

Math Sci 9: Test		Total No. 40	
Name: _____ Roll No. : _____			
Date: _____ - _____ -20 Teacher's Signature: _____			

Q.1: Tick (✓) the correct answer.

Symbol "for all" is _____ :

> (D)

< (C)

$\forall$ (B)

∇ (A)

The value of i^{10} is _____ :

$-i$ (D)

i (C)

-1 (B)

1 (A)

Conjugate of $6 + 5i$ is _____ :

$6 + 5i$ (D)

$6 - 5i$ (C)

$-6 - 5i$ (B)

$6 + 5i$ (A)

Write $\sqrt[7]{x}$ in exponential form:

$x^{7/2}$ (D)

$x^{1/7}$ (C)

x^7 (B)

x (A)

In $\sqrt[3]{35}$ the radicand is:

Any No / کوئی نہیں (D)

35 (C)

$\frac{1}{3}$ (B)

3 (A)

Imaginary part of $-i(3i + 2)$ is _____ :

-3 (D)

3 (C)

2 (B)

-2 (A)

If $x, y, z \in R$, $z < 0$, then $x < y \Rightarrow$ _____ :

Any No / کوئی نہیں (D)

$xz = yz$ (C)

$xz > yz$ (B)

$xz < yz$ (A)

Real part of $2ab(i + i^2)$ is:

$-2abi$ (D)

$2abi$ (C)

$-2ab$ (B)

$2ab$ (A)

Write $4^{2/3}$ with radical sign:

$\sqrt[4]{4^6}$ (D)

$\sqrt[2]{4^3}$ (C)

$\sqrt[4]{4^3}$ (B)

$\sqrt[3]{4^2}$ (A)

The conjugate of $5 + 4i$ is:

$5 + 4i$ (D)

$5 - 4i$ (C)

$-5 - 4i$ (B)

$-5 + 4i$ (A)

$10 \times 2 = 20$ Write short answers to any ten (10) questions.

What is terminating decimal fractions?

Express $0.\overline{23} = 0.232323.....$ in the form of p/q .

Give a rational number between $3/4$ and $5/9$.

Write any two properties of real numbers.

Simplify: $\sqrt[3]{16x^4y^5}$

Simplify radical expression: $\sqrt[4]{32}$

Simplify:

$$(2x^5y^{-4})(-8x^{-3}y^2)$$

Define a complex number with example.

Find the value of i^{50} .

Find the value of x and y if

$$x + iy + 1 = 4 - 3i$$

Write about the addition of two complex numbers.

If $z = 2 + i$ then find the value of $\overline{z}$.

$1 \times 10 = 10$ Write answer to any One (1) question.

Solve $(2 - 3i)(x + yi) = 4 + i$ for real x and y .

Solve following equation for real x and y . $(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$

سوال نمبر 1- درست جواب پر (✓) کا نشان لگائیں۔

1- "تمام کے لیے" علامت _____ ہے:

$\forall$ (B)

∇ (A)

2- i^{10} کی قیمت _____ ہے:

-1 (B)

1 (A)

3- $6 + 5i$ کا کانجوگٹ _____ ہے:

$-6 - 5i$ (B)

$6 + 5i$ (A)

4- $\sqrt[7]{x}$ کو پاور فارم میں لکھیے:

x^7 (B)

x (A)

5- $\sqrt[3]{35}$ میں ریڈیکنڈ _____ ہے:

$\frac{1}{3}$ (B)

3 (A)

6- کمپلیکس نمبر $-i(3i + 2)$ کا امیجری حصہ ہے:

2 (B)

-2 (A)

7- اگر $z < 0$ تو $x < y \Rightarrow$ _____ :

$xz > yz$ (B)

$xz < yz$ (A)

8- کمپلیکس نمبر $2ab(i + i^2)$ کا حقیقی حصہ ہے:

$-2ab$ (B)

$2ab$ (A)

9- $4^{2/3}$ کو ریڈیکل فارم میں لکھیے:

$\sqrt[4]{4^3}$ (B)

$\sqrt[3]{4^2}$ (A)

10- $5 + 4i$ کا کانجوگٹ _____ ہے:

$-5 - 4i$ (B)

$-5 + 4i$ (A)

سوال نمبر 2- کوئی سے 10 سوالات کے جوابات تحریر کیجیے۔

i- اختتام پذیر اعشاری ناطق اعداد کسے کہتے ہیں؟

ii- $0.\overline{23} = 0.232323.....$ کو p/q کی شکل میں ظاہر کریں۔

iii- اعداد $3/4$ اور $5/9$ کے درمیان ایک ناطق عدد بتائیے۔

iv- حقیقی اعداد کی کوئی دو خصوصیات تحریر کریں۔

v- مختصر کریں۔ $\sqrt[3]{16x^4y^5}$

vi- ریڈیکل شکل کو عام شکل میں لکھیے: $\sqrt[4]{32}$

vii- مختصر کیجیے:

viii- کمپلیکس عدد کی تعریف کریں بمعہ مثال۔

ix- i^{50} کی قیمت معلوم کریں۔

x- x اور y کی قیمت معلوم کریں، اگر

xi- دو غیر حقیقی اعداد کے لیے جمع کے عمل کی وضاحت کریں۔

xii- اگر $z = 2 + i$ ہو تو $\overline{z}$ کی قیمت معلوم کریں۔

نوٹ: کوئی سے ایک سوال کا جواب لکھیے۔

سوال نمبر 3- $(2 - 3i)(x + yi) = 4 + i$ کو x اور y میں حل کریں۔

سوال نمبر 4- مندرجہ ذیل مساوات کو x اور y میں حل کریں۔ $(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$

Exercise 2.1

Q.1 Identity which of the following are rational and irrational numbers?

- (i) $\sqrt{3}$ Irrational number
- (ii) $\frac{1}{6}$ Rational number
- (iii) π Irrational number
- (iv) $\frac{15}{2}$ Rational number
- (v) 7.25 Rational number
- (vi) $\sqrt{29}$ Irrational number

Q.2 Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$

Solution: $\frac{17}{25}$

$$\begin{array}{r} 0.68 \\ 25 \overline{) 170} \\ \underline{-150} \\ 200 \\ \underline{-200} \\ 0 \end{array}$$

$\frac{17}{25} = 0.68$ **Ans**

(ii) $\frac{19}{4}$

Solution: $\frac{19}{4}$

$$\begin{array}{r} 4.75 \\ 4 \overline{) 19.000} \\ \underline{16} \\ 30 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$= \frac{19}{4}$
 $= 4.75$ **Ans**

(iii) $\frac{57}{8}$

Solution: $\frac{57}{8}$

$$\begin{array}{r} 7.125 \\ 8 \overline{) 57} \\ \underline{-56} \\ 10 \\ \underline{8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$= \frac{57}{8}$
 $= 7.125$ **Ans**

(iv) $\frac{205}{18}$

Solution: $\frac{205}{18}$

$$\begin{array}{r} 11.388 \\ 18 \overline{) 205.000} \\ \underline{25} \end{array}$$

$$\begin{array}{r}
 18 \\
 70 \\
 -54 \\
 \hline
 160 \\
 -144 \\
 \hline
 160 \\
 -144 \\
 \hline
 16
 \end{array}$$

$$\begin{array}{r}
 208 \\
 18 \\
 \hline
 = 11.3888 \\
 = 11.3889 \text{ Ans}
 \end{array}$$

(v) $\frac{5}{8}$

Solution: $\frac{5}{8}$

$$\begin{array}{r}
 .625 \\
 8 \overline{) 5.000} \\
 \underline{48} \\
 20 \\
 \underline{-16} \\
 40 \\
 \underline{-40} \\
 0
 \end{array}$$

$$\begin{array}{r}
 5 \\
 8 \\
 \hline
 = 0.625 \text{ Ans}
 \end{array}$$

(vi) $\frac{25}{38}$

Solution: $\frac{25}{38}$

$$\begin{array}{r}
 0.65789... \\
 38 \overline{) 250} \\
 \underline{-228} \\
 220 \\
 \underline{-190} \\
 300 \\
 \underline{-266} \\
 340 \\
 \underline{-304} \\
 360 \\
 \underline{-342} \\
 18
 \end{array}$$

$$\begin{array}{r}
 25 \\
 38 \\
 \hline
 = 0.65789 \text{ Ans}
 \end{array}$$



Q.3 Which of the following statements are true and which are false?

(i) $\frac{2}{3}$ is an irrational number.

False

(ii) π is an irrational number.

True

(iii) $\frac{1}{9}$ is a terminating fraction.

False

(iv) $\frac{3}{4}$ is a terminating fraction.

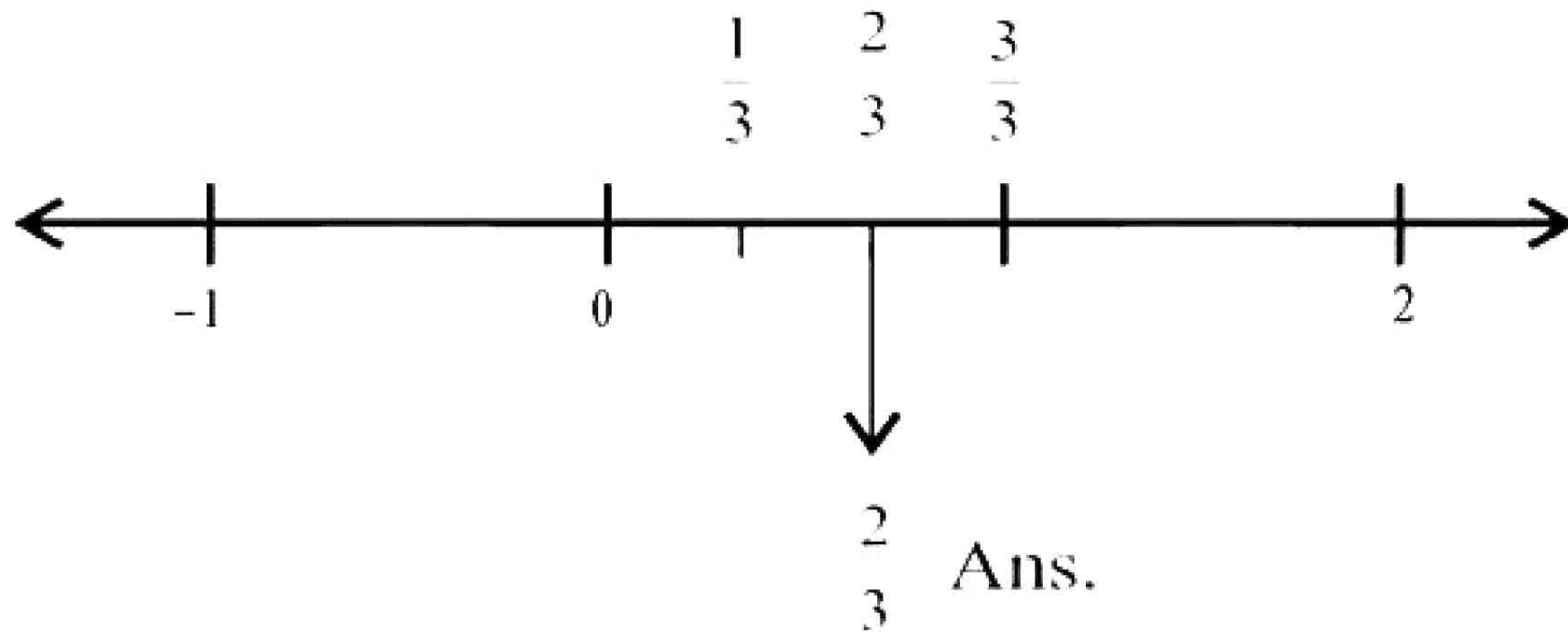
True

(v) $\frac{4}{5}$ is a recurring fraction.

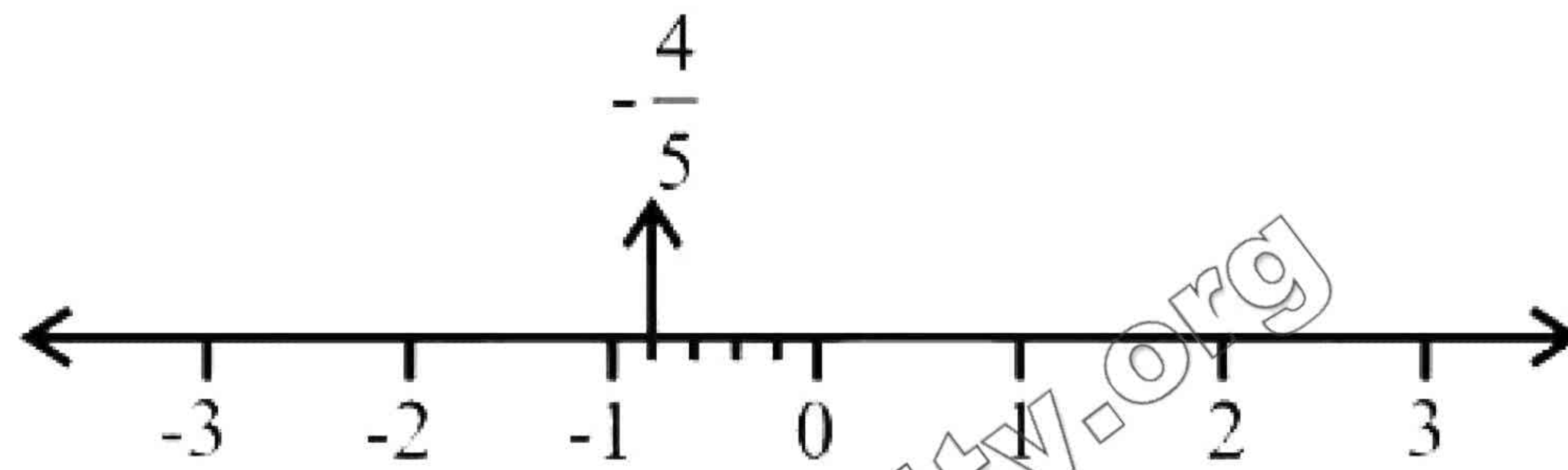
False

Q.4 Represent the following numbers on the number line.

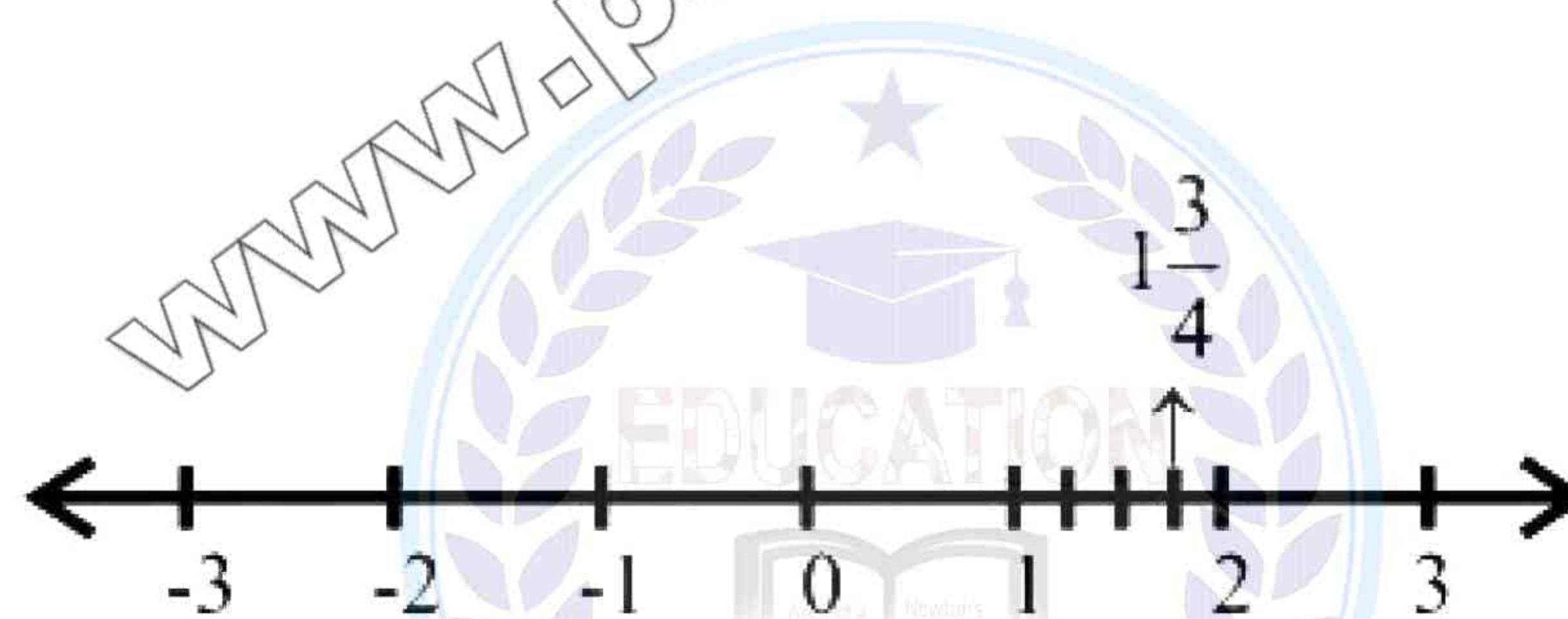
(i) $\frac{2}{3}$



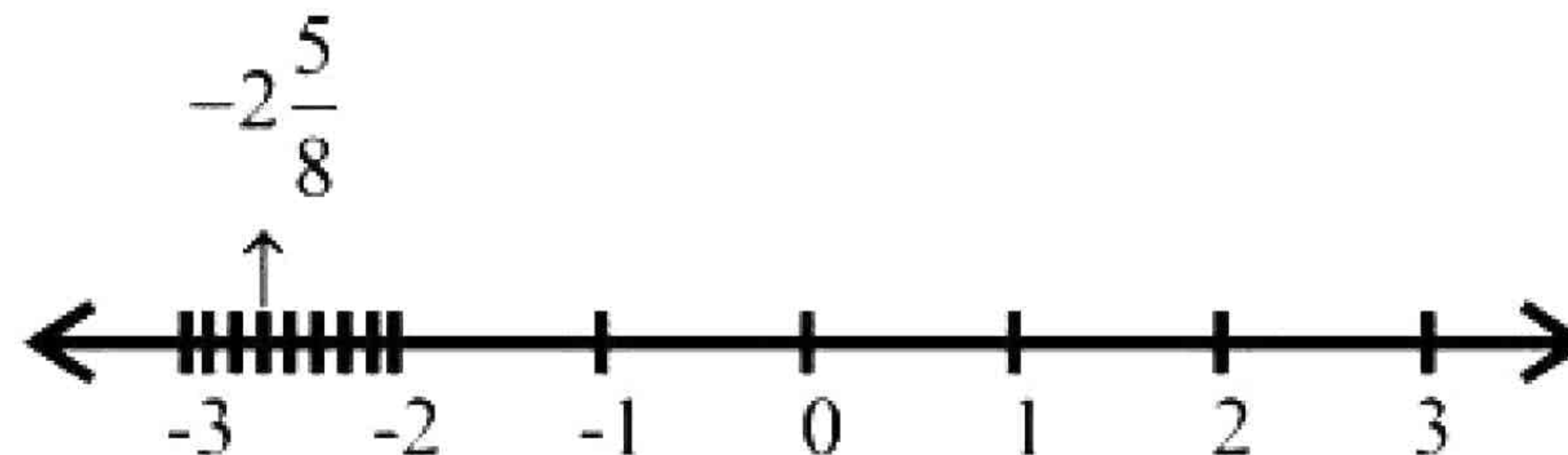
(ii) $-\frac{4}{5}$



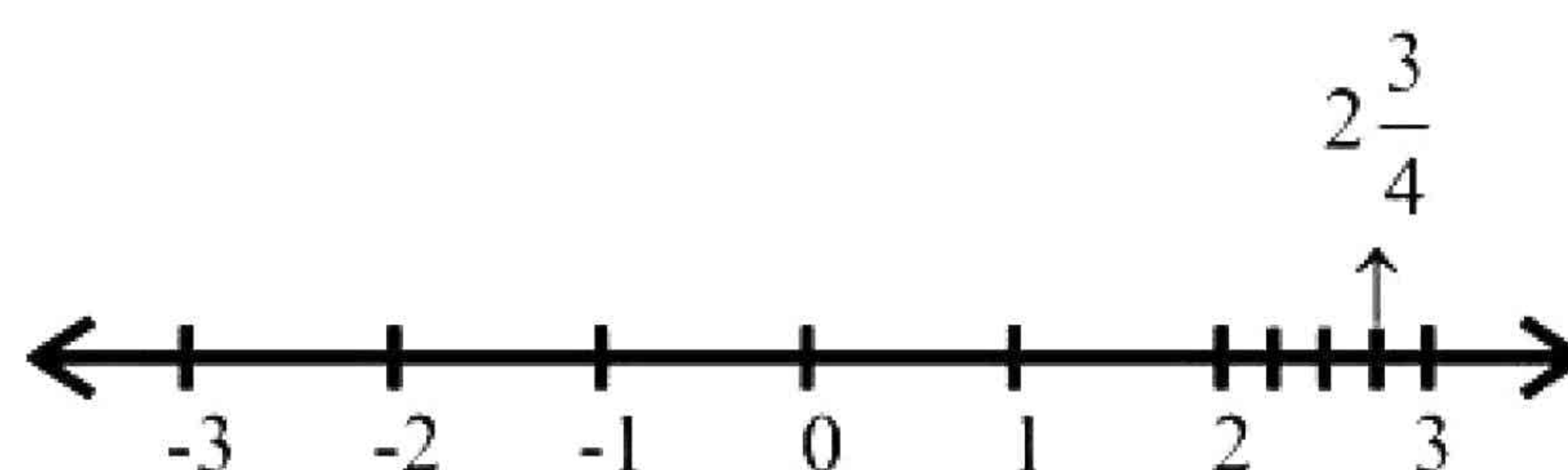
(iii) $1\frac{3}{4}$



(iv) $-2\frac{5}{8}$



(v) $2\frac{3}{4}$



(vi) $\sqrt{5}$

By Pythagoras theorem

$$(\text{Hypoteneus})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(\overline{OB})^2 = (2)^2 + (1)^2$$

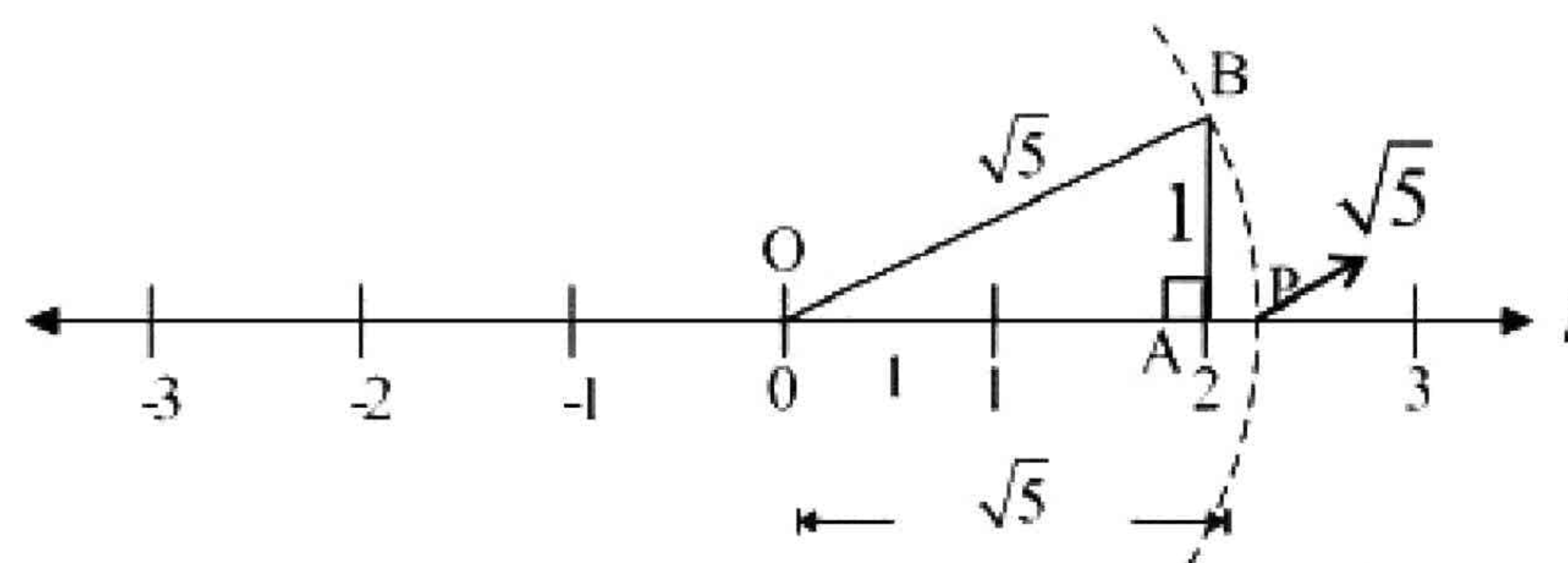
$$(\overline{OB})^2 = 4 + 1$$

$$(\overline{OB})^2 = 5$$

Taking square root on both sides

$$\sqrt{(\overline{OB})^2} = \sqrt{5}$$

$$\overline{OB} = \sqrt{5}$$

**Q.5** Give a rational number between

$$\frac{3}{4} \text{ and } \frac{5}{9}$$

Solution:

Required No between

$$\frac{3}{4} \text{ and } \frac{5}{9}$$

$$= \left[\frac{3}{4} + \frac{5}{9} \right] \div 2$$

$$= \left[\frac{27 + 20}{36} \right] \div 2$$

$$= \left[\frac{47}{36} \right] \div 2$$

$$= \frac{47}{36} \times \frac{1}{2}$$

$$= \frac{47}{72} \text{ Ans}$$

Q.6

Express the following recurring decimals as the rational number

$$\frac{p}{q} \text{ where } p, q \text{ are integer and } q \neq 0.$$
(i)

$$0.\overline{5}$$

Solution:

$$x = 0.\overline{5}$$

$$x = 0.555\ldots$$

$$10 \times x = 10 \times 0.555\ldots$$

$$10x = 5.555\ldots$$

$$10x = 5 + 0.555\ldots$$

$$10x = 5 + x$$

$$10x - x = 5$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$\therefore 0.\overline{5} = \frac{5}{9} \text{ Ans}$$

(ii) $0.\overline{13}$ **Solutions:**

Suppose

$$x = 0.\overline{13}$$

$$x = 0.131313...$$

$$100^x x = 100 \times 1.131313...$$

$$100x = 13.1313...$$

$$100x = 13 + 0.1313...$$

$$100x = 13 + x$$

$$100x - x = 13$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$\therefore 0.\overline{13} = \frac{13}{99} \text{ Ans}$$

(iii) $0.\overline{67}$ **Solutions:**

Suppose

$$x = 0.\overline{67}$$

$$x = 0.676767...$$

$$100 \times x = 100 \times 0.676767...$$

$$100x = 67.6767...$$

$$100x = 67 + 0.6767...$$

$$100x = 67 + x$$

$$100x - x = 67$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$\therefore 0.\overline{67} = \frac{67}{99} \text{ Ans}$$



Exercise 2.2

Q.1 Identify the property used in the following.

- | | | |
|--------|-------------------------------------|---|
| (i) | $a + b = b + a$ | Commutative Property w.r.t addition |
| (ii) | $(ab)c = a(bc)$ | Associative Property w.r.t multiplication |
| (iii) | $7 \times 1 = 7$ | Multiplicative Identity |
| (iv) | $x > y$ or $x = y$ or $x < y$ | Trichotomy |
| (v) | $ab = ba$ | Commutative w.r.t multiplication |
| (vi) | $a + c = b + c \Rightarrow a = b$ | Cancellation Property of addition |
| (vii) | $5 + (-5) = 0$ | Additive Inverse |
| (viii) | $7 \times \frac{1}{7} = 1$ | Multiplicative inverse |
| (ix) | $a > b \Rightarrow ac > bc (c > 0)$ | Multiplicative property |

Q.2 Fill in the following blanks by stating the properties of real numbers used.

$$\begin{aligned}
 &3x + 3(y - x) \\
 &= 3x + 3y - 3x, \dots \text{Distributive property} \\
 &= 3x - 3x + 3y, \dots \text{Commutative} \\
 &= 0 + 3y, \dots \text{Additive Inverse} \\
 &= 3y, \dots \text{Additive identity}
 \end{aligned}$$

Q.3 Give the name of property used in the following.

- | | | |
|-------|--|-----------------------------------|
| (i) | $\sqrt{24} + 0 = \sqrt{24}$ | Additive Identity |
| (ii) | $-\frac{2}{3} \left[5 + \frac{7}{2} \right] = \left[-\frac{2}{3} \right] (5) + \left[-\frac{2}{3} \right] \left[\frac{7}{2} \right]$ | Distributive Property |
| (iii) | $\pi + (-\pi) = 0$ | Additive Inverse |
| (iv) | $\sqrt{3} \cdot \sqrt{3}$ is a real number. | Closure property w.r.t $\times$. |
| (v) | $\left[-\frac{5}{8} \right] \left[-\frac{8}{5} \right] = 1$ | Multiplicative Inverse. |

Exercise 2.3

Q.1 Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[3]{-64}$
 $= (-64)^{\frac{1}{3}}$

(ii) $2^{\frac{3}{5}}$
 $= \sqrt[5]{2^3}$

(iii) $-7^{\frac{1}{3}}$
 $= -\sqrt[3]{7}$

(iv) $y^{-\frac{2}{3}}$
 $= \sqrt[3]{y^{-2}}$

Q.2 Tell whether the following statements are true or false?

(i) $5^{\frac{1}{5}} = \sqrt{5}$

(ii) $2^{\frac{2}{3}} = \sqrt[3]{4}$

(iii) $\sqrt{49} = \sqrt{7}$

(iv) $\sqrt[3]{x^{27}} = x^3$

False

True

False

False

Q.3 Simplify the following radical expression.

(i) $\sqrt[3]{-125}$

Solution:

$$\begin{aligned} &= \sqrt[3]{-125} \\ &= \sqrt[3]{-5 \times -5 \times -5} \\ &= \sqrt[3]{(-5)^3} \\ &= -5 \text{ Ans} \end{aligned}$$

(ii) $\sqrt[4]{32}$

Solutions:

$$\begin{aligned}
 &= \sqrt[4]{32} \\
 &= \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2} \\
 &= \sqrt[4]{2^4 \times 2} \\
 &= \sqrt[4]{2^4} \times \sqrt[4]{2} \\
 &= 2\sqrt[4]{2} \text{ Ans}
 \end{aligned}$$

(iii) $\sqrt[5]{\frac{3}{32}}$

Solution:

$$\begin{aligned}
 &= \sqrt[5]{\frac{3}{32}} \\
 &= \frac{\sqrt[5]{3}}{\sqrt[5]{32}} \\
 &= \frac{\sqrt[5]{3}}{\sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}} \\
 &= \frac{\sqrt[5]{3}}{\sqrt[5]{(2)^5}} \\
 &= \frac{\sqrt[5]{3}}{2} \text{ Ans}
 \end{aligned}$$

(iv) $\sqrt[3]{-\frac{8}{27}}$

Solution:

$$\begin{aligned}
 &= \sqrt[3]{-\frac{8}{27}} \\
 &= \sqrt[3]{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)} \\
 &= \sqrt[3]{\left(-\frac{2}{3}\right)^3} \\
 &= -\frac{2}{3} \text{ Ans}
 \end{aligned}$$



Exercise 2.4

Q.1 Use laws of exponents to simplify.

(i)
$$\frac{(243)^{-\frac{2}{3}} (32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

Solution:

$$\begin{aligned} & \frac{(243)^{-\frac{2}{3}} (32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}} \\ &= \frac{(243)^{-\frac{2}{3}} (32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}} \\ &= \frac{(3^5)^{-\frac{2}{3}} \times (2^5)^{-\frac{1}{5}}}{\sqrt{[(14)^2]^{-1}}} \\ &= \frac{(3)^{-\frac{10}{3}} \times 2^{-1}}{\sqrt{[(14)^{-1}]^2}} \\ &= \frac{(3)^{-\frac{10}{3}} \times 2^{-1}}{(14)^{-1}} \\ &= \frac{14}{(3)^{\frac{10}{3}} \times 2} \\ &= \frac{7}{3^{\frac{10}{3}}} \\ &= \frac{7}{\sqrt[3]{3^{10}}} \\ &= \frac{7}{\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}} \\ &= \frac{7}{\sqrt[3]{3^3 \times 3^3 \times 3^3 \times 3}} \\ &= \frac{7}{\sqrt[3]{3^3} \times \sqrt[3]{3^3} \times \sqrt[3]{3^3} \times \sqrt[3]{3}} \end{aligned}$$

$$\begin{aligned} &= \frac{7}{3 \times 3 \times 3 \times \sqrt[3]{3}} \\ &= \frac{7}{27\sqrt[3]{3}} \text{ Ans} \end{aligned}$$

(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution:

$$\begin{aligned} & (2x^5y^{-4})(-8x^{-3}y^2) \\ &= -16x^{5-3}y^{-4+2} \\ &= -16x^2y^{-2} \\ &= \frac{-16x^2}{y^2} \text{ Ans} \end{aligned}$$

(iii)
$$\left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right]^{-3}$$

Solution:

$$\begin{aligned} & \left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right]^{-3} \\ &= [x^{-2-4}y^{-1+3}z^{-4-0}]^{-3} \\ &= (x^{-6}y^{+2}z^{-4})^{-3} \\ &= (x^{-6})^{-3} (y^2)^{-3} (y^{-4})^{-3} \\ &= x^{18}y^{-6}z^{12} \\ &= \frac{x^{18}z^{12}}{y^6} \text{ Ans} \end{aligned}$$

(iv)
$$\frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)}$$

Solution:

$$\begin{aligned} & \frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)} \\ &= \frac{(3^4)^n \cdot 3^5 - 3^{4n} \cdot 3^{-1} \cdot 3^5}{(3^2)^{2n} \cdot 3^3} \\ &= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^{-1+5}}{3^{4n} \cdot 3^3} \\ &= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^4}{3^{4n} \cdot 3^3} \end{aligned}$$

$$\begin{aligned}
 &= \frac{3^{4n} \cdot 3^4 (3-1)}{3^{4n} \cdot 3^3} \\
 &= 3^{4n-4n} \cdot 3^{4-3} \cdot (2) \\
 &= 3^0 \cdot 3^1 \cdot 2 \\
 &= 1 \times 3 \times 2 \\
 &= 6 \text{ Ans}
 \end{aligned}$$

Q.2 Show that

$$\left[\frac{x^a}{x^b} \right]^{a+b} \times \left[\frac{x^b}{x^c} \right]^{b+c} \times \left[\frac{x^c}{x^a} \right]^{c+a} = 1$$

Proof:

L.H.S

$$\begin{aligned}
 &= \left[\frac{x^a}{x^b} \right]^{a+b} \times \left[\frac{x^b}{x^c} \right]^{b+c} \times \left[\frac{x^c}{x^a} \right]^{c+a} \\
 &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\
 &= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \\
 &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\
 &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\
 &= x^0 \\
 &= 1 \\
 &= \text{R.H.S Ans}
 \end{aligned}$$

Q.3 Simplify

(i)
$$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

Solution:
$$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$\begin{aligned}
 &= \frac{2^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}} \times (2 \times 2 \times 3 \times 5)^{\frac{1}{2}}}{(2 \times 2 \times 3 \times 3 \times 5)^{\frac{1}{2}} \times (2^2)^{\frac{1}{3}} \times (3^2)^{\frac{1}{4}}} \\
 &= \frac{2^{\frac{1}{3}} \times 3 (2^2)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}} \\
 &= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}
 \end{aligned}$$

$$\begin{aligned}
 &= 2^{\frac{1}{3}} \times 2^{+1} \times 2^{-1} \times 2^{\frac{+2}{3}} \times 3^1 \times 3^{\frac{1}{2}} \times 3^{-1} \times 3^{\frac{-1}{2}} \times 5^{\frac{1}{2}} \times 5^{\frac{-1}{2}} \\
 &= 2^{\frac{1}{3} + 1 - 1 + \frac{2}{3}} \times 3^{1 + \frac{1}{2} - 1 - \frac{1}{2}} \times 5^{\frac{1}{2} - \frac{1}{2}} \\
 &= 2^{\frac{1+2}{3}} \times 3^0 \times 5^0 \\
 &= 2^{\frac{1+2}{3}} \times 1 \times 1 \\
 &= 2^{\frac{3}{3}} \\
 &= 2 \text{ Ans}
 \end{aligned}$$

(ii)
$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$$

Solution:
$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(\frac{25 \cancel{100}}{\cancel{A}}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(25)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(5^2)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{5}}$$

$$= \sqrt{6^2 \times 5^{+1} \times 5^{-1}}$$

$$= \sqrt{6^2 \times 5^{+1-1}}$$

$$= \sqrt{6^2 \times 5^0}$$

$$= \sqrt{6^2 \times 1}$$

$$= \sqrt{6^2}$$

$$= 6 \text{ Ans}$$

(iii) $5^{2^3} \div (5^2)^3$

Solution: $5^{2^3} \div (5^2)^3$
 $= 5^8 \div 5^6$
 $= 5^{8-6}$
 $= 5^2$
 $= 25 \text{ Ans}$

(iv) $(x^3)^2 \div x^{3^2}, x \neq 0$

Solution: $(x^3)^2 \div x^{3^2}, x \neq 0$
 $= x^6 \div x^9$
 $= x^{6-9}$
 $= x^{-3}$
 $= \frac{1}{x^3} \text{ Ans}$



Exercise 2.5

Q.1 Evaluate

(i) i^7

Solution:

$$\begin{aligned} &= i^7 \\ &= i^6 i \\ &= (i^2)^3 i \\ &= (-1)^3 i \\ &= -1 \times i \\ &= -i \\ &= -i \text{ Ans} \end{aligned}$$

(ii) i^{50}

Solution: i^{50}

$$\begin{aligned} &= (i^2)^{25} \\ &= (-1)^{25} \\ &= -1 \text{ Ans} \end{aligned}$$

(iii) i^{12}

Solution:

$$\begin{aligned} &i^{12} \\ &= (i^2)^6 \\ &= (-1)^6 \\ &= 1 \text{ Ans} \end{aligned}$$

(iv) $(-i)^8$

Solution:

$$\begin{aligned} &(-i)^8 \\ &= i^8 \\ &= (i^2)^4 \\ &= (-1)^4 \\ &= 1 \text{ Ans} \end{aligned}$$

(v) $(-i)^5$

Solution:

$$\begin{aligned} &(-i)^5 \\ &= -i^5 \\ &= -i^4 i \\ &= -(i^2)^2 i \\ &= -(-1)^2 i \\ &= -(1)(i) \\ &= -i \text{ Ans} \end{aligned}$$

(vi) i^{27}

Solution: i^{27}

$$\begin{aligned} &= i^{26} i \\ &= (i^2)^{13} i \\ &= (-1)^{13} i \\ &= -1 i \\ &= -i \text{ Ans} \end{aligned}$$

Q.2 Write the conjugate of the following numbers.

(i) $2 + 3i$
 $= 2 - 3i$

(ii) $3 - 5i$
 $= 3 + 5i$

(iii) $-i$
 $= i$

(iv) $-3 + 4i$
 $= -3 - 4i$

(v) $-4 - i$
 $= -4 + i$

(vi) $i - 3$
 $= -i - 3$

Q.3 Write the real and imaginary part of the following numbers.

(i) $1 + i$
 Real = 1
 Imaginary = 1

(ii) $-1 + 2i$
 Real = -1
 Imaginary = 2

- (iii) $-3i + 2$
Real = 2
Imaginary = - 3
- (iv) $-2 - 2i$
Real = -2
Imaginary = - 2
- (v) $-3i$
Real = 0
Imaginary = - 3
- (vi) $2 + 0i$
Real = 2
Imaginary = 0

Q.4 Find the value of x and y if

$$x + iy + 1 = 4 - 3i$$

Solution: Given that

$$x + iy + 1 = 4 - 3i$$

$$x + iy = 4 - 3i - 1$$

$$x + iy = 3 - 3i$$

$$x = 3 \quad y = -3$$

$$x = 3, y = -3 \quad \text{Ans}$$



Exercise 2.6

Q.1 Identify the following statement as true or false.

- (i) $\sqrt{-3}\sqrt{-3} = 3$ **False**
- (ii) $i^{73} = -i$ **False**
- (iii) $i^{10} = -1$ **True**
- (iv) Complex conjugate of $(-6i + i^2)$ is $(-1 + 6i)$ **True**
- (v) Difference of a complex number $z = a + bi$ and its conjugate is a real number. **False**
- (vi) If $(a - 1) - (b + 3)i = 5 + 8i$, then $a = 6$ and $b = -11$. **True**
- (vii) Product of a complex number and its conjugate is always a non-negative real number. **True**

Q.2 Express the each complex number in the standard form $a + bi$, where a and b are real number.

(i) $(2 + 3i) + (7 - 2i)$

Solution:

$$\begin{aligned}
 &= 2 + 3i + 7 + 2i \\
 &= 2 + 7 + 3i + 2i \\
 &= 9 + i \text{ Ans}
 \end{aligned}$$

(ii) $2(5 + 4i) - 3(7 + 4i)$

Solution:

$$\begin{aligned}
 &2(5 + 4i) - 3(7 + 4i) \\
 &= 10 + 8i - 21 - 12i \\
 &= 10 - 21 + 8i - 12i \\
 &= -11 - 4i \text{ Ans}
 \end{aligned}$$

(iii) $(-3 + 5i) - (4 + 9i)$

Solution:

$$\begin{aligned}
 &(-3 + 5i) - (4 + 9i) \\
 &= -3 + 5i - 4 - 9i \\
 &= -3 - 4 - 5i - 9i \\
 &= -7 - 14i \text{ Ans}
 \end{aligned}$$

(iv) $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

Solution: $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$$= 2(-1) + 6i^2 \cdot i + 3(i^2)^8 - 6(i^2)^9 \cdot i + 4(i^2)^{12} \cdot i$$

$$= -2 + 6(-1)i + 3(-1)^8 - 6(-1)i + 4(-1)^{12} \cdot i$$

$$= -2 - 6i + 3 - 6(-1)i + 4(+1)i$$

$$= 1 - \cancel{6i} + \cancel{6i} + 4i$$

$$= 1 + 4i \text{ Ans}$$

Q.3 Simplify and write your answer in the form $a + bi$

(i) $(-7 + 3i)(-3 + 2i)$

Solution:

$$\begin{aligned}
 &(-7 + 3i)(-3 + 2i) \\
 &= -7(-3 + 2i) + 3i(-3 + 2i) \\
 &= 21 - 14i - 9i + 6i^2 \\
 &= 21 - 23i + 6(-1) \\
 &= 21 - 23i - 6 \\
 &= 15 - 23i \text{ Ans}
 \end{aligned}$$

(ii) $(2 - \sqrt{-4})(3 - \sqrt{-4})$

Solution: $(2 - \sqrt{-4})(3 - \sqrt{-4})$
 $= (2 - \sqrt{4 \times -1})(3 - \sqrt{4 \times -1})$
 $= (2 - \sqrt{4i^2})(3 - \sqrt{4i^2})$
 $= (2 - 2i)(3 - 2i)$
 $= 2(3 - 2i) - 2i(3 - 2i)$
 $= 6 - 4i - 6i + 4i^2$
 $= 6 - 10i + 4(-1)$
 $= 6 - 10i - 4$
 $= 2 - 10i$ **Ans**

(iii) $(\sqrt{5} - 3i)^2$

Solution: $(\sqrt{5} - 3i)^2$
 $= (\sqrt{5})^2 + (3i)^2 - 2(\sqrt{5})(3i)$
 $= 5 + 9i^2 - 6\sqrt{5}i$
 $= 5 + 9(-1) - 6\sqrt{5}i$
 $= 5 - 9 - 6\sqrt{5}i$
 $= -4 - 6\sqrt{5}i$ **Ans**

(iv) $(2 - 3i)(\overline{3 - 2i})$

Solution: $(2 - 3i)(\overline{3 - 2i})$
 $= (2 - 3i)(3 + 2i)$
 $= 2(3 + 2i) - 3i(3 + 2i)$
 $= 6 + 4i - 9i - 6i^2$
 $= 6 - 5i - 6(-1)$
 $= 6 - 5i + 6$
 $= 6 + 6 - 5i$
 $= 12 - 5i$ **Ans**

Q.4 Simplify and write your answer in the form $a + bi$.

(i) $\frac{-2}{1+i}$

Solution: $\frac{-2}{1+i}$
 $= \frac{-2}{1+i} \times \frac{1-i}{1-i}$
 $= \frac{-2(1-i)}{(1)^2 - (i)^2}$
 $= \frac{-2+2i}{1-i^2}$
 $= \frac{-2+2i}{1-(-1)}$
 $= \frac{-2+2i}{1+1}$
 $= \frac{-2+2i}{2}$
 $= -\frac{2}{2} + \frac{2i}{2}$
 $= -1 + i$ **Ans**

(ii) $\frac{2+3i}{4-i}$

Solution: $\frac{2+3i}{4-i}$
 $= \frac{2+3i}{4-i} \times \frac{4+i}{4+i}$
 $= \frac{(2+3i)(4+i)}{(4)^2 - (i)^2}$
 $= \frac{2(4+i) + 3i(4+i)}{16 - (-1)}$
 $= \frac{8 + 2i + 12i + 3i^2}{16 + 1}$
 $= \frac{8 + 4i + 3(-1)}{17}$
 $= \frac{8 + 4i - 3}{17}$
 $= \frac{5 + 4i}{17}$
 $= \frac{5}{17} + \frac{4}{17}i$ **Ans**

(iii) $\frac{9-7i}{3+i}$

Solution: $\frac{9-7i}{3+i}$

$$= \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(9-7i)(3-i)}{(3)^2 - (i)^2}$$

$$= \frac{9(3-i) - 7i(3-i)}{9 - (-1)}$$

$$= \frac{27 - 9i - 21i + 7i^2}{9+1}$$

$$= \frac{27 - 30i + 7(-1)}{10}$$

$$= \frac{27 - 30i - 7}{10}$$

$$= \frac{20 - 30i}{10}$$

$$= \frac{20}{10} - \frac{30i}{10}$$

$$= 2 - 3i \text{ Ans}$$

(iv) $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

Solution: $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

$$= \frac{2-6i - (4+i)}{3+i}$$

$$= \frac{2-6i-4-i}{3+i}$$

$$= \frac{-2-7i}{3+i}$$

$$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{-2(3-i) - 7i(3-i)}{(3)^2 - (i)^2}$$

$$= \frac{-6 + 2i - 21i + 7i^2}{9 - (-1)}$$

$$= \frac{-6 - 19i + 7(-1)}{9+1}$$

$$= \frac{-6 - 19i - 7}{10}$$

$$= \frac{-13 - 19i}{10}$$

$$= \frac{-13}{10} - \frac{19i}{10} \text{ Ans}$$

(v) $\left[\frac{1+i}{1-i} \right]^2$

Solution: $\left[\frac{1+i}{1-i} \right]^2$

$$= \frac{(1+i)^2}{(1-i)^2}$$

$$= \frac{(1)^2 + (i)^2 + 2ab}{(1)^2 + (i)^2 - 2ab}$$

$$= \frac{(1)^2 + (i)^2 + 2(1)(i)}{(1)^2 + (i)^2 - 2(1)(i)}$$

$$= \frac{1 + (-1) + 2i}{+1 + (-1) - 2i}$$

$$= \frac{\cancel{1} - \cancel{1} + 2i}{\cancel{1} - \cancel{1} - 2i}$$

$$= \frac{2i}{-2i} = -1$$

$$= -1$$

$$= -1 + 0i \text{ Ans}$$

(vi) $\frac{1}{(2+3i)(1-i)}$

Solution: $\frac{1}{(2+3i)(1-i)}$

$$= \frac{1}{2(1-i) + 3i(1-i)}$$

$$= \frac{1}{2 - 2i + 3i - 3i^2}$$

$$\begin{aligned}
&= \frac{1}{2+i-3(-1)} \\
&= \frac{1}{2+i+3} \\
&= \frac{1}{2+3+i} \\
&= \frac{1}{5+i} \\
&= \frac{1}{5+i} \times \frac{5-i}{5-i} \\
&= \frac{1(5-i)}{(5)^2 - (i)^2} \\
&= \frac{5-i}{25-(-1)} \\
&= \frac{5-i}{25+1} \\
&= \frac{5-i}{26} \\
&= \frac{5}{26} - \frac{1i}{26} \text{ Ans}
\end{aligned}$$

Q.5 Calculate

(a) $\bar{z}$ (b) $z + \bar{z}$ (c) $z - \bar{z}$ (d) $z\bar{z}$ for each of the following.

(i) $z = -i$

Solution: $z = -i$

(a) $\bar{z} = +i$

(b) $z + \bar{z} = -i + i$
 $= 0$

(c) $z - \bar{z} = (-i) - (i)$
 $= -2i$

(d) $z\bar{z} = (-i)(i)$
 $= -i^2$
 $= -(-1)$
 $= 1 \text{ Ans}$

(ii) $z = 2+i$

Solution: $z = 2+i$
 $z + 2i$

(a) $\bar{z} = 2-i$

(b) $z + \bar{z} = (2+i) + (2-i)$

$= 2 + \cancel{i} + 2 - \cancel{i}$

$= 2 + 2$

$= 4$

(c) $z - \bar{z} = (2+i) - (2-i)$

$= \cancel{2} + i - \cancel{2} + i$

$= i + i$

$= 2i$

(d) $z\bar{z} = (2+i)(2-i)$

$= (2)^2 - (i)^2$

$= 4 - i^2$

$= 4 - (-1)$

$= 4 + 1$

$= 5 \text{ Ans}$

(iii)

$z = \frac{1+i}{1-i}$

Solution: $z = \frac{1+i}{1-i}$

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{1(1+i) + i(1+i)}{(1-i)(1+i)}$

$= \frac{1+i+i+(-1)}{(1)^2 - (i)^2}$

$= \frac{1+2i+(-1)}{1-(-1)}$

$= \frac{\cancel{1} + 2i - \cancel{1}}{1+1}$

$= \frac{2i}{2}$

$= i$

$z = i$

(a) $\bar{z} = -i$

(b) $z + \bar{z} = i + (-i)$

$= \cancel{i} - \cancel{i}$

$= 0$

(c) $z - \bar{z} = i - (-i)$

$= i + i$

$$= 2i$$

$$(d) \quad z\bar{z} = (i)(-i)$$

$$= -i^2$$

$$= -(-1)$$

$$= +1 \quad \text{Ans}$$

$$(iv) \quad z = \frac{4-3i}{2+4i}$$

$$\text{Solution: } z = \frac{4-3i}{2+4i}$$

$$z = \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}$$

$$= \frac{4(2-4i) - 3i(2-4i)}{(2+4i)(2-4i)}$$

$$= \frac{8-16i-6i+12i^2}{(2)^2 - (4i)^2}$$

$$= \frac{8-22i+12(-1)}{4-16i^2}$$

$$= \frac{8-22i-12}{4-16(-1)}$$

$$= \frac{8-12-22i}{4+16}$$

$$= \frac{-4-22i}{20}$$

$$= -\frac{4}{20} - \frac{22}{20}i$$

$$= -\frac{1}{5} - \frac{11}{10}i$$

$$(a) \quad \bar{z} = -\frac{1}{5} + \frac{11}{10}i$$

(b)

$$z + \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) + \left(-\frac{1}{5} + \frac{11}{10}i\right)$$

$$= -\frac{1}{5} - \frac{11}{10}i - \frac{1}{5} + \frac{11}{10}i$$

$$= -\frac{1}{5} - \frac{1}{5}$$

$$= \frac{-1-1}{5}$$

$$= -\frac{2}{5}$$

(c)

$$z - \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) - \left(-\frac{1}{5} + \frac{11}{10}i\right)$$

$$= -\frac{1}{5} - \frac{11}{10}i + \frac{1}{5} - \frac{11}{10}i$$

$$= -\frac{11}{10}i - \frac{11}{10}i = \frac{-11i-11i}{10}$$

$$= -\frac{22i}{10}$$

$$= -\frac{11}{5}i$$

$$(d) \quad z\bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$$

$$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$$

$$= \frac{1}{25} - \frac{121}{100}i^2$$

$$= \frac{1}{25} - \frac{121}{100}(-1)$$

$$= \frac{1}{25} + \frac{121}{100}$$

$$= \frac{4+121}{100}$$

$$= \frac{125}{100}$$

$$= \frac{5}{4} \quad \text{Ans}$$

Q.6

If $z = 2 + 3i$ and show that.

$$(i) \quad \overline{z+w} = \bar{z} + \bar{w}$$

$$\text{Solution: } \overline{z+w} = \bar{z} + \bar{w}$$

$$z + w = 2 + 3i + 5 - 4i$$

$$= 2 + 5 + 3i - 4i$$

$$= 7 - i$$

$$\text{L.H.S} = \overline{z+w}$$

$$= \overline{7-i}$$

$$= 7+i$$

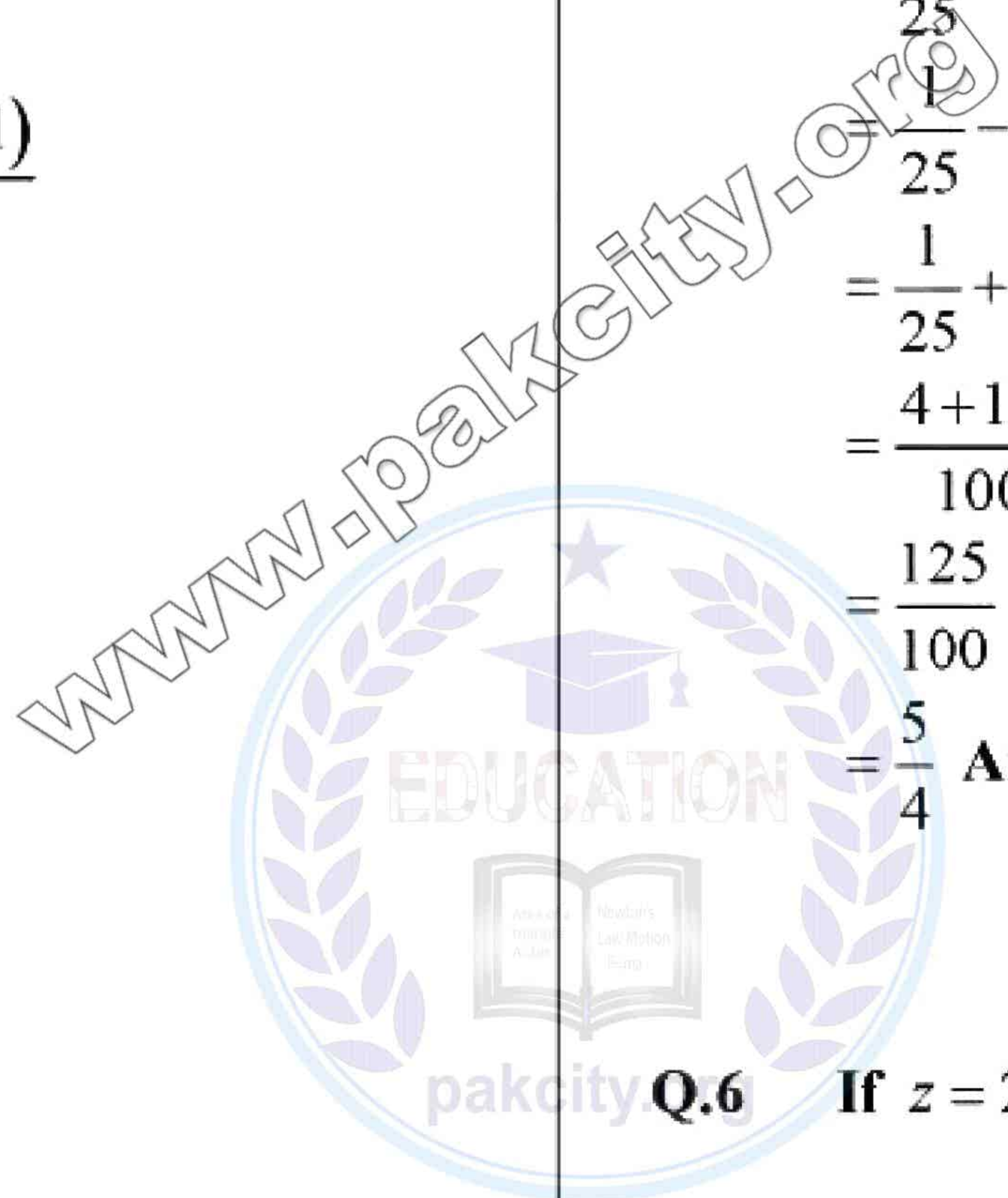
... (i)

$$\text{R. H. S} = \bar{z} + \bar{w}$$

$$= \overline{(2+3i)} + \overline{(5-4i)}$$

$$= 2-3i + 5+4i$$

$$= 2+5-3i+4i$$



$$= 7 + i$$

...

(ii)

From (i) and (ii) we get

$$\text{L.H.S} = \text{R.H.S}$$

$$\overline{z+w} = \overline{z} + \overline{w}$$

Hence proved

$$(ii) \quad \overline{z-w} = \overline{z} - \overline{w}$$

$$\text{Solution: } \overline{z-w} = \overline{z} - \overline{w}$$

$$z-w = (2+3i) - (5-4i)$$

$$= 2+3i-5+4i$$

$$= 2-5+3i+4i$$

$$= -3+7i$$

$$\text{L.H.S} = \overline{z-w}$$

$$= \overline{-3+7i}$$

$$= -3-7i$$

... (i)

$$\text{R.H.S} = \overline{z} - \overline{w}$$

$$= \overline{(2+3i)} - \overline{(5-4i)}$$

$$= 2+3i - (5+4i)$$

$$= 2-3i-5-4i$$

$$= -3-7i$$

From (i) and (ii) we get

$$\text{L.H.S} = \text{R.H.S}$$

$$\overline{z-w} = \overline{z} - \overline{w}$$

Hence proved

$$(iii) \quad \overline{zw} = \overline{z} \overline{w}$$

$$\text{Solutions: } \overline{zw} = \overline{z} \overline{w}$$

$$zw = (2+3i)(5+4i)$$

$$= 2(5-4i) + 3i(5-4i)$$

$$= 10-8i+15i-12i^2$$

$$= 10+7i-12(-1)$$

$$= 10+7i+12$$

$$= 22+7i$$

$$\text{L.H.S} = \overline{zw}$$

$$= \overline{22+7i}$$

$$= 22-7i$$

$$\text{R.H.S} = \overline{z} \overline{w}$$

$$= \overline{(2+3i)} \overline{(5-4i)}$$

$$= (2-3i)(5+4i)$$

$$= 2(5+4i) - 3i(5+4i)$$

$$= 10+8i-15i-12i^2$$

$$= 10-7i-12(-1)$$

$$= 10-7i+12$$

$$= 22-7i$$

From (i) and (ii) we get

$$\text{L.H.S} = \text{R.H.S}$$

$$\overline{zw} = \overline{z} \overline{w}$$

Hence proved

$$(iv) \quad \overline{\left[\frac{z}{w} \right]} = \frac{\overline{z}}{\overline{w}}, \text{ where } w \neq 0$$

$$\text{Solutions: } \overline{\left[\frac{z}{w} \right]} = \frac{\overline{z}}{\overline{w}}$$

$$\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$$

$$= \frac{2(5+4i) + 3i(5+4i)}{(5-4i)(5+4i)}$$

$$= \frac{10+8i+15i+12i^2}{(5)^2 - (4i)^2}$$

$$= \frac{10+23i+12(-1)}{25-16i^2}$$

$$= \frac{10+23i-12}{25-(6(-1))}$$

$$= \frac{10+23i-12}{25+16}$$

$$= \frac{-2+23i}{41}$$

$$\text{L.H.S} = \overline{\left[\frac{z}{w} \right]}$$

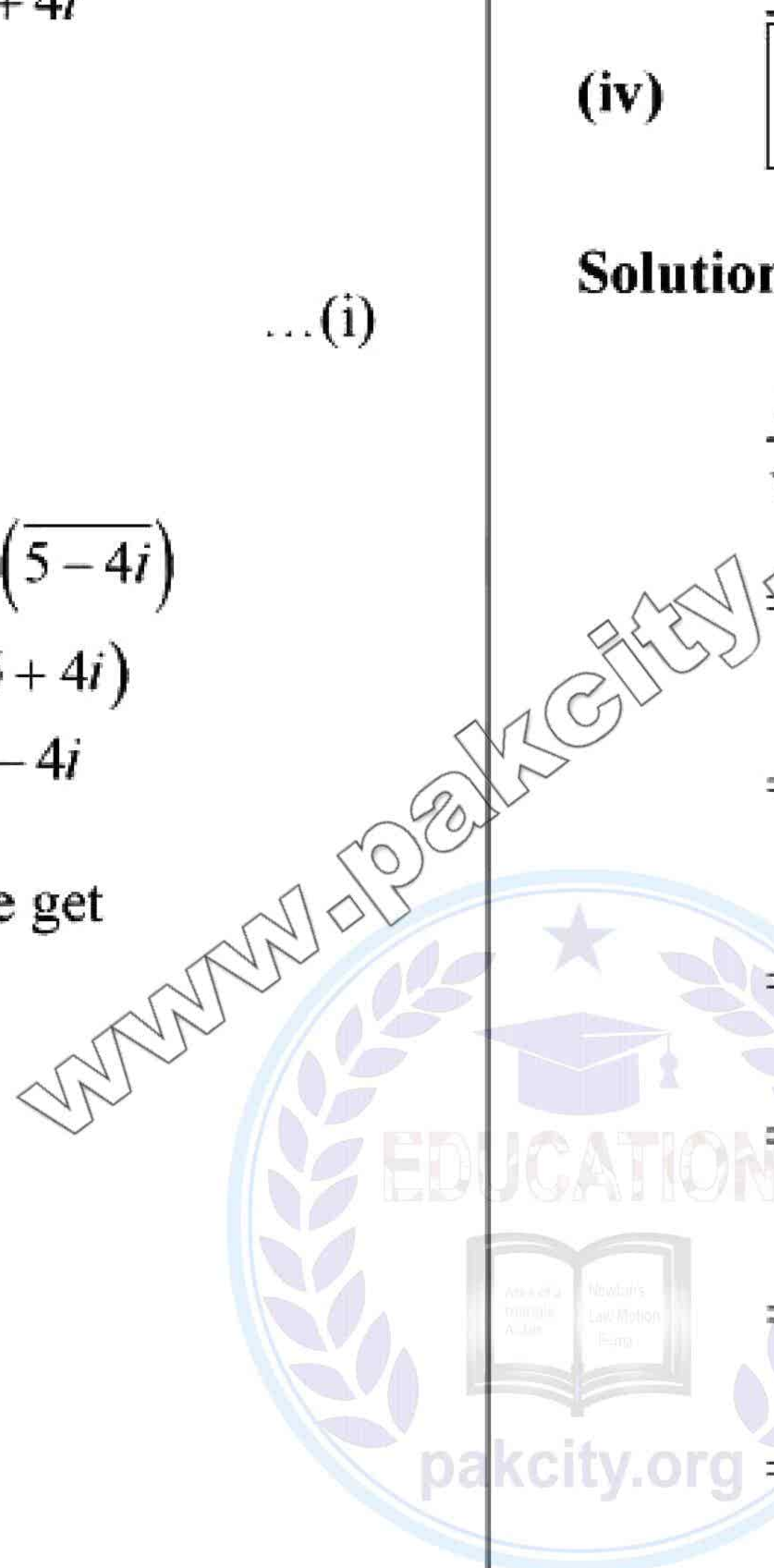
$$= \overline{\left[\frac{-2+23i}{41} \right]}$$

$$= \frac{-2}{41} - \frac{23}{41}i$$

... (i)

$$\text{R.H.S} = \frac{\overline{z}}{\overline{w}}$$

$$= \frac{\overline{(2+3i)}}{\overline{(5-4i)}}$$



$$\begin{aligned}
&= \frac{2-3i}{5+4i} \\
&= \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i} \\
&= \frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)} \\
&= \frac{10-8i-15i+12i^2}{(5)^2-(4i)^2} \\
&= \frac{10-23i+12(-1)}{25-16i^2} \\
&= \frac{10-23i+12(-1)}{25-16(-1)} \\
&= \frac{10-23i-12}{25+16} \\
&= \frac{-2-23i}{41} \\
&= \frac{-2}{41} - \frac{23}{41}i \quad \dots (ii)
\end{aligned}$$

From (i) and (ii) we get

L.H.S=R.H.S

Hence Proved

$$\overline{\left[\frac{z}{w}\right]} = \frac{\bar{z}}{\bar{w}}$$

(v) $\frac{1}{2}(z + \bar{z})$ is the real part of z .

$$\begin{aligned}
\text{Solution: } &\frac{1}{2}(z + \bar{z}) \\
&= \frac{1}{2}[(2+3i) + \overline{(2+3i)}] \\
&= \frac{1}{2}[(2+3i) + (2-3i)] \\
&= \frac{1}{2}[2 + \cancel{3i} + 2 - \cancel{3i}] \\
&= \frac{1}{2}[2+2] \\
&= \frac{1}{2}[4] \\
&= 2 = \text{Re}(z)
\end{aligned}$$

$\frac{1}{2}(z + \bar{z})$ is the real part of z . **Ans**

(vi) $\frac{1}{2}(z - \bar{z})$ is the imaginary part of z .

$$\begin{aligned}
\text{Solution: } &\frac{1}{2}(z - \bar{z}) \\
&\frac{1}{2}(z - \bar{z}) = \\
&= \frac{1}{2}[(2+3i) - \overline{(2+3i)}] \\
&= \frac{1}{2}[(2+3i) - (2-3i)] \\
&= \frac{1}{2}[\cancel{2} + 3i - \cancel{2} + 3i] \\
&= \frac{1}{2}[6i] \\
&= 3i \\
&= \text{Imaginary}(z)
\end{aligned}$$

$\frac{1}{2}(z - \bar{z})$ is the imaginary part of z . **Ans**

Q.7 Solve the following equations for real x and y .

(i) $(2-3i)(x+yi) = 4+i$

Solution: $(2-3i)(x+yi) = 4+i$

$$\begin{aligned}
x+yi &= \frac{4+i}{2-3i} \\
x+yi &= \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i} \\
&= \frac{4(2+3i) + i(2+3i)}{(2-3i)(2+3i)} \\
&= \frac{8+12i+2i+3i^2}{(2)^2-(3i)^2} \\
&= \frac{8+14i+3(-1)}{4-9i^2} \\
&= \frac{8+14i-3}{4-9(-1)} \\
&= \frac{8-3+14i}{4+9}
\end{aligned}$$

$$= \frac{5+14i}{13}$$

$$x + yi = \frac{5}{13} + \frac{14}{13}i$$

$$x = \frac{5}{13}, y = \frac{14}{13} \text{ Ans}$$

$$(ii) \quad (3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$$

Solution:

$$(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$$

$$3(x + yi) - 2i(x + yi) = 2x - 4yi + 2i - 1$$

$$3x + 3yi - 2xi - 2yi^2 = (2x - 1) + i(2 - 4y)$$

$$3x + (3x - 2x)i - 2y(-1) = (2x - 1) + i(2 - 4y)$$

$$3x + (3y - 2x)i + 2y = (2x - 1) + i(2 - 4y)$$

$$(3x + 2y) + (3y - 2x)i = (2x - 1) + (2 - 4y)i$$

Comparing the real and imaginary parts.

$$3x + 2y = 2x - 1, \quad ,$$

$$3y - 2x = 2 - 4y$$

$$3x - 2x + 2y = -1, \quad ,$$

$$3y - 2x = 2 - 4y$$

$$x + 2y = -1$$

$$-2x + 3y + 4y = 2$$

$$-2x + 7y = 2$$

$$x + 2y = -1 \quad \text{_____ (i)}$$

$$-2x + 7y = 2 \quad \text{_____ (ii)}$$

Multiply equation (i) with (2)

$$2(x + 2y) = -1 \times 2$$

$$2x + 4y = -2 \quad \text{_____ (iii)}$$

$$\cancel{2x} + 4y = \cancel{-2}$$

$$\cancel{-2x} + 7y = \cancel{2}$$

$$11y = 0$$

$$y = \frac{0}{11}$$

$$y = 0$$

Putting $y = 0$ in equation (i)

$$x + 2y = -1$$

$$x + 2(0) = -1$$

$$x + 0 = -1$$

$$x = -1 + 0$$

$$x = -1 \text{ Ans}$$

$$(iii) \quad (3 + 4i)^2 - 2(x - yi) = x + yi$$

$$\text{Solution: } (3 + 4i)^2 - 2(x - yi) = x + yi$$

$$(3 + 4i)^2 - 2(x - yi) = x + yi$$

$$9 + 24i + 16i^2 - 2x + 2yi = x + yi$$

$$9 + 24i + 16(-1) - 2x + 2yi = x + yi$$

$$9 + 24i - 16 - 2x + 2yi = x + yi$$

$$9 + 24i - 16 - 2x = x + 2yi - yi = 0$$

$$9 + 24i - 16 - 3x + yi = 0$$

$$-3x + yi = -9 - 24i + 16$$

$$-3x + yi = 16 - 9 - 24i$$

$$-3x + yi = 7 - 24i$$

Comparing the real and imaginary parts.

$$-3x = 7$$

$$y = -24$$

$$x = \frac{-7}{3}$$

$$y = -24 \text{ Ans}$$



Review Exercise 2

Q.1 Multiple choice questions. Choose the correct answer.

(i) $(27x^{-1})^{-\frac{2}{3}}$ _____

(a) $\frac{\sqrt[3]{x^2}}{9}$

(b) $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$

(d) $\frac{\sqrt{x^3}}{8}$

(ii) Write $\sqrt[7]{x}$ in the exponential form _____

(a) x

(b) x^7

(c) $x^{\frac{1}{7}}$

(d) $x^{\frac{7}{2}}$

(iii) Write $4^{\frac{2}{3}}$ with radical sing _____

(a) $\sqrt[3]{4^2}$

(b) $\sqrt[2]{4^3}$

(c) $\sqrt[2]{4^3}$

(d) $\sqrt{4^6}$

(iv) In $\sqrt[3]{35}$ the radicand is;

(a) 3

(b) $\frac{1}{3}$

(c) 35

(d) None

(v) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} =$ _____

(a) $\frac{5}{4}$

(b) $\frac{4}{5}$

(c) $-\frac{5}{4}$

(d) $-\frac{4}{5}$

(vi) The conjugate of $5 + 4i$ is _____

(a) $-5 + 4i$

(b) $-5 - 4i$

(c) $5 - 4i$

(d) $5 + 4i$

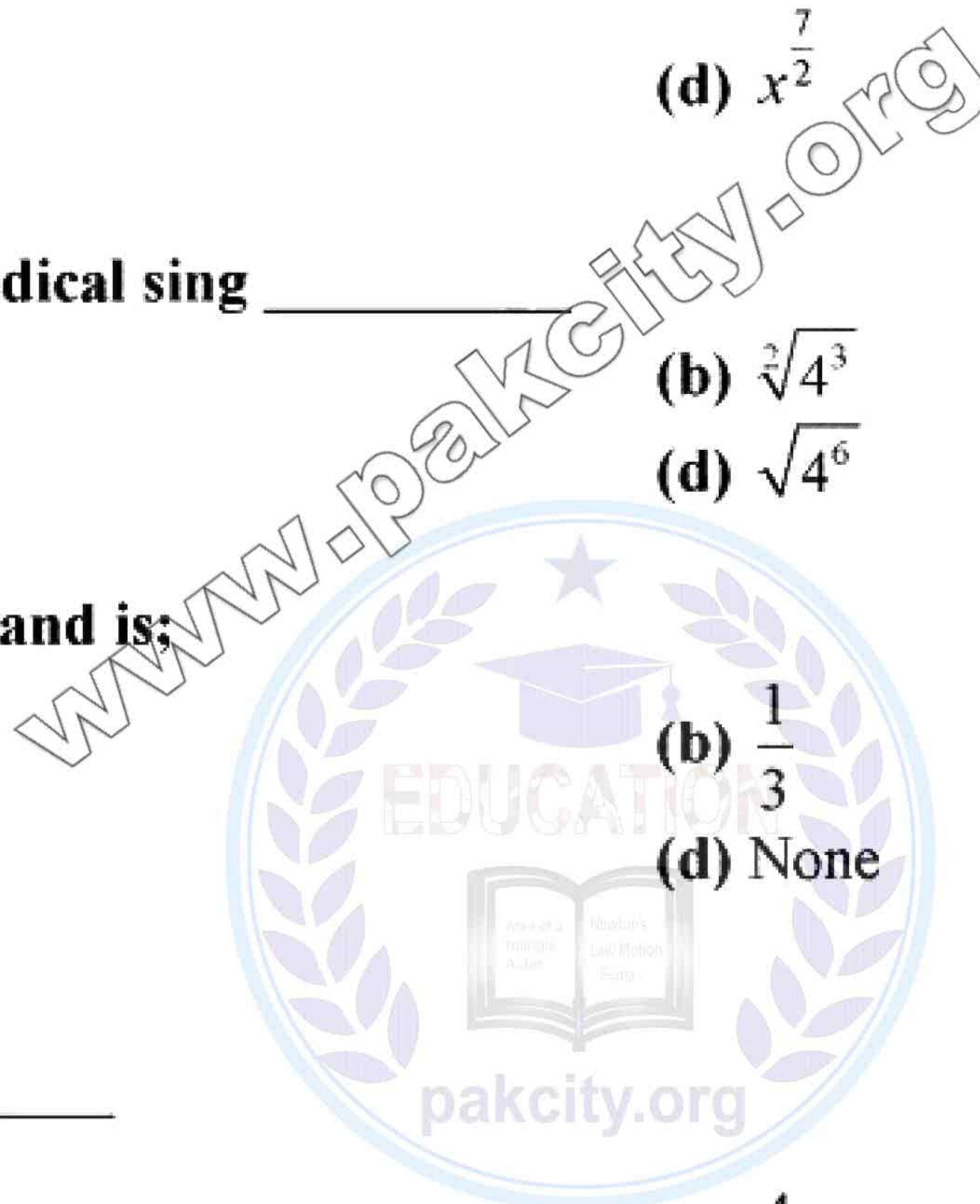
(vii) The value of i^9 is;

(a) 1

(b) -1

(c) i

(d) $-i$



- (viii) Every real number is _____
 (a) Positive integer (b) A rational number
 (c) A negative integer (d) A complex number
- (ix) Real point of $2ab(i + i^2)$ is _____
 (a) $2ab$ (b) $-2ab$
 (c) $2abi$ (d) $-2abi$
- (x) Imaginary part of $-i(3i + 2)$ is _____
 (a) -2 (b) 2
 (c) 3 (d) -3
- (xi) Which of the following sets have the closure property w.r.t addition _____
 (a) $\{0\}$ (b) $\{0, 1\}$
 (c) $\{0, 1\}$ (d) $\left\{1, \sqrt{2}, \frac{1}{2}\right\}$
- (xii) Name the property of real number used in $\left[-\frac{\sqrt{5}}{2}\right] \times 1 = -\frac{\sqrt{5}}{2}$ _____
 (a) Additive identity (b) Additive inverse
 (c) Multiplicative identity (d) Multiplicative inverse
- (xiii) If $x, y, z \in R, z < 0$, then $x < y \Rightarrow$...
 (a) $xz < yz$ (b) $xz > yz$
 (c) $xz = yz$ (d) None of these
- (xiv) IF $a, b \in R$, only one of $a = b$ or $a < b$ or $a > b$ hold is called _____
 (a) Trichotomy property (b) Transitive property
 (c) Additive property (d) Multiplicative property
- (xv) A non-terminating, non-recurring decimal represents ...
 (a) A natural number (b) A rational number
 (c) An irrational number (d) A prime number

ANSWER KEY

i	a	vi	c	xi	a
ii	c	vii	c	xii	c
iii	a	viii	d	xiii	b
iv	c	ix	b	xiv	a
v	b	x	a	xv	c

Q.2 True or False? Identity

- (i) Division is not an associative operation. **True**
 (ii) Every whole number is a natural number. **False**
 (iii) Multiplicative inverse of 0.02 is 50. **True**
 (iv) π is rational number. **False**
 (v) Every integer is a rational number. **True**
 (vi) Subtraction is a commutative operation. **False**
 (vii) Every real number is a rational number. **False**
 (viii) Decimal representation of a rational number is either terminating or recurring. **True**
 (ix) $1.\bar{8} = 1 + \frac{8}{9}$ **True**

Q.3 Simplify the following

(i) $\sqrt[4]{81y^{-12}x^{-8}}$

Solution:

$$\begin{aligned}
 &= (3^4 y^{12} x^{-8})^{\frac{1}{4}} \\
 &= 3^{4 \times \frac{1}{4}} y^{12 \times \frac{1}{4}} x^{-8 \times \frac{1}{4}} \\
 &= 3 y^3 x^{-2} \\
 \sqrt[4]{81y^{-12}x^{-8}} &= \frac{3}{y^3 x^2} \text{ Ans}
 \end{aligned}$$

(ii) $\sqrt{25x^{10}y^{8m}}$

Solution:

$$\begin{aligned}
 &= \sqrt{25x^{10n}y^{8m}} \\
 &= (5^2 x^{10n} y^{8m})^{\frac{1}{2}} \\
 &= 5^{2 \times \frac{1}{2}} x^{10n \times \frac{1}{2}} y^{8m \times \frac{1}{2}} \\
 \sqrt{25x^{10}y^{8m}} &= 5x^{5n} y^{4m} \text{ Ans}
 \end{aligned}$$

(iii) $\left[\frac{x^3 y^4 z^5}{x^{-2} y^{-1} z^{-5}} \right]^{\frac{1}{5}}$

Solution:

$$\begin{aligned}
 &= (x^{3+2} y^{4+1} z^{5+5})^{\frac{1}{5}} \\
 &= (x^5 y^5 z^{10})^{\frac{1}{5}} \\
 &= x^{5 \times \frac{1}{5}} y^{5 \times \frac{1}{5}} z^{10 \times \frac{1}{5}}
 \end{aligned}$$

$$\left[\frac{x^3 y^4 z^5}{x^{-2} y^{-1} z^{-5}} \right]^{\frac{1}{5}} = x y z^2 \text{ Ans}$$

(iv)

Solution:

$$\begin{aligned}
 &\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}} \right)^{\frac{2}{5}} \\
 &= \left(\frac{2^5 x^{-6} y^{-4} z}{5^4 x^4 y z^{-4}} \right)^{\frac{2}{5}} \\
 &= \left[\frac{2^5 z^{1+4}}{5^4 x^{4+6} y^{1+4}} \right]^{\frac{2}{5}} \\
 &= \left[\frac{2^5 z^5}{5^4 x^{10} y^5} \right]^{\frac{2}{5}} \\
 &= \frac{2^{5 \times \frac{2}{5}} \times z^{5 \times \frac{2}{5}}}{5^{4 \times \frac{2}{5}} \times x^{10 \times \frac{2}{5}} \times y^{5 \times \frac{2}{5}}} \\
 &= \frac{2^2 \times z^2}{5^{\frac{8}{5}} \times x^4 \times y^2} \\
 &= \frac{4z^2}{5^{\frac{8}{5}} \times x^4 y^2} \\
 &= \frac{4z^2}{5^{1+\frac{3}{5}} \times x^4 y^2} \\
 \left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}} \right)^{\frac{2}{5}} &= \frac{4z^2}{5 \times 5^{\frac{3}{5}} x^4 y^2} \text{ Ans}
 \end{aligned}$$

Q.4 Simplify $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Solution:

$$\begin{aligned} & \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}} \\ &= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}} \\ &= \sqrt{\frac{6^2 \times 5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}} \\ &= \sqrt{\frac{6^2 \times 5}{(5^2)^{\frac{3}{2}}}} \\ &= \sqrt{\frac{6^2 \times 5}{(5)^3}} \\ &= \sqrt{\frac{6^2}{5^3 \times 5^{-1}}} \\ &= \sqrt{\frac{6^2}{5^{3-1}}} \\ &= \sqrt{\frac{6^2}{5^2}} \\ &= \sqrt{\left(\frac{6}{5}\right)^2} \\ &= \left(\frac{6}{5}\right)^{2 \times \frac{1}{2}} \\ &= \frac{6}{5} \text{ Ans} \end{aligned}$$

Q.5 $\left(\frac{a^p}{a^q}\right)^{p+q} \times \left(\frac{a^q}{a^r}\right)^{q+r} \div 5(a^q \cdot a^r)^{p-r}$

Solution:

$$\begin{aligned} &= \frac{(a^{p-q})^{p+q} (a^{q-r})^{q+r}}{5(a^{p+r})^{p-r}} \\ &= \frac{a^{(p-q)(p+q)} a^{(q-r)(q+r)}}{5a^{(p+r)(p-r)}} \end{aligned}$$

$$\begin{aligned} &= \frac{a^{p^2-q^2} a^{q^2-r^2}}{5a^{p^2-r^2}} \\ &= \frac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}} \\ &= \frac{a^{p^2-r^2-p^2+r^2}}{5} \end{aligned}$$

$$= \frac{a^0}{5}$$

$$\begin{aligned} & \left(\frac{a^p}{a^q}\right)^{p+q} \times \left(\frac{a^q}{a^r}\right)^{q+r} \div 5(a^q \cdot a^r)^{p-r} \\ &= \frac{1}{5} \text{ Ans} \end{aligned}$$

Q.6 Simplify $\left(\frac{a^{2l}}{a^{l+m}}\right) \left(\frac{a^{2m}}{a^{m+n}}\right) \left(\frac{a^{2n}}{a^{n+2}}\right)$

Solution:

$$\begin{aligned} &= a^{2l-l-m} a^{2m-m-n} a^{2n-n-2} \\ &= a^{l-m} a^{m-n} a^{n-2} \\ &= a^{l-m+m-n+n-2} \\ &= a^0 \\ & \left(\frac{a^{2l}}{a^{l+m}}\right) \left(\frac{a^{2m}}{a^{m+n}}\right) \left(\frac{a^{2n}}{a^{n+2}}\right) = 1 \text{ Ans} \end{aligned}$$

Q.7 Simplify $\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^r}}$

Solution:

$$\begin{aligned} &= \sqrt[3]{a^{l-m}} \sqrt[3]{a^{m-n}} \sqrt[3]{a^{n-l}} \\ &= \left(a^{l-m}\right)^{\frac{1}{3}} \times \left(a^{m-n}\right)^{\frac{1}{3}} \times \left(a^{n-l}\right)^{\frac{1}{3}} \\ &= a^{\frac{l-m}{3}} \times a^{\frac{m-n}{3}} \times a^{\frac{n-l}{3}} \\ &= a^{\frac{l-m}{3} + \frac{m-n}{3} + \frac{n-l}{3}} \\ &= a^{\frac{l-m+m-n+n-l}{3}} \\ &= a^{\frac{0}{3}} \\ &= a^0 \\ &= 1 \text{ Ans} \end{aligned}$$

Unit 2: Real and Complex Numbers

Overview

Natural Numbers:

The numbers 1, 2, 3, ... which we use for counting certain objects are called natural numbers or positive integers. The set natural numbers is denoted by N .

$$\text{i.e. } N = \{1, 2, 3, \dots\}$$

Whole Numbers:

If we include 0 in the set of natural number, the resulting set is the set of whole numbers, denoted by W ,

$$\text{i.e. } W = \{0, 1, 2, 3, \dots\}$$

Integers:

The set of integers consist of positive integers, 0 and negative integers and is denoted by Z i.e. $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational Numbers:

All numbers of the form $\frac{p}{q}$ where p, q are integers and q is not zero are called rational numbers. The set of rational numbers is denoted by Q ,

$$\text{i.e. } Q = \left\{ \frac{p}{q} \mid p, q \in Z \wedge q \neq 0, (p, q) = 1 \right\}$$

Irrational Numbers:

The numbers which cannot be expressed as quotient of integers are called irrational numbers. The set of irrational numbers is denoted by Q' ,

$$\text{i.e. } Q' = \left\{ x \mid x \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0 \right\}$$

The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by R ,

$$\text{i.e. } R = Q \cup Q'$$

Types of Rational Numbers:

(i) Terminating Decimal Fractions

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction. For example $\frac{2}{5} = 0.4$ and $\frac{3}{8} = 0.375$.

(ii) Recurring and Non-terminating Decimal Fractions:

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called recurring decimal fraction.

For example $\frac{2}{9} = 0.2222\ldots$ and $\frac{4}{11} = 0.363636\ldots$

Concept of Radicals and Radicands:

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{}$ is called the radical sign, n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Base and Exponent:

In the exponential notation of (read as a to the n th power) we call ' a ' as the base and ' n ' as the exponent or the power to which the base is raised.

Definition of a Complex Number:

A number of the form $z = a + bi$ where a and b are real numbers and $i = \sqrt{-1}$, is called a complex number and is represented by z i.e., $z = a + ib$

Conjugate of a Complex Number:

If we change i to $-i$ in $z = a + bi$, we obtain another complex number $a - bi$ called the complex conjugate of z and is denoted by $\bar{z}$ (read z bar).

