Math Sci 9: Test		Total No. 40	

Name:_____ **Roll No. :**_____

Date:_____-_____-20 **Teacher's Signature:** _____

Q.1: Tick (✓) the correct answer. سوال نمبر 1۔ درست جواب پر (✓) کا نشان لگائیں۔

Antilogarithm table was prepared by ____:
1۔ ضدلوگارتھم کی جدول_____نے تیار کی:
(A) John Napier / جان نیپئر **(B)** Henry Briggs / ہنری برگز **(C)** Jobst Burgi / جابسٹ برگی **(D)** Aurther Calay / آرتھر کیلے

Scientific notation of 0.0074 **is:**
2۔ 0.0074 کی سائنسی ترقیم ہوتی ہے:
(A) 7.4×10^{-3} **(B)** 7.4×10^{-1} **(C)** 7.4×10^{-2} **(D)** 7.4×10^{-4}

$\log_a a$ **is equal to:**
3۔ $\log_a a$ کے برابر ہے:
(A) 0 **(B)** -1 **(C)** 1 **(D)** 10

The logarithm of unity to any base is:
4۔ کسی اساس پر '1' کا لوگارتھم____کے برابر ہوتا ہے:
(A) 1 **(B)** 10 **(C)** e **(D)** 0

In $\log_x 64 = 2$**, the value of** x **is:**
5۔ $\log_x 64 = 2$ میں x کی قیمت ہے:
(A) 64 **(B)** 2 **(C)** 64^2 **(D)** 8

If $a^x = n$**, then _____:**
6۔ اگر $a^x = n$ ہو تو _____:
(A) $a = \log_x n$ **(B)** $x = \log_n a$ **(C)** $x = \log_a n$ **(D)** $a = \log_n x$

$\log e =$ _____. *where* $(e \approx 2.718)$
7۔ $(e \approx 2.718).$ _____ $= \log e$
(A) 0 **(B)** 0.4343 **(C)** ∞ **(D)** 1

$\log m^n$ **can be written as _____:**
8۔ $\log m^n$ کو_____ بھی لکھا جاسکتا ہے:
(A) $(\log m)^n$ **(B)** $m\log n$ **(C)** $n\log m$ **(D)** $\log(mn)$

$\log p - \log q$ **is same as:**
9۔ _____ $= \log p - \log q$
(A) $\log(q/p)$ **(B)** $\log(p-q)$ **(C)** $\log p / \log q$ **(D)** $\log(p/q)$

Scientific notation of 0.0643 **is _____:**
10۔ 0.0643 کی سائنسی ترمیم_____ہے:
(A) 6.43×10^{-2} **(B)** 64.3×10^{-2} **(C)** 64.3×10^{-4} **(D)** 6.43×10^{-4}

$10\times2=20$ **Write short answers to any ten (10) questions.** سوال نمبر 2۔ کوئی سے 10 سوالات کے جوابات تحریر کیجیے۔

i۔ Define scientific notation. سائنسی ترقیم کی تعریف کیجیے۔
ii۔ Write each of the following in scientific notation: 30600 , 0.000058 درج ذیل اعداد کو سائنسی ترقیم میں لکھیے:
iii۔ Write in scientific notation: $83,000$ سائنسی ترقیم میں لکھیے۔
iv۔ Differentiate between common and natural logarithm? عام لوگارتھم اور قدرتی لوگارتھم میں کیا فرق ہے؟
v۔ Why Henry Briggs is famous? ہنری برگز کس لیے مشہور ہے؟
vi۔ Evaluate $\log 512$ *to the base* $2\sqrt{2}$. $\log 512$ کی اساس $2\sqrt{2}$ سے قیمت معلوم کریں۔
vii۔ Find the value of x: $\log_3 x = 4$ x کی قیمت معلوم کریں:
viii۔ Write the laws of logarithm. لوگارتھم کے قوانین لکھیں۔
ix۔ Evaluate 0.2913×0.004236. لوگارتھم کی مدد سے 0.2913×0.004236 کی قیمت معلوم کریں۔
x۔ Write in the form of single logarithm: $2\log x - 3\log y$ واحد لوگارتھم کی شکل میں لکھیے:
xi۔ Calculate: 0.8176×13.64 قیمت معلوم کریں:
xii۔ Simplify: $\sqrt[3]{25.47}$ مختصر کریں۔

$1\times10=10$ **Write answer to any One (1) question.** نوٹ: کوئی سے ایک سوال کا جواب لکھیے۔

Find the value of x in $\log_4 256 = x$. سوال نمبر 3۔ $\log_4 256 = x$ میں x کی قیمت معلوم کریں

Use log table to find the value of: $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$ سوال نمبر 4۔ لوگارتھم جدول کی مدد سے قیمت معلوم کریں:

Exercise 3.1

Q.1 Express each of the following numbers in scientific notations.

(i) 5700

$= 5.7 \times 10^3$ **Ans**

(ii) 49,800,000

$= 4.98 \times 10^7$ **Ans**

(iii) 96000000

$= 9.6 \times 10^7$ **Ans**

(iv) 416.9

$= 4.169 \times 10^2$ **Ans**

(v) 83000

$= 8.3 \times 10^4$ **Ans**

(vi) 0.00643

$= 6.43 \times 10^{-3}$ **Ans**

(vii) 0.0074

$= 7.4 \times 10^{-3}$ **Ans**

(viii) 60,000,000

$= 6 \times 10^7$ Ans

(ix) 0.00000000395

$= 3.95 \times 10^{-9}$ **Ans**

(x) $\dfrac{275000}{0.0025}$

$= \dfrac{2.75 \times 10^5}{2.5 \times 10^{-3}}$ **Ans**

Q.2 Express the following number in ordinary notation.

(i) 6×10^{-4}

$= 0.0006$ **Ans**

(ii) 5.06×10^{10}

$= 50600000000$ **Ans**

(iii) 9.018×10^{-6}

$= 0.000009018$ Ans

(iv) 7.865×10^8

$= 786500000$ **Ans**

Exercise 3.2

Q.1 Find the common logarithms of each of the following numbers.

(i) 232.92

Solution: 232.92

Suppose $x = 232.92$

Taking log

$\log x = \log 232.92$

$Ch = 2$

Mantissa = 0.3672

$\log x = 2.3672$ **Ans**

(ii) 29.326

Solution: 29.326

Suppose $x = 29.326$

Taking log

$\log x = \log 29.326$

$Ch = 1$

Mantissa = 0.4672

$\log x = 1.4672$ **Ans**

(iii) 0.00032

Solution: 0.00032

Suppose $x = 0.00032$

Taking log

$\log x = \log 0.00032$

$Ch = \bar{4}$

Mantissa = 0.5051

$\log x = \bar{4}.5051$ **Ans**

(iv) 0.3206

Solution: 0.3206

Suppose $x = 0.3206$

Taking log:

$\log x = \log 0.3206$

$Ch = \bar{1}$

Mantissa = 0.5059

$\log x = \bar{1}.5059$ **Ans**

Q.2 If log 31.09 = 1.4926, find the value of the following.

If

$\log 31.09 = 1.4926$

Then

(i) $\log 3.109 = 0.4926$

(ii) $\log 310.9 = 2.4926$

(iii) $\log 0.003109 = \bar{3}.4926$

(iv) $0.3109 = \bar{1}.4926$

Solution:

(i) $\log 3.109$

Characteristics = 0

Mantissa = 0.4926

$\log 3.109 = 0.4926$ **Ans**

(ii) $\log 310.9$

Characteristics = 2

Mantissa =0.4926

$\log 310.9 = 2.4926$ **Ans**

(iii) $\log 0.003109$

Characteristics = $\bar{3}$

Mantissa = 0.4926

$\log 0.003109 = \bar{3}.4926$ **Ans**

(iv) $\log 0.3109$

Characteristics = $\bar{1}$

Mantissa = 0.4926

$\log 0.3109 = \bar{1}.4926$ **Ans**

Q.3 Find the numbers whose common logarithms are

(i) 3.5621

Solution:

$\log x = 3.5621$

Ch=3(If ch is positive, then plus for reference point)

Mantissa = 0.5621

$x = \text{antilog } 3.5621$

$x = 3649.0$ **Ans**

(ii) $\bar{1}.7427$

Solution:

$\log x = \bar{1}.7427$

$Ch = \bar{1}$

Mantissa $= 0.7427$

$x = \text{anti}\log \bar{1}.7427$

$x = 0.5530$ **Ans**

Q.4 What replacement for the unknown in each of the following will make the true statements?

(i) $\log_3 81 = L$

Solution: $\log_3 81 = L$

Writing in exponential form.

$3^L = 81$

$3^L = 3^4$

$\because$ Bases are equal so

$L = 4$ **Ans**

(ii) $\log_a 6 = 0.5$

Solution: $\log_a 6 = 0.5$

$a^{0.5} = 6$

$a^{\frac{1}{2}} = 6$

$\sqrt{a} = 6$ Taking square on both sides

$\sqrt{(a)}^2 = (6)^2$

$a = 36$ **Ans**

(iii) $\log_5 n = 2$

Write in exponential form

$5^2 = n$

$25 = n$

Or $n = 25$ **Ans**

(iv) $10^P = 40$

Solution: $10^P = 40$

Changing into logarithmic form

$P = \log_{10} 40$

$= \log 40$

$= 1.6021$ **Ans**

Q.5 Evaluate.

(i) $\log_2 \frac{1}{128}$

Solution: $\log_2 \frac{1}{128}$

Suppose $\log_2 \frac{1}{128} = x$

Writing in exponential form.

$2^x = \frac{1}{128}$

$2^x = \frac{1}{2^7}$

$2^x = 2^{-7}$

$\because$ Bases are equal so

$x = -7$ **Ans**

(ii) log 512 to the base $2\sqrt{2}$

Solution: $\log_{2\sqrt{2}} 512 = x$

Writing in exponential form

$\left(2\sqrt{2}\right)^x = 512$

$\left(2^1 . 2^{\frac{1}{2}}\right)^x = 2^9$

$\left(2^{\frac{3}{2}}\right)^x = 2^9$

$2^{\frac{3}{2}x} = 2^9$

$\because$ Bases are equal so

$\frac{3}{2}x = 9$

$x = \frac{9 \times 2}{3}$

$x = \frac{\cancel{18}^{6}}{\cancel{3}}$

$x = 6$ **Ans**

Q.6 Find the value of x from the following statements.

(i) $\log_2 x = 5$

Solution: $\log_2 x = 5$

Write in exponential form.

$2^5 = x$

$32 = x$ **Ans**

(ii) $\log_{81} 9 = x$

Solution: $\log_{81} 9 = x$

Writing in the exponential form.

$81^x = 9$

$\left(9^2\right)^x = 9$

$9^{2x} = 9$

$2x = 1$

$x = \frac{1}{2}$ **Ans**

(iii) $\log_{64} 8 = \frac{x}{2}$

Solution: $\log_{64} 8 = \frac{x}{2}$

Writing in exponential form.

$64^{\frac{x}{2}} = 8$

$\left(8^2\right)^{\frac{x}{2}} = 8$

$8^x = 8$

$x = 1$ **Ans**

(iv) $\log_x 64 = 2$

Solution: $\log_x 64 = 2$

Writing in exponential form

$x^2 = 64$

$x^2 = 8^2$

$x = 8$ **Ans**

(v) $\log_3 x = 4$

Solution: $\log_3 x = 4$

$3^4 = x$

$81 = x$

Or $x = 81$ **Ans**

Exercise 3.3

Q.1 Write the following into sum or difference $\log(A \times B)$

(i) $\log(A \times B)$

Solution: $\log(A \times B)$

$$\log A \times B = \log A + \log B \quad \textbf{Ans}$$

(ii) $\log \frac{15.2}{30.5}$

Solution: $\log \frac{15.2}{30.5}$

$$\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5 \quad \textbf{Ans}$$

(iii) $\log \frac{21 \times 5}{8}$

Solution: $\log \frac{21 \times 5}{8}$

$$\log \frac{21 \times 5}{8} = \log(21 \times 5) - \log 8$$

$$= \log 21 + \log 5 - \log 8 \quad \textbf{Ans}$$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

Solution: $\log \sqrt[3]{\frac{7}{15}}$

$$\log \sqrt[3]{\frac{7}{15}} = \log\left(\frac{7}{15}\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\log\left(\frac{7}{15}\right)$$

$$= \frac{1}{3}(\log 7 - \log 15)$$

$$= \frac{1}{3}\log 7 - \frac{1}{3}\log 15 \quad \textbf{Ans}$$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Solution: $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

$$\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log 22^{\frac{1}{3}} - \log 5^3$$

$$= \frac{1}{3}\log 22 - 3\log 5 \quad \textbf{Ans}$$

(vi) $\log \frac{25 \times 97}{29}$

Solution: $\log \frac{25 \times 97}{29}$

$$\log \frac{25 \times 47}{29} = \log(25 \times 47) - \log 29$$

$$= \log 25 + \log 47 - \log 29 \quad \textbf{Ans}$$

Q.2 Express

$\log x - 2\log x + 3\log(x+1) - \log(x^2 - 1)$ **as a single logarithm.**

Solution:

$$\log x - 2\log x + 3\log(x+1) - \log(x^2 - 1)$$

$$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2 - 1)$$

$$= \log\left(\frac{x}{x^2}\right) + \log\frac{(x+1)^3}{x^2 - 1}$$

$$= \log\left(\frac{x}{x^2} \times \frac{(x+1)^3}{x^2 - 1}\right)$$

$$= \log\left(\frac{x(x+1)^3}{x^2(x^2 - 1)}\right)$$

$$= \log \frac{x(x+1)^2 \cancel{(x+1)}}{x \times x(x-1)\cancel{(x+1)}}$$

$$= \log \frac{(x+1)^2}{x(x-1)} \quad \textbf{Ans}$$

Q.3 Write the following in the form of a single logarithm.

(i) $\log 21+\log 5$

Solution: $\log 21+\log 5$

$=\log(21\times 5)$ **Ans**

(ii) $\log 25-2\log 3$

Solution: $\log 25-2\log 3$

$=\log 25-2\log 3$

$=\log 25-\log 3^2$

$=\log\frac{25}{3^2}$ **Ans**

(iii) $2\log x-3\log y$

Solution: $2\log x-3\log y$

$=2\log x-3\log y$

$=\log x^2-\log y^3$

$=\log\frac{x^2}{y^3}$ **Ans**

(iv) $\log 5+\log 6-\log 2$

Solution: $\log 5+\log 6-\log 2$

$=\log 5+\log 6-\log 2$

$=\log(5\times 6)-\log 2$

$=\log\frac{5\times 6}{2}$ **Ans**

Q.4 Calculate the following.

(i) $\log_3 2\times\log_2 81$

Solution: $\log_3 2\times\log_2 81$

$=\frac{\cancel{\log 2}}{\log 3}\times\frac{\log 81}{\cancel{\log 2}}$

$=\frac{\log 81}{\log 3}$

$=\frac{\log 3^4}{\log 3}$

$=\frac{4\cancel{\log 3}}{\cancel{\log 3}}$

$=4$ **Ans**

(ii) $\log_3\times\log_3 25$

Solution: $\log_3\times\log_3 25$

$=\frac{\cancel{\log 3}}{\log 5}\times\frac{\log 25}{\cancel{\log 3}}$

$=\frac{\log 25}{\log 5}$

$=\frac{\log 5^2}{\log 5}$

$=\frac{2\cancel{\log 5}}{\cancel{\log 5}}$

$=2$ **Ans**

Q.5 If $\log 2=0.3010, \log 3=0.4771$ **and** $\log 5=0.6990,$ **then find the values of the following.**

(i) $\log 32$

$=\log 2^5$

$\because$ using 3^{rd} law of logarithm

$=5\log 2$

By putting the value of $\log 2$

$=5(0.3010)$

$=1.5050$ **Ans**

(ii) $\log 24$

Solution: log 24

$=\log(2^3\times 3)$

$=\log 2^3+\log 3$

$=3\log 2+\log 3$

By putting the value of $\log 2$ and $\log 3$

$=3(0.3010)+0.4771$

$=0.9030+0.4771$

$=1.3801$ **Ans**

(iii) $\log\sqrt{3\frac{1}{3}}$

Solution: $\log\sqrt{3\frac{1}{3}}$

$=\log\left(\frac{10}{3}\right)^{\frac{1}{2}}$

$$= \frac{1}{2}\log\left[\frac{2\times 5}{3}\right]$$

$$= \frac{1}{2}(\log 2 + \log 5 - \log 3)$$

By putting the values of $\log 2, \log 3$ and $\log 5$

$$= \frac{1}{2}(0.3010 + 0.69900 - 0.4771)$$

$$= \frac{1}{2}(1 - 0.4771)$$

$$= \frac{1}{2}(0.5229)$$

$$= 0.26145 \quad \textbf{Ans}$$

(iv) $\log \frac{8}{3}$

Solution: $\log \frac{8}{3}$

$$= \log \frac{2^3}{3}$$

$$= \log 2^3 - \log 3$$

$$= 3\log 2 - \log 3$$

By putting the values of $\log 2$ and $\log 3$

$$= 3(0.3010) - 0.4771$$

$$= 0.9030 - 0.4771$$

$$= 0.4259 \quad \textbf{Ans}$$

(v) $\log 30$

Solution: $\log 30$

$$= \log(5\times 2\times 3)$$

$\because$ using first law of logarithm

$$= \log 5 + \log 2 + \log 3$$

By putting the values of $\log 2, \log 3$ $\log 5$

$$= (0.6990) + (0.3010) + (0.4771)$$

$$= 1.4771 \quad \textbf{Ans}$$

Exercise 3.4

Q.1 Use log tables to find the value of

(i) 0.8176×13.64

Solution: 0.8176×13.64

Suppose

$x=0.8176\times13.64$

Taking log on both sides

$\log x=\log(0.8176\times13.64)$

According to first law of logarithm

$\log x=\log 0.8176+\log 13.64$

$=\bar{1}.9125+\underline{1}.1348$

$\log x=-1+0.9125+1.1348$

$\log x=1.0473$

To find antilog

$x=\text{antilog}\,1.0473$

Ch $=1$

$x=1.115$

Reference point

$x=11.15$ **Ans**

(ii) $(789.5)^{\frac{1}{8}}$

Solution: $(789.5)^{\frac{1}{8}}$

Let $x=(789.5)^{\frac{1}{8}}$

Taking log on both sides

$\log x=\log(789.5)^{\frac{1}{8}}$

According to third law

$\log x=\frac{1}{8}\log(789.5)$

$\log x=\frac{1}{8}(2.8974)$

$=\frac{2.8974}{8}$

$\log x=0.3622$

To find antilog

$x=\text{antilog}\,0.3622$

Characteristics $=0$

$x=2.302$

Reference point

$x=2.302$ **Ans**

(iii) $\frac{0.678\times9.01}{0.0234}$

Solution: $\frac{0.678\times9.01}{0.0234}$

Suppose

$x=\frac{0.678\times9.01}{0.0234}$

Taking log on both sides

$\log x=\log\frac{0.678\times9.01}{0.0234}$

According to 1st and 2nd law of log

$\log x=\log 0.678+\log 9.01-\log 0.0234$

$\log x=\bar{1}.8312+0.9547-\bar{2}.3692$

$=-1+0.8312+0.9547-(-2+0.3692)$

$=2.4167$

To find antilog

$x=\text{antilog}\,2.4167$

Characteristics $=2$

$x=2.610$

$x=261.0$ **Ans**

(iv) $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

Solution: $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

$(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}$

Suppose:

$x=(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}$

Taking log on both side

$\log x=\log\left[(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}\right]$

According to law of logarithm

$\log x=\log(2.709)^{\frac{1}{5}}+\log(1.239)^{\frac{1}{7}}$

According to third law of logarithm

$\log x=\frac{1}{5}\log(2.709)+\frac{1}{7}\log(1.239)$

$\log x=\frac{1}{5}\log(2.709)+\frac{1}{7}\log(1.239)$

$$= \frac{1}{5}0.4328 + \frac{1}{7}0.0931$$

$$= \frac{0.4328}{5} + \frac{0.0931}{7}$$

$0.0866 + 0.0133$

$= 0.0999$

To find antilog

x = antilog 0.999

Characteristics = 0

$x = 1.259$

Reference point

$x = 1.259$ **Ans**

(v) $\dfrac{1.23 \times 0.6975}{0.0075 \times 1278}$

Solution: $\dfrac{1.23 \times 0.6975}{0.0075 \times 1278}$

Suppose

$$x = \frac{1.23 \times 0.6975}{0.0075 \times 1278}$$

$$\log x = \log \frac{1.23 \times 0.6975}{0.0075 \times 1278}$$

$$= \log(1.23 \times 0.6975) - \log(0.0075 \times 1278)$$

$$= \log 1.23 + \log 0.6975 - (\log 0.0075 + \log 1278)$$

$$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$$

$$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$

$$= 0.8999 + (-1 + 0.8435) - (-3 + 0.8751) + 3.1065$$

$$= -1.0482$$

$$\log x = -2 + 2 - 1.0482$$

$$\log x = 02 + 0.9515$$

$$\log x = \bar{2}.9518$$

To find antilog

x = antilog $\bar{2}.9518$

Ch = $\bar{2}$

$x = 8950$

$= 0.08950$ **Ans**

(vi) $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

Solution: $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

Let $x = \left[\dfrac{0.7214 \times 20.37}{60.8}\right]^{\frac{1}{3}}$

Taking log on both sides

$$\log x = \log\left(\frac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$$

3rd of logarithm

$$\log x = \frac{1}{3}\log\left[\frac{0.7214 \times 20.37}{60.8}\right]$$

According to first and 2nd law

$$\log x = \frac{1}{3}\left[\log 0.7214 + \log 37 - \log 60.8\right]$$

$$\log x = \frac{1}{3}\left[\bar{1}.8582 + 1.3089 - 1.7839\right]$$

$$\frac{1}{3}\left[-1 + 0.8582 + 1.3089 - 1.7839\right]$$

$$= \frac{1}{3}(-0.6168)$$

$= -0.2056$

$\log x$ is in negative, so

$\log x = -1 + 1 - 0.2056$

$= -1 + 79144$

$= \bar{1}.7944$

To find antilog

x = antilog $\bar{1}.7944$

Ch = $\bar{1}$

$x = 6229$

Reference point

0.6229 **Ans**

(vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Solution: $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Suppose: $x = \dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$$x = \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

Taking on both side

$$\log x = \log \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

According to 1st and 2nd law of log

$$\log x = \log 83 + \log (92)^{\frac{1}{3}} - \log 127 - \log (246)^{\frac{1}{5}}$$

According to third law of log

$$\log x = \log 83 + \frac{1}{3}\log 92 - \log 27 - \frac{1}{5}\log 246$$

$$\log x = (1.9191) + \frac{1}{3}(1.9638) - (2.1038)$$

$$-\frac{1}{5}(2.3909)$$

$$= 1.9191 + 0.65460 - 2.1038 - 0.47818$$
$$= 1.9191 + 0.6546 - 2.1038 - 0.47818$$
$$= -0.0083$$

$\log x$ is in negative, so

$$\log x = -1 + 1 - 0.0083$$
$$= -1 + 0.9917$$
$$= \bar{1}.9917$$

To find antilog

$x =$ antilog $\bar{1}.9917$

Ch = $\bar{1}$

$x = 9.811$

Reference point

$x = 0.9811$ **Ans**

(viii) $\dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

Solution: $\dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

Suppose: $x = \dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

$$x = \frac{(438)^3 (0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log on both side

$$\log x = \log \left(\frac{(438)^3 (0.056)^{\frac{1}{2}}}{(388)^4} \right)$$

According to 1st and 2nd law

$$\log x = \log (438)^3 + \log (0.056)^{\frac{1}{2}} - \log (388)^4$$

According to third law

$$\log x = 3\log (438) + \frac{1}{2}\log (0.056) - 4\log (38)$$

$$\log x = 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$= 7.9245 + \frac{1}{2}(-2 + 0.7482) - 10.3552$$

$$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$$

$$= 7.9245 - 0.6259 - 10.3552$$
$$= -3.0566$$

log is in negative, so

$$\log x = -4 + 4 - 3.0566$$
$$= -4 + 0.9434$$

To find antilog

$x =$ antilog $\bar{4}.9434$

Ch = $\bar{4}$

$x = 8778$

Reference point

$= 0.0008778$ **Ans**

Q.2 **A gas is expanding according to the law $pv^n = C$.**
Find C when p = 80, v = 3.1 and $n = \frac{5}{4}$.

Solution: Given that $pv^n = C$

Taking log on both sides

$\text{Log}\left(pv^n\right) = \log C$

$\text{Log } P + \log v^n = \log C$

$\text{Log } C = \log P + \log v^n$

$\text{Log } C = \log P + n \log v$

Putting P=80, v=3.1 and $n = \frac{5}{4}$

$$\text{Log } C = \log 80 + \frac{5}{4} \log 3.1$$

$$= 1.9031 + \frac{5}{4}(0.4914)$$

$$= 1.9031 + 0.6143$$

Log C=2.5174
Taking antilog both sides
C=Antilog (2.5174)
C=329.2 **Ans:**

Q.3 The formula $p = 90\,(5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution: Given that $p = 90(5)^{\frac{-q}{10}}$

Taking log on both sides

$$Log\ p = \log\left(90(5)^{\frac{-q}{10}}\right)$$

$$Log\ p = \log 90 + \log 5^{\frac{-q}{10}}$$

$$\text{Log P} = \log 90 - \frac{q}{10} \log 5$$

$$\text{Log } 18 = \log 90 - \frac{q}{10} \log 5$$

(P =18)

$$1.2553 = 1.9542 - \frac{q}{10} \times 0.6990$$

$$1.2553 - 1.9542 = -\frac{q}{10} \times 0.6990$$

$$-0.6989 \times 10 = -q \times 0.6990$$

$$-6.989 = -q \times 0.6996$$

$$6.989 = q \times 0.6996$$

$$\frac{6.989}{0.6990} = q$$

q =10 approximately
Hence 10 units will be demanded

Q.4 If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$.

Solution: Given that $A = \pi r^2$
Taking log on both sides
Log A=log πr^2
Log A=log π +log r^2
Log A=log π +2 log r
Putting $\pi = \frac{22}{7}$ and r =15

$$\text{Log A} = \log \frac{22}{7} + 2 \log 15$$

=log 22-log 7+2 log 15
=1.3424-0.8451+2(1.1761)
=0.4973+2.3522
Log A=2.8495
Taking antilog on both sides
A=antilog 2.8495
A=707.1 **Ans**

Q.5 If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and h=4.2.

Solution: Given that $V = \frac{1}{3}\pi r^2 h$

Taking log on both sides

$$Log V = \log \frac{1}{3}\pi r^2 h$$

$$= \log \frac{1}{3} + \log \pi r^2 h$$

=log 1-log 3+ log πr^2 + log h
= 0-0.4771+ log π +log r^2 + log h

$$= -0.4771 + \log \frac{22}{7} + 2 \log r + \log h$$

$$\left(\pi = \frac{22}{7}, r = 2.5 \text{ and } h = 4.2\right)$$

$$= -0.4771 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$$

$$= -0.4771 + 1.3424 - 0.8450 + 2\,x0.3979 + 0.6232$$

$$= -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$$

Log V=1.4394
Taking antilog on both sides
V=antilog 1.4394
V=27.50 **Ans**

Review Exercise 3

Q.1 Multiple choice Questions. Choose of the correct answer.

(i) If $a^x = n$, then…

(a) $a = \log_x n$ **(b)** $x = \log_n a$

(c) $x = \log_a n$ **(d)** $a = \log_n x$

(ii) The relation $y=\log_z x$ implies…

(a) $x^y = z$ **(b)** $z^y = x$

(c) $x^z = y$ **(d)** $y^z = x$

(iii) The logarithm of unity to any base is…

(a) 1 **(b)** 10

(c) e **(d)** 0

(iv) The logarithm of any number to itself as base is…

(a) 1 **(b)** 0

(c) e **(d)** 10

(v) Log e=…,where e ≈ 2.718

(a) 0 **(b)** 0.4343

(c) ∞ **(d)** 1

(vi) The value of $\log\left(\frac{p}{q}\right)$ **is…**

(a) $\log p - \log q$ **(b)** $\frac{\log p}{\log q}$

(c) $\log p + \log q$ **(d)** $\log q - \log p$

(vii) Log *p*-log *q* is same as …

(a) $\log\left(\frac{q}{p}\right)$ **(b)** $\log(p-q)$

(c) $\frac{\log p}{\log q}$ **(d)** $\log q - \log p$

(viii) Log(m^n) can be written as…

(a) $(\log m)^n$ **(b)** $m \log n$

(c) $n \log m$ **(d)** $\log(mn)$

(ix) $\log_b a \times \log_c b$ **can be written as…**

(a) $\log_a c$ **(b)** $\log_c a$

(c) $\log_a b$ **(d)** $\log_b c$

(x) **$\text{Log}_y x$ will be equal to…**

(a) $\dfrac{\log_z x}{\log_y z}$ **(b)** $\dfrac{\log_x z}{\log_y z}$

(c) $\dfrac{\log_z x}{\log_z y}$ **(d)** $\dfrac{\log_z y}{\log_z x}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii	ix	x
c	b	d	a	b	a	d	c	b	c

Q.2 Complete the following:

(i) For common logarithm, the base is…

(ii) The integral part of the common logarithm of a number is called the …

(iii) The decimal part of the common logarithm of a number is called the …

(iv) If x = log y, then y is called the… of x.

(v) If the characteristic of the logarithm of a number have…zero(s) immediately after the decimal point.

(vi) If the characteristic of the logarithm of a number is 1, that number will have digits in its integral part.

ANSWER KEY

i	ii	iii	iv	v	vi
10	Characteristic	Mantissa	Antilogarithm	One	2

Q.3 Find the value of x in the following.

(i) $\log_3 x = 5$

Solution: $\log_3 x = 5$

Write in exponential form.

$3^5 = x$

$243 = x$ **Ans**

(ii) $\log_4 256 = x$

Solution: $\log_4 256 = x$

Write in exponential form

$4^x = 256$

$4^x = 4^4$

$x = 4$

$x = 4$ **Ans**

(iii) $\log_{625} 5 = \dfrac{1}{4}x$

Solution: $\log_{625} 5 = \dfrac{1}{4}x$

Write in exponential form

$(625)^{\frac{1}{4}x} = 5$

$(625)^{\frac{x}{4}} = 5$

$(5^4)^{\frac{x}{4}} = 5$

$$5^{\frac{4x}{4}} = 5$$
$$5^{x} = 5^{1}$$
$$x = 1 \textbf{ Ans}$$

(iv) $\log_{64} x = -\frac{2}{3}$

Solution: $\log_{64} x = -\frac{2}{3}$

Write in exponential form

$$(64)^{\frac{-2}{3}} = x$$
$$\left(4^{3}\right)^{-\frac{2}{3}} = x$$
$$4^{-\frac{6}{3}} = x$$
$$4^{-2} = x$$
$$\frac{1}{4^{2}} = x$$
$$\frac{1}{16} = x \textbf{ Ans}$$

Q.4 Find the value of x in the following.

(i) $\log x = 2.4543$

Solution: $\log x = 2.4543$

$\log x = 2.4543$

$x = \text{antilog}\, 2.4543$

$\text{Ch} = 2$

$x = 284.6$ **Ans**

(ii) $\log x = 0.1821$

Solution: $\log x = 0.1821$

$\log x = 0.1821$

$x = \text{antilog}\, 0.1821$

$\text{Ch} = 0$

$x = 1.521$ **Ans**

(iii) $\log x = 0.0044$

Solution: $\log x = 0.0044$

$\log x = 0.0044$

$x = \text{antilog}\, 0.0044$

$\text{Ch} = 0$

$x = 1.010$ **Ans**

(iv) $\log x = \bar{1}.6238$

Solution: $\log x = \bar{1}.6238$

$\log x = \bar{1}.6238$

$x = \text{antilog}\, \bar{1}6333$

$\text{Ch} = \bar{1}$

$x = 0.4206$ **Ans**

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$, and $\log 5 = 0.6990$ then find the values of the following.

(i) $\log 45$

Solution: $\log 45$

$= \log(9 \times 5)$

$= \log(3^{2} \times 5)$

$= \log 3^{2} + \log 5$

$= 2\log 3 + \log 15$

$= 2(0.4771) + 0.6990$

$= 0.9542 + 0.6990$

$= 1.6532$ **Ans**

(ii) $\log \frac{16}{15}$

Solution: $\log \frac{16}{15}$

$= \log \frac{2^{4}}{3 \times 5}$

$= \log 2^{4} - \log(3 \times 5)$

$= 4\log 2 - (\log 3 + \log 5)$

$= \log 2^{4} - \log 3 - \log 5$

$= 4\log 2 - \log 3 - \log 5$

$= 4(0.3010) - 0.4771 - 0.6990$

$= 1.2040 - 0.4771 - 0.6990$

$= 0.0279$ **Ans**

(iii) $\log 0.048$

Solution: $\log 0.048$

$$= \log \frac{48}{1000}$$

$$= \log \frac{2\times2\times2\times2\times3}{2\times2\times2\times5\times5\times5}$$

$$= \log \frac{2^4\times3}{2^3\times5^3}$$

$$= \log 2^4 + \log 3 - \log 2^3 - \log 5^3$$

$$= 4\log 2 + \log 3 - 3\log 2 - 3\log 5$$

$$= 4(0.3010) + 0.4771 - 3(0.3010) - 3(0.6990)$$

$$= 1.2040 + 0.4771 - 0.9030 - 2.0970$$

$$= -1.3189$$

$$= -1 - 0.3189$$

$$= -1 - 1 + 1 - 0.3189$$

$$= -2 + 0.6811$$

$$= \bar{2}.6811 \text{ Ans}$$

Q.6 Simplify the following.

(i) $\sqrt[3]{25.47}$

Solution: $\sqrt[3]{25.47}$

Let $x = \sqrt[3]{25.74}$

$$= (25.47)^{\frac{1}{3}}$$

Taking log on both sides

$$\log x = \log(25.47)^{\frac{1}{3}}$$

$$= \frac{1}{3}\log 25.47$$

$$= \frac{1}{3}(1.4060)$$

$$\log x = 0.4687$$

$$x = \text{anti}\log 0.4687$$

$$\text{Ch} = 0$$

$$x = 2.943 \text{ Ans}$$

(ii) $\sqrt[5]{342.2}$

Solution: $\sqrt[5]{342.2}$

Let

$$x = \sqrt[5]{342.2}$$

$$x = (242.)^{\frac{1}{5}}$$

Taking log on both sides

$$\log x = (342.2)^{\frac{1}{5}}$$

$$\log x = \frac{1}{5}\log 342.2$$

$$= \frac{1}{5}(2.5343)$$

$$\log x = 0.5069$$

$$\log x = \text{antilog}\, 0.5069$$

$$\text{Ch} = 0$$

$$x = 3.213 \text{ Ans}$$

(iii) $\dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Solution: $\dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Let $x = \dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Taking log on both sides

$$\log x = \log \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$

$$= \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$$

$$= 3\log 8.97 + 2\log 3.95 - \frac{1}{3}\log 15.37$$

$$= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$$

$$= 2.8584 + 1.1932 - 0.3956$$

$$\log x = 3.656$$

$$x = \text{antilog}\, 3.656$$

$$\text{Ch} = 3$$

$$x = 4529 \text{ Ans}$$

Unit 3: Logarithms

Overview

Scientific Notation:

A number written in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer, is called the scientific notation.

Logarithm of a Real Number:

If $a^x = y$ they x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

Characteristic of logarithm of the Number:

An integral part which is positive for a number greater than 1 and negative for a number less than 1, is called the characteristic of logarithm of the number.

Mantissa of the logarithm of the Number:

A decimal part which is always positive, is called the mantissa of the logarithm of the number.

Antilogarithm:

The number whose logarithm is given is called antilogarithm.