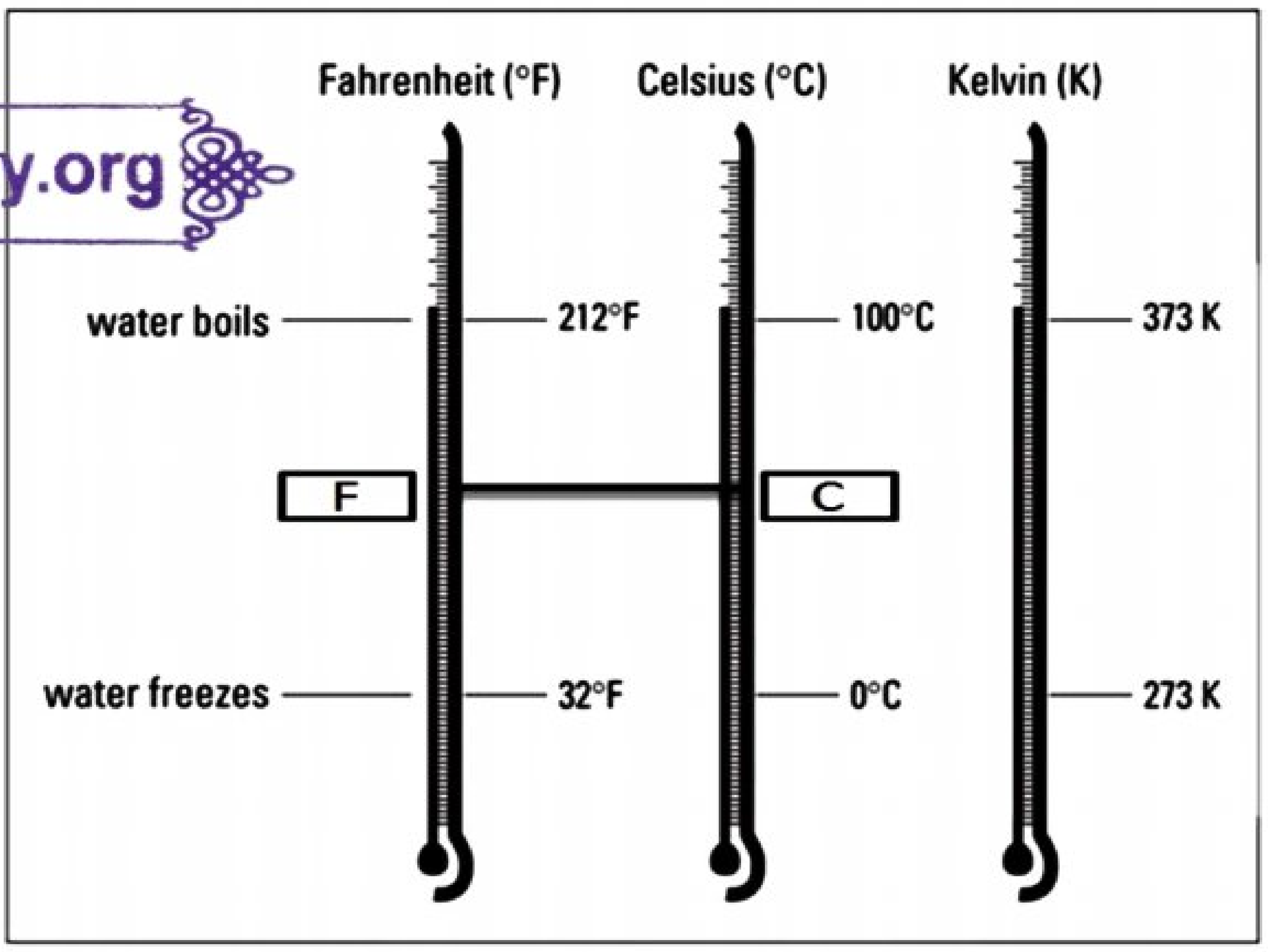
Temperature:

"The degree of hotness or coldness of an object or body is called temperature. "

OR

"In an isolated system temperature is the average translational kinetic energy of molecules or atoms."

The S.I unit of temperature is Kelvin. Other units are Fahrenheit and Centigrade.



Temperature scales:

There are three different scales of temperature

1. Celsius Scale
2. Fahrenheit Scale
3. Kelvin Scale

Relation between Kelvin scale and Celsius scale

From figure it is clear that

$$\boldsymbol{K = C + 273}$$

Relation between Fahrenheit scale and Celsius scale

Consider a common point on Fahrenheit and Celsius scale then, we have

$$\frac{C-0}{100-0} = \frac{F-32}{212-32}$$

$$\frac{C}{100} = \frac{F-32}{180}$$

$$C = \frac{100}{180}\,(F-32)$$

$$\boldsymbol{C = \frac{5}{9}\,(F-32)}$$

Law of Heat Exchange:

In an isolated system the amount of heat lost by the hot body is always equal to the amount of heat gained by the cold body.

$$\text{Heat lost by hot body} = \text{Heat gained by cold body}$$

Thermometric Property:

Property of a substance, which changes uniformly with the change in temperature, is called Thermometric property.

Thermal Expansion:

The change in physical dimension of a body (e-g length, Area or volume) with the change in temperature is called Thermal expansion.

Type of Thermal Expansion:

There are following type of thermal expansion:

1. Linear Expansion.
2. Volume Expansion.
3. Areal Expansion

Linear Expansion:

The phenomenon in which length of an object changes due to change in its temperature is called linear expansion.

Mathematical Expression:

Experimentally it is found that change in length is directly proportional to the original length L and change in temperature ΔT.

$$\Delta L \propto L$$

$$\Delta L \propto \Delta T$$

$$\boxed{\Delta L = \alpha L \Delta T} \text{ --------- (1)}$$

Where α is constant of proportionality and termed as coefficient of linear expansion. Its value changes from material to material and it may be defined as

"The change in length per unit length per unit change in temperature"

$$\boxed{\boldsymbol{\alpha = \frac{\Delta L}{L \Delta T}}}$$

Final Length (L'):

If L' is the final length then change in length can be expressed as

$$Change\ in\ length = final\ length - initial\ length$$

$$\Delta L = L' - L$$

Substitute this value in equation (1), we have

$$L' - L = \alpha L \Delta T$$

$$L' = L + \alpha L \Delta T$$

$$\boldsymbol{L' = L\,(1 + \alpha \Delta T)}$$

Volume Expansion:

The phenomenon in which volume of an object changes due to change in its temperature is called volume expansion.

Mathematical Expression:

Experimentally it is found that change in volume is directly proportional to the original volumeand change in temperature ΔT.

$$\Delta V \propto V$$

$$\Delta V \propto \Delta T$$

$$\boxed{\Delta V = \beta V \Delta T} \text{ --------- } (1)$$

Where β is constant of proportionality and termed as coefficient of volume expansion and it may be defined as

"Change in volume per unit volume per unit change in temperature."

$$\boxed{\boldsymbol{\beta = \frac{\Delta V}{V\Delta T}}}$$

Final Volume (V'):

If V' is the final volume then change in volume can be expressed as

$$Change\ in\ volume\ =\ final\ volume - initial\ volume$$

$$\Delta V = V' - V$$

Substitute this value in equation (1), we have

$$V' - V = \beta V \Delta T$$

$$V' = V + \beta V \Delta T$$

$$\boldsymbol{V' = V\,(1 + \beta \Delta T)}$$

Relation between Coefficients of linear and volume expression:

Consider a box of dimensions l, w and h. Itsvolume at some temperature T_1 is V. If the temperature changes to T_2 andits volume changes to V' then according to linear expansion

$$L' = L\,(1 + \alpha \Delta T)$$

Since each dimension of cubic box is linear in nature therefore new dimensions will be

$$l' = l(1 + \alpha \Delta T)$$

$$w' = w(1 + \alpha \Delta T)$$

$$h' = h(1 + \alpha \Delta T)$$

We know that initial volume at temperature T_1

$$V = lwh \_\_\_\_\_\_\_\_\_\_ (1)$$

After temperature change to T_2, final volume will become

$$V' = l'w'h'$$

$$V' = l(1+\alpha\Delta T)\,w(1+\alpha\Delta T)h(1+\alpha\Delta T)$$

$$V' = lwh(1+\alpha\Delta T)^3 \_\_\_\_\_ (2)$$

Using the mathematical relation

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

And $V = lwh$

Putting values in equation (2), we have

$$V' = V(1 + \alpha^3\Delta T^3 + 3\alpha\Delta T + 3\alpha^2\Delta T^2)$$

As α is very small in magnitude neglecting its higher powers we get

$$V' = V(1+3\alpha\Delta T) \text{ ----------- } (3)$$

$$V' = V + 3\alpha V\Delta T$$

$$V' - V = 3\alpha V\Delta T$$

$$\Delta V = 3\alpha V\Delta T$$

$$\frac{\Delta V}{V\Delta T} = 3\alpha$$

By definition, we know that

$$\beta = \frac{\Delta V}{V\Delta T}$$

Therefore

$$\boxed{\boldsymbol{\beta = 3\alpha}}$$

Hence, coefficient of volume expansion is three times the coefficient of linear expansion.

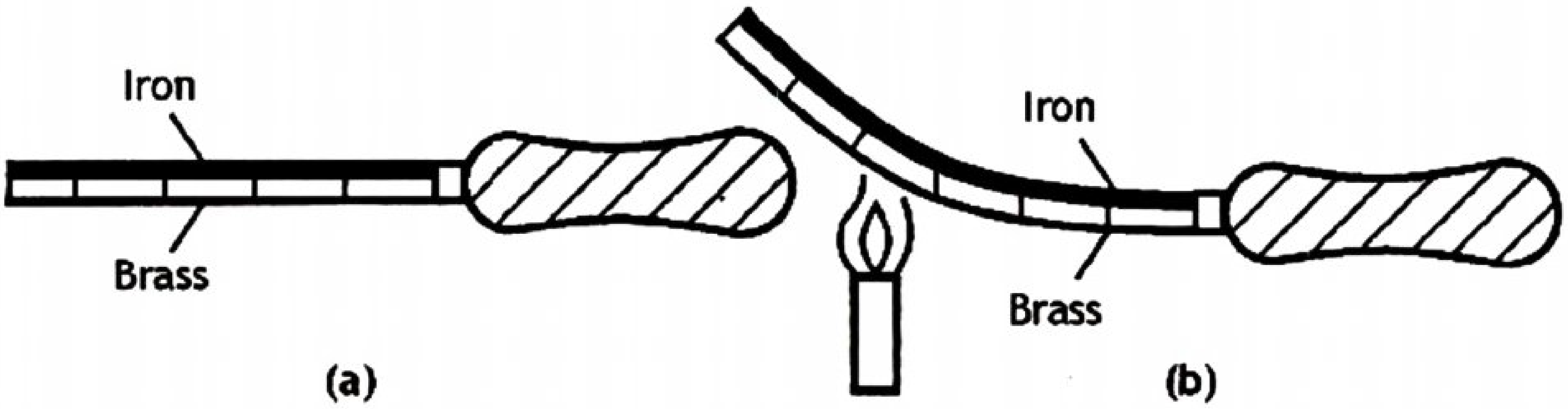
Bimetallic Strip:

A strip which is made of two different metals having different coefficient of linear expansion is called bimetallic strip.

Working Principle:

When the strip is heated the two metals having different coefficient of linear expansion expands at different rates causing the strip to bend.

$$coefficinet\ of\ linear\ expansion_{\,Brass} > coefficient\ of\ linear\ expanions_{\,Iron}$$



Bimetallic Thermostat:

Bimetallic thermostat is a device used to maintain the temperature. It works on the principle of bimetallic strip.

Working principle:

As the temperature of the stripincreases, the two metals expand by differentamounts and the strip bends. The change in shape of a bimetallic strip can make or break up electrical connection.

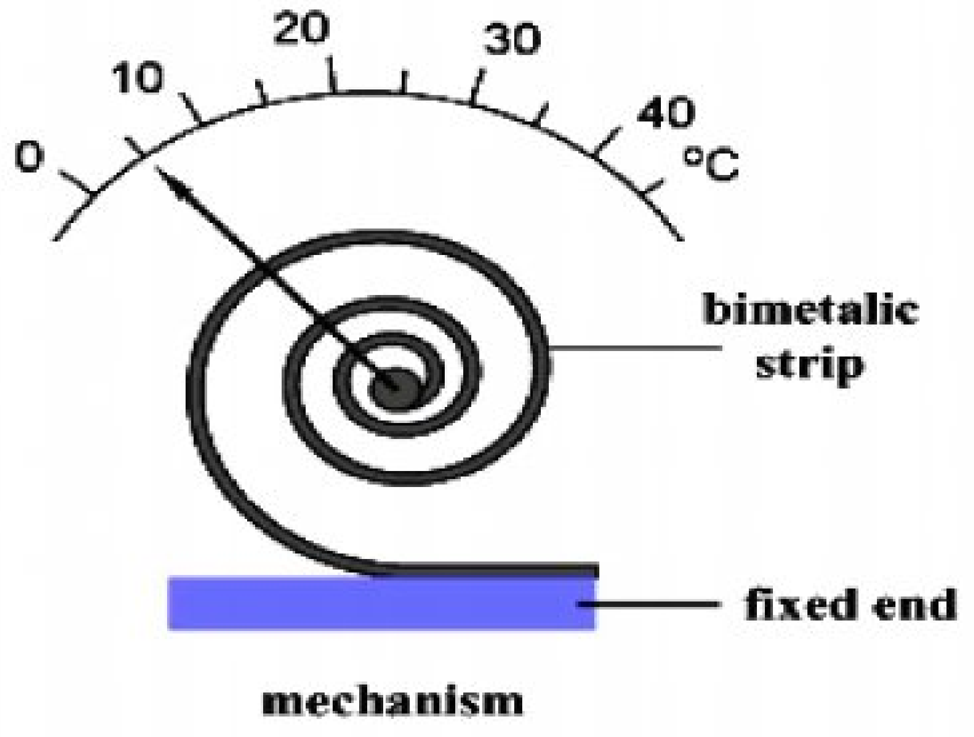
Bimetallic Thermostat

Bimetallic Thermometer:

Bimetallic thermometer is used to measure high range of temperature. It works on the principle of bimetallic strip.

Working Principle:

Thin bimetallic strip is converted into spiral whose one end is fixed and other end is movable which is attached to a pointer which moves over a scale as shown in figure. As the temperature rises the spiral tends to bend and pointer moves over a calibrated scale. The movement of pointer over a calibrated scale measures the temperature.



Kinetic Molecular Theory of Gases:

In 19th century Scientist give assumption on which they define an ideal gas. Some major points of kinetic molecular theory of gases are given below:

1. A finite volume of a gas contains very large number of molecules.
2. The distance between molecules is very large as comparable to the dimensions of molecules.
3. There is no molecular force of attraction between two molecules of a gas.
4. Molecules of a gas are in random motion.
5. The collision between two molecules or with the surface container of a gas is perfectly elastic.

Gas Laws:

Physical laws that relate pressure, volume, mass and temperature of a gas are called gas laws.

Boyles law:

Statement:

According to Boyles Law

> "For a given amount of gas, volume of a gas is inversely proportional to its pressure keeping the temperature of gas constant."

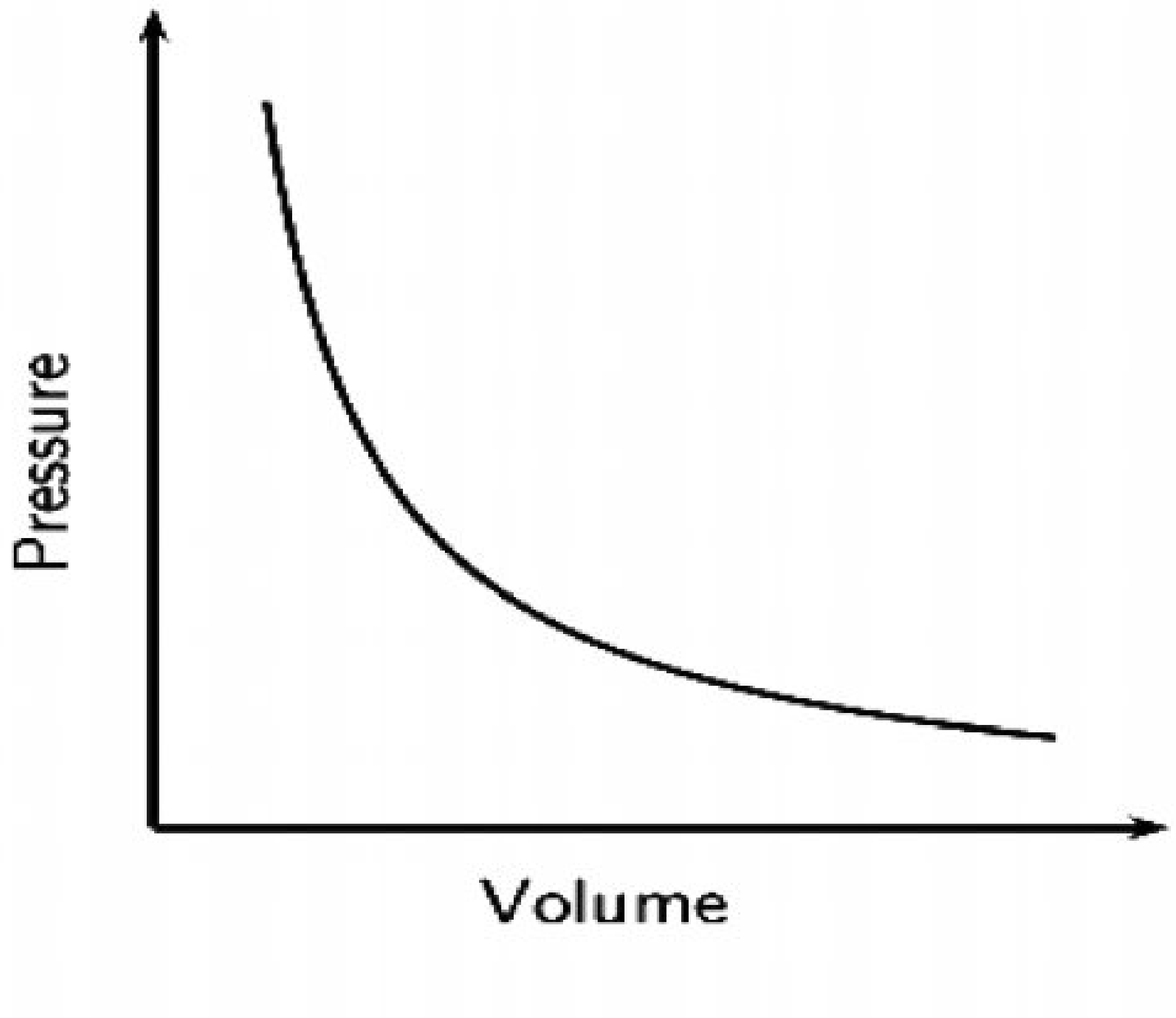
Mathematically:

$$V \propto \frac{1}{P}$$

$$V = \frac{constant}{P}$$

$$PV = constant$$

Graphical Representation:



Charles Law:

Statement:

"For a given amount of gas, volume of a gas is directly proportional to the absolute temperature keeping the pressure of gas constant."

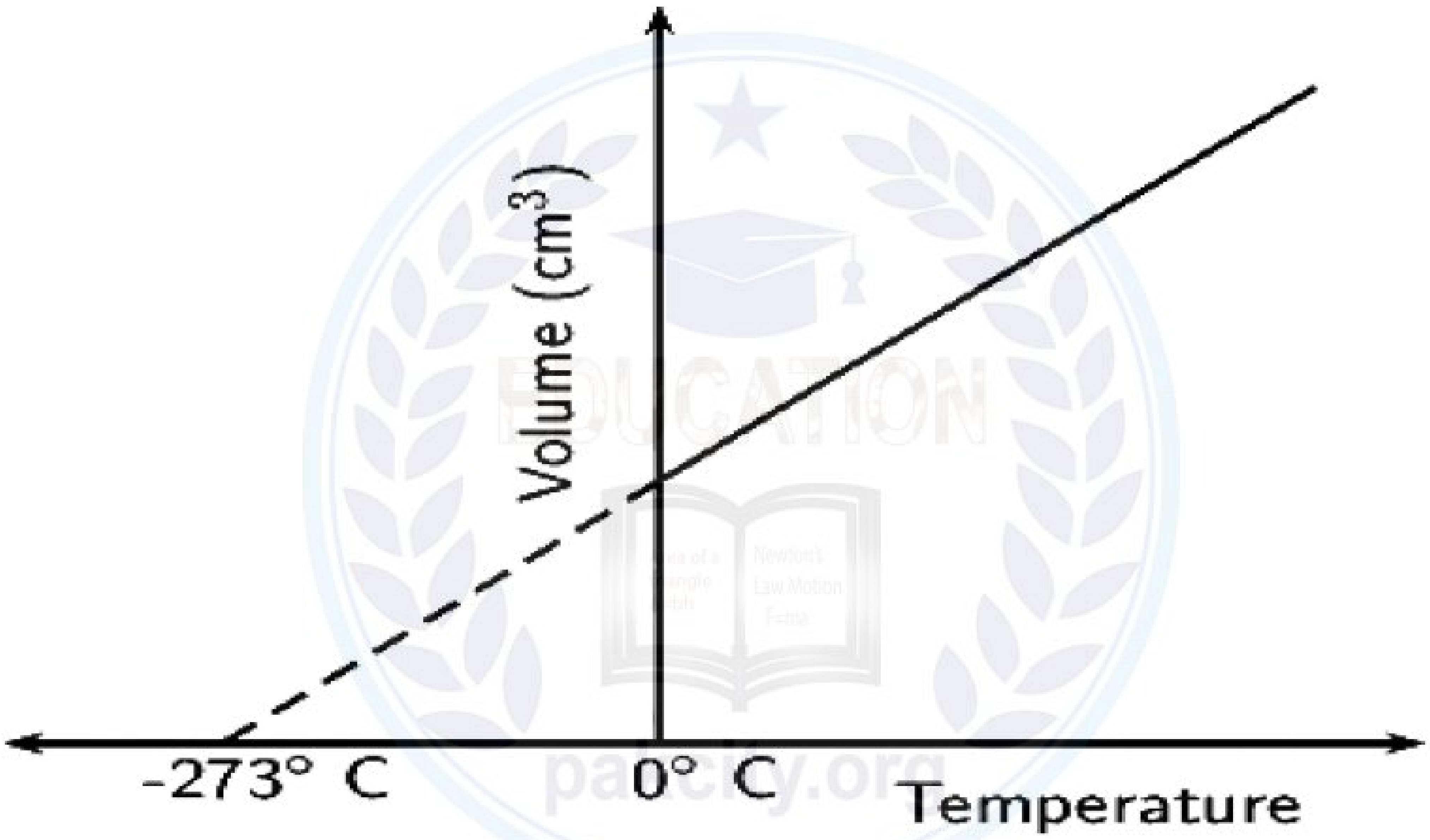
Mathematically

$$V \propto T$$

$$V = (Constant)T$$

$$\frac{V}{T} = Constant$$

Graphical Representation:



Absolute zero temperature:

Absolute zero can be defined as:

"It is the temperature, at which Volume of an ideal gas is theoretically Zero."

Hence Charles law is not applicable at Zero Kelvin because:

- At Zero Kelvin nothing can exist in gaseous state and Charles law is only applicable on gases.
- According to law of conservation of energy volume of any material cannot be zero at any temperature.

General Gas Law:

According to Boyle's Law:

"Volume of a given mass of a gas is inversely proportional to its pressure keeping the temperature of gas constant."

$$P \propto \frac{1}{V}$$

Or

$$V \propto \frac{1}{P} \quad ------(1)$$

According to Charles Law:

"Volume of given mass of a gas is directly proportional to the absolute temperature keeping the pressure of gas constant."

$$V \propto T \quad ------(2)$$

According to Avogadro Law

"Volume of given mass of a gas is directly proportional to the number of moles keeping the pressure and temperature of gas constant."

$$V \propto n \quad -------(3)$$

Combining (1), (2) and (3), we have

$$V \propto \frac{nT}{P}$$

$$V = \frac{nRT}{P}$$

$$\boxed{\boldsymbol{PV = nRT}}$$

Where

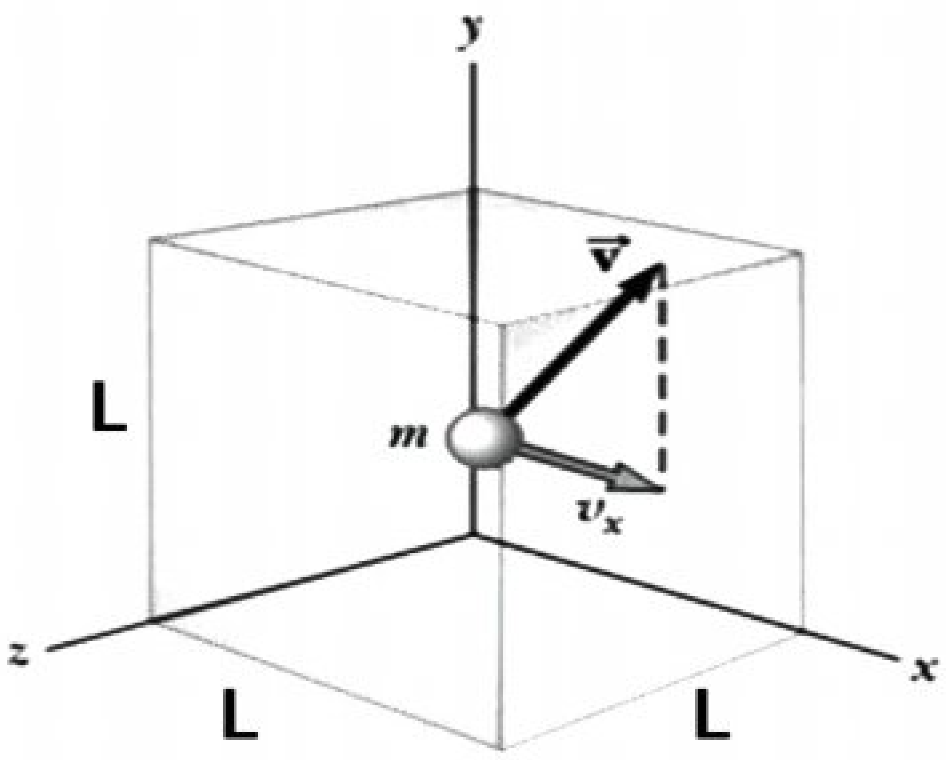
n = number of moles and

R = Universal Gas Constant

R = 8.314 J/mol^{-1} K^{-1}.

Pressure of an Ideal Gas (Pressure Equation):

Consider a cubical container of dimensions equal to "L" containing N number of molecules of an ideal gas. Each is having mass "m". Let one of the N molecules is moving in x- direction with velocity $\boldsymbol{v_x}$ and after striking the wall of the container it bounce back in opposite direction.



Change in the momentum of the molecule can be written as

Initial momentum: $p_i = -mv_x$

Final momentum: $p_f = mv_x$

Change in momentum = Final momentum - Initial momentum

$$\Delta p = p_f - p_i$$

$$\Delta p = mv_x - (-mv_x)$$

$$\Delta p = mv_x + mv_x$$

$$\Delta p = 2mv_x$$

Time taken during the complete trip can be find out as

$$S = vt$$

Here

$$S = 2L$$

$$v = v_x$$

$$t = \Delta t$$

$$2L = v_x \Delta t$$

$$\Delta t = \frac{2L}{v_x}$$

We know that the rate of change of linear momentum is equal to force, therefore

$$F = \frac{\Delta p}{\Delta t}$$

$$F_x = \frac{2mv_x}{2L/v_x}$$

$$F_x = m\frac{v_x^2}{L}$$

Force due to N molecules can be written as

$$F_x = \frac{m}{L}(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2)$$

Since pressure is defined as force per unit area

$$P = \frac{F}{A}$$

$$P_x = \frac{m}{L}(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2) \times \frac{1}{\mathrm{L}^2}$$

$$P_x = N\frac{m}{L^3}(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2)$$

Multiply by N/N

$$P_x = N\frac{m}{L^3}\frac{(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2)}{N}$$

Density of the gas "**ρ**" can be written as:

$$\rho = \frac{m}{V}$$

For N number of molecules

$$\rho = \frac{Nm}{V}$$

Since $$V = L^3$$

$$\rho = \frac{Nm}{L^3}$$

Similarly square mean of the velocities can be written as

$$\overline{v_x^2} = \frac{(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2)}{N}$$

Now pressure can be written as

$$P_x = \rho\overline{v_x^2}$$

Using Pascal principle $$P_x = P_y = P_z = P$$

$$P = \rho\overline{v_x^2} \text{------------------------------- (1)}$$

For molecule having all three components of velocity

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

For $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

$$\overline{v^2} = \overline{v_x^2} + \overline{v_x^2} + \overline{v_x^2}$$

$$\overline{v^2} = 3\overline{v_x^2}$$

$$\frac{\overline{v^2}}{3} = \overline{v_x^2}$$

Putting in equation (1) we get

$$\boxed{\boldsymbol{P = \frac{1}{3}\rho\overline{v^2}}}$$

Above equation is called Pressure equation of an ideal gas.

Kinetic Energy of Molecules of an Ideal Gas:

Consider the pressure equation for ideal gas

$$P = \frac{1}{3}\rho\overline{v^2}$$

Since

$$\rho = \frac{Nm}{V}$$

$$P = \frac{1}{3}\frac{Nm}{V}\overline{v^2}$$

$$PV = \frac{1}{3} Nm\overline{v^2}$$

Using General Gas law PV= nRT

$$nRT = \frac{1}{3}\ Nm\overline{v^2} - - - -(1)$$

Number of moles can be expressed in terms of number of molecules (N) and Avogadro number (N_A) as

$$n = \frac{N}{N_A}$$

Equation (1) becomes

$$\frac{N}{N_A} RT = \frac{1}{3} Nm\overline{v^2}$$

$$\frac{R}{N_A} T = \frac{1}{3} m\overline{v^2} - - - -(2)$$

Here

$$\frac{R}{N_A} = K_B$$

Where K_B is called Boltzmann Constant

Equation (2) becomes

$$K_B T = \frac{1}{3} m\overline{v^2}$$

$$3K_B T = m\overline{v^2}$$

Dividing both sides with 2 we get

$$\frac{3}{2} K_B T = \frac{1}{2} m\overline{v^2}$$

Since $K.E = \frac{1}{2} mv^2$

$$\boxed{K.E = \frac{3}{2} K_B T}$$

Above equation represent the kinetic energy of molecules of an ideal gas. From above equation it is clear that kinetic energy of molecule is dependent only on temperature of the gas.

$$\boldsymbol{K.E \propto T}$$

Derivation of Gas Laws from Pressure equation:

Boyles Law:

Consider the pressure equation for ideal gas

$$P = \frac{1}{3}\rho\overline{v^2}$$

Since

$$\rho = \frac{Nm}{V}, therefore$$

$$P = \frac{1}{3}\frac{Nm}{V}\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} - - - - - (1)$$

Now from relation for kinetic energy of molecule of an ideal gas

$$K.E = \frac{3}{2}K_BT$$

$$\frac{3}{2}K_BT = \frac{1}{2}m\overline{v^2}$$

$$\overline{v^2} = \frac{3K_BT}{m}$$

Putting above in equation (1) we get

$$PV = \frac{1}{3}Nm\frac{3K_BT}{m}$$

$$PV = (NK_BT)$$

$$PV = constant$$

$$P = \frac{constant}{V}$$

$$V \propto \frac{1}{P}$$

Hence volume of an ideal gas is found to be inversely proportional to its pressure keeping other factors constant which is according to Boyles Law.

Charles Law:

Consider the pressure equation for ideal gas

$$P = \frac{1}{3}\rho\overline{v^2}$$

Since

$$\rho = \frac{Nm}{V}$$

$$P = \frac{1}{3}\frac{Nm}{V}\overline{v^2}$$

$$V = \frac{1}{3P}N\, m\overline{v^2} ------(1)$$

Now

$$K.E = \frac{3}{2}K_BT$$

$$\frac{3}{2}K_BT = \frac{1}{2}m\overline{v^2}$$

Separating velocity

$$\overline{v^2} = \frac{3K_BT}{m}$$

Putting above in equation (1)

$$V = \frac{1}{3P}Nm\frac{3K_BT}{m}$$

$$V = \left(\frac{NK_B}{P}\right)T$$

Hence

$$V = (constant)T$$

Or

$$V \propto T$$

Hence volume of an ideal gas is found to be directly proportional to the absolute temperature which is according to Charles Law.

Root Mean Square Velocity of Molecule of an Ideal Gas(v_{rms}):

Consider the relation for kinetic energy of an ideal gas

$$K.E = \frac{3}{2}K_BT$$

$$\frac{3}{2}K_BT = \frac{1}{2}m\overline{v^2}$$

$$\overline{v^2} = \frac{3K_BT}{m}$$

Taking square root on both sides, we get

$$\boxed{v_{rms} = \sqrt{\frac{3K_BT}{m}}}$$

Heat Capacity:

It is the amount of heat required to raise the temperature of a substance through 1 degree Celsius or 1 degree Kelvin. It is denoted by C.

Mathematically:

$$\Delta Q \propto \Delta T$$

$$\Delta Q = (Constant)\,\Delta T$$

$$Here\ Constant\ =\ Heat\ Capacity = C$$

$$\Delta Q = C\,\Delta T$$

$$\boxed{C = \frac{\Delta Q}{\Delta T}}$$

Specific Heat Capacity:

It is the amount of heat required to raise the temperature of 1 Kg of a substance by 1 degree Celsius or 1 degree Kelvin. It is denoted by c.

Mathematically:

$$\Delta Q \propto \Delta T$$

$$\Delta Q \propto m$$

$$Combining\ both, we\ get$$

$$\Delta Q \propto m\Delta T$$

$$\Delta Q = (Constant)m\Delta T$$

$$Here\ Constant = Specific\ Heat\ Capacity = c$$

$$\Delta Q = c\,m\Delta T$$

$$\boxed{c = \frac{\Delta Q}{m\Delta T}}$$

The unit of specific heat capacity is Joules per Kilogram per Degree Kelvin $\boldsymbol{J.Kg^{-1}K^{-1}}$

Molar Specific Heat Capacity:

It is the amount of heat required to raise the temperature of 1 mole of a substance by 1 degree Celsius or 1 degree Kelvin.

Mathematically:

Consider relation for specific Heat of a substance

$$c = \frac{\Delta Q}{m\Delta T} ------(1)$$

Number of moles (n) can be defined as the ratio of mass of the substance (m) and its molecular mass (M);

$$n = \frac{m}{M}$$

$$m = nM$$

Putting in equation (1) we get

$$c = \frac{\Delta Q}{nM\Delta T}$$

$$(M)(c) = \frac{\Delta Q}{n\Delta T}$$

$$\boxed{M_C = \frac{\Delta Q}{n\Delta T}}$$

Here M_C is called molar specific heat of the substance and its unit is $\boldsymbol{J.mol^{-1}K^{-1}}$.

Molar Specific Heat Capacity at Constant Volume:

It is the amount of heat required to raise the temperature of a gas by 1 degree Celsius or 1 degree Kelvin in such a way that gas is not allowed to expand (i.e. at constant volume). It is represented by $\boldsymbol{C_V}$.

$$\Delta Q_v = nc_v\Delta T$$

Molar Specific Heat Capacity at Constant Pressure:

It is the amount of heat required to raise the temperature of a gas by 1 degree Celsius or 1 degree Kelvin at constant pressure. It is represented by $\boldsymbol{C_P}$.

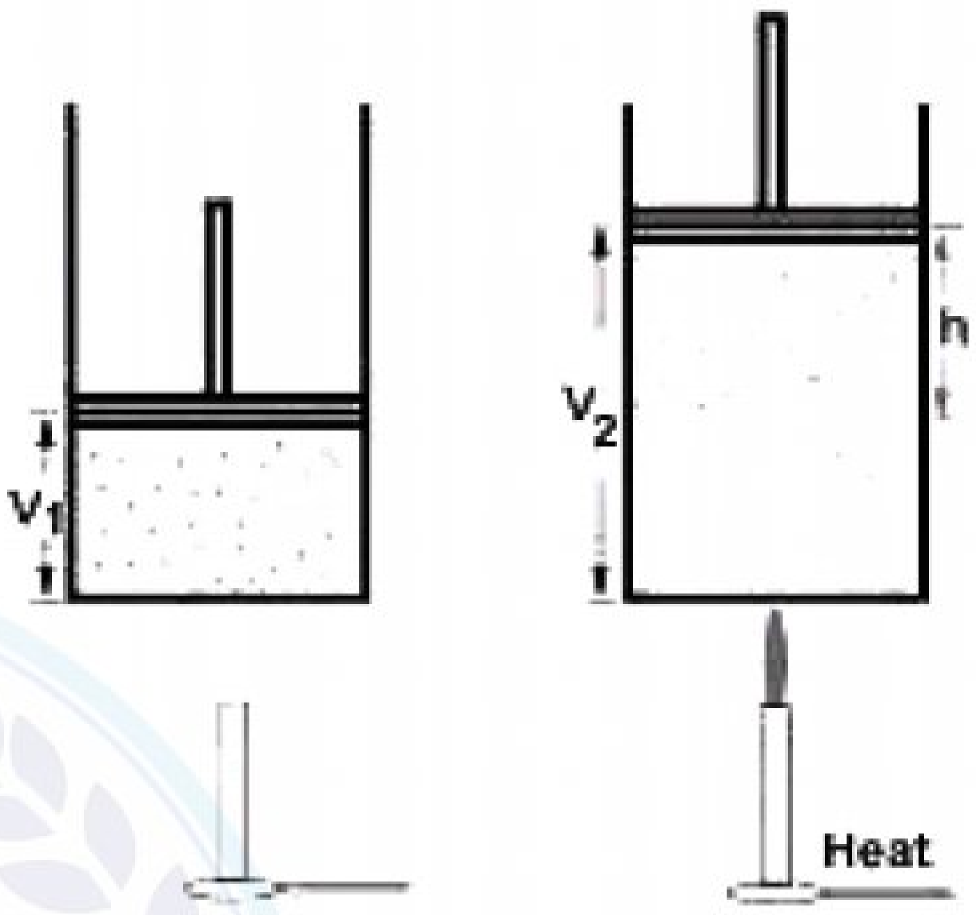
$$\Delta Q_P = nc_P\Delta T$$

Thermodynamics:

Thermodynamics is the branch of physics which deals with the transformation of heat energy into other forms of energy.

Work Done in Thermodynamics:

Consider a thermodynamic system such as a gas in a cylinder with insulated walls and fitted with a movable frictionless piston. When the gas is heated the gas in container is allowed to expand and results in increase in height of the piston (h) (due to increase in volume) to Work done by the system can be written as



$$Mechanical\ Work\ done = \Delta W$$
$$= F.d --(1)$$

Since

$$P = \frac{F}{A}$$

Or

$$F = PA\text{, put in (1)}$$

Equation (1) becomes

$$\Delta W = PA\mathrm{h}$$

Since $A\mathrm{h} = \Delta V$(change in volume), therefore

$$\boldsymbol{Mechanical\ Work\ done = \Delta W = P\Delta V}$$

Above equation, represent work done in thermodynamics. It shows that if volume is increased, the work will be positive and if volume is decreased, the work will be negative.

First Law of Thermodynamics:

Statement:

"In thermodynamic process, heat energy added to a system is equal to the increase in the internal energy stored in the system plus the work done by the system."

Mathematically:

Let

ΔQ= amount of heat energy absorbed by the system

ΔW = work done by the system

$\Delta \boldsymbol{U}$ = change in internal energy of the system

Then according to 1st law of thermodynamics:

$$\boxed{\Delta Q = \Delta U + \Delta W}$$

Sign convention:

- ΔQ is positive when heat is added to a system.
- ΔQ is negative when heat is rejected from the system.
- ΔW is positive when work is done by the system.
- ΔW is negative when work is done on the system.
- ΔU is positive when internal energy is increased.
- ΔU is negative when internal enrgy is decreased.

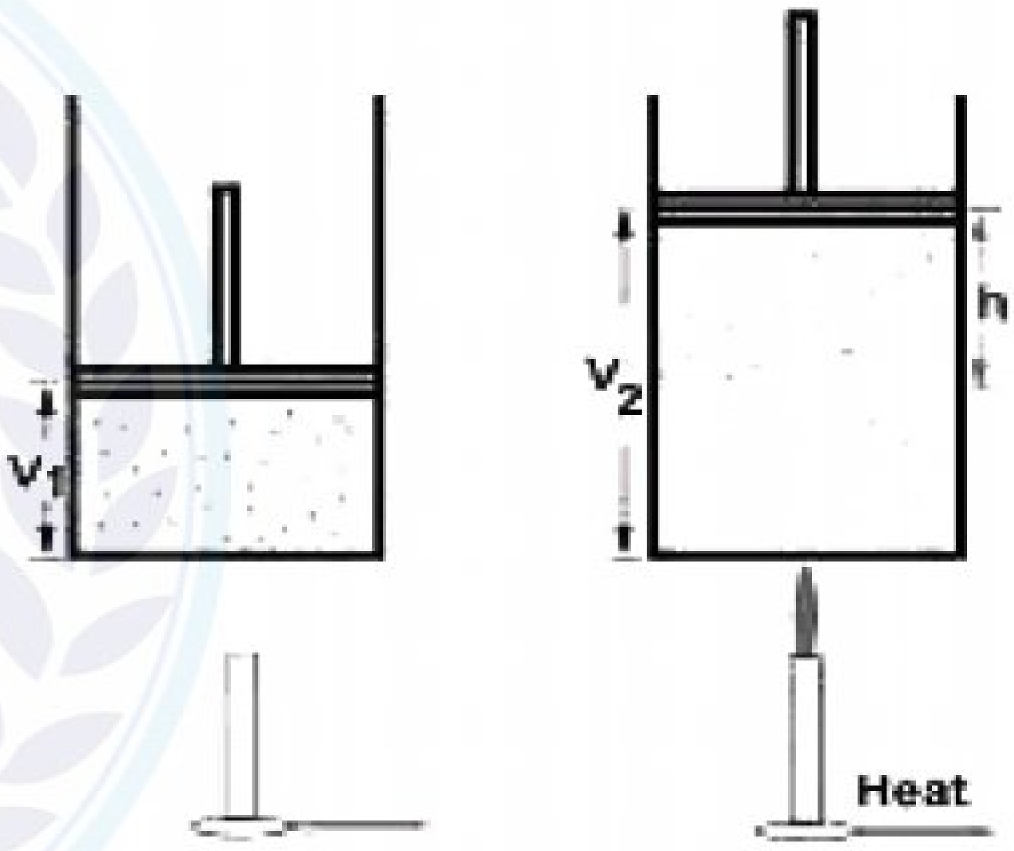
Applications of First Law of Thermodynamics:

There are four applications of thermodynamics

1. Isobaric process (constant pressure)
2. Isochoric process (constant volume)
3. Isothermal process (constant temperature)
4. Adiabatic process (heat, $\Delta Q = 0$)

Isobaric Process:

A thermodynamic process that takes place at constant pressure is called isobaric process.



Explanation:

Consider a gas cylinder with insulated walls and frictionless movable piston. If heat, ΔQ is added to the system then according to the first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U - - - (1)$$

When the gas is heated the gas in container is allowed to expand and results in increase in height of the piston (h) (due to increase in volume). The Work done by the system can be written as

$$\Delta W = F.d - -(2)$$

Since $$P = \frac{F}{A}$$

Or

$$F = PA$$

Equation (2) becomes

$$\Delta W = PA\mathrm{h}$$

Since $A\mathrm{h} = \Delta V$ (change in volume), therefore

$$\Delta W = P\Delta V$$

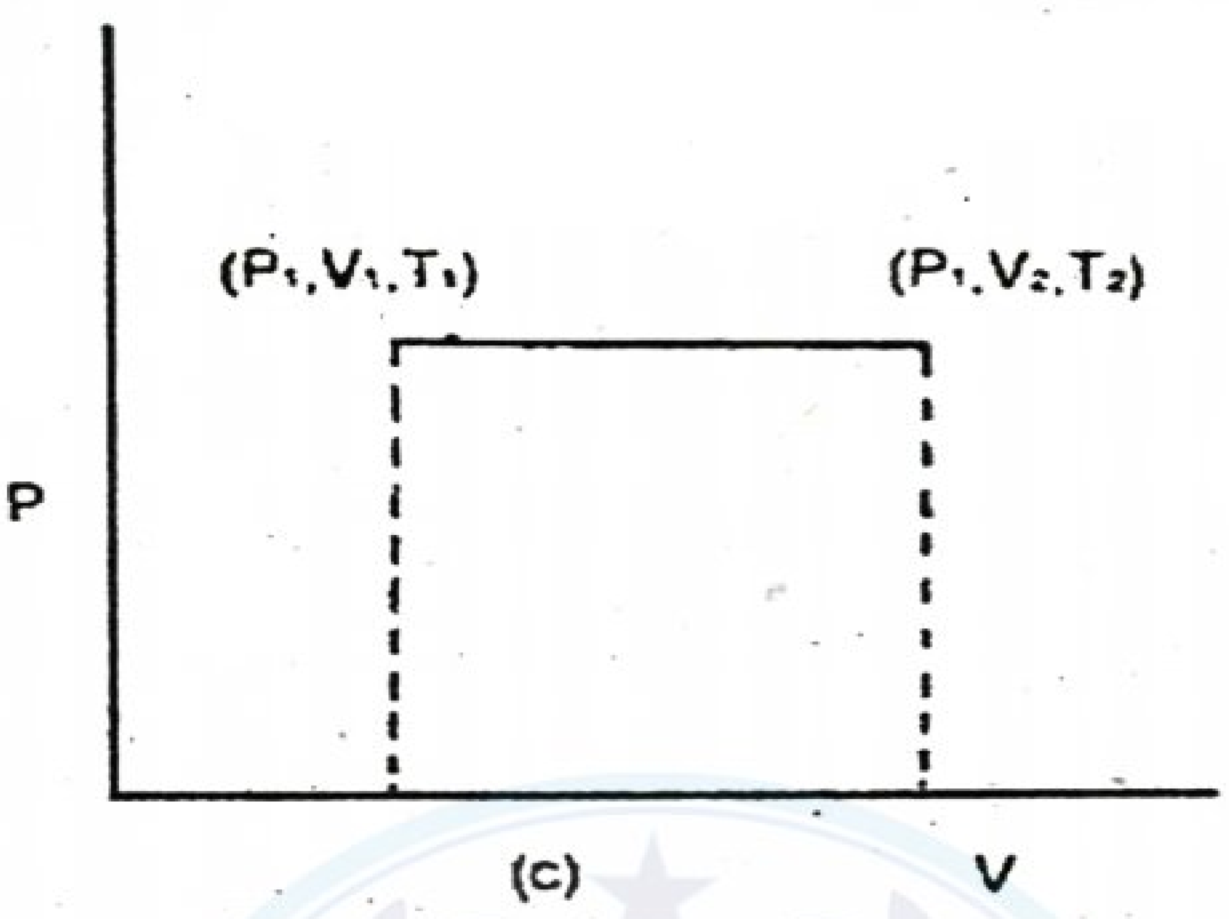
Putting value of ΔWin equation (1), we get

$$\boxed{\Delta Q = \Delta U + P\Delta V}$$

Above equation represent Isobaric process.

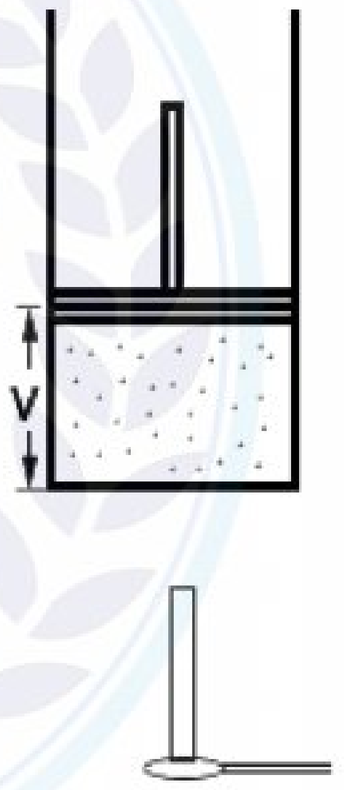
Conclusion:

1. Heat supplied to the system is used partially to change the internal energy and partially to do work.
2. Mechanical Work, $\Delta W = P\Delta V$
3. Graph between pressure and volume in isobaric process is given by

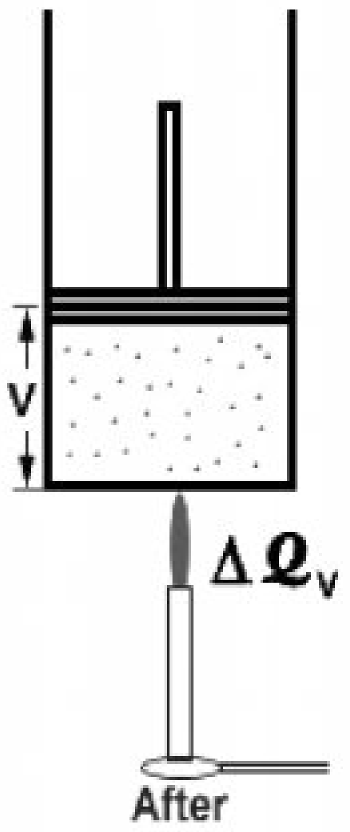


Isochoric Process:

A thermodynamic process that takes place at constant volume is called isochoric process.



Before heating



After heating

Explanation:

Consider a gas cylinder with insulated walls and fixed piston. If heat, ΔQ is added to the system then according to the first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U - - - (1)$$

Hence when the gas is heated it is not allowed to expand and no work is done. As piston is fixed and $\Delta V = 0$, therefore

$$\Delta W = P\Delta V = P\,(0) = 0$$

Now equation (1) will become

$$\boxed{\Delta Q = \Delta U}$$

Above equation represent Isochoric process.

Conclusion:

1. In isochoric process heat supplied to the system is used completely to change the internal energy.
2. Work done is zero i.e. $\Delta W = 0$
3. Graph between pressure and volume in isochoric process is given by

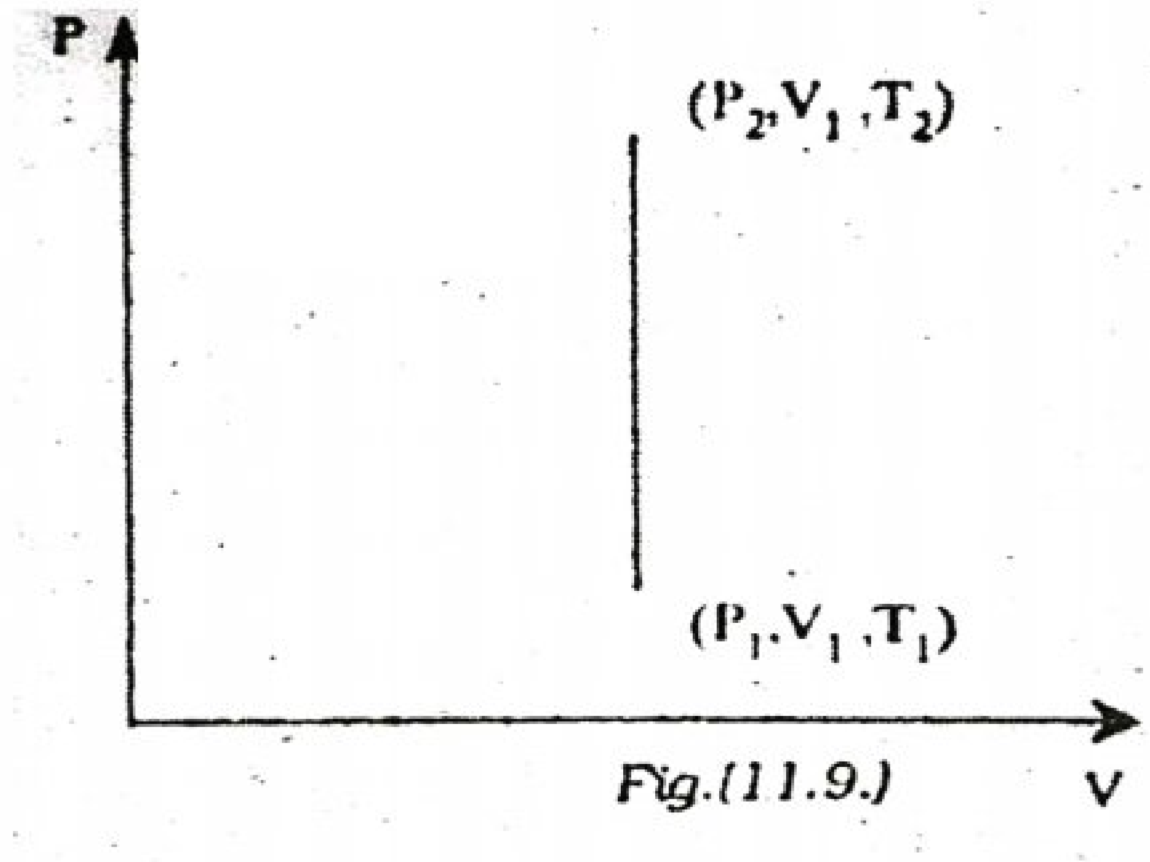
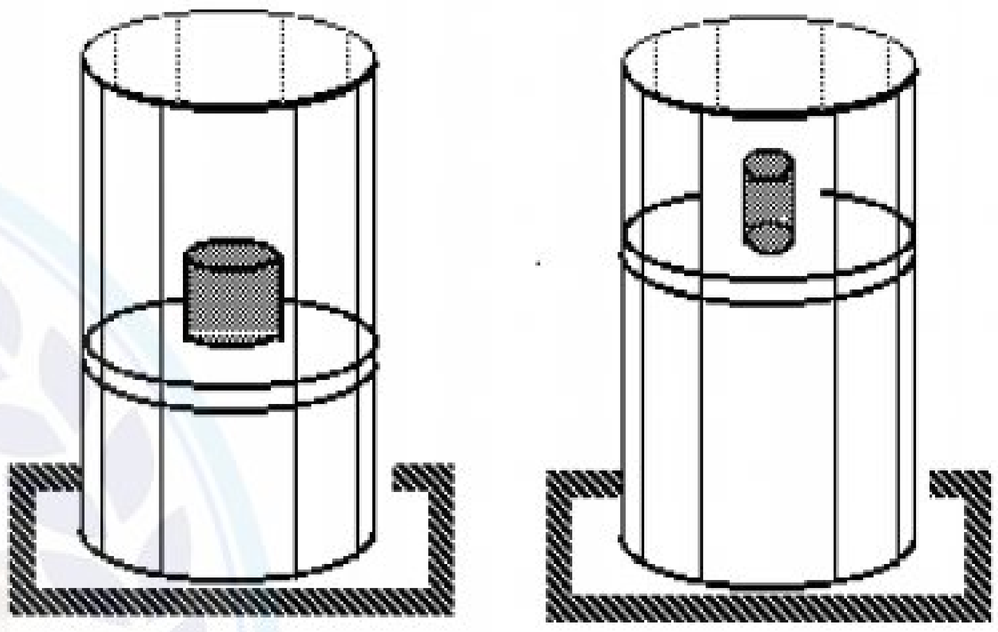


Fig.(11.9.)

Isothermal Process:

Such a thermodynamic process that takes at constant temperature is called isothermal process.

Explanation: (Isothermal expansion)



Consider a gas cylinder with insulated walls and frictionless movable piston. Let the gas is allowed to expand by decreasing the pressure. At the same time heat is added to the system by heat reservoir to maintain the constant temperature. According to 1st law of thermodynamics

$$\Delta Q = \Delta W + \Delta U - - - (1)$$

Since there is no change in temperature, therefore

$\Delta U = 0, substitute\ in\ (1)$, we have

$$\boldsymbol{\Delta Q = \Delta W}$$

Conclusion:

1. Work done is maximum in this process, $\Delta Q = \Delta W$
2. For an isothermal process all the heat energy absorbed is utilized in doing work with no change in internal energy.
3. Pressure and volume are inversely proportional at constant temperature which holds Boyle's law.
4. The graphical representation of isothermal process is given by

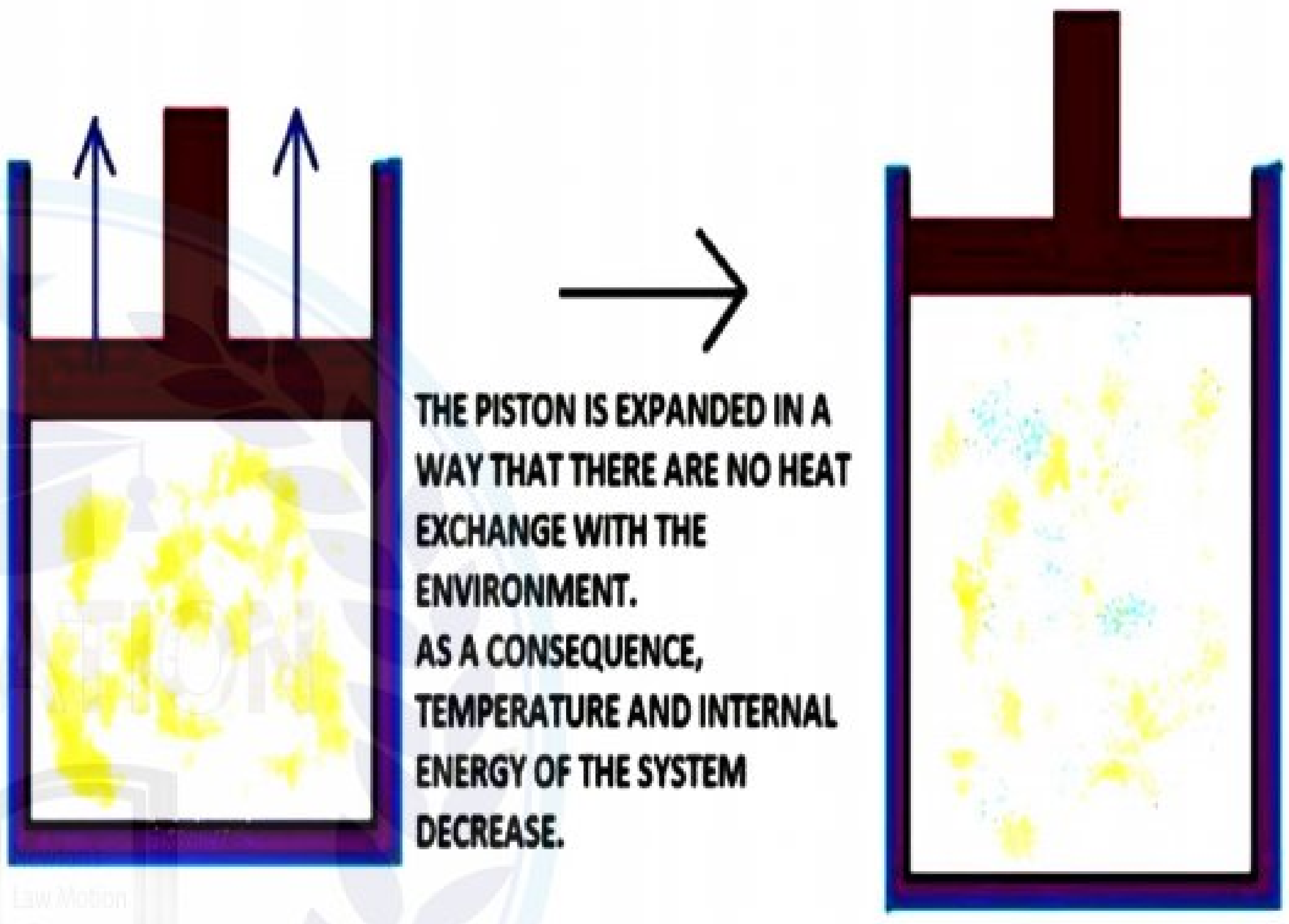
Adiabatic Process:

A thermodynamic process in which no heat flows into or out of the system is called adiabatic process.

Explanation: (Adiabatic expansion)

Consider a gas cylinder with insulated and non conducting walls. The walls and base of the container are perfectly insulated from its surrounding and no heat flow in or out of the system.

If the piston is released by decreasing the pressure then the gas will expand at the cost of internal energy of the system because no heat will flow in or out of the system.



Now according to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W \text{ --------- } (1)$$

Since no heat flow in or out, therefore $\Delta Q = 0$, put in (1), we have

$$0 = \Delta U + \Delta W$$

$$\boxed{-\Delta \boldsymbol{U} = \Delta \boldsymbol{W}}$$

Conclusion:

1. Mechanical work is done at the cost of internal energy, $\Delta W = -\Delta U$
2. $P\,V^{\gamma} = constant, w\hbar ere\ \gamma = \frac{C_P}{C_V}$
3. $Grpa\hbar ical\ representation\ of\ adiabatic\ process\ is\ given\ by$

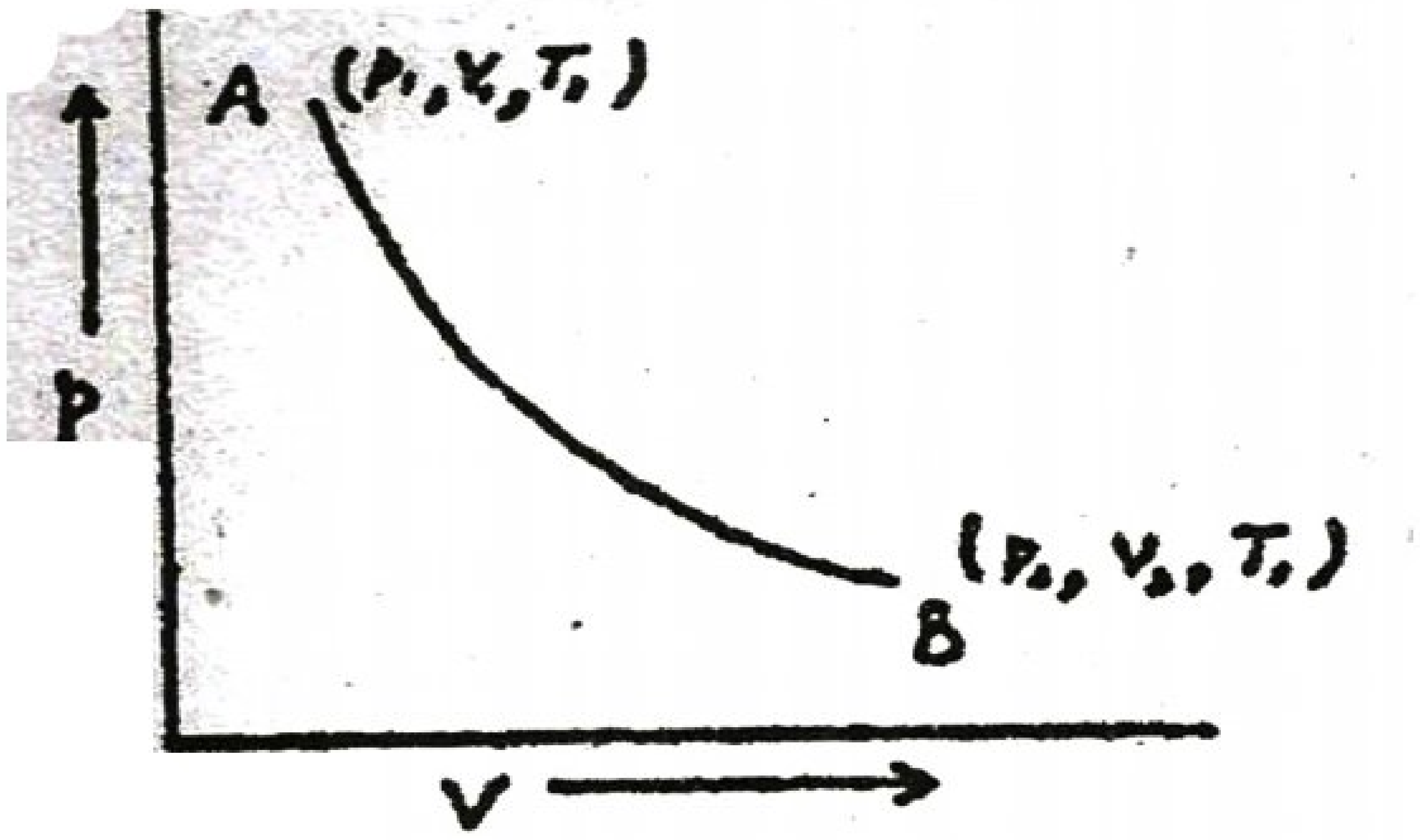
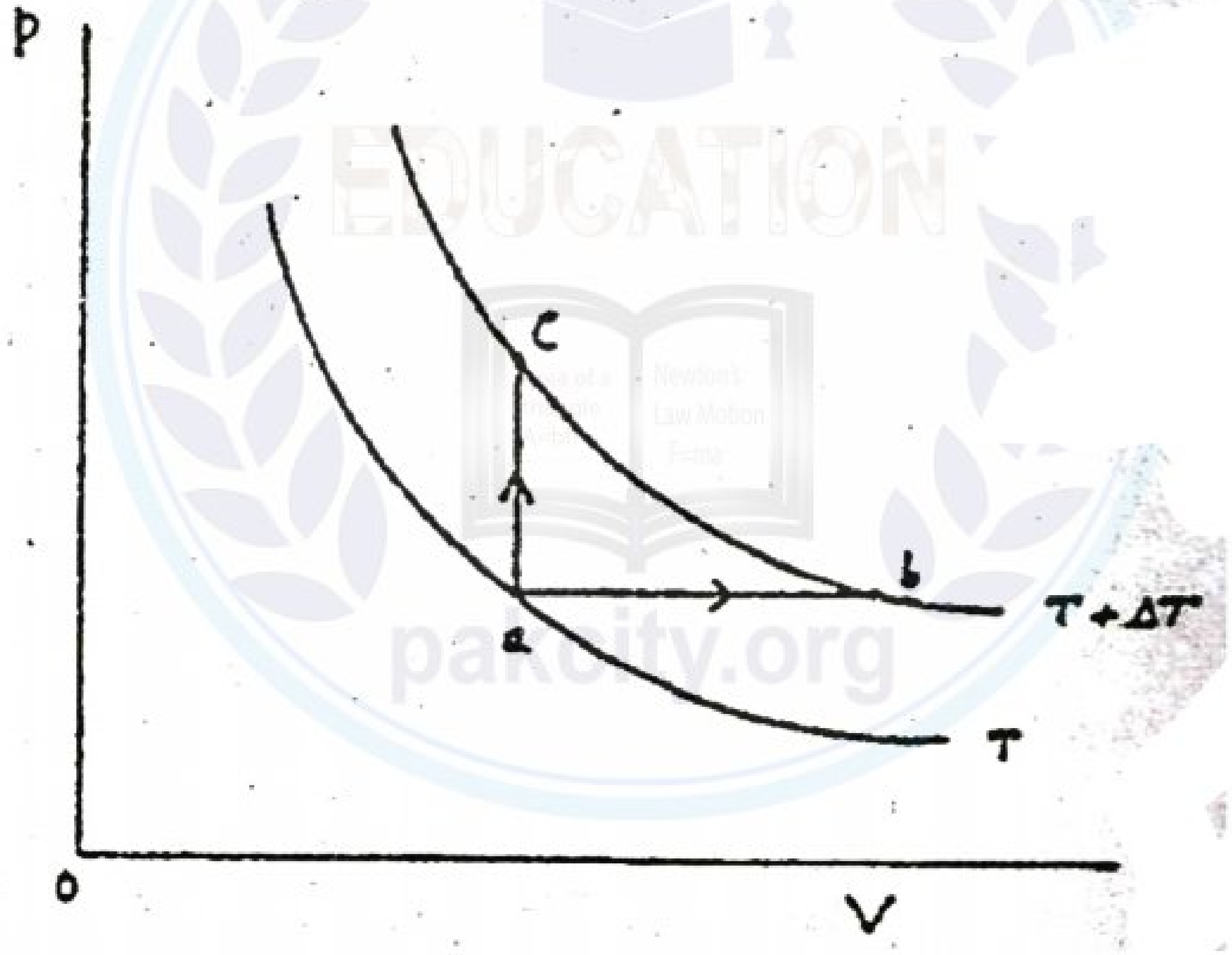


Fig.

Question: Prove that $Cp - Cv = R$

Relation between Specific Heat at Constant Pressure (C_p) and Specific Heat at Constant Volume (C_V):

Consider two isotherms of an ideal gas at temperature T and T + ΔT.



From a ⟶ b:

Suppose that heat is provided to the gas at constant pressure along path a ⟶ b. The heat transformed to the gas in this process is given by

$$\Delta Q_P = nc_P\Delta T$$

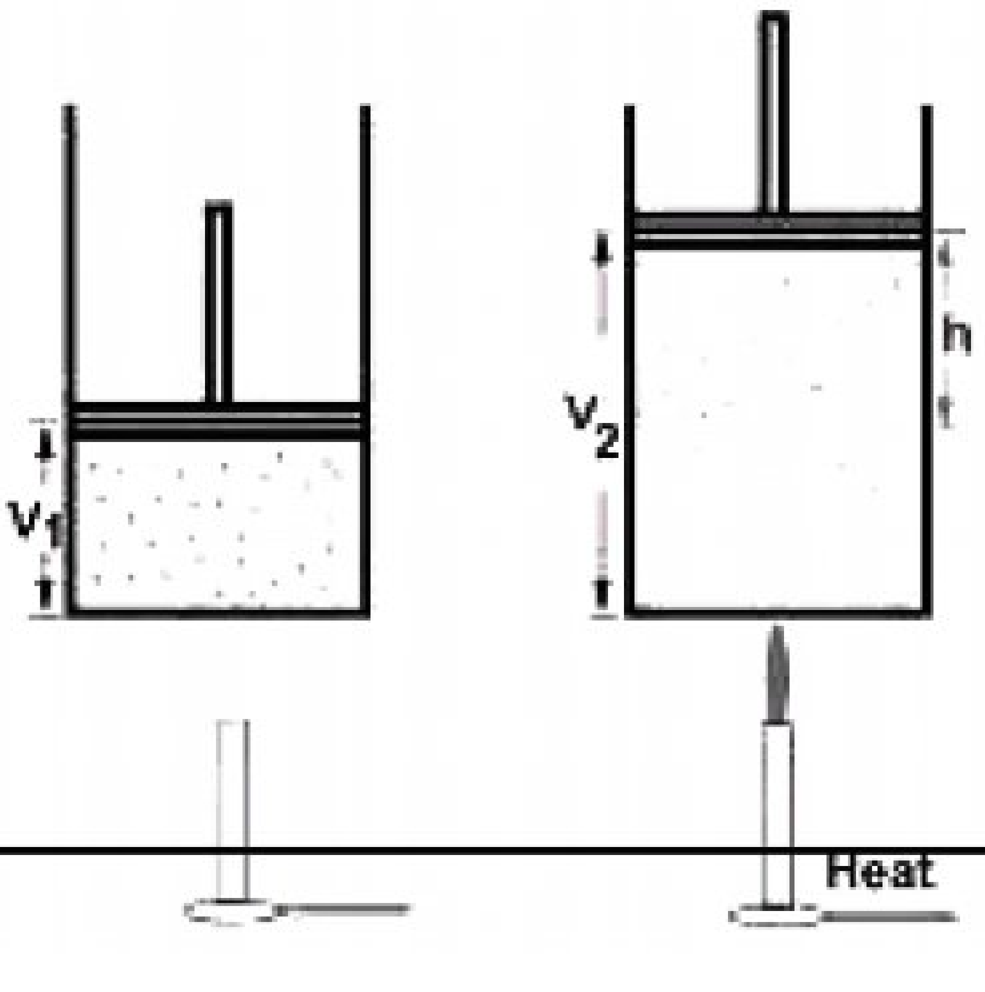
$$[since\ \Delta Q = nc\Delta T]$$

The mechanical work done in this process is given by

$$\Delta W = P\Delta V$$

According to 1st law of thermodynamics

$$\Delta Q_P = \Delta U + \Delta W ------(1)$$



Substitute values of ΔQ_P and ΔW in above equation (1), we have

$$nc_P\Delta T = \Delta U + P\Delta V\text{--------} \ (2)$$

Using general gas law

$$PV = nRT$$

$$P\Delta V = nR\Delta T$$

Putting this value in Equation (2), we get

$$nc_P\Delta T = \Delta U + nR\Delta T ------(3)$$

<u>From a → c:</u>

Now suppose that heat is provided to the gas at constant volume along path a → c as shown in graph. The heat transferred in this process is given by

$$\Delta Q_v = nc_v\Delta T$$

We know that at constant volume, $\Delta W = 0, and$

According to 1st law of thermodynamics

$$\Delta Q_v = \Delta U + \Delta W\text{---------} \ (4)$$

Substitute $\Delta W = 0 \ and \ \Delta Q_v = nc_v\Delta T$ in above equation (4), we have

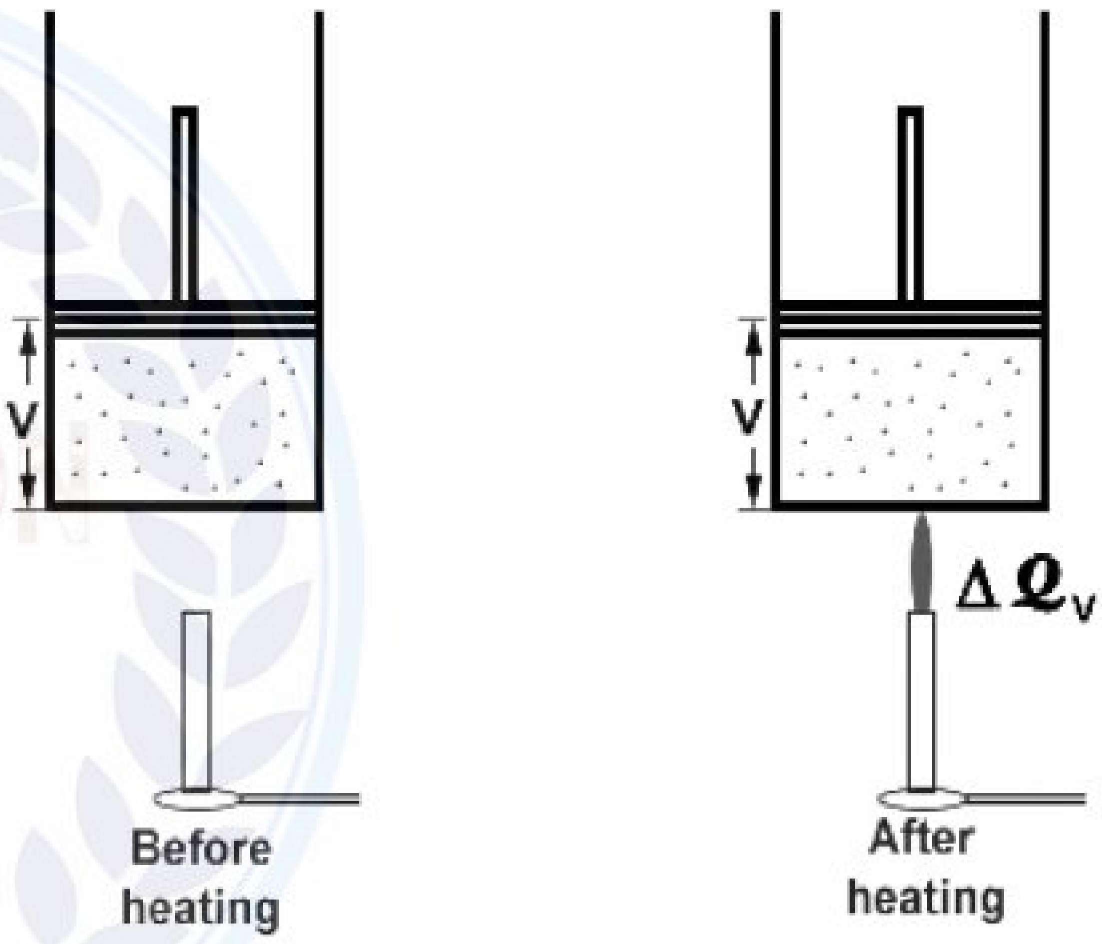
$$nc_v\Delta T = \Delta U$$

Substitute this value ΔU in equation (3), we get

$$nc_P\Delta T = nc_v\Delta T + nR\Delta T$$

$$nc_P\Delta T = n\Delta T(c_v + R)$$

$$c_P = c_v + R$$



$$\boxed{c_P - c_v = R}$$

Hence the difference of specific heat at constant pressure and specific heat at constant volume isequal to the general gas constant. Above equation also shows that the specific heat capacity of any ideal gas at constant pressure is always greater that specific heat capacity of the same gas at constant volume. I-e

$$\boxed{c_p > c_v}$$

Heat Engine:

Any device which converts heat energy into mechanical energy is called heat engine.

Essentials of Heat Engine:

Essential of heat engines are:

1. Furnace or hot body.
2. Working substance.

3. Condenser or Cold body.

Carnot Engine:

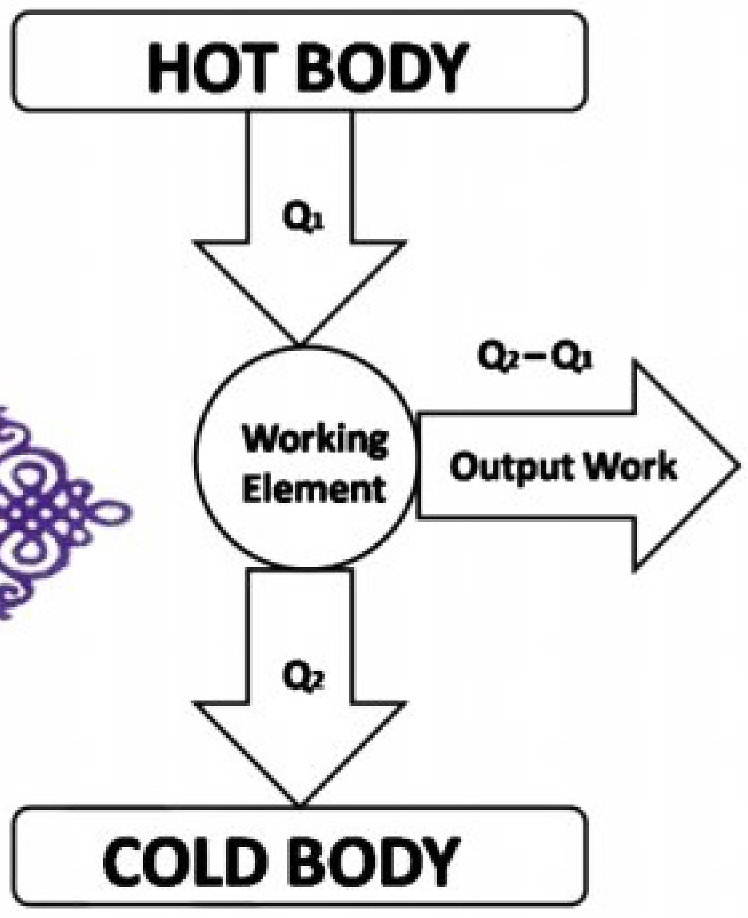
Introduction:

Carnot engine is an ideal heat engine which converts heat energy into mechanical energy. It is an imaginary heat engine free from friction & heat losses due to radiations or conduction.

Efficiency of Carnot engine:

Suppose that heat Q_1 is extracted from source (hot reservoir) of temperature T_1 by Carnot engine and heat Q_2 is rejected to sink (cold reservoir) of temperature T_2.

HEAT ENGINE



Then useful mechanic work in one cycle, $W = Q_1 - Q_2$

Input heat in each cycle = Q_1

Heat rejected in each cycle = Q_2

The ratio of output mechanical work to the input energy is called efficiency of Carnot engine.

$$Efficiency = \frac{Output\ work}{Input\ heat}$$

Therefore

$$E = \frac{W}{Q_1}$$

From figure, $W = Q_1 - Q_2$

Therefore

$$E = \frac{Q_1 - Q_2}{Q_1}$$

$$E = 1 - \frac{Q_2}{Q_1}$$

In percentage form

$$\boldsymbol{E\% = \left(1 - \frac{Q_2}{Q_1}\right) x\ 100}$$

Since $Q \propto T$

Therefore

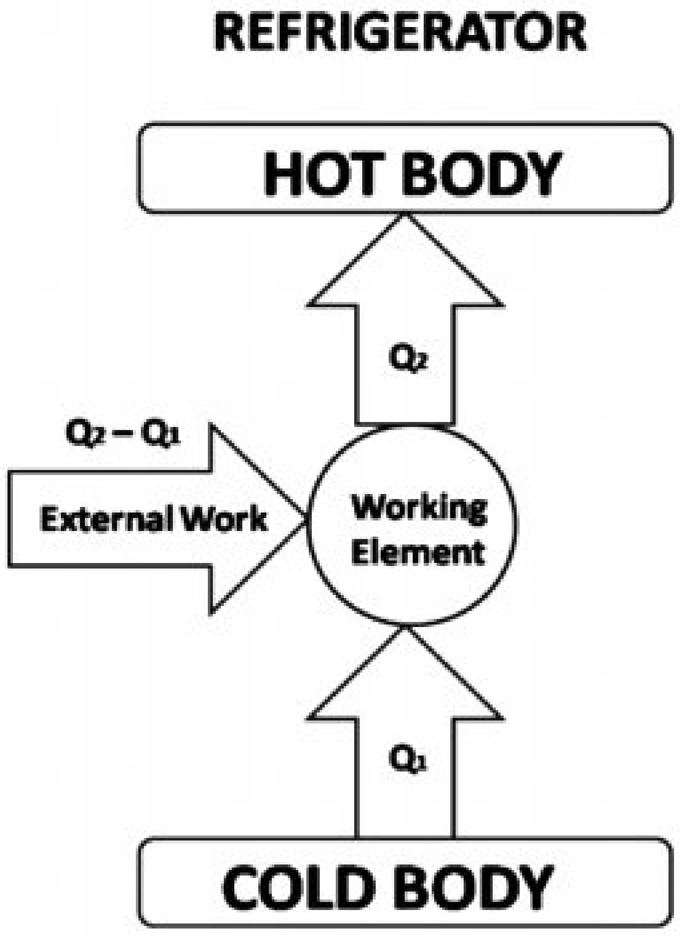
$$\boldsymbol{E\% = \left(1 - \frac{T_2}{T_1}\right) x\ 100}$$

Refrigerator:

The device that makes heat to flow from cold body to hot body is called Refrigerator.

Essential of Refrigerator:

- Condenser or cold body.
- Working substance.
- Furnace or hot body.



Second Law of Thermodynamics:

Two statements can state the second law of thermodynamics.

Kelvin Statement:

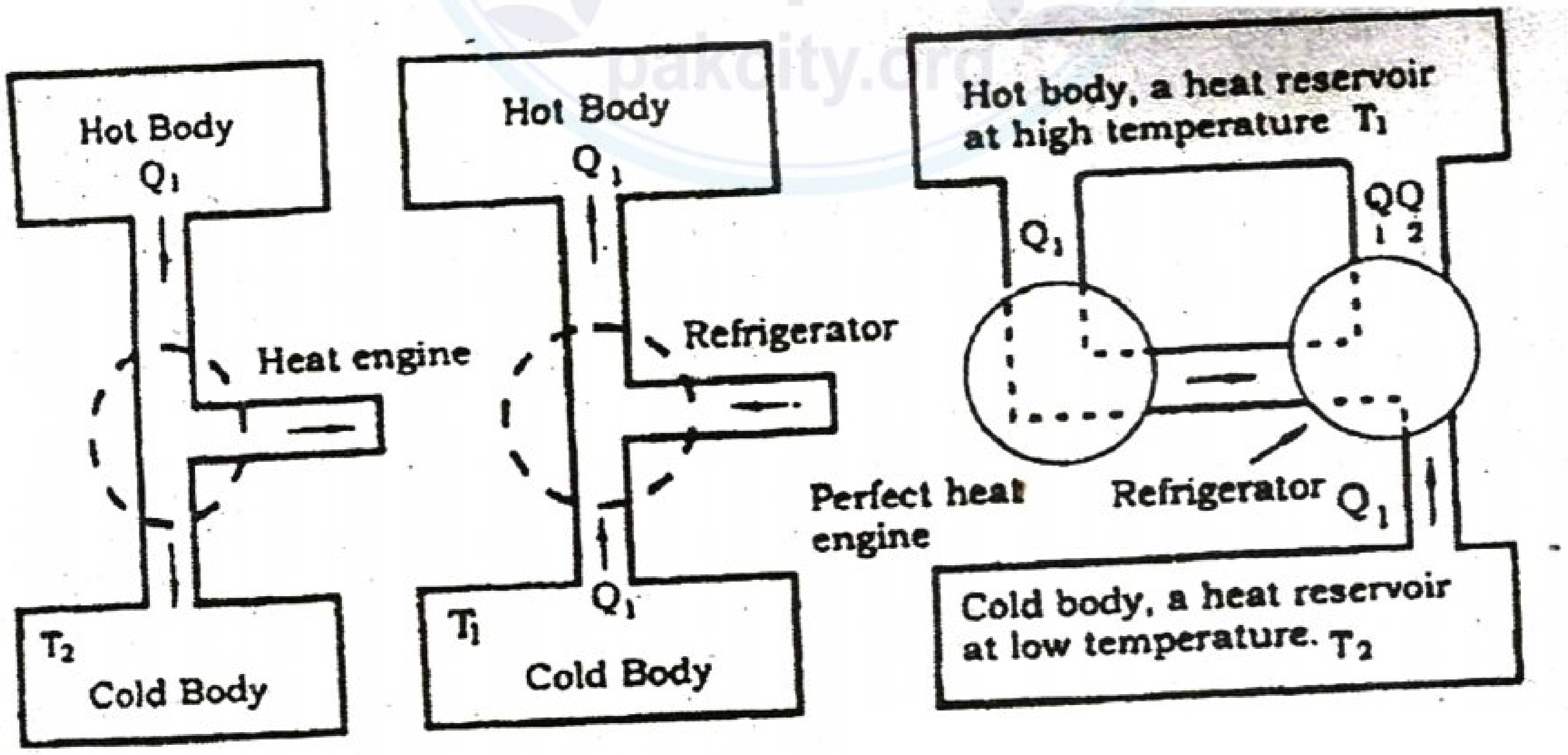
"It is impossible for a heat engine to take heat from a source and converts it completely into work."

Clausius Statement:

"It is impossible to cause heat flow from a cold body to a hot body without the expenditure of work."

Equivalency of Kelvin and Clausius Statements:

Suppose that Kelvin's statement is false then we could have a heat engine which takes heat from a source and converts it completely into work. If we connect this perfect heat engine to an ordinary refrigerator we can take heat from the hot body and convert it completely to work. This work can be used to operate the refrigerator which conveys heat from the cold body to hot body. The net result is a transfer of heat from a cold body to a hot body without the expenditure of work which is contrary to Clausius statement. Therefore the two statements are equivalent.

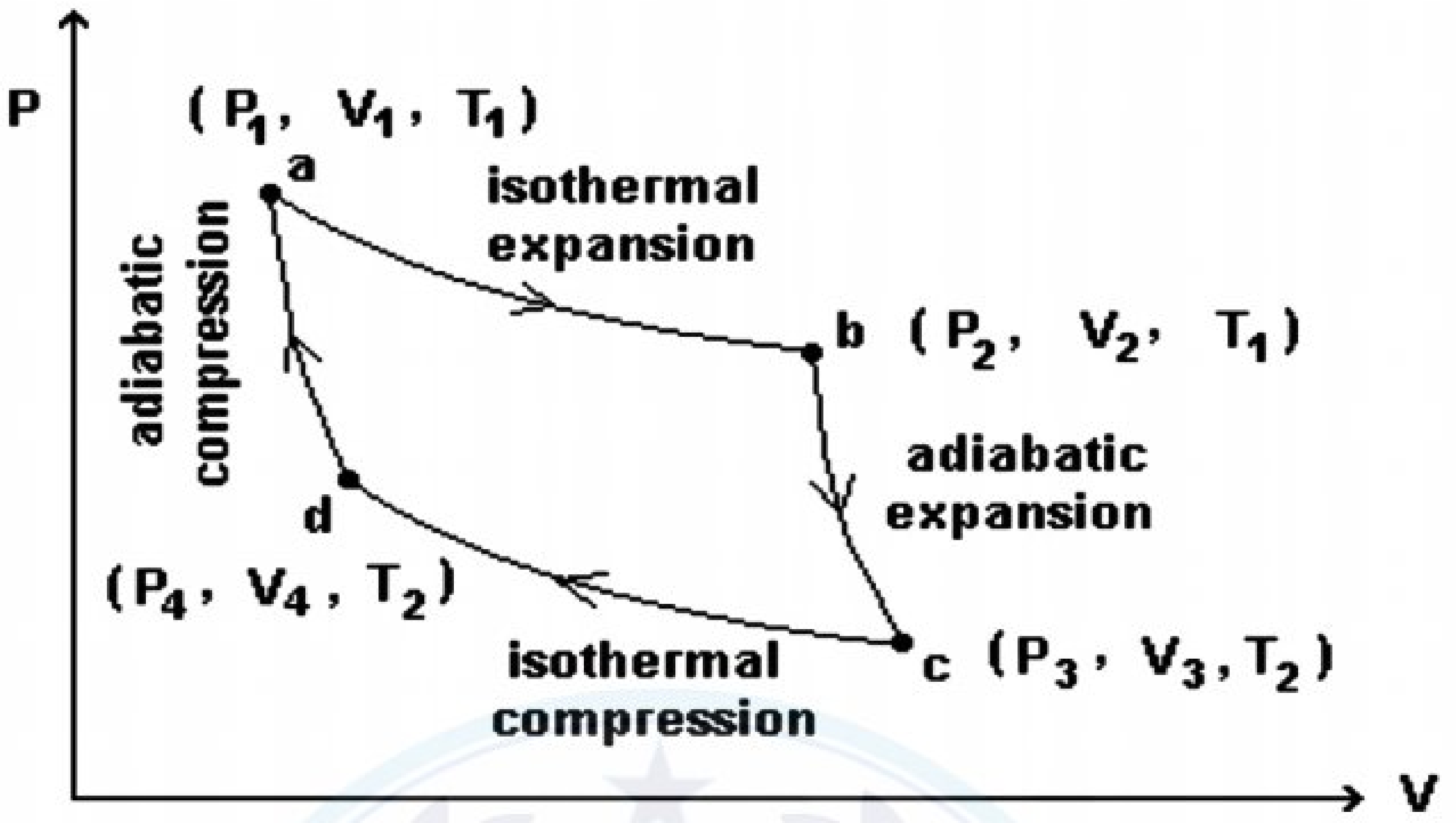


Carnot Cycle:

Sadi Carnot in 1840 described an ideal engine using only isothermal and adiabatic processes. He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperature would be the most efficient engine. Carnot cycle uses an ideal gas as working substance.

Working:

Let we have an ideal gas with initial pressure P_1, volume V_1 and temperature T_1. Working of Carnot engine consist of steps as follows



Carnot cycle

Step-1 (Isothermal expansion, a → b)

The gas is allowed to expand isothermally at constant temperature T_1, absorbing heat Q_1 from the hot reservoir. Therefore initial state (P_1, V_1, T_1) of a gas changes to final state (P_2, V_2, T_1).

Step-2 (Adiabatic expansion, b → c)

At this stage (P_2, V_2, T_1) the gas is then allowed to expand adiabatically until its temperature drops to T_2. Therefore the final state will be (P_3, V_3, T_2).

Step – 3 (Isothermal compression, c → d)

The gas at this stage is compressed isothermally at constant temperature T_2 rejecting heat Q_2 to the cold reservoir. The temperature is constant and the final state in this stage becomes (P_4, V_4, T_2).

Step – 4 (Adiabatic compression, d → a)

Finally the gas is compressed adiabatically to restore its initial state (P_1, V_1, T_1) at temperature T_1. Since there is no change in temperature in complete cycle and working substance returns to its initial state so the change in internal energy in complete cycle is zero (i.e. $\Delta U = 0$). Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible.

Entropy:

The measure of molecular disorder of a system during a process is called Entropy.

In any process entropy increases or remains constant.

Mathematically:

If ΔQ be the heat transferred to a system at constant kelvin temperature T, then

$$\boxed{\Delta S = \frac{\Delta Q}{T}}$$

The S.I unit of entropy is J/K.

Second law of thermodynamics in terms of entropy:

"When an isolated system undergoes a change, the entropy of the system either remains constant or it increases."