Chapter = 12

Electrostatics

Electrostatics:

Electrostatics is the branch of physics which deals with the study of charges in rest condition.

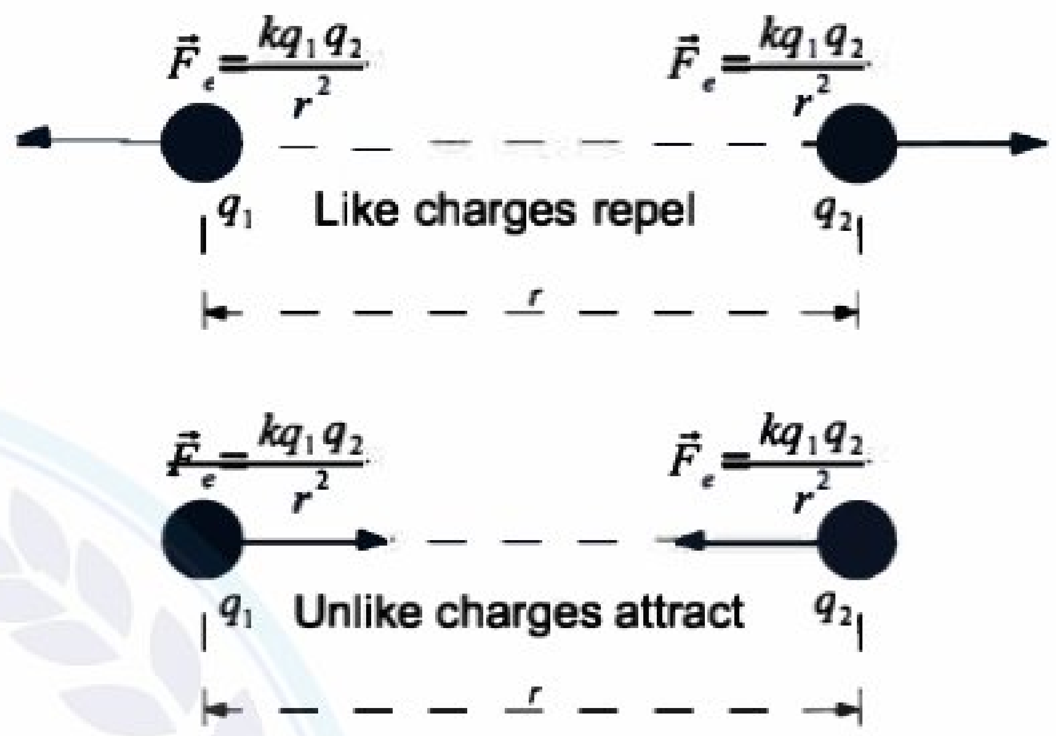
Coulomb's Law:

Statement:

"The electrostatic force of repulsion or attraction between two charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of separation of charges."

Mathematical Representation:

Consider two charges q_1 and q_2 placed at a distance r from each other. The magnitude of electric force between two charges F can be written as



$F \propto q_1q_2$

$F \propto \frac{1}{r^2}$

Combining above expressions, we get

$F \propto \frac{q_1q_2}{r^2}$

$$\boxed{F = k\frac{q_1q_2}{r^2}}$$

Where, $k =$ constant of proportionality and it depends upon the medium between charges and its value for free space is defined as

$$k = \frac{1}{4\pi\varepsilon_o}$$

Here, $\boldsymbol{\varepsilon_o}$is the permittivity of free space. Numerical value of $\boldsymbol{k}$ for free space is

$$k = 8.98755 \times 10^9\ Nm^2C^{-2} \cong 9 \times 10^9 Nm^2C^{-2}$$

Electric force can now be written as

$$\boxed{F = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1q_2}{r^2}}$$

For medium other then free space value of $\boldsymbol{k}$ can be defined as

$$k = \frac{1}{4\pi\epsilon}$$

Here, ϵ is the permittivity of medium and it is defined as

$$\epsilon = \varepsilon_o\varepsilon_r$$

Here, $\varepsilon_r$is called relative permittivity.Electric force between two charges can now be written as

$$F = \frac{1}{4\pi\epsilon} \times \frac{q_1q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \times \frac{q_1q_2}{r^2}$$

In vector form force exerted by charge $q_1$on q_2 can be written as

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \times \frac{q_1q_2}{r^2}\hat{r}}$$

Where: $\hat{r}$ = unit vector in the direction of $\overrightarrow{F}$.

Electric Field:

"The region around the charge in which another charge experiences electrostatic force is called electric field."

Test Charge:

A charge with very small magnitude (approaching to zero) so that the effect of its electric field is negligible is called test charge.

Electric Field Intensity:

The electric field intensity is defined as the electric force experienced by test charge divided by the magnitude of test charge. It is a vector quantity and its direction is same as the direction of electric force.

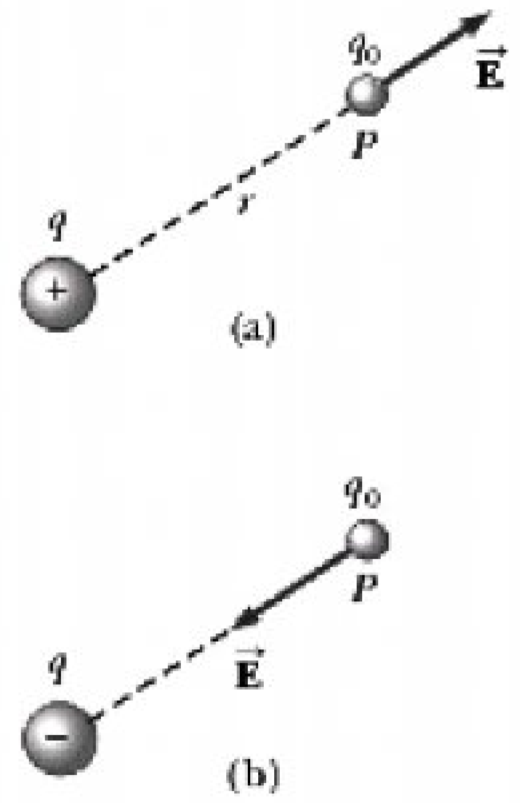
Mathematical Representation:

The electric field $\vec{E}$ intensity due a charge q at a point in space in presence of a test charge q_o can be written as

$$\vec{E} = \frac{\vec{F}}{\overrightarrow{q_0}}$$

Since

$$\vec{F} = \frac{1}{4\pi\varepsilon_o} \times \frac{qq_o}{r^2}\hat{r}$$



Therefore

$$\overrightarrow{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{qq_o}{r^2}\frac{1}{q_o}\hat{r}$$

$$\boxed{\overrightarrow{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2}\hat{r}}$$

$$\boxed{\overrightarrow{E} = \frac{kq}{r^2}\hat{r}}$$

Here $\hat{r}$ is the unit vector in the direction of electric field.

The magnitude of electric field intensity can be written as

$$\boxed{E = \frac{kq}{r^2}}$$

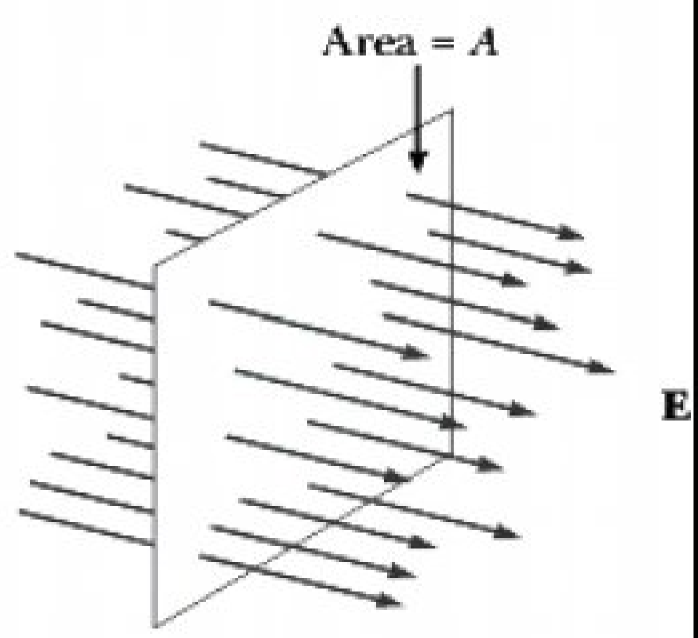
Unit:

The SI unit of electric field intensity is Newton per Coulomb N/C or volt per meter V/m.

Electric Flux:

Number of Electric lines of force passing through surface area is called electric flux.

Electric flux is a scalar quantity. The S.I unit is Nm/c.



Mathematically:

"The dot product of Electric field intensity and Vector area is called Electric Flux."

$$\boxed{\phi = \vec{E}.\vec{A}} \quad \text{Or}$$

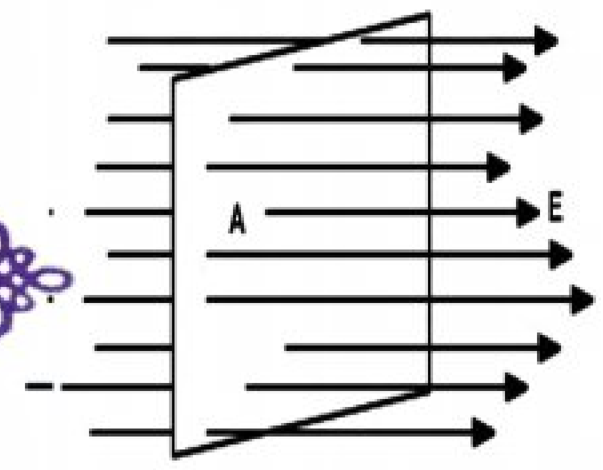
$$\boxed{\phi = EAcos\theta}$$

θ = angle between electric field intensity $\vec{E}$ and area vector $\overrightarrow{A}$.

Vector Area is defined as

"A vector Perpendicular to surface having magnitude equal to the Area of the surface."

Maximum Flux:



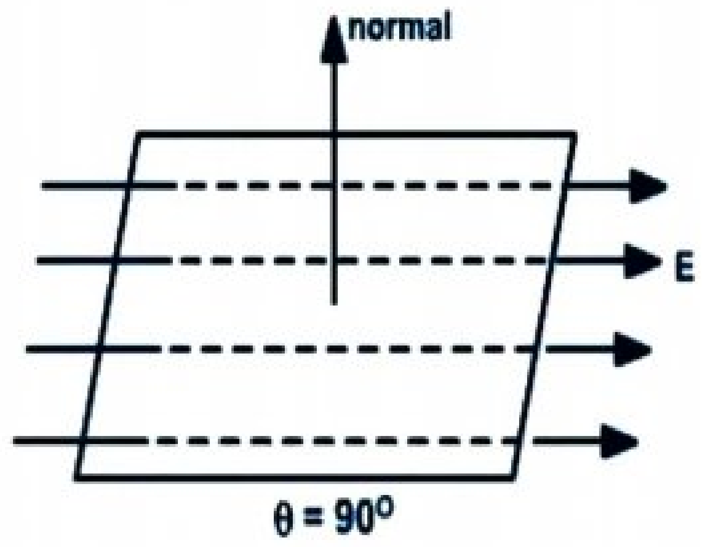
From figure electric intensity and area vector are parallel i.e. $\theta = 0^{o}$

$\boxed{\phi = EAcos\theta}$

$\phi = EA\cos(0^{0})$ $\quad$ [since $\cos(0^{0}) = 1$]

$\boxed{\phi = EA}$

Minimum Flux:



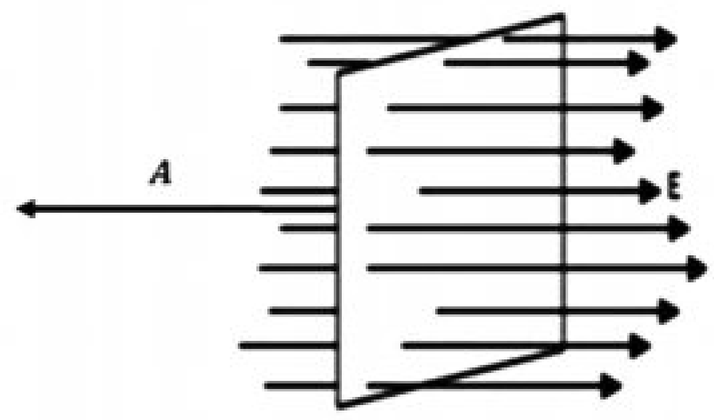
From figure electric intensity and area vector are perpendicular i.e. $\theta = 90^{o}$

$\boxed{\phi = EAcos\theta}$

$\phi = EA\cos(90^{0})$ $\quad$ [since $\cos(90^{0}) = 0$]

$\boxed{\phi = 0}$

Negative Flux:



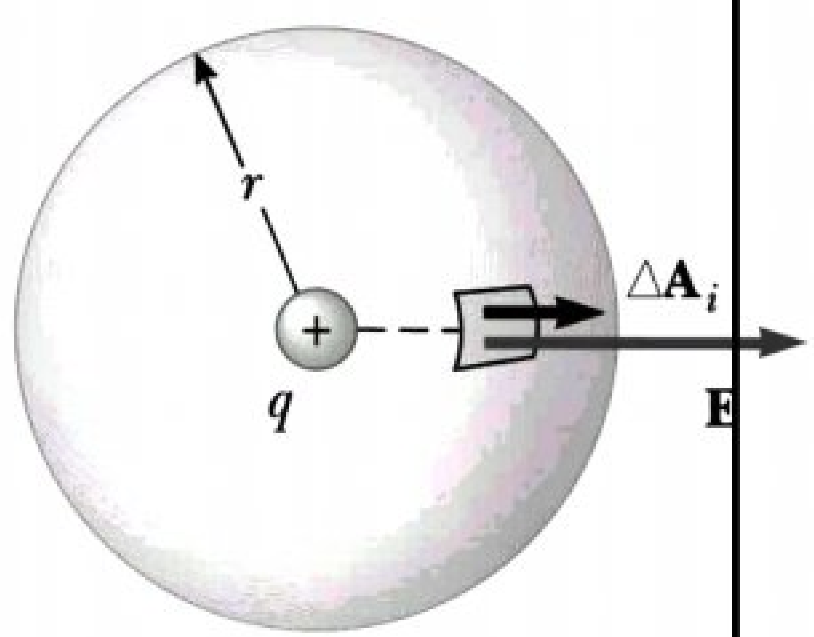
From figure electric intensity and area vector are anti parallel i.e. $\theta = 180^{o}$

$\boxed{\phi = EAcos\theta}$

$\phi = EA\cos(180^{0})$ $\quad$ [since $\cos(180^{0}) = -1$]

$\boxed{\phi = -EA}$

Electric flux due to a point charge in a closed sphere:



Consider an isolated positive point charge q placed at the center of sphere. The electric lines of forces from q will spread uniformly in space around it cutting the surface of an imaginary sphere. To determine the flux due to a point charge we divide the whole sphere into small sections such that area of each section is ΔA.

Now the total flux over the surface of sphere is the summation of individual flux of all sections.

$\phi = \Sigma\, E\Delta A\cos\theta$

As the angle between vector area and electric filed is zero for each section, $\theta = 0^{o}$

$\phi = \Sigma\, EA\cos(0)$

$\phi = \Sigma\, E\Delta A$

Since E is uniform for all sections, therefore

$\phi = E\,\Sigma\Delta A$

Where $\Sigma\Delta A$ = Area of sphere = $4\pi r^{2}$

Therefore

$\phi = E\,(4\pi r^{2})$

We know that the electric field intensity due to a point charge q

$$E = \frac{1}{4\pi\varepsilon_{o}} \times \frac{q}{r^{2}}$$

Substitute this value in above equation, we get

$$\phi = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2}(4\pi r^2)$$

$$\phi = \frac{q}{\varepsilon_0}$$

The electric flux depends on the charge enclosed by the sphere.

Gauss's Law:

Statement:

"The total electric flux in an arbitrary closed surface called Gaussian surface is equal to the total charge enclosed by that surface divided the permittivity of free space.

Mathematically:

Mathematically Gauss's Law can also be defined as

$$\boxed{\phi = \frac{q}{\varepsilon_o}}$$

If there are n positive spherical charge bodies of different enclosed in a Gaussian surface then flux due to each charge body can be written as

$$\phi_1 = \frac{q_1}{\varepsilon_o}$$

$$\phi_2 = \frac{q_2}{\varepsilon_o}$$

$$\phi_n = \frac{q_n}{\varepsilon_o}$$

If total flux diverging out of the surface is ϕ then it may be written as

$$\phi = \sum \phi_n$$

$$\phi = \sum \frac{q_n}{\varepsilon_o}$$

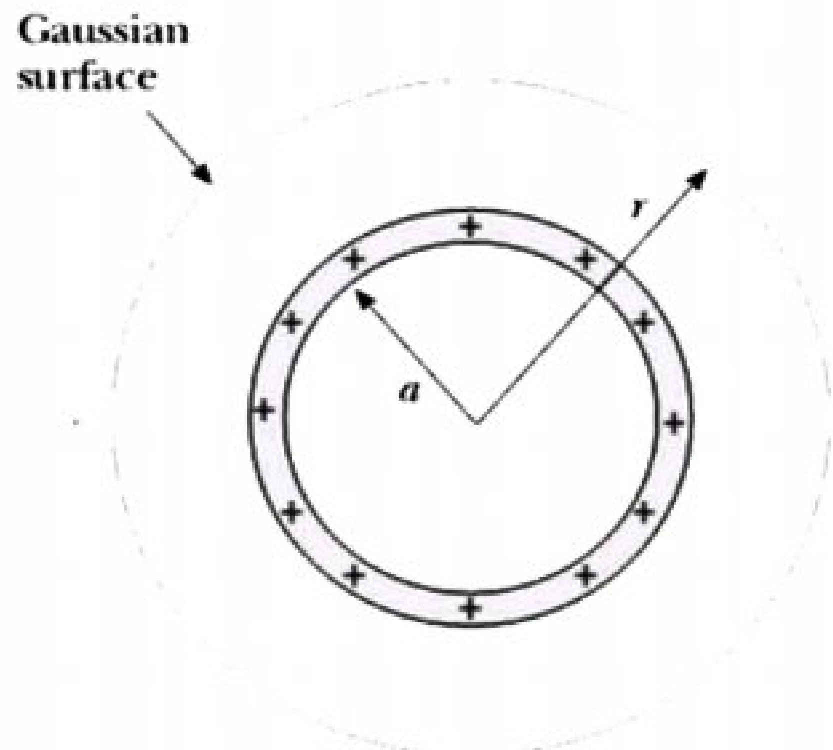
$$\phi = \frac{1}{\varepsilon_o} \sum q_n$$

Hence, total flux diverging out is equals to the $\frac{1}{\varepsilon_o}$ times the total charge enclosed q.

$$\boxed{\boldsymbol{\phi = \frac{q}{\varepsilon_o}}}$$

Application of Gauss's Law:

a) The Electric Field Due to a Thin Spherical charged Shell:



Consider a uniformly charged spherical shell having radius a as shown in figure below. Due to the symmetrical charge distribution the electric field has the same value on all the points of the surface. To calculate the electric field outside charged spherical shell considers a Gaussian's surface or radius r.

Now, Electric flux due to the charges can be written as

$$\phi = EA$$

Since according to Gauss's law

$$\phi = \frac{q}{\varepsilon_o}$$

We have

$$\frac{q}{\varepsilon_o} = EA --(1)$$

Field intensity inside the Shell:

For Field intensity inside the Shell consider Gaussian surface consider a Gaussian surface inside the shell. Since charged inside the shell is zero there fore

Equation (1) will become

$$\frac{0}{\varepsilon_o} = EA$$

$$\boxed{E = 0}$$

Field intensity on the surface of Shell:

For field intensity on the surface Equation (1) will become

$$\frac{q}{\varepsilon_o} = E(4\pi a^2)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_o a^2}}$$

Field intensity outside the Shell:

For Field intensity outside the Shell consider Gaussian surface consider a Gaussian surface outside the shell having radius r such that $r > a$.

Equation (1) will become

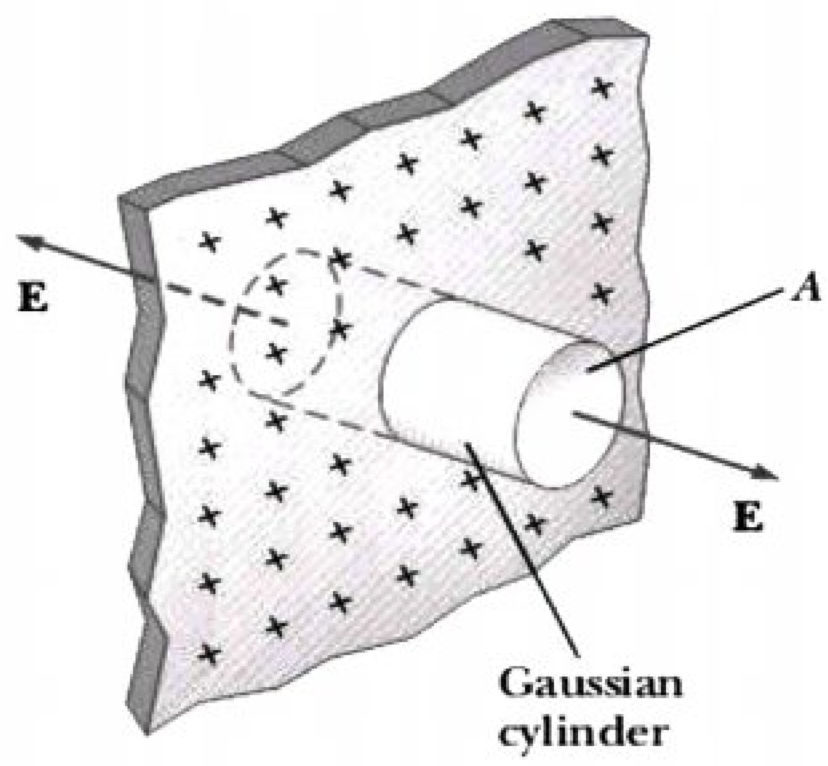
$$\frac{q}{\varepsilon_o} = E(4\pi r^2)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_o r^2}}$$

Hence Electric field intensity decreases with increase in distance from the center of shell.

b) Electric Field Due to a infinite charged Sheet:

Consider an infinite positive charged sheet as shown in figure. Consider a cylindrical Gaussian surface passing through the charged sheet as shown in figure.



Let ϕ_1 = flux at curved surface of cylinder

ϕ_2 = flux at cross-sections of cylinder

Since, there is no electric line of force through the surface area of Gaussian surface there is no electric flux through the surface area

i-e

$$\phi_1 = (0)A_1 \cos\theta$$

$$\boxed{\phi_1 = 0}$$

Electric flux through the two cross section of the Gaussian Surface

$$\phi_2 = EA\cos\theta + EA\cos\theta$$

Since the angle between electric line of forces and vector Area is 0^0

$$\phi_2 = 2EA\cos(0^0)$$

$$\boxed{\phi_2 = 2EA}$$

Total flux ϕ can be written as

$$\phi = \phi_1 + \phi_2$$
$$\phi = 0 + 2EA$$
$$\phi = 2EA$$

According to Gauss law

$$\phi = \frac{q}{\varepsilon_o}$$
$$2EA = \frac{q}{\varepsilon_o}$$
$$\boxed{E = \frac{q}{2A\varepsilon_o}} --- (1)$$

Charge density σ can be defined as

$$\sigma \equiv \frac{q}{A}$$

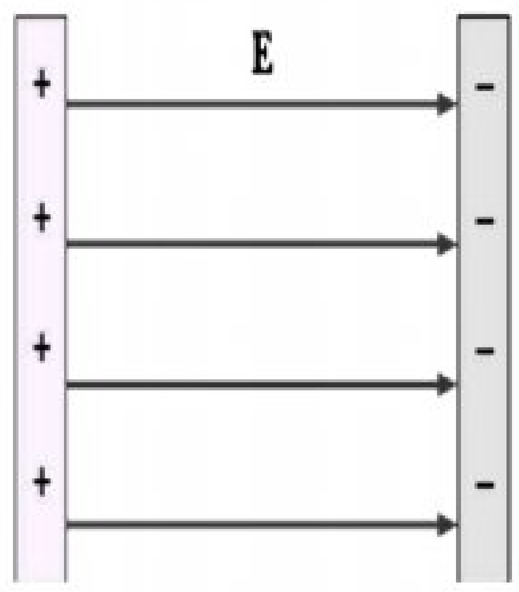
Equation (1) will become

$$\boxed{E = \frac{\sigma}{2\varepsilon_o}}$$

c) Electric Intensity Due To Two Oppositely Charged Sheets:

Consider two metal plates of same area places parallel to each other such that the distance between them is much smaller than their area. If the plates are carrying the equal amount of charges the flux leaving the positive plate is same as the flux entering the negative plate.

Electric field intensity for both plates will be same as for infinitely charged sheet i.e. $\frac{\sigma}{2\varepsilon_o}$



Net Electric field intensity at any point between the plates will be

$$E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_o}}$$

Electric Potential Energy:

The work done against the electric field is stored as energy known as electric potential energy.

Electric Potential Difference:

The work done in moving a test charge against the electric field divided by the magnitude of test charge is called electric potential difference.

Mathematical Explanation:

Consider test charge q_o placed at a point in an electric field. If the Work done in carrying the that test charge to that point is W then electric potential V can be written as

$$V = \frac{W}{q_o}$$

Unit:

The S.I unit of Electric Potential is Volts (V).

Relation between Electric Field and potential Difference:

Consider a positive test q_0 charge moved from point a to point b against the electric field. Such that distance from a to b is ΔS. Suppose that the field has the uniform value over distance ΔS.

The work done ΔW against the electric file in moving that test charge

$$\Delta W = F\Delta S cos\theta \_\_\_\_\_ (1)$$

Since electric force experienced by test charge

$$F = q_0 E$$

Therefore equation (1) becomes

$$\Delta W = q_0 E \Delta S cos\theta$$
$$\Delta W = q_0 E \Delta S cos 180^0$$
$$\Delta W = q_0 E \Delta S(-1)$$
$$\Delta W = -q_0 E \Delta S$$

Since

$$\Delta V = \frac{\Delta W}{q_o}$$
$$\Delta V = \frac{-q_0 E \Delta S}{q_o}$$
$$\Delta V = -E\Delta S$$
$$\boxed{E = -\frac{\Delta V}{\Delta S}}$$

The quantity $\Delta V/\Delta S$ is called the potential gradient.

Electron-Volt:

Electron volt is a unit of energy and it can be defined as "The work done by electron charge against the electric field such that 1volt of potential difference is obtained."

$$\boxed{1eV = 1.6 \times 10^{-19} J}$$

Capacitor:

System of two conductors separated by an insulating medium such as air or any other forms a capacitor.

Capacitor is an electrical device that can store electrical charge. The distance between the conductors should be very small as com pare to their size.

Mathematical Explanation:

If V is the potential difference between the plates of capacitor and q is the charge stored on each plate then it is found that

$$q \propto V$$
$$q = CV$$

Here, C is called the capacitance of capacitor.

Capacitance:

The capacity of capacitor to store electric charges is called capacitance.

Unit:

The unit of capacitance is Farad$(\boldsymbol{F})$

Farad is very large unit so for practical purpose convenient units are

Micro farad $\mu F = 10^{-6}\, F$

Pico farad $pF = 10^{-12} F$

Parallel Plate capacitor:

When two oppositely charged parallel plates are separated by a small distance such that the distance between the plates is very small as compare to the facing are of plates then this kind of arrangement is called capacitor.

Derivation for Capacitance:

Consider to oppositely charge metal plates having charge $\boldsymbol{q}$ placed at the distance $\boldsymbol{d}$ from each other.

If the medium between the plates is air the electric field intensity between the plates can be written as

$$E = \frac{\sigma}{\varepsilon_o}$$

Now since

$$V = E\Delta S = Ed$$

$$V = \frac{\sigma d}{\varepsilon_o}$$

Now expression for capacitance $\boldsymbol{C_o}$ can be written as

$$C_o = \frac{q}{V} \_\_\_\_\_\_ (1)$$

Since

$$\sigma = \frac{q}{A}$$

$$q = \sigma A$$

Putting values of q and V in equation (1), we get

$$\boxed{C_o = \frac{A\varepsilon_o}{d}}$$

From above expression it is clear that the capacitance of a capacitor is directly proportional to the area of the plates and inversely proportional to the distance between the plates.

If the medium between the plates is other than air the capacitance of the capacitor will be

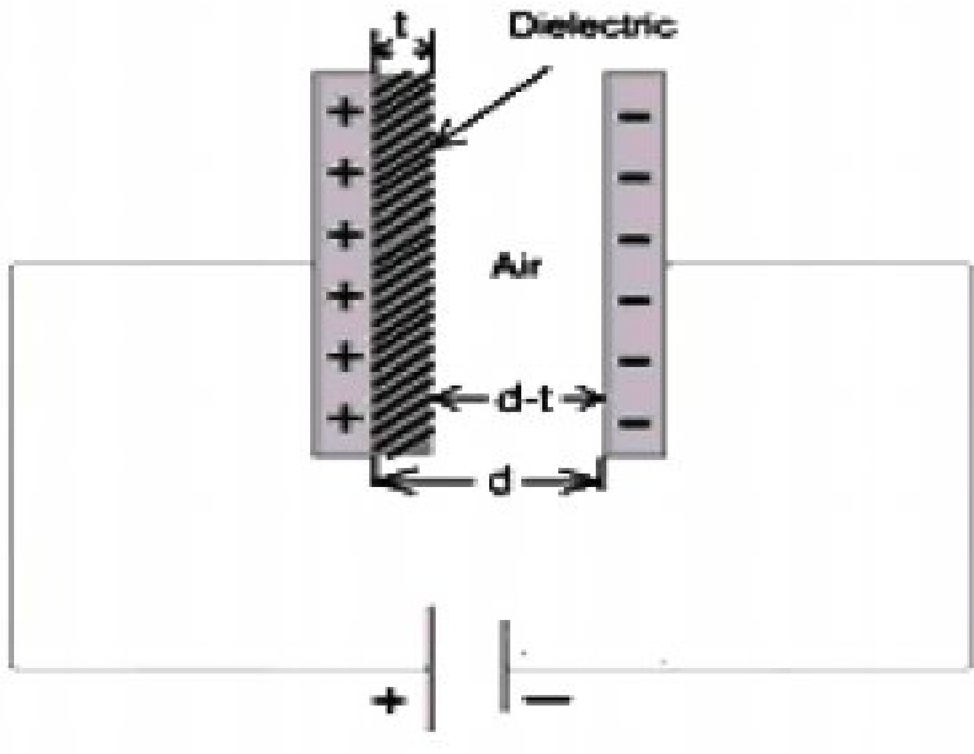
$$C = \frac{A\epsilon}{d} = \frac{A\varepsilon_o\varepsilon_r}{d}$$

$\boxed{C = C_o\varepsilon_r}$

Hence the presence of a dielectric medium increases the capacitance.

Compound Capacitor:

In parallel plate capacitor if the space b/w the plates is partially filled with the dielectric medium then it is called compound capacitor.



Derivation:

Consider a compound capacitor partially filled with dielectric medium and partially filled with air.

Let

d = Distance between the plates of capacitor

t = Thickness of dielectric medium

$d - t$ = Thickness of air

ε_r = Relative permittivity of the medium

V = Potential difference across capacitor

Electric intensity when air is present

$$E_1 = \frac{\sigma}{\varepsilon_o}$$

Electric intensity when dielectric is present

$$E_2 = \frac{\sigma}{\varepsilon_o \varepsilon_r}$$

The potential difference between plates is

$$V = V1 + V2$$

Since $V = E\Delta S$ or $V = Ed$

$$V = E_1(d - t) + E_2 t$$

$$V = \frac{\sigma(d-t)}{\varepsilon_o} + \frac{\sigma}{\varepsilon_o \varepsilon_r} t$$

$$V = \frac{\sigma}{\varepsilon_o}\left\{(d - t) + \frac{t}{\varepsilon_r}\right\}$$

$$V = \frac{\sigma}{\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

$$V = \frac{\sigma}{\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

Since charge density $\sigma = \frac{q}{A}$

$$V = \frac{q}{A\epsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

Since $q = CV$

$$V = \frac{CV}{A\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

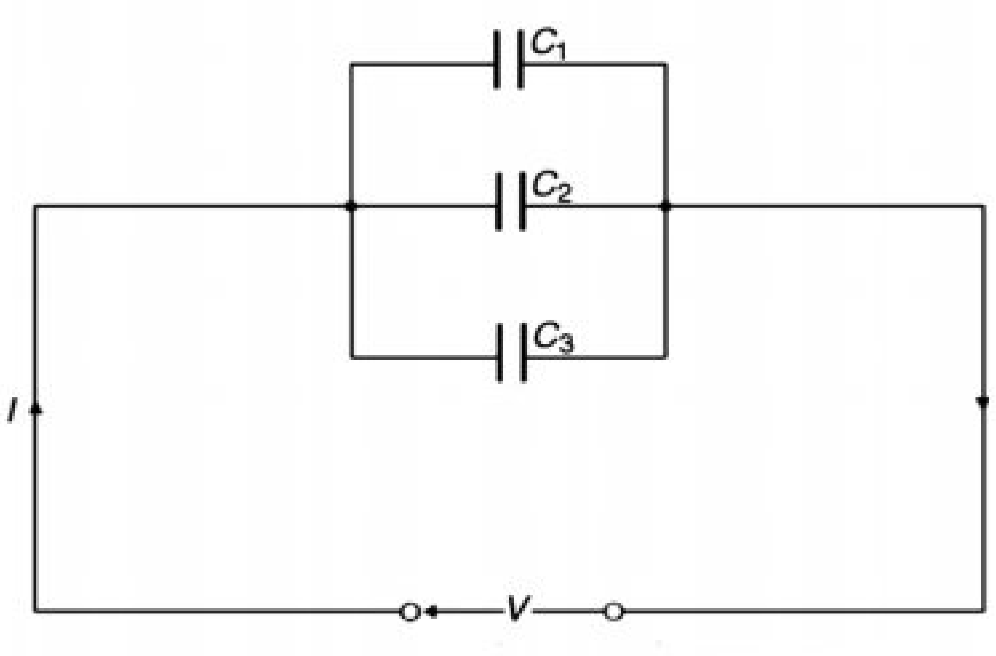
$$1 = \frac{C}{A\varepsilon_o}\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}$$

$$\boldsymbol{C = \frac{A\varepsilon_o}{\left\{d + t\left(\frac{1}{\varepsilon_r} - 1\right)\right\}}}$$

Combination of Capacitors:

Capacitors connected in parallel:

When capacitors are connected in a way such that the plates of capacitors are connected to common terminals of battery then this type of arrangement is called parallel combination.



Derivation for equivalent capacitance:

Consider three capacitors C_1, C_2, C_3 connected in parallel. A charge q given to a point divide itself resides on the plates of individual capacitor as q_1, q_2, q_3 respectively.

Since

$q = CV$

Hence, the total charge stored in parallel combination is equal to the sum of charge stored across each capacitor in combination.

$q = q_1 + q_2 + q_3$ ______(1)

$Since\ q = CV$

$Therefore, q_1 = C_1V_1, q_2 = C_2V_2, q_3 = C_3V_3$

$substituting\ all\ values\ in\ equation\ (1), we\ get$

$C_eV = C_1V_1 + C_2V_2 + C_3V_3$

$Since\ \ V = V_1 = V_2 = V_3$

$C_eV = C_1V + C_2V + C_3V$

$$C_eV = (C_1 + C_2 + C_3)V$$

$$C_e = C_1 + C_2 + C_3$$

Conclusion:

1. Equivalent capacitance is equal to the algebraic sum of the individual capacitances.
2. Equivalent capacitance is greater than the larger individual capacitance.
3. For parallel combination of $\boldsymbol{n}$ capacitors above expression will become.

$$\boldsymbol{C_e = C_1 + C_2 + C_3 + \cdots + C_n}$$

Capacitors connected in Series:

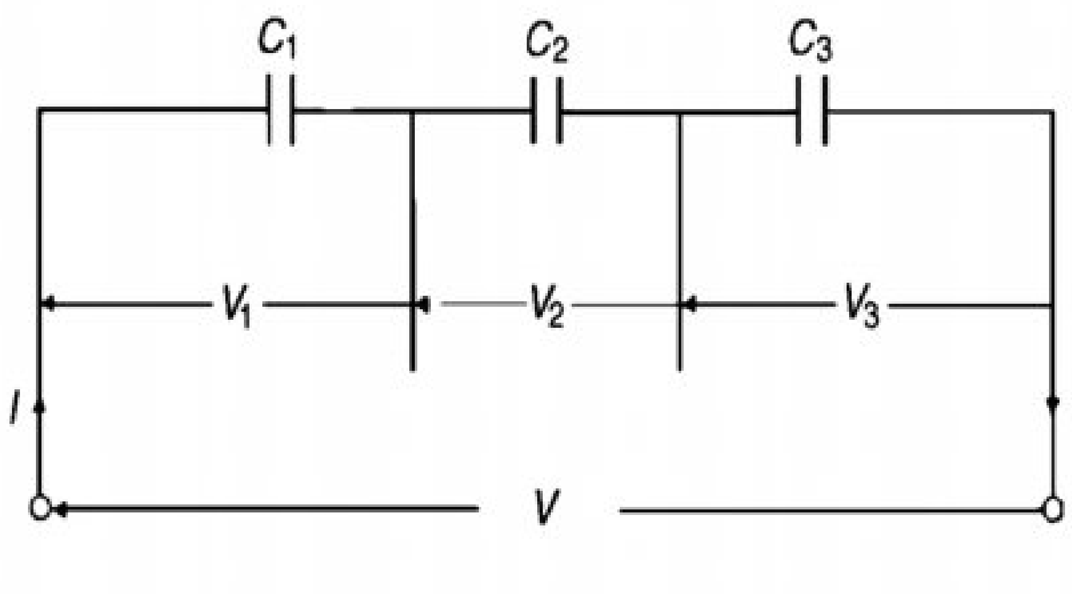
When capacitors are connected end to end then this type of arrangement is called series combinations of capacitors.

Derivation for equivalent capacitance:

Consider three capacitors C_1, C_2, C_3 are connected in series. If potential difference V is applied across combination then total voltage will divide in voltages across individual capacitors. Hence, Sum of individual voltage across each capacitor is equal to the Voltage of source.

i-e

$$V = V_1 + V_2 + V_3 \text{\_\_\_\_\_\_}(1)$$



Since

$$\frac{q}{C} = V$$

Therefore

$$\frac{q_1}{C_1} = V_1, \frac{q_2}{C_2} = V_2, \frac{q_3}{C_3} = V_3$$

Substituting all the values in equation (1), we get

$$\frac{q}{C_e} = \frac{q_1}{C_1} + \frac{q_2}{C_1} + \frac{q_3}{C_3}$$

In series combination charge is same on each capacitor, i.e.

$$q = q_1 = q_2 = q_3$$

Above expression will become

$$\frac{q}{C_e} = \frac{q}{C_1} + \frac{q}{C_1} + \frac{q}{C_3}$$

$$\frac{q}{C_e} = q\left(\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_3}$$

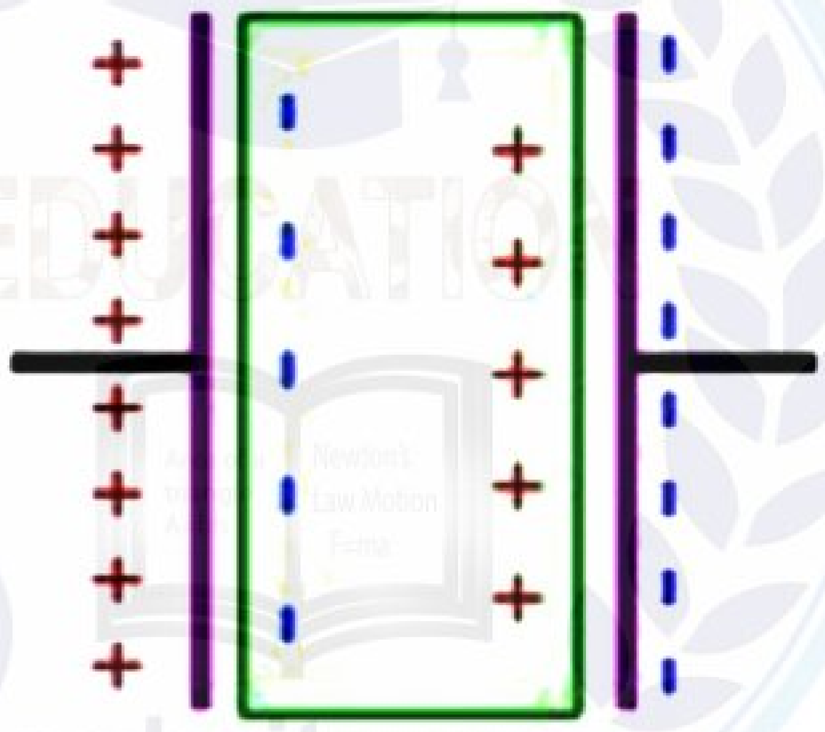
Conclusion:

1. The reciprocal of equivalent capacitance is equal to the algebraic sum of the reciprocals of individual capacitances in the network.
2. Equivalent capacitance is less than the smallest individual capacitance.
3. For series combination of n capacitors above expression will become.

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

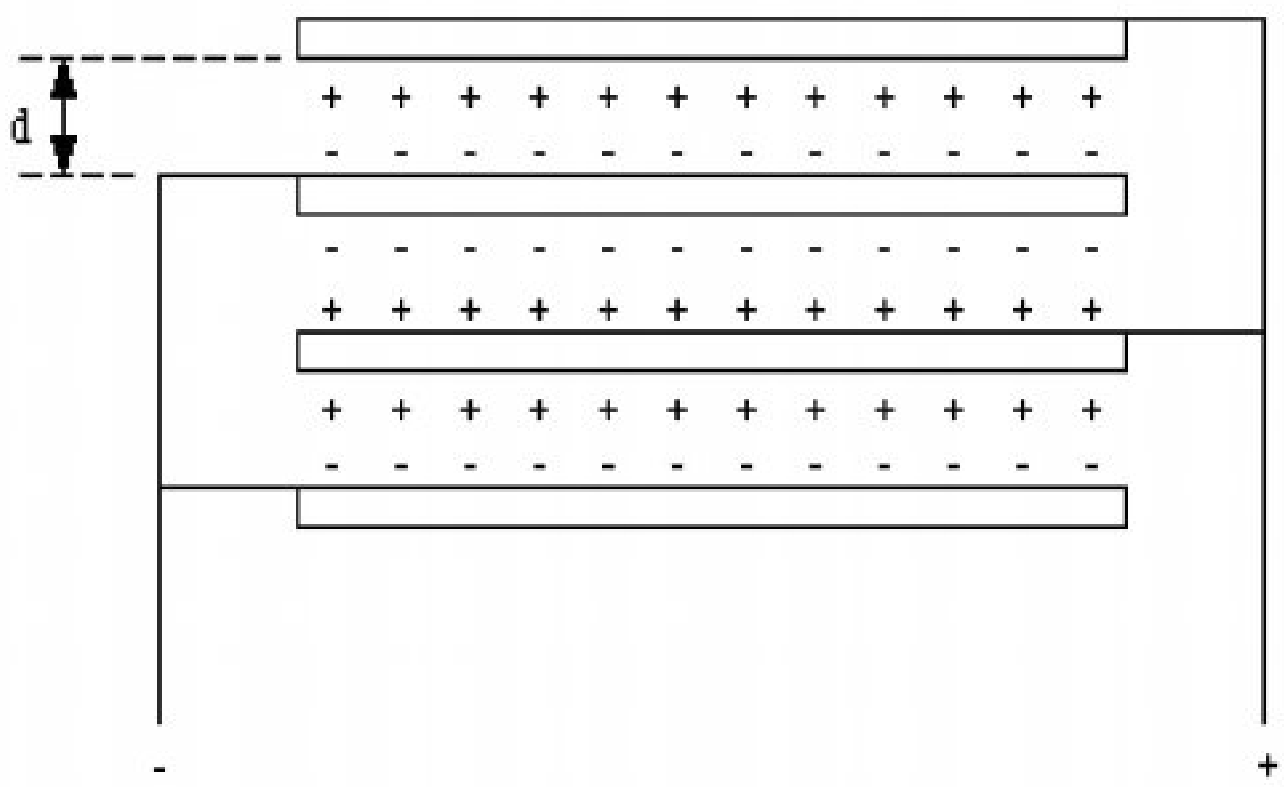
Dielectric effect in Capacitor:

In capacitor, the dielectric medium between the plates increases the capacitance. The potential difference b/w the plates distorts the molecules of the dielectric medium and hence polarized these molecules, one end becomes positive and another end becomes negative. The potential difference b/w the plates decreases and hence the capacitance of the capacitor increases.



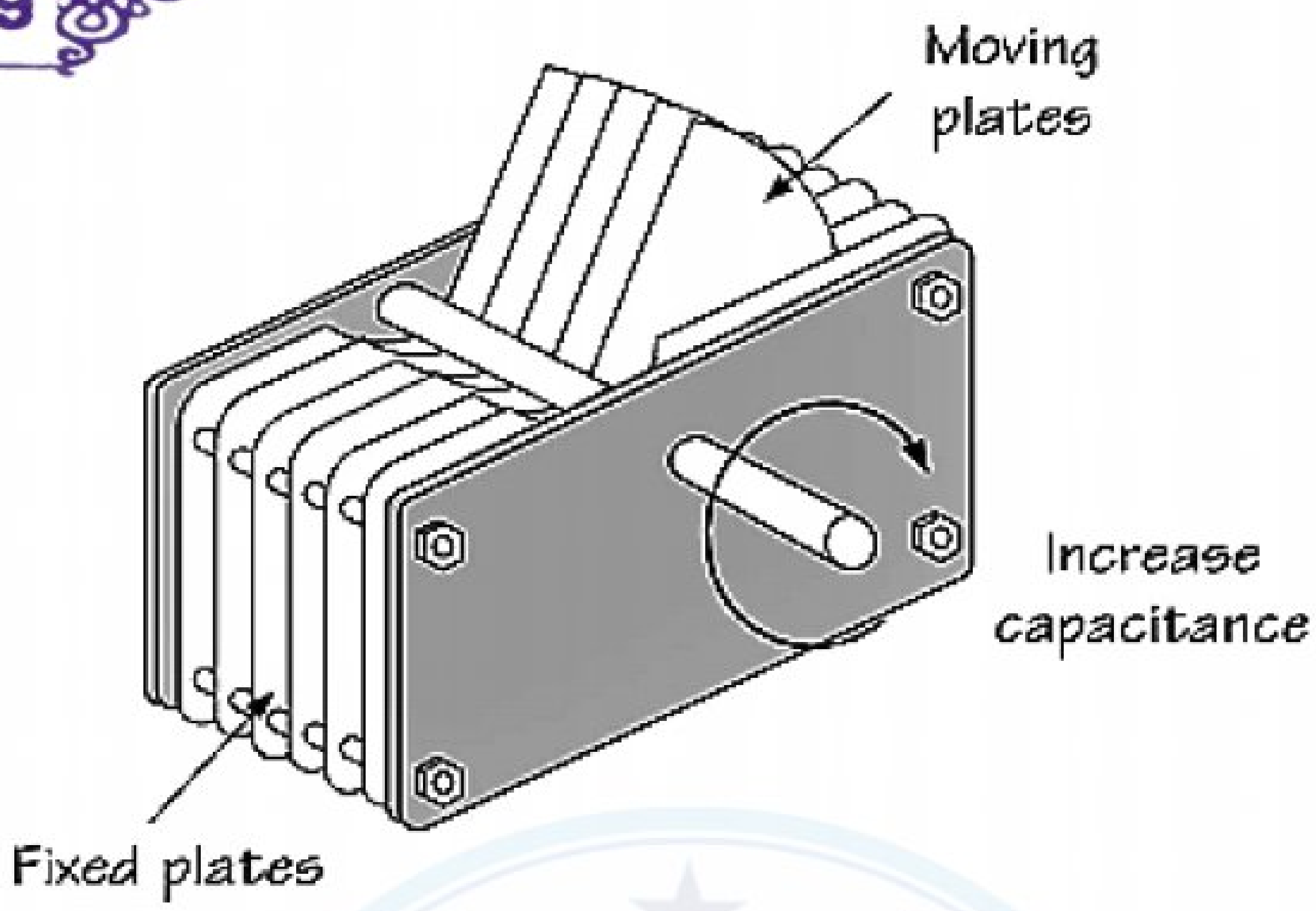
Multi plate Capacitor:

A multi plate capacitor is designed to have large capacitance. It consists of a large number of plates each of large area. If there are N plates then we have N-1 capacitors in parallel.



Variable Capacitor

Variable capacitor is a kind of capacitor used for tuning radio receiver. It consists of two sets of semi-circular plates separated by air. These plates are usually made by aluminum or brass. In order to change the capacitance, one set of plates is fixed and other is rotated by a knob to change the effective area of the plates. Thus by changing the area of the plates we can change the capacitance and hence tuning frequency of the radio receiver.



Electrolyte Capacitor:

All electrolytic capacitors are polarized capacitors whose anode (+) is made of a particular metal on which an insulating oxide layer forms by anodization, acting as the dielectric of the electrolytic capacitor. A non-solid or solid electrolyte which covers the surface of the oxide layer in principle serves as the second electrode (cathode) (-) of the capacitor.