Chapter = 12

Electrostatics

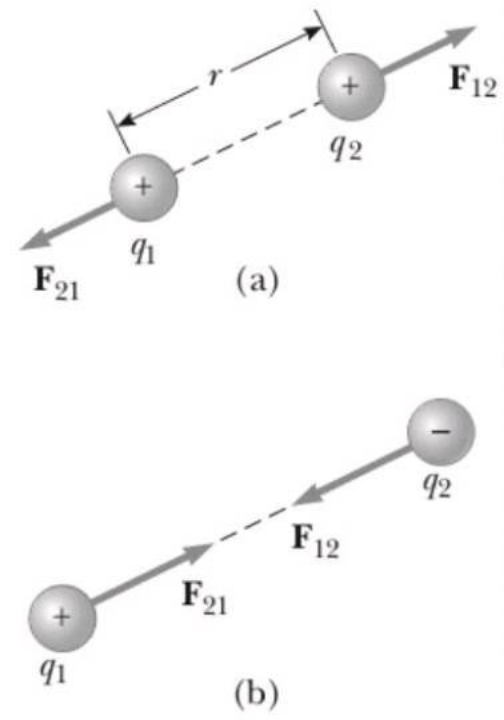
Coulomb's Law:

Statement:

"Magnitude of electric force between two charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them."

Mathematical Representation:

Consider two charges $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ placed at the distance $\boldsymbol{r}$ from each other. The magnitude of electric force between two charges $\boldsymbol{F}$ can be written as



$$F \propto q_1 q_2 - - - (a)$$

$$F \propto \frac{1}{r^2} - - - (b)$$

Combining above expressions

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{\boldsymbol{F = k\frac{q_1 q_2}{r^2}}} - - - (\mathbf{1})$$

Where: k constant of proportionality called Coulomb's constant. It depends upon the medium between charges and its value for free space is defined as

$$\boldsymbol{k = \frac{1}{4\pi\varepsilon_o}}$$

Here, $\boldsymbol{\varepsilon_o}$ is the permittivity of free space. Numerical value of $\boldsymbol{k}$ for free space is

$$\boldsymbol{k = 8.98755 \times 10^9\ Nm^2C^{-2} \cong 9 \times 10^9 Nm^2C^{-2}}$$

Electric force can now be written as

$$\boxed{\boldsymbol{F = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1 q_2}{r^2}}}$$

For medium other than free space value of $\boldsymbol{k}$ can be defined as

$$\boldsymbol{k = \frac{1}{4\pi\epsilon}}$$

Here, $\boldsymbol{\epsilon}$ is the permittivity of medium and it is defined as

$$\boldsymbol{\epsilon = \varepsilon_o \varepsilon_r}$$

Here, $\boldsymbol{\varepsilon_r}$ is called relative permittivity. Electric force between two charges can now be written as

$$\boldsymbol{F = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \times \frac{q_1 q_2}{r^2}}$$

In vector form Force exerted by charge q_1 on q_2 can be written as

$$\boxed{\boldsymbol{\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \times \frac{q_1 q_2}{r^2}\widehat{r_{12}}}}$$

Electric Field:

"Space or region surrounding an electric charge or a charged body within which another charge experiences some electrostatic force of attraction or repulsion when placed at a point is called Electric Field."

Test Charge:

A charge with very small magnitude (approaching to zero) so that the effect of its electric field is negligible is called Test Charge.

Electric Field Intensity:

A measure of the strength of an electric field at a point in space around it is called Electric field intensity.

It may also be defined as

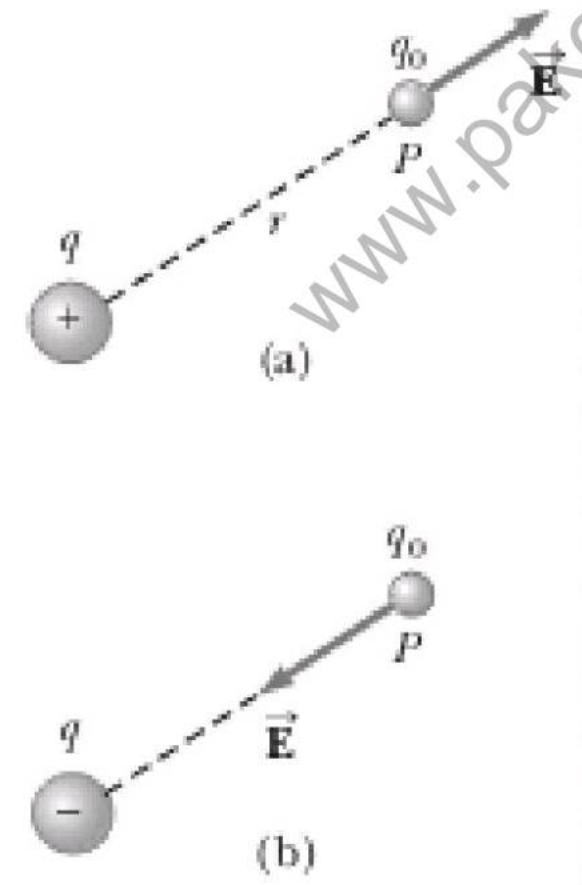
"Force experienced by a positive test charge at point in an electric field."

It is a vector quantity and its direction are same as the direction of force.

Mathematical Representation:

The electric field $\vec{E}$ intensity due a charge q at a point in space in presence of a test charge q_o can be written as

$$\boxed{\vec{E} = \frac{\vec{F}}{q_o}} - - - (a)$$



Since

$$\vec{F} = \frac{1}{4\pi\varepsilon_o} \times \frac{qq_o}{r^2}\hat{r}$$

Equation (a) will become

$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{qq_o}{r^2}\frac{1}{q_o}\hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2}\hat{r}}$$

Here $\hat{r}$ is the unit vector in the direction of force as well Electric field

The magnitude of electric field intensity can be written as

$$\boxed{E = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2}}$$

Unit:

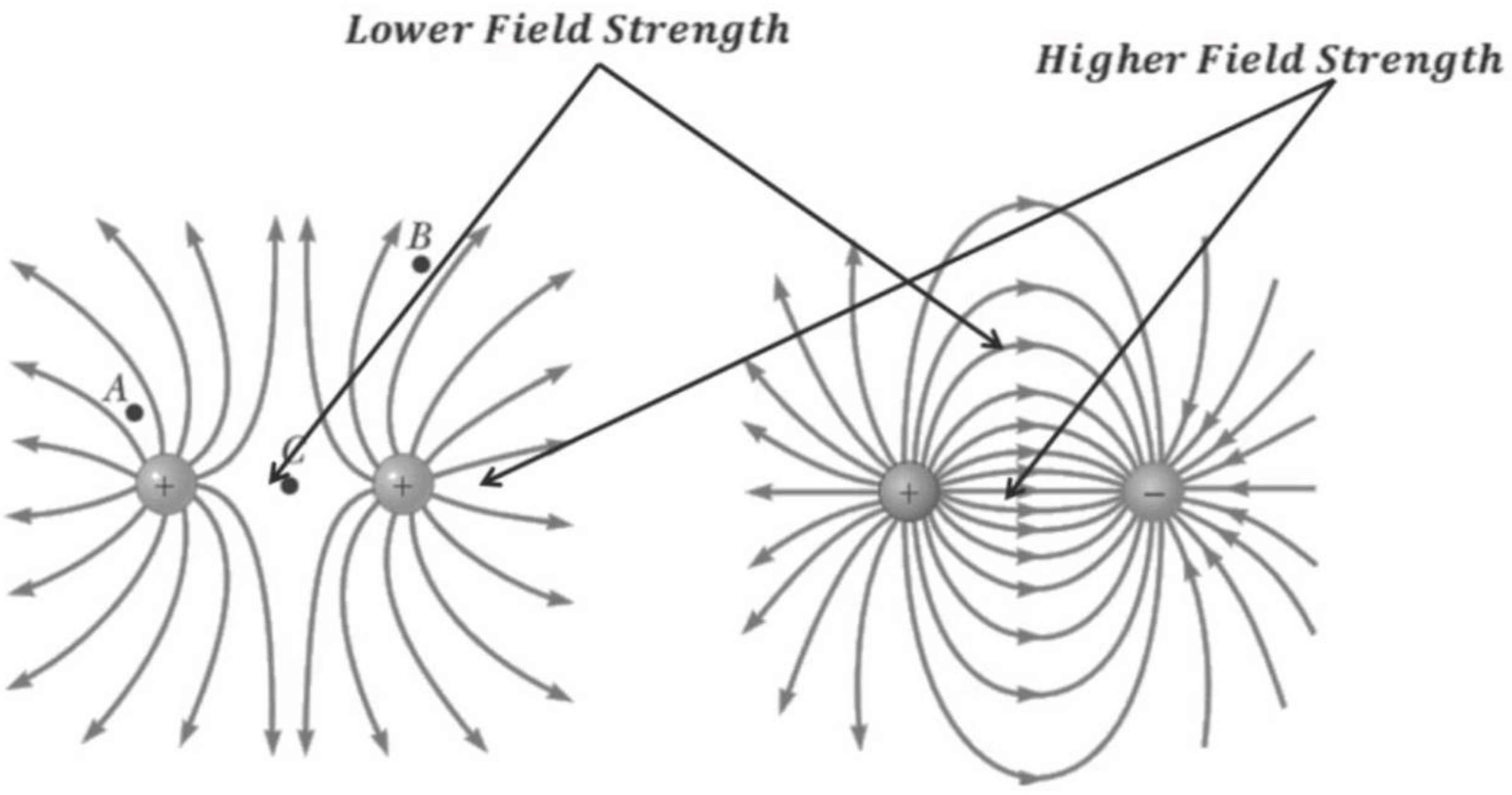
The SI unit of electric intensity is Newton per Coulomb NC^{-1} or volt per meter V/m.

Electric Lines of Force:

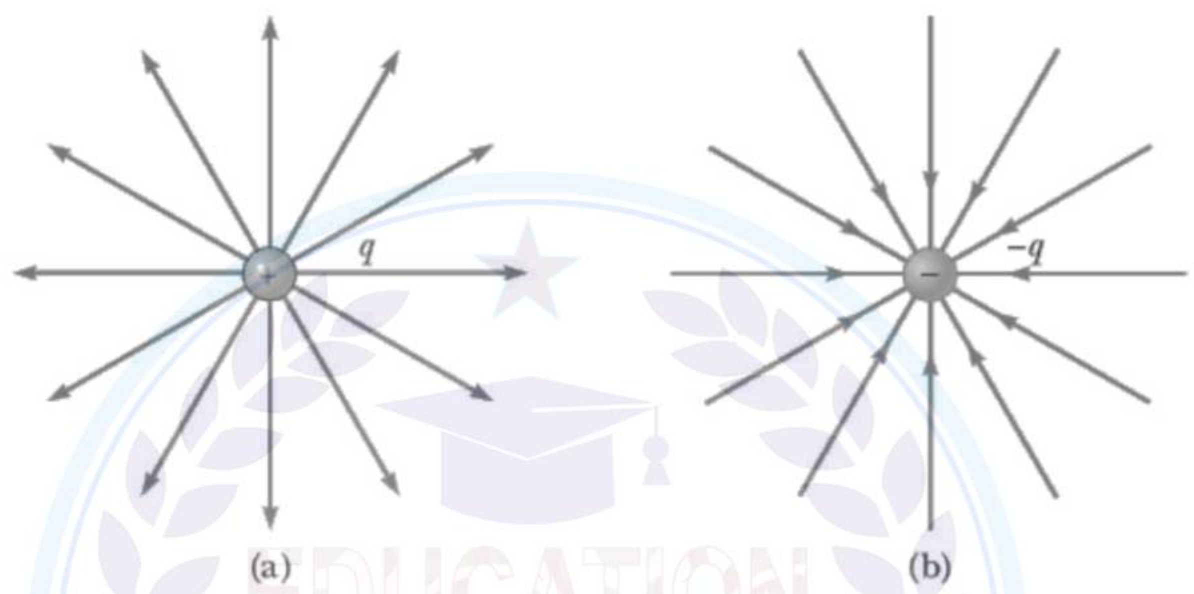
Imaginary lines drawn to represent an electric field are called Electric lines of force.

Properties of Electric Lines of force:

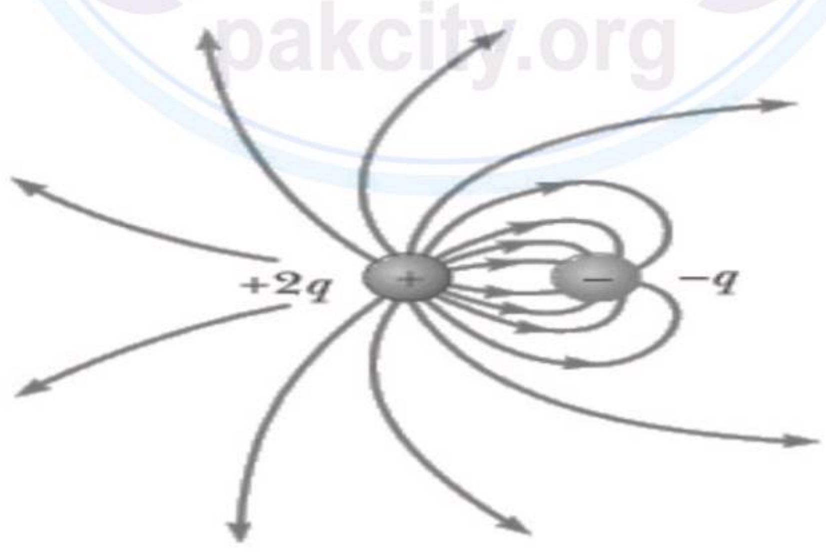
- These lines show the direction of field at each point.
- The closeness of these lines shows the field strength (closer the lines higher the field strength).



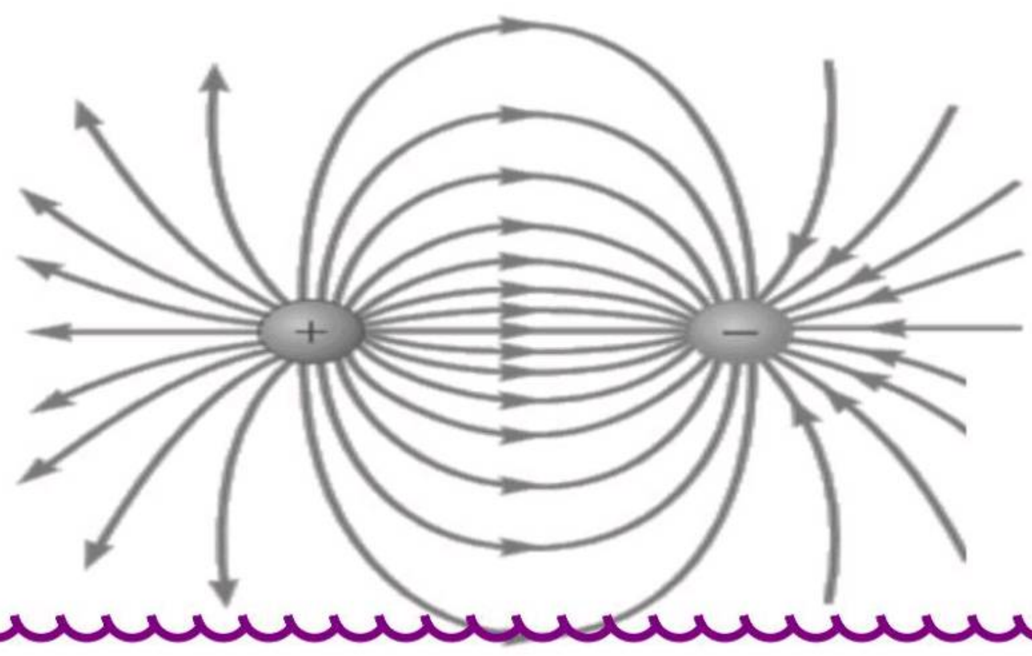
- These lines of forces are originated from Positive charges and terminated at negative charges. The direction of lines is in general the direction of force experienced by a positive test charge.



- The electric field vector $\vec{E}$ is tangent to the electric field lines at each point
- The number of lines drawn leaving a positive charge or ending on a negative charge is proportional to the magnitude of the charge.

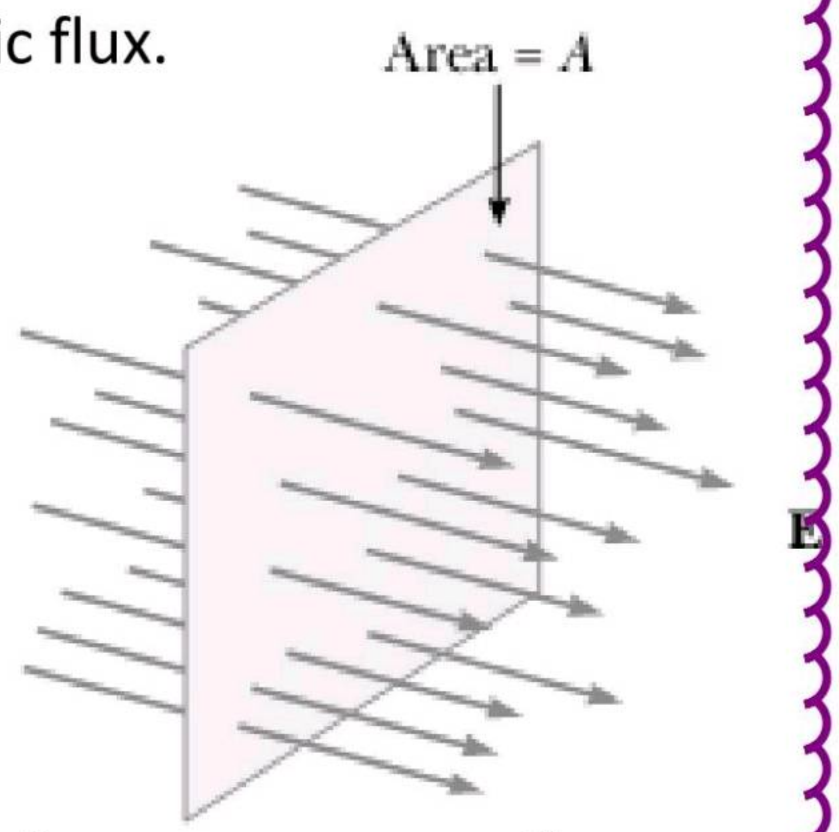


- No two field lines can cross each other.



Electric Flux:

Number of Electric lines of force passing through an area is called Electric flux.



Mathematically:

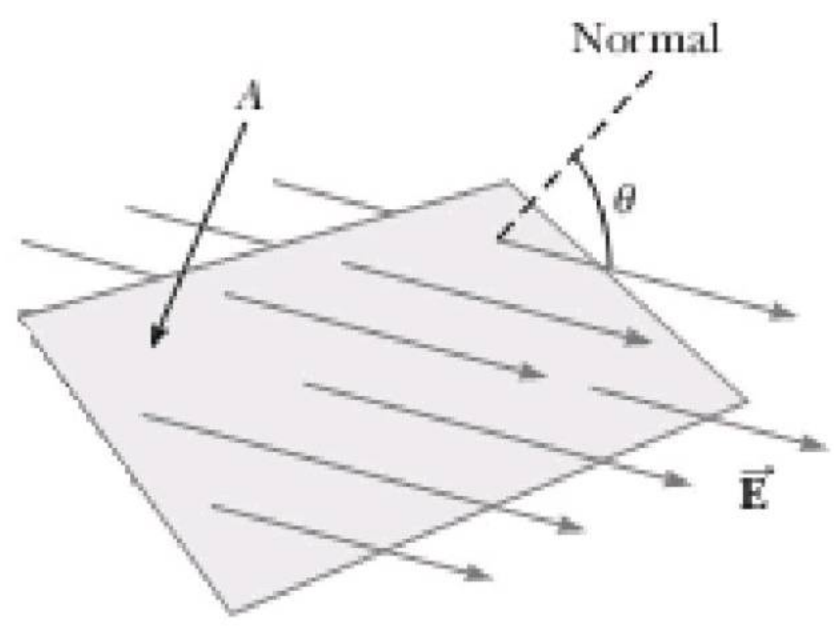
Mathematically Electric flux can be defined as

"The dot product of Electric field intensity and Vector area is called Electric Flux."

$$\boxed{\boldsymbol{\phi = \vec{E}.\vec{A}}}$$

Vector Area is defined as

"A vector Perpendicular to surface having magnitude equal to the Area of the surface is called Vector Area."



Magnitude of electric flux can be written as

$$\boldsymbol{\phi = EA\cos\theta} - -(a)$$

Here, $\boldsymbol{\theta}$ is the angle between Electric field vector and vector area.

Now,

If we divide the Area $\boldsymbol{A}$ into many small segments $\boldsymbol{\Delta A}$ such that

$$\boldsymbol{A = \sum \Delta A}$$

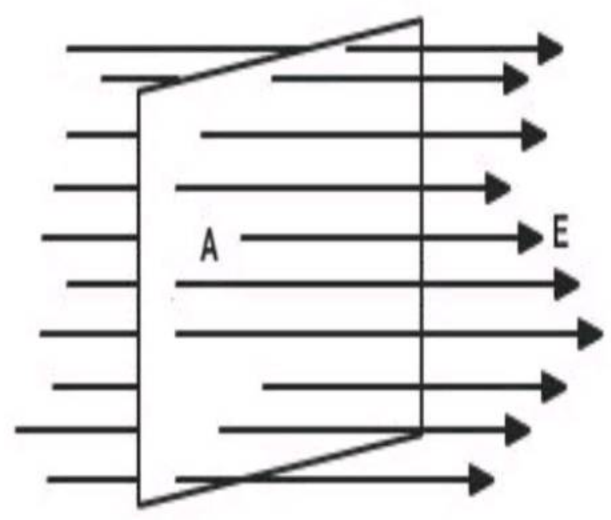
The Electric flux can now be written as

$$\boxed{\boldsymbol{\phi = \vec{E}.\sum \overrightarrow{\Delta A}}}$$

MAXIMUM FLUX:

Let $\theta = 0^o$

Equation (a) will become



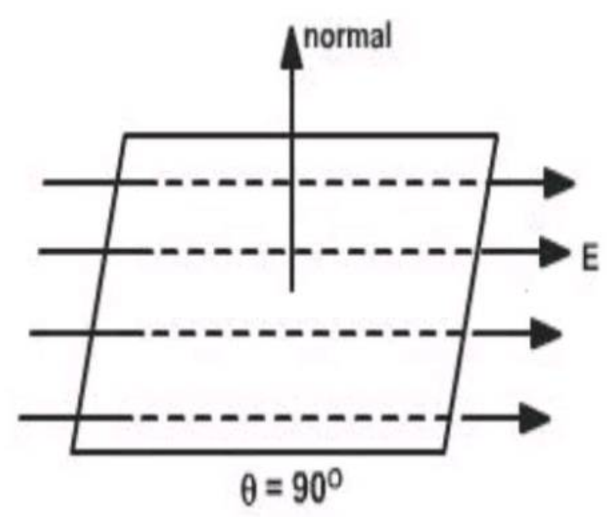
$$\phi = EA\cos(0)$$

$$\boxed{\boldsymbol{\phi = EA}}$$

ZERO FLUX:

Let $\theta = 90^o$

Equation (a) will become



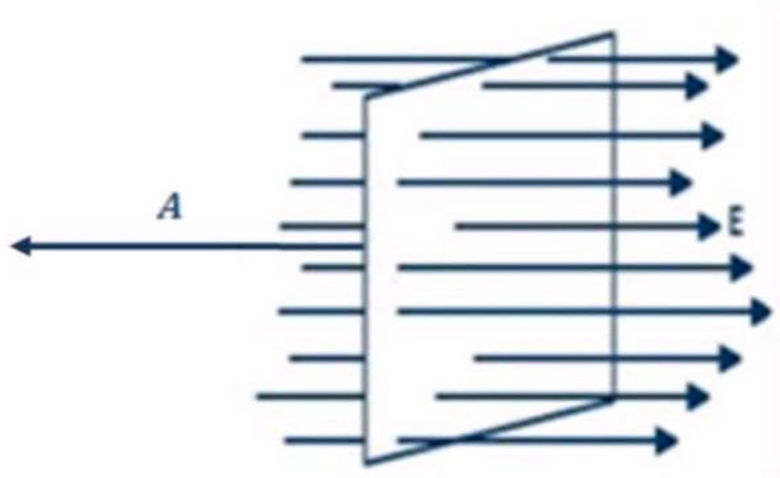
$$\phi = EA\cos(90)$$

$$\boxed{\boldsymbol{\phi = 0}}$$

MINIMUM FLUX:

Let $\theta = 180^{o}$

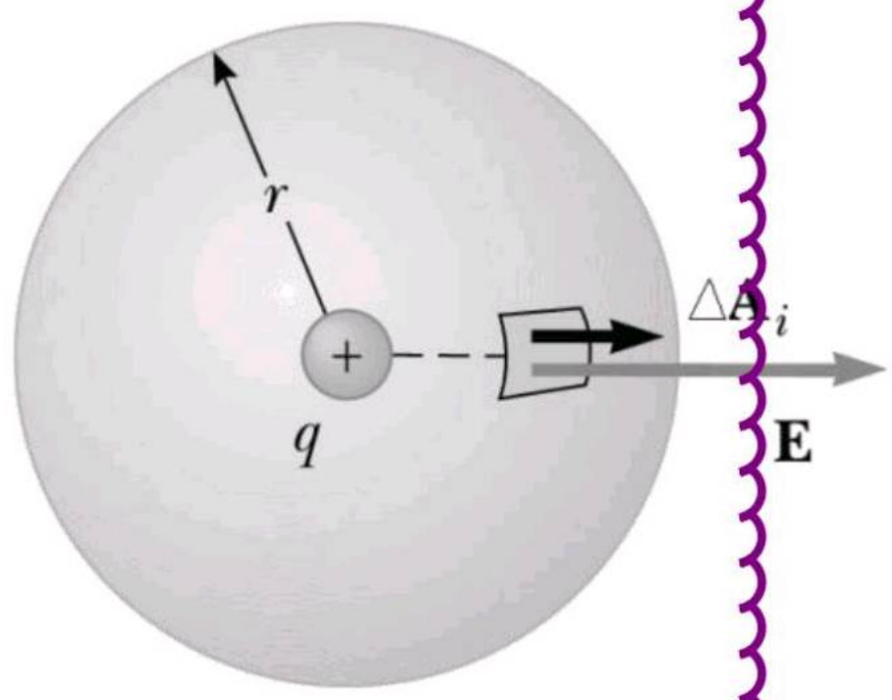
Equation (a) will become



$$\phi = EA\cos(180)$$

$$\boxed{\boldsymbol{\phi = -EA}}$$

Electric Flux due to a Point Charge in a Closed Sphere:



Consider an isolated positive point charge $\boldsymbol{q}$. The lines of forces from $\boldsymbol{q}$ will spread uniformly in space around it cutting the surface of an imaginary sphere. Now we want to find flux due to point charge. For this purpose, we divide the whole sphere into small patches. Each patch is denoted by $\Delta\boldsymbol{A}$.

Electric flux can be written as

$$\boldsymbol{\phi = EA\cos\theta}$$

As the electric field vector is parallel to each patch i-e the angle between vector area and electric filed is zero for every patch i-e $\boldsymbol{\theta = 0^{o}}$

Now

$$\phi = EA\cos(0)$$

$$\boldsymbol{\phi = EA - - - (1)}$$

Now flux through patch ΔA_1

$$\phi_1 = E\Delta A_1$$

Similarly flux through patch ΔA_2

$$\phi_2 = E\Delta A_2$$

Now following the similarly, the flux through n^{th} patch can be written as

$$\phi_n = E\Delta A_n$$

Total flux through the sphere will be equal to the algebraic sum of all these flux i.e.

$$\boldsymbol{\phi = \sum_{i=n}^{i=1} \phi_i}$$

$$\phi = \sum_{i=n}^{i=1} E.\Delta A_i$$

$$\phi = E\sum_{i=n}^{i=1} \Delta A_i - - - (2)$$

Since

$$A = \sum_{i=n}^{i=1} \Delta A_i = 4\pi r^2$$

And

$$E = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2}$$

Equation (2) will become

$$\phi = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2} \times 4\pi r^2$$

$$\boxed{\boldsymbol{\phi = \frac{q}{\varepsilon_o}}}$$

Hence, The Flux going out of the sphere is independent of radius of sphere.

Gauss's Law:

It is a quantitative relation which applies to any closed hypothetical surface called Gaussian surface. It is named after the ***Karl Gauss (1771-1855)*** who stated it first.

Statement:

"The total Electric Flux emerging out or sinking into a closed surface in an electric field is proportional to the algebraic sum of the electric charges enclosed within the surface."

Mathematically:

Mathematically Gauss's Law can also be defined as

"The total electric flux diverging out from a closed surface is equal to the product of the sum of all charges present in that closed surface and $\frac{1}{\varepsilon_o}$*."*

$$\boxed{\boldsymbol{\phi = \frac{q}{\varepsilon_o}}}$$

If there are n positive spherical charge bodies of different enclosed in a Gaussian surface then flux due to each charge body can be written as

$$\phi_1 = \frac{q_1}{\varepsilon_o}$$

$$\phi_2 = \frac{q_2}{\varepsilon_o}$$

.

.

.

$$\phi_n = \frac{q_n}{\varepsilon_o}$$

If total flux diverging out of the surface is ϕ then it may be written as

$$\phi = \sum \phi_n$$

$$\phi = \sum \frac{q_n}{\varepsilon_o}$$

$$\phi = \frac{1}{\varepsilon_o} \sum q_n$$

Hence, total flux diverging out is equals to the $\frac{1}{\varepsilon_o}$ times the total charge enclosed q.

$$\boxed{\phi = \frac{q}{\varepsilon_o}}$$

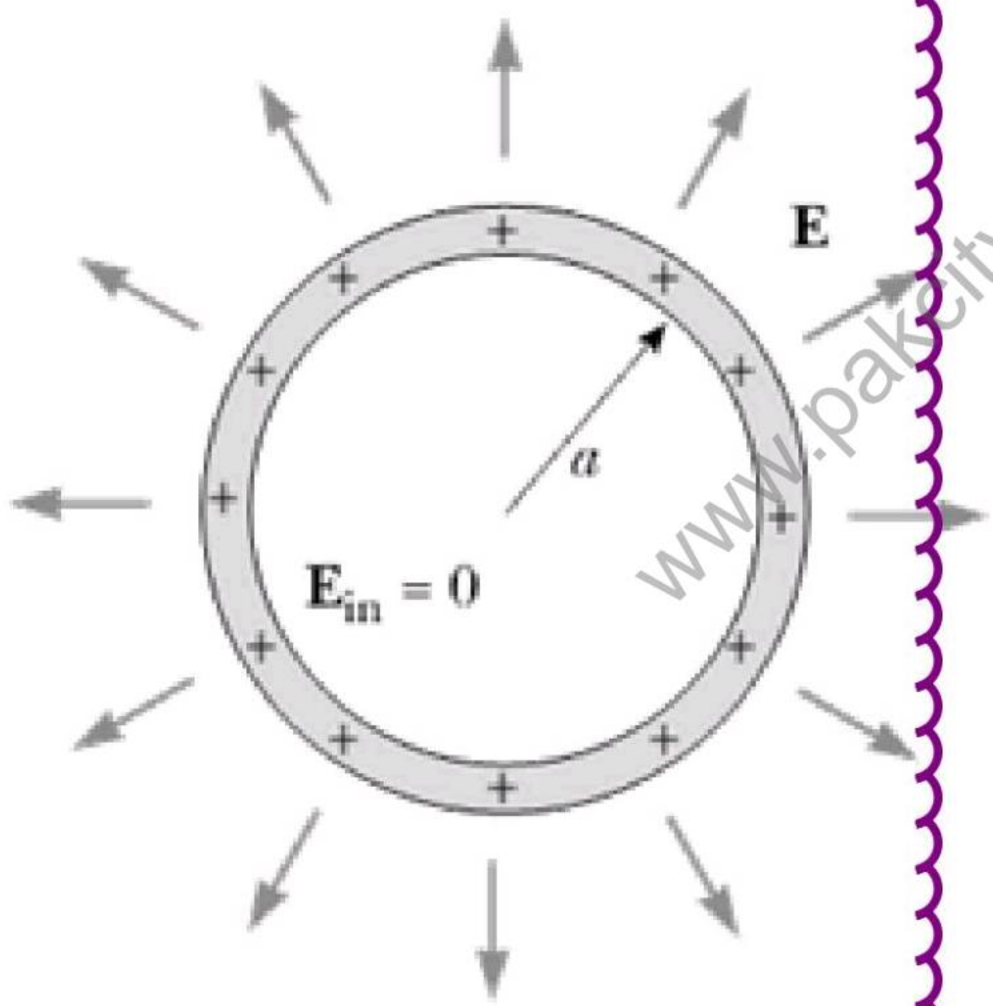
Application of Gauss's Law:

a) The Electric Field Due to a Thin Spherical charged Shell:

Consider a uniformly charged spherical shell having radius a as shown in figure below

Now, Electric flux due to the charges can be written as

$$\phi = EA$$



Since

$$\phi = \frac{q}{\varepsilon_o} \text{------------------(a)}$$

For total Flux "ϕ" For this the whole Gaussian surface is divided into small nth sections having surface area ΔA_1, ΔA_2, ΔA_3 ....ΔA_n of almost equal area "ΔA" and Electric field "E" is remain parallel and same for each section. Then the total flux through all sections of Gaussian surface/Sphere will be:

$\phi = \phi 1 + \phi 2 + \phi 3 ......+ \phi n$ ________________ eq(1)

$\phi = E\Delta A_1 \cos 0^0 + E\Delta A_2 \cos 0^0 + E\Delta A_3 \cos 0^0 + ... + E\Delta A_n \cos 0^0$

$\boxed{\cos 0^0 = 1}$

$\phi = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + ... + E\Delta A_n$

$\phi = E (\Delta A_1 + \Delta A_2 + \Delta A_3 + ... + \Delta A_n)$

$\boxed{\sum \Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3 + ... + \Delta A_n}$

$\sum \Delta A = 4\pi r^2$

$\phi = E (\sum \Delta A)$

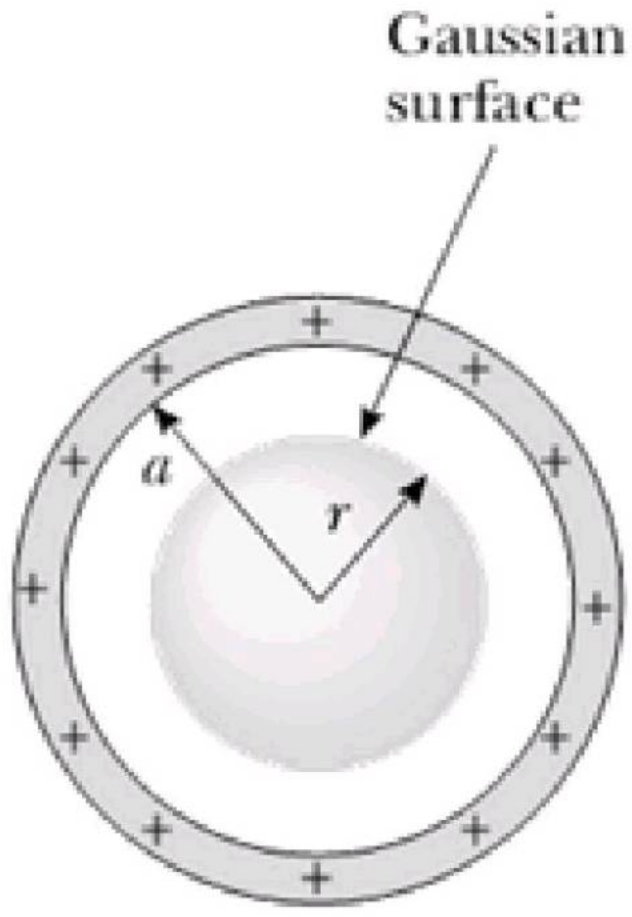
$\phi = E (4\pi r^2)$---------------(b)

Compare eq. (a) and eq. (b)

$E (4\pi r^2) = \frac{q}{\varepsilon_o}$

$$\boxed{E = \frac{q}{4\pi r^2 \varepsilon_o}}$$

Field intensity inside the Shell:



For Field intensity inside the Shell consider Gaussian surface consider a Gaussian surface inside the shell having radius r such that $r < a$.
Equation (a) will become

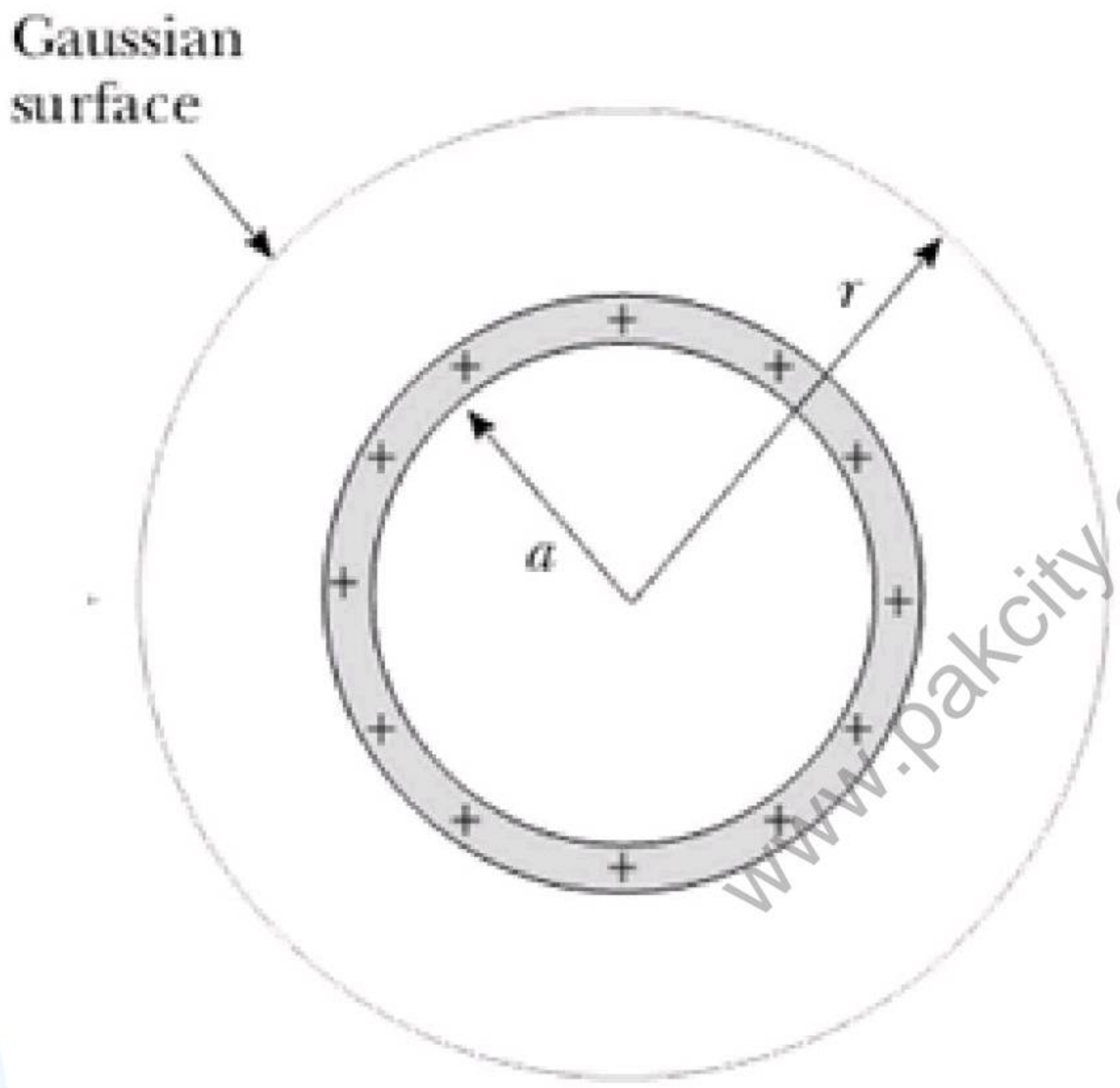
$$\frac{q}{\varepsilon_o} = E(4\pi r^2)$$

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$

Since charged inside the shell is zero there fore

$$\boxed{E = 0}$$

Field intensity on the surface of Shell:



For field intensity on the surface Equation (a) will become

$$\frac{q}{\varepsilon_o} = E(4\pi a^2)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_o a^2}}$$

Field intensity outside the Shell:

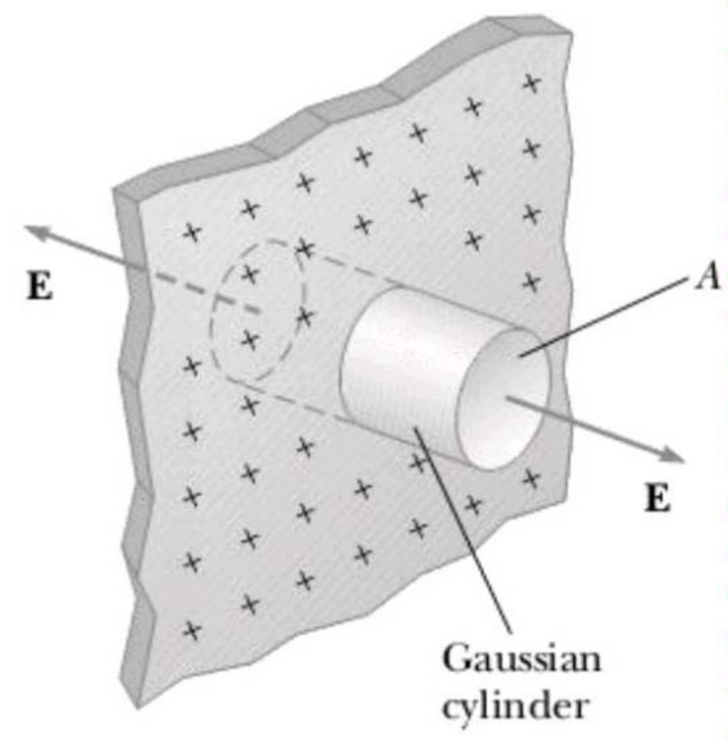
For Field intensity outside the Shell consider Gaussian surface consider a Gaussian surface inside the shell having radius r such that $r > a$.
Equation (a) will become

$$\frac{q}{\varepsilon_o} = E(4\pi r)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_o r^2}}$$

b) Electric Field Due to an infinite charged Sheet:

Consider an infinite positive charged sheet as shown in figure. The electric filed intensity due to this charged sheet can be find out by using Gauss law. Consider a cylindrical Gaussian surface passing through the charged sheet as shown in figure.



Since, there is no electric line of force through the surface area of Gaussian surface there is no electric flux through the surface area.

$$\phi_1 = (0)A_1 \cos\theta$$

$$\boxed{\boldsymbol{\phi_1 = 0}}$$

Electric flux through the two-cross section of the Gaussian Surface

$$\phi_2 = EA\cos\theta$$

$$\phi_3 = EA\cos\theta$$

Since the angle between electric line of forces and vector Area is 0^0

$$\phi_2 = EA\cos(0)$$

$$\boxed{\boldsymbol{\phi_2 = EA}}$$

For second cross section area

$$\phi_3 = EA\cos(0)$$

$$\boxed{\boldsymbol{\phi_3 = EA}}$$

Total flux $\boldsymbol{\phi}$ can be written as

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\phi = 0 + EA + EA$$

$$\boldsymbol{\phi = 2EA}$$

According to Gauss law

$$\boldsymbol{\phi = \frac{q}{\varepsilon_o}}$$

$$\boldsymbol{2EA = \frac{q}{\varepsilon_o}}$$

$$\boxed{\boldsymbol{E = \frac{q}{2A\varepsilon_o}}} ---(\boldsymbol{a})$$

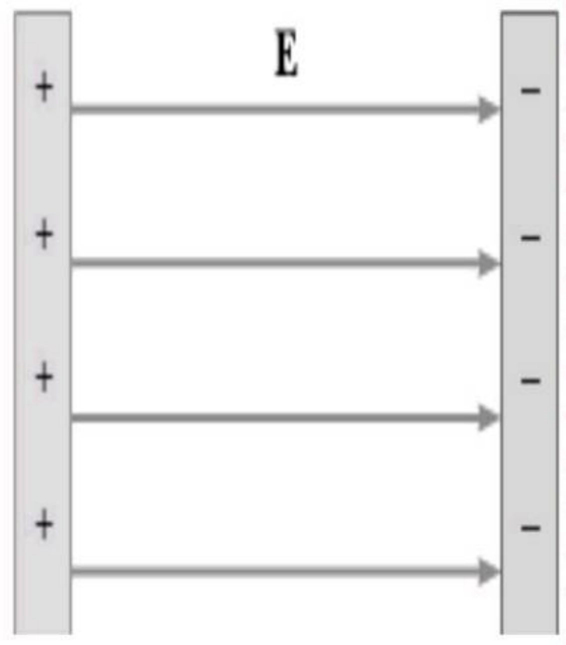
Charge density $\boldsymbol{\sigma}$ can be defined as

$$\boldsymbol{\sigma \equiv \frac{q}{A}}$$

Equation (a) will become

$$\boxed{\boldsymbol{E = \frac{\sigma}{2\varepsilon_o}}}$$

c) Electric Intensity Due to Two Oppositely Charged Sheets:



Consider two metal plates of same area places parallel to each other such that the distance between them is much smaller than their area. If the plates are carrying the equal amount of charges the flux leaving the positive plate is same as the flux entering the negative plate. Electric field intensity for both plates will be same as for infinitely charged sheet i.e $\frac{\sigma}{2\varepsilon_o}$

Net Electric field intensity at any point between the plates will be

$$E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_o}}$$

$$\boxed{\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{r}}$$

Here, $\hat{r}$ is the unit vector directed from positive plate toward the negative plate.

Electric Potential:

The electric potential in an electric field at a point is defined as:

"The amount of work done in moving a unit positive test charge from infinity to that point against the electric forces is called Electric Potential."

Mathematical Explanation:

Consider test charge q_o placed at a point in an electric field. If the Work done in carrying the that test charge to that point is W then electric potential V can be written as

$$V = -\frac{W}{q_o} - -(a)$$

Here, Negative sign indicate the work done against the field

The Electric potential energy can be written as

$$U = -W$$

Equation (a) will become

$$\boxed{V = \frac{U}{q_o}}$$

Hence electric potential can be defined as

"Potential energy per unit test charge is called the Electric Potential."

Electric Potential Difference OR Potential Difference:

"The amount of work done in moving a unit positive test charge from one point to another point in an electric field is called Electric Potential Difference or Potential Difference."

Mathematical Explanation:

Consider a positive test $q_o$charge present at point $a$where its potential energy is U_a.This test charged moved to point b within the electric fieldwhere its energy becomeU_b.

The electric potential energy at point a can be written as

$$V_a = \frac{U_a}{q_o}$$

Similarly, electric potential at point b can be written as

$$V_b = \frac{U_a}{q_o}$$

The change in electric potential or potential difference between point a and b can be written as

$$V_a - V_b = \frac{U_a}{q_o} - \frac{U_b}{q_o}$$

$$V_a - V_b = \frac{U_a - U_b}{q_o}$$

$$\boxed{\Delta V = \frac{\Delta U}{q_o}}$$

Above expression represent the potential difference between the two points in Electric field.

Relation between Electric Field and potential Difference:

Consider a positive test $q_o$charge present at point a.This test charged moved to point $b$within the electric field. Such that displacement vector from a to b is $\overrightarrow{\Delta S}$.
The work doneΔW against the electric file in moving that test charge to b can be written as

$$\Delta W = \vec{F}.\overrightarrow{\Delta S} - -(i)$$

Since$\vec{F} = q_o\vec{E}$, equation (i) becomes

$$\Delta W = q_o\vec{E}.\overrightarrow{\Delta S}$$

$$\Delta W = q_o E \Delta S \cos\theta$$

$$\Delta W = q_o E \Delta S \cos\theta$$

$$E\cos\theta = \frac{\Delta W}{q_o \Delta S}$$

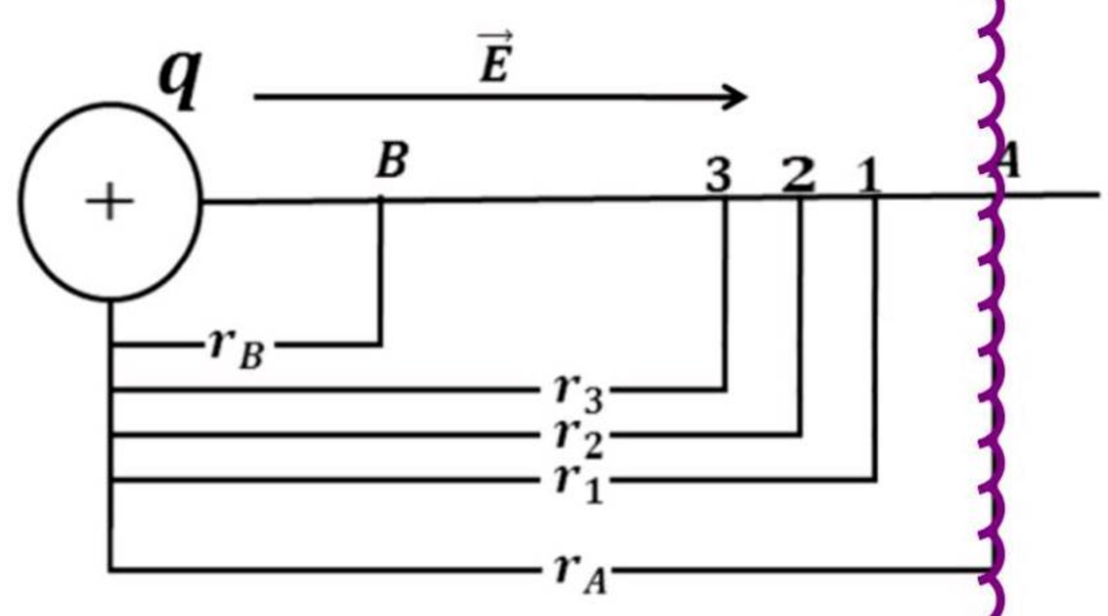
$$E\cos\theta = \frac{\Delta V}{\Delta S}$$

If the angle between the electric field and displacement vector is 180^othen Electric field intensity can be written as:

$$\boxed{E = -\frac{\Delta V}{\Delta S}}$$

The quantity$\Delta V/\Delta S$ is called the potential gradient so the electric field intensity at any point in space is ***"Negative of rate of change of potential with respect to displacement."***

Absolute Potential near an Isolated Point Charge:



Consider two points $\boldsymbol{A}$ and $\boldsymbol{B}$ in a straight line at distance $\boldsymbol{r_A}$ and $\boldsymbol{r_B}$ respectively from a point charge $\boldsymbol{q}$ as shown in figure. In order to determine the potential difference between point $\boldsymbol{A}$ and $\boldsymbol{B}$ a test charge should be moved from point $\boldsymbol{A}$ to $\boldsymbol{B}$. Since the distance between point $\boldsymbol{A}$ and $\boldsymbol{B}$ is considerably large determining potential difference in steps.

Potential difference ΔV_{A1} between point $\boldsymbol{A}$ and $\mathbf{1}$ can be written as

$$\Delta V_{A1} = -\frac{Eq_o(r_A - r_1)}{q_o}$$

$$\Delta V_{A1} = -E(r_A - r_1)$$

Since $\boldsymbol{E} = \frac{\boldsymbol{Kq}}{\boldsymbol{r^2}}$

$$\Delta V_{A1} = -\frac{Kq}{r^2}(r_A - r_1) - - - (1)$$

Here $\boldsymbol{r}$ is the geometric mean of $\boldsymbol{r_A}$ and $\boldsymbol{r_1}$ **i-e**

$$\boldsymbol{r = \sqrt{r_A r_1}}$$

$$\boldsymbol{r^2 = r_A r_1}$$

Putting in equation (1) we get

$$\Delta V_{A1} = -\frac{Kq}{r_A r_1}(r_A - r_1)$$

$$\Delta V_{A1} = -Kq\left(\frac{r_A}{r_A r_1} - \frac{r_1}{r_A r_1}\right)$$

$$\boldsymbol{\Delta V_{A1} = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right)}$$

Similarly, potential difference ΔV_{12} between point 1 and 2 can be written as

$$\boldsymbol{\Delta V_{12} = -Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

Following the same procedure potential difference ΔV_{BN} between point B and N can be written as

$$\boldsymbol{\Delta V_{BN} = -Kq\left(\frac{1}{r_N} - \frac{1}{r_B}\right)}$$

The total potential difference $\boldsymbol{\Delta V_{BA}}$ between point $\boldsymbol{A}$ and $\boldsymbol{B}$ will be the sum of the potential difference of each step i-e

$$\Delta V_{BA} = -\boldsymbol{Kq}\left(\frac{\mathbf{1}}{\boldsymbol{r_1}} - \frac{\mathbf{1}}{\boldsymbol{r_A}}\right) - \boldsymbol{Kq}\left(\frac{\mathbf{1}}{\boldsymbol{r_2}} - \frac{\mathbf{1}}{\boldsymbol{r_1}}\right) + \cdots - \boldsymbol{Kq}\left(\frac{\mathbf{1}}{\boldsymbol{r_N}} - \frac{\mathbf{1}}{\boldsymbol{r_B}}\right)$$

$$\Delta V_{BA} = -Kq\left(\frac{1}{r_A} - \frac{1}{r_1} + \frac{1}{r_1} - \frac{1}{r_2} + \cdots + \frac{1}{r_N} - \frac{1}{r_B}\right)$$

$$\boxed{\Delta \boldsymbol{V_{BA}} = -\boldsymbol{Kq}\left(\frac{\mathbf{1}}{\boldsymbol{r_A}} - \frac{\mathbf{1}}{\boldsymbol{r_B}}\right)}$$

$$\boldsymbol{V_B} - \boldsymbol{V_A} = \boldsymbol{Kq}\left(\frac{\mathbf{1}}{\boldsymbol{r_B}} - \frac{\mathbf{1}}{\boldsymbol{r_A}}\right) - - - (2)$$

Now to find the absolute potential $\boldsymbol{V_B}$at point$\boldsymbol{B}$ let the point$\boldsymbol{A}$to be at infinity so that potential$\boldsymbol{V_A}$ at point$\boldsymbol{A}$ is Zero.

Equation (2) will become

$$\boldsymbol{V_B} - \mathbf{0} = \boldsymbol{Kq}\left(\frac{\mathbf{1}}{\boldsymbol{r_B}} - \frac{\mathbf{1}}{\infty}\right)$$

$$\boldsymbol{V_B} = \frac{\boldsymbol{Kq}}{\boldsymbol{r_B}}$$

In general potential $\boldsymbol{V}$at a distance $\boldsymbol{r}$from the charge $\boldsymbol{q}$can be written as

$$\boxed{V = \frac{Kq}{r}}$$

Electron-Volt:

Consider the expression for electric potential due to an electron

$$\boldsymbol{V} = \frac{\boldsymbol{U}}{\boldsymbol{e}}$$

$$\boldsymbol{U} = \boldsymbol{eV}$$

Putting the units, we get

$$\boldsymbol{Joules} = \boldsymbol{Charge\ of\ electron} \times \boldsymbol{Volt}$$

Hence, we get a unit of energy called the ***Electron-Volt (eV)***.

Relation between electron volts and potential energy can be expressed as

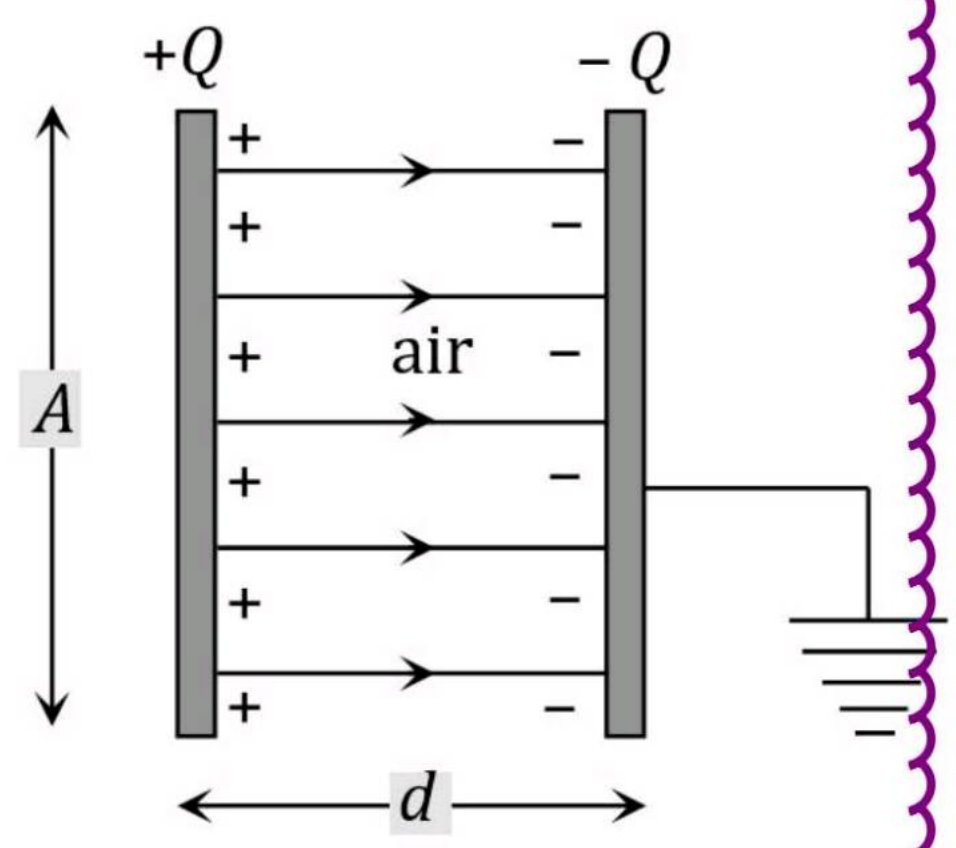
$$\boxed{\mathbf{1}\boldsymbol{eV} = \mathbf{1.6} \times \mathbf{10^{-19}}\boldsymbol{J}}$$

Electron volt is very useful unit. In atomic and semiconductor physics energies are frequently expressed in Electron-Volts.

Capacitor:

The capacitor is an electrical device that can store electrical charge, thereby creating an electric field that, in turn, stores energy. It is also called Condenser.

Construction:



System of two conductors separated at a distance **"d"** by an insulating medium such as air or any other forms a capacitor. The conductors could be of any size but the distance between them should be very small as compare to their size.

Mathematical Explanation:

Since the energy stored in capacitor is dependent on the electric field intensity between the conductors which depends on the charge stored on each conductor the more the charge stored higher the potential difference between the conductors.

Therefore, the potential difference between the plates is directly proportional to the charge stored on each plate. i-e

$$q \propto V$$

$$\boldsymbol{q = CV}$$

Here, ***C*** is called the capacitance of capacitor or Capacitance which may be defined as

"The ability of a system of conductors and insulator (i-e capacitor) to store electric charge is called Capacitance."

It may also be defined as

"Charge stored per unit voltage developed across it is called capacitance."

Capacitance of a device is dependent on the Geometric arrangement of conductors and the insulated medium between them.

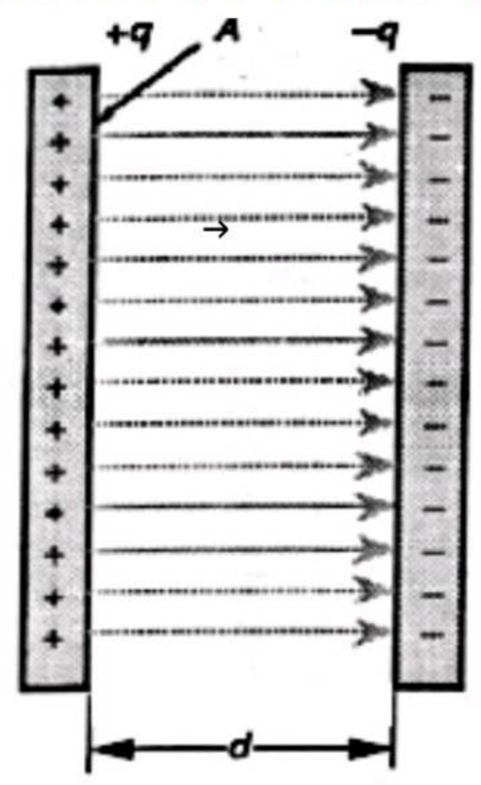
Unit:

The unit of capacitance is Farad$(\boldsymbol{F})$

Farad is very large unit so for practical purpose convenient units are

Micro farad $\mu F = 10^{-6}\ F$

Pico farad $pF = 10^{-12} F$



Parallel Plate capacitor:

A capacitor made by the placing two conducting metal sheets parallel to each other such that the distance between the plates is very small as compare to the facing area of plates is called Parallel plate capacitor.

Expression for Capacitance:

Consider to oppositely charge metal plates having charge $\boldsymbol{q}$ placed at the distance $\boldsymbol{d}$ from each other. If the medium between the plates is air the electric field intensity between the plates can be written as

$$\boldsymbol{E = \frac{\sigma}{\varepsilon_o} --- (1)}$$

Since

$$\sigma = \frac{q}{A}$$

$$\boldsymbol{q = \sigma A}$$

Now since

$$V = E\Delta S = Ed$$

$$\boldsymbol{V = \frac{\sigma d}{\varepsilon_o}}$$

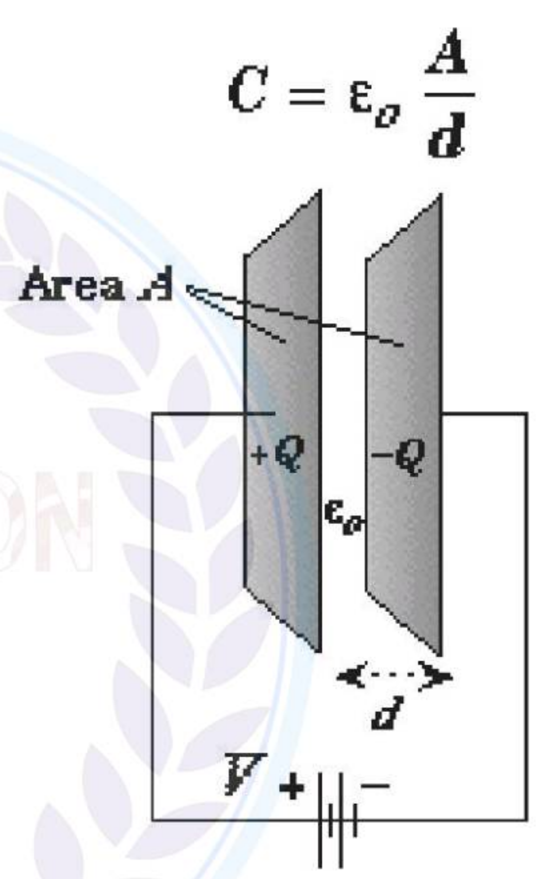
Now expression for capacitance $\boldsymbol{C_o}$ can be written as

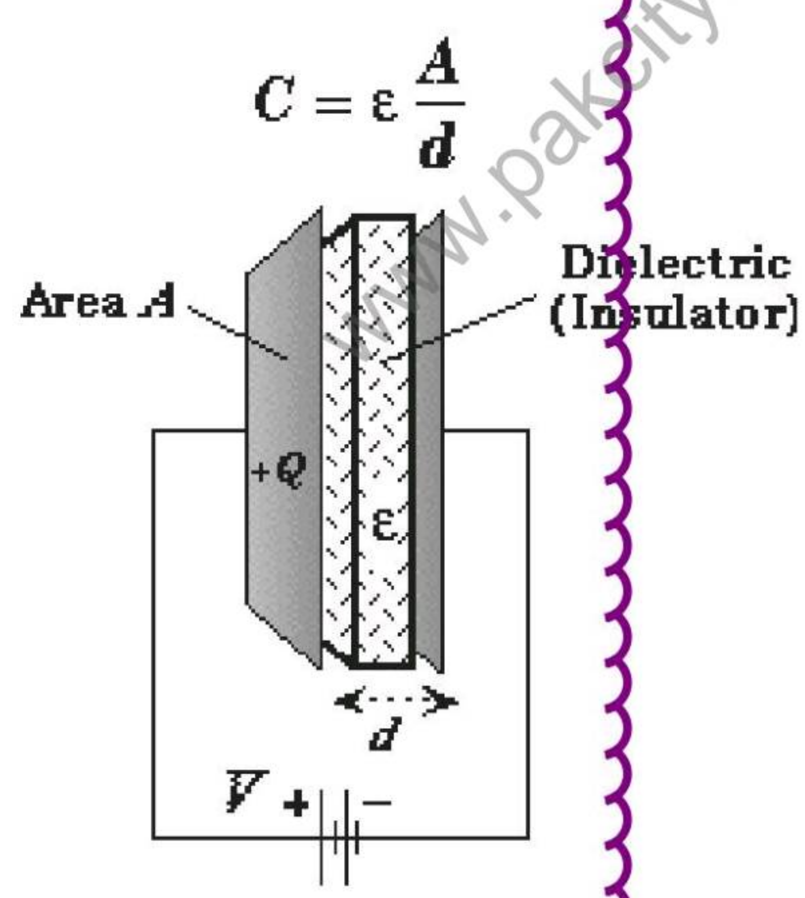
$$\boldsymbol{C_o = \frac{q}{V}}$$

Putting values, we get

$$C_o = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_o}}$$

$$\boxed{\boldsymbol{C_o = \frac{A\varepsilon_o}{d}}}$$





From above expression it is clear that the capacitance of a capacitor is dependent on the geometric properties of capacitor and the medium separating the plates.

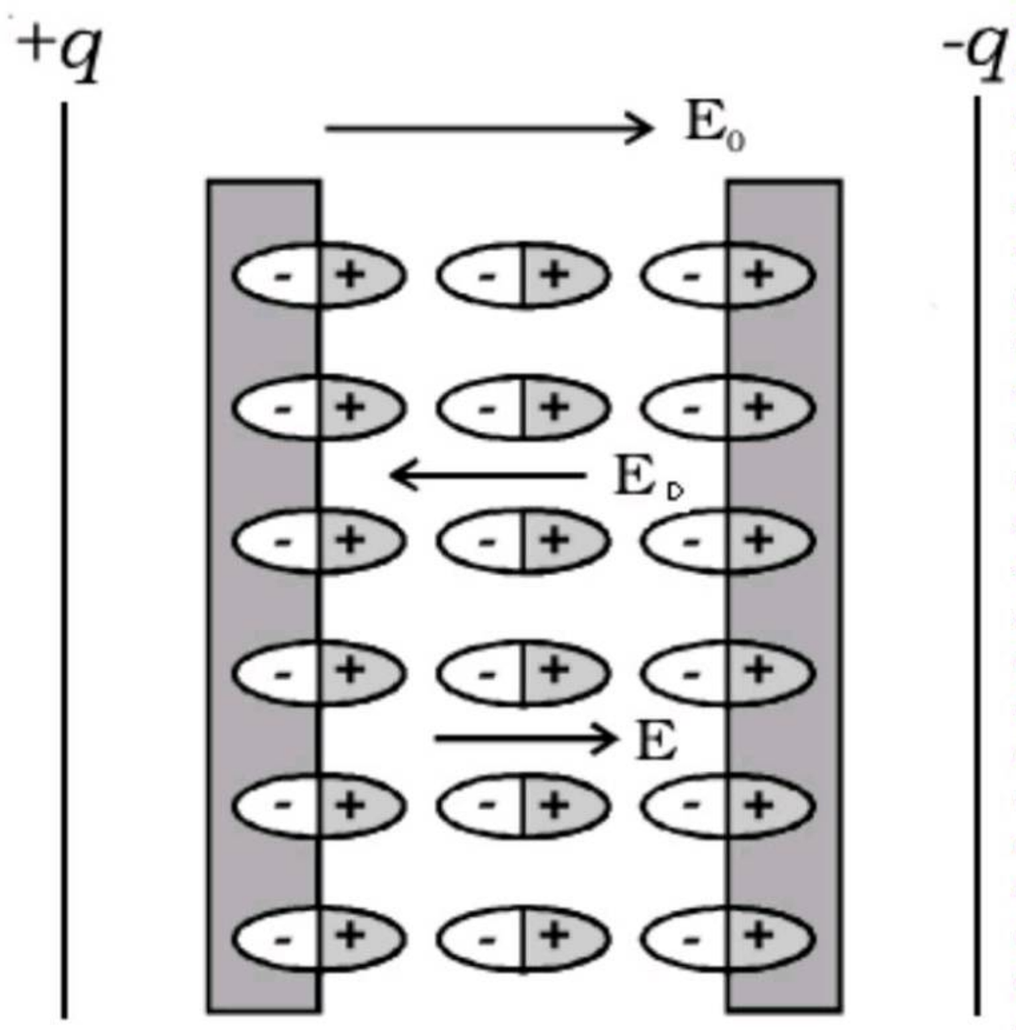
If the medium between the plates is other than air the capacitance of the capacitor will be

$$\boldsymbol{C = \frac{A\epsilon}{d} = \frac{A\varepsilon_o\varepsilon_r}{d}}$$

$$\boxed{\boldsymbol{C = C_o\varepsilon_r}}$$

Hence the presence of a dielectric medium increases the capacitance.

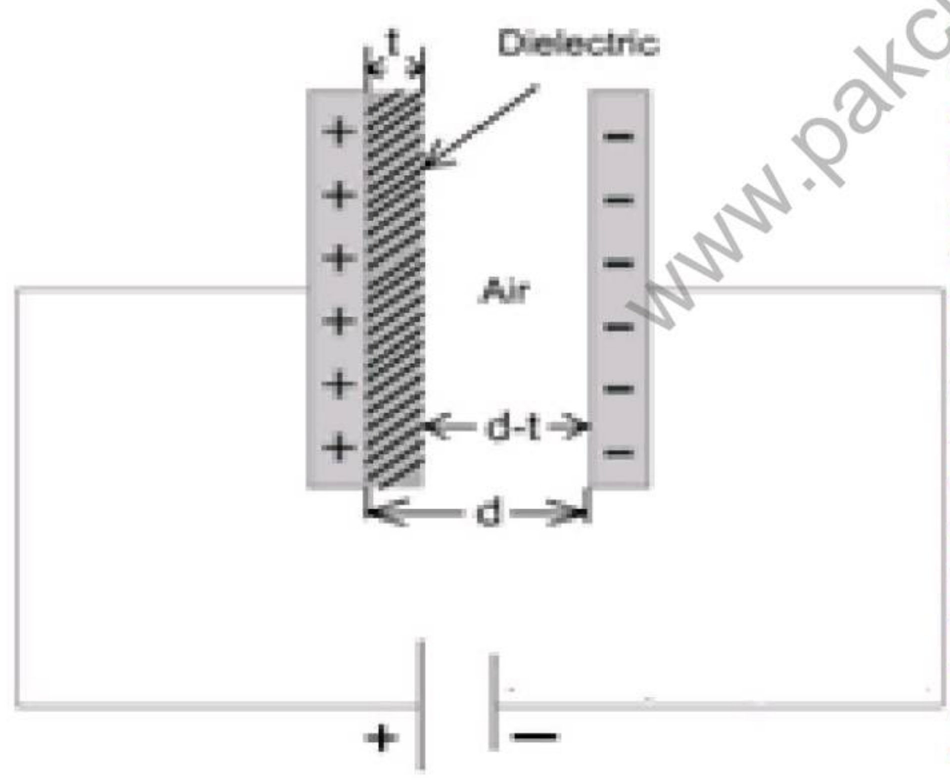
Effect of Dielectric on Capacitance:



Consider an air-filled capacitor charged at a certain voltage so that an electric field E_o is set up between the plates of capacitor.

Now if we introduce a dielectric medium between the plates due to the potential difference between the plates of capacitor the Molecules of Dielectric medium get polarized as shown in figure and an electric field E_D is set up due to the polarized molecules of the dielectric medium since the Electric field set up by the molecules is opposite to Electric field E_o the net electric field E become less than E_o which in response decreases the potential difference between the plates and hence more charge has to be stored on plates to mention the Voltage across the plates. So, the charge per unit Volt is increased which is defined to be capacitance.

Capacitance of a Parallel Plate Capacitor When Dielectric is Completely Filled between the Plates



Let the space between the plates of capacitor is filled with a dielectric of relative permittivity ε_r. The presence of dielectric reduces the electric intensity by ε_r times and thus the capacitance increases by **εr** times.

$$C = C_o\varepsilon_r$$

$$C = \frac{A\epsilon}{d} = \frac{A\varepsilon_o\varepsilon_r}{d}$$

When both air & dielectric is present between plates:

Electric intensity when air is present

$$E_1 = \frac{\sigma}{\varepsilon_o}$$

Electric intensity when air & dielectric both are present

$$E_2 = \frac{\sigma}{\varepsilon_o\varepsilon_r}$$

The potential difference between plates is

$$V = V_1 + V_2$$

But $V = E\Delta S$

$$V = E_1(d-t) + E_2 t$$

$$V = \frac{\sigma(d-t)}{\varepsilon_o} + \frac{\sigma}{\varepsilon_o\varepsilon_r}t$$

$$V = \frac{\sigma}{\varepsilon_o}\left\{(d-t) + \frac{t}{\varepsilon_r}\right\}$$

$$V = \frac{\sigma}{\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

$$V = \frac{\sigma}{\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

But, $\sigma = \frac{q}{A}$

$$V = \frac{q}{A\epsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

Since $q = CV$

$$V = \frac{CV}{A\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

$$1 = \frac{C}{A\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

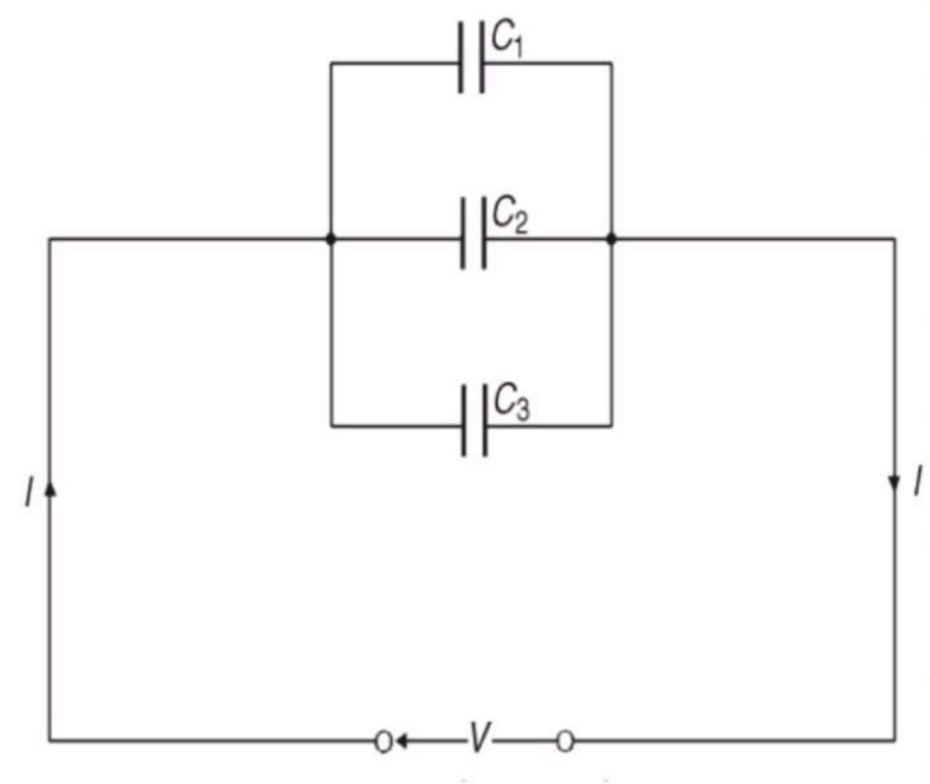
$$C = \frac{A\varepsilon_o}{\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}}$$

Combination of Capacitors:

Capacitors connected in parallel:

Consider three capacitors having C_1, C_2, C_3 capacitance respectively are connected in parallel. We can replace them by an equivalent capacitor having capacitance C_e. A charge q given to a point divide itself resides on the plates of individual capacitor as q_1, q_2, q_3 respectively.
Since

$$\boldsymbol{q = CV}$$



Hence, the total charge stored in parallel combination is equal to the sum of charge stored across each capacitor in combination.

$$q = q_1 + q_2 + q_3$$

$$C_e V = C_1 V_1 + C_2 V_2 + C_3 V_3$$

Since $V = V_1 = V_2 = V_3$

$$C_e V = C_1 V + C_2 V + C_3 V$$

$$C_eV = (C_1 + C_2 + C_3)V$$

$$C_e = C_1 + C_2 + C_3$$

Hence, the equivalent capacitance of capacitor connected in parallel is the algebraic sum of the capacitances of each capacitor in the network. For parallel combination of $\boldsymbol{n}$ capacitors above expression will become.

$$\boldsymbol{C_e = C_1 + C_2 + C_3 + \cdots + C_n}$$

Capacitors connected in Series:

Consider three capacitors havingC_1,C_2 , C_3capacitance respectively are connected in parallel. We can replace them by an equivalent capacitor having capacitance C_e.

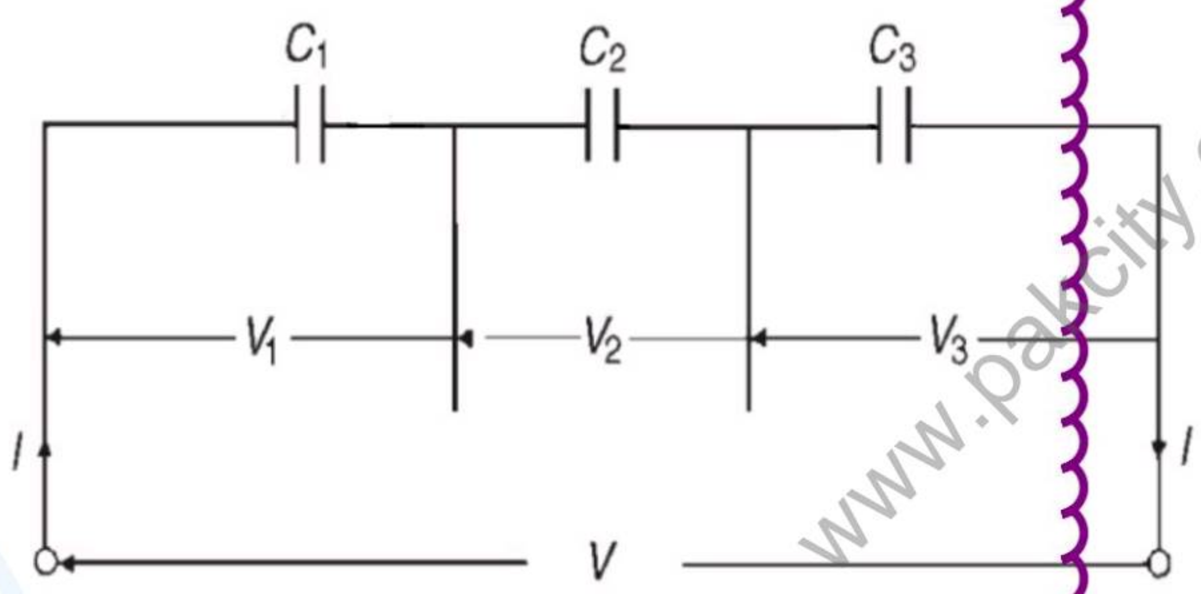
Since

$$\boldsymbol{\frac{q}{C} = V}$$

Hence, Sum of individual voltage across each capacitor is equal to the Voltage of source.

i-e

$V = V_1 + V_2 + V_3$



$$\frac{q}{C_e} = \frac{q_1}{C_1} + \frac{q_2}{C_1} + \frac{q_3}{C_3}$$

Since the charge developed across each capacitor is same

$$q = q_1 = q_2 = q_3$$

Above expression will become

$$\frac{q}{C_e} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{q}{C_e} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Hence, the reciprocal of equivalent capacitance of capacitor connected in series the algebraic sum of the reciprocal of capacitances of each capacitor in the network. For series combination of $\boldsymbol{n}$ capacitors above expression will become.

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

CHAPTER-12

NUMERICALS
PAST PAPER

1990

Q.4. (c) An oil drop having a mass of 0.002kg and charge equal to 6 electron's charge is suspended stationary in a uniform electric field. Find the intensity of electric field. (Charge of electron = 1.6×10^{-19}C) **(2.04×10^{16} V/m)**

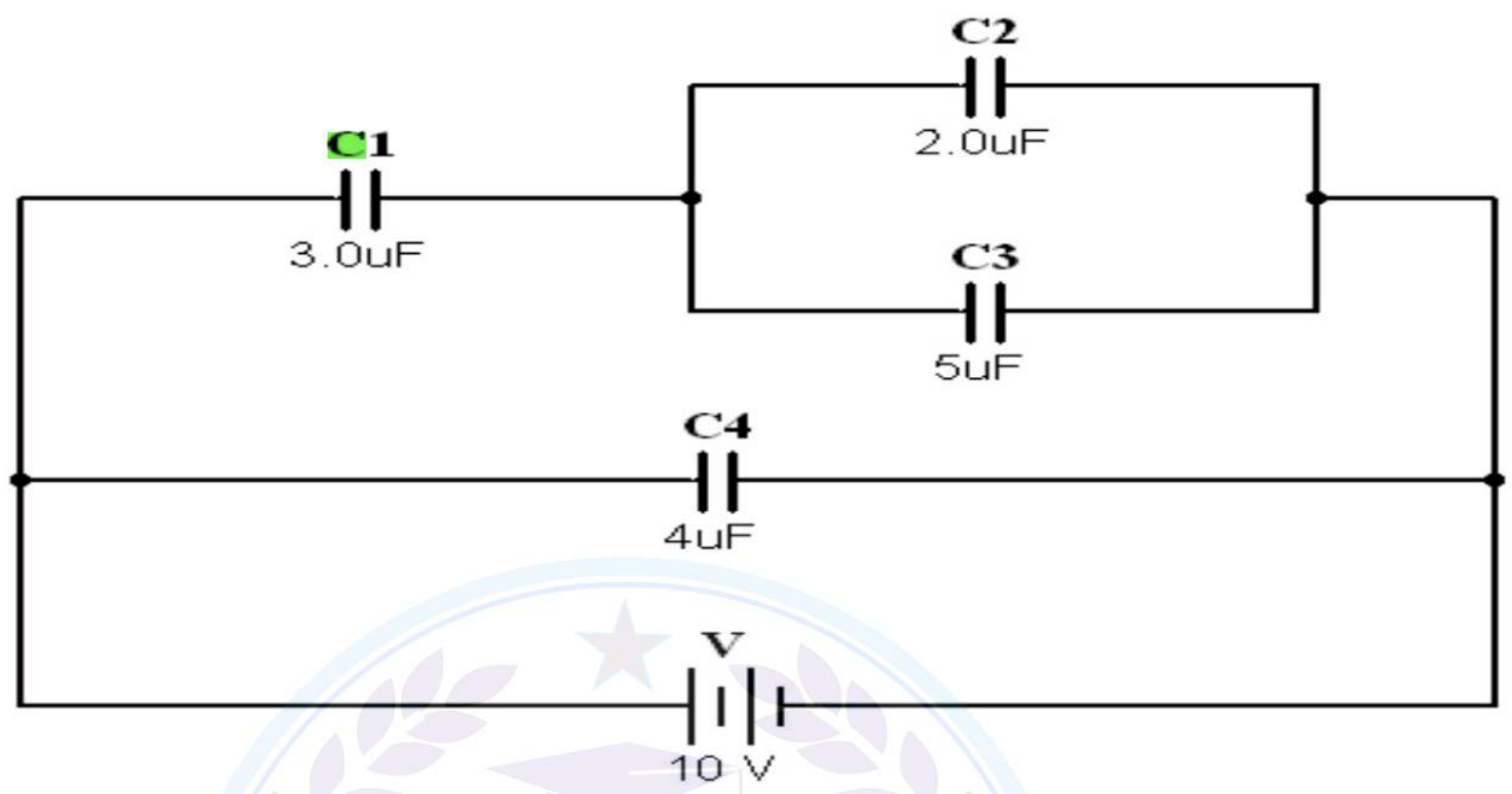
1991

Q.4. (c) Calculate the potential difference between two plates when they are separated by a distance of a 0.005m and are able to hold an electron motionless between them. (Mass of electron = 9.1×10^{-31} Kg) **(2.79×10^{-13} volts)**

1993

Q.3 (c) Calculate the equivalent capacitance and charge on 5µF capacitor as show in the figure

C_1 = 3µ F
C_2 = 2µ F
C_3 = 5µ F
C_4 = 4µ F
V = 10 volts



(6.1µF)

1994

Q.4. (c) Two horizontal parallel metallic plates, separated by a distance of 0.5cm are connected with a battery of 10 volts. Find:

1. The electric field intensity between the plates.
2. The force on a proton placed between the plates. **(2000V/m, 3.2×10^{-16} N)**

1995

Q.4. (c) Two capacitors of capacitance 400µF and 600µF are charged to the potential difference of 300volts & 400volts respectively. They are then connected in parallel. What will be the resultant potential difference and charge on each capacitor? **(360V, 0.144C, 0.216C)**

1996

Q.3. (c) A thin sheet of positive charge attracts a light charged sphere having a charge -5×10^{-6} C with a force 1.69N. Calculate the surface charge density of the sheet. ($\epsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2) **(5.98×10^{-6}coul/m^2)**

Q.4. (c) A particle of mass 0.5g and charge 4×10^{-6}C is held motionless between two oppositely charged horizontal metal plates. If the distance between them is 5mm, find:

(i) The electric intensity (ii) The potential difference between the plates.

(1225V/m, 6.125 V)

1997

Q.3. (c) A capacitor of 200 pF is charged to a P.D. of 100 volts. Its plates are then connected in parallel to another capacitor and are found that the P.D. between the plates falls to 60 volts. What is the capacitance of the second capacitor? **(133.33pF)**

1998

Q.3. (c) Calculate the force of repulsion on $+2 \times 10^{-8}$ coulomb charge. If it is placed before a large vertical charged plate whose charge density is $+ 20 \times 10^{-4}$ coulombs/m^2.**(2.26 N)**

1999

Q.3. (c) Two capacitors of 2.0 μF and 8.0μF capacitance are connected in series and a potential difference of 200 volts is applied. Find the charge and the potential difference for each capacitor. **(3.2×10^{-4}Coul, 160V, 40V)**

2000

Q.3. (c) A charged particle of –17.7 μC is close to a positively charged thin sheet having surface charge density 2×10^{-8}Coul/m^2. Find the magnitude and direction of force acting on the charged particle. **(0.02N, towards the –ve charge)**

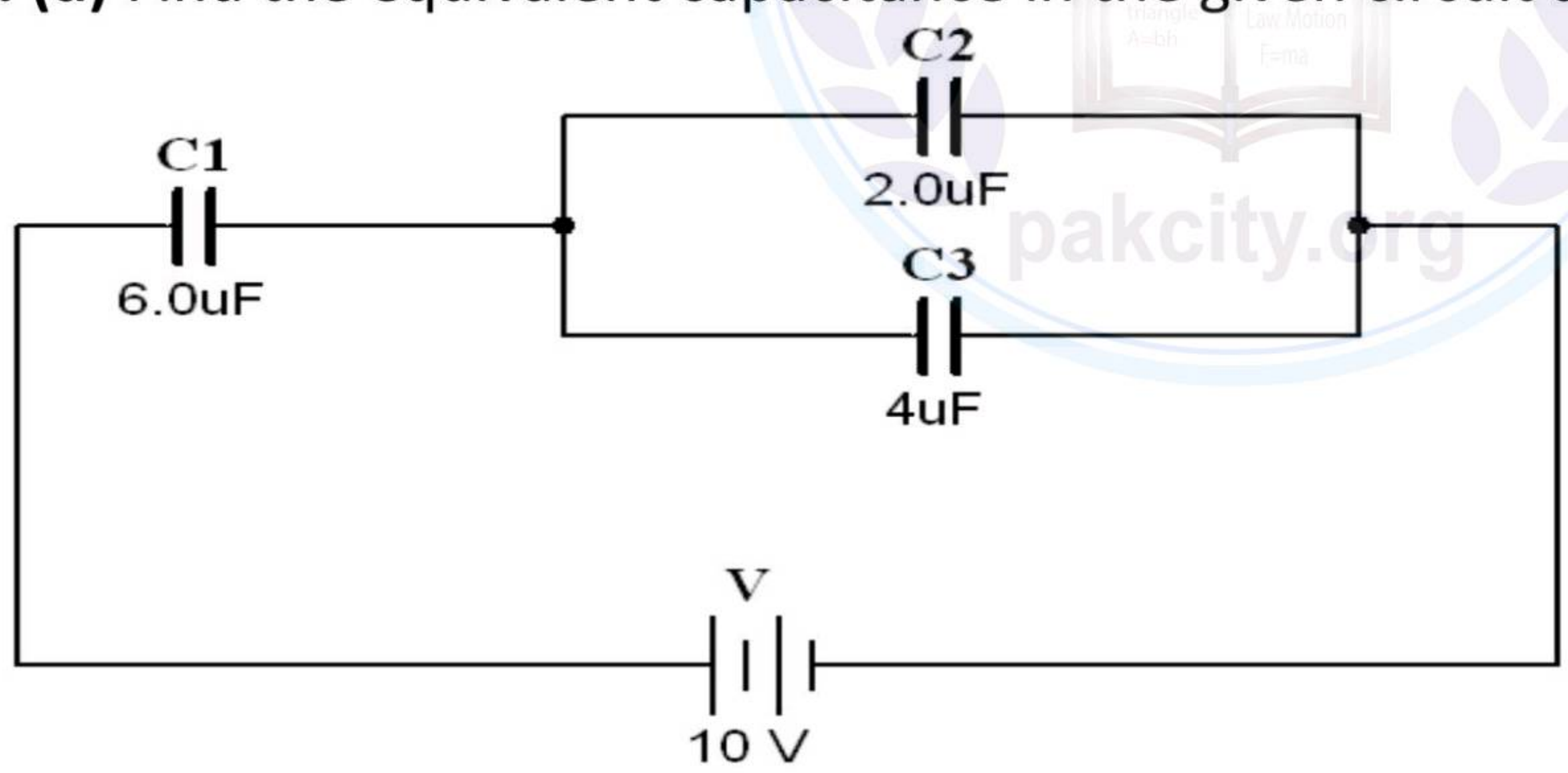
2001

Q.3. (c) A parallel plate capacitor has the plates 10cm x 10cm separated by a distance of 2.5cm. It is initially filled with air, what will be the increase in its capacitance if a dielectric slab of the same area and thickness 2.5cm is placed between the two plates? (Take $\epsilon_R = 2$)

(3.54pF)

2002 (Pre-Med. group)

Q.3. (d) Find the equivalent capacitance in the given circuit and charge on each capacitor:



(3μF, 30 μC, 10 μC, 20 μC)

Q.4. (d) A small sphere of weight 5×10^{-3}N is suspended by a silk thread 50mm long which is attached to a point on a large charge insulating plane. When a charge of 6×10^{-8}C is placed on the ball the thread makes an angle of 30° with the vertical; find the charge density of the plane. **(8.515×10^{-7} C/m^2)**

2002 (Pre Engg. group)

Q.5. (d) A proton of mass 1.67 x 10^{-27} kg and a charge of 1.6 x 10^{-19}C is to be held motionless between two horizontal parallel plates 10cm apart: find the voltage required to be applied between the plates. (**1.0228 x 10^{-8} V**)

2003 (Pre Med. group)

Q.5. (d) A particle carrying a charge of 10^{-5}C starts from rest in a uniform electric field of intensity 50Vm^{-1} Find the force on the particle and the kinetic energy it acquires when it is moved 1m. (**5x10^{-4}N, 5x10^{-4}J**)

2004

Q.3. (d) An electron has a speed of 10^6 m/s. Find its energy in electron volts. (**2.8125eV**)

2006

Q.3. (d) How many electrons should be removed from each of the two similar spheres, each of 10 gm, so that electrostatic repulsion is balanced by the gravitational force? (**5.39 x 10^6 electrons**)

2007

Q.3. (d) How many excess electrons must be placed on each of the two small spheres placed 3.0cm apart if the force of repulsion between the spheres is 10^{-19}N? (**625 electrons**)

2008

Q.3. (d) A capacitor of 12 μF is charged to a potential difference 100V. Its plates are hen disconnected from the source and are connected parallel to another capacitor. The potential difference in this combination comes down to 60V. What is the capacitance of the second capacitance? (**8μF**)

2009

Q.3. (d) A proton of mass 1.67 x 10^{-27} kg and charge 1.6 x10^{-19} C is to be held motionless between two horizontal parallel plates 6cm apart; find the voltage required to be applied between the plates. (**6.13725 nV**)

2010 Q2. (x) How many electrons should be removed from each of two similar spheres each of 10 gm so that electrostatic repulsion may be balanced by gravitational force **(e = 1.602 x 10^{-19} C)?** (**5.4 x 10^6 electrons**)

2011

2. (vii) A proton of mass 1.67x10^{-27} kg and charge 1.6 x10^{-19}C is to be held motionless between two parallel horizontal plates. Find the distance between the plates when the potential difference of 6 x 10^{-9} volts is applied across the plates. (**5.86 cm**)

2012

Q2(xiii) Two-point charges of +2 x 10^{-4} C and -2 x 10^{-4} C are placed at 40 cm from each other. A charge of +5 x 10^{-5} C is placed midway between them. What is the magnitude and direction of force on it?

2013

Q2(xi) The surface charge density on a vertical metal plate is $25\text{x}10^{-6}$ *C/m*2. find the force experienced by a charge of $2 \text{ x } 10^{-10} C$ placed in front close to the sheet.

($\varepsilon_o = 8.85 \text{ x } 10^{-12} C/Nm^2$)

2014

Q2(viii) A 10μF capacitor is charged to a potential difference of 220V. It is then disconnected from the battery. Its plates are connected in parallel to another capacitor and it is found that the potential difference falls to 100V. What is the capacitance of the second capacitor?

2015

Q2(x) A thin sheet of positive charge attracts a light charged sphere having a charge $-5\text{x}10^{-6}$ C with a force 1.69N. Calculate the surface charge density of the sheet.

($\epsilon_o = 8.85\text{x } 10^{-12} \text{ C}^2/\text{Nm}^2$) (**$5.98\text{x}10^{-6}$coul/m^2**)

Q2(xiv) How many electrons should be removed from each of two similar spheres each of 10 gm so that electrostatic repulsion may be balanced by gravitational force (Gravitational constant = $G = 6.67 \text{ x } 10^{-11}\text{Nm}^2/\text{Kg}^2$ and $K = 9 \times 10^{911}\text{Nm}^2/\text{C}^2$)? (**$5.4 \text{ x } 10^6$ electrons**)